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Testing for Common Values
in Canadian Treasury Bill Auctions

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Abstract

We develop a test for common values in auctions in which some bidders possess information about rivals' bids. Information about a rival's bid causes a bidder to bid differently when she has a private value than when her value depends on rivals' information. In a divisible good setting, such as treasury bill auctions, bidders with private values who obtain information about rivals' bids use this information only to update their prior about the distribution of residual supply. In the model with a common value component, they also update their prior about the value of the good being auctioned. We use these differential updating effects to construct our test. The proposed test displays good performance in Monte Carlo studies. We then apply it to data from Canadian treasury bill market, where some bidders have to route their bids through dealers who also submit bids on their own. We cannot reject the null hypothesis of private values in our data for 3-months treasury bills, but we reject private values for 12-months treasury bills. Furthermore, we use the structural model to estimate the value of customer order flow to a dealer. We find that the extra information contained in customers' bids leads on average to an increase in payoff equal to about 0.5 of a basis point, or 32% of the expected surplus of dealers from participating in these auctions.

Keywords: multiunit auctions, treasury auctions, structural estimation, nonparametric identification and estimation, test for common value

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1 Introduction

Is the private or common valuation component more important in treasury bill auctions? Can we use data to provide an answer? These are the two questions that we attempt to address in this paper. Our empirical strategy is based on observations of bidders' responses to information revealed about their rivals: a bidder should augment her bidding strategy in a different way when her valuation is private and when the valuation has a common component.

The question of finding a way to distinguish between the common and private valuation paradigms is not new to economics literature. Milgrom and Weber's (1982) theory of equilibrium bidding in different auction environments motivated empirical researchers to develop formal techniques to help them decide which valuation paradigm fit their application better. For single object auctions, Gilley and Karels (1981) were first to propose a reduced form testing approach based on examining how bids vary with the number of participants. For second-price sealed-bid and English auctions, Paarsch (1991) and Bajari and Hortaçsu (2003) employed tests for CV using standard regression techniques: under the maintained null hypothesis of independent private values and (weakly dominant strategy) truthful bidding, bids should not respond to information about the number of participants. Pinkse and Tan (2005) establish, however, that such a reduced form test cannot distinguish unambiguously a CV from PV model in first price auctions. Paarsch's (1992) seminal paper showed that a more detailed structural model can achieve the goal of distinguishing CV and PV in first-price auctions. Paarsch's method, however, relies on parametric assumptions about the distribution of bidder's private information, and hence it is hard to disentangle the influence of the parametric assumptions on the actual outcomes of the testing procedure. Our approach, instead, will be nonparametric, building on the closely related paper by Haile, Hong and Shum (2003) (henceforth HHS) . HHS propose a nonparametric test for common value in first-price auctions making use of variation in the number of bidders across auctions. They use nonparametric techniques developed in recent empirical auctions literature (e.g., Laffont and Vuong (1995), and Guerre, Perrigne and Vuong (2002)) to estimate the distribution of valuations given the observed bids, under the null hypothesis of private values. In particular, their theory predicts a certain ordering between the distribution of bids under common valuation paradigm as the number of

bidders varies, while the expected value of the object conditional on winning should not vary with the number of participants under PV.

The methodological contribution of this paper is to extend the structural testing approach from single-unit auctions to the setting of multi-unit auctions, with which government debt is auctioned around the world, and to exploit a different source of variation in our data to base our test on. Specifically, our data set from Canadian treasury bill auctions allows us to observe the modifications that a subset of bidders (*dealers*), make to their submitted bids upon observing the bids of some of their competitors (*customers*). Thus, we are able to observe how bids change *within* an auction in response to new information about competition. In contrast, the testing strategy utilized in the literature so far focuses on *across-auction* responses to changes in the number of competing bidders. Our *within-auction* testing strategy makes our results less susceptible to the presence of unobserved heterogeneity across auctions.

As an example for our testing approach, consider a situation in which bidder i is about to submit her bid (demand) function y_i, but before submitting y_i she observes a bid actually submitted by bidder j. With private valuations bidder i obtains better information about the location and shape of residual supply she will be facing in the upcoming auction. Using this additional information, she revises her initial bid y_i and submits an alternative bid y_i'. In an auction with interdependent values or a common value component, on top of the additional information about the location and shape of the residual supply curve, she also obtains new important information about the underlying valuation. Therefore she submits a new bid y_i'' taking into account both of these two pieces of new information. In general, the way she will revise her bid y_i will differ under the two scenarios and this distinction motivates our test.

This paper is also the first attempt to conduct an econometric test of a common value component in government securities auctions. Analysts of these auctions have expressed widely differing beliefs as to whether the bidding environment is best described as a private value as opposed to a common value environment. Whereas the earliest theoretical analyses, by Vickrey (1961), Vernon Smith (1966) have pursued the private value model, later modelling attempts by Wilson (1979), Kyle

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(1985), Bikhchandani and Huang (1989) emphasized the common value component, citing the existence of a liquid resale market. The structural econometric approach to government securities auctions is also similarly divided between common and private value paradigms: while Fevrier, Preguet and Visser (2002) and Armantier and Sbai (2006) base their econometric models on a pure common value specification, other studies, including Hortaçsu (2002), Kastl (2006a), Kang and Puller (2006), utilize independent private value models.[1] The latter set of papers argue that for short term debt the private valuation component is probably more important, because most investors hold these papers in their portfolios until maturity so that there is almost no resale. However, there is still some controversy in modelling auctions of government debt using private valuation models. In particular, for example due to different expectations of some global risk, say of interest rate fluctuations, there might still be an important common valuation component involved. It therefore remains a matter of taste as to which model to apply.

Our paper also provides a structural econometric method to analyze the value of observing "order flow." In particular, we use our estimated model to quantify the extra rents that dealers enjoy from observing the information contained in customers' orders. Consistently with views of the practioners we find that the customers' order flow contributes significantly to dealers' overall profits from participating in primary auctions.

The remainder of the paper is organized as follows. In Section 2 we lay out the model of a discriminatory auction of a perfectly divisible unit good and characterize the necessary conditions for equilibrium bidding under private values and restricted strategy sets. We use these necessary conditions to conduct structural estimation of bidders' marginal valuations under the null hypothesis. We describe the actual test for common values in Section 3. To evaluate the performance of the proposed test, we conduct a Monte Carlo simulation in Section 4. In Section 5 we describe two data sets of Canadian treasury bill auctions (3-months and 12 months maturities) and present the results of our estimation in Sections 6 and 7. Finally, Section 8 concludes.

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[1]A related set of papers model bidding in electricity auctions (e.g., Wolak (2003, 2005), Hortaçsu and Puller (2007)). These researchers share the view that the private value framework seems like an appropriate setting for these auctions. These papers are relevant to our work as they provide important contribution to current estimation methods of private value divisible good auctions.

2   The Model and Test Description

The basic model underlying our analysis is based on the share auction model of Wilson (1979) with private information, in which both quantity and price are assumed to be continuous. There are N bidders, who are bidding for a share of a perfectly divisible good. Each bidder receives a private (possibly multidimensional) signal, s_i, which is the only private information about the underlying value of the auctioned goods. The joint distribution of the signals will be denoted by $F(\mathbf{s})$.

Assumption 1 *Bidder i's signal s_i is drawn from a common support $[0,1]^M$ according to an atomless marginal d.f. $F_i(s_i)$ with strictly positive density $f_i(s_i)$.*

Winning q units of the security is valued according to a marginal valuation function $v_i(q, s_i, s_{-i})$. In the special case of independent private values (IPV), the s_i's are distributed independently across bidders, and bidders' valuations do not depend on private information of other bidders, i.e., the valuation has the form $v_i(q, s_i)$. At the estimation stage we will not impose full symmetry, since we will allow for different groups, within which the signal is distributed identically across bidders. We will impose the following assumptions on the marginal valuation function $v(\cdot, \cdot, \cdot)$:

Assumption 2 *$v_i(q, s_i, s_{-i})$ is measurable and bounded, strictly increasing in (each component of) s_i $\forall (q, s_{-i})$ and weakly decreasing in q $\forall (s_i, s_{-i})$.*

We will denote by $V_i(q, s_i, s_{-i})$ the gross utility: $V_i(q, s_i, s_{-i}) = \int_0^q v_i(u, s_i, s_{-i}) \, du$. Throughout the paper we will distinguish between private values and other valuation structures, where bidders' valuations could be interdependent (for example could have a common value component). The following definition states what we understand under these terms using our notation.

Definition 1 *(i) Bidders have private values when $\forall i : v_i(q, s_i, s_{-i}) = v_i(q, s_i,)$.*
(ii) Bidders have interdependent values if $\forall i, j$ and $a.e.s_i$ $\exists S'_j, S''_j : S'_j \cap S''_j = \emptyset$ such that $\Pr\left(S'_j\right) > 0, \Pr\left(S''_j\right) > 0$ and $\mathbb{E}_{s_{-i}}\left(v_i(q, s_i, s_{-i}) | s_j \in S'_j, s_i\right) \neq \mathbb{E}_{s_{-i}}\left(v_i(q, s_i, s_{-i}) | s_j \in S''_j, s_i\right)$.

Our definition of interdependent values simply states that each bidder possesses with positive probability some private information that is relevant for valuation of each of his rivals. In particular, in the context of our empirical application it implies that at least some information that banks (customers) possess is useful for primary dealers' estimates of the valuation of the underlying security.

Bidders' pure strategies are mappings from private signals to bid functions: $\sigma_i : S_i \rightarrow \mathcal{Y}$, where the set $\mathcal{Y}$ includes all possible functions $y : \mathbb{R}^+ \rightarrow [0, 1]$. A bid function for type s_i can thus be summarized by a function, $y_i(\cdot|s_i)$, which specifies for each price p, how big a share $y_i(p|s_i)$ of the securities offered in the auction (type s_i of) bidder i demands. Q will denote the amount of T-bills for sale, i.e., the good to be divided between the bidders. Q might itself be a random variable if it is not announced by the auctioneer ex ante. In the auctions we study, the Government of Canada has the right to cancel the auction or restrict the announced supply. We assume that the distribution of Q is common knowledge among the bidders.

Furthermore, the number of potential bidders participating in an auction, denoted by N, is also commonly known. This assumption is reasonable in the context of our empirical application as all participants have to register with the auctioneer before the auction and the list of registered participants is publicly available. The natural solution concept to apply in this setting is Bayesian Nash Equilibrium. The expected utility of type s_i of bidder i who employs a strategy $y_i(\cdot|s_i)$ in a discriminatory auction given that other bidders are using $\{y_j(\cdot|\cdot)\}_{j \neq i}$ can be written as:

$$EU_i(s_i) = E_{Q, s_{-i}|s_i} \left[\begin{array}{c} \int_0^{q_i^c(Q, \mathbf{s}, \mathbf{y}(\cdot|s))} v_i(u, s_i) \, du \\ - \sum_{k=1}^K \mathbf{1}(q_i^c(Q, \mathbf{s}, \mathbf{y}(\cdot|s)) > q_k)(q_k - q_{k-1})b_k \\ - \sum_{k=1}^K \mathbf{1}(q_k \geq q_i^c(Q, \mathbf{s}, \mathbf{y}(\cdot|s)) > q_{k-1})(q_i^c(Q, \mathbf{s}, \mathbf{y}(\cdot|s)) - q_{k-1})b_k \end{array} \right] \quad (1)$$

where $q_i^c(Q, \mathbf{s}, \mathbf{y}(\cdot|s))$ is the (market clearing) quantity bidder i obtains if the state (bidders' private information and the supply quantity) is $(\mathbf{s}, Q)$ and bidders bid according to strategies specified in the vector $\mathbf{y}(\cdot|s) = [y_1(\cdot|s_1), ..., y_N(\cdot|s_N)]$, and similarly $p^c(Q, \mathbf{s}, \mathbf{y}(\cdot|s))$ is the market clearing price associated with state $(\mathbf{s}, Q)$. The first term in (1) is the gross utility the type s_i enjoys from his allocation, the second term is the total payment for all units allocated at steps at which the

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type s_i was not rationed and the final term is the payment for units allocated during rationing. A Bayesian Nash Equilibrium in this setting is thus a collection of functions such that almost every type s_i of bidder i is choosing his bid function so as to maximize his expected utility: $y_i(\cdot|s_i) \in \arg\max EU_i(s_i)$ for a.e. s_i and all bidders i.

2.1   Equilibrium strategy of a bidder in a private value auction

In this subsection we describe equilibrium behavior of a bidder in a private value setting. The discriminatory auction version of Wilson's model with private values has been previously studied in Hortaçsu (2002a). Kastl (2006b) extends this model to empirically relevant setting, in which bidders are restricted to use step functions with limited number of steps as their bidding strategies. He proves that an equilibrium in this game exists and provides its characterization via a set of necessary conditions:

Proposition 1 *Suppose values are private, rationing is pro-rata on-the-margin, and bidders can use at most K steps. Then in any Bayesian Nash Equilibrium of a Discriminatory Auction, for almost all s_i, with a bidder of type s_i submitting $\hat{K}(s_i) \leq K$ steps, every step k in the equilibrium bid function $y_i(\cdot|s_i)$ has to satisfy:*
(i) $\forall k \leq \hat{K}(s_i)$ such that $v(q, s_i)$ is continuous in a neighborhood of q_k for a.e. s_i:

$$v(q_k, s_i) = b_k + \frac{\Pr(b_{k+1} \geq p^c|s_i)}{\Pr(b_k > p^c > b_{k+1}|s_i)}(b_k - b_{k+1}) \tag{2}$$

(ii) if $v(q, s_i)$ is a step function in q such that $v(q, s_i) = v_k$ $\forall q \in (q_{k-1}, q_k]$ for a.e. s_i, then

$$v_k = b_k + \frac{\Pr(b_k > p^c|s_i)}{\frac{\partial \Pr(b_k > p^c|s_i)}{\partial b_k}} \tag{3}$$

Notice that if signals were independent, the probabilities in (2) and (3) would not be conditional on s_i, but would still be of course a function of the submitted bid curve.

2.2 Estimation of marginal valuations

Using the necessary conditions (2) and (3), and assuming either continuity of marginal valuation function in q or assuming $v(\cdot, s_i)$ is a step function, we can obtain point estimates of marginal valuations at submitted quantity-steps nonparametrically as described in Hortaçsu (2002) and Kastl (2006a). The resampling method that we employ in these papers is based on simulating different possible states of the world (realizations of the vector of private information) using the data available to the econometrician and thus obtaining an estimator of the distribution of the market clearing prices. Specifically, in the case where *all N bidders are ex-ante symmetric* and the data is generated by a symmetric Bayesian Nash equilibrium, the resampling method operates as follows:

Fix a bidder. From the observed data, draw (with replacement) $N-1$ actual bid functions submitted by bidders. This simulates one possible state of the world from the perspective of the fixed bidder, a possible vector of private information, and thus results in one potential realization of the residual supply. Intersecting this residual supply with the fixed bidder's bid we obtain a market clearing price. Repeating this procedure large number of times we obtain an estimate of the full distribution of the market clearing price conditional on the fixed bid. Using this estimated distribution of market clearing price, we can obtain our estimates of valuation at each step submitted by the bidder whose bid we fixed using (2) or (3) depending on the assumption on the marginal valuation function we are willing to impose.

We now discuss how we can adapt this method into the present context where some bidders (the "dealers") observe the bids of others ("customers"). Suppose there are two classes of bidders: N_d potential *dealers* (in index set $\mathcal{D}$) and N_c potential *customers* (in index set $\mathcal{C}$). Customers observe private signals, s, which are iid across customers, with marginal distribution $F^C(s)$. Each dealer also observes a private signal, s. Dealers also observe an additional piece of "orderflow" information, z. z may equal the customer bid observed by that dealer, or is null when the dealer does not observe any additional information. We assume that z is not observed by the dealer's competitors, but is observed by the econometrician. Let $F^D(s_1, ..., s_{N_d}, z_1, .., z_{N_d})$ be the joint distribution of dealer signals and orderflow information. We assume that (s_i, z_i) are iid across dealers $i \in \mathcal{D}$.

Given the symmetry assumptions, we will assume that the bidding data is generated by a Bayesian Nash equilibrium of the game in which customers submit bid functions that are symmetric up to their private signals, i.e. $y_i(p, s_i) = y^C(p, s_i)$, $i \in \mathcal{C}$. Dealers' bid functions are also symmetric, but up to their private signal *and* order flow information, i.e. $y_i(p, s_i, z_i) = y^D(p, s_i, z_i)$, $i \in \mathcal{D}$.

With this equilibrium assumption, the distribution of the market clearing price, as observed by a dealer, i, for whom $z_i = \emptyset$ is given by:

$$\Pr(b_{k+1} \geq p | s_i, z_i) = E_{\left\{s_{j \in \mathcal{C} \cup \mathcal{D} \setminus i}, z_{k \in \mathcal{D} \setminus i}\right\}} I \left(Q - \sum_{j \in \mathcal{C}} y^C(p, s_j) - \sum_{k \in \mathcal{D} \setminus i} y^D(p, s_k, z_k) \geq y^D(p, s_i, \emptyset) \right) \quad (4)$$

where $E_{\{.\}}$ is the expectation over the relevant random variables, and $I(.)$ is the indicator function.

If dealer i instead observes customer m's bid function, i.e. $z_i = \{y^C(p, s_m), \forall p\}$,

$$\Pr(b_{k+1} \geq p | s_i, z_i) = E_{\left\{s_{j \in \mathcal{C} \setminus m \cup \mathcal{D} \setminus i}, z_{k \in \mathcal{D} \setminus i}\right\}} I \left(Q - \sum_{j \in \mathcal{C} \setminus m} y^C(p, s_j) - \sum_{k \in \mathcal{D} \setminus i} y^D(p, s_k, z_k) \geq y^D(p, t_i, z_i) + y^C(p, s_m) \right)$$
$$(5)$$

The resampling algorithm used by Hortaçsu (2002) and Kastl (2006a) should thus be modified in the following manner: to estimate the probability in (4), we draw N_c customer bids from the set of observed customer bids (to which we have added zero bids for non-participating customers). Conditional on the customer bid drawn, draw a dealer's bid as follows: (i) If a zero customer bid is drawn, draw from the pool of dealers' bids, which have been submitted without observing any bid by the customers, or (ii) If a non-zero customer bid is drawn, draw from the pool of dealers' bids, which have been submitted having observed the *same* customer bid. After drawing N_c customer bids, continue drawing from the pool of bids submitted by uninformed dealers until N_d dealer bids are drawn. Obtain the market clearing price and repeat.

Performing such a conditional drawing procedure, unfortunately, has the limitation that customer bids are typically unique within or across auctions. Thus, the "conditional" draws often consist of repeatedly drawing the same customer and "informed" dealer pair. Of course, asymptotically, we expect the number of dealer bids corresponding to a given customer bid to increase;

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however, in small samples, this is rarely true. We may thus consider a "smoothing" strategy similar to nonparametric regression techniques: instead of drawing informed dealer bids that exactly correspond to a given customer bid, draw bids from dealers who saw customer bids that are "close" to the given customer bid.

Indeed, an extreme case of this "smoothing" strategy, which serves as a robustness check to the "unsmoothed" algorithm we provide above, is to perform an *unconditional* simulation: when drawing from dealer bids, we first flip a coin to determine whether this dealer has hypothetically seen a customer's bid or not, where the coin is biased such that it reflects the actual probability of a dealer observing a customer's bid. If the coin determines that a customer bid has been seen, then we draw (uniformly) from the set of updated informed dealer bids, otherwise we draw from the set of "uninformed" dealer bids. This is performed for each of N_d potential dealer draws, i.e., independently of the customer bids actually drawn in a given simulation round.

To estimate the probability in (5), we modify the above procedure slightly: fixing a dealer, who has seen a customer bid, we draw $N_c - 1$, not N_c, customer bids, and take the observed customer bid along with the dealer's own bid as given when calculating the market clearing price.

Applying the conditional resampling procedure for a dealer that we observe both before and after receiving a customer bid therefore results in two sets of marginal valuation estimates – and our test will be based on comparing the two sets of marginal valuation estimates. One caveat involved in constructing this test is that the bids before and after the information about rival's bid arrives are not necessarily submitted for the same quantities(even though for a subset of bidders in our sample updated bids are submitted at the same quantities), and hence we will face an inference problem of how to compare the two sets of estimates. We will discuss these issues and the solutions in Section 3 of the paper which deals with the test specification.

Note that the necessary conditions described in Proposition 1 apply only to dealers, whose bids are not revealed to anyone. We base our test exclusively on the behavior of dealers. Of course, customers may have a (potentially complex) strategic response to the fact their bids are observed by dealers. This may generate an alternative testing strategy: the equilibrium effect of how a bidder adjusts her bid when she knows that her bid will be observed by her rival is likely to differ when

values are private and when values are interdependent. We do not pursue this approach, however, and only utilize the (empirical) distribution of customer bids to characterize dealers' best-responses.

2.3 Asymptotic Distribution of the Estimates

As suggested above our test will be based on comparing two sets of estimates. Therefore, we have to be able to account for the sampling error when constructing our test statistic and deriving its asymptotic distribution. Let us first look at the asymptotic behavior of the estimates of marginal valuation. It is easy to see from equation (2) that these estimates are a non-linear function of the distribution of the market clearing price, which is estimated by the resampling method described above. Let us rewrite (2) as

$$v\left(q_k, s_i\right) = b_k + \frac{H\left(b_{k+1}\right)}{G\left(b_k\right) - H\left(b_{k+1}\right)}\left(b_k - b_{k+1}\right)$$

where $H\left(X\right)$ (resp. $G\left(X\right)$) is the probability that market clearing price is weakly (resp. strictly) lower than X.

To establish the asymptotic distribution of the resampling estimator $\hat{H}^R$, let us first focus on the case where we have data from T auctions with N symmetric bidders, where the observed vector of bid functions $\{y_{it}(p, s_{it}), i = 1, .., N, t = 1, ..T\}$ are iid draws from the same distribution.

Proposition 2 *Let $\hat{H}^R\left(X\right)$ denote the resampling estimator. N number of bidders in an auction, T the number of auctions. Suppose data from the T auctions are iid as above, and let $\Phi\left(y_1, ..., y_{N-1}; X\right) = I\left(Q - \sum_{j=1}^{N-1} y_j\left(X|s_j\right) \geq y_i\left(X|s_i\right)\right)$, then*

$$\sqrt{T}\left(\hat{H}^R\left(X\right) - H\left(X\right)\right) \to N\left(0, \frac{\left(N-1\right)^2}{N}\zeta\right) \tag{6}$$

where $\zeta = E_{s_{-i}}\left[\left(\Phi\left(y_1, ..., y_{N-1}; X\right)\right)^2\right] - \left(\binom{NT}{N-1}^{-1}\sum_{(1,1)\leq\alpha_1<\alpha_2<...<\alpha_{N-1}\leq(T,N-1)} \Phi\left(y_{\alpha_1}, ..., y_{\alpha_{N-1}}, X\right)\right)^2$ and where that last summation is taken over all combinations of $N-1$ indices $\alpha_i \in \{(1,1), (1,2), ..., (1, N-1), ..., (T, N-1)\}$ such that $\alpha_1 < \alpha_2 < ... < \alpha_{N-1}$.[2]

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[2]The asymptotic distribution of the resampling estimator $\hat{G}^R$ can be established analogously, by replacing the weak inequality in the definition of $\Phi(\cdot)$ by a strict one.

Proof. Consider the following statistic based on all subsamples of size $(N-1)$ from the full sample of NT datapoints:

$$\theta\left(\hat{F};c\right) = \binom{NT}{N-1}^{-1} \sum_{1 \leq \alpha_1 < \alpha_2 < ... < \alpha_{N-1} \leq NT} \Phi\left(y_{\alpha_1}, ..., y_{\alpha_{N-1}}, c\right)$$

where $\hat{F}$ is the empirical distribution of bid functions. θ is a *U-statistic* and the result thus follows from applying Theorem 7.1 of Hoeffding (1948) which provides a useful version of a central limit theorem for this class. A sufficient condition for asymptotic normality is the existence of the second moment of the kernel of the functional θ, which in our case is equivalent to finiteness of $E\left[\Phi\left(\cdot\right)^2\right]$, which is satisfied since $\Phi\left(\cdot\right)$ is an indicator function. The resampling estimator, as it is implemented by Hortaçsu (2002) and Kastl (2006a), is a slightly modified version of θ, and is given by the *V-statistic*:

$$\tilde{\theta}\left(\hat{F};c\right) = \frac{1}{(NT)^{N-1}} \sum_{\alpha_1=1}^{NT} ... \sum_{\alpha_{N-1}=1}^{NT} \Phi\left(y_{\alpha_1}, ..., y_{\alpha_{N-1}}, c\right)$$

where the averaging is over every permutation of the NT observations (since this is not computationally feasible, we approximate this average by Monte Carlo sampling). Lehmann (1999, Theorem 6.2.2, p.388) shows that the asymptotic distribution of this V-statistic is identical to that of the U-statistic. ∎

The resampling estimator that we described for the probabilities in equations (4) and (5) are slightly different in that the bid functions y_{it} are drawn from asymmetric distributions (but $\Phi(\cdot)$ is still a symmetric function of its elements). Hoeffding (1948), Theorem 8.1, extends the asymptotic normality result to the case where all y_{it} are allowed to have different distributions. This extension requires a slightly stronger condition on the third moment of $\Phi(\cdot)$ to use the Liapunoff Central Limit Theorem, but this condition is still satisfied since $\Phi(\cdot)$ is an indicator function which is uniformly bounded.

Using the asymptotic variance of the distribution of the market clearing prices, $H(X)$ (and $G(X)$), we can use the delta-method to derive the asymptotic variance of the estimates of the marginal valuations, i.e., $Var\left(\hat{v}_k\right) = J_v' \Sigma J_v$, where J_v is the matrix of partial derivatives with

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respect to $H(b_{k+1})$, $H(b_k)$ and $G(b_{k+1})$ and Σ is the asymptotic variance/covariance matrix for those estimates. How do we obtain the asymptotic covariance matrix of $\{H(c_1), H(c_2), G(c_3)\}$ at particular three values of c? An advantage of Hoeffding's theorem is that it applies also to vector-valued random variables and the off-diagonal elements of the asymptotic variance/covariance matrix are the asymptotic covariances between two corresponding U-statistics.

Instead of using the asymptotic normal approximation, we can also use bootstrap confidence intervals, which are readily generated by iteraring the resampling scheme used for point estimates on bootstrap samples of the bid data. The following proposition establishes the validity of bootstrap in our setting.

Proposition 3 *Let $\hat{F}$ denote the empirical distribution of the bid functions and let F^b denote its bootstrap approximation. Then*

$$T^{\frac{1}{2}}\left(\tilde{\theta}\left(F^b; c\right) - \tilde{\theta}\left(\hat{F}; c\right)\right) \rightarrow N\left(0, \frac{(N-1)^2}{N}\zeta\right)$$

where ζ is as defined in Proposition 2.

Proof. The result follows from Theorem 3.1 of Bickel and Freedman (1981) using the fact that the variance and any covariances of our kernel $\Phi(y_1, ..., y_{N-1})$ in the U-statistic $\theta(F; c)$ are bounded.
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2.4 Equilibrium strategy of a bidder in an auction with interdependent or common values

If the valuation of a bidder has a common value component, then we will not be able to replicate the updating process of this bidder as new information becomes available to him. While the updating part due to better information about the location and shape of the residual demand is still the same as in the private value setting, there is a second updating component due to the additional information about the signal of a rival and hence about the underlying value. In particular, the necessary condition for optimality at k^{th} step in an interdependent value environment

is (if $v\left(q, s_i, s_{-i}\right)$ is continuous in a neighborhood of q_k for a.e. $\left(s_i, s_{-i}\right)$):

$$\Pr\left(b_k > p^c > b_{k+1}\right)\left[E\left[v\left(q_k, s_i, s_{-i}\right) | \{b_k, q_k\}_{k=1}^K\right] - b_k\right] =$$
$$= \Pr\left(b_{k+1} \geq p^c\right)\left(b_k - b_{k+1}\right) + \gamma\left(\{b_k, q_k\}_{k=1}^K, s_i\right)$$

In other words, we have the familiar trade-off in a discriminatory auction that occurs even with private values: marginally shading the quantity demanded at k^{th} step results on the one hand in loss of expected surplus of $\left[E\left[v\left(q_k, s_i, s_{-i}\right) | \{b_k, q_k\}_{k=1}^K\right] - b_k\right]$ in the states that exactly that quantity would be won in. On the other hand it results in saving of $b_k - b_{k+1}$ whenever the market clearing price is lower than the bid at the next quantity step. But now, because of the interdependent values, there is an additional effect summarized by the function $\gamma(\cdot)$: marginally shading the quantity at k^{th} step can lead to a different slope of expected market clearing price in the region where k^{th} quantity demand effects the market clearing price or allocation and thus it can effect the way inference is drawn from the market clearing price realization on the unknown valuation (through updated information about rival's signals).

Since we do not know enough about $E\left[v\left(q_k, s_i, s_{-i}\right) | \{b_k, q_k\}_{k=1}^K\right]$, we cannot identify $v\left(q_k, s_i, s_{-i}\right)$ non-parametrically without imposing more structure as without knowing more about $v\left(q_k, s_i, s_{-i}\right)$, we cannot identify the additional updating effect captured in $\gamma(\cdot)$.

Thus, we cannot unambiguously sign the bias in the estimate of the marginal valuation if we were to use the identification equation (1) from the private value model and the true model were interdependent values. This is in contrast to Haile, Hong and Shum, who can establish a relationship (under the alternative hypothesis) between the estimated distributions of private valuations in terms of stochastic dominance as the number of bidders increases. Therefore our testing strategy will focus on the null hypothesis of private value setting.

3   Test Specification

The fact that in the treasury bill auction of the Canadian government a subset of bidder (*dealers*) submit bids on behalf of other bidders (*customers*) and that these bids are visible to the econo-

metrician provides a unique environment for testing for the presence of a common component in bidders' valuations which are not observed by the econometrician. In particular, dealers sometimes submit their own bids, but after fulfilling a request of one or more of their customers to submit a bid on their behalf, they decide to adjust their previously submitted bid. Since we observe the bid both before and after the additional information was made available to the dealer, the private value assumption becomes testable. In a pure private valuation setting any such bid adjustment should be driven solely by more information about the residual supply that this bidder will be facing in the actual auction. In a setting with a common valuation component, the adjustment reflects both more information about the residual supply *and* more information about the common valuation component, and hence these two adjustment results should be different. As mentioned earlier, in Hortaçsu (2002) and Kastl (2006a) we proposed nonparametric methods based on simulating rivals' strategies for estimation of marginal valuations in private value divisible good auctions. We will utilize these methods to estimate the marginal valuation schedules implied by the initial bid, and by the updated bid, taking into account the new information about the residual supply. In other words, as described earlier we are able to mimic the bid updating process under private values hypothesis, but we are not be able to mimic it under common values. Therefore, under the null hypothesis of private valuation setting the estimates before and after the additional information should coincide. Should we find that the two marginal valuation schedules are significantly different, we will reject the null and conclude that the common valuation component plays an important role in these auctions.

We now discuss several practical challenges that need to be addressed in formulating our testing strategy.

3.1 Accounting for the Discreteness of Bid Functions

One important practical challenge arises from the fact that bids in multiunit auctions are submitted as discrete price-quantity pairs. Unfortunately, we can only obtain point-identification for marginal valuations at the discrete price-quantity points, and since bidders may change the discrete bid steps they submit after they receive extra information, we face the challenge of testing the equality of

non-point identified parameters. In the subsequent section we discuss two alternative tests that can be performed without requiring non-parametric identification of the whole level-curve. The first is based on testing for monotonicity of the estimated marginal valuation function and the second is based on comparing the two sets of estimates of marginal valuations, $k = BI, AI$, in situation where a bid has been submitted for the same quantity. For completeness, in appendix A we also include a test which would be applicable if the researcher believed that the level curves of the marginal valuation function can be identified non-parametrically.

3.1.1 Nonparametric Test for Monotonicity

Possibly the most natural way to think about the asymptotics is to consider the number of steps and thus the number of quantities at which the marginal value can be estimated as fixed, and let just the number of auctions increase, which is necessary for these estimates to be consistent. Then we could test for monotonicity of the estimated marginal values at quantities submitted before and after the additional information as follows: Fix bidder i and pool together the vectors of quantity demands submitted by this bidder i before and after the information arrival (i.e., steps of her bid functions) and the associated estimated marginal valuations, $\left(\{q_{ik}^{BI}, \hat{v}_{ik}^{BI}\}_{k=1}^{K^{BI}}, \{q_{ik}^{AI}, \hat{v}_{ik}^{AI}\}_{k=1}^{K^{AI}} \right)$ and order the resulting pooled vector by quantity requested at each step: $q_{i1} < q_{i2} < ... < q_{i(K^{BI}+K^{AI})}$ and let $\hat{v}_{i1}, ..., \hat{v}_{i(K^{BI}+K^{AI})}$ denote the associated estimated marginal values, which are thus ordered by quantity. Consider the following test statistic:

$$S_i = \max_j \{\hat{v}_{ij+1} - \hat{v}_{ij}, 0\} \tag{7}$$

Clearly, when monotonicity is satisfied at all quantities, then $\hat{v}_{ij} \geq \hat{v}_{ij+1}$ and hence $S_i = 0$. On the other hand we could get violations of monotonicity due to the sampling error in a finite sample and hence $S_i > 0$ could be consistent with the null hypothesis. The major advantage of this approach is that it does not restrict the class of possible marginal valuation functions in any other way than that it be non-increasing in quantity.

We obtain the critical value for this test statistic using bootstrap.[3] Using B bootstrap draws,

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[3]Notice that the test statistic S_i is simply the maximum violation of monotonicity for a given set of estimates for

the critical values are computed as follows:

$$\tilde{c}_{1-\alpha} = \inf\left\{ x : \frac{1}{B}\sum_{b=1}^{B} 1\left\{\tilde{S}_b \leq x\right\} \geq 1 - \alpha\right\} \tag{8}$$

For each bootstrap draw of the test statistic, the marginal valuation is re-estimated by the resampling method described earlier, where a new sample of bid functions from which this resampling is performed is drawn. To construct a bootstrap sample of bid functions, we have to follow a procedure similar to the conditional resampling. In constructing these bootstrap samples we need to include also the 'zero' bids for those potential bidders that do not end up actually submitting a bid. We start by drawing N_c customer bids with replacement giving $\frac{1}{TN_c}$ probability to each (where $T \geq 1$ is the number of auctions which we pooled together for resampling). Conditional on having drawn a non-zero customer's bid, we draw from the observed sample N_d^c dealer bids submitted following the same customer's bid with replacement giving $\frac{1}{N_d^c}$ probability to each such dealer bid. Conditional on drawing a zero customer bid, we draw from dealers' bids submitted without knowledge of any customer's bid putting equal probability on each.

Before adopting this testing approach, however, we should discuss an important shortcoming. Since our testing for monotonicity via the test statistic S falls into the framework of partial identification, there could be situations in which the true model in fact has a common value component and thus the level curves of (expected) marginal valuation are different before and after the information is revealed, but our monotonicity test fails to reject the null hypothesis[4]. In other words, the monotonicity test proposed above might not be consistent against all alternatives. However, its special case discussed below **is** consistent against all alternatives provided that there is some dependence of a bidder's payoff on rivals' information such as the one required in Definition 1. More specifically, asymptotic consistency requires that the common valuation component depends non-trivially on the information of those bidders whose bid a dealer gets to observe.

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bidder i, and not an estimate of a maximum hence bootstrap can be used to approximate its distribution.

[4] The test may fail to reject possibly even asymptotically as no more points from the level curves might be identified due to bidding strategies being step functions.

3.1.2 Equality Test

A special case of the above described monotonicity test can be performed if bids are submitted at the same quantities before and after the information becomes available. In that case, under private values the two estimates of marginal valuations should coincide asymptotically and thus could differ in a finite sample only due a sampling error. Consider the test statistic:

$$T_i(q) = \left| \hat{v}_i^{BI}(q, s_i) - \hat{v}_i^{AI}(q, s_i) \right| \tag{9}$$

where $\hat{v}_i^{BI}(q, s_i)$ is the estimated marginal valuation for share q before the information was revealed and similarly $\hat{v}_i^{AI}(q, s_i)$ is the estimated marginal valuation for share q after the additional information arrived. The following proposition reveals an appealing feature of this test.

Proposition 4 *(Asymptotic consistency)*
Under H_1 of interdependent values, $\Pr\left(T_i(q) > T_i^B(q)\right) \to 1$ as (the number of auctions) $T \to \infty$ where T^B is the critical value obtained by bootstrap.

This special case of our test is thus consistent against all alternatives, since if under any alternative (interdependent or common values) $E_{s_{-i}}\left[\hat{v}_i^{BI}(q, s_i, s_{-i}) \,|p, s_i\right] = E_{s_{-i}}\left[\hat{v}_i^{AI}(q, s_i, s_{-i}) \,|p, s_i\right]$ with probability 1, then the additional knowledge of s_j would not contribute any additional information about the value at q, which is not consistent with the basic assumption of the interdependent value model that the (expected) value depends on rivals' signals.

3.2 Joint hypothesis test

So far we discussed how we can test for common values based on a set of estimated values for a given bidder, whom we observe submitting a bid before and after information arrives. In our application, however, we observe multiple such bidders. Therefore, the proper way to test whether the null hypothesis of private values can be rejected is to test this null jointly for all bidders. The most direct way to test a joint hypothesis is to adjust the desired p-value to satisfy the Bonferroni inequality, i.e., instead of comparing each individual hypothesis to a p-value of 0.05, compare it to a p-value of $\frac{0.05}{\#\ of\ hypotheses}$. This method, while controlling the familywise error rate (type-I errors),

is usually regarded as being too conservative.[5] An alternative way is to construct a test statistic which take into account all of the tested hypotheses at once. For example, motivated by a χ^2 test, we construct the following "sum of squares" of the test statistics S_i and T_i:

$$SQ_S \;\; = \;\; \sum_i S_i^2 \tag{10}$$

$$SQ_T \;\; = \;\; \sum_i T_i^2 \tag{11}$$

We also construct studentized versions of these test statistics by scaling S_i and T_i by their (bootstrap) standard deviations.

Another test statistic that takes into account all of the hypotheses at once is the maximum (first-order statistic), among bidders i, of S_i and T_i:

$$FOS_S \;\; = \;\; \max_{i \in \mathcal{D}} S_i \tag{12}$$

$$FOS_T \;\; = \;\; \max_{i \in \mathcal{D}} T_i \tag{13}$$

We approximate the asymptotic distribution of these test statistics again by bootstrap.[6] For example, in the case of SQ, the null hypothesis of private values is rejected on level α, when $\hat{SQ} > SQ_{1-\alpha}$, where $\hat{SQ}$ is the sample value of the statistic, and $SQ_{1-\alpha}$ is the $(1-\alpha)^{th}$ quantile of the boostrap distribution of the test statistic.

3.3 Affiliated values

As in Haile, Hong and Shum (2003), our test is, in principle, able to distinguish between a private value model (with or without independent signals) and a model with a common valuation component. The estimation procedure would have to be adapted in the affiliate private values case, however, since with affiliated signals we would have to resample other bids conditional on signal bidder i observes (i.e., resample whole vectors of bids by other bidders submitted in auctions in which i submitted the same bid). While this would not cause problem for the asymptotic behavior,

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[5]See Romano, Shaikh and Wolf (2008) for more detailed discussion on multiple testing.

[6]Note that in FOS_S and FOS_T, the maximum is taken over the number of dealers, which is fixed as the data set gets larger. Thus bootstrap is not problematic.

in a finite sample this test would greatly reduce the number of states (vectors of private information) that can be simulated since almost no bidder would be observed submitting the exact same bid in multiple auctions. Therefore, in our empirical application we will specify our null hypothesis as independent private values and the alternative would thus encompass both a model with a common value component and a private value model with affiliated signals.

3.4 Unobservable heterogeneity across auctions

Another practical challenge in implementing the testing procedure is the presence of auction-level covariates that are observed by the bidders, but not by the econometrician. For example, the test for common values in a single unit setting proposed in Haile, Hong and Shum (2003) relies on the ability of the econometrician to observe repetitions of the same experiment over time, where the number of competing bidders varies exogenously across auctions. If there are auction characteristics that are unobserved by the econometrician, but observed by (potential) bidders, this may lead to biased test results when bidders' participation decisions are driven by such omitted factors.

Our testing strategy is based on looking at modification of bids by a given bidder, *within* the same auction. Thus, at least in principle, we do not have to rely on across auction information to construct our test statistic. However, our estimates of marginal valuations (under the null of independent private values) will be more precise if we can pool bid data across auctions. Pooling data across auctions, however, may lead to biases in our estimation of bid shading if auction-level unobservables are present. We will therefore experiment with different levels of data pooling.

A more subtle concern regarding omitted variable bias may arise in our application if the changes in dealer bids are caused by innovations to the bidders' information set that are not observed by the econometrician. The presence of such omitted pieces of information biases our estimates of bidders' optimal bid shading. We can test for the presence of such omitted information flows if the information is *public*, i.e. observed by multiple bidders. In our data, as we observe exact time of each bid submission, we can distinguish a change in bid due to more information coming from a customer from a change in bid due to some new public information. In the latter case, conditional on some small time window, all adjustments by dealers should be positively correlated, whereas in

the former case they should be independent. Therefore if we subject to our test only those changing bids that are not accompanied by similar changes in rival's bids, we can be more confident that no commonly observed (but unobserved by us) piece of information is biasing our test. We will be more explicit about addressing this issue in Section 5 when we discuss the institutional background.

4   Monte Carlo Study

Our ability to test the performance of the above described testing procedure in multiunit auctions is limited by the fact that in most general cases we do not have closed form solutions for equilibrium strategies, either in the private or in the affiliated values settings. We circumvent this problem by conducting a set of Monte Carlo exercises in a first price auction with independent private values, with interdependent values and pure common values, where we endow some bidders with the ability of observing a rival's bid. As mentioned earlier, the fact that some bidders' bids might be observed by their rivals is likely to have an equilibrium effect on the formation of these bids to begin with. Since we want to focus on the updating of bids associated with gaining information contained in a rival's bid, we instead consider Monte Carlo exercises where we shut down this equilibrium adjustment effect caused by the bid being revealed to a rival. In particular, in all examples, we generate the data from an equilibrium model of bidding with 3 bidders, where bidder 1 is a strategic player while bidders 2 and 3 are automatons playing as in a sealed bid first price auction with 3 bidders. We non-parametrically estimate the marginal values (of bidder 1) implied by the bids using Guerre, Perrigne and Vuong (2002) (henceforth GPV). In line with the spirit of the test we propose we assume that bidder 1 observes bidder 2's bid and submits an updated bid which supersedes her original bid. We generate the data from an equilibrium bidding function, which of course differs from the regular FPSB auction with 3 bidders. We again estimate the implied values of (informed) bidder 1 as if he were facing one less rival using GPV which assumes private values. We construct our test statistics described in the previous section and bootstrap the critical values.

4.1 First Price Auction with Informed Bidders

4.1.1 Independent private values (IPV)

The first exercise we consider is a first price auction with 3 bidders, valuations $v(s_i) = s_i$ and signals distributed uniformly on [0,1]. The unique equilibrium in strictly increasing differentiable strategies when all bidders are uninformed is $b^U(s_i) = \frac{2}{3}s_i$. Now consider the case that bidder 1 would be able to observe bidder 2's bid and bidders 2 and 3 are automatons that continue to bid as in a regular FPSB auction. In that case the optimal bid by the informed bidder would be:

$$b^I(s_1, s_2) = \begin{cases} \frac{s_1}{2} \, if \, \frac{s_1}{2} > \frac{2s_2}{3} \\ \frac{2s_2}{3} \, if \, s_1 > \frac{2s_2}{3} > \frac{s_1}{2} \end{cases}$$

Figure 1 compares the estimated valuations of a bidder before and after she is informed. The figure suggests that except at the boundaries of the support of the bid/valuation distribution, the valuation estimated with and without conditioning on observed information is likely to be very close. To correct the behavior at the boundaries, some adjustment would be necessary due to the bias in the kernel estimation, which we do by trimming $\frac{1}{9}$ of the estimated valuations on the bottom and $\frac{2}{9}$ of the estimated valuations on the top.[7]

Of course, this figure depicts what happens only in one randomly chosen data set on bids. We then implement a joint test of the null hypothesis of the private values using randomly drawn data sets. We begin with testing for the equality of the median of estimated valuation distributions before and after information is received. In particular, in the first column of Table 1, we calculate the difference in the median of the distributions of valuations, $Med(\hat{v}_{informed}) - Med(\hat{v}_{uninformed})$, in each Monte Carlo sample and construct the 2.5th and 97.5th bootstrap quantiles (using 400 resamples of the Monte Carlo data set) of these differences (re-centering the boostrap distribution by the test statistic computed on the original sample). The null hypothesis is rejected when the sample test statistic is not within this confidence interval. To make use of the fact that we observe two bids by the same bidder type, we also construct the "sum of squared dif-

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[7]As the figure suggests, the estimation bias arises mostly in the upper tail of the distribution and therefore we focus the trimming there. Trimming at the lower tail does not affect the results of our Monte Carlo experiments.

ferences" test, $SQ = \sum_i \left(\hat{v}_{i,informed} - \hat{v}_{i,uninformed}\right)^2$, and the "first order statistic" test, $FOS = \max_i \left(\hat{v}_{i,informed} - \hat{v}_{i,uninformed}\right)$, both of which we described earlier. The rejection probabilities are reported in the second and third columns of table 1. In order to understand how sampling error affects the rejection performance, we replicated the exercise for data sets of size 50, 100, and 200 – which are data sets of similar size to the empirical exercise. The (true) null of IPV is rejected less than 10% for all the cases for the median and SQ test statistics, with the SQ test statistic displaying particularly good performance. The FOS appears to overreject the null, and gives particularly bad results when the data is not trimmed.

Finally, to evaluate an joint hypothesis test , we also report the results of the pointwise test, in which we compare the absolute value of the difference between each $\hat{v}_{informed}$ and $\hat{v}_{uninformed}$ pair, to the difference at a given quantile of the (re-centered) bootstrap distribution of the difference between *that pair*. In particular, we report the percentage of points where we obtain rejection where each pointwise difference is compared to the 95th percentile, and to the quantile correponding to the Bonferroni's method, $100 - \frac{5}{\# \ of \ hypotheses}$.

The null rejection frequencies of this alternative testing procedure is displayed in the first two columns of Table 2. While the Bonferroni method is generally regarded as too conservative, in our setting it still would lead to a wrongful rejection of the null hypothesis, even though it (unsurprisingly) leads to a large reduction in rejection rates.

4.1.2 First price auction with interdependent values and independent signals (IIV)

In the second exercise we look at a first price auction with interdependent values and independent signals (IIV). The valuation function is $v\left(s_i, s_{-i}\right) = \frac{s_i}{2} + \frac{\sum_{j \neq i} s_j}{2(n-1)}$ where $s_i \sim U[0,1]$. The unique symmetric equilibrium in strictly increasing differentiable strategies involved bidding according to $b^U(s_i) = \frac{7}{12} s_i$. In the appendix we show that the equilibrium strategy of an informed bidder who observes a bid of his rival (and thus for practical purposes another signal S_2) is bidding according to:

$$b_1^I(s_1, s_2) = \begin{cases} \frac{7}{11}\left(\frac{s_1}{2} + \frac{s_2}{4}\right) & if \quad s_1 > \frac{4}{3}s_2 \\ \frac{7}{12}s_2 & if \quad \frac{4}{3}s_2 \geq s_1 \geq \frac{25}{48}s_2 \end{cases}$$
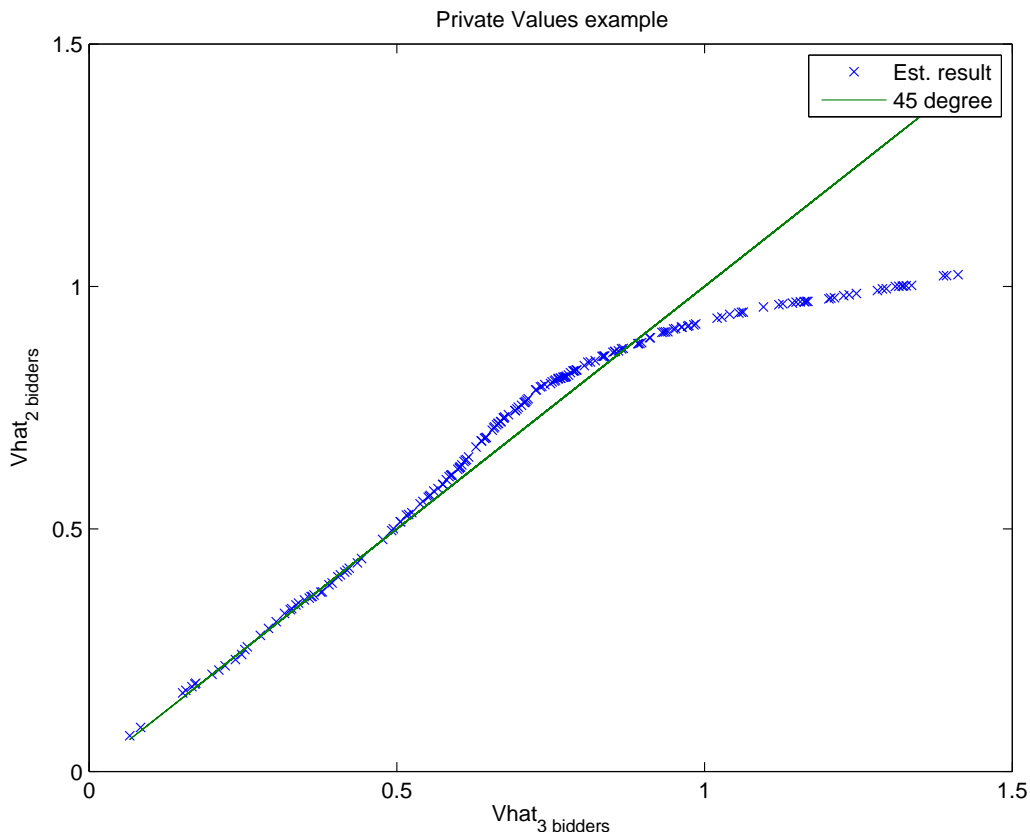
Figure 1: Estimated values for informed and uninformed bidders in a FPA with private values

Figure 2 depicts the results of estimating the implied values using GPV for a randomly selected data set. The null rejection frequencies of the testing procedures utilized in the IPV example are displayed in Tables 1 and 2. Observe that in this case, the FOS test appears to perform the best, in that for data sets of size exceeding 100, the test appears to work very well in that it rejects the null with close to 95% probability. The median and especially the SQ tests have lower power for smaller data sizes, though with $N = 200$, their performance increases dramatically. A similar pattern is observed in the pointwise tests.

In Table 3, we also report the studentized versions of the SQ and FOS test statistics, where the individual test statistics are scaled by their standard deviation. It appears that studentization increases the power of both FOS and SQ for all sample sizes – allowing FOS to reject the alternative in more than 92% of the time for all sample sizes considered.
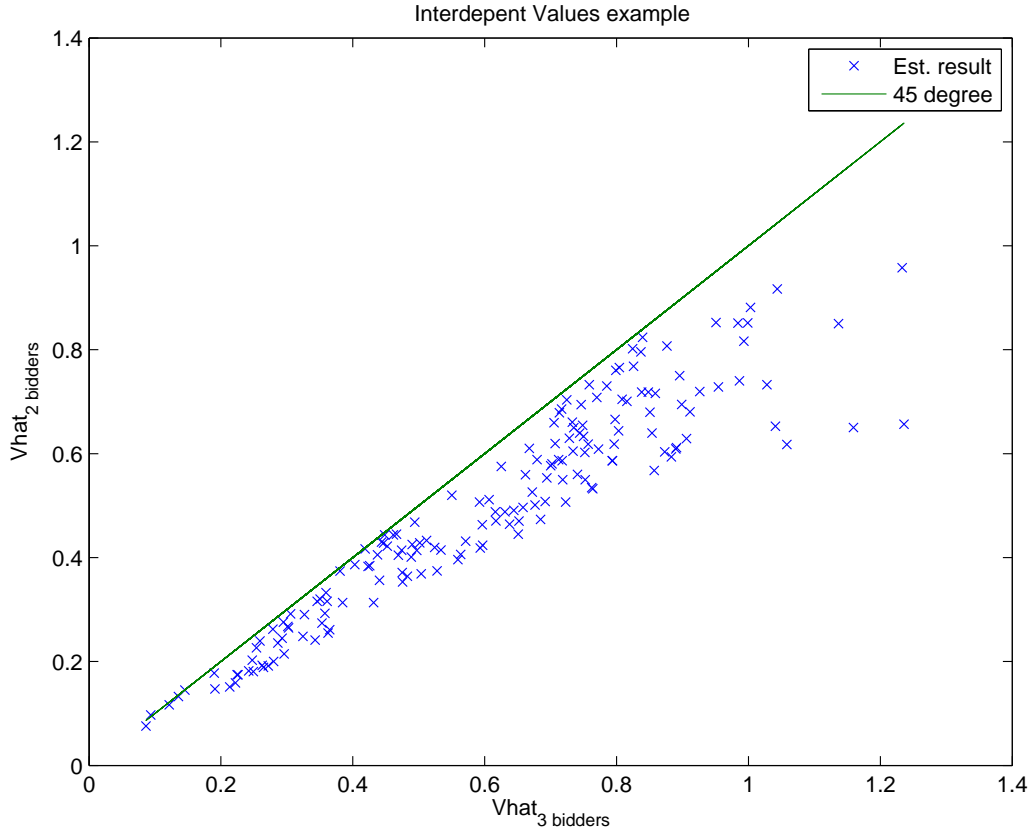
24

Figure 2: Estimated values for informed and uninformed bidders in a FPA with interdependent values

4.1.3 First price auction with pure common values

In our third exercise we examine a first price auction with pure common values described in Matthews (1984). Let the utility be $u_i(s_i) = v$ where $v \sim \text{Pareto}(\alpha): g(v) = \alpha v^{-(\alpha+1)}$ for $1 \leq v \leq \infty$ and $F(s|v) = \frac{s}{v}$.

In this case Matthews shows that there is a unique (symmetric) equilibrium in differentiable strictly increasing strategies of the form:

$$b(s) = \left(\frac{(N-1) + [\max\{1, s\}]^{-(N-1)-1}}{(N-1) + 1} \right) \hat{v}(s, N)$$

where

$$\hat{v}(s,N) = \frac{N+\alpha}{N+\alpha-1}\max\{1,s\}$$

is the expected valuation conditional on winning. Notice that for $s \geq 1$ we have

$$b(s) = \frac{(N+\alpha)s\left[(N-1)+s^{-N}\right]}{N(N+\alpha-1)}$$

Now if bidder 1 were again to observe bidder 2's bid, two cases can occur: either he can infer s_2 or that $s_2 < 1$.

Suppose that $s_2 \leq s_1$. Then the optimal bid is as before since no such signal is informative about realized v conditional on winning ($s_{\max}$ is a sufficient statistic of the sample $(s_1, ..., s_N)$ for v). On the other hand, if $s_2 > s_1$, then the optimal bid becomes:

$$b(s_1,s_2) = \frac{(N+\alpha)s_2\left[(N-1)+s_2^{-N}\right]}{N(N+\alpha-1)}$$

In other words, bidder uses just the highest signal he observes to base his bid upon and updates the prior on the distribution of v using the winning event.

We once again generated data for an informed and an uninformed bidder using the above described bidding strategies and used GPV to estimate the implied valuations under the null hypothesis of private values. The results, for a randomly chosen data set, are displayed in Figure 3.

In contrast to the IIV case, the median test appears to perform the best in this case. Figure 3 sheds some light into what might drive this result: it appears that boundary effects are particularly important in this example. Thus, trimming is particularly effective in increasing the power of SQ and FOS tests. Note that studentization also helps increase the power of SQ and FOS tests, as displayed in table 3.

4.1.4 Monte Carlo Summary

Our Monte Carlo exercises reveal several important observations regarding the joint test statistics FOS and SQ: due to the bias of the kernel estimates, without restricting attention to a subset of

Table 1: Monte Carlo Exercises: Joint test

	IPV			IIV			CV		
N	Median[a]	SQ[b]	FOS[c]	Median	SQ	FOS	Median	SQ	FOS
50	0.04	0.00	0.32	0.34	0.00	0.54	0.92	0.04	0.02
100	0.04	0.00	0.66	0.40	0.04	0.98	0.99	0.00	0.00
200	0.10	0.00	0.94	0.72	0.68	0.99	0.99	0.00	0.00
50: trimmed[d]	0.06	0.00	0.08	0.40	0.02	0.80	0.96	0.10	0.66
100: trimmed	0.02	0.00	0.18	0.64	0.12	0.99	0.99	0.44	0.72
200: trimmed	0.04	0.00	0.24	0.74	0.98	0.99	0.99	0.78	0.92

[a] Test based on difference in medians of distributions.
[b] Test based on sum of squares of individual test statistics.
[c] Test based on the first-order statistic of individual test statistics.
[d] $\frac{1}{9}$ of estimated values discarded at the lower tail, $\frac{2}{9}$ in the upper tail.

Table 2: Monte Carlo Exercises: Pointwise test

	IPV		IIV		CV	
N	Ptwise^a_{95}	Ptwise^b_{Bonf}	Ptwise_{95}	Ptwise_{Bonf}	Ptwise_{95}	Ptwise_{Bonf}
50	0.14	0.01	0.46	0.10	0.79	0.21
100	0.17	0.01	0.60	0.20	0.81	0.53
200	0.21	0.02	0.72	0.43	0.85	0.71
50: trimmed[c]	0.07	0.01	0.54	0.18	0.95	0.37
100: trimmed	0.09	0.01	0.69	0.33	0.98	0.71
200: trimmed	0.11	0.01	0.76	0.52	0.99	0.92

[a] Comparing each hypothesis with 95th percentile of the corresponding asymptotic distribution.
[b] Comparing each hypothesis with $\left(1 - \frac{0.05}{N}\right)^{th}$ percentile of the corresponding asymptotic distribution.
[c] $\frac{1}{9}$ of estimated values discarded at the lower tail, $\frac{2}{9}$ in the upper tail.

Table 3: Monte Carlo Exercises: Studentized joint test

	IPV		IIV		CV	
N	SQ^a_{Stud}	FOS^b_{Stud}	SQ_{Stud}	FOS_{Stud}	SQ_{Stud}	FOS_{Stud}
50	0.04	0.40	0.08	0.92	0.20	0.96
100	0.00	0.46	0.32	0.99	0.68	0.99
200	0.02	0.84	0.92	0.99	0.99	0.98
50: trimmed[c]	0.00	0.20	0.12	0.92	0.26	0.99
100: trimmed	0.00	0.16	0.40	0.99	0.88	0.99
200: trimmed	0.00	0.22	0.86	0.99	0.99	0.99

[a] Test based on sum of squares of studentized individual test statistics.
[b] Test based on the first-order statistic of studentized individual test statistics.
[c] $\frac{1}{9}$ of estimated values discarded at the lower tail, $\frac{2}{9}$ in the upper tail.
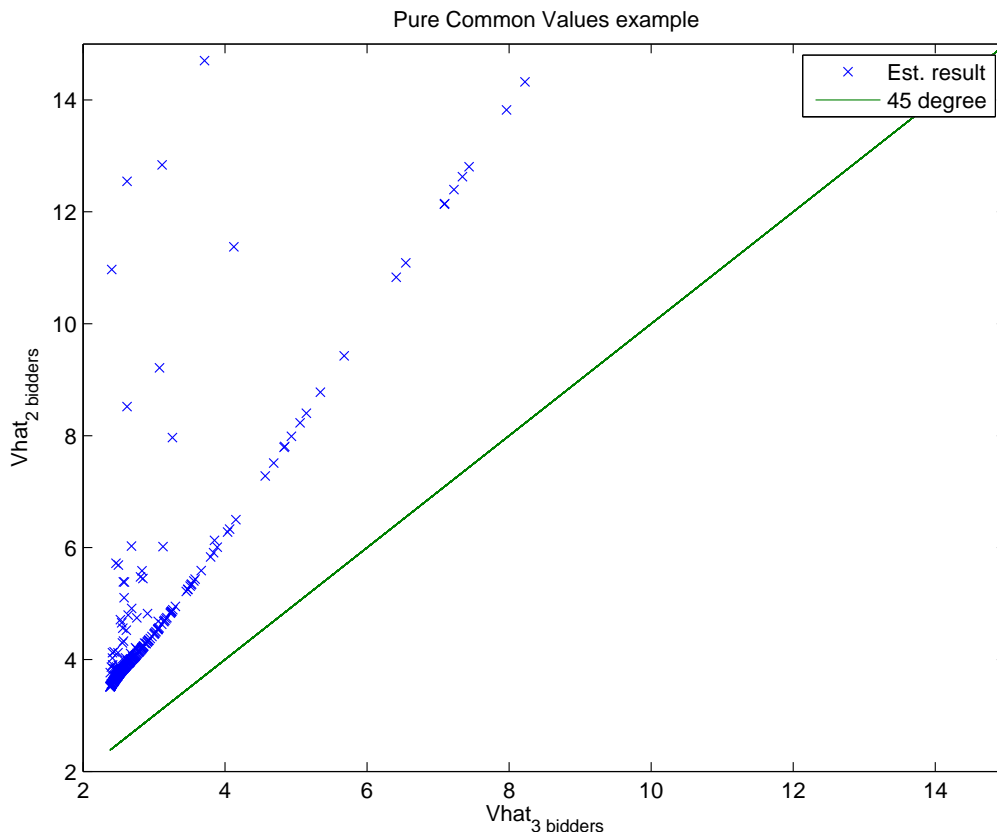
Figure 3: Estimated values for informed and uninformed bidders in a FPA with pure common values

hypotheses which consider bidders with estimated valuations that are likely far enough from the boundary of the support, the tests can have much lower power in smaller samples than desired. While the joint hypothesis test based on the first-order statistic (FOS) seems to have higher power against the alternatives considered here, it also over-rejects the correct null unless the extent to which we trim the estimated marginal values increases substantially. The test based on sum of squares, SQ, on the other hand, does not over-reject the null, but has lower power against the alternatives, especially when no trimming is applied. Both tests' power against the alternative appears to improve if they are studentized, though studentization leads to a slight amount of overrejection of the null. We also found that joint hypothesis testing based on multiple independent hypotheses and the Bonferroni correction does not generate significantly better performance than

FOS or SQ.

Given that the various testing approaches appear to have different strengths and weaknesses, we will present results of all three main testing approaches in our application: (i) individual hypothesis tests, $\{S_i, T_i\}_{i=1}^N$, where N is the number of hypotheses, (ii) sum of squares (unstudentized and studentized), (SQ_T, SQ_S), and (iii) first-order statistic test (unstudentized and studentized), (FOS_S, FOS_T).

5   Data and Institutional Background

Treasury bills and other Bank of Canada securities are issued in the primary market through sealed-bid discriminatory auctions. Bids are submitted electronically and can be revised at any point before the submission deadline. There are two major groups of potential bidders: government securities distributors (*dealers*) and *customers*. The customers are typically not individual investors. Many of them are actually large banks that for some reason choose not to be registered as a dealers, but whose demands are sufficiently important for the Bank of Canada to require their separate identification. The major distinction between customers and dealers, however, is that customers cannot bid on their own account in the auction, but have to route their bids through one of the dealers. The dealers are required to identify bids submitted by customers in the electronic bidding system.

Our sample consists of all submitted bids in 116 auctions of 3-months treasury bills of the Canadian government issued between 10/29/1998 and 3/27/2003. The auctions were conducted as sealed-bid discriminatory price auctions. Along with the set of bids taken into consideration when making the final allocation, we also have the entire record of electronic bid submissions by dealers (under their own bidder ID and their customers' IDs) during the bid submission period. Thus, we are able to observe any modifications made by the dealers to their own bids up until the bidding deadline. Each electronic submission has a time stamp, thus we are able to observe whether a dealer's bid modification was preceded by the entry of a customer bid.

One may wonder why dealers submit their bids before the bidding deadline, and do not wait until they have seen the bids of their customers. One answer is provided by Hortaçsu and Sareen

(2006), who report that customer bids typically come in a very narrow window before the bid submission deadline. They also report that some dealers' modifications to their own bids in response to these late customer bids narrowly missed the bid submission deadline, and that such missed bid modification opportunities had a negative impact on dealers' ex-post profits. Moreover, although some customers appear to be in long-term relationships with their dealers, some of them appear to change dealers frequently; which may render some customer bids as a surprise. Since entering bid modifications is not a very costly activity, the uncertainty in the arrival of customer bids may render it optimal for the dealers to follow the strategy of entering their best response conditional on their information set.

Hortaçsu and Sareen (2006) also report various descriptive measures suggesting that obtaining customer information has a causal impact on dealers' bidding patterns. They find that the direction of changes in a dealer's (quantity-weighted price) bid typically follows the direction of discrepancy between the dealer's pre-customer information bid, and the customer's bid. Hortaçsu and Sareen point out that both common value and private value models are consistent with their descriptive patterns, however, and do not conduct tests to distinguish between these informational environments.

An important potential caveat regarding our testing strategy is that privately observed customer bids *per se* are not the causal drivers of observed changes in dealer bids, and that customer bids are correlated with other unobservable information flows driving modifications to dealer bids. The presence of such unobservable information flows would confound our testing strategy, since these information flows may affect the dealer's marginal valuation, and/or allow them to observe an extra piece of information regarding the auction environment that we are not able to account for in our marginal valuation estimation procedure.

One source of unobservable information flows maybe in the form of news announcements or market movements during the bidding period that are observed by all dealers, but not the econometrician. To examine the plausibility of such unobserved public information flows, we examined the timing of changes in dealer bids in our data set. If information flows are publicly observed across dealers, we should observe some amount of clustering in the timing of bid modifications in

our data set. We failed to find an important degree of clustering in this dimension – within any 5 minute window around a particular bid updating event, there was at most one other dealer changing his/her bid (and such a dealer was only found in 40 instances out of the total 213 updated bids in our sample). This suggests that it is unlikely that customer bids were driven by or accompanied with important public information releases that are unobservable to us. As a complement to this finding, Hortaçsu and Sareen (2006) report that unobservable public information releases by official sources are highly unlikely, as Bank of Canada and Treasury pay careful attention to avoid public disclosures during the bidding period.

Along with customer bids, dealers may also adjust their net long-short positions during the bidding period in the when-issued market. The dealer's net long/short position may shift his/her bid and/or her marginal valuation curve independent of the information contained in customers bid; thus this is a potential confounding factor. Fortunately, the Bank of Canada requires dealers to report their net positions (short or long) in the when-issued market along with their bids. In our data, only 13 out of 216 updated bids in 3-month T-bill auctions were accompanied by a change in the net position of the dealer in the when-issued market. In 12-month T-bill auction, only 7 dealers (out of 275 updates) also changed their net position. Given that such a small fraction of bids are subject to this potential confound, we do not think that developments in the when-issued market affect our test results.

In our analysis, we will use data from both 3 month and 12 month maturity treasury bill auctions. These securities are sold at the same time in parallel auctions, and are the most frequently auctioned securities in our data. Moreover, our prior is that the role played by common vs. private value components is likely to be different across these securities. In particular, we expect the common value component for 12-month treasury bills to be relatively more important. 12 month treasury bills could be used as a hedging instrument in a larger array of transactions, hence have a more active resale market. Moreover, bidders forecasts' regarding the economic environment over the next 12 months period is likely to have higher variance than their forecasts over 3 months. Hence, there may be higher value in learning other bidders' forecasts to assess the fundamental value of a 12 month security, as compared to valuing a 3 month security.

5.1 Summary Statistics

Table 4 offers some summary statistics of our data set, which encompasses all submitted bids in 116 auctions of 3-months treasury bills of the Canadian government issued between 10/29/1998 and 3/27/2003.

Table 4: Data Summary

Summary Statistics for 3 month T-bill auctions				
Auctions	116			
	Mean	St.Dev.	Min	Max
Dealers	12.34	1.64	9	15
Customers	4.66	2.3	0	12
Participants	17	2.83	11	23
Submitted steps	2.88	1.69	1	7
Price bid	989,353	3,244	984,515	994,969
Quantity bid	0.092	0.07	0.00023	0.2512
Issued Amount (billions C\$)	3.881	0.552	2.8	5

On average, 12 dealers and less than 5 customers participate in every auction. An average bid curve constists of less than 3 steps and the average quantity demand is for about 10% of the supply.

As usual in most government securities auctions, bids can be submitted both as competitive tenders and as noncompetitive tenders. Each participant is allowed to submit a single noncompetitive tender. A noncompetitive tender specifies a quantity that the bidder wishes to purchase at the price at which the auction clears. As Table 5 reveals, in our data, there are on average 3.6 noncompetitive tenders in an auction of the preannounced amount for sale. The non-competitive bids which simply operate by shifting the available supply at any price to the left are not as important in Canadian treasury bill auctions as in other settings. In particular, while the average non-competitive bid is for about 4.4% of the supply, most of this is driven by non-competitive bids that were placed by the central bank itself. The average non-competitive bid conditional on being placed by a dealer or a customer is for less than 0.06% of the supply and hence quite negligible. In our estimation approach we thus resample a non-competitive bid from the central bank and then we also resample either 2 or 3 non-competitive bids from regular participants. All these resampled bids then shift the available supply to the left.

Table 5: Summary of Noncompetitive Bids

Auctions with NC bid	116			
	Mean	St.Dev.	Min	Max
Number of NC bids	3.6	1.1	2	7
NC bid	0.044	0.08	0.00003	0.27

Table 6 presents the summary statistics for the 12 month T-bill auctions. Relative to auctions of 3-months treasury bills (which are sold in parallel auctions), there is less participation both by dealers and especially by customers. Price bids exhibit larger variation. The amount offered for sale in each auction is also significantly lower.

Table 6: Data Summary

Summary Statistics for 12 month T-bill auctions				
Auctions	116			
	Mean	St.Dev.	Min	Max
Dealers	5.69	0.76	4	8
Customers	2.84	1.78	0	8
Participants	8.53	1.93	5	15
Submitted steps	2.91	1.69	1	7
Price bid	958,997	10,972	940,004	979,928
Quantity bid	0.096	0.074	0.00028	0.25
Issued Amount (billions C$)	1.67	0.194	1.3	2

6 Test results from 3 month T-bill auctions

In the 116 3 month T-bill auctions in our sample, we observed 216 dealer bids that were updated after a customer bid arrived. Figure 4 depicts updating of a bid by one dealer. After observing a relatively low bid by one customer, the dealer submits a new bid which is uniformly weakly below his original bid.[8]

Before updating, these 216 bids consisted of 802 bidsteps (price-quantity pairs) and after updating they consisted of 859 bidsteps. We focus on these updated bids to conduct our tests. We construct a bootstrap sample of 400 replications of the test statistics (using always 5000 resampling

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[8]This "parallel shift" of the updated bid is not a general feature of the data, however. Some updated bids cross the original bids.
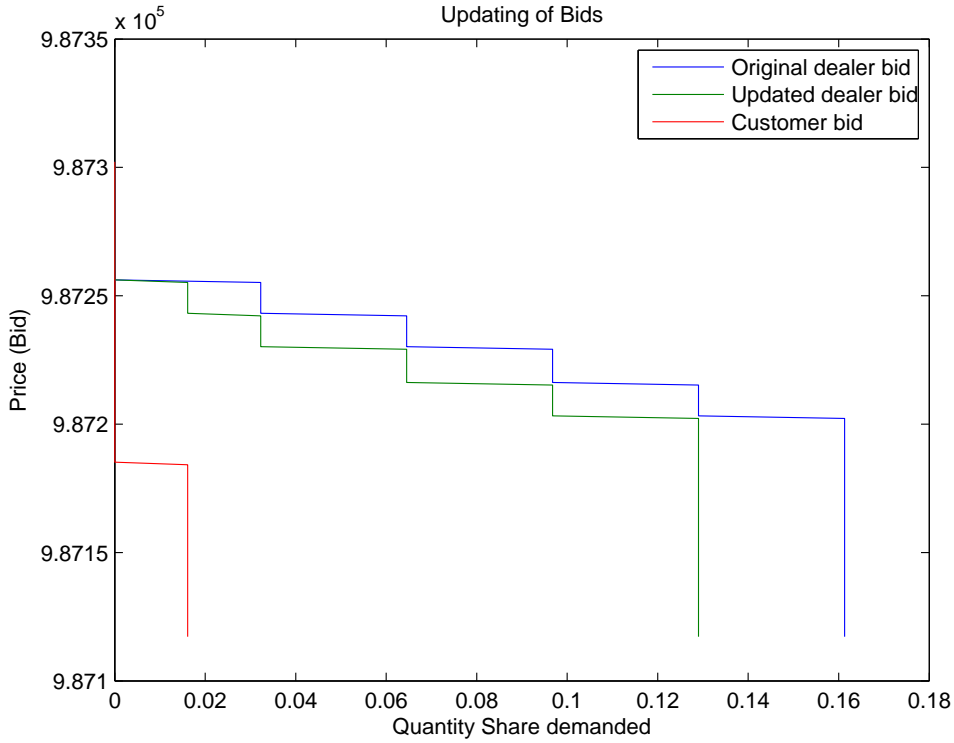
Figure 4: Updating of a dealer's bid

draws for estimating each bidder's marginal valuation) for each of these bidders as defined by (7) and construct the corresponding critical values given by (8).

To illustrate the marginal valuation estimation procedure, figures 5 and 6 depict the marginal valuation estimation results for two dealers. In figure 6 we depict a bidder for whom the two estimates of marginal valuations are statistically indistinguishable, while in figure 5 we depict a bidder for whom the equality of estimated marginal valuations when information is taken into account can be rejected at one of his steps. We do not use (or plot) the estimates of the marginal value at the last step of the bid curve since as proposition 1 shows bid at that step depends on the properties of the marginal valuation function.

To construct the various testing statistics, we first estimated bidder's marginal valuations under different data pooling schemes. We first used resamples of bids from the same auction only (the "1 auction" case). This does not pool bid data across auctions, and hence minimizes a potential
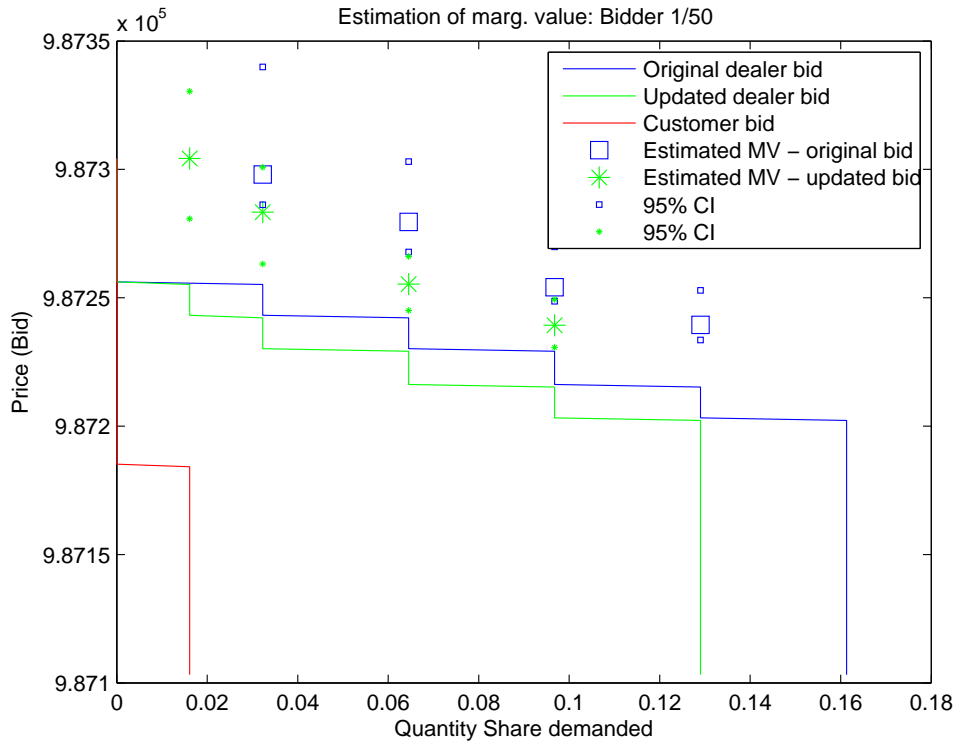
34

Figure 5: Estimated valuations - dealer 1

unobserved heterogeneity problem. However, since the data set used to estimate marginal valuations is small, the estimation error is potentially large, and, as in our Monte Carlo exercises, the power of our tests might be lower than desired. Resampling from a pooled set of auctions that are similar may decrease estimation error, but unobserved heterogeneity across auctions may result in rejections of the null hypothesis for other reasons. To explore this tradeoff, we report the same estimates that are obtained if we pool data across 2 and 4 consecutive auctions respectively (this assumes that the economic environment is stable across 2 and 6 week periods, respectively).

Our Monte Carlo exercise suggested that the various testing approaches utilize have complementary size-power tradeoffs in small samples, thus we will display results across an array of test statistics. In Table 7, we report results from hypothesis tests on individual bidders' updating behavior. First, we report results based on the monotonicity tests statistic (7) computed separately for each updated bid (the critical values were obtained using bootstrap). We find that we are
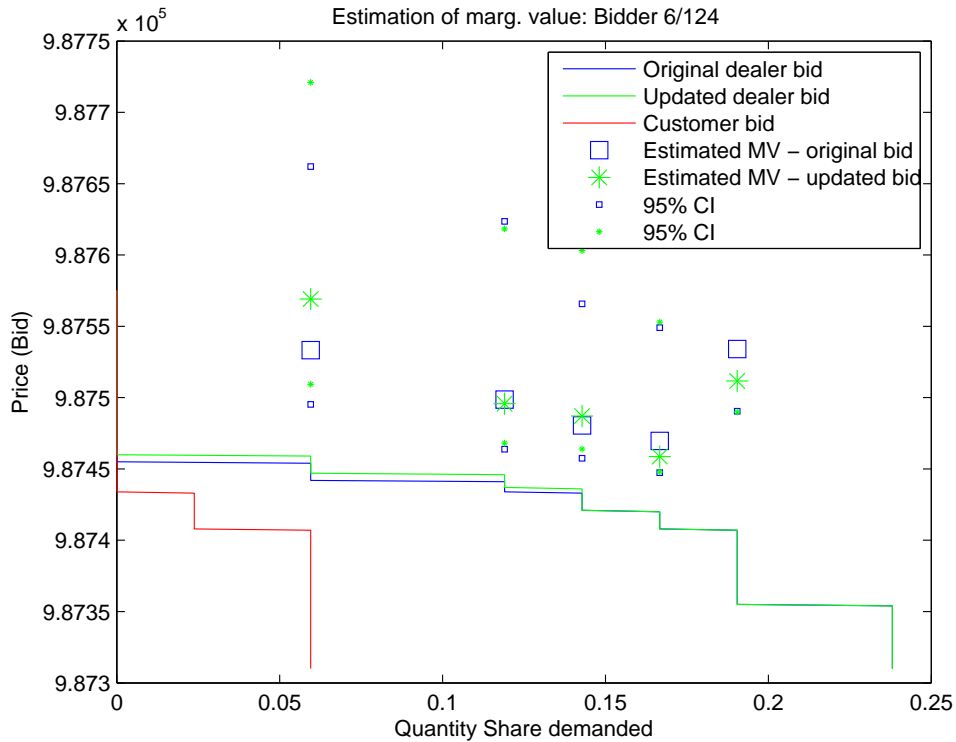
35

Figure 6: Estimated valuations - dealer 6

able to reject the null (at the 5% level) only in 1% of the individual hypotheses when we estimate marginal valuations using data from a single auction. When we increase the number of auctions in used to estimate marginal valuations, we are able to reject more of the individual hypotheses: 6% of the individual hypotheses are rejected when we resample from 2 neighboring auctions, and 15% are rejected when we resample from 4 neighboring auctions.

The individual hypothesis tests suggest that the null of private values is not overwhelmingly rejected. It appears that as we increase the size of the sample used to estimate marginal valuations, the rejection rates increase. However, as we noted earlier, increasing sample size may lead to overrejection of the null due to the introduction of unobserved heterogeneity as well. We should also recall from the Monte Carlo exercise that, in finite samples, this testing approach may lead to overrejection of the null even without the presence of unobserved heterogeneity.

In table 8, we report results from the tests based on the joint test statistics defined in Section 3.2.

Once again, we estimate marginal valuations using 1, 2, and 4 neighboring auctions. Especially in the case where we were resampling from a single auction, we found that the unstudentized test statistics were affected by large outliers resulting from marginal value estimation error. We thus decided to studentize the test statistics.[9] Recall that the Monte Carlo exercise suggested that studentization leads to a slight degree of overrejection of the null hypothesis, but, overall, increases power against the alternatives.

For each case, we use the bootstrap to calculate the critical values of the studentized first-order test statistic (FOS) and the sum-of-squared differences statistic (SQ).[10] Based on the studentized joint hypothesis tests and using the treasury bill with 3-months maturity, table 8 shows that we reject the null hypothesis only when resampling from 4 auctions.[11]

7 Test results from 12 month T-bill auctions

Tables 7 and 8 also include the results of our tests using updated dealer bids from 12 month auctions. There were 275 updated dealer bids in this sample, comprising of consisted of 937 bidsteps (price-quantity pairs) and after updating they consisted of 996 bidsteps.

We suggested above that the common value component maybe more important relative to the 3-months treasury bills. Our data provides some evidence supporting this hypothesis.

On the full sample, the median value of the test statistic for monotonicity violation using resampling from 4 auctions is 86.33 while the median critical value is 164.94. The median value of the test statistic for the equality of marginal valuations is 55.56 while the median critical value is 67.79. Overall, table 7 shows that based on individual hypothesis testing the test of equal marginal

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[9]We found similar results when we constructed our test statistics using marginal value estimates with variance below certain cutoffs. However, the choice of these cutoffs was subjective; we thus decided to focus on the studentized test statistic.

[10]For example, the joint hypothesis test based on the first order statistic is constructed by first dividing each individual test statistic evaluated on the sample by its standard deviation estimated by bootstrap and then taking the maximum of these.

[11]There is one additional rejection based on the first order statistic and resampling from only 1 auction. This is due to one outlier. In particular, there is one hypothesis that has a very small standard deviation of the bootstrap distribution of the associated test statistic, which leads to high values of the studentized test statistic. The value of the studentized test statistic associated with this hypothesis then becomes the first order statistic both on the sample, and also in almost all bootstrap iterations, and thus after re-centering the bootstrap distribution we reject the null. When we test all other hypotheses jointly (after eliminating the outlying one), all of our tests fail to reject the null of private values.

valuations rejects for 19% of steps and the monotonicity test rejects 20% of the hypotheses for treasury bill with 12-months maturity.

The joint hypothesis tests based on studentized test statistics, results of which are reported in table 8, show that virtually all performed tests result in lower critical values and larger values of the test statistic in case of t-bills with 12-months maturity than for t-bills with 3-months maturity. In particular, for resampling from only two auctions, we never reject the null of private values based on the joint hypothesis test for 3-months t-bills, whereas we always reject private values for 12-months t-bills.

Table 7: Individual Hypothesis Test Results

Used for resampling	1 auction		2 auctions		4 auctions	
Maturity	3-m	12-m	3-m	12-m	3-m	12-m
Monotonicity						
# of hypotheses	174	214	193	240	196	240
Rejection rate	0.01	0.04	0.06*	0.15*	0.15*	0.20*
Equality						
# of hypotheses	431	421	448	458	452	461
Rejection rate	0.03	0.07*	0.07*	0.15*	0.15*	0.19*

* Null hypothesis of private values is rejected at 5% level.

Table 8: Joint Hypothesis Studentized Test Results

Used for resampling	1 auction		2 auctions		4 auctions	
Maturity	3-m	12-m	3-m	12-m	3-m	12-m
Monotonicity						
Critical value (SQ[a])	513.03	331.72	569.02	557.18*	187.84*	535.37*
Sample statistic (SQ)	13.77	132.25	107.56	706.82*	1055.77*	1130.97*
Critical value (FOS[b])	15.48	7.43	14.95	3.26*	2.71*	2.23*
Sample statistic (FOS)	2.53	6.52	4.98	11.32*	15.53*	12.19*
Equality						
Critical value (SQ)	1158.02	852.37	1346.62	928.10*	858.00*	672.16*
Sample statistic (SQ)	483.20	500.86	555.81	1527.45*	1751.11*	2212.38*
Critical value (FOS)	3.14*	1.45*	10.06	2.68*	7.35*	2.45*
Sample statistic (FOS)	16.76*	12.70*	9.89	18.60*	12.42*	12.92*

[a] Test based on sum of squares.
[b] Test based on first-order statistic.
* Null hypothesis of private values is rejected at 5% level.

7.1 Value of Information

Given that several of our tests failed to reject private values for 3-months treasury bills, in what follows we will use our estimates of marginal valuations generated by assuming the private values paradigm to estimate the value of information. In particular, we try to answer the question what is the effect of the additional information on a dealer's interim (expected) and ex post utility. Let $U_d^{EP}\left(B^I\right)$ denote the ex post utility of a dealer d, when submitting a bid B^I where the superscript I denotes information. In particular, as before $I = AI$ denotes additional information is incorporated, i.e., when the updated bid is used to compute the utility, and $I = BI$ when the original bid is used instead.

We can measure the value of information in terms of this notation as follows:

$$VI_d^{EP} = U_d^{EP}(B^{AI}) - U_d^{EP}(B^{BI}) \tag{14}$$

To compute the value of information given by equation (14), for each bidder we take the upper envelope of her estimated marginal valuations and the actual realized residual supply this bidder faced in the auction. We then evaluate this bidder's ex-post utility using the original bid (submitted before the arrival of the customer's bid) to obtain $U_d^{EP}(B^{BI})$ and then evaluate her utility using the updated bid to obtain $U_d^{EP}(B^{AI})$. Performing this exercise for each bidder whom we observe updating her bid, we obtain a full distribution of the value of information, the mean of which we call value of information. Using these estimates we find that that the ex post value of information, VI^{EP}, is on average about 0.44 of a basis point per T-bill for sale[12]. Since the average ex post utility amounts to 1.36 basis points, the extra information contained in customers' order flow generates about 32% of the payoff of the dealers. We conclude from this analysis that consistently with views of professionals from the financial industry the information that the primary dealers obtain from the order flow from their customers is an important source of valuable private information that enables the primary dealers to extract more information rents from participating in primary auctions even when values are private.

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[12]The standard deviation of ex post payoff is slightly over 2.5 basis points.

8 Conclusion

In this paper we proposed a novel nonparametric test for common values. The test can be applied universally in both single-unit first-price auctions and multiunit auctions. On the other hand a necessary condition for the test to be applicable is the ability of the researcher to distinguish between more and less informed bidders, who are ex ante symmetric. The test is based on comparing two estimated distribution of valuations, which should coincide under the null hypothesis of private values. Our Monte Carlo experiments suggest that the test performs well. We also apply our test to data from Canadian treasury bill auctions. While we cannot conclusively reject the null hypothesis of private values for 3-month treasury bills, in the majority of our test specifications, we can reject the null of private values for 12 month treasury bills.

Since we compare two estimates of valuations within an auction, our test is less susceptible to unobserved heterogeneity of individual auctions than the recent alternative test for common values proposed in Haile, Hong and Shum (2003). We also used our structural model to quantify the benefits accruing to the dealers from observing customers' order flow. Using our valuation estimates we estimated the value of information by comparing the (expected) utility of a dealer with customer information to that of the same dealer without this information. We found that information coming from customers' order flow is responsible for about 32% of dealer's surplus.

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A   Appendix

A.1   Test for Equality of Nonparametric Regressions

Because there is an estimation error in the estimates of marginal valuations, we can write the estimated marginal valuation function of bidder i as:

$$v_{ik}^I = f^I\left(q_{ik}^I\right) + \varepsilon_{ik}^I \quad \text{for} \quad I = BI, AI$$

where BI and AI stands for "before information" and "after information" respectively and ε_{ik}^I is the estimation error in marginal valuation estimates, i.e., $v_i\left(q_{ik}, \bar{s}\right) = \hat{v}_i\left(q_{ik}, \bar{s}\right) + \varepsilon_{ik}$ with $\hat{v}$ being the estimates of marginal valuation from our resampling procedure. Since our estimate of marginal value at quantity q_i is consistent, $E\left(\varepsilon|q\right) = 0$, and hence the level curve of the marginal valuation function at $\bar{s}$, $f^I\left(q_i^I\right)$ would be nonparametrically identified whenever the number of observed

bids at different quantities for this particular signal level would go to infinity. Unfortunately, it is reasonable to believe that this assumption which is necessary for consistency of nonparametric regression might be violated in practice, since the quantity demands at submitted bids are not a random selection from the support of quantities. Therefore, we might not observe bids at some quantities even asymptotically.

Suppose that the whole level curves of the marginal valuation function are non-parametrically identified. Under the null hypothesis of private values, $f_{BI} = f_{WI}$ and hence we can simply test for equality of two nonparametric regressions. Few of such tests have been proposed in the statistics literature on treatment evaluations (e.g., Koul and Schick, 1997).

Consider the statistic

$$T = \sqrt{\frac{n_B n_W}{n_B + n_W}} \frac{1}{n_B n_W} \sum_{i=1}^{n_B} \sum_{j=1}^{n_W} \frac{1}{2} \left(\eta \left(q_{B,i} \right) + \eta \left(q_{W,j} \right) \right) \rho \left(v_{B,i} - v_{W,j} \right) w_a \left(q_{B,i} - q_{W,j} \right)$$

where a is a small positive number depending on the sample sizes. H_0 is rejected for large values of T. Koul and Schick call this test a covariate-matched test.

The test statistic considered above assumes that for any given level curve of the marginal valuation function $v \left(q, \bar{s} \right)$ at an unobserved signal $\bar{s}$, the set of quantities at which the value is estimated grows asymptotically, so that $v \left(q, \bar{s} \right)$ can be identified nonparametrically. The test then rejects H_0 if the two estimated regression curves are sufficiently different. As mentioned in the text, in practice, however, the number of steps in the observed bids is very low and there is no compelling reason to believe that it would vary much as the number of observed auctions increases (for a given unobserved signal realization $\bar{s}$). One possibility to obtain the asymptotic behavior consistent with the construction above is to assume private cost c per bidpoint as in Kastl (2006a), which is drawn independently of s. As $c \downarrow 0$, bidders would submit bids with more and more steps (a continuous function in the limit of zero cost) for any s and thus $v \left(q, \bar{s} \right)$ would again be nonparametrically identified.

B   Appendix

Here we present the derivation of the closed form solution for bidding used to generate data in our Mont Carlo studies with 3 bidders.

B.1   First price auction with independent private values

Let the utility function be:

$$u_i = x_i$$

In this case bidder 1 maximizes $\Pr(b_1 > max\{b_2, b_3\})(x_1 - b_1)$ which implies that the symmetric equilibrium bidding function is:

$$b(x) = \frac{2}{3}x$$

If he observed 2's bid, he would bid in 2 cases (assuming any tie is broken in 1's favor and bidders 2 and 3 continue using the strategies given above) using the rule:

$$b(x_1, x_2) = \begin{cases} \frac{x_1}{2} \ if \ \frac{x_1}{2} > \frac{2x_2}{3} \\ \frac{2x_2}{3} \ if \ x_1 > \frac{2x_2}{3} > \frac{x_1}{2} \end{cases}$$

where the second case occurs whenever bid of bidder 1 using the rule for the first case would be lower than 2's bid, but bidder 1 would prefer to win the object.

B.2   First price auction with interdependent values

Let the utility function be:

$$u_i = \frac{x_i}{2} + \frac{\sum_{j \neq i} x_j}{2(n-1)}$$

where

$$x_i \sim U[0, 1]$$

With 3 bidders there exists a unique symmetric equilibrium in differentiable strictly increasing strategies:

$$b(x) = \frac{7}{12}x$$

Now suppose that bidders 2 and 3 follow these strategies. Suppose bidder 1 can observe bidder 2's bid. Since 2's strategy is strictly increasing, bidder 1 can recover the signal s_2. In this case, bidder 1's expected payoff when getting a signal s_1, observing s_2 and bidding b is:

$$\int_0^{\frac{12b}{7}} \left(\frac{s_1}{2} + \frac{s_2}{4} + \frac{\alpha}{4} - b \right) d\alpha$$

Maximizing this expression w.r.t. the bid b results in:

$$b^* (s_1, s_2) = \frac{7}{11} \left(\frac{s_1}{2} + \frac{s_2}{4} \right)$$

Finally, this bid is weakly higher than $b_2 = \frac{7}{12} s_2$ if and only if $s_1 \geq \frac{4}{3} s_2$. In the case that b_2 is higher than $b^* (s_1, s_2)$, bidder 1 will still prefer to win if $s_1 \geq \frac{25}{48} s_2$.