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[1] We have benefited from comments by Michael Lechner and Christoph Rothe.

Abstract

In the presence of an endogenous treatment and a valid instrument, causal effects are (nonparametrically) point identified only for the subpopulation of compliers, given that the treatment is monotone in the instrument. Further populations of likely policy interest have been widely ignored in econometrics. Therefore, we use treatment monotonicity and/or stochastic dominance assumptions to derive sharp bounds on the average treatment effects of the treated population, the entire population, the compliers, the always takers, and the never takers. We also provide an application to labor market data and briefly discuss testable implications of the instrumental exclusion restriction and stochastic dominance.

Keywords

Instrument, noncompliance, principal stratification, nonparametric bounds.

JEL Classification

C14, C31, C36

1 Introduction

Endogeneity of the (binary) treatment variable and noncompliance to the treatment assignment in randomized experiments are widespread phenomenons in the evaluation of treatment effects, see for instance Bloom (1984). In the presence of an instrumental variable (IV) such as random treatment assignment, Imbens and Angrist (1994) (see also Angrist, Imbens, and Rubin, 1996) show that average treatment effects are point identified only for a subpopulation, given that the treatment is monotone in the instrument. This local average treatment effect (LATE) refers to the so called compliers, whose treatment status reacts on a change in the instrument. If both the treatment and the instrument are binary and monotonicity is positive, the compliers are those with the treatment value always being equal to the instrument value.

Whether the LATE is a relevant parameter heavily depends on the empirical context and has been controversially discussed in the literature, see for instance Imbens (2009), Deaton (2010), and Heckman and Urzúa (2010). In many applications, one prefers to make inference for further or different populations. E.g., applications in the program evaluation literature typically want to learn about the average treatment effects (ATE) on the treated or on the entire population. Note that these parameters are themselves weighted averages of the ATEs on several subpopulations, including the always takers (always treated irrespective of the instrument) and the never takers (never treated irrespective of the instrument). So far, any of these groups has been widely ignored in the econometric literature.

An exception is Frölich and Lechner (2010) who also point identify the ATEs on the always takers and never takers. Therefore, they invoke both IV and selection on observables (or conditional independence, see for instance Imbens, 2004) assumptions. This identification strategy is, however, in contrast to virtually all other IV applications, where an instrument is used exactly for the reason that no other source of identification (such as selection on observables) is available. Then, point identification for the never takers, always takers, the treated, and the entire population is not feasible in a nonparametric framework (unless the complier share is 100 %).

The main contribution of this paper is to show that informative nonparametric bounds on the ATEs for populations other than the compliers can be attained under relatively weak assumptions. In many empirical applications the identification of a reasonable set of values for the ATE on, for instance, the treated might be preferable to the point identification of the LATE, which may suffer from decreased external validity. The assumptions invoked in this paper are (i) monotonicity of the treatment in the instrument and (ii) stochastic dominance of the potential outcomes of one subpopulation over the others. The identifying power of these restrictions is investigated both separably and jointly using the principal stratification framework suggested by Frangakis and Rubin (2002). We derive sharp bounds for the always takers, never takers, the treated, and the entire population. As a further contribution, we find testable implications of the IV exclusion restriction and stochastic dominance when monotonicity is invoked.

Partial identification of economic parameters in general goes back to Manski (1989, 1994) and Robins (1989). Previous work on nonparametric bounds under treatment endogeneity, which is the problem considered here, has exclusively focused on the ATE on the entire population,[1] but neglected further populations. Manski (1990) bounds the ATE by invoking only mean independence between the potential outcomes and the instrument. Considering binary outcomes, Balke and Pearl (1997) derive sharp bounds under the same statistical independence considered in Imbens and Angrist (1994) and under monotonicity (see also Dawid, 2003) of the treatment in the instrument. Also Shaikh and Vytlacil (2010) bound the ATE on the entire population in the binary outcome case under monotonicity, see Bhattacharya, Shaikh, and Vytlacil (2005) for an application. In addition, Shaikh and Vytlacil (2010) also consider the so-called "monotone treatment response" assumption of Manski (1997), which a priori restricts the direction of the treatment effect. This appears unattractive given that the latter is unknown and has to be estimated and will, therefore, not be considered here. Cheng and Small (2006) extend the results for binary outcomes to three treatments (in contrast to the standard binary treatment framework considered here) under particular forms of (one-sided) noncompliance.

[1]For the derivation of semiparametric bounds on the ATE on the entire population, see Chiburis (2010) and the references therein.

In contrast to much of the epidemiologic literature, Heckman and Vytlacil (2001) and Kitagawa (2009) allow for both discrete and continuous outcomes. Kitagawa partially identifies the potential outcome distributions for the entire population under (various forms of) the exclusion restriction and monotonicity and derives bounds on the ATE. Heckman and Vytlacil (2001) invoke a nonparametric threshold crossing model for the treatment instead of monotonicity for deriving the bounds. However, both approaches are equivalent by the results of Vytlacil (2002). One interesting finding of Heckman and Vytlacil (2001) and Kitagawa (2009) is that the width of their bounds is the same as those of Manski (1990) (see also Balke and Pearl, 1997, for binary outcomes), given that the monotonicity/threshold crossing model assumptions are satisfied.

The present work adds to the literature on nonparametric bounds under endogeneity by considering more populations and an extended set of identifying assumptions. The identifying power of jointly imposing monotonicity and stochastic dominance is demonstrated in an empirical application to the U.S. Job Training Partnership Act. Using experimental data previously analyzed by Abadie, Angrist, and Imbens (2002), we find (in addition to the point identified complier effect) a significantly positive ATE on the earnings of the treated that lies within reasonably tight bounds. Monotonicity and stochastic dominance have also been considered in a different context, namely under non-random sample selection and attrition, see for instance Zhang and Rubin (2003), Lechner and Melly (2007), Zhang, Rubin, and Mealli (2008), and Lee (2009), and Huber and Mellace (2010).

The remainder of this paper is organized as follows. Section 2 characterizes the endogeneity/noncompliance problem based on principal stratification. Section 3 discusses the identifying assumptions and derives bounds on the ATEs for various populations. Section 4 very briefly presents the estimators. In Section 5, we consider an empirical application to experimental labor market data. Section 6 concludes.

2 Using principal stratification to characterize noncompliance

Suppose that we want to estimate the effect of a binary treatment $T \in \{1, 0\}$ (e.g., a training activity) on an outcome Y (e.g., labor market success such as employment or earnings) evaluated at some point in time after the treatment. We will use the experimental framework to motivate the problems of endogeneity and noncompliance. Assume that individuals are randomly assigned into treatment or non-treatment according to the binary assignment variable $Z \in \{1, 0\}$, which will serve as instrument. Denote by $T(z)$ the potential treatment state for $Z = z$, and by $Y_i(1)$ and $Y_i(0)$ the potential outcomes (see for instance Rubin, 1974) of individual i under treatment and non-treatment.

Throughout the discussion we will rule out interference between individuals as well as general equilibrium effects of the treatment, which is implied by the "Stable Unit Treatment Value assumption" (SUTVA), see for instance Rubin (1990). Furthermore, we will assume that mere assignment does not have any direct effect on the potential outcomes other than through the treatment. Taking assignment to a training as an example, this rules out that individuals change their labor market state as a reaction of being assigned to the training. What matters is whether the training is actually received. This exclusion restriction and the SUTVA are formalized in Assumption 1 (see also Angrist, Imbens, and Rubin, 1996):

Assumption 1:

$Y(t, Z) = Y(t) \ \forall \ t \in \{0, 1\}$ (exclusion restriction),

and

$Y_i(t_i) \perp t_j$ and $T_i(z_i) \perp z_j \ \forall j \neq i$ (SUTVA).

Of course, the individual effect $Y_i(1) - Y_i(0)$ can never be evaluated as individual i is either treated or not treated, but cannot be observed in both states. However, under particular assumptions aggregate parameters such as the average treatment effect (ATE) $\Delta = E[Y(1)] - E[Y(0)]$ can be identified.

Our second assumption restricts Z to be independent of the potential values of any post-assignment variables, which holds by random assignment. Analogous to Imbens and Angrist (1994), we make the following joint independence assumption:

Assumption 2:

$Z \perp T(1), T(0), Y(1), Y(0)$ (joint independence),

where "$\perp$" denotes independence. Alternatively, one may assume that independence only holds conditional on some observed pre-assignment variables X, such that Assumption 2 changes to $Z \perp T(1), T(0), Y(1), Y(0) | X = x, \quad \forall x \in \mathcal{X}$, where $\mathcal{X}$ denotes the support of X. This is closely related to the framework of Frölich (2007) who shows point identification of the LATE given a conditionally valid instrument (given X). In the further discussion, conditioning on X will be kept implicit, such that all results either refer to the experimental framework or to an analysis within cells of X. Assumption 2 is equivalent to the "random assignment restriction" in Kitagawa (2009) who, as an alternative, also considers the following, weaker assumption: $Z \perp Y(1), Y(0)$. This case would allow for dependence between the instrument and the potential treatment states. It is not considered in this paper, as it is not consistent with (successful) randomization, which implies independence of Z and any potential post-treatment variables.

Note that experimental compliance is perfect if $T(1) = 1$ and $T(0) = 0$ for all individuals. In this case and under Assumptions 1 and 2, $E[Y|Z = 1] - E[Y|Z = 0] = E[Y|T = 1] - E[Y|T = 0] = E[Y(1)] - E[Y(0)] = \Delta$, where the first equality follows from perfect compliance and the second from random assignment. As all individuals are compliers, the ATE is identified. However, if post-assignment complications occur such that $T(z) \neq z$ for some subpopulation, selection bias may flaw the validity of the evaluation in spite of the randomization of the assignment. This is due to the potential threat that individuals systematically select themselves into the treatment according to their potential outcomes.

Using the principal stratification framework advocated by Frangakis and Rubin (2002), the population can be divided into four principal strata, denoted by G, according to the choice of T

as a reaction of Z. As outlined in Angrist, Imbens, and Rubin (1996), the four groups are the compliers, the always takers, who are always treated irrespective of the assignment, the never takers, who are never treated irrespective of the assignment, and the defiers, who are treated when not assigned, but not treated when assigned:

Table 1: Principal strata

Principal strata (G)	T(1)	T(0)	Notion
11	1	1	Always takers
10	1	0	Compliers
01	0	1	Defiers
00	0	0	Never takers

It is obvious that we cannot directly observe the principal stratum an individual belongs to as either $T(1)$ or $T(0)$ is known. Let $G_i \in \{11, 10, 01, 00\}$ represent the principal stratum to which subject i belongs. By Assumption 2, G_i is not affected by the instrument and can be regarded as a covariate that is only partially observed in the data. Independence implies that $Y(0), Y(1) \perp z | T(0), T(1)$ (or $Y(0), Y(1) \perp z | T(0), T(1), X$ under conditional independence). Thus, potential outcomes are independent of the instrument given the principal stratum. Therefore, any effect of Z on Y conditional on a principal stratum is well defined. It is yet not causal, as the instrument has no direct effect on the outcome by Assumption 1. Any change in Y following a change in Z must be triggered by a change in T. Hence, if the effect of Z on Y can be scaled by the effect of Z on T within a stratum, the ATE of T on Y can be recovered for the respective subpopulation. This is exactly how the LATE on the compliers is identified under the monotonicity assumption discussed further below.

Table 2: Observed subgroups and principal strata

Observed subgroups $o(Z,T)$	principal strata
$o(1,1) = \{i : Z_i = 1, T_i = 1\}$	subject i belongs either to 11 or to 10
$o(1,0) = \{i : Z_i = 1, T_i = 0\}$	subject i belongs either to 01 or to 00
$o(0,1) = \{i : Z_i = 0, T_i = 1\}$	subject i belongs either to 11 or to 01
$o(0,0) = \{i : Z_i = 0, T_i = 0\}$	subject i belongs either to 10 or to 00

However, without the imposition of further assumptions, neither the principal strata proportions nor the distribution of Y within any stratum is identified. To see this, note that the observed values of Z and T generate four observed subgroups, denoted as $o(Z, T)$, which are all mixtures of two principal strata, see Table 2. Therefore, also the probability to belong to an observed subgroup is a mixture of principal strata proportions, henceforth denoted as $\pi_{tt'} \equiv \Pr(T(1) = t, T(0) = t')$. Let $P_{t|z}$ represent the observed treatment probability conditional on assignment, $\Pr(T = t|Z = z)$, in the population of interest. Under Assumption 2, which ensures that the strata proportions conditional on the instrument are equal to the unconditional strata proportions, the relation between the observed $P_{t|z}$ and the latent $\pi_{tt'}$ is as displayed in Table 3.

Table 3: Observed probabilities and principal strata proportions

Observed cond. selection prob.	princ. strata proportions
$P_{1\|1} \equiv \Pr(T = 1\|Z = 1)$	$\pi_{11} + \pi_{10}$
$P_{0\|1} \equiv \Pr(T = 0\|Z = 1)$	$\pi_{01} + \pi_{00}$
$P_{1\|0} \equiv \Pr(T = 1\|Z = 0)$	$\pi_{11} + \pi_{01}$
$P_{0\|0} \equiv \Pr(T = 0\|Z = 0)$	$\pi_{10} + \pi_{00}$

Thus, point identification of causal effects can only be obtained by invoking further assumptions. E.g., under monotonicity of T in Z and effect homogeneity, the ATE on the entire population is identified. Albeit invoked in much of the IV literature, effect homogeneity is a very unattractive assumption given the rich empirical evidence on effect heterogeneity in the field of treatment evaluation. Under monotonicity and effect heterogeneity, the LATE on the compliers is identified, but this effect may be "too local" to be of policy interest. Fortunately, assumptions as monotonicity and stochastic dominance also bear identifying power for further populations and may yield informative bounds, as discussed in the next section.

3 Assumptions and interval identification

The strategies for the partial identification of ATEs on various populations which we are going to present in this section are based on the fact that each of the four observed conditional outcomes

comes from a mixture of two principal strata:

$$\begin{aligned} E(Y|Z=0,T=1) &= \frac{\pi_{11}}{\pi_{11}+\pi_{01}} \cdot E(Y|Z=0,T=1,G=11) \\ &+ \frac{\pi_{01}}{\pi_{11}+\pi_{01}} \cdot E(Y|Z=0,T=1,G=01), \end{aligned} \tag{1}$$

$$\begin{aligned} E(Y|Z=1,T=1) &= \frac{\pi_{11}}{\pi_{11}+\pi_{10}} \cdot E(Y|Z=1,T=1,G=11) \\ &+ \frac{\pi_{10}}{\pi_{11}+\pi_{10}} \cdot E(Y|Z=1,T=1,G=10), \end{aligned} \tag{2}$$

$$\begin{aligned} E(Y|Z=0,T=0) &= \frac{\pi_{10}}{\pi_{00}+\pi_{10}} \cdot E(Y|Z=0,T=0,G=10) \\ &+ \frac{\pi_{00}}{\pi_{00}+\pi_{10}} \cdot E(Y|Z=0,T=0,G=00), \end{aligned} \tag{3}$$

and

$$\begin{aligned} E(Y|Z=1,T=0) &= \frac{\pi_{01}}{\pi_{00}+\pi_{01}} \cdot E(Y|Z=1,T=0,G=01) \\ &+ \frac{\pi_{00}}{\pi_{00}+\pi_{01}} \cdot E(Y|Z=1,T=0,G=00). \end{aligned} \tag{4}$$

Horowitz and Manski (1995) have shown that whenever it is possible to bound the mixing probability, sharp bounds can be obtained on any parameter that respects stochastic dominance of the mixture components. We will use this fact to derive bounds for the ATEs on different populations. Note that similar arguments can be used to obtain bounds on other parameters respecting stochastic dominance, as for instance the quantile treatment effect (QTE), see for instance Abadie, Angrist, and Imbens (2002) and Frölich and Melly (2008).

3.1 Worst case bounds

Assume that the support $\mathcal{Y}$ of the outcome variable Y is bounded, i.e., $\mathcal{Y} = [y^{LB}, y^{UB}]$. This condition will rule out infinite upper or lower bounds on the ATE of any population. Without imposing any restrictions other than Assumptions 1 and 2, we obtain the following equations by Table 3:

$$\begin{aligned} P_{1|0} - \pi_{01} = \pi_{11} \quad &\Rightarrow \quad \pi_{01} \leq P_{1|0}, \\ P_{0|1} - \pi_{01} = \pi_{00} \quad &\Rightarrow \quad \pi_{01} \leq P_{0|1}, \\ P_{1|1} - P_{1|0} + \pi_{01} = \pi_{10} \quad &\Rightarrow \quad \pi_{01} \geq P_{1|0} - P_{1|1}, \end{aligned} \tag{5}$$

and thus, the lower and upper bounds on the defiers' proportion are

$$\pi_{01} \in [\max(0, P_{1|0} - P_{1|1}), \min(P_{1|0}, P_{0|1})]. \tag{6}$$

We denote by $\pi_{01}^{\min} = \max(0, P_{1|0} - P_{1|1})$ and $\pi_{01}^{\max} = \min(P_{1|0}, P_{0|1})$ the minimum and maximum of admissible values for π_{01}. The remaining strata proportions can be bounded analogously.

In order to bound the ATEs on the four populations, we introduce some additional notation. We define $\bar{Y}_{z,t} \equiv E(Y|Z = z, T = t)$ to be the conditional mean of Y given $Z = z$ and $T = t$. Furthermore, denote by $F_{Y_{z,t}}(y) \equiv \Pr(Y \leq y|Z = z, T = t)$ the conditional cdf of Y given $Z = z$ and $T = t$. Let $q_{z,t}^{G}$ denote the share of individuals belonging to stratum G in the observed subgroup $o(z,t)$. If necessary, we will denote by $q_{z,t}^{G,\pi_{01}^{\max}}$ and $q_{z,t}^{G,\pi_{01}^{\min}}$, the value of $q_{z,t}^{G}$ when π_{01} is equal to $\pi_{01}^{\max}$ or $\pi_{01}^{\min}$, respectively. Then, $F_{Y_{z,t}}^{-1}(q_{z,t}^{G}) \equiv \inf\{y : F_{Y_{z,t}}(y) \geq q_{z,t}^{G}\}$ is the conditional quantile function of Y given $Z = z$ and $T = t$. Finally, let $\bar{Y}_{z,t}(\min|q_{z,t}^{G}) \equiv E(Y|Z = z, T = t, y \leq F_{Y_{z,t}}^{-1}(q_{z,t}^{G}))$ and $\bar{Y}_{z,t}(\max|q_{z,t}^{G}) \equiv E(Y|Z = z, T = t, y \geq F_{Y_{z,t}}^{-1}(1 - q_{z,t}^{G}))$.

Using this notation, the upper and lower bounds, denoted by Δ_{10}^{UB} and Δ_{10}^{LB}, for the ATE on the compliers, Δ_{10}, are respectively,

$$\begin{aligned}
\Delta_{10}^{UB} &= \max_{\pi_{01}} \Bigg[\frac{P_{1|1} \cdot \bar{Y}_{1,1} - (P_{1|0} - \pi_{01}) \cdot \max\left(\bar{Y}_{1,1}(\min|q_{1,1}^{11}), \bar{Y}_{0,1}(\min|q_{0,1}^{11})\right)}{P_{1|1} - P_{1|0} + \pi_{01}} \\
&- \frac{P_{0|0} \cdot \bar{Y}_{0,0} - (P_{0|1} - \pi_{01}) \cdot \min\left(\bar{Y}_{0,0}(\max|q_{0,0}^{00}), \bar{Y}_{1,0}(\max|q_{1,0}^{00})\right)}{P_{1|1} - P_{1|0} + \pi_{01}} \Bigg], \\
\Delta_{10}^{LB} &= \min_{\pi_{01}} \Bigg[\frac{P_{1|1} \cdot \bar{Y}_{1,1} - (P_{1|0} - \pi_{01}) \cdot \min\left(\bar{Y}_{1,1}(\max|q_{1,1}^{11}), \bar{Y}_{0,1}(\max|q_{0,1}^{11})\right)}{P_{1|1} - P_{1|0} + \pi_{01}} \\
&- \frac{P_{0|0} \cdot \bar{Y}_{0,0} - (P_{0|1} - \pi_{01}) \cdot \max\left(\bar{Y}_{0,0}(\min|q_{0,0}^{00}), \bar{Y}_{1,0}(\min|q_{1,0}^{00})\right)}{P_{1|1} - P_{1|0} + \pi_{01}} \Bigg],
\end{aligned} \tag{7}$$

where $q_{1,1}^{11} = \frac{P_{1|0} - \pi_{01}}{P_{1|1}}$ (the share of always takers among those with $Z = 1$ and $T = 1$), $q_{0,1}^{11} = \frac{P_{1|0} - \pi_{01}}{P_{1|0}}$ (the share of always takers among those with $Z = 0$ and $T = 1$), $q_{1,0}^{00} = \frac{P_{0|1} - \pi_{01}}{P_{0|1}}$ (the share of never takers among those with $Z = 0$ and $T = 1$), and $q_{0,0}^{00} = \frac{P_{0|1} - \pi_{01}}{P_{0|0}}$ (the share of never takers among those with $Z = 0$ and $T = 0$). The proofs for the sharpness of these bounds as well as for any bounds proposed below are provided in the appendix.

Four points are worth noting concerning the derivation of the bounds. First of all, they follow from the application of the results of Horowitz and Manski (1995): since we are able to bound the strata proportions, we can also bound the mean potential outcomes of the compliers under treatment and non-treatment by using trimmed means that come from the observed subgroups. Secondly, (7) has to be optimized w.r.t. admissible defier proportions, defined by (6). Thirdly, the exclusion restriction (see Assumption 1) gives rise to the maximum and minimum operators. Note that in the first (third) line in (7) one computes the upper (lower) bound of the compliers' mean potential outcome under treatment by subtracting the lower (upper) bound of the mean potential outcome of the always takers. As their lower (upper) bound under treatment is not affected by the value of Z due to the exclusion restriction, the lower (upper) bound is the maximum (minimum) of the always takers' lower (upper) bounds for $Z = 1$ and $Z = 0$. An analogous result holds for lines 2 and 4 w.r.t. the bounds on the potential mean outcomes under non-treatment of the never takers. Finally, these bounds are defined only if $P_{1|0} < P_{1|1}$. This is equivalent to $\pi_{10} > \pi_{01}$, saying that the share of compliers is larger than the share of defiers.

In a symmetric way one obtains the sharp upper and lower bounds on the ATE of the defiers, Δ_{01}:

$$\begin{aligned}\Delta_{01}^{UB} &= \max_{\pi_{01}}\Bigg[\frac{P_{1|0}\cdot\bar{Y}_{0,1}-(P_{1|0}-\pi_{01})\cdot\max\left(\bar{Y}_{1,1}(\min|q_{1,1}^{11}),\bar{Y}_{0,1}(\min|q_{0,1}^{11})\right)}{\pi_{01}}\\ &- \frac{P_{0|1}\cdot\bar{Y}_{1,0}-(P_{0|1}-\pi_{01})\cdot\min\left(\bar{Y}_{0,0}(\max|q_{0,0}^{00}),\bar{Y}_{1,0}(\max|q_{1,0}^{00})\right)}{\pi_{01}}\Bigg],\\ \Delta_{01}^{LB} &= \min_{\pi_{01}}\Bigg[\frac{P_{1|0}\cdot\bar{Y}_{0,1}-(P_{1|0}-\pi_{01})\cdot\min\left(\bar{Y}_{1,1}(\max|q_{1,1}^{11}),\bar{Y}_{0,1}(\max|q_{0,1}^{11})\right)}{\pi_{01}}\\ &- \frac{P_{0|1}\cdot\bar{Y}_{1,0}-(P_{0|1}-\pi_{01})\cdot\max\left(\bar{Y}_{0,0}(\min|q_{0,0}^{00}),\bar{Y}_{1,0}(\min|q_{1,0}^{00})\right)}{\pi_{01}}\Bigg],\end{aligned} \tag{8}$$

These bounds are only defined if $P_{1|0} > P_{1|1}$, i.e., if there are more defiers than compliers. This condition together with the previous discussion on the compliers implies that without imposing monotonicity of the treatment w.r.t. the instrument as outlined below, bounds are informative either for the defiers or for the compliers, but never for both populations.[2] This also means that unless $P_{1|1} - P_{1|0} = 0$, either positive (if $P_{1|1} - P_{1|0} > 0$) or negative (if $P_{1|0} - P_{1|1} > 0$) monotonicity of T in Z is consistent with the data, but not both at the same time. See also the discussion in the next subsection.

Concerning the always takers, note that their outcome is only observed under treatment in both $o(1,1)$ and $o(0,1)$. The shares of always takers in $o(1,1)$ and $o(0,1)$ are, respectively, $\pi_{11}/(\pi_{11}+\pi_{10}) = (P_{1|0}-\pi_{01})/P_{1|1}$ and $\pi_{11}/(\pi_{11}+\pi_{01}) = (P_{1|0}-\pi_{01})/P_{1|0}$. Therefore, we can bound the upper and lower values of the mean potential outcome under treatment for this population by $\min\left(\bar{Y}_{1,1}(\max|q_{1,1}^{11,\pi_{01}^{\max}}),\bar{Y}_{0,1}(\max|q_{0,1}^{11,\pi_{01}^{\max}})\right)$ and $\max\left(\bar{Y}_{1,1}(\min|q_{1,1}^{11,\pi_{01}^{\max}}),\bar{Y}_{0,1}(\min|q_{0,1}^{11,\pi_{01}^{\max}})\right)$, respectively. As already discussed, the intuition for the optimization over different values of the instrument is that Z does not have a direct effect on the mean potential outcomes. Therefore, the set of admissible potential outcomes for $T = 1$ is the intersection of possible values under $Z = 0$ and $Z = 1$.

Since the outcomes of the always takers are never observed under non-treatment, we have

[2] An equivalent result for the sample selection framework is discussed in Huber and Mellace (2010).

to rely on the theoretical upper and lower bounds (at the theoretical ends of the support of Y) denoted by y^{UB} and y^{LB}, respectively. The sharp upper and lower bounds for the ATE on the always takers Δ_{11}, are:

$$\begin{aligned} \Delta_{11}^{UB} &= \min\left(\bar{Y}_{1,1}(\max|q_{1,1}^{11,\pi_{01}^{\max}}), \bar{Y}_{0,1}(\max|q_{0,1}^{11,\pi_{01}^{\max}})\right) - y^{LB}, \\ \Delta_{11}^{LB} &= \max\left(\bar{Y}_{1,1}(\min|q_{1,1}^{11,\pi_{01}^{\max}}), \bar{Y}_{0,1}(\min|q_{0,1}^{11,\pi_{01}^{\max}})\right) - y^{UB}, \end{aligned} \tag{9}$$

The bounds are only defined if $P_{1|0} > P_{0|1} \Rightarrow \pi_{11} > \pi_{00}$, i.e., if the share of always takers is larger than the share of never takers.

Analogously, the outcomes of never takers are only observed under non-treatment in both $o(0,0)$ and $o(1,0)$. The shares of never takers in $o(0,0)$ and $o(1,0)$ are, respectively, $\pi_{00}/(\pi_{00} + \pi_{10}) = (P_{0|1} - \pi_{01})/P_{0|0}$ and $\pi_{00}/(\pi_{00} + \pi_{01}) = (P_{0|1} - \pi_{01})/P_{0|1}$. Therefore, the upper and lower bounds on the mean potential outcome under treatment are $\min\left(\bar{Y}_{1,0}(\max|q_{1,0}^{00,\pi_{01}^{\max}}), \bar{Y}_{0,0}(\max|q_{0,0}^{00,\pi_{01}^{\max}})\right)$ and $\max\left(\bar{Y}_{1,0}(\min|q_{1,0}^{00,\pi_{01}^{\max}}), \bar{Y}_{0,0}(\min|q_{0,0}^{00,\pi_{01}^{\max}})\right)$, respectively. For the never takers, only the outcomes under non-treatment are observed, which again requires invoking the theoretical lower and upper bounds. The sharp upper and lower bounds on the ATE of the never takers, Δ_{00}, are, respectively:

$$\begin{aligned} \Delta_{00}^{UB} &= y^{UB} - \max\left(\bar{Y}_{1,0}(\min|q_{1,0}^{00,\pi_{01}^{\max}}), \bar{Y}_{0,0}(\min|q_{0,0}^{00,\pi_{01}^{\max}})\right), \\ \Delta_{00}^{LB} &= y^{LB} - \min\left(\bar{Y}_{1,0}(\max|q_{1,0}^{00,\pi_{01}^{\max}}), \bar{Y}_{0,0}(\max|q_{0,0}^{00,\pi_{01}^{\max}})\right). \end{aligned} \tag{10}$$

The bounds are defined if $P_{1|0} < P_{0|1}$, i.e., if there are more never takers than always takers in the population.

In the program evaluation literature, the population that received the treatment is often most interesting. To derive the ATE on the treated population (denoted as $\Delta_{T=1}$), note that it is a weighted average made up by three populations: the always takers, the compliers, and the defiers.

The shares of these populations are, respectively, given by

$$\begin{aligned} \frac{2 \cdot \pi_{11}}{2\pi_{11} + \pi_{10} + \pi_{01}} &= \frac{2 \cdot (P_{1|0} - \pi_{01})}{P_{1|1} + P_{1|0}}, \\ \frac{\pi_{10}}{2\pi_{11} + \pi_{10} + \pi_{01}} &= \frac{P_{1|1} - P_{1|0} + \pi_{01}}{P_{1|1} + P_{1|0}}, \\ \frac{\pi_{01}}{2\pi_{11} + \pi_{10} + \pi_{01}} &= \frac{\pi_{01}}{P_{1|1} + P_{1|0}}. \end{aligned} \tag{11}$$

Assuming the upper bound of the mean potential outcome under treatment and $Z = 1$ for the always takers, $\bar{Y}_{1,1}(\max|q_{1,1}^{11})$, implies assuming the lower bound of the mean potential outcome under treatment for the compliers, $\bar{Y}_{1,1}(\min|q_{1,1}^{10})$, and vice versa, as the weighted average of both must always yield $\bar{Y}_{1,1}$. For the same reason, assuming the upper bound of the mean potential outcome under treatment and $Z = 0$ for the always takers, $\bar{Y}_{0,1}(\max|q_{0,1}^{11})$, is equivalent to assuming the lower bound of the mean potential outcome under treatment for the defiers, $\bar{Y}_{0,1}(\min|q_{0,1}^{01})$.

Note that concerning the always takers, $\bar{Y}_{0,1}(\max|q_{0,1}^{11})$ is part of the upper bound for the treated if $\bar{Y}_{1,1}(\max|q_{1,1}^{11}) \geq \bar{Y}_{0,1}(\max|q_{0,1}^{11})$, because the upper bound for the always takers is $\min(\bar{Y}_{1,1}(\max|q_{1,1}^{11}), \bar{Y}_{0,1}(\max|q_{0,1}^{11}))$, see the previous discussion on the exclusion restriction. Thus, for the values of π_{01}, for which $\bar{Y}_{1,1}(\max|q_{1,1}^{11}) \geq \bar{Y}_{0,1}(\max|q_{0,1}^{11})$, the upper bound is given by

$$\begin{aligned} \Delta_{T=1}^{UB1} &= \max_{\pi_{01}} \Bigg[\frac{P_{1|0} - \pi_{01}}{P_{1|1} + P_{1|0}} \cdot \bar{Y}_{0,1}(\max|q_{0,1}^{11}) - \frac{2 \cdot (P_{1|0} - \pi_{01})}{P_{1|1} + P_{1|0}} \cdot y^{LB} \\ &+ \frac{P_{1|1} - P_{1|0} + \pi_{01}}{P_{1|1} + P_{1|0}} \cdot \Delta_{10}^{UB} + \frac{P_{1|0}}{P_{1|1} + P_{1|0}} \cdot \bar{Y}_{0,1} \\ &- \frac{P_{0|1} \cdot \bar{Y}_{1,0} - (P_{0|1} - \pi_{01}) \cdot \min\left(\bar{Y}_{0,0}(\max|q_{0,0}^{00}), \bar{Y}_{1,0}(\max|q_{1,0}^{00})\right)}{P_{1|1} + P_{1|0}} \Bigg], \end{aligned} \tag{12}$$

while for values of π_{01} for which $\bar{Y}_{1,1}(\max|q_{1,1}^{11}) < \bar{Y}_{0,1}(\max|q_{0,1}^{11})$ the maximization problem is

$$\begin{aligned}\Delta_{T=1}^{UB2} &= \max_{\pi_{01}}\left[\frac{P_{1|0}-\pi_{01}}{P_{1|1}+P_{1|0}}\cdot\bar{Y}_{1,1}(\max|q_{1,1}^{11}) - \frac{2\cdot(P_{1|0}-\pi_{01})}{P_{1|1}+P_{1|0}}\cdot y^{LB}\right. \\ &+ \frac{\pi_{01}}{P_{1|1}+P_{1|0}}\cdot\Delta_{01}^{UB} + \frac{P_{1|1}}{P_{1|1}+P_{1|0}}\cdot\bar{Y}_{1,1} \\ &- \left.\frac{P_{0|0}\cdot\bar{Y}_{0,0}-(P_{0|1}-\pi_{01})\cdot\min\left(\bar{Y}_{0,0}(\max|q_{0,0}^{00}),\bar{Y}_{1,0}(\max|q_{1,0}^{00})\right)}{P_{1|1}+P_{1|0}}\right].\end{aligned} \tag{13}$$

Finally, the upper bound is the maximum of both conditions, $\Delta_{T=1}^{UB} = \max\left(\Delta_{T=1}^{UB1},\Delta_{T=1}^{UB2}\right)$.

An analogous result holds for the lower bound. For values of π_{01} with $\bar{Y}_{1,1}(\min|q_{1,1}^{11}) \leq \bar{Y}_{0,1}(\min|q_{0,1}^{11})$, it follows that

$$\begin{aligned}\Delta_{T=1}^{LB1} &= \min_{\pi_{01}}\left[\frac{P_{1|0}-\pi_{01}}{P_{1|1}+P_{1|0}}\cdot\bar{Y}_{0,1}(\min|q_{0,1}^{11}) - \frac{2\cdot(P_{1|0}-\pi_{01})}{P_{1|1}+P_{1|0}}\cdot y^{UB}\right. \\ &+ \frac{P_{1|1}-P_{1|0}+\pi_{01}}{P_{1|1}+P_{1|0}}\cdot\Delta_{10}^{LB} + \frac{P_{1|0}}{P_{1|1}+P_{1|0}}\cdot\bar{Y}_{0,1} \\ &- \left.\frac{P_{0|1}\cdot\bar{Y}_{1,0}-(P_{0|1}-\pi_{01})\cdot\max\left(\bar{Y}_{0,0}(\min|q_{0,0}^{00}),\bar{Y}_{1,0}(\min|q_{1,0}^{00})\right)}{P_{1|1}+P_{1|0}}\right],\end{aligned} \tag{14}$$

while for other values of π_{01},

$$\begin{aligned}\Delta_{T=1}^{LB2} &= \min_{\pi_{01}}\left[\frac{P_{1|0}-\pi_{01}}{P_{1|1}+P_{1|0}}\cdot\bar{Y}_{1,1}(\min|q_{1,1}^{11}) - \frac{2\cdot(P_{1|0}-\pi_{01})}{P_{1|1}+P_{1|0}}\cdot y^{UB}\right. \\ &+ \frac{\pi_{01}}{P_{1|1}+P_{1|0}}\cdot\Delta_{01}^{LB} + \frac{P_{1|1}}{P_{1|1}+P_{1|0}}\cdot\bar{Y}_{1,1} \\ &- \left.\frac{P_{0|0}\cdot\bar{Y}_{0,0}-(P_{0|1}-\pi_{01})\cdot\max\left(\bar{Y}_{0,0}(\min|q_{0,0}^{00}),\bar{Y}_{1,0}(\min|q_{1,0}^{00})\right)}{P_{1|1}+P_{1|0}}\right].\end{aligned} \tag{15}$$

Therefore, $\Delta_{T=1}^{LB} = \min\left(\Delta_{T=1}^{LB1},\Delta_{T=1}^{LB2}\right)$.

Interestingly, the bounds on $\Delta_{T=1}$ are always informative despite the fact that either the bounds for the compliers or for the defiers are not informative. The bounds on $\Delta_{T=1}$ can be regarded as a weighted average of the bounds on the always takers, compliers, and defiers, however, taking account for the requirement that assuming the upper bound of one population may imply the lower bound for another one. One needs to consider all feasible combinations of upper and lower bounds to obtain the results (12), (13), (14), and (15). This is discussed in more detail in the appendix. In a symmetric way one can bound the ATE on the non-treated, $\Delta_{T=0}$, which is not reported here.

Finally, we derive the bounds for the ATE on the entire population, denoted as Δ. This effect is a weighted average based on a mixture of all four populations (always takers, compliers, defiers, and never takers). From Table 3, it follows that

$$\begin{aligned}\pi_{11} &= P_{1|0} - \pi_{01}, \\ \pi_{10} &= P_{1|1} - P_{1|0} + \pi_{01}, \\ \pi_{00} &= P_{0|1} - \pi_{01}.\end{aligned} \tag{16}$$

Furthermore, assuming the lower bound of the mean potential outcome given $Z = 0$ for the never takers implies assuming the upper bound of the mean potential outcome for the compliers, and vice versa. The same argument holds for the outcomes of the never takers and defiers given $Z = 1$. Analogously to $\Delta_{T=1}$, the bounds on Δ are a function of the intersection of admissible potential outcomes for $T = 1$ under $Z = 0$ and $Z = 1$ for the always takers and of the intersection for $T = 0$ under $Z = 0$ and $Z = 1$ for the never takers. Therefore, four possible combinations for $\bar{Y}_{1,1}(\max|q^{11}_{1,1}), \bar{Y}_{0,1}(\max|q^{11}_{0,1})$ and $\bar{Y}_{1,0}(\min|q^{00}_{1,0}), \bar{Y}_{0,0}(\min|q^{00}_{0,0})$, respectively, arise. However, as we will show in the appendix, all four combinations yield the same upper and lower bounds, which are

$$\begin{aligned}\Delta^{UB} &= P_{1|0} \cdot \bar{Y}_{0,1} - (P_{1|0} - \pi^{\min}_{01}) \cdot y^{LB} + P_{1|1} \cdot \bar{Y}_{1,1} \\ &- (P_{1|0} - \pi^{\min}_{01}) \cdot \max\left(\bar{Y}_{1,1}(\min|q^{11,\pi^{\min}_{01}}_{1,1}), \bar{Y}_{0,1}(\min|q^{11,\pi^{\min}_{01}}_{0,1})\right) \\ &+ (P_{0|1} - \pi^{\min}_{01}) \cdot \min\left(\bar{Y}_{0,0}(\max|q^{00,\pi^{\min}_{01}}_{0,0}), \bar{Y}_{1,0}(\max|q^{00,\pi^{\min}_{01}}_{1,0})\right) \\ &- P_{0|1} \cdot \bar{Y}_{1,0} + (P_{0|1} - \pi^{\min}_{01}) \cdot y^{UB} - P_{0|0} \cdot \bar{Y}_{0,0}\ ,\end{aligned} \tag{17}$$

and

$$\begin{aligned}
\Delta^{LB} &= P_{1|0} \cdot \bar{Y}_{0,1} - (P_{1|0} - \pi_{01}) \cdot y^{UB} + P_{1|1} \cdot \bar{Y}_{1,1} \\
&- (P_{1|0} - \pi_{01}) \cdot \min\left(\bar{Y}_{1,1}(\max|q_{1,1}^{11,\pi_{01}^{\min}}), \bar{Y}_{0,1}(\max|q_{0,1}^{11,\pi_{01}^{\min}})\right) \\
&+ (P_{0|1} - \pi_{01}) \cdot \max\left(\bar{Y}_{0,0}(\min|q_{0,0}^{00,\pi_{01}^{\min}}), \bar{Y}_{1,0}(\min|q_{1,0}^{00,\pi_{01}^{\min}})\right) \\
&- P_{0|1} \cdot \bar{Y}_{1,0} + (P_{0|1} - \pi_{01}) \cdot y^{LB} - P_{0|0} \cdot \bar{Y}_{0,0} \,.
\end{aligned} \tag{18}$$

Again, these bounds are always informative no matter whether the bounds for the compliers or those for the defiers are not informative.

Note that Δ^{LB}, Δ^{UB} might be narrower than the IV bounds derived by Manski (1990). The reason is that we assume joint independence of the instrument and any potential post-treatment variable (see Assumption 2) due to randomization of the treatment assignment, whereas Manski only imposes mean independence of the potential outcomes: $E(Y(t)|Z = 1) = E(Y(t)|Z = 0)$ for $t \in \{0, 1\}$. For the same reason, also Balke and Pearl (1997) find their bounds to be sharper than the Manski bounds. However, their result refer to binary outcomes, whereas the findings presented here also apply to continuous outcomes.

In order to see the difference between our bounds and those of Manski, consider the case that the lower bound on the defier share which is consistent with the data is zero, $\pi_{01}^{\min} = 0$. The appendix shows that Δ^{UB}, Δ^{LB} are maximized and minimized, respectively, for $\pi_{01} = \pi_{01}^{\min}$, just as stated in (17) and (18). Therefore, if $\pi_{01}^{\min} = 0$, the worst case bounds coincide with those under positive monotonicity of T in Z, which imposes $\pi_{01} = 0$ and is discussed in Section 3.2. Heckman and Vytlacil (2001) show that under (an assumption which is equivalent to) monotonicity, Manski's bounds simplify to

$$\begin{aligned}
\Delta_{Ma}^{UB} &= \left(P_{1|z}^{\max} \cdot \bar{Y}_{z,1} + (1 - P_{1|z}^{\max}) \cdot y^{UB}\right) - \left(P_{0|z}^{\max} \cdot \bar{Y}_{z,0} + (1 - P_{0|z}^{\max}) \cdot y^{LB}\right), \\
\Delta_{Ma}^{LB} &= \left(P_{1|z}^{\min} \cdot \bar{Y}_{z,1} + (1 - P_{1|z}^{\min}) \cdot y^{LB}\right) - \left(P_{0|z}^{\min} \cdot \bar{Y}_{z,0} + (1 - P_{0|z}^{\min}) \cdot y^{UB}\right),
\end{aligned}$$

where $P_{t|z}^{\max}$ and $P_{t|z}^{\min}$ denote the maximum and minimum values of $P_{t|z}$ w.r.t. Z. $\pi_{01}^{\min} = 0$ implies that $P_{1|1} > P_{1|0}$ and $P_{0|0} > P_{0|1}$. Then, Manski's upper bound becomes

$$\Delta_{Ma}^{UB} = \left(P_{1|1} \cdot \bar{Y}_{1,1} + P_{0|1} \cdot y^{UB}\right) - \left(P_{0|0} \cdot \bar{Y}_{0,0} + P_{1|0} \cdot y^{LB}\right).$$

On the other hand, since $\bar{Y}_{0,1}(\min|q_{0,1}^{11,0}) = \bar{Y}_{0,1}$ and $\bar{Y}_{1,0}(\min|q_{1,0}^{00,0}) = \bar{Y}_{1,0}$, our upper bound becomes

$$\begin{aligned}\Delta^{UB} &= \Big(P_{1|1} \cdot \bar{Y}_{1,1} + P_{1|0} \cdot \bar{Y}_{0,1} - P_{1|0} \cdot \max(\bar{Y}_{1,1}(\min|q_{1,1}^{11,0}), \bar{Y}_{0,1}) + P_{0|1} \cdot y^{UB}\Big) \\ &- \Big(P_{0|0} \cdot \bar{Y}_{0,0} + P_{0|1} \cdot \bar{Y}_{1,0} - P_{0|1} \cdot \min(\bar{Y}_{0,0}(\max|q_{0,0}^{00,0}), \bar{Y}_{1,0}) + P_{1|1} \cdot y^{LB}\Big).\end{aligned}$$

After some simple algebra it is easy to see that $\Delta^{UB} < \Delta_{Ma}^{UB}$ if either $\bar{Y}_{1,1}(\min|q_{1,1}^{11,0}) > \bar{Y}_{0,1}$ or $\bar{Y}_{0,0}(\max|q_{0,0}^{00,0}) < \bar{Y}_{1,0}$. This suggests that Manski's and our upper bounds are the same unless the difference in the potential outcomes of compliers and defiers is sufficiently "large" such that at least one of the two conditions above is met. Equivalent arguments and results follow for the lower bound and for the case in which $\pi_{01}^{\min} = P_{1|0} - P_{1|1}$.

We conclude the discussion on the worst case bounds by noting that they are likely to be very wide for the entire population and for most other groups. Therefore, they are often not helpful for obtaining meaningful results in empirical applications. The following subsections will introduce further assumptions that appear plausible in many empirical problems and might entail considerably tighter bounds.

3.2 Monotonicity

This subsection discusses the identifying power of monotonicity of the treatment in the instrument. (Weak) monotonicity of T in Z implies that the treatment state under $Z = 1$ is at least as high as under $Z = 0$ for all individuals.

Assumption 3:

$\Pr(T(1) \geq T(0)) = 1$ (montonicity).

As the potential treatment state never decreases in the instrument, the existence of the defiers (stratum 01) is ruled out. A symmetric result is obtained by assuming $\Pr(T(0) \geq T(1)) = 1$ which implies that stratum 10 does not exist. Note that assuming $\Pr(T(1) \geq T(0)) = 1$ (positive monotonicity) is only consistent with the data if $P_{1|1} - P_{1|0} \geq 0$, otherwise stratum 01 must necessarily exist. Similarly, $\Pr(T(0) \geq T(1)) = 1$ (negative monotonicity) requires that $P_{1|0} - P_{1|1} \geq 0$, see Table 3. Even though these are necessary conditions for the respective monotonicity assumption, they are not sufficient. Due to the symmetry of positive and negative monotonicity, we will only focus on Assumption 3 (positive monotonicity) in the subsequent discussion.

In their seminal paper on the identification of the local average treatment effect (LATE), Imbens and Angrist (1994) (see also Angrist, Imbens, and Rubin, 1996) show that Δ_{10} is point identified under Assumptions 1, 2, and 3. I.e., the worst case bounds collapse to a single point given that π_{01} is equal to zero:

$$\begin{aligned}
\Delta_{10} &= \left(\frac{P_{1|1}}{P_{1|1} - P_{1|0}} \cdot \bar{Y}_{1,1} - \frac{P_{1|0}}{P_{1|1} - P_{1|0}} \cdot \bar{Y}_{0,1}\right) - \left(\frac{P_{0|0}}{P_{1|1} - P_{1|0}} \cdot \bar{Y}_{0,0} - \frac{P_{0|1}}{P_{1|1} - P_{1|0}} \cdot \bar{Y}_{1,0}\right) \\
&= \frac{(P_{1|1} \cdot \bar{Y}_{1,1} + P_{0|1} \cdot \bar{Y}_{1,0}) - (P_{1|0} \cdot \bar{Y}_{0,1} + P_{0|0} \cdot \bar{Y}_{0,0})}{P_{1|1} - P_{1|0}} \\
&= \frac{\Pr(T=1|Z=1) \cdot E(Y|Z=1,T=1) + \Pr(T=0|Z=1) \cdot E(Y|Z=1,T=0)}{\Pr(T=1|Z=1) - \Pr(T=1|Z=0)} \\
&- \frac{\Pr(T=1|Z=0) \cdot E(Y|Z=0,T=1) + \Pr(T=0|Z=0) \cdot E(Y|Z=0,T=0)}{\Pr(T=1|Z=1) - \Pr(T=1|Z=0)} \\
&= \frac{E(Y|Z=1) - E(Y|Z=0)}{E(T|Z=1) - E(T|Z=0)}. \qquad (19)
\end{aligned}$$

The final equation yields the well known result that the ATE on the compliers is just the ratio of two differences in conditional expectations, namely the intention to treat effect divided by the share of compliers.

There is a further difference to the worst case scenario worth noting because it allows constructing tests for the instrumental exclusion restriction. Under monotonicity the observed subgroup $o(0,1)$ (not instrumented but treated) consists of always takers only, such that $\bar{Y}_{0,1}$ immediately gives the mean potential outcome under treatment for the always takers. An optimization similar to the worst case bounds of the kind $\max\left(\bar{Y}_{1,1}(\min|q^{11}_{1,1}), \bar{Y}_{0,1})\right)$ and $\min\left(\bar{Y}_{1,1}(\max|q^{11}_{1,1}), \bar{Y}_{0,1})\right)$ (with $\pi_{01} = 0$) is, thus, not required here. Note, however, that this comparison gives a testable implication for the exclusion restriction. If it holds, $\bar{Y}_{1,1}(\min|q^{11}_{1,1}) \leq \bar{Y}_{0,1} \leq \bar{Y}_{1,1}(\max|q^{11}_{1,1})$, otherwise Z has a direct effect on the outcomes of the always takers. Similarly, $\bar{Y}_{1,0}$ is the mean potential outcome under non-treatment for the never takers. Therefore, another testable implication of the exclusion restriction is $\bar{Y}_{0,0}(\min|q^{00}_{0,0}) \leq \bar{Y}_{1,0} \leq \bar{Y}_{0,0}(\max|q^{00}_{0,0})$.[3]

In the absence of defiers the bounds for the always takers and never takers (Δ_{11} and Δ_{00}) simplify to

$$\begin{aligned} \Delta^{UB}_{11} &= \bar{Y}_{0,1} - y^{LB}, \\ \Delta^{LB}_{11} &= \bar{Y}_{0,1} - y^{UB}, \end{aligned} \tag{20}$$

and

$$\begin{aligned} \Delta^{UB}_{00} &= y^{UB} - \bar{Y}_{1,0}, \\ \Delta^{LB}_{00} &= y^{LB} - \bar{Y}_{1,0}. \end{aligned} \tag{21}$$

These bound are sharp because $E(Y|T=1, G=11)$ and $E(Y|T=0, G=00)$ are now point identified by $\bar{Y}_{0,1}$ and $\bar{Y}_{1,0}$ (if the exclusion restriction holds). However, monotonicity does not impose any restrictions on the distributions of $Y|T=0, G=11$ and $Y|T=1, G=00$ such that the worst case bounds y^{LB}, y^{UB} have to be assumed.

Under the monotonicity assumption, the bounds on the ATEs of the treated ($\Delta_{T=1}$) and the

[3]Kitagawa and Hoderlein (2009) propose a formal test for the validity of instruments. They test for both violations of the exclusion restriction and the monotonicity assumption jointly by checking for negative densities of the compliers' treated or non-treated outcomes, whereas the testable implications considered here only refer to the exclusion restriction.

entire population (Δ) are trivially derived as a linear combination of the ATE on the compliers and the bounds for the always takers and never takers, respectively. These components are weighted by the share of the respective population, which is point identified in the absence of defiers. Therefore, we obtain

$$\begin{aligned}
\Delta^{UB}_{T=1} &= \frac{2 \cdot P_{1|0}}{P_{1|1} + P_{1|0}} \cdot \Delta^{UB}_{11} + \frac{P_{1|1} - P_{1|0}}{P_{1|1} + P_{1|0}} \cdot \Delta_{10}, \\
\Delta^{LB}_{T=1} &= \frac{2 \cdot P_{1|0}}{P_{1|1} + P_{1|0}} \cdot \Delta^{LB}_{11} + \frac{P_{1|1} - P_{1|0}}{P_{1|1} + P_{1|0}} \cdot \Delta_{10},
\end{aligned} \tag{22}$$

and

$$\begin{aligned}
\Delta^{UB} &= P_{1|0} \cdot \Delta^{UB}_{11} + (P_{1|1} - P_{1|0}) \cdot \Delta_{10} + P_{0|1} \cdot \Delta^{UB}_{00}, \\
\Delta^{LB} &= P_{1|0} \cdot \Delta^{LB}_{11} + (P_{1|1} - P_{1|0}) \cdot \Delta_{10} + P_{0|1} \cdot \Delta^{LB}_{00}.
\end{aligned} \tag{23}$$

These bounds are sharp because the bounds on the always takers and never takers are sharp.

Interestingly, Balke and Pearl (1997), Heckman and Vytlacil (2001), and Kitagawa (2009) show that under monotonicity, their bounds of the ATE on the entire population coincide with the worst case bounds of Manski (1990) who only imposes the exclusion restriction. I.e., Assumption 3 does, if it is satisfied, not bring any additional identifying power for Δ. As already mentioned in Section 3.1, this is also the case for our bounds. It is easy to show that for $\pi^{\min}_{01} = 0$ (positive monotonicity), the worst case bounds 17 and 18 collapse to those in 23. This is due to the fact that under monotonicity, the exclusion restriction implies that $\bar{Y}_{z,t}(\min|q^G_{z,t}) \leq \bar{Y}_{z,t} \leq \bar{Y}_{z,t}(\max|q^G_{z,t})$.

3.3 Stochastic dominance

Stochastic dominance states that the potential outcome evaluated at any rank in the distribution of one population is at least as high as that of some other population. This rules out the crossing of potential outcomes of two populations. The stochastic dominance assumption has been used in the sample selection framework by Zhang and Rubin (2003), Lechner and Melly (2007), Zhang, Rubin, and Mealli (2008), Lee (2009), and Huber and Mellace (2010). We will show that it also bears identifying power in the IV framework.

Assumption 4:

$\Pr(Y(t)|G = 10 \leq y) \leq \Pr(Y(t)|G = g \leq y) \quad \forall\, t \in \{0, 1\},\ g \in \{11, 00\}, y$ in the support of Y (stochastic dominance).

Assumption 4 states that at any rank, the potential outcomes of the compliers are at least as the high as those of the always takers and the never takers. When considering the ATE, stochastic dominance is only required to hold w.r.t. the mean. The latter condition is weaker than the way Assumption 4 is stated, which, for instance, also restricts the variance across different populations. However, Assumption 4 would be required when considering quantile treatment effects, too. Furthermore, note that the kind of stochastic dominance considered here is only one out of many possible relations between the potential outcomes of various populations. Its plausibility has to be judged in the light of the empirical application and theoretical considerations. Fortunately and as discussed in the next subsection, stochastic dominance has testable implications if it is jointly assumed with monotonicity. In the application presented in Section 5 we will test Assumption 4 and show that it is not rejected at any reasonable significance level.

Under stochastic dominance alone, the ATE on the compliers is bounded by

$$\begin{aligned}
\Delta_{10}^{UB} &= \max_{\pi_{01}} \left[\frac{P_{1|1} \cdot \bar{Y}_{1,1} - (P_{1|0} - \pi_{01}) \cdot \max\left(\bar{Y}_{1,1}(\min|q^{11}_{1,1}), \bar{Y}_{0,1}(\min|q^{11}_{0,1})\right)}{P_{1|1} - P_{1|0} + \pi_{01}} \right. \\
&- \left. \frac{P_{0|0} \cdot \bar{Y}_{0,0} - (P_{0|1} - \pi_{01}) \cdot \min\left(\bar{Y}_{0,0}, \bar{Y}_{1,0}(\max|q^{00}_{1,0})\right)}{P_{1|1} - P_{1|0} + \pi_{01}} \right], \\
\Delta_{10}^{LB} &= \min_{\pi_{01}} \left[\frac{P_{1|1} \cdot \bar{Y}_{1,1} - (P_{1|0} - \pi_{01}) \cdot \min\left(\bar{Y}_{1,1}, \bar{Y}_{0,1}(\max|q^{11}_{0,1})\right)}{P_{1|1} - P_{1|0} + \pi_{01}} \right. \\
&- \left. \frac{P_{0|0} \cdot \bar{Y}_{0,0} - (P_{0|1} - \pi_{01}) \cdot \max\left(\bar{Y}_{0,0}(\min|q^{00}_{0,0}), \bar{Y}_{1,0}(\min|q^{00}_{1,0})\right)}{P_{1|1} - P_{1|0} + \pi_{01}} \right].
\end{aligned} \tag{24}$$

The intuition of this result is that the compliers' mean potential outcome under treatment cannot be lower than that of the always takers, while under non-treatment it cannot be lower than the one of the never takers. Therefore, we now have the minimization problems $\min\left(\bar{Y}_{0,0}, \bar{Y}_{1,0}(\max|q^{00}_{1,0})\right)$ and $\min\left(\bar{Y}_{1,1}, \bar{Y}_{0,1}(\max|q^{11}_{0,1})\right)$, as $\bar{Y}_{0,0}$ is the lower bound on the compliers' mean potential outcome in the mixed population with the defiers and $\bar{Y}_{1,1}$ in the mixed group with the always takers. The sharpness of these bounds and all other bounds under stochastic dominance proposed below follows from the fact that they are special cases of the worst case bounds and that we can apply Lemma 1 in Huber and Mellace (2010) to formally prove their sharpness.

The bounds for the defiers are the same as in the worst case scenario, since we do not impose any stochastic dominance assumption w.r.t. the potential outcomes of this population. The bounds for the ATEs on the always takers and never takers are, respectively,

$$\begin{aligned}
\Delta^{UB}_{11} &= \min\left(\bar{Y}_{1,1}, \bar{Y}_{0,1}(\max|q^{11,\pi^{\max}_{01}}_{0,1})\right) - y^{LB}, \\
\Delta^{LB}_{11} &= \min_{\pi_{01}}\Big[\max\left(\bar{Y}_{1,1}(\min|q^{11}_{1,1}), \bar{Y}_{0,1}(\min|q^{11}_{0,1})\right) \\
&- \frac{P_{0|0}\cdot\bar{Y}_{0,0} - (P_{0|1}-\pi_{01})\cdot\max\left(\bar{Y}_{0,0}(\min|q^{00}_{0,0}), \bar{Y}_{1,0}(\min|q^{00}_{1,0})\right)}{P_{1|1}-P_{1|0}+\pi_{01}}\Big],
\end{aligned} \tag{25}$$

and

$$\begin{aligned}
\Delta^{UB}_{00} &= \max_{\pi_{01}}\Big[\frac{P_{1|1}\cdot\bar{Y}_{1,1} - (P_{1|0}-\pi_{01})\cdot\max\left(\bar{Y}_{1,1}(\min|q^{11}_{1,1}), \bar{Y}_{0,1}(\min|q^{11}_{0,1})\right)}{P_{1|1}-P_{1|0}+\pi_{01}} \\
&- \max\left(\bar{Y}_{1,0}(\min|q^{00}_{1,0}), \bar{Y}_{0,0}(\min|q^{00}_{0,0})\right)\Big], \\
\Delta^{LB}_{00} &= y^{LB} - \min\left(\bar{Y}_{1,0}(\max|q^{00,\pi^{\max}_{01}}_{1,0}), \bar{Y}_{0,0}\right).
\end{aligned} \tag{26}$$

$\bar{Y}_{1,1}$, $\bar{Y}_{0,0}$ are now the upper bounds on the mean potential outcome under treatment and the mean potential outcome under non-treatment for the always takers and the never takers, respectively. Moreover, stochastic dominance implies that the always takers' upper bound under non-treatment cannot be higher than the compliers' upper bound under non-treatment and that the never takers' upper bound under treatment cannot be higher than the compliers' upper bound under treatment. This is a considerable improvement over the worst case bounds, as it allows us to replace the theoretical upper bound y^{UB} by observed quantities. Note that the substitution of the theoretical upper bounds by the compliers' bounds requires the optimization over all possible values of π_{01}.

The upper bound on the ATE of the treated is now a function of $\bar{Y}_{1,1}$ and $\bar{Y}_{0,1}(\max|q_{0,1}^{11})$. For those values of π_{01} for which $\bar{Y}_{1,1} \geq \bar{Y}_{0,1}(\max|q_{0,1}^{11})$, the upper bound is

$$\begin{aligned}\Delta_{T=1}^{UB1} &= \max_{\pi_{01}} \left[\frac{P_{1|0} - \pi_{01}}{P_{1|1} + P_{1|0}} \cdot \bar{Y}_{0,1}(\max|q_{0,1}^{11}) - \frac{2 \cdot (P_{1|0} - \pi_{01})}{P_{1|1} + P_{1|0}} \cdot y^{LB} \right. \\ &+ \frac{P_{1|1} - P_{1|0} + \pi_{01}}{P_{1|1} + P_{1|0}} \cdot \Delta_{10}^{UB} + \frac{P_{1|0}}{P_{1|1} + P_{1|0}} \cdot \bar{Y}_{0,1} \\ &- \left. \frac{P_{0|1} \cdot \bar{Y}_{1,0} - (P_{0|1} - \pi_{01}) \cdot \min\left(\bar{Y}_{0,0}, \bar{Y}_{1,0}(\max|q_{1,0}^{00})\right)}{P_{1|1} + P_{1|0}} \right],\end{aligned} \tag{27}$$

while for the remaining values,

$$\begin{aligned}\Delta_{T=1}^{UB2} &= \max_{\pi_{01}} \left[\frac{(P_{1|1} + P_{1|0} - \pi_{01}) \cdot \bar{Y}_{1,1} - 2 \cdot (P_{1|0} - \pi_{01}) \cdot y^{LB} + \pi_{01} \cdot \Delta_{01}^{UB}}{P_{1|1} + P_{1|0}} \right. \\ &- \left. \frac{P_{0|0} \cdot \bar{Y}_{0,0} - (P_{0|1} - \pi_{01}) \cdot \min\left(\bar{Y}_{0,0}, \bar{Y}_{1,0}(\max|q_{1,0}^{00,\min})\right)}{P_{1|1} + P_{1|0}} \right].\end{aligned} \tag{28}$$

As in the worst case scenario, the maximum of both conditions yields the upper bound: $\Delta_{T=1}^{UB} = \max\left(\Delta_{T=1}^{UB1}, \Delta_{T=1}^{UB2}\right)$. Concerning the lower bound, if π_{01} is such that $\bar{Y}_{1,1}(\min|q_{1,1}^{11}) \leq \bar{Y}_{0,1}(\min|q_{0,1}^{11})$, then

$$\begin{aligned}\Delta_{T=1}^{LB1} &= \min_{\pi_{01}} \left[\frac{P_{1|0} - \pi_{01}}{P_{1|1} + P_{1|0}} \cdot \bar{Y}_{0,1}(\min|q_{0,1}^{11}) - \frac{2 \cdot (P_{1|0} - \pi_{01}) \cdot P_{0|0}}{(P_{1|1} + P_{1|0}) \cdot (P_{1|1} - P_{1|0} + \pi_{01})} \cdot \bar{Y}_{0,0} \right. \\ &+ \frac{P_{1|1} - P_{1|0} + \pi_{01}}{P_{1|1} + P_{1|0}} \cdot \Delta_{10}^{LB} + \frac{P_{1|0}}{P_{1|1} + P_{1|0}} \cdot \bar{Y}_{0,1} \\ &- \left. \frac{P_{0|1} \cdot \bar{Y}_{1,0} - (P_{1|0} - \pi_{01}) \cdot \max\left(\bar{Y}_{0,0}(\min|q_{0,0}^{00}), \bar{Y}_{1,0}(\min|q_{1,0}^{00})\right)}{P_{1|1} + P_{1|0}} \right],\end{aligned} \tag{29}$$

otherwise,

$$
\begin{aligned}
\Delta^{LB2}_{T=1} &= \min_{\pi_{01}} \Bigg[\frac{P_{1|0} - \pi_{01}}{P_{1|1} + P_{1|0}} \cdot \bar{Y}_{1,1}(\min |q^{11}_{1,1}) + \frac{\pi_{01}}{P_{1|1} + P_{1|0}} \cdot \Delta^{LB}_{01} \\
&- \frac{P_{0|0} \cdot \bar{Y}_{0,0} - (P_{1|0} - \pi_{01}) \cdot \max\left(\bar{Y}_{0,0}(\min |q^{00}_{0,0}), \bar{Y}_{1,0}(\min |q^{00}_{1,0})\right)}{P_{1|1} + P_{1|0}} \Bigg].
\end{aligned} \tag{30}
$$

Therefore, $\Delta^{LB}_{T=1} = \min\left(\Delta^{LB1}_{T=1}, \Delta^{LB2}_{T=1}\right)$

For the ATE on the entire population stochastic dominance implies the following bounds:

$$
\begin{aligned}
\Delta^{UB} &= \max_{\pi_{01}} \big[P_{1|0} \cdot \bar{Y}_{0,1} - (P_{1|0} - \pi_{01}) \cdot y^{LB} + P_{1|1} \cdot \bar{Y}_{1,1} \\
&- (P_{1|0} - \pi_{01}) \cdot \max\left(\bar{Y}_{1,1}(\min |q^{11}_{1,1}), \bar{Y}_{0,1}(\min |q^{11}_{0,1})\right) \\
&+ \left(P_{0|1} - \pi_{01}\right) \cdot \min\left(\bar{Y}_{0,0}, \bar{Y}_{1,0}(\max |q^{00}_{1,0})\right) - P_{0|1} \cdot \bar{Y}_{1,0} \\
&+ (P_{0|1} - \pi_{01}) \cdot \frac{P_{1|1} \cdot \bar{Y}_{1,1} - (P_{1|0} - \pi_{01}) \cdot \max\left(\bar{Y}_{1,1}(\min |q^{11}_{1,1}), \bar{Y}_{0,1}(\min |q^{11}_{0,1})\right)}{P_{1|1} - P_{1|0} + \pi_{01}} \\
&- P_{0|0} \cdot \bar{Y}_{0,0} \big],
\end{aligned} \tag{31}
$$

and

$$
\begin{aligned}
\Delta^{LB} &= \min_{\pi_{01}} \big[P_{1|0} \cdot \bar{Y}_{0,1} + P_{1|1} \cdot \bar{Y}_{1,1} \\
&- (P_{1|0} - \pi_{01}) \cdot \frac{P_{0|0} \cdot \bar{Y}_{0,0} - (P_{0|1} - \pi_{01}) \cdot \max\left(\bar{Y}_{0,0}(\min |q^{00}_{0,0}), \bar{Y}_{1,0}(\min |q^{00}_{1,0})\right)}{P_{1|1} - P_{1|0} + \pi_{01}} \\
&- (P_{1|0} - \pi_{01}) \cdot \min\left(\bar{Y}_{1,1}, \bar{Y}_{0,1}(\max |q^{11}_{0,1})\right) \\
&+ (P_{0|1} - \pi_{01}) \cdot \max\left(\bar{Y}_{0,0}(\min |q^{00}_{0,0}), \bar{Y}_{1,0}(\min |q^{00}_{1,0})\right) \\
&- P_{0|1} \cdot \bar{Y}_{1,0} + (P_{0|1} - \pi_{01}) \cdot y^{LB} - P_{0|0} \cdot \bar{Y}_{0,0} \big].
\end{aligned} \tag{32}
$$

In contrast to the worst case bounds, we have to optimize w.r.t. π_{01} under stochastic dominance because some of the theoretical bounds have been substituted by the compliers' bounds.

While invoking either monotonicity or stochastic dominance might tighten the bounds considerably (or even lead to point identification for the compliers), a joint imposition of both assumptions is likely shrink the identified sets even further. Therefore, the next section discusses the joint identifying power of monotonicity and stochastic dominance.

3.4 Monotonicity and stochastic dominance

In this subsection we derive the bounds under both monotonicity (Assumption 3) and stochastic dominance (Assumption 4). Since Δ_{10} is point identified under Assumptions 1 to 3, Assumption 4 does not bring any further improvement w.r.t. the compliers. For all other populations, the bounds become tighter by invoking both assumptions.

The upper and lower bounds of the ATE on the always takers are now

$$\begin{aligned} \Delta_{11}^{UB} &= \bar{Y}_{0,1} - y^{LB}, \\ \Delta_{11}^{LB} &= \bar{Y}_{0,1} - \left(\frac{P_{0|0}}{P_{1|1} - P_{1|0}} \cdot \bar{Y}_{0,0} - \frac{P_{0|1}}{P_{1|1} - P_{1|0}} \cdot \bar{Y}_{1,0} \right). \end{aligned} \tag{33}$$

As under stochastic dominance, the upper bound of the always takers' mean potential outcome under non-treatment cannot be higher than the compliers' upper bound under non-treatment. Furthermore, monotonicity implies that the latter is point identified by $\frac{P_{0|0}}{P_{1|1}-P_{1|0}} \cdot \bar{Y}_{0,0} - \frac{P_{0|1}}{P_{1|1}-P_{1|0}} \cdot \bar{Y}_{1,0}$. Again, Δ_{11}^{LB} is sharp by Lemma 1 in Huber and Mellace (2010). Similarly, the bounds for the never takers tighten to

$$\begin{aligned} \Delta_{00}^{UB} &= \left(\frac{P_{1|1}}{P_{1|1} - P_{1|0}} \cdot \bar{Y}_{1,1} - \frac{P_{1|0}}{P_{1|1} - P_{1|0}} \cdot \bar{Y}_{0,1} \right) - \bar{Y}_{1,0}, \\ \Delta_{00}^{LB} &= y^{LB} - \bar{Y}_{1,0}. \end{aligned} \tag{34}$$

By the monotonicity assumption, $\frac{P_{1|1}}{P_{1|1}-P_{1|0}} \cdot \bar{Y}_{1,1} - \frac{P_{1|0}}{P_{1|1}-P_{1|0}} \cdot \bar{Y}_{0,1}$ is the compliers' mean potential outcome under treatment. Under stochastic dominance, this is an upper bound for the never takers' mean potential outcome under treatment.

As under monotonicity, the bounds for the treated and the entire population are weighted averages of the bounds on the various subgroups:

$$\begin{aligned}\Delta^{UB}_{T=1} &= \frac{2\cdot P_{1|0}}{P_{1|1}+P_{1|0}}\cdot\Delta^{UB}_{11}+\frac{P_{1|1}-P_{1|0}}{P_{1|1}+P_{1|0}}\cdot\Delta_{10},\\ \Delta^{LB}_{T=1} &= \frac{2\cdot P_{1|0}}{P_{1|1}+P_{1|0}}\cdot\Delta^{LB}_{11}+\frac{P_{1|1}-P_{1|0}}{P_{1|1}+P_{1|0}}\cdot\Delta_{10},\end{aligned} \tag{35}$$

and

$$\begin{aligned}\Delta^{UB} &= P_{1|0}\cdot\Delta^{UB}_{11}+(P_{1|1}-P_{1|0})\cdot\Delta_{10}+P_{0|1}\cdot\Delta^{UB}_{00},\\ \Delta^{LB} &= P_{1|0}\cdot\Delta^{LB}_{11}+(P_{1|1}-P_{1|0})\cdot\Delta_{10}+P_{0|1}\cdot\Delta^{LB}_{00}.\end{aligned} \tag{36}$$

As a final remark it is worth noting that under Assumptions 1 to 3, Assumption 4 (stochastic dominance) is testable. Recall that the always takers' potential outcome distribution is identified by $Y|Z=0,T=1$. Therefore, stochastic dominance of the compliers can be tested by comparing the distributions of $Y|Z=0,T=1$ and $Y|Z=1,T=1$, which also encounters compliers and, therefore, has to stochastically dominate. Equivalently, the distribution of $Y|Z=1,T=0$, which are the never takers' potential outcomes under non-treatment, must be dominated by the distribution of $Y|Z=0,T=0$, which contains never takers and compliers. The intuition is that since the distribution of the potential outcomes of always takers (never takers) is not affected by Z under the exclusion restriction, the observed subgroup consisting of both compliers and always takers (never takers) stochastically dominates the observed subgroup with always takers (never takers) alone. The respective null hypotheses to be tested are $\Pr(Y|Z=1,T=1\leq y)\leq \Pr(Y|Z=0,T=1\leq y)$ and $\Pr(Y|Z=0,T=0\leq y)\leq \Pr(Y|Z=1,T=0\leq y)$ for all y in the support of Y, see Section 5 for the application of stochastic dominance tests.

4 Estimation

Estimators can be constructed by using the sample analogs of the bounds derived under the various assumptions, which is straightforward. To this end, we define the following sample parameters:

$$
\begin{aligned}
\hat{P}_{1|1} &\equiv \frac{\sum_{i=1}^{n} T_i \cdot Z_i}{\sum_{i=1}^{n} Z_i}, \quad \hat{P}_{0|1} \equiv 1 - \frac{\sum_{i=1}^{n} T_i \cdot Z_i}{\sum_{i=1}^{n} Z_i}, \\
\hat{P}_{1|0} &\equiv \frac{\sum_{i=1}^{n} T_i \cdot (1-Z_i)}{\sum_{i=1}^{n} (1-Z_i)}, \quad \hat{P}_{0|0} \equiv 1 - \frac{\sum_{i=1}^{n} T_i \cdot (1-Z_i)}{\sum_{i=1}^{n} (1-Z_i)}, \\
\hat{\bar{Y}}_{1,1} &\equiv \frac{\sum_{i=1}^{n} Y_i \cdot T_i \cdot Z_i}{\sum_{i=1}^{n} T_i \cdot Z_i}, \quad \hat{\bar{Y}}_{0,1} \equiv \frac{\sum_{i=1}^{n} Y_i \cdot T_i \cdot (1-Z_i)}{\sum_{i=1}^{n} T_i \cdot (1-Z_i)}, \\
\hat{\bar{Y}}_{1,0} &\equiv \frac{\sum_{i=1}^{n} Y_i \cdot (1-T_i) \cdot Z_i}{\sum_{i=1}^{n} (1-T_i) \cdot Z_i}, \quad \hat{\bar{Y}}_{0,0} \equiv \frac{\sum_{i=1}^{n} Y_i \cdot (1-T_i) \cdot (1-Z_i)}{\sum_{i=1}^{n} (1-T_i) \cdot (1-Z_i)}, \\
\hat{\bar{Y}}_{t,s}(\max|q_{t,s}^G) &\equiv \frac{\sum_{i=1}^{n} Y_i \cdot I\{T_i = s\} \cdot I\{Z_i = t\} \cdot I\{Y \geq \hat{y}_{1-q_{t,s}^G}\}}{\sum_{i=1}^{n} I\{T_i = s\} \cdot I\{Z_i = t\} \cdot I\{Y \geq \hat{y}_{1-q_{t,s}^G}\}}, \\
\hat{\bar{Y}}_{t,s}(\min|q_{t,s}^G) &\equiv \frac{\sum_{i=1}^{n} Y_i \cdot I\{T_i = s\} \cdot I\{Z_i = t\} \cdot I\{Y \leq \hat{y}_{q_{t,s}^G}\}}{\sum_{i=1}^{n} I\{T_i = s\} \cdot I\{Z_i = t\} \cdot I\{Y \leq \hat{y}_{q_{t,s}^G}\}}, \\
\hat{y}_{q_{t,s}^G} &\equiv \min\left\{ y : \frac{\sum_{i=1}^{n} T_i \cdot D_i \cdot I\{Y_i \leq y\}}{\sum_{i=1}^{n} T_i \cdot D_i} \geq q_{t,s}^G \right\}, \\
\hat{y}^{LB} &\equiv \min(Y), \quad \hat{y}^{UB} \equiv \max(Y)
\end{aligned}
$$

where $I\{\cdot\}$ is the indicator function. Using these expressions instead of the population parameters in the various formulas for the bounds immediately yields feasible estimators. $\sqrt{n}$-consistency and asymptotic normality of the estimators under monotonicity or monotonicity and stochastic dominance follows from the results by Lee (2009) and its discussion is, therefore, omitted.

5 Application

We apply the methods outlined in the last sections to a randomized experiment that was conducted within the Job Training Partnership Act (JTPA), a large publicly-funded U.S. training program taking place in the 1980s and 1990s. The largest JTPA component was training for economically disadvantaged individuals, the so called “Title II”. The latter covered roughly 1 million participants per year in the early 1990s, see for instance Orr, Bloom, Bell, Doolittle, Lin, and Cave (1996) for more details. The sample we investigate has been previously analyzed by Abadie, Angrist, and Imbens (2002) and consists of 11,204 adults that had applied for Title II between

November 1987 and September 1989. Applicants were randomly assigned to be offered a training and those not receiving an offer were excluded from Title II for 18 months. We evaluate the average effect of the training on the sum of earnings in the 30-months after training assignment, which, according to Abadie, Angrist, and Imbens (2002), is a suitable measure of the program's lasting economic impact on participants.

Table 4: Observed strata proportions

Conditional treatment probability	estimate	standard error
$P_{1\|1} \equiv \Pr(T = 1\|Z = 1)$	0.642	(0.006)
$P_{0\|1} \equiv \Pr(T = 0\|Z = 1)$	0.358	(0.006)
$P_{1\|0} \equiv \Pr(T = 1\|Z = 0)$	0.015	(0.002)
$P_{0\|0} \equiv \Pr(T = 0\|Z = 0)$	0.985	(0.002)

Let Z denote the random assignment indicator, Y the earnings outcome, and T the actual training state. As shown in Table 4, compliance with the treatment assignment was not perfect. Only 64.2 % of those who were offered a training actually took advantage of the offer, while 35.8 % did not. 98.5 % of the individuals that were randomized out did not receive the training, but 1.5 % participated anyway. Table 5 reports the estimated bounds on the strata proportions in the worst case scenario and the respective point estimates under Assumption 3 (monotonicity).

Table 5: Estimated (bounds on the) proportions of latent strata

Latent strata	Worst case bounds	Proportions under monotonicity
Always takers	[0.000, 0.015]	0.015
Compliers	[0.627, 0.642]	0.627
Never takers	[0.343, 0.358]	0.358
Defiers	[0.000, 0.015]	-

We estimate bounds on the ATEs of the compliers, the always takers, the never takers, the treated, and the total population under the worst case, stochastic dominance, and/or monotonicity. Whenever optimization over the share of defiers is required, we use an equidistant grid of 100 values within the minimum (0) and maximum(0.015) possible share. We do not estimate defier effects because $\hat{P}_{1|1} > \hat{P}_{0|1}$, implying that the bounds for the defiers are not informative in the worst case scenario. Furthermore, defiers are ruled out under monotonicity (and under both monotonicity and stochastic dominance). Finally, note that also the bounds for the always takers are not informative in the worst case scenario, because $\hat{P}_{1|0} < \hat{P}_{0|1}$ such that the share of always takers is smaller than the share of never takers. However, under monotonicity and/or stochastic dominance, informative bounds can be obtained for this population.

Concerning inference, we compute the 95% confidence intervals for the respective ATE (i.e., for the parameters of interest, not for its bounds) based on the method described in Imbens and Manski (2004):

$$\left(\hat{\Delta}^{LB} - 1.645 \cdot \hat{\sigma}^{LB}, \hat{\Delta}^{UB} + 1.645 \cdot \hat{\sigma}^{UB}\right),$$

where $\hat{\Delta}^{LB}, \hat{\Delta}^{UB}$ are the estimated bounds and $\hat{\sigma}^{LB}, \hat{\sigma}^{UB}$ denote their respective estimated standard errors. The latter are obtained by bootstrapping the original sample 1999 times and estimating $\hat{\Delta}^{LB}, \hat{\Delta}^{UB}$ in each bootstrap replication in order to estimate their distributions. As worst case bounds y^{UB} and y^{LB}, we take the maximum and minimum cumulative earnings observed in the data, which are 155,760 and 0, respectively.

Table 6: ATE estimates and confidence intervals

Assumptions	Compliers	Always takers	Never takers	Treated	Entire pop.
Worst case	[628, 1942]	[-155760, 65536]	[-13980, 143943]	[-4569, 2408]	[-5924, 52163]
	(-114, 2686)	Not informative	(-14453, 162095)	(-6152, 3122)	(-6599, 58939)
Stoch. dom.	[637, 1942]	[-16803, 17440]	[-13980, 5624]	[-521, 2408]	[-4405, 2673]
	(-102, 2686)	(-17452, 17851)	(-14452, 6221)	(-1284, 3121)	(-4858, 3192)
Monoton.	[1849, 1849]	[-142245, 13515]	[-13980, 141780]	[-4532, 2365]	[-5917, 52163]
	(945, 2753)	(-160550, 16390)	(-14453, 159859)	(-6114, 3081)	(-6590, 58939)
Both	[1849, 1849]	[-2167, 13515]	[-13980, 3551]	[1671, 2365]	[-3882, 2628]
	(945, 2753)	(-4990, 16390)	(-14453, 4131)	(933, 3081)	(-4333, 3151)

Bounds in square brackets and 95 % confidence intervals in round brackets.
Confidence intervals are based on 1999 bootstraps. Average treatment effects are rounded.

Table 6 presents the results under the various assumptions. The bounds of the ATE estimates are given in square brackets, the 95% confidence intervals are in round brackets. The worst case bounds are not informative for the always takers and very wide for all other populations apart from the compliers. For the latter, we find a positive ATE, even without imposing any further assumptions on top of the exclusion restriction. However, the effect is not statistically significant on the 5% level. Imposing stochastic dominance narrows the bounds for the compliers only slightly and the effect is again not significant. Even though the assumption has more identifying power for the other populations, the identification region of any group other than the compliers still includes a zero effect.

As discussed before, monotonicity of T in Z (such that defiers are ruled out) entails point identification of the ATE on the compliers. The estimate, which is highly significant, suggests that on average, training generates 1849 USD of additional earnings in the 30-months after training assignment for those participating in the training if assigned and not participating if not. The bounds for any other group, however, still include the possibility of a zero effect on earnings. When invoking both monotonicity and stochastic dominance, the results suggest that the ATE on the treated, which often represent the most interesting (from a policy perspective) population, is significantly positive. Both its upper and lower bound are not too far from the point estimate for the compliers, suggesting that the effects on both populations might be comparable in the application considered.

As mentioned in Section 3.4, stochastic dominance of the compliers' potential outcomes has testable implications if monotonicity holds. We test the null hypotheses of stochastic dominance w.r.t. the always takers and the never takers, namely that $\Pr(Y|Z=1,T=1 \leq y) \leq \Pr(Y|Z=0,T=1 \leq y)$ and $\Pr(Y|Z=0,T=0 \leq y) \leq \Pr(Y|Z=1,T=0 \leq y)$ for all y in the support of Y, by means of one-sided Kolmogorov-Smirnov tests for two samples. The test statistics are reported in Table 7 and are not significant, suggesting that stochastic dominance cannot be rejected in either case.

Table 7: Stochastic dominance tests

	H_0: Compliers dominate always takers	H_0: Compliers dominate never takers
KS test statistic	0.026	0.006
p-value	0.931	0.896

Finally, Table 8 gives the estimates separately for the 6,102 females and 5,102 males in the sample. In general, the bounds for the females tend to be tighter the the ones for the males.

Furthermore, the former provide more evidence in favor of a positive effect of the training on the cumulative earnings. The ATE on the compliers is significantly positive even in the worst case scenario. Under monotonicity and stochastic dominance, also the lower bound for the always takers is positive (in addition to the complier effect and the lower bound of the ATE on the treated, which are positive in all samples), albeit not statistically significant.

Table 8: ATE estimates and confidence intervals by gender

	Females				
Assumptions	Compliers	Always takers	Never takers	Treated	Entire pop.
Worst case	[928, 2075]	[-114739, 65536]	[-11934, 104756]	[-3426, 2598]	[-4612, 36704]
	(53, 2949)	Not informative	(-12582, 120490)	(-5070, 3496)	(-5350, 42402)
Stoch. dom	[957, 2075]	[-13267, 14224]	[-11934, 4244]	[21, 2567]	[-3234, 2348]
	(77, 2869)	(-14053, 14640)	(-12465, 5078)	(-916, 3362)	(-3832, 2977)
Monoton.	[1942, 1942]	[-101576, 13163]	[-11934, 102805]	[-3391, 2520]	[-4612, 36704]
	(906, 2977)	(-117690, 17106)	(-12582, 118567)	(-5045, 3398)	(-5350, 42402)
Both	[1942, 1942]	[852, 13163]	[-11934, 2320]	[1886, 2520]	[-2832, 2266]
	(906, 2977)	(-3299, 17106)	(-12582, 3129)	(981, 3398)	(-3412, 2901)
	Males				
Assumptions	Compliers	Always takers	Never takers	Treated	Entire pop.
Worst case	[606, 1881]	[-155760, 50316]	[-16216, 141489]	[-3306, 2292]	[-6579, 53906]
	(-978, 3440)	Not informative	(-17079, 157783)	(-5699, 3802)	(-7686, 60166)
Stoch. dom.	[612, 1881]	[-20909, 21521]	[-16216, 7254]	[-563, 2292]	[-5580, 3353]
	(-969, 3440)	(-22510, 22379)	(-17079, 8428)	(-2218, 3803)	(-6512, 4425)
Monoton.	[1825, 1825]	[-141596, 14164]	[-16216, 139544]	[-3222, 2260]	[-6579, 53906]
	(-45, 3696)	(-157341, 18261)	(-17079, 155721)	(-5609, 3769)	(-7686, 60166)
Both	[1825, 1825]	[-5666, 14164]	[-16216, 5439]	[1562, 2260]	[-5063, 3326]
	(-45, 3696)	(-9937, 18261)	(-17079, 6601)	(8, 3769)	(-5987, 4401)

Note: Bounds in square brackets and 95 % confidence intervals in round brackets.
Confidence intervals are based on 1999 bootstraps. Average treatment effects are rounded.

6 Conclusion

This paper sheds light on the question of what can be learnt about the average treatment effects (ATE) on various populations under endogeneity/noncompliance when a valid instrumental variable (IV) is at hand. Since the work by Imbens and Angrist (1994) it is well known that a local ATE (LATE) on the compliers (who take the treatment if instrumented, but do not if not) is point identified under monotonicity of the treatment in the instrument. Even though point identification is not feasible for other groups, we show that informative bounds can be obtained for the always takers (treated irrespective of the instrument), the never takers (not treated irrespective of the instrument), the treated population, and the entire population. We also investigate the identifying power of stochastic dominance of the potential outcomes of the compliers over those of the always takers and never never takers.

The main contribution is the derivation of sharp bounds under monotonicity, under stochastic dominance, and under both assumptions. Using estimators that are based on the sample analogs of our identification results, we also provide an application to the U.S. Job Training Partnership Act using experimental data previously analyzed by Abadie, Angrist, and Imbens (2002). We find (on top of the complier effect) a significantly positive ATE on the earnings of the treated, a group of major policy interest. As valuable "by-products" of our identification results we also obtain testable implications of the IV exclusion restriction (no direct effect of the instrument on the outcome) and of stochastic dominance, respectively, when monotonicity is assumed. The statistical power of these implications might be investigated in future research.

A Appendix

A.1 Worst case scenario

We will only show the sharpness of the upper bounds, the proofs for the lower bounds are symmetric.

A.1.1 Proof of the sharpness of the bounds for the compliers

First of all, note that if w is a random variable which is distributed as a two components mixture

$$f(w) = p \cdot f(w_1) + (1-p) \cdot f(w_2) \quad p \in [0,1],$$

then

$$E(w) = p \cdot E(w_1)^{UB} + (1-p) \cdot E(w_2)^{LB}, \tag{A.1}$$

where $E(w_1)^{UB}$ is the upper bound of $E(w_1)$ and $E(w_2)^{LB}$ is the lower bound of $E(w_2)$. By Assumption 1 and equations (2) and (3) we have

$$\begin{aligned} E(Y|T=1,G=10)^{UB} &= \frac{P_{1|1}}{P_{1|1}-P_{1|0}+\pi_{01}} \cdot \bar{Y}_{1,1} \\ &- \frac{P_{1|0}-\pi_{01}}{P_{1|1}-P_{1|0}+\pi_{01}} \cdot E(Y|T=1,G=11)^{LB}, \end{aligned} \tag{A.2}$$

and

$$\begin{aligned} E(Y|T=0,G=10)^{LB} &= \frac{P_{0|0}}{P_{1|1}-P_{1|0}+\pi_{01}} \cdot \bar{Y}_{0,0} \\ &- \frac{P_{0|1}-\pi_{01}}{P_{1|1}-P_{0|1}+\pi_{01}} \cdot E(Y|T=0,G=00)^{UB}. \end{aligned} \tag{A.3}$$

Lemma 1 together with Proposition 1 in Imai (2008) implies that $\bar{Y}_{1,1}(\min|q^{11}_{1,1})$ is the sharp lower bound of $E(Y|Z=1,T=1,G=11)$ and that $\bar{Y}_{0,1}(\min|q^{11}_{0,1})$ is the sharp lower bound of $E(Y|Z=0,T=1,G=11)$. By Assumption 1, $E(Y|Z=1,T=1,G=11) = E(Y|Z=0,T=1,G=11) = E(Y|T=1,G=11)$. Thus, $E(Y|T=1,G=11) \geq \bar{Y}_{1,1}(\min|q^{11}_{1,1})$ and $E(Y|T=1,G=11) \geq \bar{Y}_{0,1}(\min|q^{11}_{0,1})$. This implies that

$$E(Y|T=1,G=11)^{LB} = \max\left(\bar{Y}_{1,1}(\min|q^{11}_{1,1}), \bar{Y}_{0,1}(\min|q^{11}_{0,1})\right). \tag{A.4}$$

Similarly, Lemma 1 together with Proposition 1 in Imai (2008) implies that $\bar{Y}_{0,0}(\max|q^{00}_{0,0})$ is the sharp upper bound of $E(Y|Z=0,T=0,G=00)$ and that $\bar{Y}_{1,0}(\max|q^{00}_{1,0})$ is the sharp upper bound of $E(Y|Z=1,T=$

$0, G = 00)$. Again, by Assumption 1 we have $E(Y|T = 0, G = 00) \leq \bar{Y}_{1,0}(\max|q^{00}_{1,0})$ and $E(Y|T = 0, G = 00) \leq \bar{Y}_{0,0}(\max|q^{00}_{0,0})$. Therefore,

$$E(Y|T = 0, G = 00)^{UB} = \min\left(\bar{Y}_{1,0}(\max|q^{00}_{1,0}), \bar{Y}_{0,0}(\max|q^{00}_{0,0})\right). \tag{A.5}$$

This shows that $\Delta^{UB}_{10} = E(Y|T = 1, G = 10)^{UB} - E(Y|T = 0, G = 10)^{LB}$ is the sharp upper bound of Δ_{10} for a given value of π_{01}, which is unknown. Therefore, Δ^{UB}_{10} is obtained by maximizing w.r.t. π_{01}.

A.1.2 Proof of the sharpness of the bounds for the defiers

By Assumption 1, (A.1), (1) and (4) we have

$$\begin{aligned} E(Y|T = 1, G = 01)^{UB} &= \frac{P_{1|0}}{\pi_{01}} \cdot \bar{Y}_{0,1} \\ &- \frac{P_{1|0} - \pi_{01}}{\pi_{01}} \cdot E(Y|T = 1, G = 11)^{LB}, \end{aligned} \tag{A.6}$$

and

$$\begin{aligned} E(Y|T = 0, G = 01)^{LB} &= \frac{P_{0|1}}{\pi_{01}} \cdot \bar{Y}_{1,0} \\ &- \frac{P_{0|1} - \pi_{01}}{\pi_{01}} \cdot E(Y|T = 0, G = 00)^{UB}. \end{aligned} \tag{A.7}$$

Thus, by (A.4) and (A.5), $\Delta^{UB}_{01} = E(Y|T = 1, G = 01)^{UB} - E(Y|T = 0, G = 01)^{LB}$ is the sharp upper bound of Δ_{01} for a given value of π_{01}, which is unknown. Therefore, Δ^{UB}_{01} is obtained by maximizing w.r.t. π_{01}.

A.1.3 Proof of the sharpness of the bounds for the always takers

Lemma 1 together with Proposition 1 in Imai (2008) implies that $\bar{Y}_{1,1}(\max|q^{11}_{1,1})$ is the sharp upper bound of $E(Y|Z = 1, T = 1, G = 11)$ and that $\bar{Y}_{0,1}(\max|q^{11}_{0,1})$ is the sharp upper bound of $E(Y|Z = 0, T = 1, G = 11)$. By Assumption 1, $E(Y|T = 1, G = 11) \leq \bar{Y}_{1,1}(\max|q^{11}_{1,1})$ and $E(Y|T = 1, G = 11) \leq \bar{Y}_{0,1}(\max|q^{11}_{0,1})$. This implies that

$$E(Y|T = 1, G = 11)^{UB} = \min\left(\bar{Y}_{1,1}(\max|q^{11}_{1,1}), \bar{Y}_{0,1}(\max|q^{11}_{0,1})\right). \tag{A.8}$$

Since the sampling process does not impose any restriction on $Y|T = 0, G = 11$ for a fixed value of π_{01}, $\Delta^{UB}_{11} = E(Y|T = 1, G = 11)^{UB} - y^{LB}$ is the sharp upper bound of Δ_{11}. Since both $\bar{Y}_{1,1}(\max|q^{11}_{1,1})$ and $\bar{Y}_{0,1}(\max|q^{11}_{0,1})$ are increasing in π_{01}, Δ^{UB}_{11} is maximized for $\pi_{01} = \pi^{\max}_{01}$.

A.1.4 Proof of the sharpness of the bounds for the never takers

Lemma 1 together with Proposition 1 in Imai (2008) implies that $\bar{Y}_{0,0}(\min|q^{00}_{0,0})$ is the sharp lower bound of $E(Y|Z=0,T=0,G=00)$ and that $\bar{Y}_{1,0}(\min|q^{00}_{1,0})$ is the sharp lower bound of $E(Y|Z=1,T=0,G=00)$. Again, by Assumption 1 we have that $E(Y|T=0,G=00) \geq \bar{Y}_{1,0}(\min|q^{00}_{1,0})$ and $E(Y|T=0,G=00) \geq \bar{Y}_{0,0}(\min|q^{00}_{0,0})$. Therefore,

$$E(Y|T=0,G=00)^{LB} \quad = \quad \max\left(\bar{Y}_{1,0}(\min|q^{00}_{1,0}), \bar{Y}_{0,0}(\min|q^{00}_{0,0})\right). \tag{A.9}$$

Since the sampling process does not impose any restriction on $Y|T=1,G=00$ for a fixed value of π_{01}, $\Delta^{UB}_{00} = y^{UB} - E(Y|T=0,G=00)^{LB}$ is the sharp upper bound of Δ_{00}. Since both $\bar{Y}_{0,0}(\min|q^{00}_{0,0})$ and $\bar{Y}_{1,0}(\min|q^{00}_{1,0})$ are decreasing in π_{01}, Δ^{UB}_{00} is maximized for $\pi_{01} = \pi^{\max}_{01}$.

A.1.5 Proof of the sharpness of the bounds for the treated

We will only show the sharpness of the upper bound given that $\bar{Y}_{1,1}(\max|q^{11}_{1,1}) \geq \bar{Y}_{0,1}(\max|q^{11}_{0,1})$. The proof for the case that $\bar{Y}_{1,1}(\max|q^{11}_{1,1}) < \bar{Y}_{0,1}(\max|q^{11}_{0,1})$ is symmetric and the proof for the lower bound could be derived in an analogous way to the upper bound and is, therefore, omitted. Note that

$$\Delta_{T=1} = \frac{2\cdot(P_{1|0}-\pi_{01})}{P_{1|1}+P_{1|0}}\cdot\Delta_{11} + \frac{P_{1|1}-P_{1|0}+\pi_{01}}{P_{1|1}+P_{1|0}}\cdot\Delta_{10} + \frac{\pi_{01}}{P_{1|1}+P_{1|0}}\cdot\Delta_{01}. \tag{A.10}$$

For the upper bound, substituting Δ_{11} by Δ^{UB}_{11}, Δ_{10} by Δ^{UB}_{10} and Δ_{01} by Δ^{UB}_{01} in (A.10) would give a sharp upper bound on $\Delta_{T=1}$. However, such a bound would contradict (A.1) since it is impossible to have the upper bounds for the always takers and the defiers at the same time in the mixture. This, however, shows that the admissible sharp upper bound would be the maximum of

$$\begin{aligned}
\Delta^{UB1}_{T=1} \quad = \quad & \frac{2\cdot(P_{1|0}-\pi_{01})\cdot\left(\bar{Y}_{0,1}(\max|q^{11}_{0,1})-y^{LB}\right)}{P_{1|1}+P_{1|0}} \\
+ \quad & \frac{P_{1|1}-P_{1|0}+\pi_{01}}{P_{1|1}+P_{1|0}}\cdot\Delta^{UB}_{10} \\
+ \quad & \frac{P_{1|0}\cdot\bar{Y}_{0,1}-(P_{1|0}-\pi_{01})\cdot\bar{Y}_{0,1}(\max|q^{11}_{0,1})}{P_{1|1}+P_{1|0}} \\
- \quad & \frac{P_{0|1}\cdot\bar{Y}_{1,0}-(P_{0|1}-\pi_{01})\cdot\min\left(\bar{Y}_{0,0}(\max|q^{00}_{0,0}),\bar{Y}_{1,0}(\max|q^{00}_{1,0})\right)}{P_{1|1}+P_{1|0}}
\end{aligned} \tag{A.11}$$

and

$$\begin{aligned}
\Delta_{T=1}^{UB2} &= \frac{2\cdot(P_{1|0}-\pi_{01})\cdot\left(\max\left(\bar{Y}_{1,1}(\min|q_{1,1}^{11}),\bar{Y}_{0,1}(\min|q_{0,1}^{11})\right)-y^{LB}\right)}{P_{1|1}+P_{1|0}} \\
&+ \frac{P_{1|1}-P_{1|0}+\pi_{01}}{P_{1|1}+P_{1|0}}\cdot\Delta_{10}^{UB} \\
&+ \frac{P_{1|0}\cdot\bar{Y}_{0,1}-(P_{1|0}-\pi_{01})\cdot\max\left(\bar{Y}_{1,1}(\min|q_{1,1}^{11}),\bar{Y}_{0,1}(\min|q_{0,1}^{11})\right)}{P_{1|1}+P_{1|0}} \\
&- \frac{P_{0|1}\cdot\bar{Y}_{1,0}-(P_{0|1}-\pi_{01})\cdot\min\left(\bar{Y}_{0,0}(\max|q_{0,0}^{00}),\bar{Y}_{1,0}(\max|q_{1,0}^{00})\right)}{P_{1|1}+P_{1|0}}.
\end{aligned} \tag{A.12}$$

After some simple algebra (A.13) and (A.12) can be rewritten as

$$\begin{aligned}
\Delta_{T=1}^{UB1} &= \frac{P_{1|0}-\pi_{01}}{P_{1|1}+P_{1|0}}\cdot\bar{Y}_{0,1}(\max|q_{0,1}^{11})-\frac{2\cdot(P_{1|0}-\pi_{01})}{P_{1|1}+P_{1|0}}\cdot y^{LB} \\
&+ \frac{P_{1|1}-P_{1|0}+\pi_{01}}{P_{1|1}+P_{1|0}}\cdot\Delta_{10}^{UB}+\frac{P_{1|0}}{P_{1|1}+P_{1|0}}\cdot\bar{Y}_{0,1} \\
&- \frac{P_{0|1}\cdot\bar{Y}_{1,0}-(P_{0|1}-\pi_{01})\cdot\min\left(\bar{Y}_{0,0}(\max|q_{0,0}^{00}),\bar{Y}_{1,0}(\max|q_{1,0}^{00})\right)}{P_{1|1}+P_{1|0}},
\end{aligned} \tag{A.13}$$

and

$$\begin{aligned}
\Delta_{T=1}^{UB2} &= \frac{P_{1|0}-\pi_{01}}{P_{1|1}+P_{1|0}}\cdot\max\left(\bar{Y}_{1,1}(\min|q_{1,1}^{11}),\bar{Y}_{0,1}(\min|q_{0,1}^{11})\right)-\frac{2\cdot(P_{1|0}-\pi_{01})}{P_{1|1}+P_{1|0}}\cdot y^{LB} \\
&+ \frac{P_{1|1}-P_{1|0}+\pi_{01}}{P_{1|1}+P_{1|0}}\cdot\Delta_{10}^{UB}+\frac{P_{1|0}}{P_{1|1}+P_{1|0}}\cdot\bar{Y}_{0,1} \\
&- \frac{P_{0|1}\cdot\bar{Y}_{1,0}-(P_{0|1}-\pi_{01})\cdot\min\left(\bar{Y}_{0,0}(\max|q_{0,0}^{00}),\bar{Y}_{1,0}(\max|q_{1,0}^{00})\right)}{P_{1|1}+P_{1|0}}.
\end{aligned} \tag{A.14}$$

To prove that $\Delta_{T=1}^{UB1}\geq\Delta_{T=1}^{UB2}$, it is sufficient to show that

$$\bar{Y}_{0,1}(\max|q_{0,1}^{11})\geq\max\left(\bar{Y}_{1,1}(\min|q_{1,1}^{11}),\bar{Y}_{0,1}(\min|q_{0,1}^{11})\right),$$

which is always satisfied since $\bar{Y}_{0,1}(\max|q_{0,1}^{11})\geq\bar{Y}_{1,1}(\min|q_{1,1}^{11})$ by Assumption 1. Thus, $\Delta_{T=1}^{UB1}$ is the sharp upper bound of $\Delta_{T=1}$, given that $\bar{Y}_{1,1}(\max|q_{1,1}^{11})\geq\bar{Y}_{0,1}(\max|q_{0,1}^{11})$ and for a given value of π_{01}, which is unknown. Therefore, we have to maximize $\Delta_{T=1}^{UB1}$ w.r.t. π_{01} to obtain the upper bound.

A.1.6 Proof of the sharpness of the bounds for the entire population

We will only prove the sharpness of the upper bound. The proof for the lower bound is analogous and is omitted for this reason. First of all, we will show that all four possible combinations of $\bar{Y}_{1,1}(\max|q_{1,1}^{11}),\bar{Y}_{0,1}(\max|q_{0,1}^{11})$ and $\bar{Y}_{1,0}(\min|q_{1,0}^{00}),\bar{Y}_{0,0}(\min|q_{0,0}^{00})$ yield the same upper bound. To see this, suppose the contrary. Then, we would compute the upper bound for the entire population in a similar way as the one for the treated. Thus, for the values of π_{01} for which $\bar{Y}_{1,1} \geq \bar{Y}_{0,1}(\max|q^{11}_{0,1})$ and $\bar{Y}_{1,0}(\min|q^{00}_{1,0}) \geq \bar{Y}_{0,0}(\min|q^{00}_{0,0})$, the upper bound would be

$$\begin{aligned}
\Delta^{UB1} &= \max_{\pi_{01}}\Big[P_{1|0}\cdot\bar{Y}_{0,1} - (P_{1|0}-\pi_{01})\cdot y^{LB} + (P_{1|1}-P_{1|0}+\pi_{01})\cdot\Delta^{UB}_{10} \\
&+ (P_{0|1}-\pi_{01})\cdot\frac{P_{1|1}\cdot\bar{Y}_{1,1}-(P_{1|0}-\pi_{01})\cdot\max\big(\bar{Y}_{1,1}(\min|q^{11}_{1,1}),\bar{Y}_{0,1}(\min|q^{11}_{0,1})\big)}{P_{1|1}-P_{1|0}+\pi_{01}} \\
&- P_{0|1}\cdot\bar{Y}_{1,0}\Big] \\
&= \max_{\pi_{01}}\Big[P_{1|0}\cdot\bar{Y}_{0,1} - (P_{1|0}-\pi_{01})\cdot y^{LB} + P_{1|1}\cdot\bar{Y}_{1,1} \\
&- (P_{1|0}-\pi_{01})\cdot\max\big(\bar{Y}_{1,1}(\min|q^{11}_{1,1}),\bar{Y}_{0,1}(\min|q^{11}_{0,1})\big) \\
&+ \big(P_{0|1}-\pi_{01}\big)\cdot\min\big(\bar{Y}_{0,0},\bar{Y}_{1,0}(\max|q^{00}_{1,0})\big) - P_{0|1}\cdot\bar{Y}_{1,0} \\
&+ (P_{0|1}-\pi_{01})\cdot\frac{P_{1|1}\cdot\bar{Y}_{1,1}-(P_{1|0}-\pi_{01})\cdot\max\big(\bar{Y}_{1,1}(\min|q^{11}_{1,1}),\bar{Y}_{0,1}(\min|q^{11}_{0,1})\big)}{P_{1|1}-P_{1|0}+\pi_{01}} \\
&- P_{0|1}\cdot\bar{Y}_{1,0}\Big].
\end{aligned} \tag{A.15}$$

Secondly, for the values of π_{01} for which $\bar{Y}_{1,1} \leq \bar{Y}_{0,1}(\max|q^{11}_{0,1})$ and $\bar{Y}_{1,0}(\min|q^{00}_{1,0}) \leq \bar{Y}_{0,0}(\min|q^{00}_{0,0})$, we would get

$$\begin{aligned}
\Delta^{UB2} &= \max_{\pi_{01}}\Big[P_{1|1}\cdot\bar{Y}_{1,1} - (P_{1|0}-\pi_{01})\cdot y^{LB} + \pi_{01}\cdot\Delta^{UB}_{01} \\
&+ (P_{0|1}-\pi_{01})\cdot\frac{P_{1|1}\cdot\bar{Y}_{1,1}-(P_{1|0}-\pi_{01})\cdot\max\big(\bar{Y}_{1,1}(\min|q^{11}_{1,1}),\bar{Y}_{0,1}(\min|q^{11}_{0,1})\big)}{P_{1|1}-P_{1|0}+\pi_{01}} \\
&- P_{0|0}\cdot\bar{Y}_{0,0}\Big] \\
&= \max_{\pi_{01}}\Big[P_{1|0}\cdot\bar{Y}_{0,1} - (P_{1|0}-\pi_{01})\cdot y^{LB} + P_{1|1}\cdot\bar{Y}_{1,1} \\
&- (P_{1|0}-\pi_{01})\cdot\max\big(\bar{Y}_{1,1}(\min|q^{11}_{1,1}),\bar{Y}_{0,1}(\min|q^{11}_{0,1})\big) \\
&+ \big(P_{0|1}-\pi_{01}\big)\cdot\min\big(\bar{Y}_{0,0},\bar{Y}_{1,0}(\max|q^{00}_{1,0})\big) - P_{0|1}\cdot\bar{Y}_{1,0} \\
&+ (P_{0|1}-\pi_{01})\cdot\frac{P_{1|1}\cdot\bar{Y}_{1,1}-(P_{1|0}-\pi_{01})\cdot\max\big(\bar{Y}_{1,1}(\min|q^{11}_{1,1}),\bar{Y}_{0,1}(\min|q^{11}_{0,1})\big)}{P_{1|1}-P_{1|0}+\pi_{01}} \\
&- P_{0|1}\cdot\bar{Y}_{1,0}\Big].
\end{aligned} \tag{A.16}$$

For the values of π_{01} for which $\bar{Y}_{1,1} \geq \bar{Y}_{0,1}(\max|q^{11}_{0,1})$ and $\bar{Y}_{1,0}(\min|q^{00}_{1,0}) \leq \bar{Y}_{0,0}(\min|q^{00}_{0,0})$, we would have

$$\begin{aligned}
\Delta^{UB3} &= \max_{\pi_{01}}\Big[P_{1|0}\cdot\bar{Y}_{0,1} - (P_{1|0}-\pi_{01})\cdot y^{LB} + P_{1|1}\cdot\bar{Y}_{1,1} \\
&- (P_{1|0}-\pi_{01})\cdot\max\big(\bar{Y}_{1,1}(\min|q^{11}_{1,1}),\bar{Y}_{0,1}(\min|q^{11}_{0,1})\big) \\
&+ \big(P_{0|1}-\pi_{01}\big)\cdot\min\big(\bar{Y}_{0,0},\bar{Y}_{1,0}(\max|q^{00}_{1,0})\big) - P_{0|1}\cdot\bar{Y}_{1,0} \\
&+ (P_{0|1}-\pi_{01})\cdot\frac{P_{1|1}\cdot\bar{Y}_{1,1}-(P_{1|0}-\pi_{01})\cdot\max\big(\bar{Y}_{1,1}(\min|q^{11}_{1,1}),\bar{Y}_{0,1}(\min|q^{11}_{0,1})\big)}{P_{1|1}-P_{1|0}+\pi_{01}} \\
&- P_{0|1}\cdot\bar{Y}_{1,0}\Big].
\end{aligned} \tag{A.17}$$

Finally, for the values of π_{01} for which $\bar{Y}_{1,1}(\max|q^{11}_{1,1}) \leq \bar{Y}_{0,1}(\max|q^{11}_{0,1})$ and $\bar{Y}_{1,0}(\min|q^{00}_{1,0}) \geq \bar{Y}_{0,0}(\min|q^{00}_{0,0})$, we would obtain

$$\begin{aligned}
\Delta^{UB4} &= \max_{\pi_{01}} \Big[P_{1|0} \cdot \bar{Y}_{0,1} - (P_{1|0} - \pi_{01}) \cdot y^{LB} + P_{1|1} \cdot \bar{Y}_{1,1} \\
&- (P_{1|0} - \pi_{01}) \cdot \max\left(\bar{Y}_{1,1}(\min|q_{1,1}^{11}), \bar{Y}_{0,1}(\min|q_{0,1}^{11})\right) \\
&+ \left(P_{0|1} - \pi_{01}\right) \cdot \min\left(\bar{Y}_{0,0}, \bar{Y}_{1,0}(\max|q_{1,0}^{00})\right) - P_{0|1} \cdot \bar{Y}_{1,0} \\
&+ (P_{0|1} - \pi_{01}) \cdot \frac{P_{1|1} \cdot \bar{Y}_{1,1} - (P_{1|0} - \pi_{01}) \cdot \max\left(\bar{Y}_{1,1}(\min|q_{1,1}^{11}), \bar{Y}_{0,1}(\min|q_{0,1}^{11})\right)}{P_{1|1} - P_{1|0} + \pi_{01}} \\
&- P_{0|1} \cdot \bar{Y}_{1,0} \Big] .
\end{aligned} \tag{A.18}$$

A closer inspection of these bounds shows that they are algebraically equal: $\Delta^{UB1} = \Delta^{UB2} = \Delta^{UB3} = \Delta^{UB4} = \Delta^{UB}$.

In order to see that Δ^{UB} is sharp, note that

$$\Delta = (P_{1|0} - \pi_{01}) \cdot \Delta_{11} + (P_{1|1} - P_{1|0} + \pi_{01}) \cdot \Delta_{10} + \pi_{01} \cdot \Delta_{01} + (P_{0|1} - \pi_{01}) \cdot \Delta_{00}. \tag{A.19}$$

For the upper bound, substituting Δ_{11} by Δ_{11}^{UB}, Δ_{10} by Δ_{10}^{UB}, Δ_{01} by Δ_{01}^{UB} and Δ_{00} by Δ_{00}^{UB} in (A.19) would give a sharp upper bound on Δ. However, such a bound would contradict (A.1), since it is impossible to have the upper bounds for the always takers and the defiers and the lower bounds for the never takers and the defiers at the same time in the mixtures. Because of the symmetry of the problem, we will just consider the case in which $\bar{Y}_{1,1}(\max|q_{1,1}^{11}) \geq \bar{Y}_{0,1}(\max|q_{0,1}^{11})$ and $\bar{Y}_{1,0}(\min|q_{1,0}^{00}) \geq \bar{Y}_{0,0}(\min|q_{0,0}^{00})$. In this case we can directly substitute Δ_{10} by Δ_{10}^{UB} in (A.19), without contradicting (A.1).

Given (A.19) and (A.1), the sharp upper bound under $\bar{Y}_{1,1}(\max|q_{1,1}^{11}) \geq \bar{Y}_{0,1}(\max|q_{0,1}^{11})$ and $\bar{Y}_{1,0}(\min|q_{1,0}^{00}) \geq \bar{Y}_{0,0}(\min|q_{0,0}^{00})$ is the maximum of the following four admissible upper bounds:

$$\begin{aligned}
\Delta^{UB1} &= \max_{\pi_{01}} \Big[(P_{1|0} - \pi_{01}) \cdot (\bar{Y}_{0,1}(\max|q_{0,1}^{11}) - y^{LB}) + (P_{1|0} - \pi_{01}) \cdot \Delta_{10}^{UB} \\
&+ P_{1|0} \cdot Y_{0,1} - (P_{1|0} - \pi_{01}) \cdot \bar{Y}_{0,1}(\max|q_{0,1}^{11}) \\
&- P_{0|1} \cdot Y_{1,0} + (P_{0|1} - \pi_{01}) \cdot \bar{Y}_{1,0}(\min|q_{1,0}^{00}) \\
&+ (P_{0|1} - \pi_{01}) \cdot (y^{UB} - \bar{Y}_{1,0}(\min|q_{1,0}^{00})) \Big] \\
&= \max_{\pi_{01}} \Big[(P_{1|0} - \pi_{01}) \cdot \Delta_{10}^{UB} - (P_{1|0} - \pi_{01}) \cdot y^{LB} \\
&+ P_{1|0} \cdot Y_{0,1} - P_{0|1} \cdot Y_{1,0} + (P_{0|1} - \pi_{01}) \cdot y^{UB} \Big] ,
\end{aligned} \tag{A.20}$$

$$\begin{aligned}
\Delta^{UB2} &= \max_{\pi_{01}} \Big[(P_{1|0} - \pi_{01}) \cdot (\bar{Y}_{0,1}(\max|q_{0,1}^{11}) - y^{LB}) + (P_{1|0} - \pi_{01}) \cdot \Delta_{10}^{UB} \\
&+ P_{1|0} \cdot Y_{0,1} - (P_{1|0} - \pi_{01}) \cdot \bar{Y}_{0,1}(\max|q_{0,1}^{11}) \\
&- P_{0|1} \cdot Y_{1,0} + (P_{0|1} - \pi_{01}) \cdot \bar{Y}_{1,0}(\max|q_{1,0}^{00}) \\
&+ (P_{0|1} - \pi_{01}) \cdot (y^{UB} - \bar{Y}_{1,0}(\max|q_{1,0}^{00}))\Big] \\
&= \max_{\pi_{01}} \Big[(P_{1|0} - \pi_{01}) \cdot \Delta_{10}^{UB} - (P_{1|0} - \pi_{01}) \cdot y^{LB} \\
&+ P_{1|0} \cdot Y_{0,1} - P_{0|1} \cdot Y_{1,0} + (P_{0|1} - \pi_{01}) \cdot y^{UB}\Big],
\end{aligned} \tag{A.21}$$

$$\begin{aligned}
\Delta^{UB3} &= \max_{\pi_{01}} \Big[(P_{1|0} - \pi_{01}) \cdot (\bar{Y}_{0,1}(\min|q_{0,1}^{11}) - y^{LB}) + (P_{1|0} - \pi_{01}) \cdot \Delta_{10}^{UB} \\
&+ P_{1|0} \cdot Y_{0,1} - (P_{1|0} - \pi_{01}) \cdot \bar{Y}_{0,1}(\min|q_{0,1}^{11}) \\
&- P_{0|1} \cdot Y_{1,0} + (P_{0|1} - \pi_{01}) \cdot \bar{Y}_{1,0}(\min|q_{1,0}^{00}) \\
&+ (P_{0|1} - \pi_{01}) \cdot (y^{UB} - \bar{Y}_{1,0}(\min|q_{1,0}^{00}))\Big] \\
&= \max_{\pi_{01}} \Big[(P_{1|0} - \pi_{01}) \cdot \Delta_{10}^{UB} - (P_{1|0} - \pi_{01}) \cdot y^{LB} \\
&+ P_{1|0} \cdot Y_{0,1} - P_{0|1} \cdot Y_{1,0} + (P_{0|1} - \pi_{01}) \cdot y^{UB}\Big],
\end{aligned} \tag{A.22}$$

$$\begin{aligned}
\Delta^{UB4} &= \max_{\pi_{01}} \Big[(P_{1|0} - \pi_{01}) \cdot (\bar{Y}_{0,1}(\min|q_{0,1}^{11}) - y^{LB}) + (P_{1|0} - \pi_{01}) \cdot \Delta_{10}^{UB} \\
&+ P_{1|0} \cdot Y_{0,1} - (P_{1|0} - \pi_{01}) \cdot \bar{Y}_{0,1}(\min|q_{0,1}^{11}) \\
&- P_{0|1} \cdot Y_{1,0} + (P_{0|1} - \pi_{01}) \cdot \bar{Y}_{1,0}(\max|q_{1,0}^{00}) \\
&+ (P_{0|1} - \pi_{01}) \cdot (y^{UB} - \bar{Y}_{1,0}(\max|q_{1,0}^{00}))\Big] \\
&= \max_{\pi_{01}} \Big[(P_{1|0} - \pi_{01}) \cdot \Delta_{10}^{UB} - (P_{1|0} - \pi_{01}) \cdot y^{LB} \\
&+ P_{1|0} \cdot Y_{0,1} - P_{0|1} \cdot Y_{1,0} + (P_{0|1} - \pi_{01}) \cdot y^{UB}\Big].
\end{aligned} \tag{A.23}$$

Since $\Delta^{UB1} = \Delta^{UB2} = \Delta^{UB3} = \Delta^{UB4} = \Delta^{UB}$, Δ^{UB} is the sharp upper bound for Δ. Finally we have have to show that Δ^{UB} is maximized for $\pi_{01} = \pi_{01}^{\min}$. It is sufficient to see that the first derivative of Δ^{UB} w.r.t. π_{01} is always negative, let

$$Y^{11,\min} \equiv \max\left(\bar{Y}_{1,1}(\min|q_{1,1}^{11}), \bar{Y}_{0,1}(\min|q_{0,1}^{11})\right),$$

and

$$Y^{00,\max} \equiv \min\left(\bar{Y}_{0,0}(\max|q^{00}_{0,0}), \bar{Y}_{1,0}(\max|q^{00}_{1,0})\right)$$

we have

$$Y^{11,\min} - Y^{00,\max} + (P_{1|0} - \pi_{01}) \cdot \frac{\partial Y^{00,\max}}{\partial \pi_{01}} - (P_{1|0} - \pi_{01}) \cdot \frac{\partial Y^{11,\min}}{\partial \pi_{01}} < y^{UB} - y^{LB},$$

which is always satisfied.

A.2 Monotonicity

A.2.1 Proof of the sharpness of the bounds for the always takers

Under monotonicity and the IV exclusion restriction $E(Y|T=1, G=11)$ is identified by $\bar{Y}_{0,1}$. Since monotonicity does not impose any restrictions on the distribution of $Y|T=0, G=11$ the worst case bounds y^{LB}, y^{UB} have to be assumed. This implies that Δ^{UB}_{11} and Δ^{LB}_{11} are the sharp upper and lower bounds of Δ_{11}.

A.2.2 Proof of the sharpness of the bounds for the never takers

Under monotonicity and the IV exclusion restriction $E(Y|T=0, G=00)$ is identified by $\bar{Y}_{1,0}$. Since monotonicity does not impose any restrictions on the distribution of $Y|T=1, G=00$ the worst case bounds y^{LB}, y^{UB} have to be assumed. This implies that Δ^{UB}_{00} and Δ^{LB}_{00} are the sharp upper and lower bounds of Δ_{00}.

A.2.3 Proof of the sharpness of the bounds for the treated

These bounds are linear combinations of Δ_{10} and the bounds on Δ_{11}. Therefore, they are sharp.

A.2.4 Proof of the sharpness of the bounds for the entire population

These bounds are linear combinations of Δ_{10} and the bounds on Δ_{11} and Δ_{00}. Therefore, they are sharp.

A.3 Stochastic dominance

Lemma 1 in Huber and Mellace (2010) shows that under stochastic dominance, the upper bounds of $E(Y|T=1, Z=1, G=11)$ and $E(Y|T=0, Z=0, G=00)$ are $\bar{Y}_{1,1}$ and $\bar{Y}_{0,0}$, respectively. Moreover, stochastic dominance implies that $E(Y|T=0, G=11) \leq E(Y|T=0, G=10)$ and $E(Y|T=1, G=00) \leq E(Y|T=1, G=10)$. All bounds are special cases of the worst case bounds under the restrictions just mentioned. Therefore, all of them are sharp.

A.4 Monotonicity and stochastic dominance

A.4.1 Proof of the sharpness of the bounds for always takers

Under monotonicity and the IV exclusion restriction $E(Y|T=1,G=11)$ is identified by $\bar{Y}_{0,1}$. Stochastic dominance implies that $E(Y|T=0,G=11) \leq E(Y|T=0,G=10) = \frac{P_{0|0}\cdot\bar{Y}_{0,0}-P_{0|1}\cdot\bar{Y}_{1,0}}{P_{1|1}-P_{1|0}}$. Thus, Δ_{11}^{LB} is the sharp lower bound of Δ_{11}.

A.4.2 Proof of the sharpness of the bounds for never takers

Under monotonicity and the IV exclusion restriction $E(Y|T=0,G=00)$ is identified by $\bar{Y}_{1,0}$. Stochastic dominance implies that $E(Y|T=1,G=00) \leq E(Y|T=1,G=10) = \frac{P_{1|1}\cdot\bar{Y}_{1,1}-P_{1|0}\cdot\bar{Y}_{0,1}}{P_{1|1}-P_{1|0}}$. Thus, Δ_{00}^{UB} is the sharp upper bound of Δ_{00}.

A.4.3 Proof of the sharpness of the bounds for the treated

These bounds are linear combinations of Δ_{10} and the bounds on Δ_{11}. Therefore, they are sharp.

A.4.4 Proof of the sharpness of the bounds for the entire population

These bounds are linear combinations of Δ_{10} and the bounds on Δ_{11} and Δ_{00}. Therefore, they are sharp.

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