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Unit Root Testing

By Jürgen Wolters and Uwe Hassler *

Summary: The occurrence of unit roots in economic time series has far reaching consequences for univariate as well as multivariate econometric modelling. Therefore, unit root tests are nowadays the starting point of most empirical time series studies. The oldest and most widely used test is due to Dickey and Fuller (1979). Reviewing this test and variants thereof we focus on the importance of modelling the deterministic component. In particular, we survey the growing literature on tests accounting for structural shifts. Finally, further applied aspects are addressed how to get the size correct and obtain good power at the same time.

Keywords: Dickey-Fuller, size and power, deterministic components, structural breaks. JEL C22.

1. Introduction

A wide variety of economic time series is characterized by trending behaviour.This raises the important question how to statistically model the long-run component. In the literature, two different approaches have been used. The so-called trend stationary (TS) model assumes that the long-run component follows a time polynomial, which is often assumed to be linear, and added to an otherwise stationary autoregressive moving average (ARMA) process. The difference stationary (DS) model assumes that differencing is required to obtain stationarity, i.e. that the first difference of a time series follows a stationary and invertible ARMA process[1]. This implies that the level of the time series has a unit root in its autoregressive (AR) part. Unit root processes are also called integrated of order 1, I(1).

Since the seminal paper by Nelson and Plosser (1982) economists know that modelling the long-run behaviour by TS or DS models has far-reaching consequences for the economic interpretation. In a TS model the effects of shocks are only temporary implying that the level of the variable is not influenced in the long run. In contrast a shock has permanent effects in a DS model, meaning that the level of the variable will be shifted permanently after the shock has occurred.

Traditional econometrics assumes stationary variables (constant means and time-independent autocorrelations). This is one of the reasons why applied economists very often transform non-stationary variables into stationary time series. According to the two above-mentioned models this can be done by eliminating deterministic trends in the case of a TS model or by taking first differences in the case of a DS model. But what happens if the wrong transformation is applied? The papers by Chan, Hayya and Ord (1977), Nelson and Kang (1981) and Durlauf and Phillips (1988) investigate

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1 For ARMA processes see e.g. Box and Jenkins (1976).

this problem. Eliminating the non-stationarity in a TS model by taking first differences has two effects: one gets rid of the linear trend, but the stationary stochastic part is overdifferenced, implying spurious short-run cycles. If, on the other hand, it is tried to eliminate the non-stationarity in a DS model by taking the residuals of a regression on a constant and on time as explanatory variables, spurious long-run cycles are introduced. These depend on the number of observations used in the regression. In this case artificial business cycles are produced that lead to wrong economic interpretations.

Moreover, regressing independent DS processes on each other leads to the problem of spurious regressions as Granger and Newbold (1974) have demonstrated in a simulation study. Later on Phillips (1986) gave the theoretical reasoning for this phenomenon: The usual t-statistics diverge to infinity in absolute value, while the R^2 does not converge to zero, hence indicating spurious correlation between independent DS processes. Granger (1981) and Engle and Granger (1987) offered a solution to the spurious regression problem by introducing the concept of cointegration.

The above discussion clearly indicates that the analysis of non-stationary time series requires a serious investigation of the trending behaviour. Therefore, formal tests are needed which allow to distinguish between trend stationary and difference stationary behaviour of time series. Such tests have first been developed by Fuller (1976) and Dickey and Fuller (1979, 1981) (DF test, or augmented DF test, ADF). In the meantime a lot of extensions and generalizations have been published which also are presented in different surveys such as Dickey, Bell and Miller (1986), Diebold and Nerlove (1990), Campbell and Perron (1991), Hassler (1994), Stock (1994) and Phillips and Xiao (1998).

Due to page limitations we will present here only the (augmented) Dickey-Fuller approach for testing the null hypothesis of difference stationarity. The related semi-parametric approach developed by Phillips (1987) and Phillips and Perron (1988) is not presented. Furthermore, we do not deal with tests for seasonal unit roots as proposed e.g. by Hylleberg, Engle, Granger and Yoo (1990), or tests having stationarity in the maintained hypothesis as Kwiatkowski, Phillips, Schmidt and Shin (1992). We rather focus on modelling the deterministic part of the time series under investigation. This is very important in case of structural breaks, since neglecting deterministic shifts may result in misleading conclusions.

The paper is structured as follows. In Section 2, Dickey-Fuller unit root tests are described and discussed. The third section deals with important applied aspects around size and power. Section 4 turns to the handling of structural breaks.

2. Dickey-Fuller unit root tests

2.1. Model. For the rest of the paper we assume the following data generating process (DGP):

$$y_t = d_t + x_t\,, \quad t = 1, \ldots, T\,. \tag{1}$$

The observed variable (y_t) is composed of a deterministic component d_t and a purely stochastic component x_t. The deterministic part may consist of a constant, seasonal dummy variables, a linear trend or a step dummy. The stochastic component is assumed to be a zero mean AR(p) process,

$$x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + ... + \alpha_p x_{t-p} + u_t \, , \qquad (2)$$

with $\alpha_p \neq 0$ and u_t being white noise. The AR(p) model in (2) can be reparameterized as

$$x_t = \rho x_{t-1} + \sum_{i=1}^{p-1} a_i \Delta x_{t-i} + u_t \qquad (3)$$

with

$$\rho = \sum_{j=1}^{p} \alpha_j \quad \text{and } a_i = - \sum_{j=i+1}^{p} \alpha_j \, , i = 1, ..., p-1.$$

If the lag polynomial of x_t

$$1 - \alpha_1 z - \alpha_2 z^2 - ... - \alpha_p z^p = 0$$

has a unit root, then it holds

$$\rho = \sum_{j=1}^{p} \alpha_j = 1. \qquad (4)$$

Substituting (3) in (1) we get the following expression for the observable variable y_t:

$$y_t = d_t - \rho d_{t-1} - \sum_{i=1}^{p-1} a_i \Delta d_{t-i} + \rho \, y_{t-1} + \sum_{i=1}^{p-1} a_i \Delta y_{t-i} + u_t.$$

Subtracting y_{t-1} on both sides of this equation, we obtain the Augmented Dickey-Fuller (ADF) regression:

$$\Delta y_t = d_t - \rho d_{t-1} - \sum_{i=1}^{p-1} a_i \Delta d_{t-i} + (\rho - 1) \, y_{t-1} + \sum_{i=1}^{p-1} a_i \Delta y_{t-i} + u_t. \qquad (5)$$

Note that in addition to the (lagged) level of the deterministic part its lagged changes are included, too.

In the case that $d_t = c$, a constant, the ADF regression is given as

$$\Delta y_t = a + (\rho - 1) y_{t-1} + \sum_{i=1}^{k} a_i \Delta y_{t-i} + u_t \, , \quad t = k+2, \ldots, T \, , \qquad (6)$$

with $a = (1-\rho)c$, meaning that under the null hypothesis the process is I(1) without drift. When the stochastic component x_t follows an AR(p) process then $k = p-1$ in (6). More generally, however, x_t may be an ARMA process with invertible moving average component. In that case, Said and Dickey (1984) propose to approximate the ARMA stucture by autoregressions of order k where the lag length k has to grow with the sample size.

In the case of a linear trend, $d_t = c + mt$, we get from (5) as ADF regression

$$\Delta y_t = a + bt + (\rho - 1)y_{t-1} + \sum_{i=1}^{k} a_i \Delta y_{t-i} + u_t\,, \quad t = k+2, \dots, T\,, \tag{7}$$

with

$$a = c(1-\rho) + \rho m - m \sum_{i=1}^{k} a_i \text{ and } b = m(1-\rho).$$

Under the null of a unit root ($\rho = 1$) the trend term in (7) vanishes, while the constant term contains not only the slope parameter m but also the coefficients a_i of the short run dynamic. Under the null it holds that $E(\Delta y_t) \neq 0$, and hence y_t displays a stochastic trend, I(1), as well as a deterministic one because $E(y_t)$ grows linearly with t. Such series are called integrated with drift.

2.2. Distribution. The null hypothesis to be tested is that x_t and hence y_t is integrated of order one. Under the null hypothesis there is a unit root in the AR polynomial and we have because of (4):

$$H_0 : \rho - 1 = 0.$$

This hypothesis can be tested directly by estimating (5) with least squares and using the t-statistic of $\hat{\rho} - 1$. It is a one-sided test that rejects in favour of stationarity if $\hat{\rho} - 1$ is significantly negative. The limiting distribution was discovered by Fuller (1976) and Dickey and Fuller (1979). It turned out that it is not centered around zero (but rather shifted to the left) and not symmetrical. In particular, limiting standard normal theory is invalid for $\rho = 1$, while it does apply in case of stationarity ($|\rho| < 1$). Similarly, the t-distribution is not a valid guideline for unit root testing in finite samples. The limiting distribution is very sensitive to the specification of d_t. Fuller (1976) and Dickey and Fuller (1979) consider three cases: No deterministics ($d_t = 0$), just a constant, and a constant and a linear trend[2]. Critical values for those cases have first been provided by simulation in Fuller (1976, Table 8.5.2, p.373). Nowadays, somewhat more precise critical values are widely employed, which have been derived using more intensive simulations by MacKinnon (1991, p.275). For a test with $T = 100$ at the 5 % level the critical values are -2.89 and -3.46, respectively, if d_t contains only a constant

[2] Critical values for polynomials in t up to order 5 are derived in Ouliaris, Park and Phillips (1989). Further, Kim and Schmidt (1993) established experimentally that conditionally heteroskedastic errors have little effect on DF tests as long as there is only moderate heteroskedasticity.

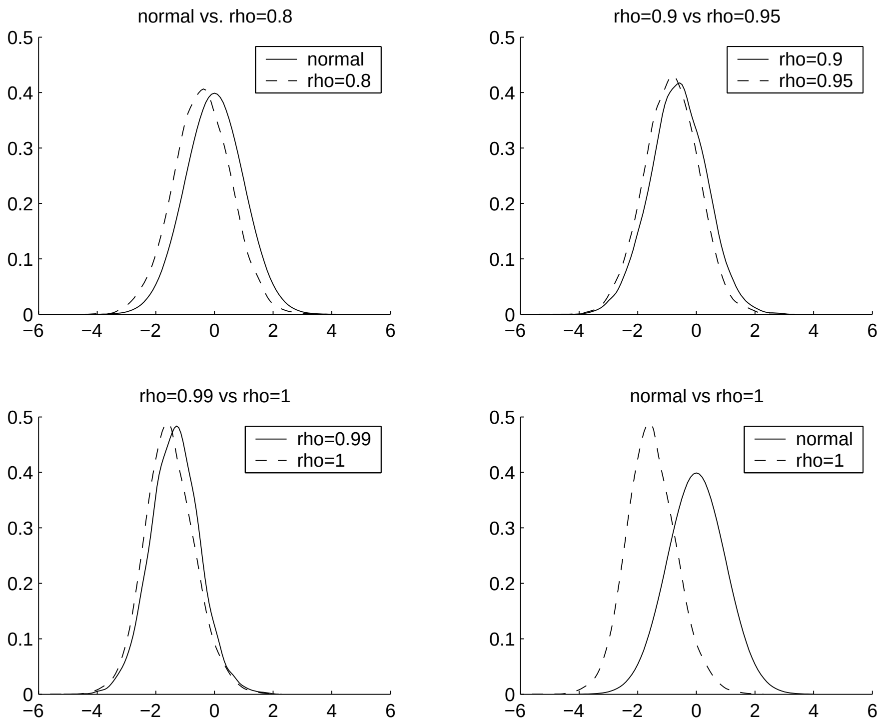

FIGURE 1. Estimated density functions ($T = 100$) and standard normal.

and if d_t includes a constant and a linear trend. These values are larger in absolute terms than the corresponding critical value of the t-distribution which is -1.65. Using the incorrect t-distribution the null hypothesis would be rejected much too often. The decision would wrongly be in favour of stationarity or trendstationarity despite the fact that the time series contains an I(1) component.

Theoretically, $\rho = 1$ is a singularity in the parameter space in that for any $|\rho| < 1$ limiting normality holds true, while $\rho = 1$ results in the non-normal Dickey-Fuller distributions. Some economists blame this as being artificial and claim that the true ρ equals 1 with probability zero. According to this, the normal approximation should be applied throughout. In pratice however, and that means in finite samples, the distinction between stationarity and I(1) is not so clear-cut[3]. Evans and Savin (1981, p.763) observed that for ρ "*near but below unity this distribution function is very poorly approximated by the limiting normal even for large values of* T". Similar evidence has been collected by Evans and Savin (1984) and Nankervis and Savin (1985). Evans and Savin (1981, 1984) considered a normalization different from the t-statistic, while Nankervis and Savin (1985) considered the usual

[3] In fact, Phillips (1987a) presented a unifying local-to-unity theory bridging the gap from stationarity to I(1) by introducing a time dependent ρ, $\rho_T = \exp\left(\frac{c}{T}\right)$.

studentization but did not provide graphs for the t-statistics. Therefore, we want to add the results of a small Monte Carlo study here[4]. The true DGP is (with $y_0 = 0$)

$$y_t = \rho y_{t-1} + u_t, \quad u_t \sim \mathcal{N}(0,1),$$

for $t = 1, ..., T = 100$. Relying on

$$\Delta y_t = \hat{a} + (\hat{\rho} - 1) y_{t-1} + \hat{u}_t,$$

the usual t-statistic is computed. Figure 1 displays density estimates that are constructed from 10000 replications by smoothing with a normal kernel. From those results we learn that even for ρ considerably smaller than 1 the standard normal approximation may be a very bad guideline in finite samples.

3. Size and power considerations

Since the work by Schwert (1989) it has been documented in several papers that the DF test may be over-sized in situations of practical importance. Hence, proposals how to control for the probability of a type I error have attracted a lot of attention in the last decade. At the same time DF tests are blamed for poor power[5], and many papers tackled the problem to increase power. Several aspects related to those topics are addressed next.

3.1. Lag length selection.

The lag length k in (6) and (7) has to be chosen to ensure that the residuals empirically follow a white noise process. Said and Dickey (1984) prove that the ADF test in (5) is valid if the true DGP is an ARMA process of unknown order, provided that the lag length k in the autoregression increases with the sample size but at a lower rate. A proof under more general conditions was recently provided by Chang and Park (2002). In practice, the choice of k is a crucial and difficult exercise. On the one hand, a growing number of lags reduces the effective sample while the number of estimated parameters is increased, and this reduction in degrees of freedom will result in a loss of power. On the other hand, k has to be large enough for the residuals to be approximately uncorrelated in order for the limiting theory to be valid. Empirical researchers often start with a maximum lag length k_{max} and follow a sequential general-to-specific strategy, i.e. reduce lags until reaching significance at a prespecified level. Here, significance testing builds on a limiting standard normal distribution. Alternatively, k may be determined relying on information criteria. The performance of the two strategies in finite samples has been investigated by Ng and Perron (1995). In particular, information criteria tend to choose k

[4] All programming was done by Mu-Chun Wang in MATLAB.

[5] Gonzalo and Lee (1996), however, illustrated by means of Monte Carlo experiments that testing for $\rho = 1$ results in rejection frequencies very similar to those available if $|\rho| < 1$.

too small to get the size correct. Therefore, Ng and Perron (2001) proposed adequately modified criteria.

The AR approximation is particularly poor in case of moving average roots close to one. Consider as DGP

$$\Delta y_t = a + u_t - \theta u_{t-1}, \quad |\theta| < 1.$$

With θ close to one the polynomial $1 - \theta L$ almost cancels with $\Delta = 1 - L$, and the true null of integration will be rejected frequently, resulting in a test where the empirical size is above the nominal one, cf. Schwert (1989). In such a situation one may use the procedure proposed by Said and Dickey (1985) that explicitly takes into account the MA component of a series.

3.2. Deterministic components. When performing unit root tests an appropriate specification of the deterministics in (5) is of crucial importance. First, consider the case where the true DGP is trend stationary. If the ADF regression (6) without detrending is applied, then the test has asymptotically no power, which was shown by West (1987) and Perron (1988). In finite samples the null hypothesis of a unit root is rarely rejected, and is never rejected in the limit. Hence, we propose to include a linear trend as in (7) whenever a series is suspicious of a linear trend upon visual inspection[6]. Notice that the decision about time as regressor may not build on the standard t-statistic of the estimate $\hat{b}$. Second, assume the other way round that a detrended test is performed from (7) while the data does not contain a linear trend. In this situation a test from (6) without detrending would be more powerful. The effect of ignoring eventual mean shifts in the DGP when specifying d_t will be discussed in the following section, while the effect of neglected seasonal deterministics will be touched upon in the next subsection.

The treatment of the deterministic component plays a major role when it comes to power of unit root tests. Elliott, Rothenberg and Stock (1996) and Hwang and Schmidt (1996) proposed point optimal unit root tests with maximum power against a given local alternative $\rho_T = 1 - \frac{c}{T}$ for some specified constant $c > 0$. Power gains are obtained by efficiently removing the deterministic component under the alternative (using Generalized Least Squares, GLS). Use of GLS, however, amounts to the following procedure, see also Xiao and Phillips (1998). First, compute quasi-differences of the observed variable and the deterministic regressors,

$$\Delta_c y_t = y_t - \left(1 - \frac{c}{T}\right) y_{t-1}\,, \quad \Delta_c z_t = z_t - \left(1 - \frac{c}{T}\right) z_{t-1}\,,$$

where z_t is a deterministic vector such that d_t from (1) is parameterized as $d_t = \gamma' z_t$. Second, estimate the vector γ by simply regressing $\Delta_c y_t$ on $\Delta_c z_t$. Third, apply the ADF test without deterministics to the residuals from the

[6] Ayat and Burridge (2000) investigated a more rigorous sequential procedure to determine the appropriate deterministics when testing for unit roots.

second step. The distribution and hence critical values depend on the choice of c. Yet another approach to obtain more powerful unit root tests has been advocated by Shin and So (2001). They proposed to estimate γ by $\hat{\gamma}_t$ with information only up to t, and remove the deterministic component from y_t as follows: $\tilde{y}_t = y_t - \hat{\gamma}'_{t-1} z_{t-1}$. Applying an ADF type regression to $\tilde{y}_t$ results in a limiting distribution again different from Dickey-Fuller. Critical values have been provided by Shin and So (2001, Table II) for the simplest case of a constant where $z_t = 1$.

3.3. Span vs. frequency. In practice, it often happens that data are available only for a fixed time span due to some structural breaks or institutional changes. In such a situation it is often recommended to use data with higher frequency to increase the number of observations and hence the power of tests. To that end people often work with monthly data instead of quarterly or annual observations. Perron (1989) has analyzed the power of some unit root tests when the sampling interval is varied but the time span is hold fixed. A general outcome of his computer experiments is that tests over short time span have low power, which is not significantly enhanced by choosing a shorter sampling interval. For related results see also Shiller and Perron (1985).

Moreover, in many cases higher frequency of observations comes at the price of additional seasonal dynamics that have to be modelled. In case of deterministic seasonal pattern it is important to remove the seasonality by including seasonal dummies in the regression. Dickey, Bell and Miller (1986) prove that the inclusion of seasonal dummies instead of a constant does not affect the limiting distribution of DF tests, while Demetrescu and Hassler (2004) demonstrate that neglecting seasonal deterministic results in tests with low power and bad size properties at the same time. Another way of removing seasonal deterministics is simply to work with seasonal differences, which, however, can not be recommended in general. Hassler and Demetrescu (2005) argue that seasonal differencing may introduce artificial persistence into a time series and may hence create spurious unit roots.

Given a fixed time span of data the purpose of unit root testing is not to investigate the true nature of some abstract economic process but to describe the degree of persistence in a given sample. Even if difference stationarity is not a plausible theoretical model as $T \to \infty$ for economic series such as inflation rates, interest rates or unemployment rates, the unit root hypothesis may still provide an empirically valid description. In that sense the significance against $H_0 : \rho = 1$ may be understood as strength of mean-reversion in a given sample. Similarly, Juselius (1999, pp.264) argues that "*the order of integration of a variable is not in general a property of an economic variable but a convenient statistical approximation to distinguish between the short-run, medium-run and long-run variation in the data.*"

4. Structural breaks

4.1. Ignoring breaks. Consider for the moment the regression with a constant only,

$$\Delta y_t = \hat{a} + (\hat{\rho} - 1) y_{t-1} + \hat{u}_t \,. \tag{8}$$

It is now assumed that the regression (8) is misspecified: The true process is I(0), but it displays a break in the mean at time λT,

$$y_t = \begin{cases} x_t \,, & t < \lambda T \\ x_t + \mu \,, & t \geq \lambda T \end{cases} , \quad x_t \sim I(0), \tag{9}$$

where $\lambda \in (0,1)$, and $\mu \neq 0$. Given (9), neither the null nor the alternative hypothesis of the DF test holds true. Perron (1990) proves that $\hat{\rho}$ converges to a value that approaches 1 as the break $|\mu| > 0$ is growing. A corresponding result is found in Perron (1989a) in case of the detrended version of the DF test,

$$\Delta y_t = \hat{a} + \hat{b}\, t + (\hat{\rho} - 1) y_{t-1} + \hat{u}_t. \tag{10}$$

This means that for a considerable break $\mu \neq 0$ the wrong null of a unit root is hardly rejected. A high probability of a type II error arises because the DF regression is misspecified in that it does not account for the structural break in the data. See also the intuitive discussion in Rappoport and Reichlin (1989). Moreover, Perron (1989a) investigates the situation of trend stationary series with a break:

$$y_t = \begin{cases} x_t + \delta\, t \,, & t < \lambda T \\ x_t \; (+\mu) + (\delta + \tau)\, t \,, & t \geq \lambda T \end{cases} , \quad x_t \sim I(0).$$

For $\tau \neq 0$ we have a break in the slope of the trend, while there may be an additional shift in the level ($\mu \neq 0$) or not. With this assumption Perron (1989a) investigates $\hat{\rho}$ from the detrended DF test (10). His asymptotic formulae were corrected by Montañés and Reyes (1998), but without changing the empirically relevant fact: Given a trend stationary series with a break in the linear trend there is little chance to reject the false null hypothesis of integration. Those results opened a new research avenue aiming at the discrimination between unit roots and structural shifts as potential causes of economic persistence.

Quite surprisingly, the opposite feature to that discovered by Perron (1989a, 1990) has been established by Leybourne, Mills and Newbold (1998): If a unit root process is subject to a mean shift, then the probability of rejecting the null of a unit root is not equal to the level of the test[7]. The

[7] This, however, is only true in finite samples or if the break is growing with the number of observations. If in contrast the break is finite, then the level of the DF test is not affected asymptotically, see Amsler and Lee (1995). Still, power considerations suggest a correction for potential breaks.

authors assume

$$y_t = \begin{cases} x_t\,, & t < \lambda T \\ x_t + \mu\,, & t \geq \lambda T \end{cases}, \quad x_t \sim I(1),$$

with $\mu \neq 0$. Leybourne, Mills and Newbold (1998) prove that the limiting distribution of the DF statistic depends on λ if μ is growing with T. Experimentally, they establish that the empirical level is well above the nominal one in case of an early break, $\lambda \leq 0.1$; if, however, the break occurs in the second half of the sample, then the DF test is conservative in the sense that the rejection frequency is below the nominal level. More generally, Kim, Leybourne, and Newbold (2004) have shown that the results of the ADF test (6) including only a constant term is highly unpredictable, if the true deterministic is a broken trend.

4.2. Correcting for breaks. To avoid spurious unit roots due to structural breaks, Perron (1989a, 1990) suggested to test for integration after removing structural breaks[8]. Unfortunately, this changes the limiting distributions depending on the break fraction $\lambda \in (0,1)$. To correct for a break in the level we need the step dummy

$$s_t(\lambda) = \begin{cases} 0\,, & t < \lambda T \\ 1\,, & t \geq \lambda T \end{cases}.$$

Now, all the deterministic components are removed from the observed series in a first step by an OLS regression[9],

$$y_t = \tilde{a} + \tilde{\mu}\, s_t(\lambda)\ (+\tilde{b}\, t\,) + \tilde{x}_t. \tag{11}$$

Next, the zero mean residuals $\tilde{x}_t$ are tested for a unit root. However, the validity of the asymptotic percentiles requires the inclusion of the impulse dummy

$$\Delta s_t(\lambda) = s_t(\lambda) - s_{t-1}(\lambda).$$

The necessity to include (lagged values of) $\Delta s_t(\lambda)$ has been recognized only by Perron and Vogelsang (1992, 1993) although it is well motivated by (5):

$$\Delta\tilde{x}_t = (\hat{\rho} - 1)\tilde{x}_{t-1} + \sum_{i=1}^{k} \hat{a}_i \Delta\tilde{x}_{t-i} + \sum_{i=0}^{k} \hat{\alpha}_i \Delta s_{t-i}(\lambda) + \hat{u}_t. \tag{12}$$

[8] Perron (1994) provides a very accessible survey.

[9] Lütkepohl, Müller and Saikkonen (2001) and Saikkonen and Lütkepohl (2001) suggested to remove the deterministic components efficiently in the sense of Elliott, Rothenberg and Stock (1996). This approach of working with quasi-differences has the advantage of yielding limiting distributions independent of λ. The same property has the test by Park and Sung (1994).

Critical values when testing for $\rho = 1$ in (12) are found in Perron (1989a, 1990). They depend on the break fraction λ that is assumed to be known. Similarly, Perron (1989a) considered modifications of the detrended DF test allowing for a shift in the slope of the linear time trend.

If λ is not known the break point can be estimated from the data. Zivot and Andrews (1992) e.g. suggested to vary λ and to compute the test statistic $ADF(\lambda)$ for each regression. Then the potential break point can be determined as $\hat{\lambda} = \arg\min_\lambda ADF(\lambda)$. Confer also Banerjee, Lumsdaine and Stock (1992) and Christiano (1992).

4.3. Smooth transitions and several breaks. When defining the step dummy $s_t(\lambda)$ we assumed a sudden change at $t = \lambda T$. But even if the cause of a change occurs instantaneously, its effect most likely evolves gradually over a period of transition. To account for that effect Perron (1989a) proposed the so-called innovational outlier model, which assumes that "the economy responds to a shock to the trend function the same way as it reacts to any other shock", Perron (1989a, p.1380). This amounts to adding a step dummy variable to the augmented Dickey-Fuller regression instead of applying the ADF test after removing all deterministics. Leybourne, Newbold and Vougas (1998) considered unit root testing in the presence of more general deterministic smooth transition functions, see also Lin and Teräsvirta (1994), however without providing asymptotic theory. Limiting results under smooth transitions have been established in Saikkonen and Lütkepohl (2001) for known breakpoint, and in Saikkonen and Lütkepohl (2002) in case the date of the break is not known a priori. For a comparison of related tests see also Lanne, Lütkepohl and Saikkonen (2002).

Lumsdaine and Papell (1997) allowed for two break points where both break dates are assumed to be unknown. Park and Sung (1994) dealt with the case of several breaks, however at known time. Kapetanios (2005) combined both features, i.e. more than two breaks occuring at unknown break points. Our personal opinion, however, is that the data-driven estimation of m break dates for a given series should not be a recommended strategy unless it is possible to reveal an economic event or institutional change behind each eventual break.

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