Liesenfeld, Roman; Hogrefe, Jens; Aßmann, Christian

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by Christian Aßmann, Jens Hogrefe and
Roman Liesenfeld

C|A|U
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The Decline in German Output Volatility: A Bayesian Analysis

Christian Aßmann[*]
Department of Economics, Christian-Albrechts-Universität Kiel, Germany

Jens Hogrefe
Department of Economics, Christian-Albrechts-Universität Kiel, Germany

Roman Liesenfeld
Department of Economics, Christian-Albrechts-Universität Kiel, Germany

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Abstract

Empirical evidence suggests a sharp volatility decline of the growth in U.S. gross domestic product (GDP) in the mid-1980s. Using Bayesian methods, we analyze whether a volatility reduction can also be detected for the German GDP. Since statistical inference for volatility processes critically depends on the specification of the conditional mean we assume for our volatility analysis different time series models for GDP growth. We find across all specifications evidence for an output stabilization around 1993, after the downturn following the boom associated with the German reunification. However, the different GDP models lead to alternative characterizations of this stabilization: In a linear AR model it shows up as smaller shocks hitting the economy, while regime switching models reveal as further sources for a stabilization, a narrowing gap between growth rates during booms and recessions or flatter trajectories characterizing the GDP growth rates. Furthermore, it appears that the reunification interrupted an output stabilization emerging already around 1987.

Keywords: business cycle models; Gibbs sampling; Markov Chain Monte Carlo; regime switching models; structural breaks

[*]Corresponding author. Tel.: +49-431-8803399; fax: +49-431-8807605. *E-mail address:* christian.assmann@stat-econ.uni-kiel.de (C. Aßmann)

1. Introduction

Recent empirical literature documents strong evidence for a sharp volatility reduction of the growth in post-war U.S.-output. While the matter of increased output stability was already risen by Burns (1960), Kim and Nelson (1999) and McConnell and Perez-Quiros (2000) were the first to present explicit empirical evidence for a sharp reduction of output volatility manifested as a structural break occurring in the mid-1980s. Confirmed by numerous empirical studies, including those of Chauvet and Potter (2001), Stock and Watson (2002) and Kim et al. (2004), this phenomenon now rates as a stylized fact. Possible explanations for the volatility decline discussed in the literature are improvements of macroeconomic and monetary policy, a better inventory management or simply a reduction in size of the random shocks hitting the economy; see Stock and Watson (2002) for a comprehensive review of this literature. In the meantime, the matter of output stabilization has been widened to an international context and the corresponding empirical evidence suggests that the volatility reduction is not a U.S.-specific phenomenon. In particular, the output volatility in most industrialized countries declined over the post-war period, even though the timing of the reduction differs across countries (see, e.g., van Dijk et al., 2002, Mills and Wang, 2003 and Stock and Watson, 2003).

In contrast to the U.S., the volatility reduction of the German output has not attracted a lot of research so far. Exceptions are the cross-country comparisons of Mills and Wang (2003) and Stock and Watson (2003), analyzing the German output within a panel of G-7 countries, as well as the studies of Buch et al. (2004) and Fritsche and Kuzin (2005), focussing exclusively on the German output. Moreover, the empirical results of these studies regarding timing and stochastic characterization of the volatility reduction in the German output are mixed. Using a Markov-switching framework for the German GDP growth rate, Mills and Wang (2003) find a structural break around 1974 in form of a reduced conditional variance. In contrast, Buch et al. (2004) report a reduction of the conditional variance since the early-1990s, an empirical result which is based upon a linear autoregressive (AR) representation for the output gap (measuring the cycle component of output). This result is in accordance with those of Stock and Watson (2003), which are obtained under an AR model with random coefficients and stochastic volatility, indicating that the conditional variance of GDP growth experienced a sharp decline towards more stabilization around 1993. Buch et al. (2004) link their results with the German reunification and argue that the implied adjustment processes led to less clear cut evidence for a break in output volatility compared with the U.S. Finally, the results of Fritsche and Kuzin (2005), which are somewhat at odds with those of Buch et al. and Stock and Watson, suggest that the output stabilization is due to a change in the persistence predicted under an AR model, leading to a lower unconditional variance of GDP growth. The corresponding reported change occurred during the mid-1970s.

In this paper, we revisit the volatility reduction in the German GDP and focus on the robustness of the evidence for such a reduction as well as on the identification of its timing. For this purpose, we consider different alternative time series models, that are used in the literature to characterize the conditional mean of GDP growth rates since, as argued, for example, by Kim and Nelson (1999), the inference with respect to the (in)stability of the volatility critically depends on the assumed specification of the conditional mean for the growth rate. Furthermore, they point out that an observed volatility reduction might be due to a (structural) change in the conditional mean or in the variance of the innovations. Accordingly, they study the U.S. volatility reduction using a flexible Markov-switching (MS) model of GDP growth and allow for a structural break in the variance of innovations as well as in the gap between average growth rates during booms and recessions.

Here we analyze the volatility reduction using a linear AR model for GDP growth (as it is assumed in the studies of McConnell and Perez-Quiros, 2000, Stock and Watson, 2002 and Kim et al., 2004) as well as more flexible non-linear regime-switching specifications. In particular, we consider a MS model a là Hamilton (1989) and, in addition, a modified version of the regime-switching model recently introduced by DeJong et al. (2006a) (DLR model hereafter). The MS model (as it is used by Kim and Nelson, 1999) characterizes booms and recessions as switches in the growth rate between high and low states, governed by a latent Markov process. In contrast, under the DLR model of DeJong et al., where GDP growth follows trajectories that fluctuates stochastically between periods of acceleration and deceleration, switches are triggered by an observable 'tension index'. A further important feature of the DLR model is that the trajectories are allowed to differ across regimes.

Following the above cited literature, we investigate structural breaks in the error variance of the GDP growth rate equations under the considered business cycle models. As additional sources of a volatility reduction, we consider a narrowing gap between growth rates during booms and recessions (under the MS model) and lower slopes for the growth trajectories (under the DLR model). The volatility analysis is carried out using a Bayesian model comparison framework as proposed in Kim and Nelson (1999) and Kim et al. (2004). Such a Bayesian analysis has the following advantages over classical approaches for evaluating structural breaks. In contrast to classical test procedures, a Bayesian approach delivers as an immediate byproduct estimates for the (posterior) distribution of the unknown break date. Furthermore, a Bayesian analysis of a structural break based on a comparison of marginal likelihoods explicitly incorporates in a coherent way sample information about the unknown timing of the structural break. This information is typically ignored in a classical test framework.

The outline of the paper is as follows. In Section 2 we describe the data. The stochastic models for the GDP growth rate are introduced in Section 3. This section also provides a description of the corresponding Bayesian inference. The estimation results are given in Section 4 and Section 5 concludes.

2. Data

Data used is quarterly, seasonally adjusted German real GDP spanning 1970:I trough 2003:IV. [1] The data were obtained from the data base of the International Monetary Fund. We compute the output growth, denoted by g_t, as differences of log real GDP, annualized by multiplying the differences by 400. In order to account for the German reunification, we use up to 1991 growth rates of the West German GDP and after 1991 the corresponding growth rates for reunified Germany, while the rate at the matching point (1991:II) is set equal to the average growth rate of the surrounding ten observations. The resulting time-series is illustrated in the top panel of Figure 1. It suggests that output volatility is lower towards the end of the sample period than during the first half of the sample. This is confirmed by the fact that prior to 1994:I, which is the break date identified using the break test procedures described below, the standard deviation of the growth rates is 4.59, whereas after 1994:I the standard deviation decreases to 2.07.

Table 1 reports results of standard classical structural break tests for unknown break dates. The results are based upon an AR(4) model of the form (more parsimonious specifications yield similar results):

$$g_t = \mu + \phi_1 g_{t-1} + \cdots + \phi_4 g_{t-4} + \epsilon_t, \quad \mathrm{E}(\epsilon_t) = 0, \quad \mathrm{E}(\epsilon_t^2) = \sigma^2, \quad t : 1 \to T. \tag{1}$$

In order to test stability in the mean and variance parameters given by $\mu, \phi_1, .., \phi_4$ and σ^2, respectively, we compute the Wald form of the Quandt (1960) statistic with a heteroscedasticity consistent covariance matrix. In particular, we use the largest Wald statistic (sup-Wald statistic) for a structural break test over all potential break dates ℓ between $T_1 = [0.2T]$ and $T_2 = [0.8T]$. Asymptotic critical values for the sup-Wald statistic are provided by Andrews (1993), and asymptotic p-values

[1] GDP data are available from 1960 onwards. However, in order to avoid problems that may occur due to the change in the German national accounting standards in 1968, we do not include data of the period before 1970.

by Hansen (1997). In addition to the sup-Wald statistic we also use the exp-Wald statistic proposed by Andrews and Ploberger (1994) which is given by $\exp W = \ln[1/(T_2 - T_1 + 1)] \sum_{\ell=T_1}^{T_2} \exp\{W(\ell)/2\}$, where $W(\ell)$ is the Wald test statistic for a potential break date ℓ.

According to the results given in Table 1, the tests yield no evidence of a break in the mean parameters except for ϕ_4. While the p-values for ϕ_4 are given by 0.03 (sup-Wald) and 0.07 (exp-Wald), indicating slight evidence of a break, the p-values of the remaining mean parameters are all above 0.34. In contrast to the mean parameters, the tests yield very strong evidence of a structural break in the error variance σ^2 and the estimate of the break date, based on the largest Wald statistic, is 1994:I. The sequence of Wald statistics for the break test of σ^2 together with the time series of the AR(4) residuals are plotted in Figure 1. The sequence of Wald statistics reveals that, despite the strong evidence of the occurrence of a structural break, there is some uncertainty associated with the exact date of the break. In particular, the Wald statistic becomes significant at the 5 percent level as early as 1987 and reaches its maximum 1994, while it is fairly flat at a high level between 1993 and 1996.

In summary, the classical test results suggest that under a linear AR model the conditional variance of the GDP growth underwent a structural break, while there is only slight evidence for a change in the conditional mean. However, while the implemented classical test procedures are effective in detecting structural breaks, they are not very precise for identifying the timing of breaks in variance parameters (see, DeJong et al., 2006b). Furthermore, the underlying linear model ignores the asymmetric nature of business cycles which could bias the inference results. In order to address these issues, we perform a Bayesian analysis typically improving the precision of the inference regarding the timing of a volatility break and consider non-linear business cycle models in order to reach more robust conclusions.

3. Model Specifications and Bayesian Inference

Our further investigation of the possible volatility reduction in the German GDP growth rate and its timing is based on a Bayesian analysis of three alternative times series specifications that are used in the literature to characterize GDP growth rates. In addition to a simple linear AR model of the form given in Equation 1, we consider a MS model and the DLR model with stochastic switches between periods of acceleration and deceleration. These alternative models are chosen in order to account for possible sensitivity of the results for the volatility to the specification of the conditional mean.

3.1 Linear Autoregressive Model

The employed AR(k) model with a structural break in the error variance is given by

$$g_t = \mu + \sum_{\ell=1}^{k} \phi_\ell g_{t-\ell} + \epsilon_t, \qquad \epsilon_t \sim \mathcal{N}(0, \sigma^2_{d_t}) \tag{2}$$

with

$$\sigma^2_{d_t} = \sigma^2_0(1-d_t) + \sigma^2_1 d_t, \quad d_t = \begin{cases} 0, & t \le t^* \\ 1, & t > t^* \end{cases} . \tag{3}$$

The variable d_t represents a latent state variable that governs the unknown date of the structural break in the error variance t^*. Following Kim and Nelson (1999), d_t is assumed to follow a restricted two-state Markov chain characterized by the probabilities

$$P(d_{t+1} = 0 | d_t = 0) = \upsilon, \quad P(d_{t+1} = 1 | d_t = 1) = 1, \quad \upsilon \in (0, 1). \tag{4}$$

Note that this specification allows for only one break with a break probability in period t given by $P(d_{t+1} = 1 | d_t = 0) = 1 - \upsilon$. After a break has occurred, say in $t = t^*$, the state variable remains in state $d_t = 1$ with probability one. In order to test for a structural change in the error variance, this AR specification with a break is compared with the restricted specification without a break, obtained by setting $\sigma^2_0 = \sigma^2_1$.[2]

A Bayesian analysis of these models requires appropriate prior distributions for the parameter vector $\Theta = (\mu, \phi_1, ..., \phi_k, \sigma^2_0, \sigma^2_1, \upsilon)$ and relies upon the corresponding posterior distributions given the data $G_T = (g_1, ..., g_T)'$. To implement the Bayesian analysis, the parameter vector Θ is augmented to include the latent state variables $D_T = (d_1, ..., d_T)'$. Then the Gibbs sampling technique is used to sample from the joint posterior distribution $f(\Theta, D_T | G_T)$. This technique is based upon appropriate conditional posterior distributions in order to construct a Markov Chain whose equilibrium distribution is the joint posterior $f(\Theta, D_T | G_T)$ (see, e.g., Koop, 2003). The parameters in Θ are estimated by reporting appropriate statistics for their simulated values.

In our application below we assume natural conjugate priors leading to conditional posteriors from the same family as the prior distributions from which draws can be easily generated. In particular, we use for $(\mu, \phi_1, ..., \phi_k)$ a multivariate Normal prior, for $1/\sigma^2_0$ and $1/\sigma^2_1$ Gamma priors and for υ a Beta prior, leading to conditional posteriors given by a multivariate normal distribution, Gamma distributions and a Beta distribution, respectively. The particular selection of the hyper-parameters

[2]Since the preliminary results for the AR model in Section 2 indicate only slight evidence for a break in the mean parameters, we do not consider this possibility within the following Bayesian analysis of the AR model.

in the priors will be discussed below. The full conditional posterior distribution for the state variable D_T is implicitly given by the conditional posterior of the break date, which has the form:

$$P(t^* = \tau | G_T, \Theta) \propto (1 - \upsilon)\upsilon^{\tau - T_1 - 1} f(G_T | \Theta, t^* = \tau), \quad T_1 < \tau < T_2, \tag{5}$$

where $f(G_T|\Theta, t^* = \tau)$ denotes the conditional likelihood given a break date t^* in period τ, while T_1 and T_2 define the range of possible break dates. In our application below, we use $T_1 = [0.1T]$ and $T_2 = [0.9T]$.

For a Bayesian approach to hypotheses testing and model comparison one can use the ratio of the marginal likelihoods (Bayes factor) for the hypotheses or models to be compared. For a comparison of the AR model with a break ($\mathcal{M}_{AR,1}$) with the corresponding AR model without a break ($\mathcal{M}_{AR,0}$) the Bayes factor is

$$B_{AR,10} = \frac{f(G_T|\mathcal{M}_{AR,1})}{f(G_T|\mathcal{M}_{AR,0})}, \tag{6}$$

where $f(G_T|\mathcal{M}_{AR,i})$, $i = 0, 1$ represents the marginal likelihood under the model $\mathcal{M}_{AR,i}$. The marginal likelihood is obtained as the corresponding integrating constant of the posterior distribution for Θ. Following Kim and Nelson (1999), we use the procedure of Chib (1995) in order to evaluate that likelihood. This procedure is based on the following representation of the marginal likelihood

$$f(G_T|\mathcal{M}_{AR,i}) = \frac{f_i(G_T|\Theta)\pi_i(\Theta)}{f_i(\Theta|G_T)}, \qquad i = 0, 1, \tag{7}$$

where $f_i(G_T|\Theta)$, $\pi_i(\Theta)$ and $f_i(\Theta|G_T)$ denote the likelihood, the prior and the posterior density, respectively. While for a given value of Θ (here we use its posterior mean), the likelihood and the assumed prior densities can be evaluated directly, the computation of the joint posterior of Θ, which is typically not available in an analytical closed-form solution, requires Monte Carlo techniques. For this purpose, the joint posterior of the parameters $f(\Theta|G_T)$ is factorized as $f(\Theta|G_T) = f(\Theta_1|G_T)f(\Theta_1|\Theta_2, G_T) \cdots f(\Theta_P|\Theta_1, ..., \Theta_{P-1}, G_T)$ according to an appropriate partitioning of the parameter vector $\Theta = (\Theta_1, \Theta_2, ..., \Theta_P)'$. Then the individual factors of the joint posterior are evaluated by Monte Carlo integration based on Gibbs draws of the parameters (for details, see Kim and Nelson, 1999).

For a comparison of the specifications based upon the Bayes factor we use the scale proposed by Jeffreys (1961). According to this scale $\ln B \leq 0$ is interpreted as evidence for the specification under H_0, while $0 < \ln B \leq 1.15$ indicates very slight evidence against H_0, $1.15 < \ln B \leq 2.3$ slight evidence, $2.3 < \ln B \leq 4.6$ strong to very strong evidence, and $4.6 < \ln B$ decisive evidence against the H_0-specification.

3.2 Markov Switching Model

It has long been recognized that successful modeling of GDP growth rates hinges critically on the ability to account for the asymmetric and non-linear nature of business cycles. A popular class of models which explicitly takes into account such features of business cycles are the MS models with different behavior in economic contractions and expansions.

Following Kim and Nelson (1999), we consider a MS specification which allows in its most general version for a structural break in the error variance as well as in the gap between average growth rates during expansions and contractions. In particular, we specify the growth rates as[3]

$$g_t = \bar{\mu}_{s_t} + \sum_{\ell=1}^{k} \phi_\ell g_{t-\ell} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma^2_{d_t}), \tag{8}$$

where the error variance $\sigma^2_{d_t}$ is specified as under the linear AR model by equations (3) and (4), while the assumed specification of the intercept is

$$\begin{aligned} \bar{\mu}_{s_t} &= \bar{\mu}_{0d_t}(1-s_t) + \bar{\mu}_{1d_t} s_t, \qquad \bar{\mu}_{0d_t} < \bar{\mu}_{1d_t} \\ \bar{\mu}_{0d_t} &= \mu_{00}(1-d_t) + \mu_{01} d_t \\ \bar{\mu}_{1d_t} &= \mu_{10}(1-d_t) + \mu_{11} d_t. \end{aligned} \tag{9}$$

Here d_t is the state variable indicating a structural break as defined in equations (3) and (4), while s_t is a latent variable that indicates the recurrent business cycle phase (with intercepts during expansions and contractions given by $\bar{\mu}_{1d_t}$ and $\bar{\mu}_{0d_t}$). It is assumed that s_t follows a two-state Markov process characterized by the transition probabilities

$$\begin{aligned} P(s_{t+1}=1|s_t=1) &= \quad p, \qquad P(s_{t+1}=0|s_t=0) = \ q, \qquad p,q \in (0,1), \\ P(s_{t+1}=0|s_t=1) &= \ 1-p, \qquad P(s_{t+1}=1|s_t=0) = 1-q. \end{aligned} \tag{10}$$

Note that under this MS model the error variance $\sigma^2_{d_t}$ and the two intercepts $\bar{\mu}_{0d_t}$ and $\bar{\mu}_{1d_t}$ are allowed to undergo simultaneously a structural break with an unknown break date at t^*. Hence, it allows for two sources of a volatility reduction: a decline in the error variance as well as a narrowing gap between the mean growth rates during expansions and contractions.

In order to investigate the output stabilization, we compare the following three versions of the MS model (8)-(10), (3), (4): The model without any structural break ($\mathcal{M}_{MS,0}$), the model with a

[3] In contrast to Kim and Nelson (1999), who use a specification with regime switches in the unconditional mean, we assume a MS model with switches in the intercept, because preliminary experimentation indicates that the latter specification provides a slightly better description of the German GDP.

structural break just in the error variance ($\mathcal{M}_{MS,1}$), and the model with a break in both the intercept and the error variance ($\mathcal{M}_{MS,2}$).[4] These specifications are obtained as

$$\begin{aligned} \mathcal{M}_{MS,0} &: \mu_{00} = \mu_{01} \quad \text{and} \quad \mu_{10} = \mu_{11} \quad \text{and} \quad \sigma_0^2 = \sigma_1^2 \\ \mathcal{M}_{MS,1} &: \mu_{00} = \mu_{01} \quad \text{and} \quad \mu_{10} = \mu_{11} \quad \text{and} \quad \sigma_0^2 \neq \sigma_1^2 \\ \mathcal{M}_{MS,2} &: \mu_{00} \neq \mu_{01} \quad \text{and} \quad \mu_{10} \neq \mu_{11} \quad \text{and} \quad \sigma_0^2 \neq \sigma_1^2. \end{aligned}$$

A Bayesian analysis of the MS specifications can be performed analogously to that for the AR model discussed above. In particular, the parameters summarized in the vector $\Theta = (\mu_{00}, \mu_{01}, \mu_{10}, \mu_{11}, \phi_1, ..., \phi_k, \sigma_0^2, \sigma_1^2, \upsilon, p, q)$ are augmented to include the sequences of latent state variables D_T and $S_T = (s_1, ..., s_T)'$ and then the Gibbs sampling procedure is used to simulate from the joint posterior $f(\Theta, D_T, S_T|G_T)$. As for the AR model, we assume natural conjugate priors for the parameters in Θ. Accordingly, we employ for the additional set of parameters in the MS model, given by $(\mu_{00}, \mu_{01}, \mu_{10}, \mu_{11})$ and (p, q), Normal and Beta priors, respectively. Furthermore, to simulate the state variables S_T from their full conditional posterior distribution, we utilize the fact that it has the form of a multinomial Bernoulli distribution characterized by (see, Albert and Chib, 1993a)

$$f(s_t|G_T, D_T, S_{T\backslash t}, \Theta) \propto f(s_t|s_{t-1})f(s_{t+1}|s_t)f(g_t|G_{t-1}, s_t, D_t, \Theta), \tag{11}$$

where $f(s_t|s_{t-1})$ and $f(s_t|s_{t-1})$ denote the probability density functions associated with the transition probabilities in equation (10), while $f(g_t|G_{t-1}, s_t, D_t, \Theta)$ is the corresponding conditional density of g_t, and $S_{T\backslash t}$ denotes S_T minus its tth element. The full conditional posterior for the variance states D_T has the same form as under the AR model (see, Equation 5). Finally, in order to evaluate the marginal likelihoods for the MS models to be compared, the Chib (1995) procedure, implemented for the AR-models and described above is adapted. For a detailed description of the implementation of the Gibbs sampler and of the computation of the marginal likelihoods for the MS specifications, see also Kim and Nelson (1999).

3.3 Switching Trend Model

An alternative non-linear regime-change model for GDP growth rates is the DLR model of DeJong et al. (2006a), which is designed to account for the observed heterogeneity in the behavior of growth rates across the business cycles. Under this model, growth rates follow trajectories that fluctuate

[4]Results for a specification with a break in intercepts only are not reported here. Preliminary analysis showed that such a specification cannot be favored against the specification without a break.

stochastically between alternative periods of general acceleration and deceleration. Furthermore, the trajectories are allowed to differ across regimes. Regime changes are triggered stochastically by an observable "tension index" constructed as the geometric sum of deviations of observed GDP growth from a corresponding "sustainable" growth rate interpreted as the growth of potential GDP. Let g_t^* denote the sustainable rate and y_t the deviations $y_t = (g_t - g_t^*)$, the tension index is given by

$$h_t = \sum_{\ell=1}^{\infty} \delta^{\ell} y_{t-\ell}, \tag{12}$$

where the parameter $\delta \in (0,1)$ governs the persistence of past deviations on current h_t. Here we specify g_t^* as the sample mean of g_t and set δ equal to 0.575, but the estimation results presented below are robust to alternative specifications of g_t^* (like, e.g., a Hodrick-Prescott filtered trend fitted to g_t) as well as to alternative values of δ between 0.5 and 0.95.

The resulting tension index is plotted in the top panel of Figure 2 together with the business cycle peaks and troughs as identified by Artis et al. (2004) using a business cycle dating procedure based on absolute declines and rises of Hodrick-Prescott filtered GDP. Observe that the h_t series tends to pass between phases of general expansions and general contractions in which g_t tends to outstrip and fall short of g_t^*, respectively. Moreover, peaks in h_t typically precede the marked business cycle peaks, and troughs tend to precede or coincide with business cycle troughs.

Under the interpretation of the DLR model neither accelerating nor decelerating phases are sustainable and both produce tension buildups that lead to corresponding regime changes. These are characterized by the following probit specification

$$P(r_{t+1} = -r_t | r_t, h_t) = \Phi(\beta_0 + \beta_1 r_t h_t), \tag{13}$$

with

$$r_t = \begin{cases} 1, & \text{if } t \text{ is in an accelerating regime} \\ -1, & \text{if } t \text{ is in a decelerating regime} \end{cases},$$

where Φ denotes a $\mathcal{N}(0,1)$-distribution function. The switching process is restricted to allow only for switches from accelerating to decelerating regimes and vice versa. With respect to the length of a regime, we assume a-priori a regime to prevail at least three periods. The specification of the growth rates in terms of their deviations from g_t^*, allowing for a break in the error variance is given by

$$y_t = m_t + \nu h_{t-1} + \gamma y_{t-1} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma_{d_t}^2), \tag{14}$$

where $\sigma^2_{d_t}$ is specified by Equations (3) and (4). The variable m_t represents a regime drift characterized by linear (local) trends of the form[5]

$$m_t = a_j + b_j r_t[t - t(j-1) - 1], \quad b_j > 0, \quad t : [t(j-1)+1] \rightarrow t(j), \tag{15}$$

where the index j ($j : 1 \rightarrow J$) denotes the regime prevailing in period t, and $t(j)$ is the regime change period from regime j to regime $j+1$ (i.e. the last period under regime j), with $t(0) \equiv 0$. The first component a_j represents the value of the regime drift in the first period of regime j, while the second part $b_j r_t[t - t(j-1) - 1]$ directs the slope of the linear m_t-trajectory during regime j. In order to account for the fact that no two business cycles are alike, the a_js and b_js are allowed two vary across regimes, leading to a corresponding variation in the properties of the drift m_t across the J regimes. In particular, we treat the intercept and slope parameters a_j and b_j as unknown parameters.[6] The heterogeneity across business cycles captured by the resulting regime drift component is illustrated by the time series plot of the m_t series computed at the estimates of the drift parameters and the regime change periods (see bottom panel of Figure 2).

Under this DLR model (12)-(15), (3), (4), a volatility reduction can be generated by a decline in the error variance $\sigma^2_{d_t}$ and/or by decreasing slopes of the trend component characterized by low values of the slope parameters b_j. Accordingly, in order to investigate the output stabilization under the DLR model, we compare the specification with a break in the error variance ($\mathcal{M}_{DLR,1}$) and that without a break ($\mathcal{M}_{DLR,0}$), and compare the estimated slopes of successive regimes. Note that while lower slopes under the DLR model and a narrowing gap between mean growth rates during expansions and contractions under a MS model have similar effects, they have slightly different interpretations: Lower slopes are part of the structure of the DLR model whereas a narrowing gap presents a change in the structure of the MS model.

For a Bayesian analysis of the DLR model the parameter vector $\Theta = (\nu, \gamma, a_1, b_1, ..., a_J, b_J, \sigma^2_0, \sigma^2_1, \beta_0, \beta_1)'$ is augmented to include the variance states D_T and the regime indicators $R_T = (r_1, ..., r_T)'$. Observe that given R_T and D_T, the growth rates follow a standard linear regression model. As for the previous models we assume natural conjugate priors for the parameters, with multivariate Normals for (ν, γ) and (β_0, β_1), truncated multivariate Normals for $(a_j, b_j > 0)$, and Gamma priors

[5]The linear trend specification assumed here represents a special case of the specification used by DeJong et al. (2006a) for the post-war US GDP growth, allowing for non-linear trends.

[6]In contrast to the "fixed-effects" specification assumed here, DeJong et al. (2006a) specify a_j and b_j as latent random variables leading to a more parsimonious "random-effects" version. Preliminary experimentations with both specification showed that there is no significant difference in the goodness of fit. However, the fixed effect model is much easier to analyze.

for $1/\sigma_0^2$ and $1/\sigma_1^2$. The corresponding Gibbs sampling algorithm which we employ to simulate from the joint posterior $f(\Theta, D_T, R_T|G_T)$, given a fixed number of regime changes $J = \bar{J}$ has the following structure:

(i) Simulate from $f(\nu, \gamma|\Theta_{\backslash\nu,\gamma}, D_T, R_T, G_T)$, which is a bivariate normal distribution;

(ii) Simulate from $f(a_j, b_j|\Theta_{\backslash a_j,b_j}, D_T, R_T, G_T)$, $(j : 1 \rightarrow \bar{J})$ which are truncated bivariate normal distributions with the truncation $b_j > 0$;

(iii) Simulate from $f(1/\sigma_k^2|\Theta_{\backslash\sigma_k^2}, D_T, R_T, G_T)$, $(k : 1, 2)$ which are Gamma distributions;

(iv) Simulate from $f(\beta_0, \beta_1|\Theta_{\backslash\beta_0,\beta_1}, D_T, R_T, G_T)$ based on an augmentation step including the latent variables underlying the probit specification, say $\{r_t^*\}$. They are simulated from the appropriately truncated N(0, 1)-distribution (see, Albert and Chib, 1993b). Given $\{r_t^*\}$, the corresponding conditional posterior of (β_0, β_1) to be simulated is a bivariate normal distribution;

(v) Simulate the variance state variables D_T according to the conditional posterior of the break date $P(t^* = \tau|G_T, R_T, \Theta)$ which has the form as under the AR model (see Equation 5);

(vi) Simulate the regime indicators R_T by drawing the $\bar{J}$ regime change dates $T_{\bar{J}} = \{t(1), ..., t(\bar{J})\}$ based on the Gibbs sequence characterized by the conditional posteriors of the individual regime-change periods $t(j)$ given the remaining regime change dates $T_{\bar{J}\backslash t(j)}$. These conditional posteriors are multinomial distributions of the form

$$P(t(j) = \tau|T_{\bar{J}\backslash t(j)}, G_T, D_T, \Theta) \propto f(G_T, t(j) = \tau|\Theta, D_T, T_{\bar{J}\backslash t(j)}), \quad j : 1 \rightarrow \bar{J}.$$

The l.h.s. represents the corresponding conditional joint likelihood for the growth rates G_T and the jth regime change date occurring at period τ. Note that the simulated regime change dates lead to corresponding simulated regime indicators R_T from the set of R_Ts associated with $\bar{J}$ regime changes $\mathcal{S}_{\bar{J}} = \{R_T(T_J) : J = \bar{J}\}$.

The results presented below are based on $\bar{J} + 1 = 8$ different regimes, a number which was selected using the Bayes factor to compare specifications under alternative Js.

In order to compare the DLR model with a break in the error variance ($\mathcal{M}_{DLR,1}$) and that without a break ($\mathcal{M}_{DLR,0}$), we evaluate the Bayes factor based on marginal likelihoods computed using (as for the AR and MS model) the procedure of Chib (1995). The unconditional likelihood

$f_i(G_T|\Theta)$ for model i, $(i = 0, 1)$, is obtained by integrating the regime indicators R_T out of the joint likelihood $f_i(G_T, R_T|\Theta)$ using Monte Carlo integration. In particular, we use an importance-sampling procedure, which utilizes the MCMC approximation to the posterior distribution of R_T (see, Richard, 1995 and Stern, 1997 for detailed descriptions of importance sampling techniques). The importance sampler we use is given by a truncated multivariate Bernoulli distribution with a probability density function

$$m(R_T) = \left[\prod_{t=1}^{T} \hat{p}_t^{r_t}(1-\hat{p}_t)^{1-r_t}\right] \mathcal{I}_{\{\mathcal{S}_{\bar{J}}\}}(R_T)/P(R_T \in \mathcal{S}_{\bar{J}}),$$

where $\hat{p}_t$ represents the estimated probability for $r_t = 1$ according to the simulations from the conditional posterior of R_T (see Step vi above), and $\mathcal{I}_{\{\mathcal{S}_{\bar{J}}\}}$ is an indicator function of the set $\mathcal{S}_{\bar{J}}$. The corresponding MC approximation of the unconditional likelihood is

$$f_i(G_T|\Theta) \doteq \frac{1}{K}\sum_{k=1}^{K} \frac{f_i(G_T, \tilde{R}_T^{(k)}|\Theta)}{m_i(\tilde{R}_T^{(k)})}, \qquad i = 0, 1, \tag{16}$$

where $\{\tilde{R}_T^{(k)}\}_{k=1}^{K}$ represents a sequence of K draws of the vector of regime indicators simulated using the importance sampler $m_i(\cdot)$.

4. Empirical Results

In this section, we present the empirical results obtained for the three alternative business cycle models. All inferences are based on 10,000 Gibbs iterations, where the initial 2,000 Gibbs draws were discarded in order to mitigate the effect of the initial conditions. For each of the three models, the Bayesian analysis is performed under different sets of prior specifications, which allows to evaluate the sensitivity of the results to prior believes. These sets are summarized in Table 2.

Tables 3 through 6 provide the Bayesian estimation results for the model specifications allowing for a structural break in the conditional variance, i.e, the specifications $\mathcal{M}_{AR,1}$, $\mathcal{M}_{MS,1}$, $\mathcal{M}_{MS,2}$, $\mathcal{M}_{DLR,1}$. The reported posterior moments and log-Bayes factors are obtained under the set (I) of prior distributions specified in Table 2. The sensitivity of the marginal likelihoods to the alternative prior assumptions are summarized in Table 7. Furthermore, Figure 3 plots the corresponding posterior distributions of the break date for the conditional variance.

4.1 AR-model

The Bayesian results of the AR-model $\mathcal{M}_{AR,1}$, provided in Table 3 indicate that the posterior mean of the error variance decreases from σ_0^2 to σ_1^2 by 72%. This reduction is of the same order of magnitude as that obtained by Kim et al. (2004) under an AR-model for the U.S. economy. Furthermore, the log of the Bayes factor given by 3.08 indicates according to the scale of Jeffreys (1961) strong evidence that this reduction is due to a structural break in the variance parameter. The posterior mean of the break date is 1993:III, nearly a decade latter than the break point of 1984 estimated by Kim et al. (2004) for the U.S. The top panel of Figure 3 shows that the posterior distribution of the break date is tightly dispersed around two modes which are located closely to each other, one in period 1993:I and the other in 1994:I. Under the alternative sets of prior distributions II and III specified in Table 2, the posterior moments (not reported here) are almost identical. This robustness with respect to the prior specification for the AR model shows also up in the fairly stable values of the log marginal likelihood across the different prior specifications (see Table 7).

Hence, these results suggest an output stabilization after the downturn following the boom period in the early-1990s associated with the German reunification. This finding confirms the result of Buch et al. (2004) who report evidence for a structural break in the volatility of the German output gap at the beginning of the 1990s – a result which is obtained using classical test procedures.

4.2 Markov Switching Model

The results of the Bayesian estimates of the MS model with a structural break in the variance parameter and that with a simultaneous break in the variance as well as in the shift parameters are summarized in Tables 4 and 5, respectively. The values of the log Bayes factors suggest that the specification without any structural break ($\mathcal{M}_{MS,0}$) is slightly dominated by the model with a structural break in the variance parameter ($\mathcal{M}_{MS,1}$) and strongly dominated by the model with a joint break in the variance and shift parameters ($\mathcal{M}_{MS,2}$). This preference in favor of specifications allowing for a break also holds for the two alternative sets of prior assumptions (see Table 7). A direct comparison of the models $\mathcal{M}_{MS,1}$ and $\mathcal{M}_{MS,2}$ indicates that the latter is strongly preferred for the set of priors (I), while this preference is less pronounced under the sets (II) and (III).

According to the posterior means in Table 4 and 5, the variance parameter experienced after the break a significant reduction of 84% for the model $\mathcal{M}_{MS,1}$ and of 69% for $\mathcal{M}_{MS,2}$. Under the model $\mathcal{M}_{MS,2}$ this reduction is accompanied by a notable decrease in the gap between the average growth rate during booms and recession from $\mu_{10} - \mu_{00}$ to $\mu_{11} - \mu_{01}$ by 43%. The posterior means of the break dates for the $\mathcal{M}_{MS,1}$ and $\mathcal{M}_{MS,2}$ model are close together and are given by 1993:II and 1993:I, respectively. The second and third panel of Figure 3 show that the corresponding posterior distributions for the break date are very similar to that under the AR model.

Figure 4, which plots the posterior recession probabilities for all three MS specifications, illustrate the effect of the structural break on the classification of booms and recessions. The plots show that for the quarters after the estimated break point around 1993 the classifications of expansions and contractions are much sharper for the specification with a break in the variance parameter than for the model without a break. In contrast, for the periods before the break these classifications are - though the difference is only marginal - less precise for the model with a break. Comparing the model with a joint break in the variance and intercept parameters with that without a break shows that the posterior probabilities for the former are sharper after the break and also, at least slightly, more pronounced before the break. Hence, these results underscore that not only a reduction in the conditional variance parameter but also a break in the intercepts seems to be relevant for the description of the output stabilization.

In summary, the results for the MS models confirm the conclusion from the AR model that there has been a structural break in the German economy around 1993, leading to less volatile GDP growth rates. Furthermore, in addition to a decrease of the shocks hitting the economy, a narrowing gap between growth during economic expansions and contractions, indicating changes in the structure of the economy, seems to contribute to this output stabilization. Finally, we notice that this characterization of the decline in German output volatility is similar to the results for the U.S. economy reported by Kim and Nelson (1999), even though the decline in U.S. output volatility emerged according to their results nearly a decade later.

4.3 Switching Trend Model

The estimates of the DLR model including a break in the conditional variance are given in Table 6. Estimates of the error variance indicate a reduction of 69%, with non overlapping 95% bands of the posteriors. The corresponding mean of the break point posterior is 1990:III, which is about three years earlier than in the AR and MS model. Furthermore, the posterior standard deviation of the break date, given by 12 quarters, is notably larger than under the AR and MS specifications, where the standard deviation ranges from 6 to 8 quarters. The bottom panel of Figure 3 shows the posterior distribution of the break date under the DLR model. It indicates that the posterior mean does not provide a sensible estimate for the location of the break. In fact, the posterior exhibits a pronounced bimodal behavior with modes (at 1987:I and 1992:I) which are fairly far away from each other. Notice, that these modes coincide with the significant decrease of the GDP in the first quarter of 1987, and the last quarter of the boom period associated with the German reunification process with a large growth rate (see Figure 1 upper panel)[7]. Interestingly, the first mode at 1987:I coincides with a rise of the Wald statistic above its critical value (see Figure 1 lower panel).

Despite the large reduction in the posterior mean of the error variance, the log Bayes factor given by 0.53 indicates only very slight evidence in favor of a structural break in the variance parameter. This low evidence for a break, which holds across all sets of priors (see Table 7), seems to reflect the comparably large uncertainty concerning the possible timing of the break date discussed above. Figure 5 illustrates the effect of the structural break in the error variance on the estimated probabilities of expansions and contractions under the DLR model. For the period after the second mode of the break date distribution (1992:I) the inference about the contraction probabilities are sharper under the model with a break than under the one without a break. In contrast, for the period before the first mode (1987:I) the probabilities of the model with a break are less pronounced than those obtained without a break. However, the differences are only marginal, which is consistent with the result that the marginal likelihood values of both specifications are close together.

As mentioned above, a further possible source of output stabilization within the DLR models is a decrease in the slope coefficients b_j of the local trends. A comparison of the estimates for the b_js across the 8 identified regimes provided in Table 6 reveals that the local trends of GDP growth are significantly flatter for the last two regimes than for the previous ones. In particular, the estimate of b_8 (given by 0.25) is the lowest value for all decelerating regimes and that of b_7 (given by 0.06) the lowest for all accelerating regimes. The estimates of the slope coefficients are visualized in the bottom panel of Figure 2 plotting the estimated trend component m_t. It illustrates that the last two regimes with estimated ranges from 1993:II to 1999:IV and 2000:I to 2003:IV, respectively, are characterized by flatter local trends than the previous regimes, suggesting a stabilization of the growth rates. This empirical result closely corresponds to the narrowing gap between growth rates during booms and recessions obtained under the MS model.

Furthermore note that the estimated regime change date (1993:I) at the beginning of the flat-trajectory episode within the DLR model is in close accordance with the timing of the structural break obtained under the AR and MS model around 1993. However, the associated interpretations

[7] The large decrease of GDP in 1987:I is mainly due to a large decrease of German exports induced by a significant revaluation of the Deutsche Mark against the U.S. Dollar and to a particularly cold winter, see the annual economic report of the Federal Ministry of Economics (1988).

of the output stabilization are slightly different: While under the AR and MS model the stabilization around 1993 is captured by a change in the structure of the model, a stabilization due to flattened trajectories is part of the structure defining the DLR-model. Furthermore, a decrease in the DLR slope coefficients leads to a gradual reduction of output volatility, whereas the estimated structural breaks within the AR and MS model immediately triggers an output stabilization.

Compared to the AR and MS model, the DLR specification, permitting a variation of the trend behavior across regimes, exhibits a greater flexibility in the mean equation, which allows to capture some of the observed heterogeneity across business cycles. This heterogeneity with comparably flat trajectories since the early-1990s seems to be one feature of the output stabilization. However, note that this flexibility in the mean equation of the DLR model comes along with a corresponding large uncertainty concerning the existence and timing of an additional reduction in the error variance. On the other hand, the flexibility of the DLR model allows for a more detailed characterization of the stabilization of the German economy than under the AR and MS model. In particular, the first mode of the break date distribution for the error variance under the DLR model suggests that a decline in the output volatility emerged already around 1987. In the early-1990s, this emerging stabilization was interrupted by the extraordinary boom associated with the German reunification and the opening of Eastern Europe and by the ensuing severe downturn. The reunification boom shows up in a sequence of growth rates which are significantly above the local trend (see bottom panel of Figure 2) and a corresponding large increase in the tension index triggering a regime change (see top panel of Figure 2). After this reunification episode, finally, the interrupted process of output stabilization seems to continue with flattened growth trajectories and a smaller error variance. In contrast to this characterization obtained under the DLR model, the period of a volatility decline predicted by the AR and MS model is concentrated only on the quarters following the reunification episode.

5. Summary and Conclusion

In this paper we analyze whether the volatility of the growth in German output has declined over the past decades like in most industrialized countries. Our analysis is based upon a Bayesian analysis of three different business cycle models for the GDP growth rates. In addition to a simple linear autoregressive (AR) model, we consider a Markov-switching (MS) model and a regime switching (DLR) model based on local trends as proposed by DeJong et al. (2006a). Within the AR model we allow for an output stabilization via a change in the error variance. Within the MS and DLR model we consider as additional sources for a stabilization a narrowing gap between the average growth rates during economic expansions and contractions, and flattened local trends characterizing the GDP growth rates, respectively.

Our empirical results based on quarterly data from 1970 to 2004 indicate a stabilization of the German output in early-1990s, in particular after the downturn following the boom period associated with the German reunification and the opening of Eastern Europe. This stabilization shows up in all estimated business cycle models. While under the AR model the stabilization is reflected only by a significant decrease in the shocks hitting the economy, the estimation results for the MS and DLR model suggest that corresponding changes in the properties of the regimes also contributes to the decline in the output volatility. In fact, we find within the MS model a notable reduction in the gap between growth rates during booms and recessions, and under the DLR model a significant decrease of the slopes characterizing the local trends for the GDP growth. The result that reduced output volatility can be traced back to a change in mean behavior is in line with the finding of Fritsche and Kuzin (2005).

A further result of our analysis is that under the AR and MS specification the posterior distribution for the date of the volatility decline is tightly concentrated on the quarters around the downturn following the reunification. In contrast under the DLR model the distribution of the break date for the error variance is much more dispersed with two pronounced modes, one after and one before the reunification episode. Hence, viewed through the lens of the DLR model, it seems that the German reunification interrupted an output stabilization emerging already before this extraordinary event hitting the German economy, an argument in line with Buch et al. (2004). The retarding moment of the German reunification on output volatility is also found by Stock and Watson (2003). In contrast Mills and Wang (2003) time the break in error variance in the early-1970ies which can hardly be compared to the findings presented here due to the fact that the the considered data sets differ in the sample length. Mills and Wang (2003) use data from 1960 onwards.

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Table 1. Classical Structural Break Tests in an AR(4) model

	H$_0$: parameter is constant					
Parameter	μ	ϕ_1	ϕ_2	ϕ_3	ϕ_4	σ^2
sup-Wald	3.91 (.34)	2.31 (.64)	2.29 (.65)	2.59 (.58)	9.35 (.03)	22.58 (.00)
exp-Wald	.63 (.34)	.23 (.70)	.30 (.61)	.44 (.46)	1.81 (.07)	8.80 (.00)
Estimated Break Date	−−	−−	−−	−−	1992 : I	1994 : I

NOTE: Asymptotic p-values are in parentheses.

Table 2. Prior Specifications

	Sets of priors		
Parameters	(I)	(II)	(III)
		AR-model	
μ	$\mathcal{N}(0,10)$	$\mathcal{N}(0,10)$	$\mathcal{N}(0,5)$
$(\phi_1,...,\phi_4)'$	$\mathcal{N}(0,10\cdot I)$	$\mathcal{N}(0,10\cdot I)$	$\mathcal{N}(0,5\cdot I)$
$1/\sigma_0^2, 1/\sigma_1^2$	$\mathcal{G}(0.02,5)$	$\mathcal{G}(0.05,2)$	$\mathcal{G}(0.01,1)$
υ	$\mathcal{B}(8,0.2)$	$\mathcal{B}(8,0.1)$	$\mathcal{B}(8,0.1)$
		MS-model	
μ_{10},μ_{11}	$\mathcal{N}(2,10)\mathcal{I}_{(0.5,\infty)}(\cdot)$	$\mathcal{N}(2,10)\mathcal{I}_{(0.5,\infty)}(\cdot)$	$\mathcal{N}(2,5)\mathcal{I}_{(0.5,\infty)}(\cdot)$
μ_{00},μ_{01}	$\mathcal{N}(0,10)\mathcal{I}_{(-\infty,0.5)}(\cdot)$	$\mathcal{N}(0,10)\mathcal{I}_{(-\infty,0.5)}(\cdot)$	$\mathcal{N}(0,5)\mathcal{I}_{(-\infty,0.5)}(\cdot)$
$(\phi_1,...,\phi_4)'$	$\mathcal{N}(0,10\cdot I)$	$\mathcal{N}(0,10\cdot I)$	$\mathcal{N}(0,5\cdot I)$
p,q	$\mathcal{B}(6,2)$	$\mathcal{B}(6,2)$	$\mathcal{B}(6,2)$
$1/\sigma_0^2, 1/\sigma_1^2$	$\mathcal{G}(0.02,5)$	$\mathcal{G}(0.05,2)$	$\mathcal{G}(0.01,1)$
υ	$\mathcal{B}(8,0.2)$	$\mathcal{B}(8,0.1)$	$\mathcal{B}(8,0.1)$
		DLR-model	
$(\nu,\gamma)'$	$\mathcal{N}(0,10\cdot I)$	$\mathcal{N}(0,10\cdot I)$	$\mathcal{N}(0,5\cdot I)$
$\{(a_j,b_j)'\}_{j=1}^8$	$\mathcal{N}(0,10\cdot I)\mathcal{I}_{(0,\infty)}(b_j)$	$\mathcal{N}(0,10\cdot I)\mathcal{I}_{(0,\infty)}(b_j)$	$\mathcal{N}(0,5\cdot I)\mathcal{I}_{(0,\infty)}(b_j)$
$(\beta_0,\beta_1)'$	$\mathcal{N}(0,10\cdot I)$	$\mathcal{N}(0,10\cdot I)$	$\mathcal{N}(0,5\cdot I)$
$1/\sigma_0^2, 1/\sigma_1^2$	$\mathcal{G}(0.02,5)$	$\mathcal{G}(0.05,2)$	$\mathcal{G}(0.01,1)$
υ	$\mathcal{B}(8,0.2)$	$\mathcal{B}(8,0.1)$	$\mathcal{B}(8,0.1)$

NOTE: $\mathcal{B}(.,.)$ and $\mathcal{G}(.,.)$ refer to a Beta and a Gamma distribution, respectively, I denotes an identity matrix and $\mathcal{N}(.,.)\mathcal{I}_{(l,h)}(z)$ represents a truncated Normal distribution where the range of z is restricted to the interval (l,h).

Table 3. Bayesian Estimates of the AR-model with a Break in the Conditional Variance

	Prior		Posterior		
Parameter	Mean	Std. dev.	Mean	Std. dev.	95%-bands
μ	0	$\sqrt{10}$	1.174	.403	[.378 ; 1.953]
ϕ_1	0	$\sqrt{10}$	$-.054$	.087	[$-.223$; .117]
ϕ_2	0	$\sqrt{10}$	.124	.086	[$-.042$; .298]
ϕ_3	0	$\sqrt{10}$	.107	.087	[$-.067$; .277]
ϕ_4	0	$\sqrt{10}$	.136	.087	[$-.034$; .308]
$1/\sigma_0^2$	.100	.707	.055	.009	[.039 ; .073]
$1/\sigma_1^2$	.100	.707	.205	.058	[.109 ; .325]
σ_0^2			18.778	3.060	[13.744 ; 25.707]
σ_1^2			5.316	1.617	[3.123 ; 9.236]
υ	.976	.051	.988	.011	[.958 ; 0.999]
Break date			1993:III	7.247	[1988:I ; 1997:II]

Log marginal likelihood $\ln f(G_T \mid \mathcal{M}_{AR,1})$	-373.17
Log Bayes factor $\ln[f(G_T \mid \mathcal{M}_{AR,1})/f(G_T \mid \mathcal{M}_{AR,0})]$	3.08

NOTE: The estimated model ($\mathcal{M}_{AR,1}$) is given by Equations (2)-(4). The posterior moments are based on 10,000 Gibbs iterations, where the first 2,000 Gibbs draws are discarded. The prior specifications used for estimation are that of set (I) given in Table 2. The log marginal likelihood under the prior specifications (II) and (III) are given by -371.91 and -369.00, respectively.

Table 4. Bayesian Estimates of the MS-model with a Break in the Conditional Variance

	Prior		Posterior		
Parameter	Mean	Std. dev.	Mean	Std. dev.	95%-bands
μ_{10}	3.499	2.248	3.157	.808	[1.456 ; 4.744]
μ_{00}	-2.397	1.943	$-.362$	.455	[-1.481 ; .177]
ϕ_1	0	$\sqrt{10}$	$-.148$	.094	[$-.330$; .039]
ϕ_2	0	$\sqrt{10}$	.076	.092	[$-.104$; .255]
ϕ_3	0	$\sqrt{10}$	.108	.091	[$-.070$; .286]
ϕ_4	0	$\sqrt{10}$	.137	.088	[$-.030$; .315]
$1/\sigma_0^2$	.100	.707	.068	.014	[.045 ; .102]
$1/\sigma_1^2$	.100	.707	.470	.287	[.172 ; 1.254]
σ_0^2			15.188	3.213	[9.734 ; 22.207]
σ_1^2			2.452	1.351	[.763 ; 5.892]
υ	.976	.051	.988	.011	[.961 ; 1.000]
p	.750	.144	.713	.105	[.482 ; .892]
q	.750	.144	.735	.110	[.476 ; .910]
Break date			1993:II	6.726	[1989:II ; 1997:I]

Log marginal likelihood $\ln f(G_T \vert \mathcal{M}_{MS,1})$	-378.54
Log Bayes factor $\ln[f(G_T \vert \mathcal{M}_{MS,1})/f(G_T \vert \mathcal{M}_{MS,0})]$	.12

NOTE: The estimated model ($\mathcal{M}_{MS,1}$) is given by Equations (3), (4), (8)-(10) with $\mu_{00} = \mu_{11} = 0$. The posterior moments are based on 10,000 Gibbs iterations, where the first 2,000 Gibbs draws are discarded. The prior specifications used for estimation are that of set (I) given in Table 2. The log marginal likelihood under the prior specifications (II) and (III) are given by -375.73 and -372.09, respectively.

Table 5. Bayesian Estimates of the MS-model with a joint Break in the Conditional Variance and in the Intercept Coefficients

	Prior		Posterior		
Parameter	Mean	Std. dev.	Mean	Std. dev.	95%-bands
μ_{10}	3.499	2.248	3.773	1.034	[1.516 ; 5.598]
μ_{00}	-2.397	1.943	-1.230	.932	[-3.374 ; .038]
μ_{11}	3.499	2.248	2.203	.831	[.604 ; 3.770]
μ_{01}	-2.397	1.943	$-.671$	.736	[-2.497 ; .071]
ϕ_1	0	$\sqrt{10}$	$-.181$	.103	[$-.376$; .023]
ϕ_2	0	$\sqrt{10}$	.068	.096	[$-.124$; .251]
ϕ_3	0	$\sqrt{10}$	.107	.093	[$-.079$; .287]
ϕ_4	0	$\sqrt{10}$	.132	.088	[$-.041$; .303]
$1/\sigma_0^2$	.100	.707	.077	.022	[.047 ; .130]
$1/\sigma_1^2$	.100	.707	.266	.136	[.120 ; .636]
σ_0^2			13.921	3.649	[7.883 ; 22.023]
σ_1^2			4.299	1.776	[1.676 ; 8.284]
υ	.976	.051	.988	.011	[.960 ; .999]
p	.750	.144	.684	.110	[.444 ; .872]
q	.750	.144	.792	.109	[.545 ; .958]
Break date			1993:I	7.911	[1989:I ; 1995:II]

Log marginal likelihood $\ln f(G_T \mid \mathcal{M}_{MS,2})$	-375.34
Log Bayes factor $\ln[f(G_T \mid \mathcal{M}_{MS,2})/f(G_T \mid \mathcal{M}_{MS,0})]$	3.32
Log Bayes factor $\ln[f(G_T \mid \mathcal{M}_{MS,2})/f(G_T \mid \mathcal{M}_{MS,1})]$	3.20

NOTE: The estimated model ($\mathcal{M}_{MS,2}$) is given by Equations (3), (4), (8)-(10). The posterior moments are based on 10,000 Gibbs iterations, where the first 2,000 Gibbs draws are discarded. The prior specifications used for estimation are that of set (I) given in Table 2. The log marginal likelihood under the prior specifications (II) and (III) are given by -374.22 and -371.27, respectively.

Table 6. Bayesian Estimates of the DLR-model with a Break in the Conditional Variance

	Prior		Posterior		
Parameter	Mean	Std. dev.	Mean	Std. dev.	95%-bands
γ	0	$\sqrt{10}$	$-.261$	.149	$[-.554\ ;\ .030]$
ν	0	$\sqrt{10}$	$-.121$	.142	$[-.400\ ;\ .160]$
a_1	0	$\sqrt{10}$	.798	1.715	$[-2.762\ ;\ 3.959]$
a_2	0	$\sqrt{10}$	1.374	2.159	$[-2.735\ ;\ 5.686]$
a_3	0	$\sqrt{10}$	.109	1.915	$[-4.249\ ;\ 3.333]$
a_4	0	$\sqrt{10}$	1.124	2.347	$[-3.250\ ;\ 5.506]$
a_5	0	$\sqrt{10}$	-1.763	1.697	$[-5.312\ ;\ 1.223]$
a_6	0	$\sqrt{10}$	3.629	2.552	$[-1.334\ ;\ 8.517]$
a_7	0	$\sqrt{10}$	-1.648	.770	$[-3.311\ ;\ -.268]$
a_8	0	$\sqrt{10}$	$-.892$	1.473	$[-3.259\ ;\ 2.449]$
b_1	2.523	1.906	.463	.282	[.035 ; 1.099]
b_2	2.523	1.906	1.344	.570	[.356 ; 2.482]
b_3	2.523	1.906	.290	.890	[.017 ; 3.266]
b_4	2.523	1.906	.473	.670	[.064 ; 2.791]
b_5	2.523	1.906	.188	.077	[.042 ; .330]
b_6	2.523	1.906	1.175	.401	[.402 ; 2.058]
b_7	2.523	1.906	.063	.070	[.004 ; .176]
b_8	2.523	1.906	.252	.148	[.021 ; .574]
β_0	0	$\sqrt{10}$	-2.814	.591	$[-4.183\ ;\ -1.869]$
β_1	0	$\sqrt{10}$	.243	.078	[.104 ; .411]
$1/\sigma_0^2$	.100	.707	.067	.012	[.045 ; .093]
$1/\sigma_1^2$	.100	.707	.216	.073	[.132 ; .380]
σ_0^2			15.353	2.931	[10.813 ; 22.041]
σ_1^2			4.744	1.302	[2.633 ; 7.551]
υ	.976	.051	.987	.012	[.954 ; .999]
Break date			1990:III	11.557	[1988:IV ; 1996:I]

Log marginal likelihood $\ln[f(G_T\|\mathcal{M}_{DLR,1})]$	-371.96
Log Bayes factor $\ln[f(G_T\|\mathcal{M}_{DLR,1})/f(G_T\|\mathcal{M}_{DLR,0})]$	.53

NOTE: The estimated model ($\mathcal{M}_{DLR,1}$) is given by Equations (3), (4), (12)-(15). The prior specifications used for estimation are that of set (I) given in Table 2. The posterior moments are based on 10,000 Gibbs iterations, where the first 2,000 Gibbs draws are discarded. The log marginal likelihood under the prior specifications (II) and (III) are given by -369.86 and -366.68, respectively.

Table 7. Log Marginal Likelihoods under Alternative Priors

		Prior (I)	Prior (II)	Prior (III)
AR-model	$\ln[f(G_T \mid \mathcal{M}_{AR,0})]$	-376.25	-375.43	-372.46
	$\ln[f(G_T \mid \mathcal{M}_{AR,1})]$	-373.17	-371.93	-369.56
MS-model	$\ln[f(G_T \mid \mathcal{M}_{MS,0})]$	-378.66	-377.49	-374.90
	$\ln[f(G_T \mid \mathcal{M}_{MS,1})]$	-378.54	-375.73	-372.09
	$\ln[f(G_T \mid \mathcal{M}_{MS,2})]$	-375.34	-374.22	-371.27
DLR-model	$\ln[f(G_T \mid \mathcal{M}_{DLR,0})]$	-372.49	-371.55	-367.51
	$\ln[f(G_T \mid \mathcal{M}_{DLR,1})]$	-371.96	-369.86	-366.68

NOTE: The set of prior distributions (I)-(III) are specified in Table 2.

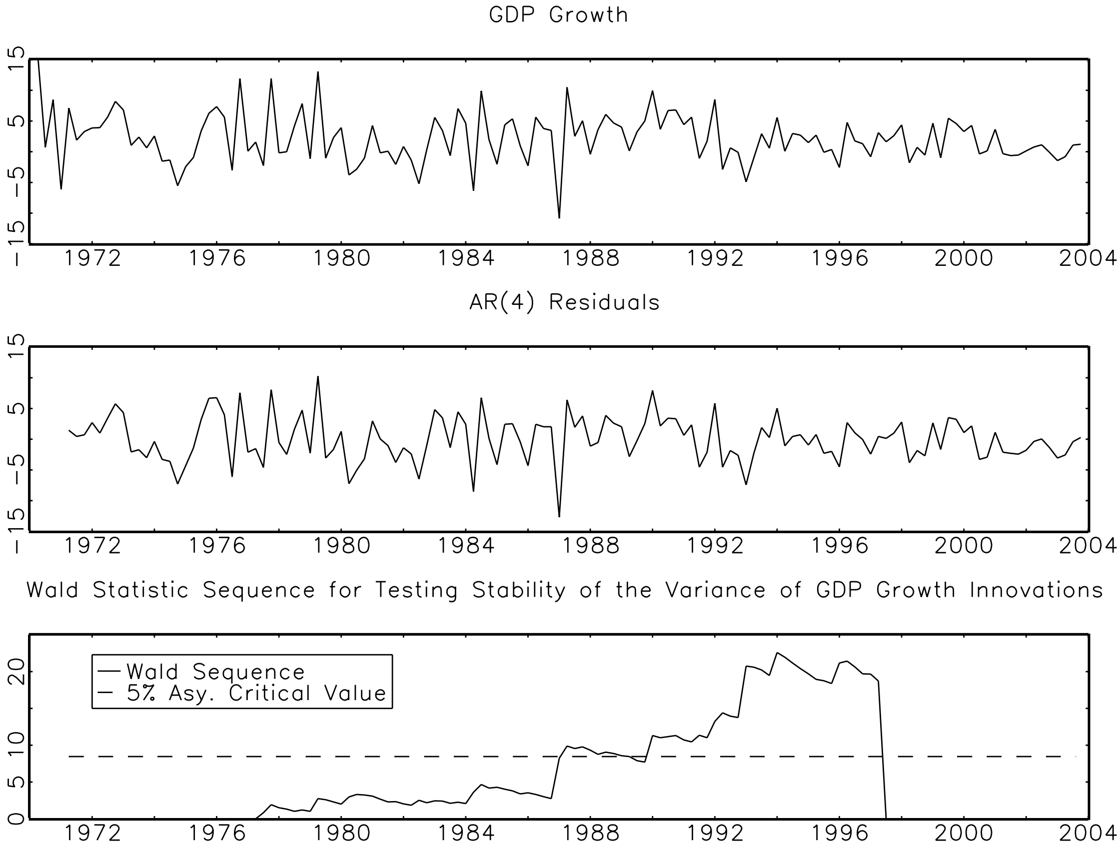

Fig. 1. German GDP growth from 1970:I to 2003:IV, measured in logged differences in quarterly GDP, annualized by multiplying by 400 (top panel); residuals from a AR(4) model (middle panel); sequence of the Wald statistics for testing for a break in the error variance of the AR(4) fitted to the GDP growth (bottom panel).

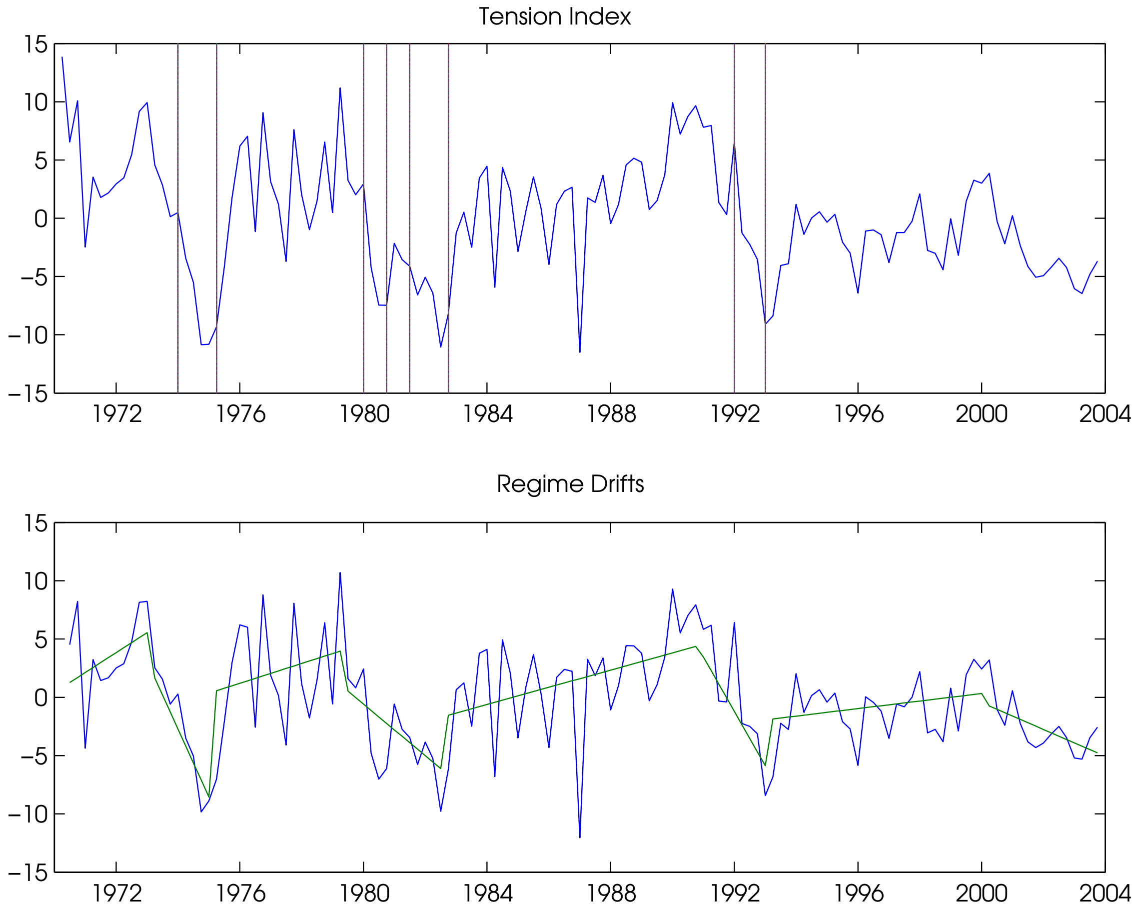

Fig. 2. Tension index h_t based on $\delta = 0.575$ and the dates of business cycle peaks and troughs (top panel); estimated stochastic regime drift m_t and deviations of growth from the estimated AR component $y_t - \gamma y_{t-1} - \nu h_{t-1}$ (bottom panel). The parameters are set equal to their posterior mean for the set of priors (I) given in Table 2.

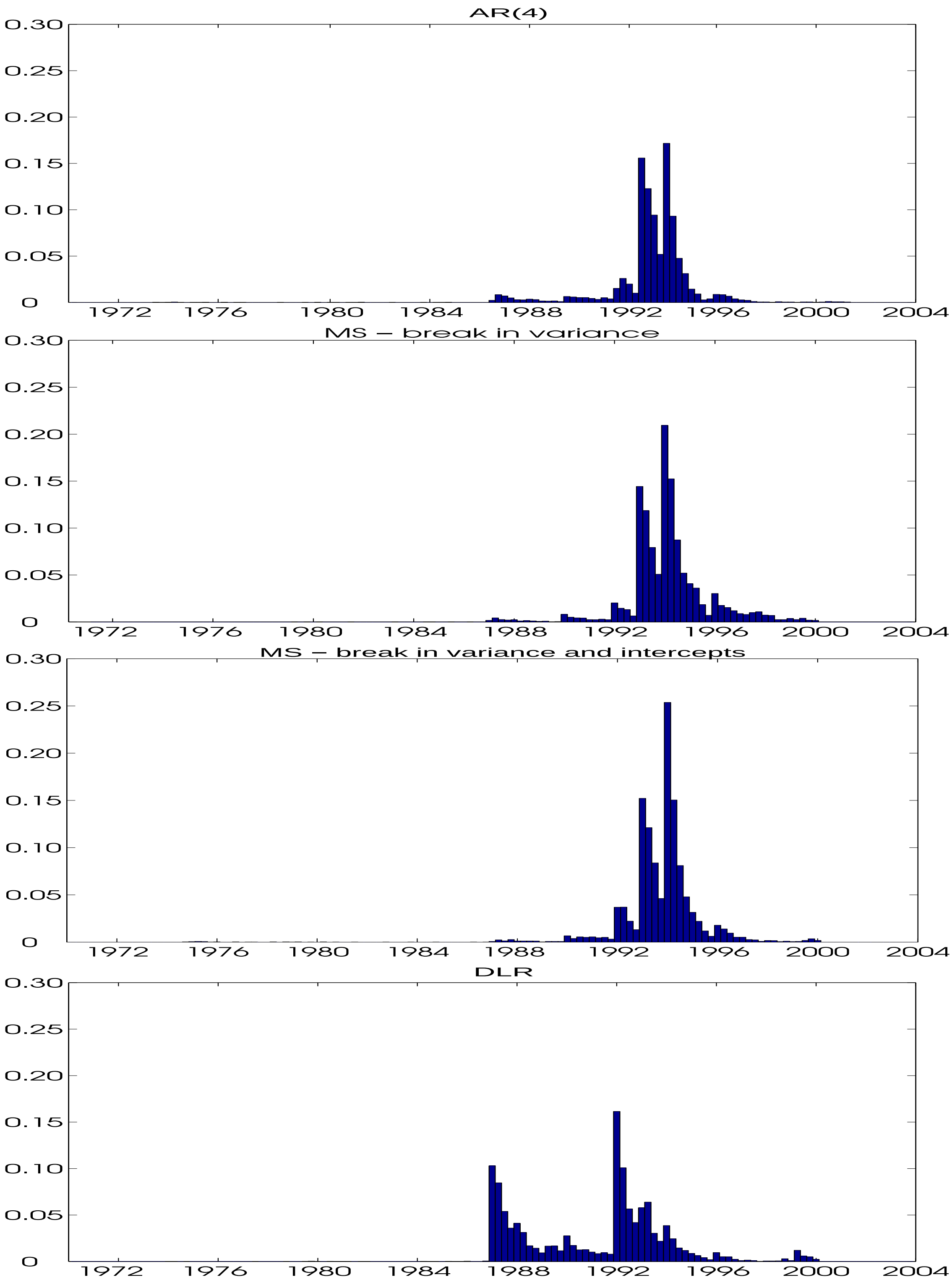

Fig. 3. Histograms of sampled break points: AR(4) model (top panel); MS model with a break in variance (second panel); MS model with a break in variance and intercepts (third panel); DLR model (bottom panel). The results are obtained for the sets of priors (I) given in Table 2.

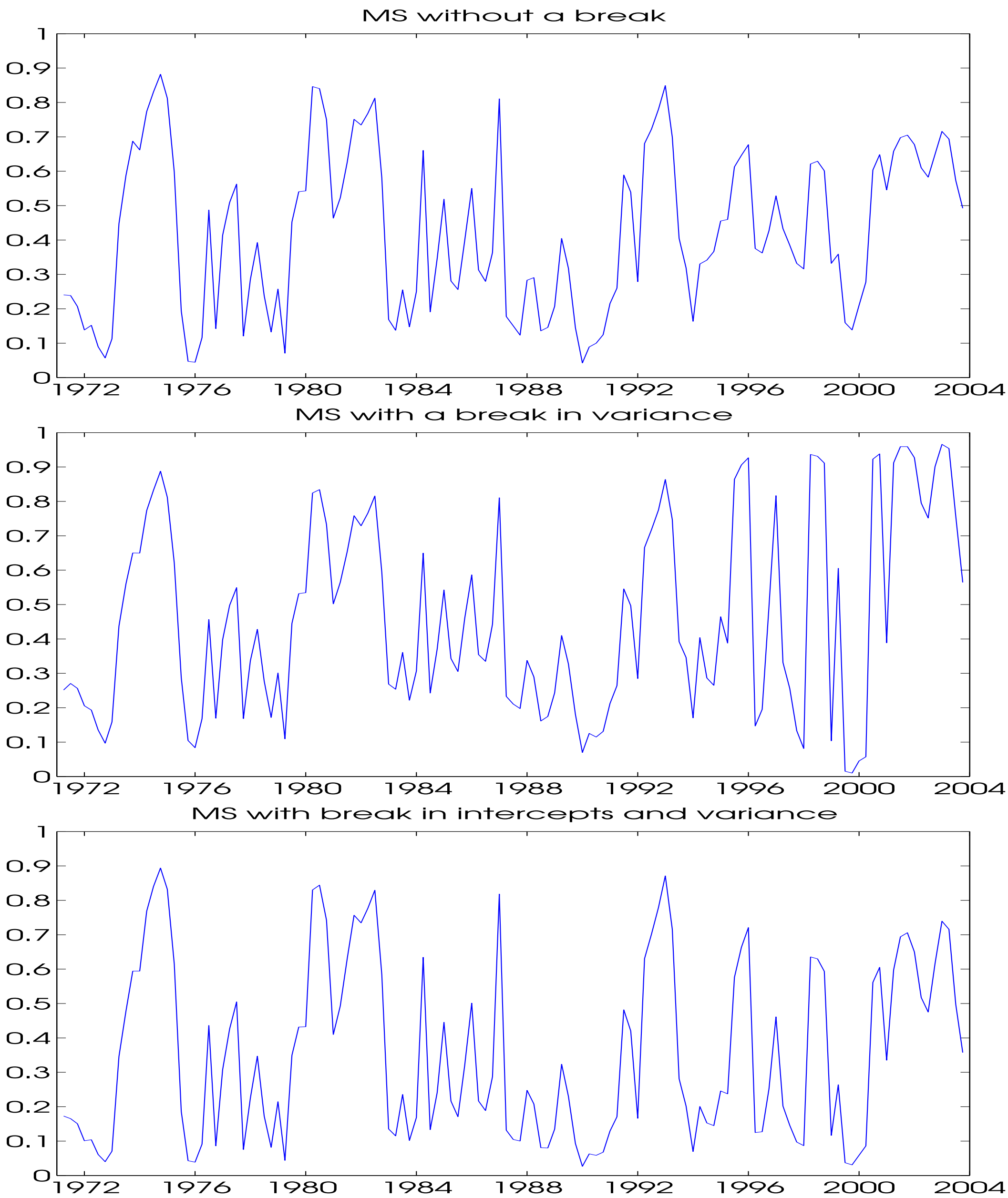

Fig. 4. Probabilities for a contraction: MS model without break (top panel); MS model with a break in variance (second panel); MS model with a break in variance and intercepts (bottom panel). The results are obtained for the sets of priors (I) given in Table 2.

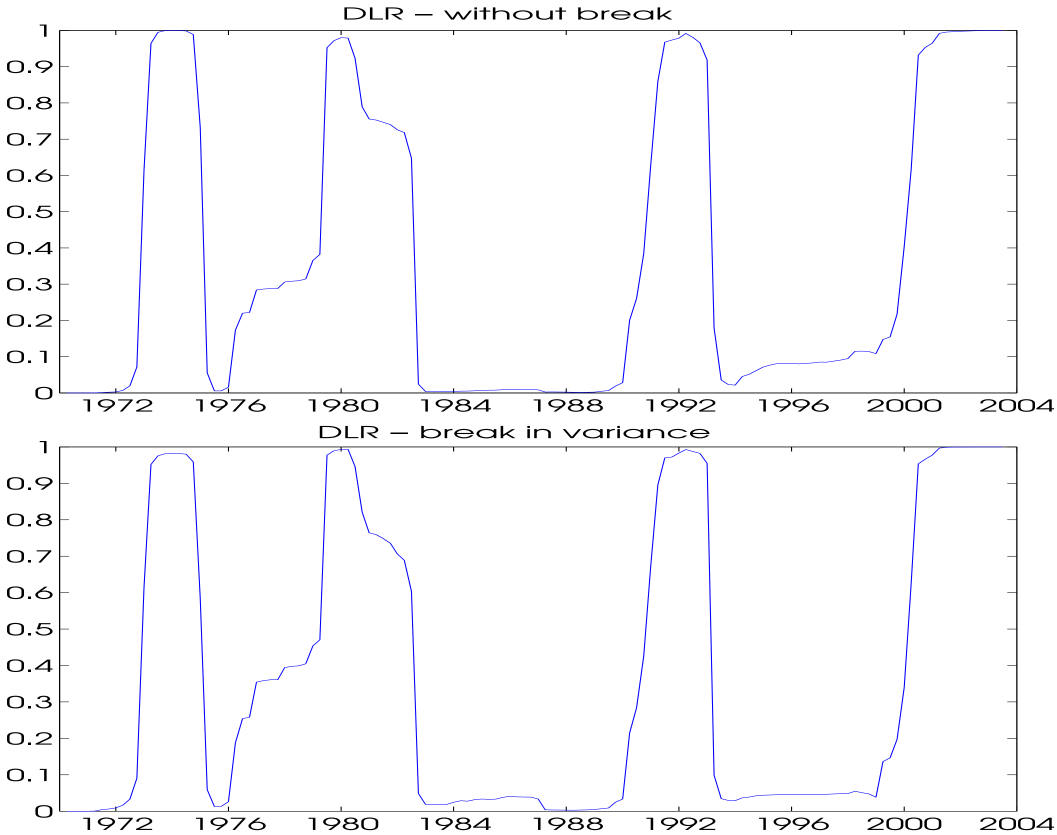

Fig. 5. Probabilities for a deceleration: DLR model without break (top panel); DLR model with a break in variance (bottom panel). The results are obtained for the set of priors (I) given in Table 2.