A sequential approach to testing seasonal unit roots in high frequency data

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Abstract

In this paper we introduce a sequential seasonal unit root testing approach which explicitly addresses its application to high frequency data. The main idea is to see which unit roots at higher frequency data can also be found in temporally aggregated data. We illustrate our procedure to the analysis of monthly data, and we find, upon analyzing the aggregated quarterly data, that a smaller amount of test statistics can sometimes be considered. Monte Carlo simulation and empirical illustrations emphasize the practical relevance of our method.

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*This work was initiated when the second author visited Lisbon (Portugal) in 2000. The data and the Gauss programs used to estimate the parameters can be obtained from the authors. Address for correspondence: P.H. Franses, Econometric Institute, Erasmus University Rotterdam, P.O. Box 1738, NL-3000 DR Rotterdam, The Netherlands, `e-mail: franses@few.eur.nl`.

1 Introduction

The use of seasonally unadjusted data in empirical applications is increasing and so is the need to adequately characterize the properties of those series, see for example Franses (1996) for an overview. As many data in economics display trends and also signs of stochastic seasonality, this has led to the development of a large number of seasonal unit root tests over the last two decades, see Dickey, Hasza and Fuller (1984), Hylleberg, Engle, Granger and Yoo [HEGY] (1990), Osborn, Chui, Smith and Birchenhall (1988), among many others. Among the tests proposed, the most widely used procedure is the HEGY test.

A characteristic common to unit root tests, as well as to seasonal unit root tests, is their poor power performance in small samples, see for example Ghysels, Lee and Noh (1994) and Rodrigues and Osborn (1999) for Monte Carlo evidence on the performance of quarterly and monthly seasonal unit root tests, respectively. Also, the power deteriorates, the more unit roots one has to examine. For instance, in a simple test regression with no deterministic variables, the HEGY test procedure in the quarterly context requires the estimation of four parameters, whereas in a monthly context this number increases to twelve.

In this paper, we propose a sequential procedure for higher frequency data, which should have better power. For this, we use information on the properties of a series under different periodicities, like quarterly and monthly, to devise more parsimonious test regressions. We show that the characteristics of a quarterly series, for example, can be useful to justify the use of more parsimonious test regressions at a monthly level. Naturally, other periodicities can be considered, but for the purpose of presentation, in this paper we focus only on quarterly and monthly data.

The paper is organized as follows. Section 2 shows how seasonal unit roots in monthly data appear in their temporally aggregated quarterly data. We use extensive simulation experiments to substantiate the results. Section 3 introduces new test procedures, their asymptotic limits and critical values. In Section 4 an empirical application is presented, and, finally, Section 5 concludes the paper.

2 Temporal Aggregation

We first determine the implications of monthly seasonal unit roots when the observations are aggregated to quarterly series. That is, we identify relationships between monthly and quarterly unit roots.

2.1 Preliminaries

Consider the monthly autoregressive process

$$\alpha(L)x_{t_M} = \varepsilon_{t_M}, \tag{1}$$

where $\varepsilon_{t_M} \sim iid(0, \sigma^2)$, $\alpha(L)$ is a polynomial of order twelve and the index t_M refers to monthly data. The (seasonal) unit roots allowed for in $\alpha(L)$ for monthly data are those given in the first panel of Table 1, while the second panel gives the unit roots for the quarterly data. When all unit roots occur simultaneously, $\alpha(L) = (1-L^{12}) \equiv \Delta_{12}$ (or $\alpha(L) = (1 - L^4) \equiv \Delta_4$), one has, what is called, a seasonal random walk.

From Table 1, we use the length of the cycles corresponding to the roots in the quarterly and monthly context, to establish that several relationships can be identified. Obviously, there is a link between the monthly and the quarterly zero frequency roots. Next, there is a link between the monthly root occurring at frequency $2\pi/3$ and the quarterly $\pi/2$ frequency root. Finally, there is a link between the monthly root at frequency $\pi/3$ and the quarterly π frequency root.

2.2 Seasonal unit root test

In order to identify other potential, but not immediately obvious, relationships between monthly and quarterly roots, as well as to confirm the ones put forward above, we resort to an application of the HEGY seasonal unit root test procedure.

The HEGY approach was originally derived for quarterly data, and has been extended to the monthly case by Franses (1991) and Beaulieu and Miron (1993). The HEGY test is based on a root decomposition of a general polynomial of at least order S, where S represents the periodicity of the data, and it has the Δ_S filter as its overall underlying null hypothesis.

Following Smith and Taylor (1999), linearizing Δ_S around the unit roots at different frequencies yields the test regression

$$\Delta_S x_{t_S} = \pi_0 x_{0,t_S-1} + \pi_{S/2} x_{S/2,t_S-1} + \sum_{k=1}^{S^*} \left(\pi_{\alpha,k} x^{\alpha}_{k,t_S-1} + \pi_{\beta,k} x^{\beta}_{k,t_S-1} \right) + \varepsilon_{t_S}, \tag{2}$$

where $\pi_{S/2} x_{S/2,t_S-1}$ needs to be omitted if S is odd, and where

$$x_{0,t_S} \equiv \sum_{j=0}^{S-1} x_{t_S-j}, \tag{3}$$

$$x_{S/2,t_S} \equiv \sum_{j=0}^{S-1} \cos[(j+1)\pi] x_{t_S-j}, \tag{4}$$

$$x^{\alpha}_{k,t_S} \equiv \sum_{j=0}^{S-1} \cos[(j+1)\omega_k] x_{t_S-j} \tag{5}$$

$$x^{\beta}_{k,t_S} \equiv -\sum_{j=0}^{S-1} \sin[(j+1)\omega_k] x_{t_S-j}, \tag{6}$$

$k = 1, ..., S^*$, where $S^* = (S/2) - 1$ (if S is even), while it is $[S/2]$ (if S is odd), with $[\cdot]$ denoting the integer part of its argument. Finally, $\Delta_S x_{Sn+s} \equiv x_{Sn+s} - x_{S(n-1)+s}$.

For the purpose of the present paper, concerning quarterly and monthly data, the relevant transformations are for $S = 4$ given by

$$x_{0,t_Q} \equiv (1 + L + L^2 + L^3) x_{t_Q},$$

$$x_{2,t_Q} \equiv -\left(1 - L + L^2 - L^3\right) x_{t_Q},$$

$$x^{\alpha}_{1,t_Q} \equiv -L(1 - L^2) x_{t_Q}$$

$$x^{\beta}_{1,t_Q} \equiv -(1 - L^2) x_{t_Q},$$

and for $S = 12$ they are

$$\begin{aligned}
x_{0,t_M} &\equiv (1 + L + L^2 + L^3 + L^4 + L^5 + L^6 + L^7 + L^8 + L^9 + L^{10} + L^{11})x_{t_M}, \\
x_{2,t_M} &\equiv -\left(1 - L + L^2 - L^3 + L^4 - L^5 + L^6 - L^7 + L^8 - L^9 + L^{10} - L^{11}\right) x_{t_M}, \\
x^{\alpha}_{1,t_M} &\equiv -(L - L^3 + L^5 - L^7 + L^9 - L^{11})x_{t_M} \\
x^{\beta}_{1,t_M} &\equiv -(1 - L^2 + L^4 - L^6 + L^8 - L^{10})x_{t_M} \\
x^{\alpha}_{2,t_M} &\equiv -\frac{1}{2}(\sqrt{3} - L + L^3 - \sqrt{3}L^4 + 2L^5 - \sqrt{3}L^6 + L^7 - L^9 + \sqrt{3}L^{10} - 2L^{11})x_{t_M} \\
x^{\beta}_{2,t_M} &\equiv \frac{1}{2}(1 - \sqrt{3}L + 2L^2 - \sqrt{3}L^3 + L^4 - L^6 + \sqrt{3}L^7 - 2L^8 + \sqrt{3}L^9 - L^{10})x_{t_M} \\
x^{\alpha}_{3,t_M} &\equiv \frac{1}{2}(\sqrt{3} + L - L^3 - \sqrt{3}L^4 - 2L^5 - \sqrt{3}L^6 - L^7 + L^9 + \sqrt{3}L^{10} + 2L^{11})x_{t_M} \\
x^{\beta}_{3,t_M} &\equiv -\frac{1}{2}(1 + \sqrt{3}L + 2L^2 + \sqrt{3}L^3 + L^4 - L^6 - \sqrt{3}L^7 - 2L^8 - \sqrt{3}L^9 - L^{10})x_{t_M} \\
x^{\alpha}_{4,t_M} &\equiv -\frac{1}{2}(1 + L - 2L^2 + L^3 + L^4 - 2L^5 + L^6 + L^7 - 2L^8 + L^9 + L^{10} - 2L^{11})x_{t_M} \\
x^{\beta}_{4,t_M} &\equiv -\frac{\sqrt{3}}{2}(1 - L + L^3 - L^4 + L^6 - L^7 + L^9 - L^{10})x_{t_M} \\
x^{\alpha}_{5,t_M} &\equiv \frac{1}{2}(1 - L - 2L^2 - L^3 + L^4 + 2L^5 + L^6 - L^7 - 2L^8 - L^9 + L^{10} + 2L^{11})x_{t_M} \\
x^{\beta}_{5,t_M} &\equiv -\frac{\sqrt{3}}{2}(1 + L - L^3 - L^4 + L^6 + L^7 - L^9 - L^{10})x_{t_M}.
\end{aligned}$$

Before we turn to a discussion of temporal aggregation, a few remarks can be made. The precise transformations of the regressors of the HEGY test regression depend on the periodicity of the data. For quarterly data ($S = 4$) these transformations can be found in Hylleberg *et al.* (1990), for the monthly case ($S = 12$) in Franses (1991) and Beaulieu and Miron (1993), and for the general case see Smith and Taylor (1999).

Typically, with the HEGY approach, the roots in monthly data are tested as follows. The zero frequency unit root and the bi-monthly unit root ($\pi_0 = 0$ and $\pi_6 = 0$) are tested using left-sided t-statistics, whereas the complex roots, $\pi_{\alpha,k} = \pi_{\beta,k} = 0$ (for $k = 1, ..., 5$) are tested using joint tests. These t and F type tests have nonstandard distributions.

Additionally, in the context of quarterly data, Ghysels *et al.* (1994) propose two new F-statistics. One of these concerns the hypothesis that $\pi_0 = \pi_2 = \pi_{\alpha,1} = \pi_{\beta,1} = 0$, and hence provides an overall test of the null hypothesis of seasonal integration.

The other test concerns unit roots at all seasonal frequencies ($\pi_2 = \pi_{\alpha,1} = \pi_{\beta,1} = 0$). These test statistics have been extended to the monthly case by Taylor (1998).

A potential problem with the HEGY test, which is inherent to the multiple testing procedure, is related to the size of the test. That is, the large number of tests of individual parameters used with this procedure has a substantial impact on the significance level implied for the overall null hypothesis. Consequently, one major difficulty is the choice of an appropriate level of significance for each of the individual tests mentioned above. On the other hand, the advantage of the HEGY test over other available seasonal unit root test procedures is that more general processes are considered under the alternative hypothesis, by allowing the possibility that some but not all unit roots implied by Δ_S could be present.

Finally, given the orthogonality of the regressors, the joint tests computed from HEGY test regressions are equivalent to averages of squared t-statistics, see for example, Chan and Wei (1988), Ghysels, Lee and Noh (1994), and others for details.

2.3 From monthly to quarterly data

To understand the implications of monthly unit roots on aggregated quarterly data, we carry out a Monte Carlo investigation. The DGP considered is

$$\alpha(L)x_{t_M} = \varepsilon_{t_M}, \tag{7}$$

where $\varepsilon_{t_M} \sim nid(0,1)$, $\alpha(L) = (1-\alpha_1 L)(1+\alpha_2 L)(1+\alpha_3 L^2)(1+\sqrt{3}\alpha_4 L+\alpha_4^2 L^2)(1-\sqrt{3}\alpha_5 L+\alpha_5^2 L^2)(1+\alpha_6 L+\alpha_6^2 L^2)(1-\alpha_7 L+\alpha_7^2 L^2)$ and $\alpha_i \in (0,1]$, $i = 1, ..., 7$.

Based on the aggregation of data generated from (7), quarterly data are obtained such that

$$x_{t_Q} = (1+L+L^2)x_{t_M}.$$

The resulting quarterly observations do not have overlapping monthly observations as they are systematically sampled.

In order to determine the implications that the monthly roots have on quarterly data, we consider the application of a HEGY test regression with $S = 4$, on samples of 500 quarterly observations obtained through the aggregation of 1500 monthly observations. For each case, we use 20000 replications.

Table 2 summarizes the findings of our extensive Monte Carlo study. From this table, we can draw the following conclusions. With regard to the zero frequency unit root, there is a direct relationship between the monthly and quarterly root. The earlier expected effects of temporal aggregation anticipated this result. Next, the monthly root at frequency $\pi/3$ impacts on the quarterly π frequency root as expected. Moreover, also the monthly π frequency root affects the quarterly π frequency root. Finally, the quarterly $\pi/2$ frequency root emerges when monthly unit roots at frequencies $5\pi/6$, $\pi/6$ and $\pi/2$ are present in the DGP. And, the monthly unit root at frequency $2\pi/3$ seems to affect the phase of the quarterly $\pi/2$ root.

Based on these outcomes, we see opportunities to design more parsimonious monthly unit root test regressions, which are based on the outcomes of tests for monthly data when first aggregated to quarterly data.

3 A sequential testing procedure

The results in the previous section conveyed that an application of the quarterly HEGY procedure could be informative for the presence of unit roots at different frequencies in the monthly polynomial denoted as $\phi_M(L)$. Consequently, the following can be considered:

1) If a zero, bi-annual and an annual unit root are detected in the quarterly data, then, in the monthly context one should consider the overall null hypothesis that $\phi_M^1(L) \equiv \Delta_{12}$ is adequate. Hence, one should apply the procedure proposed by Beaulieu and Miron (1993), and others.

2) If only a zero frequency and π frequency unit root are detected in the quarterly data, then for the monthly data one only has to consider the filter $\phi_M^2(L) \equiv (1 - L)(1 + L)(1 - L + L^2) \equiv 1 - L + L^3 - L^4$.

3) If a long-run (zero frequency) and an annual root ($\frac{\pi}{2}$ frequency) are present in the quarterly series, then, in the monthly context, the filter to be considered

is $\phi_M^3(L) \equiv (1-L)(1+L^2)(1+\sqrt{3}L+L^2)(1-\sqrt{3}L+L^2)(1+L+L^2) \equiv 1-L^3+L^6-L^9$.

In sum, in cases 2) and 3), one can save on variables (and hence parameters) in the monthly test regression, and hence increase power.

3.1 Test Procedures

To derive the necessary test regressions, the following DGP will be considered,

$$\phi_M^\kappa(L)x_{t_M} = \varepsilon_{t_M} \tag{8}$$

where $\kappa = 1, 2, 3$, refers to the three scenarios mentioned above. For each case, one should consider the following test regressions.

1) When $\kappa = 1$, the adequate test regression is the one proposed by for example Beaulieu and Miron (1993). In the notation of Smith and Taylor (1999), the adequate test regression is

$$\begin{aligned}\phi_M^1(L)y_{t_M} = \pi_0 x_{0,t_M-1} + \pi_{S/2}x_{S/2,t_M-1} \\ + \sum_{k=1}^{5}\left(\pi_{\alpha,k}x^{\alpha}_{k,t_M-1} + \pi_{\beta,k}x^{\beta}_{k,t_M-1}\right) + \varepsilon_{t_M}\end{aligned} \tag{9}$$

where the regressors correspond to the linear combinations provided earlier; see Section 2.

2) When $\kappa = 2$, $\phi_M^2(L) \equiv 1-L+L^3-L^4$, the corresponding test regression is

$$\phi_M^2(L)y_{t_M} = \pi_0 x_{0,t_M-1} + \pi_6 x_{6,t_M-1} + \left(\pi_{\alpha,1}x^{\alpha}_{1,t_M-1} + \pi_{\beta,1}x^{\beta}_{1,t_M-1}\right) + \varepsilon_{t_M} \tag{10}$$

where

$$\begin{aligned} x_{0,t_M} &\equiv \left(1+L^3\right)x_{t_M}, \\ x_{6,t_M} &\equiv -\left(1-2L+2L^2-L^3\right)x_{t_M}, \\ x^{\alpha}_{1,t_M} &\equiv -\frac{1}{2}\left(1-2L-L^2+2L^3\right)x_{t_M} \\ x^{\beta}_{1,t_M} &\equiv -\frac{\sqrt{3}}{2}\left(-1+L^2\right)x_{t_M}. \end{aligned}$$

3) When $\kappa = 3$, $\phi_M^3(L) \equiv 1 - L^3 + L^6 - L^9$ and the relevant test regression in this case is

$$\phi_M^2(L) y_{t_M} = \pi_0 x_{0,t_M-1} + \sum_{k=1}^{4} \left(\pi_{\alpha,k} x_{k,t_M-1}^{\alpha} + \pi_{\beta,k} x_{k,t_M-1}^{\beta} \right) + \varepsilon_{t_M} \tag{11}$$

where the test regression variables are defined as

$$\begin{aligned}
x_{0,t_M} &\equiv \left(1 + L + L^2 + L^6 + L^7 + L^8\right) x_{t_M}, \\
x_{1,t_M}^{\alpha} &\equiv \left(L - L^3 - L^4 + L^5 + L^6 - L^8\right) x_{t_M}, \\
x_{1,t_M}^{\beta} &\equiv \left(1 - L^2 - L^3 + L^4 + L^5 - L^7\right) x_{t_M}, \\
x_{2,t_M}^{\alpha} &\equiv \frac{1}{2}\left(-2L^8 + \sqrt{3}L^7 - L^6 + 2L^5 + \left(1 - \sqrt{3}\right) L^4 + \left(1 - \sqrt{3}\right) L^3 - L + \sqrt{3}\right) x_{t_M} \\
x_{2,t_M}^{\beta} &\equiv -\frac{1}{2}\left(-L^7 + L^6\sqrt{3} - 2L^5 + \left(\sqrt{3} + 1\right) L^4 + \left(-\sqrt{3} - 1\right) L^3 + 2L^2 - \sqrt{3}L + 1\right) x_{t_M} \\
x_{3,t_M}^{\alpha} &\equiv -\frac{1}{2}\left(2L^8 + L^7\sqrt{3} + L^6 - 2L^5 + \left(-1 - \sqrt{3}\right) L^4 + \left(-1 - \sqrt{3}\right) L^3 + L + \sqrt{3}\right) x_{t_M} \\
x_{3,t_M}^{\beta} &\equiv \frac{1}{2}\left(-L^7 - \sqrt{3}L^6 - 2L^5 + \left(1 - \sqrt{3}\right) L^4 + \left(\sqrt{3} - 1\right) L^3 + 2L^2 + \sqrt{3}L + 1\right) x_{t_M} \\
x_{4,t_M}^{\alpha} &\equiv \frac{1}{2}\left(1 + L - 2L^2 + L^6 + L^7 - 2L^8\right) x_{t_M} \left(root \frac{2\pi}{3} \right) \\
x_{4,t_M}^{\beta} &\equiv -\frac{\sqrt{3}}{2}\left(1 - L + L^6 - L^7\right) x_{t_M}
\end{aligned}$$

Hence, cases 2) and 3) have a smaller amount of variables.

3.2 Asymptotic Results

In order to derive the asymptotic results of the test procedures presented, we will refer to Chan and Wei (1988) and Smith and Taylor (1999).

Theorem 1 *Assuming that the DGP is $\phi_M^{\kappa}(L) x_{t_M} = \varepsilon_{t_M}$ with $\kappa = 1, 2, 3$ and setting the initial values to zero, the t-ratios obtained from (9), (10) and (11) can be shown to converge in each case to 1) when $\kappa = 1$, the distributions can be found in Beaulieu and Miron (1993)*

2) when $\kappa = 2$, we observe that

$$i) \qquad t_j \;\Rightarrow\; \frac{\int_0^1 W_j(r)dW_j(r)}{\left(\int_0^1 W_j^2(r)dr\right)^{1/2}} \equiv \mathcal{J}_j, \quad j = 0, 6$$

$$ii) \qquad t_{\alpha,1} \;\Rightarrow\; \frac{\int_0^1 W_{\alpha,1}(r)dW_{\alpha,1}(r) + \int_0^1 W_{\beta,1}(r)dW_{\beta,1}(r)}{\left(\int_0^1 W_{\alpha,1}^2(r)dr + \int_0^1 W_{\beta,1}^2(r)dr\right)^{1/2}} \equiv \mathcal{J}_{\alpha,1}$$

$$iii) \qquad t_{\beta,1} \;\Rightarrow\; \frac{\int_0^1 W_{\beta,1}(r)dW_{\alpha,1}(r) - \int_0^1 W_{\alpha,1}(r)dW_{\beta,1}(r)}{\left(\int_0^1 W_{\alpha,1}^2(r)dr + \int_0^1 W_{\beta,1}^2(r)dr\right)^{1/2}} \equiv \mathcal{J}_{\beta,1}$$

3) When $\kappa = 3$, it can be established that

$$i) \qquad t_0 \;\Rightarrow\; \frac{\int_0^1 W_0(r)dW_0(r)}{\left(\int_0^1 W_0^2(r)dr\right)^{1/2}} \equiv \mathcal{J}_0$$

$$ii) \qquad t_{\alpha,k} \;\Rightarrow\; \frac{\int_0^k W_{\alpha,k}(r)dW_{\alpha,k}(r) + \int_0^k W_{\beta,k}(r)dW_{\beta,k}(r)}{\left(\int_0^k W_{\alpha,k}^2(r)dr + \int_0^k W_{\beta,k}^2(r)dr\right)^{k/2}} \equiv \mathcal{J}_{\alpha,k}$$

$$iii) \qquad t_{\beta,k} \;\Rightarrow\; \frac{\int_0^k W_{\beta,k}(r)dW_{\alpha,k}(r) - \int_0^k W_{\alpha,k}(r)dW_{\beta,k}(r)}{\left(\int_0^k W_{\alpha,k}^2(r)dr + \int_0^k W_{\beta,k}^2(r)dr\right)^{k/2}} \equiv \mathcal{J}_{\beta,k}$$

where $W_i(r)$ are standard Brownian motions.

Proof: The results follow from Lemmas 3.3.1 and 3.3.2 in Chan and Wei (1988). Given the orthogonality of the regressors (see Chan and Wei (1988), Osborn and Rodrigues (1998) and Smith and Taylor (1999)), we can analyse the asymptotic behaviour of the test statistics component wise. In other words, we can consider each test statistic independently of the others.

Furthermore, the following theorem regarding relevant joint tests can also be provided.

Theorem 2 *Under the same assumptions of Theorem 1 and given the independence of the test statistics, the distributions of the joint tests are*

1) For $\kappa = 1$, see Beaulieu and Miron (1993) and others;

2) For $\kappa = 2$, we have

$$\begin{aligned} F_{0,6,\alpha,\beta} &\Rightarrow \frac{1}{4}\left[\mathcal{J}_0 + \mathcal{J}_6 + \mathcal{J}_{\alpha,1} + \mathcal{J}_{\beta,1}\right] \\ F_{6,\alpha,\beta} &\Rightarrow \frac{1}{3}\left[\mathcal{J}_6 + \mathcal{J}_{\alpha,1} + \mathcal{J}_{\beta,1}\right] \\ F_{\alpha,\beta} &\Rightarrow \frac{1}{2}\left(\mathcal{J}_{\alpha,1} + \mathcal{J}_{\beta,1}\right) \end{aligned}$$

3) For $\kappa = 3$, it can be established that

$$\begin{aligned} F_{0,\alpha1,\beta1,\ldots,\alpha4,\beta4} &\Rightarrow \frac{1}{9}\left[\mathcal{J}_0 + \sum_{k=1}^{4}\left(\mathcal{J}_{\alpha,k} + \mathcal{J}_{\beta,k}\right)\right] \\ F_{\alpha1,\beta1,\ldots,\alpha4,\beta4} &\Rightarrow \frac{1}{8}\sum_{k=1}^{4}\left(\mathcal{J}_{\alpha,k} + \mathcal{J}_{\beta,k}\right) \\ F_{\alpha,\beta} &\Rightarrow \frac{1}{2}\left(\mathcal{J}_{\alpha,k} + \mathcal{J}_{\beta,k}\right) \end{aligned}$$

Proof: The proof of this theorem follows straightforwardly from the asymptotic orthogonality of the regressors. Tables 3, 4 and 5 contain the relevant critical values for the newly developed test statistics. The critical values for the statistics presented in Theorems 1 and 2 are derived, based on data generated from a data generation process (DGP) such as (8) with $\kappa = 2, 3$ and $u_t \sim nid(0, 1)$. The RNDN function in GAUSS for Windows NT/95 Version 3.2.38 is used. The critical values are derived for samples of 120, 300 and 600 observations (10, 25 and 50 years, respectively). In each case, 20000 replications are used.

3.3 Empirical size and power

We now contrast the performance of the new procedures proposed with that of the monthly version of the HEGY test ($\kappa = 1$) via Monte Carlo simulations. Two data generation processes (DGPs) are considered, that is,

a) DGP$_A$: $\kappa = 2$

$$(1-\phi L)(1+\phi L)(1-\phi L+\phi^2 L^2)y_t = u_t$$

b) DGP$_B$: $\kappa = 3$

$$(1-\phi L)(1+\phi L^2)(1+\sqrt{3}\phi L+\phi^2 L^2)(1-\sqrt{3}\phi L+\phi^2 L^2)(1+\phi L+\phi^2 L^2)y_t = u_t$$

where $\varepsilon_t \sim nid(0, 1)$ and $\phi = \{0.7, 0.8, 0.9, 0.95, 1.00\}$.

The procedures are applied on artificial samples of 120 and 300 observations and in each case 20000 replications are used. The results are summarized in Tables 6 and 7. The overall conclusion from the results in these tables is that the sequential procedure has higher power in most instances, and in particular in smaller samples.

4 Illustration

In this section, we illustrate our sequential procedure and compare it with the standard HEGY approach for five monthly series. These series are five US industrial production series, concerning Automotive products, Food and Tobacco, Clothing, Fuels and Energy. The data are downloaded from www.economagic.com. The data for the first two series start in 1947.01 and end in 2001.03, while the last two series start in 1954.01. These series are selected out of 14 production series considered (on a quarterly basis) in Franses and Paap (2003), as it is found that the quarterly series have the following unit roots, that is, Automotive products has -1, Food and Tobacco, Clothing and Energy products have 1, i and $-i$ and Fuels has 1 and -1. So we have three times the situation with $\kappa = 2$ and twice that $\kappa = 3$.

Application of the HEGY tests, where we include seasonal dummies and a trend in each auxiliary regression, leads to the results summarized in Table 8. Note that we decide on the lag structure using LM tests for 1 to 12-th order serial correlation. Clearly, the differences between the test results are not major for most series, except for the Food and Tobacco series, for which our method does not indicate that there are seasonal unit roots at frequency $\frac{5\pi}{6}$. On the other hand, for Clothing, we do not reject the presence of the seasonal unit roots at the frequency $\frac{2\pi}{3}$, whereas the standard HEGY approach does.

5 Conclusion

In this paper we proposed a sequential approach to testing for seasonal unit roots in high frequency data. Our ideas were framed in the context of monthly data, but of course, it can be extended to any frequency, like for example daily data within a year. Through simulations we showed that our new approach has more power than the standard HEGY method, especially in small samples. We expect these outcomes to hold a fortiori when considering higher frequency data.

Table 1: Factors and Roots of Seasonal Random Walks

Monthly Seasonal Random Walk				
Factors	Roots	Freq.	Cycles	Length
$(1-L)$	1	0	0	
$(1+L)$	-1	π	6/12	2 months
$(1+L^2)$	i $-i$	$\pi/2$ $3\pi/2$	3/12 9/12	4 months 1.33 months
$(1+\sqrt{3}L+L^2)$	$-\frac{1}{2}\sqrt{3}+\frac{1}{2}i$ $-\frac{1}{2}\sqrt{3}-\frac{1}{2}i$	$5\pi/6$ $7\pi/6$	5/12 7/12	2.4 months 1.7 months
$(1-\sqrt{3}L+L^2)$	$\frac{1}{2}\sqrt{3}+\frac{1}{2}i$ $\frac{1}{2}\sqrt{3}-\frac{1}{2}i$	$11\pi/6$ $\pi/6$	11/12 1/12	1.09 months 12 months
$(1+L+L^2)$	$-\frac{1}{2}+\frac{1}{2}i\sqrt{3}$ $-\frac{1}{2}-\frac{1}{2}i\sqrt{3}$	$2\pi/3$ $4\pi/3$	4/12 8/12	3 months 1.5 months
$(1-L+L^2)$	$\frac{1}{2}-\frac{1}{2}i\sqrt{3}$ $\frac{1}{2}+\frac{1}{2}i\sqrt{3}$	$5\pi/3$ $\pi/3$	10/12 2/12	1.2 months 6 months

Quarterly Seasonal Random Walk				
Factors	Roots	Freq.	Cycles	Length
$(1-L)$	1	0	0	0
$(1+L)$	-1	π	2/4	2 quarters
$(1+L^2)$	$\pm i$	$\pi/2$	1/4	1 quarter

Table 2: Summary of Monte Carlo experiments

Monthly Unit Roots	t_{π_1}	t_{π_2}	t_{π_3}	t_{π_4}	F_{1-4}	F_{2-4}	F_{34}
π		x					
$\frac{\pi}{2}$			x	x			x
$\frac{\pi}{3}$		x					
$\frac{2\pi}{3}$				x			
$\frac{\pi}{6}$			x	x			x
$\frac{5\pi}{6}$			x	x			x
$0, \pi, \frac{\pi}{2}$	x	x	x				
$0, \frac{2\pi}{3}, \frac{\pi}{3}$	x	x		x			
$\pi, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{\pi}{6}$		x	x	x		x	x
$\frac{5\pi}{6}, \frac{\pi}{6}, \frac{2\pi}{3}, \frac{\pi}{3}$		x	x	x		x	x
$\pi, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{\pi}{6}, \frac{2\pi}{3}, \frac{\pi}{3}$		x	x	x		x	x
$0, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{\pi}{6}, \frac{2\pi}{3}, \frac{\pi}{3}$	x	x	x	x	x	x	x
$0, \pi, \frac{5\pi}{6}, \frac{\pi}{6}, \frac{2\pi}{3}, \frac{\pi}{3}$	x	x	x	x	x	x	x
$0, \pi, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{\pi}{6}, \frac{\pi}{3}$	x	x	x	x	x	x	x
$0, \pi, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{\pi}{6}, \frac{2\pi}{3}$	x	x	x	x	x	x	x
$0, \pi, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{\pi}{6}, \frac{2\pi}{3}, \frac{\pi}{3}$	x	x	x	x	x	x	x

Note: An x indicates that the results obtained for the test statistics are close to their nominal levels (1%, 5% and 10%).

Table 3: Critical Values of test statistics obtained from test regression (9); $\kappa = 1$.

		t_0			t_6		
$12N$		1%	5%	10%	1%	5%	10%
120	$\xi = 0$	-3.203	-2.652	-2.357	-3.221	-2.650	-2.366
	$\xi = 1$	-3.767	-3.169	-2.868	-3.218	-2.650	-2.360
300	$\xi = 0$	-3.367	-2.780	-2.480	-3.317	-2.782	-2.490
	$\xi = 1$	-3.855	-3.301	-3.034	-3.317	-2.780	-2.491
600	$\xi = 0$	-3.390	-2.838	-2.543	-3.383	-2.822	-2.526
	$\xi = 1$	-3.964	-3.376	-3.083	-3.387	-2.823	-2.525

		$F_{0,6,\alpha,\beta}$			$F_{6,\alpha,\beta}$			$F_{\alpha,\beta}$		
$12N$		1%	5%	10%	1%	5%	10%	1%	5%	10%
120	$\xi = 0$	5.492	4.492	4.037	5.503	4.490	4.045	7.774	5.724	4.768
	$\xi = 1$	5.754	4.710	4.260	5.465	4.474	4.024	7.789	5.728	4.759
300	$\xi = 0$	5.193	4.446	4.052	5.276	4.479	4.087	8.359	6.227	5.264
	$\xi = 1$	5.466	4.658	4.262	5.266	4.479	4.080	8.390	6.217	5.272
600	$\xi = 0$	5.100	4.422	4.075	5.218	4.480	4.088	8.573	6.445	5.461
	$\xi = 1$	5.339	4.640	4.270	5.215	4.473	4.085	8.578	6.441	5.459

Note: $\xi = 0$ and $\xi = 1$ indicate critical values obtained from test regressions with monthly seasonal dummies only and with monthly seasonal dummies and time trend, respectively. N represents de number of years considered.

Table 4: Critical Values of test statistics obtained from test regression (10); $\kappa = 2$.

			t_0			t_6	
$12N$		1%	5%	10%	1%	5%	10%
120	$\xi = 0$	-3.281	-2.676	-2.387	-3.273	-2.698	-2.393
	$\xi = 1$	-3.736	-3.182	-2.902	-3.272	-2.690	-2.394
300	$\xi = 0$	-3.325	-2.788	-2.495	-3.395	-2.785	-2.491
	$\xi = 1$	-3.844	-3.305	-3.047	-3.387	-2.785	-2.488
600	$\xi = 0$	-3.393	-2.829	-2.534	-3.366	-2.813	-2.517
	$\xi = 1$	-3.906	-3.372	-3.075	-3.369	-2.813	-2.516

			$F_{0,6,\alpha,\beta}$			$F_{6,\alpha,\beta}$			$F_{\alpha,\beta}$	
$12N$		1%	5%	10%	1%	5%	10%	1%	5%	10%
120	$\xi = 0$	6.484	5.106	4.426	7.201	5.403	4.651	8.233	5.991	5.023
	$\xi = 1$	7.247	5.725	5.014	7.173	5.363	4.622	8.134	5.955	4.984
300	$\xi = 0$	6.742	5.336	4.677	7.437	5.698	4.927	8.598	6.364	5.405
	$\xi = 1$	7.465	6.031	5.343	7.437	5.685	4.924	8.601	6.347	5.386
600	$\xi = 0$	6.857	5.435	4.758	7.543	5.841	5.017	8.867	6.570	5.516
	$\xi = 1$	7.632	6.129	5.421	7.524	5.839	5.014	8.835	6.567	5.514

Note: See Table 3.

Table 5: Critical Values of test statistics obtained from test regression (11); $\kappa = 3$.

		t_0		
$12N$		1%	5%	10%
120	$\xi = 0$	-3.236	-2.647	-2.359
	$\xi = 1$	-3.731	-3.147	-2.867
300	$\xi = 0$	-3.333	-2.780	-2.500
	$\xi = 1$	-3.856	-3.318	-3.042
600	$\xi = 0$	-3.387	-2.804	-2.517
	$\xi = 1$	-3.893	-3.353	-3.077

		$F_{0,\alpha1,\beta1,\ldots,\alpha4,\beta4}$			$F_{\alpha1,\beta1,\ldots,\alpha4,\beta4}$			$F_{\alpha k,\beta k}$		
$12N$		1%	5%	10%	1%	5%	10%	1%	5%	10%
120	$\xi = 0$	5.595	4.562	4.085	5.662	4.600	4.114	8.046	5.884	4.925
	$\xi = 1$	5.821	4.862	4.367	5.640	4.572	4.086	7.961	5.856	4.891
300	$\xi = 0$	5.471	4.607	4.184	5.620	4.686	4.245	8.553	6.336	5.394
	$\xi = 1$	5.774	4.904	4.465	5.589	4.673	4.234	8.523	6.320	5.372
600	$\xi = 0$	5.430	4.600	4.180	5.587	4.699	4.257	8.606	6.516	5.504
	$\xi = 1$	5.736	4.896	4.477	5.570	4.699	4.254	8.600	6.516	5.504

Note: See note under Table 3.

Table 6: Size and power of new procedure versus the standard HEGY approach: DGP a

$12N = 120$

ϕ		$t_0^{\kappa=2}$	$t_0^{\kappa=1}$	$t_6^{\kappa=2}$	$t_6^{\kappa=1}$	$F_{com}^{\kappa=2}$	$F_{com}^{\kappa=1}$
1.00	$\xi = 0$	.050	.059	.050	.055	.050	.058
	$\xi = 1$	.050	.067	.050	.055	.050	.059
0.95	$\xi = 0$	.139	.140	.131	.115	.203	.166
	$\xi = 1$	.097	.107	.133	.116	.204	.173
0.90	$\xi = 0$	.381	.264	.380	.218	.630	.365
	$\xi = 1$	.235	.177	.384	.219	.628	.375
0.80	$\xi = 0$	.876	.478	.910	.412	.995	.684
	$\xi = 1$	.678	.323	.913	.415	.995	.696
0.70	$\xi = 0$	.981	.601	.996	.541	1.00	.823
	$\xi = 1$	.908	.423	.996	.542	1.00	.834

$12N = 300$

ϕ		$t_0^{\kappa=2}$	$t_0^{\kappa=1}$	$t_6^{\kappa=2}$	$t_6^{\kappa=1}$	$F_{com}^{\kappa=2}$	$F_{com}^{\kappa=1}$
1.00	$\xi = 0$	.050	.053	.053	.052	.052	.056
	$\xi = 1$	.050	.053	.050	.053	.050	.054
0.95	$\xi = 0$	.600	.461	.601	.437	.881	.719
	$\xi = 1$	.377	.280	.595	.437	.877	.710
0.90	$\xi = 0$	.993	.862	.995	.851	1.00	.991
	$\xi = 1$	.934	.659	.994	.852	1.00	.990
0.80	$\xi = 0$	1.00	1.00	1.00	1.00	1.00	1.00
	$\xi = 1$	1.00	.944	1.00	.992	1.00	1.00
0.70	$\xi = 0$	1.00	1.00	1.00	1.00	1.00	1.00
	$\xi = 1$	1.00	.986	1.00	.999	1.00	1.00

Note: $\xi = 0$ and $\xi = 1$ indicate the percentage of rejection of the test statistics obtained from test regressions with monthly seasonal dummies only and with monthly seasonal dummies and time trend, respectively.

Table 7: Size and power of new procedure versus the standard HEGY approach: DGP b

$12N = 120$

ϕ		$t_0^{\kappa=3}$	$t_0^{\kappa=1}$	$F_{com1}^{\kappa=3}$	$F_{com1}^{\kappa=1}$	$F_{com2}^{\kappa=3}$	$F_{com2}^{\kappa=1}$	$F_{com3}^{\kappa=3}$	$F_{com3}^{\kappa=1}$
1.00	$\xi = 0$	.05	.05	.05	.98	.05	.99	.05	.04
	$\xi = 1$	.05	.05	.05	1.0	.05	1.0	.05	.05
0.95	$\xi = 0$	.11	.18	.10	1.0	.48	1.0	.34	.27
	$\xi = 1$	.11	.11	.49	1.0	.51	1.0	.21	.20
0.90	$\xi = 0$	.31	.36	.20	1.0	.92	1.0	.79	.48
	$\xi = 1$	.22	.20	.96	1.0	.96	1.0	.56	.50
0.80	$\xi = 0$	.78	.60	.55	1.0	1.0	1.0	.98	.81
	$\xi = 1$	.44	.38	1.0	1.0	1.0	1.0	.93	.86
0.70	$\xi = 0$	.95	.69	.79	1.0	1.0	1.0	.99	.99
	$\xi = 1$	.53	.46	1.0	1.0	1.0	1.0	.98	.90

$12N = 300$

ϕ		$t_0^{\kappa=3}$	$t_0^{\kappa=1}$	$F_{com1}^{\kappa=3}$	$F_{com1}^{\kappa=1}$	$F_{com2}^{\kappa=3}$	$F_{com2}^{\kappa=1}$	$F_{com3}^{\kappa=3}$	$F_{com3}^{\kappa=1}$
1.00	$\xi = 0$	.05	.05	.05	1.0	.05	1.0	.05	.05
	$\xi = 1$	.05	.05	.05	1.0	.05	1.0	.05	.05
0.95	$\xi = 0$	.58	.59	1.0	1.0	1.0	1.0	.87	.83
	$\xi = 1$	.35	.34	1.0	1.0	1.0	1.0	.87	.82
0.90	$\xi = 0$	.98	.96	1.0	1.0	1.0	1.0	1.0	1.0
	$\xi = 1$	.87	.79	1.0	1.0	1.0	1.0	1.0	1.0
0.80	$\xi = 0$	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	$\xi = 1$	1.0	.98	1.0	1.0	1.0	1.0	1.0	1.0
0.70	$\xi = 0$	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	$\xi = 1$	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Table 8: F test results for seasonal unit roots using the standard HEGY approach for monthly data (top panel) and our sequential approach (bottom panel)

Variables	n	lags	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	$\frac{\pi}{6}$	$\frac{2\pi}{3}$	$\frac{\pi}{3}$
			Standard HEGY approach				
Automotive products	614	25	5.944	4.208	7.915	5.663	4.182
Food and Tobacco	615	24	11.271	6.345	3.583	11.535	14.708
Clothing	615	24	4.618	2.014	5.502	9.355	3.582
Energy Products	534	21	5.754	5.879	4.151	14.437	2.916
Fuels	543	12	17.755	19.373	10.901	30.936	10.510
			Sequential HEGY approach				
Automotive products ($\kappa = 2$)	614	33					4.714
Food and Tobacco ($\kappa = 3$)	626	16	17.320	14.432	4.715	7.827	
Clothing ($\kappa = 3$)	606	36	5.834	1.405	2.672	3.981	
Energy Products ($\kappa = 3$)	534	24	5.909	5.886	1.566	4.359	
Fuels ($\kappa = 2$)	553	10					8.781

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