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A Note on Credit-Worthiness Tests

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Abstract

Banks often have to determine the credit worthiness, i.e. the ability to repay the loan, of their customers ex-ante. According to the theory of imperfect diagnosis, it can be rational not to use an informative diagnosis result, even though it can be acquired without costs. We call this non-contingent behavior: The decision whether to grant a loan or not is independent of the diagnosis result. Hence, it can be rational to grant a loan to a particular applicant, even if the specific diagnosis result is negative.
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1. Introduction

Banks often face the problem to have to determine whether a firm that applies for a loan will later be able to repay its debt or not. In a very influential paper[1], BROECKER (1990) presents a model where this problem is treated as a binomial decision problem; the bank is able to generate an informative signal about the ability to repay before it has to make its decision. This signal helps to assign the applicants to two risk classes: the high risks versus the low risks.

BROECKER uses this setting to analyze the problem of Bertrand competition between banks. In this note, we go one step back and discuss some assumptions and results of the starting point of his oligopoly model: the problem of a price-taking bank how to determine the credit worthiness of its potential customers[2]. In establishing the diagnosis problem as the basis of his oligopoly model, BROECKER (1990, 429 f.) makes two statements we find worth to analyze. We propose to make use of the insights of a new diagnosis theory that is based on the theory of imperfect decision making[3]. We are convinced that, taking our analysis of the basic diagnosis model into account, the results of the oligopolistic interaction would also be influenced.

The first claim of BROECKER (1990, 429) is his result that a "(...) bank's credit department will use any relevant information it has available to assign the applicant to a certain risk class." Our model shows that it can be rational for a bank to disregard a signal on the creditworthiness of an applicant, even though the diagnosis is informative and can be acquired without costs. We call this kind of behavior a *non-contingent* reaction: The bank's decision whether to grant a loan or not is independent of the diagnosis result. A *contingent* reaction, on the other hand, takes the diagnosis result into account.

The second claim of BROECKER (1990, 430) is his assumption that "a bank will not provide credit to firms" that are assigned the risky class. Our model shows that this behavior may not be rational. If non-contingent behavior is optimal for the bank, it will grant loans to firms even if the diagnosis has indicated the high-risk class.

The literature on credit-worthiness tests uses to disregard the possibility of non-contingent behavior being rational[4]. LONGHOFER (1996) presents a model in which lenders may find it beneficial to ignore a signal if its variance is too high. This model is

[1]According to the Social Sciences Citation Index, BROECKER (1990) was cited in a considerable row of papers, among them DENNERT (1993), DIETSCH (1993), VANDAMME (1994), GISCHER (1994), FLANNERY (1996), BAYE/KOVENOCK/DEVRIES (1996), THAKOR (1996), SMITH (1998), QIAN/XU (1998), SHAFFER (1998), SCHNITZER (1999), VILLASBOAS/SCHMIDT-MOHR (1999), DELLARICCIA/FRIEDMAN/MARQUEZ (1999).

[2]We exclude the possible principle-agent problems between the bank and its credit department from our analysis; see e.g. FEESS/SCHIEBLE (1999) on this.

[3]See HEINER (1983), HEINER (1986), and HEINER (1990). KIRSTEIN/SCHMIDTCHEN (1997) analyzed the imperfect decision-making of civil judges. On Diagnosis Theory, see GREEN/SWETS (1966) or SWETS (1988).

[4]This assumption is implicitly made by, e.g., BROECKER (1990), GEHRIG (1998, 6), FEESS/SCHIEBLE (1999, 6). In the model of GEHRIG (1998), equilibria exist in which the competing banks do not produce signals.

the closest one to our approach, yet our model does not refer to the variance of the test result, but only the expected outcome.

2. A simple diagnosis model

2.1 Loans and credit-worthiness

Consider a risk-neutral bank that faces a set of applicants for loans. The set consists of two types of applicants, one of which will be able to repay the loan plus interest (we call this one type b), and the other one will be not be able to pay back anything (type a)[5]. Let the loan be L and the interest rate be i.

The bank is unable to directly observe the type of a particular applicant. However, the distribution of the two types is known. Let $\pi \in]0,1[$ be the probability that an applicant is of type b, hence $1-\pi$ is the probability of type a. Table 1 presents the payoffs of the bank, depending on the true type of the applicant and the banks's decision whether to grant the loan, denoted as (g), or not (n).

Table 1: Payoffs of the price-taking bank

type / credit	b	a
(g)	$(1+i)L$	0
(n)	L	L

If the bank grants the credit and the applicant is of the good type, then the bank receives $(1+i)L$.[6] If the applicant is of type a, then the payment to the bank is zero. If the bank does not grant the loan, then in both cases it simply retains the amount L.

2.2 Imperfect diagnosis

We now introduce diagnosis. We assume that the bank's credit department has an opportunity to generate a costless signal that is perfectly correlated with the true, but unknown, type of the applicant. One can think of market studies concerning the chances

[5]In doing so, we simplify the model of Broecker (1990), who assumes that type $j \in \{a; b\}$ pays back the loan at a probability p_j, with $0 \le p_a < p_b \le 1$. To focus in our argument, we here make the stronger assumptions $p_a = 0$ and $p_b = 1$ and will relax them in the discussion, see section 3.

[6]Broecker (1990) also takes into account that the client my go bankrupt, thus he pays back the minimum of $(1+i)L$ or the realized return of his project, denoted as X. For simplification we assume that each successful project is profitable, hence $X > (1+i)L$. This assumption will also be relaxed in the discussion, see section 3.

of the product the investor intends to sell, or of the personal impression the applicant makes.

In any case, the credit manager is only imperfectly able to interprete this signal, because his knowledge of, e.g., the future demand for a good can only be limited. Hence, the diagnosis result is subject to two types of errors[7]: the first type of error is the assignment of the high risk-class to a low-risk applicant, whereas the credit manager commits the second type of error if he wrongly assigns the low- risk class to a high-risk applicant.

The imperfect diagnosis can lead to two possible assignments of risk classes to the applicant under scrutiny: Let B be the low-risk class and A the high-risk class. We denote the probability of result B if the applicant actually is of type b as r:

$$r := \Pr\{B|b\}$$

On the other hand, let w be the probability that the result is B, given the applicant is of type a:

$$w := \Pr\{B|a\}$$

Thus, $1 - r = \Pr\{A|b\}$ and $1 - w = \Pr\{A|a\}$. $1 - r$ is the probability of an error of first type, and w is the probability of the second-type error. Table 2 presents the conditional probabilities of diagnosis results, given the true but unknown type ot the applicant, and the prior distribution over the type.

Table 2: Diagnosis parameters

type / diagnosis result	b	a
B	r	w
A	$1 - r$	$1 - w$
prior	π	$1 - \pi$

The parameters r and w provide a measure for the bank's credit department's diagnosis skill. With $r = 1$ and $w = 0$, the bank's credit department has *perfect* diagnosis skill. Diagnosis is free of errors in this case. In case of $r = w$, the department is said to have *zero diagnosis skill*: the diagnosis result is independent of the applicant's true type. Most interesting is the case in between, with $0 < w < r < 1$. This is called an *positive, but imperfect* diagnosis skill. The bank is not entirely blind, but has to take into account

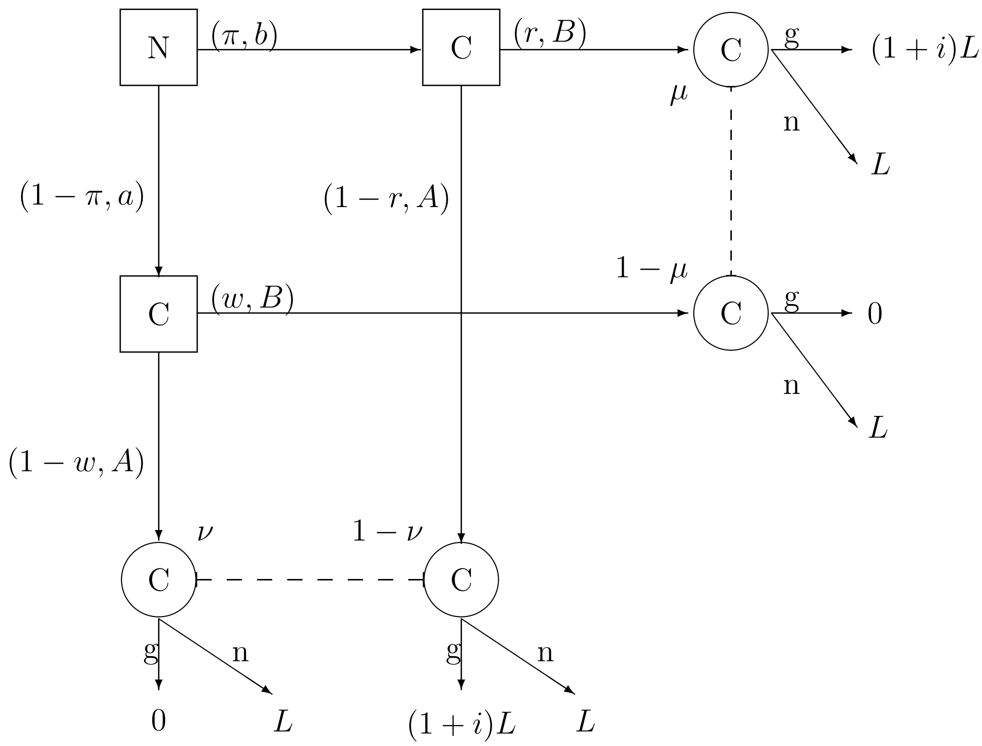
[7]See Kirstein (1999) on an extensive discussion of this *imperfect diagnosis skill* approach. Alternatively, it could be assumed that the signal is imperfectly correlated, whereas the diagnosis skill is perfect. The mathematical results are similar for both of these approaches, but the interpretation of the model would be different. Imperfect diagnosis skill points to a bounded rationality interpretation, whereas perfect processing of an imperfect signal belongs to information economics.

that errors occur occasionally[8].

Figure 1 shows the decision problem of a single bank: first, the Nature (denoted as N; the square nodes in figure 1 represent random events) draws the type of the applicant, which is unobserveable for the bank. Afterwards, the bank's credit department (labelled as C in square nodes) examines the case and produces a diagnosis result; the probabilities r and w of its two possible realizations are conditioned on the true type of the applicant. C knows the result of the diagnosis process, but not the true type of the applicant, and finally has to decide (at the circle nodes) whether to grant the loan (option g) or not (option n).

At the information set at the upper right hand of figure 1 the bank has observed realization B, and updates beliefs according to Bayes' rule to the ex-post beliefs $(\mu, 1-\mu)$, with $\mu := \Pr\{b|B\}$. At the lower information set, the bank has observed A; bayesian update leads to the ex-post beliefs $(\nu, 1-\nu)$, with $\nu := \Pr\{a|A\}$.

Figure 1: Imperfect Diagnosis



According to the Bayes formula, the diagnosis results lead to the follwing updates:

$$\mu = \frac{r\pi}{r\pi + w(1-\pi)}$$

[8]Only for simplicity, we exclude the case of a misleading diagnosis result ($r < w$). However, such a diagnosis result would be as informative as one with $r > w$ and could easily be treated in a symmetrical way.

and

$$\nu = \frac{(1-w)(1-\pi)}{(1-w)(1-\pi)+(1-r)\pi}$$

Note that perfect diagnosis skill leads to $\mu = \nu = 1$, whereas zero diagnosis skill implies $\mu = 1-\pi$ and $\nu = \pi$. In the latter case, the credit manager's posterior distribution over the types of applicants is the same as the prior, hence the diagnosis result is not informative. Now we define a parameter T as

$$T := \frac{1-\pi}{i\pi}$$

T is called the reliability threshold[9]. The threshold T represents the relevant environment or the decision-maker, whereas the paramters r and w represent his own abilities to cope with this environment. Using these definitions, we state our first result as a proposition:

Proposition 1: *The decision to grant the loan (g) is optimal if, and only if,*

a) the diagnosis result is B and $\frac{r}{w} > T$ *or*

b) the diagnosis result is A and $\frac{(1-r)}{(1-w)} > T$.

Proof: *In case a), result B has been observed. In this case, (g) is preferred if, and only if,* $\mu(1+i)L + (1-\mu)0 > L$. *This is equivalent to* $r\pi(1+i)L > r\pi L + w(1-\pi)L \Leftrightarrow ri\pi > w(1-\pi)$*: q.e.d. Case b) is to be proven in a similar way.*

Part b) of this proposition illustrates the second claim we have made in the introduction: Under certain circumstances, it can be perfectly rational for a bank's credit department to grant a loan to an applicant even if he is assigned the higher risk class (result A) by an imperfect diagnosis tool. The intuition of this result is straightforward: Among those applicants who were assigned the high-risk class, there are some who actually deserve it. However, due to errors of the first type, this set of applicants contains as well low-risk customers. Granting credit to the applicants that were assigned the high-risk class hence may bring some losses from the customers of the high-risk type, but also some gains from those who are actually low-risk types. If the latter gains outweigh the former losses, it is reasonable to grant credit even if the diagnosis result did not recommend this decision.

[9]See Heiner (1983), (1986), (1990). T is the ratio of the loss caused by granting a loan wrongly (if the applicant is of high risk type; this loss is $-L$) times the probability of the high risk type $(1-\pi)$, divided by the gain from a granted loan to a low risk type, i.e. $(1+i)L - L$, times the probability of this type (π). Note that T is independent of the amount L the applicant asks for.

2.3 Reaction strategies

The decision problem of figure 1 can be restated equivalently as a decision in *reaction strategies*. A (pure) reaction strategy is a pair of actions $(\eta\theta)$, with $\eta, \theta \in \{g; n\}$, where η denotes the action to be carried out if the diagnosis result is B, and θ denotes the reaction on A[10]. Obviously, the credit department has four reaction strategies:

- The non-contingent reaction strategies (gg) and (nn). The action to be carried out is independent of the diagnosis result; the former is the plan to grant the loan, whereas the latter one leads to the rejection of the applicant.
- The contingent reaction strategies (gn) and (ng): The action carried out depends on the diagnosis result. The former reaction strategy models the plan to grant the loan if, and only if, the result is B. According to the latter one, the loan is granted if, and only if, the result is A.

Using these definitions, we can derive our next result as a corollary to proposition 1:

> **Proposition 2:** *Given the decision problem of figure 1, and values of the parameters r, w, with $r \geq w$, and T, then the optimal reaction strategy of the bank is*
>
> *(gg) if, and only if,* $\frac{r}{w} > T$ *and* $\frac{1-r}{1-w} > T$
>
> *(nn) if, and only if,* $\frac{r}{w} < T$ *and* $\frac{1-r}{1-w} < T$
>
> *(gn) if, and only if,* $\frac{r}{w} > T$ *and* $\frac{1-r}{1-w} < T$*,*
>
> *whereas (ng) is never optimal.*

The reaction strategy (ng) would be optimal if, and only if, $r/w < T$ and $(1-r(/)1-w) > T$, which is excluded by $r \geq w$. Thus, given positive diagnosis skill, the reaction strategy (ng) can never be optimal. The only remaining contingent reaction strategy (gn) is optimal if

$$\frac{r}{w} > T > \frac{1-r}{1-w}$$

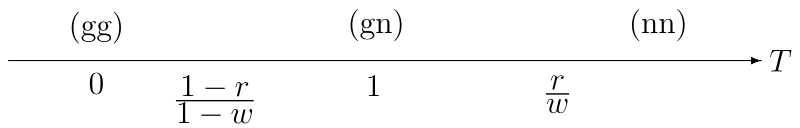
holds. We call this expression the *extended reliability condition*[11]. The r-w-terms provide a measure for the reliability of the bank's diagnosis skill. The better the diagnosis skill, the greater is r/w and the smaller is $(1-r)/(1-w)$. The parameter T, on the other hand, reflects the environment conditions for the bank's decision.

[10] It can easily be shown that, as long as diagnosis costs and budget constraints are not binding, mixed reaction strategies are irrelevant, since only corner solutions can be optimal, see KIRSTEIN (1999). Therefore, the attention is limited to pure reaction strategies.

[11] See KIRSTEIN (1999). HEINER (1983) did coin the term "reliability condition", yet focussed only the left hand side of this expression.

Proposition 2 supports the first claim we have made in the introduction: it can be rational for a bank to neglect the diagnosis result, even though it is informative ($r > w$) and available at zero costs. The proposition is illustrated by figure 2; note that, due to the assumption $r \geq w$, $(1-r)/(1-w)$ is not greater than r/w.

Figure 2: Optimal reaction strategies for different values of T



The horizontal axis in figure 2 shows the optimal reaction strategy for different values of T. If T is in the left interval (small or even negative values of T), then (gg) is the optimal strategy; if T is in the right interval (very high values), then (nn) is optimal; only in intermediate interval, (gn) is the best choice. T depends on the expected payoff parameters i and π. Due to the result that is represented in figure 2, marginal changes of these paramters may have no impact on the observeable behavior at all - this is the case when T remains within the same interval - or may change the observeable behavior rather dramatically (if T switches from one interval to another).

3. Discussion

Deviating from the exposition in BROECKER (1990), we have made two simplifying assumptions that were helpful to focus on the point to be made. Now we relax these assumptions and allow for a positive success probability even in case of higher risk types and for bankruptcy. These modifications have a systematical influence on the threshold value T, yet not on the diagnosis skill paramters r and w.

First we introduce $0 \leq p_a < p_b \leq 1$. p_a and p_b are independent of each other. Under this assumption, the bank's expected gain from granting credit to a low risk customer is $p_b(1+i)L - L$ (this type occurs with probability π). On the other hand, a credit for a high risk customer (the probability is $1-\pi$) leads to $p_a(1+i)L - L$, which now can even be positive. This has an impact on the threshold value T we introduced to state our propositions above. This threshold is now

$$T^* = \frac{1-\pi}{\pi} \cdot \frac{1-(1+i)p_a}{(1+i)p_b - 1}$$

Using this threshold, Propositions 1 and 2 still hold: (gn) is the optimal reaction strategy if, and only if, $r/w > T^* > (1-r)/(1-w)$. If p_a is greater than $1/(1+i)$, then the threshold value T is even negative. In this case, r/w and $(1-r)/(1-w)$ both are greater than T^*. Thus, (gg) is optimal: it is profitable for the bank always to grant credit (even to the high-risk types).

In cases with $p_a < 1/(1+i) < p_b$, granting a credit to the high risk customer would lead to a loss for a bank. Then T^* is decreasing in p_a and increasing in p_b. The smaller the difference between these two parameters, the lower is the threshold value T^*.

Recall that, for $T^* < 1$, a lower threshold value means that (gg) is the optimal reaction strategy for a larger set of $r-w-$combinations. Or, less formally put, the higher the difference between p_b and p_a, the more attractive it is to grant the credit blindly instead of obeying the diagnosis result.

On the other hand, as long as $T^* > 1$ is satisfied, the set of $r-w-$combinations that make (gn) optimal grows with a decrease in T^*. Hence, the smaller the difference between p_b and p_a, the more attractive it is to obey the diagnosis result instead of rejecting the customer blindly.

We additionally could take into account that even the return of a successful project (denoted as X) might be insufficient to cover the contractual obligation of the client, i.e. that he might go bankrupt. Then the payment the bank expects is client of type $j \in \{a; b\}$ is only $X < (1+i)L$ with probability p_j, and zero with probability $1-p_j$. (Recall that the probability of the type b is π.) This modification would again have an impact on the threshold value T that has to be taken into account when deriving the bank's optimal reaction strategy. We denote this threshold in this case as T^{**}, with

$$T^{**} = \frac{1-\pi}{\pi} \cdot \frac{L - p_a X}{p_b X - L}$$

Obviously, taking into account these modifications would lead to results that only differ quantitatively, yet not qualitatively, from those derived in the previous section, and our propositions would still hold: even with positive diagnosis skill and an informative diagnosis result, it can be rational for a bank to disregard this result and to make an non-contingent decision. If the values of r and w are given, one only needs to derive the threshold value that best describes the situation of the imperfect decision-making bank, and then the optimal reaction strategy of the bank can be found in figure 2.

4. Conclusion

We have analyzed the behavior of a price-taking bank that is faced the problem to determine the credit-worthiness of loan applicants. Proposition 2 shows that it can be rational to neglect diagnosis results even though the diagnosis is informative and costless. Furthermore, according to Proposition 1, it can be rational for the bank to provide credit to an applicant even if the diagnosis has assigned him the high-risk class.

Since our results will have an impact on the predicted behavior of price-taking banks, we expect the optimal behavior of banks in oligopolistic competition to be different from the way as described in Broecker (1990). Taking diagnosis theory into account when deriving the equilibrium behavior would be an interesting next step in this line of research.

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