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Abstract

Property rights are undoubtedly among the most important institutions for economic efficiency. Still, by looking at reality we usually see property rights only imperfectly enforced. In this paper we identify uncertainty faced by an enforcer to be sufficient to explain this observation. This result is independent of the enforcer's risk preferences or enforcement ability. While perfectly enforced property rights may readily be established in all possible scenarios under complete information, introducing any amount of exogenous uncertainty leads to imperfect enforcement. Further, even under complete information, we show how an appropriator may create uncertainty endogenously by means of strategic restraint. Although this means that appropriation is always possible, this situation Pareto dominates perfect enforcement.

Keywords

Property Rights, Deterrence, Uncertainty

JEL Classification

D02, D82, P14

1 Introduction

Among all rules and laws those that are designed to assign and enforce property rights are probably the most important from an economic point of view. Property rights, if properly enforced, are the rights to control economic goods and resources, as has been stressed by Rodrik (2000), and are as such fundamental for all market transactions. Maybe most importantly, property rights allow the holder to obtain income from a good or resource. As an example, a person's incentives to work depend very much on the expected share of earned income that is obtained. This is not very surprising and there is a lot of empirical evidence for this claim, see for example Field (2007). Similarly, the incentives to invest hinge to a great deal on the expected share of returns obtained, as has been shown for example in a recent study by Hornbeck (2010). This has of course also important implications for economic development. Mehlum *et al.* (2005) show in a theoretical analysis how the lack of well enforced property rights dampens incentives to invest and so might lead to poverty traps. Similarly, property rights enforcement is important for the internalization of externalities, as has been noted already in the classical works of Coase (1960) and Demsetz (1964, 1967). Overall, property rights are very important to shape incentives in various settings efficiently. Nevertheless, property rights are usually only imperfectly enforced.

In this paper we identify the presence of uncertainty as sufficient for imperfect enforcement. We interpret the process of enforcement as a contest game between an appropriator and an individual trying to defend his property, the defender. If the defender deters the appropriator from appropriation property rights are perfectly enforced. Our point of departure is a situation absent incomplete information, where deterrence may be possible if the defender is able to commit to enforcement effort. As soon as we introduce any amount of uncertainty about relevant characteristics of the appropriator, perfect enforcement breaks down and enforcement is always imperfect. The actual magnitude of this uncertainty does not play a role, and neither does the defender's attitude towards risk or his strength. Also, the introduction of the state does not alter this finding, even if the state is able to generate economies of scale in the process of enforcement. Further, we show how an appropriator is able to create uncertainty endogenously by strategically restraining himself from appropriation. The creation of this endogenous strategic uncertainty appeases the defender and leads to imperfect enforcement. In fact, creating strategic uncertainty endogenously is to the benefit of both the appropriator and the defender and leads to a Pareto improvement relative to perfect enforcement.

The creation or enforcement of property and other rights has been the subject of study for many researchers in the social sciences during the past decades. The early literature started with the seminal works of Coase (1960), Demsetz (1964) and Becker (1968). While the first two articles are mainly concerned with the effect of imperfect property rights enforcement and when individuals may have an incentive to create property rights, Becker tries to answer from a normative point of view what is the optimal extent of enforcement if there was a social planner. About a decade later another literature developed with the aim to explain why and how property rights may actually emerge absent any enforcing institution as the state. The point of departure is usually a state of anarchy.[1] Because in anarchy there do not exist rules or institutions by definition, it is possible to study the prerequisites for rules and institutions, like property rights, to emerge. Those results are important for our understanding of modern societies. After all, enforcement of property rights is, even in the presence of a state, to a great deal the responsibility of individuals or firms. To quote Friedman (1994), "*legal rules are in large part a superstructure erected upon an underlying structure of self-enforcing rights*". For example, households buy lockable doors, bike locks, or erect walls and fences around their houses, department stores hire security personnel, buy video surveillance systems or burglar alarms. For a self-interested individual there is no reason to respect others' possession. If an individual can further its goals by stealing from others it will probably do exactly that. As a consequence, property rights are inherently challenged and the possessor and appropriator engage in a game of conflict or contest. Early contributions to this literature are those of Umbeck (1977, 1981), who analyzed how claims to property evolved during the California gold rush and showed that violence was a major impediment for the creation of secure property rights. Other important contributions include the articles of Grossman & Kim (1995) and Grossman (2001). There it is shown that in a contest for possession the possessor might be able to secure his possession perfectly, if he is sufficiently strong and able to commit to some level of defensive effort, for example by moving first and building a wall or fence. A very interesting paper is also that of Muthoo (2004), in which he shows how secure property rights may emerge over time, what is similar to the results in Hafer (2006). More recent

[1]To our knowledge, the first paper studying the implications of anarchy is Bush & Mayer (1974). Influential articles analyzing economic behavior under anarchy are for example Skaperdas (1992) or Hirshleifer (1995). For an overview consult Skaperdas (2006) or Garfinkel & Skaperdas (2007).

contributions are Kolmar (2008), and Hoffmann (2010). Kolmar (2008) shows that even if secure property rights emerge, production incentives are nevertheless distorted. Hoffmann (2010) analyzes how property rights may emerge in a barter economy. A common theme in these papers is that perfect enforcement is possible if the enforcer is sufficiently strong. However, in all of those paper uncertainty does not play a role. In this paper we highlight the importance of taking uncertainty into account.

We analyze a game of conflict in which a defender has the opportunity to publicly commit to a particular level of defensive effort or fortification to fend off an appropriator. Because there are appropriators of different abilities and the defender does not know which appropriator is trying to steal from him, the enforcer faces exogenous uncertainty. This seems quite realistic, since, loosely speaking, people usually do not know in advance who is going to try to break into their houses, or whether somebody will attempt to break in at all. We show that this simple structure is sufficient to destroy the possibility of secure property rights as an equilibrium outcome of the conflict game. The presence of uncertainty about the appropriator's type gives rise to either insecure or imperfect property rights in equilibrium. The deeper reason for this finding is, metaphorically speaking, that all possible appropriators will stand in front of the same wall. In order to defend property at lowest possible costs, the defender builds the wall only slightly higher than the appropriator can climb. When there is uncertainty about the appropriator's ability it is not clear what is the optimal height and the defender builds a wall that is optimal in expectation, and thus deters only some average appropriator. We show that it will never be in the interest of the defender to build the wall high enough to deter all types of appropriators. If the wall was built high enough to deter also the most able appropriator, it would always be beneficial to decrease the height of the wall slightly, since this ensures first order gains against all less able types, whereas losses are approximately equal to zero at the margin against the highest type. In a first step we analyze the conflict game using the canonical lottery contest success function (CSF) to model the outcome. We provide exact conditions for different degrees of security of property rights to emerge and derive a formula which yields the ex-ante probability of successfully defending the good or resource. We interpret this probability as the degree of enforcement of property rights. We then proceed and show that the result of imperfect enforcement holds quite generally and does not depend on the uncertainty preferences or the absence of the state.

Finally, we show how an appropriator or criminal may restrain himself in the conflict game to create uncertainty strategically. This has the effect of appeasing the defender, and so lowers his defensive effort. As a consequence, both the appropriator and defender are better of in expectation and therefore decreasing the security of property leads to a Pareto improvement. We therefore develop a theoretical rationale for imperfect property rights enforcement.

A consequence of this result is that externalities are unlikely to be completely internalized, implying the existence of a fundamental inefficiency. Also, as noted before, this causes distortions in many important dimensions such as the decision to work, how much to invest, or to buy durable consumption goods, a finding which is in line with the works of Muthoo (2004) and Kolmar (2008). Therefore, there are two major inefficiencies related to property rights. First, in order to enforce property rights valuable resources have to be spent. Second, because of the nature of the problem of enforcement, contingent security investment is not possible and thus heterogeneity creates strategic uncertainty and thus leads to imperfectly enforced property rights. This causes distorted incentives in many important dimensions. However, our analysis also suggests that perfectly enforced property rights are not efficient either, what is in line with Becker (1968).

While the primary objective of this paper is to highlight the importance of uncertainty for property rights enforcement, we also contribute to the literature on the theory of contests. First, we provide a characterization of the equilibrium of a ratio-form contest with sequential moves and arbitrary type uncertainty. We thereby generalize the analysis of Linster (1993) along many lines. Our main result suggests that in our setting of type uncertainty the probability to win the contest may be negatively correlated with the expected relative strength of a player, which seems counterintuitive at first glance. Finally, we show how a player may use strategic restraint as a means to create uncertainty for his own benefit, a mechanism similar to that in Epstein & Nitzan (2004). Other papers dealing with informational asymmetries in contests and conflict are for example Hurley & Shogren (1998), Wärneryd (2003), Slantchev (2010), and Denter & Sisak (2011). Hurley & Shogren (1998) look at rent-seeking games with one-sided asymmetric information and show how different beliefs about the strength of the opponent alter equilibrium behavior. Wärneryd (2003) analyzes contest games between two players that are ignorant about their own type in the setting of a common-value conflict (that is, both contestants value the prize equally). Slantchev (2010) analyzes how choosing a position in a bargaining game prior to a possible conflict functions as a signal about a player's actual strength. He finds that it is often optimal to appeal to Sun Tzu's principle of "feigning weakness". Denter & Sisak (2011) study players' incentives to voluntarily exchange information. They show that the famous dictum "know thy enemy" is not necessarily true. Another recent related article is due to Chassang & i Miquel (2010). Here the authors highlight the importance of uncertainty for the stability of peace, respectively for the outbreak of overt conflict or war. They show that uncertainty generates incentives to start preemptive conflict, and thereby uncertainty makes deterrence less likely. This is in line with our findings in this paper.

The paper is organized as follows: In the next section we describe the basic model and define different degrees of security of property, relating to the extent that property rights are enforced. Section 3 analyzes a simple game of self-enforcement of property rights to develop an intuition for our main result and discusses comparative static results. Section 4 studies a generalized model and shows that the developed intuition is quite general. Section 5 shows that also in the presence of a state our results do not change, even though the state may generate economies of scale in the process of enforcement. Section 6 demonstrates how strategic uncertainty may be created endogenously in our framework. Section 7 discusses our results in relation to the literature on information economics. Most proves are relegated to the appendix.

2 The model

2.1 Basic set-up

Consider two players $i \in \{D, A\}$, where the letters are mnemonics for defender and appropriator respectively. D is in possession of a good which is his property. A wants to appropriate or steal this good. Enforcement of property rights is in the responsibility of D, which may for example be due to a weak state. In the context of property rights this assumption is quite realistic. Clearance rates of property crimes are consistently low in most countries. For example, the average clearance rate of burglaries in the U.S. in 2002 was some 14 percent, if victims reported the theft. However, it is estimated that only every third victim actually reports burglary (U.S. Department of Justice, 2002). There is also evidence that burglars do not think about the consequences they face in the event of being caught. The deterrence effect of punishment is insignificant (U.S. Department of Justice, 2002). That being said for the U.S., public enforcement of property rights is in most other countries even lower.[2] Hence, the major impediment for burglary and related crimes is usually private spending for self-enforcement.

D and A may be heterogeneous. For specificity, we assume they are heterogeneous in their valuation of the good v_i. D's valuation for the good is common knowledge and is given by v_D, whereas A has private information about his valuation v_A. v_A is drawn from $G(v_A)$, which has the compact and strictly positive support $[\underline{v}_A, \overline{v}_A] \subset \mathbb{R}_+$ and is differentiable. All this is common knowledge. Throughout the paper we contrast the incomplete information scenario to one in which there is complete information. In the latter case deterrence and therefore secure property rights are the outcome if v_D is sufficiently high. As will become apparent later on this does not hold true under incomplete information.

Note that although we model heterogeneity of players by emphasizing their different utilities derived from the good, our results are identical if heterogeneity in other dimensions is added. In fact, it is uncertainty about A's willingness to put in effective effort that matters, which is also influenced by other factors like marginal costs and effectiveness of effort.

Each player's probability p_i to succeed in the conflict is a function of both players' efforts. Specifically, we consider

$$p_D(x_D, x_A) = \begin{cases} 1 & \text{if } x_A = 0, \\ \frac{x_D}{x_D + x_A} & \text{else.} \end{cases} \tag{1}$$

and $p_A = 1 - p_D$. In Section 4 we show that our results hold more generally. $p_A(x_D, 0) = 1$ reflects that the good is in D's possession in the first place, and A needs to spend at least some resources to steal it. Each player i chooses effort x_i, and costs of effort are $C_i(x_i) = x_i$. We look at sequential moves where D moves first, respectively is able to make a commitment. This is a natural assumption in our context. Fortification or target hardening is usually a durable good and we usually see somebody erecting a wall or a fence around his house before

[2] The Heritage Foundation and the Wall Street Journal rank countries according to the degree of publicly enforced property rights on a 0 to 100 scale. For example, a score of 40 suggests "[t]he court system is highly inefficient, and delays are so long that they deter the use of the court system" and "expropriation is possible". Countries with a score equal to or lower than 40 include Argentina, Paraguay, and Bolivia in South America or Croatia, Serbia, and Russia in Europe. See `http://www.globalpropertyguide.com/`.

somebody else tries to break in.

Given the above assumptions D's objective is

$$\max_{x_D \in \mathbb{R}_+} U^D(x_D, x_A) = v_D \int_{\underline{v}_A}^{\overline{v}_A} p\left(x_D, x_A(v_A; x_D)\right) \, dG(v_A) - x_D. \tag{2}$$

For A, who holds full information after observing D's effort, utility maximization corresponds to

$$\max_{x_A \in \mathbb{R}_+} U^T(x_D, x_A) = v_A \left(1 - p(x_D, x_A)\right) - x_A. \tag{3}$$

Because D does not know the type of A we have an incomplete information game and the equilibrium concept we employ is Bayesian Nash equilibrium.

2.2 Concerning property rights

The aim of this paper is to give a rationale for imperfectly enforced property rights and to show that strategic uncertainty is sufficient to impede perfect enforcement. To facilitate the analysis we now define different levels of property rights enforcement.

The appropriator spends effort x_A in the contest. If $x_A(v_A; x_D) > 0$ type v_A is active and challenges D's property. If $x_A(v_A; x_D) = 0$ type v_A stays passive. Define π to be the equilibrium probability of observing $x_A(v_A; x_D) > 0$. We can use this convention to define three different regimes of security of property:

Definition 1. *Property rights are*

1. ***secure**, if the equilibrium probability of appropriative activity is zero, that is the good is definitively unchallenged and* $\pi = 0$ *in equilibrium. Property rights are perfectly enforced.*

2. ***imperfect**, if the equilibrium probability of appropriative activity is strictly between zero and one, and* $0 < \pi < 1$ *in equilibrium.*

3. ***insecure**, if the equilibrium probability of appropriative activity is equal to one,* $\pi = 1$.

The definition of secure property rights is identical to those in, for example, Grossman (2001) or Kolmar (2008). However, while it is usually assumed that property rights are either insecure or secure, we introduce a hybrid, imperfect property. From an empirical point of view this is probably the most relevant category. Ex-ante there is a strictly positive probability to suffer from attempted theft, and thus the probability to lose possession is also positive; however, ex-post it could well be that no theft was tried. Therefore, it may turn out that property remains unchallenged even though property is not secure according to the definition. If a theft attempt happens for sure property rights are insecure.

3 Equilibrium

3.1 Complete information: the benchmark

To establish a benchmark we let $G(v_A)$ converge to one point, such that $v_A = \underline{v}_A = \overline{v}_A$. In this case there is no uncertainty and hence there is complete information. The equilibrium of this game is our benchmark.

Proposition 1. *The defender enforces his property right with probability*

$$p_D^*(x_D^*, x_A^*) = \min\left\{\frac{v_D}{2v_A}, 1\right\}.$$

Proof. See for example Linster (1993). □

From this it follows that A does not spend any effort in equilibrium if and only if $v_D \geq 2v_A$. Therefore, if D is sufficiently strong there is deterrence and property rights are secure. Otherwise, property rights are insecure. Imperfect property rights cannot emerge. We now turn to the incomplete information game.

3.2 Incomplete information

In this section we look at how strategic uncertainty affects the level of security of property in equilibrium. If D moves first or is able to make a commitment he considers A's optimal reaction while optimizing. A's reaction function is easily found:

$$x_A(v_A; x_D) = \max\{0, \sqrt{x_D\, v_A} - x_D\}. \tag{4}$$

It follows immediately that the appropriator tries to appropriate only when $x_D < v_A$. For all higher efforts x_D there is deterrence. For property rights to be secure this has to hold for all possible appropriators. Let $\theta(\tilde{v}) \equiv \int_{\tilde{v}}^{\overline{v}_A} v_A^{-1/2} dG(v_A)$, where $\tilde{v}$ is the marginal type of A participating actively in the contest, $\tilde{v} = \max\{\underline{v}_A, x_D\}$. Note that $\theta(\tilde{v}) = \theta(x_D)$, since for all $x_D < \underline{v}_A$ the density of v_A is zero. To determine D's optimization problem this formulation is useful. Plugging (4) in (2) and using $\theta(x_D)$ it is then easily verified that D's optimization problem corresponds to:

$$\max_{x_D \in \mathbb{R}_+} v_D \left[G(x_D) + \sqrt{x_D}\,\theta(x_D)\right] - x_D.$$

A simply reacts according to his reaction function (4). Proposition 2 shows the equilibrium.

Proposition 2. *D's unique equilibrium effort in the sequential game is (implicitly) determined by*

$$x_D^+ = \left(\frac{v_D\,\theta\,(\tilde{v})}{2}\right)^2 > 0, \tag{5}$$

where $\tilde{v} = \max\{\underline{v}_D, x_D^+\}$. *Player A's equilibrium effort as the follower (F) follows immediately and is*

$$x_A^+ = \max\left\{\sqrt{v_A\,x_D^+} - x_D^+, 0\right\}.$$

Proof. See appendix. ☐

D's effort in equilibrium is monotonically increasing in v_D and always strictly positive. However, this is not true for A, who might stay passive. If all types of A stay passive we would have secure property in equilibrium. If to the contrary all types of A are active property is insecure. We next look for a condition that tells us when this is the case. It is obvious from (4) that the appropriator with the lowest valuation stays passive in the conflict whenever $x_D^+ \geq \underline{v}_A$. It is this type who is deterred first and therefore this type's equilibrium effort determines whether property is insecure or not.

Corollary 1. *All types of appropriators are active in the equilibrium of the conflict game if and only if*

$$v_D \leq \hat{v}_D \equiv \frac{2\sqrt{\underline{v}_A}}{\theta(\underline{v}_A)}.$$

Accordingly, if this condition is met property rights are insecure and $\pi = 1 - G(\underline{v}_A) = 1$. *If this condition is not met some types of A stay passive and property rights are imperfect or* *secure.*

Proof. Follows immediately from Proposition 2 and (4). □

The corollary tells us that when D gets too strong relative to A he spends so much effort in equilibrium that some types of A prefer to stay inactive in equilibrium, that is partial deterrence occurs. In order to have secure property, however, full deterrence is necessary. To answer whether property can be secure in an equilibrium we need to look more closely at x_D^+. In particular, property is secure if and only if $x_D^+ \geq \overline{v}_A$.

Corollary 2. *In equilibrium D always chooses effort*

$$x_D^+ < \overline{v}_A, \tag{6}$$

implying $\pi = 1 - G(x_D^+) > 0$ and hence property rights are never secure.

Proof. If we let $x_D^+ \to \overline{v}_A$ on the RHS of (5), $\theta(\tilde{v})$ gets zero in the limit, and so does the RHS. This yields a contradiction since then $0 = x_D^+ = \overline{v}_A > 0$. □

The corollary shows that strategic uncertainty is sufficient to explain imperfectly enforced property rights. Note that the result does not depend on the possible range of v_A, since any interval $[\underline{v}_A, \overline{v}_A]$ with positive measure suffices. Only in the limit, when $\underline{v}_A = \overline{v}_A = v_A$, secure property can be sustained in an equilibrium.

What is the intuition for this result? Assume $v_D \geq 2\overline{v}_A$. Absent strategic uncertainty D would spend exactly $x_D = v_A \in [\underline{v}_A, \overline{v}_A]$, hence deter A and enforce property rights perfectly. But when there is strategic uncertainty he prefers not to do that and enforces only imperfectly. To see the intuition behind the result it is illustrative to look at a simple example. Assume $v_D = 8$ and $v_A \in \{2, 4\}$ with equal probabilities. Without uncertainty D would deter both types of A and secure property rights perfectly. In the presence of asymmetric information D faces strategic uncertainty in his decision making. By spending $x_D = 4$ he would deter both types of A, however, he would also spend too much with a probability of 50 percent. Effort higher than 2 has, with a 50 percent probability, no benefit at all. By increasing effort the expected marginal benefit from effort decreases faster in the presence of uncertainty, because at the same time the marginal benefit of effort and the probability that effort has a benefit at all decrease. Because by increasing effort the fraction of deterred types of A increases, the

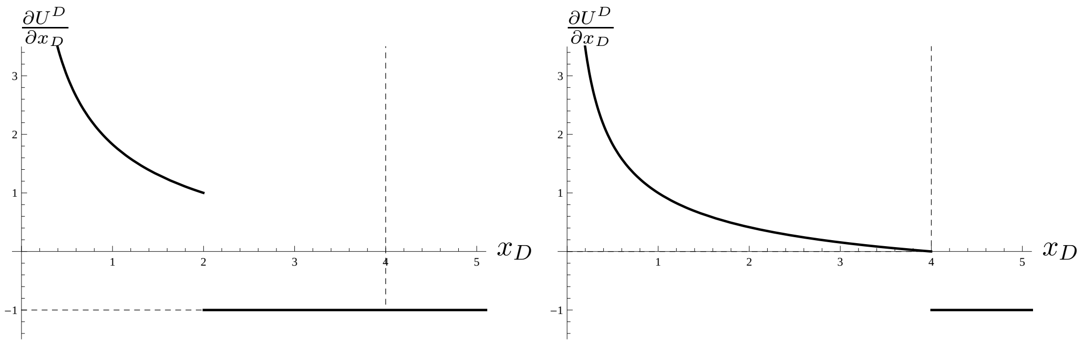

Figure 1: Derivative of D's utility function with respect to x_D against both players with $v_D = 8$, $\overline{v}_A = 4$, and $\underline{v}_A = 2$. In the right panel we see the derivative of D's objective function in case $v_A = 4$ is zero at $x_D = 4$. In the left panel we see that at this effort the derivative in case of $v_A = 2$ is negative, implying the overall derivative has to be negative, too, and therefore the first order condition for an optimum cannot hold.

marginal benefit of effort decreases not only due to diminishing marginal products but also as a consequence of a decreasing probability that effort has a positive benefit at all. However, efforts have always to be paid in full, whether or not effort is effective. In our example the marginal benefit of effort at $x_D = 4$ is with equal probability zero and one, while marginal costs are equal to one for sure. Hence, the first order condition for an optimum does not hold but effort has to be lower in equilibrium. This intuition is also graphically illustrated in Figure 1. In the case of a continuous distribution as described in the model, the probability of deterrence is continuously increasing in x_D in the interval $[\underline{v}_A, \overline{v}_A]$. Therefore, as $x_D \to \overline{v}_A$ the probability of deterrence approaches one, and therefore the expected marginal benefit of effort approaches zero.

Having shown that property rights are not secure in equilibrium, it is of interest to look more closely at the ex-ante probability that D actually keeps the good. If the probability to lose possession is economically not significant the implications of uncertainty for an individual are rather negligible. There are again two different sources of security of property: first, some types of A might be deterred, and this increases the probability of keeping the good. Second, those types that are not deterred do not succeed for sure but get the good only with some probability, which is determined by the CSF. In other words, because of deterrence, there is some probability not to encounter an appropriator at all, and there is also some probability to keep the good conditional on having a conflict. We need to consider both probabilities, what simply corresponds to integrating over the CSF, given the equilibrium strategies:

Corollary 3. *The ex-ante probability that D keeps the good is*

$$\begin{aligned} \phi &= \int_{\underline{v}_A}^{\overline{v}_A} \frac{x_D^+}{x_D^+ + x_A^+(\cdot,\cdot)}\, dG(v_A) \\ &= G(x_D^+) + \frac{\theta(x_D^+)^2 v_D}{2}, \end{aligned}$$

where x_D^+ and x_A^S are defined as in Proposition 2.

Proof. This follows immediately from the definition of the CSF in (2) and Proposition 2. □

To get an idea of the magnitude of the impact of uncertainty it is instructive to proceed with an example distribution. For specificity, assume the appropriator's valuation is uniformly distributed on $[(1-\sigma)v, v]$, $\sigma \in [0,1]$, and assume $v_D = 2v$. This ensures that the defender would perfectly secure property rights against all realizations of the appropriator's valuation if he had perfect information. σ is a spread parameter and may vary between zero and one, the former being the limit when uncertainty vanishes. When σ increases uncertainty gets larger. The expected valuation of the appropriator, however, decreases. The following formula establishes the connection between the spread and the ex-ante probability to keep possession in our example:

$$\phi^u(\sigma) = \frac{\sigma(\sigma+3)+4}{(\sigma+2)^2}. \tag{7}$$

A nice property is that ϕ^u is independent of v, what makes it quite easy to analyze. If there was no uncertainty, $\sigma = 0$, we get $\phi^u(0) = 1$, as we should have expected. As uncertainty increases ϕ^u decreases. In Figure 2 we plot ϕ^u against σ. An interesting question is now how uncertainty translates into the degree of enforcement of property rights. For example, if we are interested in finding the value of σ necessary to generate an ex-ante chance of one or 5 % to lose possession we find $\sigma^{1\%} = 0.04168$ or $\sigma^{5\%} = 0.25403$ respectively. Therefore, if uncertainty is about 25 % relative to v, which is less than 13 % relative to v_D, the probability to lose is some 5 %. However, ϕ^u cannot fall below $8/9 \approx 88.9\%$, reflecting that D is still the stronger player. Actually, as σ increases the expected valuation of A decreases, and as a consequence D becomes relatively stronger in expectation.

Note that in the example the probability to win the contest is actually negatively correlated to the ratio of expected valuation, $\kappa \equiv E[v_D]/E[v_A] = v_D/E[v_A]$, something that never happens absent imperfect information. The intuition is that higher uncertainty benefits D

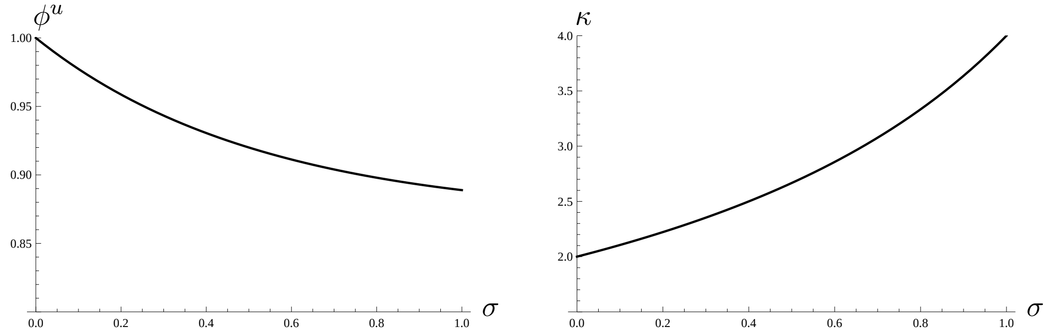

Figure 2: Left panel: The ex-ante probability ϕ^u to lose property as a function of σ, if $v_A \in [(1-\sigma)v, v]$ and $v_D = 2v$. Right panel: Expected relative valuation $\kappa \equiv E[v_D]/E[v_A] = \frac{4}{2-\sigma}$ as a function of σ. While ϕ^u is strictly monotonically decreasing in σ the opposite is true for κ. Therefore, in the presence of uncertainty κ may be a poor measure of strength in a contest, contrary to complete information contests.

because with a given level of effort the fraction of deterred appropriators is increasing. This makes it attractive to lower effort to save costs, but this necessarily brings some formerly deterred types back into the game. As a consequence, D is better off but loses more often. A consequence of this finding is that standard measures of strength in contests, as for example the valuation, marginal costs, or effort productivity, are not necessarily meaningful in the presence of uncertainty.

4 A general model

In this section we now show that our main result in Corollary 2 does not depend on the particular CSF we employed above to model the conflict, nor on the assumption of uncertainty neutrality. We proceed by first proving that deterrence is generally possible, and then show that under strategic uncertainty full deterrence cannot be an equilibrium outcome.

Each player $i \in \{D, A\}$ values the contested good by v_i. Both derive utility from wealth, which is given by

$$w_i = \begin{cases} v_i - x_i & \text{if } i \text{ wins in the contest,} \\ -x_i & \text{else.} \end{cases}$$

A player's utility is defined over wealth and given by $u_i = u_i(w_i)$ where u_i' is strictly positive and finite and $u_i'' \leq 0$. So we allow for risk aversion. Both players put effort $x_i \geq 0$. D's probability of winning the contest is

$$p_D(x_D, x_A) = \begin{cases} \frac{f(x_D)}{f(x_D)+f(x_A)} & \text{if } f(x_D) + f(x_A) > 0, \\ 1 & \text{else,} \end{cases} \tag{8}$$

where $f(0) = 0$, $f' > 0$ and $f'' \leq 0$.[3] A wins with a probability $p_A = 1 - p_D$. (8) is a standard CSF which has been axiomatized by Skaperdas (1996). For a micro foundation of it see for example Baye & Hoppe (2003). If we let $f(x) = x$ this is the CSF we employed before.

Player i's von Neumann–Morgenstern utility function is

$$U_i(x_i, x_j) = p_i(x_i, x_j)u_i(v_i - x_i) + (1 - p_i(x_i, x_j))u_i(-x_i). \tag{9}$$

As before D is able to publicly commit effort, for example by moving first. The following lemma is now important to prove our main result later on.

Lemma 1. *Assume each player maximizes a von Neumann–Morgenstern expected utility function as defined in (9) and probabilities are determined by (8). Denote A's best response by $x_A(v_A; x_D)$. Then there exists $\mu(v_A)$ such that*

1. *if $x_D \geq \mu(v_A)$ the optimal response is $x_A(v_A; x_D) = 0$,*
2. *if $x_D < \mu(v_A)$ the optimal response is $x_A(v_A; x_D) > 0$,*
3. *and $\mu(v_A)$ is increasing in v_A, $\mu'(v_A) > 0$.*

Proof. See appendix. ☐

The lemma states that in the above described framework for each type of A there exists a unique finite level of effort x_D that is necessary to deter A and so to secure property rights. This threshold level is increasing in the valuation of A. The lemma is important because it states that deterrence is technologically possible, so we do not assume the result of the following proposition by making deterrence infeasible:

Proposition 3. *Let the distribution of types of A be $G(v_A)$ with density $g(v_A)$ and support $[\underline{v}_A, \overline{v}_A] \subset \mathbb{R}_+$, where $\underline{v}_A < \overline{v}_A$ and denote D's optimal effort by x_D^+. Then in equilibrium it*

[3] In an earlier version of this paper we show that our results also hold for a more general class of CSFs that allow for deterrence under full information.

always holds that $x_D^+ < \mu(\overline{v}_A)$ and as a consequence thereof there is never full deterrence and property rights are never perfectly enforced.

Proof. See appendix. □

The result that a very small amount of incomplete information suffices to guarantee imperfect enforcement is quite general. Apart from property rights enforcement this is also interesting in other circumstances. For example, in international relations piling stocks of defensive weapons is often used as a deterrent for other countries to declare overt conflict or war. However, in the presence of uncertainty deterrence is possible only probabilistically, as we have shown. This result is similar to a recent result of Chassang & i Miquel (2010), who show that a little amount of incomplete information can impede deterrence and hence lead to overt conflict, because uncertainty may create incentives for preemptive attacks.

5 When there is a state

The above reasoning immediately carries over to the case of state enforced property rights, or property rights which are partly state and partly self-enforced, as for example in Konrad & Skaperdas (2010). Even if the state is able to generate economies of scale in the process of enforcement, this does not change our results as long as marginal costs of enforcement are positive. To see this assume the following simple extension of the model in Section 2. As before, D may spend x_D for self-enforcement of property rights. However, now total effective enforcement effort is $e(\gamma_1, \gamma_2, x_D) \equiv \gamma_1 x_D + \gamma_2$, where the γ_1 and γ_2 are enforcement instruments under the control of the state. γ_2 may be interpreted as a guaranteed minimum level of enforcement effort, whereas γ_1 influences D's marginal costs of effective enforcement effort, for example because the state subsidizes lockable doors and burglar alarms ($\gamma_2 > 1$). As a consequence, the probability of enforcement is

$$\int_{\underline{v}_A}^{\overline{v}_A} \frac{e(\gamma_1, \gamma_2, x_D)}{e(\gamma_1, \gamma_2, x_D) + x_A(v_A; e(\gamma_1, \gamma_2, x_D))} dG(v_A).$$

Let the state's cost functions be $C_1(\gamma_1)$ and $C_2(\gamma_2)$ with strictly positive marginal costs. The sequence of events is the following:

1. The state determines γ_1 and γ_2.

2. D determines x_D.

3. A determines x_A.

We assume the state moves first because usually states are longer lived than individuals and an individual takes usually as given the level of state enforcement. That D moves second follows from the same reasoning as above. The state then maximizes

$$\max_{\gamma_1,\gamma_2} \int_{\underline{v}_A}^{\overline{v}_A} \frac{e(\gamma_1,\gamma_2,x_D)}{e(\gamma_1,\gamma_2,x_D)+x_A(v_A;e(\gamma_1,\gamma_2,x_D))} dG(v_A)\, v_S - C_1(\gamma_1) - C_2(\gamma_2),$$

taking into account the optimal reactions of both D and A. It is straightforward to show that for A and D this situation is now strategically equivalent to the one in Section 3, except that D' marginal costs of effort are now $1/\gamma_1$ and he is constrained to choose effort $e \geq \gamma_2$. Hence, he will always choose $x_D < (\overline{v}_A - \gamma_2)/\gamma_1$ unless $\gamma_1 = \infty$ or $\gamma_2 = \overline{v}_A$. In those cases, however, the state is the de facto single enforcer of property rights. So it suffices to focus on the state's behavior without taking into account D's optimal reaction to study whether there may be full deterrence. The state is in the identical position as D in our baseline model above with the only difference being the cost function. But since marginal costs are strictly positive, it follows immediately from the earlier discussions that the state does not fully deter A, either. The derivative of the state's objective function at $\gamma_2 = \overline{v}_A$ is $-C_2(\overline{v}_A) < 0$, and at $\gamma_1 = \infty$ is $-C_1(\infty) < 0$. Therefore, the state will not fully deter A, either, even though it may be able to generate economies of scale in the process of enforcement.

6 Endogenous strategic uncertainty

While we showed above that for arbitrary uncertainty or heterogeneity property rights enforcement is imperfect or insecure, we now attempt to explain why there is uncertainty in the first place. First, of course, individuals differ for example in their tastes or their abilities. This is one source of uncertainty, which we may term as exogenous uncertainty. There is, however, another dimension which may be created in the game and we refer to it as endogenous strategic uncertainty henceforth.

Remember that in a deterrence equilibrium the appropriator gets nothing. If there is uncertainty, however, he gets a positive payoff in expectation. Therefore, the appropriator has an incentive to create uncertainty endogenous. He could do this for example by creating uncertainty about his ability. However, to do this a defender should know the identity of a thief, but this is arguably not the typical case. Another option A has is to influence instead of D's belief about his type D's probability to become a victim of attempted theft, and to create uncertainty in this way. One source of this endogenously created uncertainty is related to the number of possible victims of theft. Another source is related to strategic restraint. We now present a simple three player model to develop the intuition. In Appendix B we show that the intuition carries over easily to more general cases.

Assume there are two individuals Di, $i = 1, 2$, each possessing a good of value v_{Di}. Further, there is an appropriator with valuation v_A. All players' characteristics are common knowledge. As before, Di is able to commit to a particular level of defensive effort. The conflict is decided according to the ratio form CSF in (1). The new element is the probability with which A tries to steal from i, q_i. This probability is a function of two decisions: First, A may decide not to steal at all, what happens with a probability $1 - r$ to which he is able to commit. This assumption is not unrealistic. One explanation for this comes from a dynamic game, in which Di bases his decision on how much to spend for fortification on a historical measure of property crime in his neighborhood. If the appropriator decides to attempt more break-ins, that is to increase r, the overall level of crime increases and as a consequence the level of protection will increase, too. Therefore, A will choose this probability dynamically optimal, and this probability may well be smaller than one. Second, returning to our above burglary example, burglars usually do not limit their activity to burglary or property crimes, but engage in a wider range of criminal activities such as violent and drug related crimes (Rengert & Wasilchick, 2000). That means there is a valuable outside option and a burglar may well restrain himself from burglary for a while so as to give a proprietor no incentive to increase security investments. Hence, our setup can be interpreted as a reduced form of a richer model of conflict in which commitment may arise endogenously. Second, if A steals he has to decide from whom to steal. Let m_1 be the probability that he steals from $D1$, and $m_2 = 1 - m_1$ the probability that he steals from $D2$. Then we have $q_i = r\, m_i$. We may interpret $1 - r$ and m_i as different measures of strategic restraint of A. Restraining himself decreases the probability that $D1$ or $D2$ need to protect their property and hence decrease their incentives to invest in protection.

We solve the game using backwards induction. In the last stage, provided he tries to steal from i, A chooses effort x_{Ti}. As before, this is given by his reaction function $x_{Ti}(x_{Di}, v_A) = \sqrt{v_A x_{Di}} - x_{Di}$. In stage 2 Di takes this into account to maximize

$$v_{Di}\left(q_i \frac{x_{Di}}{x_{Di}+x_{Ti}} + (1-q_i)\right) - x_{Di} = v_{Di}\left(q_i \sqrt{\frac{x_{Di}}{v_A}} + (1-q_i)\right) - x_{Di}.$$

The unique equilibrium level of fortification is easily found and given by

$$x^+_{Di} = \frac{q_i^2 v_{Di}^2}{4v_A}. \tag{10}$$

We can now use this to determine A's expected equilibrium utility in the subgame in which he tries to steal from i, taking as given q_i. This is

$$\tilde{u}^+_{Ti} = \frac{(q_i v_{Di} - 2v_A)^2}{4v_A}.$$

In equilibrium A has to randomize between stealing from $D1$ or $D2$. To see this assume he goes with certainty to $D1$. This immediately implies he will never go to $D2$, so that $q_2 = 0$. As a consequence, $D2$ will optimally choose $x_{D2} = 0$ (see (10)). But then it would be optimal to increase m_2 slightly, since he wins there for sure and makes first order gains. As a consequence, he must randomize in equilibrium.

Lemma 2. *In equilibrium A randomizes whether to steal from $D1$ or $D2$ according to a non-degenerate mixed strategy, which is given by*

$$m^+_i = \frac{v_{Dj}}{v_{D1}+v_{D2}},$$

where $i, j = 1, 2$ and $i \neq j$.

Proof. This follows immediately from the above discussion and equating A's subgame payoffs $\tilde{u}^+_{T1}$ and $\tilde{u}^+_{T2}$, taking into account that $q_i = r \cdot m_i$.[4] □

The lemma shows that A approaches a stronger player with a lower probability. Because he needs to appease the stronger player more, this is also intuitive. It also implies that if a

[4] There are also other values of m_i equating A's payoffs. However, this is the only solution satisfying $0 < m_i < 1$ for all parameter values. As discussed above, this is condition has to hold in an equilibrium.

player increases his effectiveness of defensive effort, say $D1$ increases v_{D1}, he inflicts a negative externality on $D2$, since then A would approach him more often which decreases his utility. For a discussion of this argument see de Meza & Gould (1992).

We now show how A chooses r optimally to complete the analysis. From the above reasoning it follows that he chooses r to maximize

$$u_A = r\left(\frac{\left(r\frac{v_{D1}v_{D2}}{v_{D1}+v_{D2}} - 2v_A\right)^2}{4v_A}\right). \tag{11}$$

The next lemma gives the solution to this problem:

Lemma 3. *The optimal probability to attempt theft is*

$$r^+ = \min\left\{\frac{2v_A(v_{D1}+v_{D2})}{3\,v_{D1}v_{D2}}, 1\right\} \in (0,1].$$

Proof. See appendix. □

From inspection of r^+ it is easily verified that for v_A sufficiently small A profits from strategically restraining himself. If he is relatively weak he gains from appeasing $D1$ and $D2$ due to strategic restraint, or, to put it differently, by keeping the level of crime low. From Lemmas 2 and 3 it follows that for each individual Di the probability of attempted theft is

$$q_i^+ = \min\left\{\frac{2}{3}\frac{v_A}{v_{Di}}, \frac{v_{Dj}}{v_{Di}+v_{Dj}}\right\} < 1.$$

The probability to steal from Di is increasing in the valuation of A, but bounded from above by $v_j/(v_i + v_j)$. Thus, there always remains uncertainty. In general, the weaker A relative to both $D1$ and $D2$, the more strategic uncertainty he chooses, that is the lower is r. In addition, because he randomizes between $D1$ and $D2$, he further increases this uncertainty. After all, the probability that A does not try to steal from Di is $1 - q_i = 1 - m_i\, r$, which is decreasing in both r and m_i. The important point we want to make is now captured by this proposition:

Proposition 4. *If A is sufficiently weak it is in his interest to create strategic uncertainty* *by restraining himself from stealing. This appeases Di and dilutes his incentives to invest in fortification. As a consequence, the probability that Di successfully defends against an attempt of A to steal is strictly below 1 and equal to*

$$\psi_i^+ = \psi^+ = \min\left\{\frac{v_{D1}\, v_{D2}}{2v_A(v_{D1}+v_{D2})}, \frac{1}{3}\right\}.$$

From the point of view of the society, the ex-ante probability of enforcement as defined in Corollary 3, that is, the probability that ex-post everybody is still in possession of his belongings, is

$$\phi^+ = 1 - r(1-\psi^+) < 1.$$

As a consequence, enforcement is always imperfect.

Proof. This follows from the above discussion. □

Even without heterogeneity or uncertainty, A is able to endogenously create uncertainty that leads to imperfect enforcement. He has an incentive to strategically restrain himself from stealing to increase the chances of success when he tries. Strategic restraint, as we modeled it, can happen in two ways. First, the appropriator can decrease the overall probability of theft. This appeases all individuals he could steal from. Second, he randomizes about from whom to steal. This decreases for each individual the probability of being a victim and therefore leads to additional appeasement. The magnitude of this effect is increasing in the size of the population, so that in a large neighborhood or society for each individual the probability that somebody steals from her is lower, holding r fixed. This allows A to increase r and so to gain. Of course, this also would make it more profitable to become an appropriator. In Appendix B you can find a discussion in which we show that this result holds true also in a larger society with n_A appropriators and $n_D = n - n_A$ possible victims, where "profession choice" is endogenous, as in for example Konrad & Skaperdas (2010) and Dal Bó & Dal Bó (2009).

It is worth noting that, although enforcement is imperfect, all individuals gain from uncertainty. For the appropriator this is easily seen, because he improves his payoff from zero to something positive. But also for the proprietors this is true. They could always simply copy their behavior in the absence of uncertainty. However, they do not, but decrease their efforts. Their payoff conditional on being approached by A would remain identical, but the probability of such an event is lower. As a consequence, they must be better of, too.

Strategic restraint in our model is similar to the strategic restraint discussed in Epstein & Nitzan (2004). However, in their article restraint means demanding less, or strategically decreasing the prize one fights for. Still, the effect is similar: strategic restraint is a means to appease the opponent and so to increase one's payoff.

7 Discussion

Property rights and other rules and laws will not be perfectly enforced in an equilibrium. But what is the deeper reason for that? In our framework we found that inframarginal losses against all already deterred types drive the result. However, on a more fundamental level the reason is that because of uncertainty it is not possible to make contingent defensive efforts. Uncertainty about the appropriator's ability or strategic uncertainty created by the appropriator force defenders to built the same defense for all possible contingencies or states of the world, and it is then not optimal to deter all appropriative activities. This enables offenders always to make positive expected rents, which would not be the case without uncertainty. This parallels many findings from the economics of information. For example, it is precisely the impossibility to write type contingent contracts that drive the results in the seminal work of Akerlof (1970). Because sellers have to charge the market price for all types of cars, leading to a race to the bottom in quality, only bad quality cars survive, yielding another application of Gresham's law. In our example it is not possible to built up type or state contingent fortification. Note that this holds true for any number of policy instruments, as long as contingent instruments are not feasible.

Moreover, appropriators and other violators gain from uncertainty, and thus receive information rents. We have seen this best in Section 6, where the appropriator endogenously created uncertainty optimally. That uncertainty benefits some parties and allows them to receive information rents is something we find quite often in the contract theory and principal-agent literature. For example, in a Baron & Myerson (1982) regulation problem, where the authority does not know the cost function of the monopolist, we usually find the low cost types to benefit from the possibility of high cost types. Another example for such information rents is second degree price discrimination.

Finally, on a more technical note, for deterrence to be technically possible appropriators' and violators' reaction functions need to be downward sloping and must cross the abscissa at some point. Equivalently, efforts need to be strategic substitutes over a given range of the reaction function. This is what we find in standard oligopoly models with quantity competition. In this respect Cournot oligopoly models and standard (imperfectly discriminating) contest models are similar, as has been noted by Mehlum & Moene (2002) and others. However, while in Cournot competition we find strategic substitutes over the whole range of the strategy space, this is not true in contest models, in which we usually find hump-shaped best responses. For low values of the opponent's effort we find strategic complements and strategic substitutes only later (Dixit, 1987). However, if we want to analyze deterrence this is exactly the region we are interested in. So, because we find property rights are never secure in a contest with uncertainty, exogenously or endogenously determined, that directly implies that in Cournot oligopoly models with uncertainty a Stackelberg leader will never deter all possible types of competing firms, too. This is in line with Gal-Or (1987), who showed that in an environment with uncertainty "*the preemptive capabilities of a Stackelberg leader are reduced*".

A Mathematical appendix

Proof of Proposition 2. D maximizes

$$\begin{aligned} & v_D \int_{\underline{v}_A}^{\overline{v}_A} \left[\frac{x_D}{x_D + x_A(v_A; x_D)} \right] dG(v_A) - x_D \\ s.t. \quad & x_A = \max\left\{ \sqrt{v_A\, x_D} - x_D, 0 \right\}, \end{aligned}$$

or equivalently

$$G(\tilde{v}) v_D + v_D \sqrt{x_D} \theta(\tilde{v}) - x_D,$$

where $\tilde{v} = \max\{x_D^+, \underline{v}_A\}$ and $\theta(\tilde{v}) \equiv \int_{\tilde{v}}^{\overline{v}_A} v_A^{-1/2} dG(v_A)$ as defined above in Section 3. To prove existence of a maximum it is sufficient to note that this function is continuous (since it is differentiable) on the compact and convex set $[0, \overline{v}_D]$, and therefore it follows from the extreme value theorem that a maximum exists. Uniqueness and the applicability of the first order condition follow from its strict concavity, the second order condition is

$$-\frac{v_D\theta(x_D)}{4x_D^{3/2}} - \frac{v_D G'(x_D)}{2x_D} < 0.$$

The first order condition reads

$$\frac{\theta(\tilde{v})\, v_D}{2\sqrt{x_D^+}} + v_D\, G'(\tilde{v}) - v_D\sqrt{\frac{1}{x_D}}\,\sqrt{x_D}\, G'(\tilde{v}) - 1 \stackrel{!}{=} 0.$$

Simple manipulations reveal that

$$x_D^+ = \left(\frac{\theta(\tilde{v})\, v_D}{2}\right)^2.$$

□

Proof of Lemma 1. Given the above assumptions A maximizes

$$\begin{aligned} Eu_A &= p_A(x_A, x_D)\, u_A(v_A - x_A) + (1 - p_A(x_A, x_D))\, u_A(-x_A) \\ &= \frac{f(x_A)}{f(x_A) + f(x_D)} u_A(v_A - x_A) + \frac{f(x_D)}{f(x_A) + f(x_D)} u_A(-x_A) \end{aligned} \tag{12}$$

It is easy to see that his utility from staying passive in the contest, that is from spending effort $x_A = 0$, is $u_0 \equiv u_A(0)$. Moreover, all efforts $x_A > v_A$ are strictly dominated. Denote the utility level from putting $x_A = v_A$ by $u_{v_A} \equiv p_A(x_A, x_D)u_A(0) + (1 - p_A(x_A, x_D))u_A(-v_A) \leq u_0$.

The first derivative of (12) is

$$\begin{aligned} \Delta = \frac{\partial Eu_A}{\partial x_A} &= \underbrace{\frac{f'(x_A)\, f(x_D)}{(f(x_A) + f(x_D))^2} u_A(v_A - x_A) - \frac{f(x_A)}{f(x_A) + f(x_D)} u'(v_A - x_A)}_{\equiv I} \\ &- \underbrace{\frac{f'(x_A)\, f(x_D)}{(f(x_A) + f(x_D))^2} u_A(-x_A) - \frac{f(x_D)}{f(x_A) + f(x_D)} u'(-x_A)}_{\equiv II}. \end{aligned}$$

Except for the first term all of those terms are negative in sign. The first two terms are the impact of increasing effort marginally on winning, while the third and fourth term describe the effect on losing. Increasing effort has two opposing effects on the winning utility. First, winning becomes more likely, what is positive. Second, if he wins he gets less, what is negative. The third and fourth term are both negative, reflecting that increasing effort decreases the probability to lose and at the same time decreases the utility in the event of losing. It is straightforward to verify that I is strictly decreasing in x_A. That directly implies $f(x_A)/(f(x_A)+f(x_D))u_A(v_A-x_A)$ is strictly concave and is hence either a monotonic function in x_A, increasing or decreasing, or its graph is inverse U-shaped. Because II is negative Eu_A must also be either monotonic in x_A, increasing or decreasing, or it is inverse U-shaped. However, we know that $u_0 > u_{v_A}$, implying that it cannot be monotonically increasing and must be decreasing on some interval. Of course, if Eu_A is inverse U-shaped Δ must be zero at some point, at which utility is strictly higher than $\overline{u}_A$. Then the best response must be positive, $x_A(v_A;x_D) > 0$. If the derivative is strictly decreasing A's optimal choice is $x_A(v_A;x_D) = 0$. To find out which of those two possible solutions is the correct one we have to evaluate Δ at $x_A = 0$:

$$\Delta|_{x_A=0} = \frac{f'(0)}{f(x_D)}\left[u_A(v_A) - u_A(0)\right] - u'_A(0).$$

It is now easily verified by inspection of $\Delta|_{x_A=0}$ that this is a monotonic function in x_D and that there exists a threshold value $\bar{x}_D = \mu_A(v_A)$, such that

$$Sign[\Delta|_{x_A=0}] = \begin{cases} + & \text{if } x_D < \mu_A(v_A), \\ 0 & \text{if } x_D = \mu_A(v_A), \\ - & \text{if } x_D > \mu_A(v_A). \end{cases}$$

If $x_D \geq \mu_A(v_A)$ it is true that $\Delta|_{x_A=0}$ is either zero or negative. Otherwise $\Delta|_{x_A=0}$ is positive. From the above discussion it follows immediately that $x_A(v_A;x_D) > 0$ if and only if $x_D < \mu(v_A)$. The threshold is given by

$$\mu(v_A) = f^{-1}\left(\frac{f'(0)}{u'_A(0)}\left[u_A(v_A) - u_A(0)\right]\right),$$

from which it immediately follows that $\mu(v_A)$ is continuously differentiable and $\mu'(v_A) > 0$. $\square$

Proof of Proposition 3. In Lemma 1 we found that there exists a threshold level of effort x_D denoted by $\mu(\overline{v})$, which is strictly increasing and differentiable. Therefore, its inverse

$v_A = \mu^{-1}(x_D)$ exists and is also continuously differentiable. We then define

$$\omega(x_D) \equiv \mu^{-1}(x_D).$$

$\omega(x_D)$ is the marginal type v_A that is deterred given effort x_D. If $\omega(x_D) < \underline{v}_A$ all thieves are active. $\omega(x_D) \geq \overline{v}_A$ implies all thieves are deterred.

We are now able to define D's expected utility function

$$\begin{aligned}
Eu_D &= \int_{\underline{v}_A}^{\overline{v}_A} p(x_D, x_A(v_A; x_D))\, u_D(v_D - x_D) + (1 - p(x_D, x_A(v_A; x_D)))\; u_D(-x_D)\, dG(v_A) \\
&= \left(G(\omega(x_D)) + \int_{\omega(x_D)}^{\overline{v}_A} p(x_D, x_A(v_A; x_D)) dG(v_A) \right) u_D(v_D - x_D) \\
&+ \left(1 - G(\omega(x_D)) - \int_{\omega(x_D)}^{\overline{v}_A} p(x_D, x_A(v_A; x_D)) dG(v_A) \right) u_D(-x_D)
\end{aligned}$$

The derivative with respect to x_D is the following:

$$\begin{aligned}
\Gamma &= u_D(v_D - x_D) \\
&\times \left(\int_{\omega(x_D)}^{\overline{v}_A} \left(\frac{\partial p_D(\cdot,\cdot)}{\partial x_A} \frac{\partial x_A(\cdot,\cdot)}{\partial x_D} + \frac{\partial p_D(\cdot,\cdot)}{\partial x_D} \right) dG(v_A) - \omega'(\cdot) g(\omega(\cdot)) p_D(\cdot,\cdot) + \omega'(\cdot) g(\omega(\cdot)) \right) \\
&- u_D(-x_D) \\
&\times \left(\int_{\omega(x_D)}^{\overline{v}_A} \left(\frac{\partial p_D(\cdot,\cdot)}{\partial x_A} \frac{\partial x_A(\cdot,\cdot)}{\partial x_D} + \frac{\partial p_D(\cdot,\cdot)}{\partial x_D} \right) dG(v_A) - \omega'(\cdot) g(\omega(\cdot)) p_D(\cdot,\cdot) + \omega'(\cdot) g(\omega(\cdot)) \right) \\
&- u_D'(v_D - x_D) \left(G(\omega(x_D)) + \int_{\omega(x_D)}^{\overline{v}_A} p_D(\cdot,\cdot)\, dG(v_A) \right) \\
&- u_D'(-x_D) \left(1 - G(\omega(x_D)) - \int_{\omega(x_D)}^{\overline{v}_A} p_D(\cdot,\cdot)\, dG(v_A) \right)
\end{aligned}$$

Fortunately, we are able to boil down this expression significantly. To show that x_D cannot optimally be equal to or larger than $\mu(\overline{v}_A)$, it is sufficient to show that $\Gamma|_{x_D=\mu(\overline{v}_A)}$ is negative. Therefore, using $p_D(x_D, x_T) = f(x_D)/(f(x_D) + f(x_T))$, let $x_D = \mu(\overline{v}_A)$ and note that $\omega(\mu(\overline{v}_A)) = \overline{v}_A$ and $G(\overline{v}_A) = 1$ by definition. This simplifies the derivative already significantly:

$$\begin{aligned}
\Gamma|_{x_D=\mu(\overline{v}_A)} &= u_D(v_D - \mu(\overline{v}_A)) \left(g(\overline{v}_A)\omega'(\mu(\overline{v}_A)) - \frac{f(\mu(\overline{v}_A)) g(\overline{v}_A) \omega'(\mu(\overline{v}_A))}{f(x_A(\overline{v}_A, \mu(\overline{v}_A))) + f(\mu(\overline{v}_A))} \right) \\
&+ u(-\mu(\overline{v}_A)) \left(\frac{f(\mu(\overline{v}_A)) g(\overline{v}_A) \omega'(\mu(\overline{v}_A))}{f(x_A(\overline{v}_A, \mu(\overline{v}_A))) + f(\mu(\overline{v}_A))} - g(\overline{v}_A)\omega'(\mu(\overline{v}_A)) \right) \\
&- u_D'(v_D - \mu(\overline{v}_A)).
\end{aligned}$$

Now we can use that $x_A(v_A; \mu(v_A)) = 0$ by definition of $\mu(v_A))$ as well as $f(0) = 0$ to see that the only remaining term is

$$\Gamma|_{x_D=\mu(\overline{v}_A)} = -u'_D(v_D - \mu(\overline{v}_A)) < 0.$$

This is the marginal cost of effort weighted by the probability to encounter a thief with a valuation lower than $\overline{v}_A$. The derivative is negative, and therefore the necessary condition for a maximum is not fulfilled. Hence, it cannot be optimal to deter type $\overline{v}_A$. Moreover, continuity of Γ implies this is true not only for this type, who has measure zero, but for some interval of types $[v'_A, \overline{v}_A]$ with strictly positive measure. Therefore, property rights enforcement must be imperfect. □

Proof of Lemma 3. Recall from (11) that

$$u_A = r\left(\frac{\left(r\frac{v_{D1}v_{D2}}{v_{D1}+v_{D2}} - 2v_A\right)^2}{4v_A}\right).$$

Taking the derivative with respect to r yields the first order condition

$$\frac{\partial u_A}{\partial r} = \frac{(r\frac{v_{D1}v_{D2}}{v_{D1}+v_{D2}} - 2v_A)^2}{4v_A} + 2r\frac{v_{D1}v_{D2}}{v_{D1}+v_{D2}}\frac{(r\frac{v_{D1}v_{D2}}{v_{D1}+v_{D2}} - 2v_A)}{4v_A} \overset{!}{=} 0,$$

which after simple manipulations reveals that

$$r^+ = \frac{2v_A}{3}\frac{v_{D1}+v_{D2}}{v_{D1}v_{D2}}.$$

The second derivative with respect to r is

$$\frac{\partial^2 u_A}{\partial r^2} = \frac{v_{D1}v_{D2}(3rv_{D1}v_{D2} - 4v_A(v_{D1}+v_{D2}))}{2v_A(v_{D1}+v_{D2})^2}.$$

If we evaluate this at r^+ we get

$$\frac{\partial^2 u_A}{\partial r^2}|_{r=r^+} = -\frac{v_{D1}v_{D2}}{v_{D1}+v_{D2}} < 0,$$

such that the second order condition for a maximum is fulfilled. Accordingly, r^+ is A's optimal choice.

□

B Strategic restraint and free entry in the market for appropriators

Assume in the population there are n individuals, n_D of which defend their possession against the remaining $n_A = n - n_D$ appropriators. For simplicity, we assume both groups D and A are comprised of homogeneous individuals. Each appropriator's expected payoff conditional on attempting to break in house j is

$$\frac{(r_i m_{ij} \rho_{ij} v_D - 2v_A)^2}{4v_A},$$

where ρ_{ij} is the probability of i being the first appropriator trying to break in at house j. If there is more than one appropriator at the same house each is first with equal probability. We focus on a symmetric equilibrium. As a consequence, $m_{ij} = m = 1/n_D$. Then ρ_{ij} is given by

$$\begin{aligned}\rho_{ij}(n, n_A, \tilde{r}) &= \sum_{k=0}^{n_A-1} \frac{1}{1+k} \binom{n_A-1}{k} \left(\frac{\tilde{r}}{n-n_A}\right)^k \left(1 - \frac{\tilde{r}}{n-n_A}\right)^{n_A-k-1} \\ &= \frac{(n-n_A)\left(1-\frac{\tilde{r}}{n-n_A}\right)^{n_A}\left(\left(\frac{n-n_A}{n-n_A-\tilde{r}}\right)^{n_A}-1\right)}{n_A \tilde{r}}.\end{aligned}$$

$\tilde{r}$ is the probability with which appropriators $j \neq i$ attempt theft, which i takes as given. i's optimal choice of r is then

$$r_i^+ = \min\left\{\frac{2\,(n-n_A)\,v_A}{3\rho_{ij}(n, n_A, \tilde{r})v_D}, 1\right\} = \min\left\{\frac{2(n-n_A)}{3\rho_{ij}(n, n_A, \tilde{r})\kappa}, 1\right\},$$

which follows from the reasoning in Section 6 and where we let $v_D = \kappa v_A$. The parameter κ is a measure of the valuation of group D relative to group A, or this group's relative strength. We focus on $\kappa \geq 2$, since this implies without uncertainty property rights are perfectly enforced in equilibrium. In a symmetric equilibrium we have $r_i^+ = \tilde{r}$ and thus $\rho_{ij} = \rho$. Therefore,

$$r_i^+ = r^+ = \min\left\{\frac{2(n-n_A)}{3\rho(n, n_A, r^+)\kappa}, 1\right\}.$$

If $r^+ = 2(n - n_A)/(3\rho(n, n_A, r^+)\kappa) < 1$ in equilibrium each appropriator restrains himself strategically. To see whether such an equilibrium exists we look at

$$\frac{2(n-n_A)n_A r}{3\kappa(n-n_A)\left(1-\frac{r}{n-n_A}\right)^{n_A}\left(\left(\frac{n-n_A}{n-n_A-r}\right)^{n_A}-1\right)} - r = 0.$$

Because in equilibrium it cannot be the case that $r = 0$ and $n - n_A = 0$ we can cancel terms and rearrange to get

$$\left(1-\frac{r}{n-n_A}\right)^{n_A}\left(\left(\frac{n-n_A}{n-n_A-r}\right)^{n_A}-1\right) = \frac{2n_A}{3\kappa}.$$

The actual value of r^+ depends on the parameter values. Note though that at $r = 0$ the left-hand side is zero. The derivative of the left-hand side with respect to r is equal to $n_A\left(1-\frac{r}{n-n_A}\right)^{n_A}/(n-n_A-r) > 0$ and hence it is increasing in r for $r \leq 1$ for sure, and depending on $n - n_A$ also thereafter. The right-hand side is decreasing in κ. Therefore, there exists at least one value of κ for which $r^+ = 1$. By increasing κ further r^+ decreases, stays however positive unless $\kappa \to \infty$. Hence for all n and n_A there always exists a range of values of κ such that the strategic restraint equilibrium exists.

What would happen now if there was open entry into the market for appropriators? Let the payoff of an appropriator be the equilibrium utility from the contest plus some constant l, which we call leisure. The payoff of a proprietor or defender is his expected utility in the contest. Assume the utility of a defender in an equilibrium in which he enforces property rights perfectly is $\beta > l$. The assumption we just made implies that the defender needs to produce the good, which costs him time, before the appropriator can steal it. As a consequence, the appropriator has more leisure, l, than the proprietor. This assumption guarantees that at least some individuals choose to become appropriators in equilibrium. The second assumption states that this leisure utility is not enough to dominate the utility of being a proprietor with perfectly enforced property. This assumption guarantees that at least some individuals choose to become proprietors and to produce a good in equilibrium.

The equilibrium number of appropriators is such that a no-arbitrage condition between becoming an appropriator or proprietor holds. In equilibrium individuals will sort themselves into appropriators and defenders until the relative mass of appropriators is such that the expected utility from becoming an appropriator is equal to the utility of a proprietor. Note that with perfect property rights the outside option is strictly positive, $\beta > 0$ and an appropriator gets nothing but l. This can never be an equilibrium. On the other hand a situation with only proprietors cannot be an equilibrium as well. No one would invest in enforcement and hence there is an incentive to become an appropriator. The only possible equilibrium is one where enforcement is imperfect and hence appropriators earn a positive rent which makes them exactly indifferent to becoming proprietors. Hence, in any equilibrium property rights enforcement needs to be imperfect.

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