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The Timing of Asset Trade and Optimal Policy in
Dynamic Open Economies

Ozge Senay*
University of St Andrews

Alan Sutherland†
University of St Andrews
and CEPR

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ABSTRACT

Using a standard open economy DSGE model, it is shown that the timing of asset trade relative to policy decisions has a potentially important impact on the welfare evaluation of monetary policy at the individual country level. If asset trade in the initial period takes place before the announcement of policy, a national policymaker can choose a policy rule which reduces the work effort of households in the policymaker's country in the knowledge that consumption is fully insured by optimally chosen international portfolio positions. But if asset trade takes place after the policy announcement, this insurance is absent and households in the policymaker's country bear the full consumption consequences of the chosen policy rule. The welfare incentives faced by national policymakers are very different between the two cases. Numerical examples confirm that asset market timing has a significant impact on the optimal policy rule.

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* School of Economics and Finance, University of St Andrews, St Andrews, KY16 9AL, UK. E-mail: os12@st-and.ac.uk. Tel: +44 1334 462422 Fax: +44 1334 462444.

† School of Economics and Finance, University of St Andrews, St Andrews, KY16 9AL, UK and CEPR. E-mail: ajs10@st-and.ac.uk. Tel: +44 1334462446 Fax: +44 1334 462444.

1 Introduction

In the literature on optimal monetary policy in open economies it is common practice to model international asset markets in the form of trade in a complete set of Arrow-Debreu securities.¹ This makes it possible to side-step explicit analysis of portfolio allocation problems. But new solution techniques (Devereux and Sutherland (2008a) and Tille and van Wincoop (2010)) now make it possible to conduct a much more explicit analysis of asset market structure and its implications for optimal policy. These techniques can be used to consider explicitly the way in which asset markets affect the transmission mechanism between monetary policy and welfare. In this paper, we use these new techniques to examine in detail the role of the *timing of asset trade relative to the timing of policy decisions* in determining the way monetary policy affects welfare in an open economy.

We show that the incentives faced by a policymaker can be very different depending on whether asset trade takes place before or after monetary policy is determined. If asset trade takes place *before* policy is determined, equilibrium international portfolio positions provide a high degree of insurance against the effects of policy. This insurance is not present if asset trade takes place *after* policy is determined. In effect, if asset trade takes place before policy is determined, the insurance provided by asset portfolios creates an international spillover effect which implies that some of the negative welfare impact of policy in one country falls on foreign households. The insurance against the effect of policy, and thus the spillover effect, is absent when asset trade takes place after policy is determined. We show that the presence or absence of the insurance, and thus the presence or absence of the spillover effect, has a significant impact on the welfare maximising choice of monetary policy by national policymakers.²

The general principles of the argument just stated are straightforward to describe and understand. The details of how these mechanisms operate within a fully specified dynamic stochastic general equilibrium model with international trade in multiple assets

¹See, for instance, Gali and Monacelli (2005), Devereux and Engel (2003), Benigno and Benigno (2006), Pappa (2004), Faia and Monacelli (2008) and De Paoli (2009a 2009b). Some significant contributions to the open economy literature, however, are not based on trade in Arrow-Debreu assets. For instance, Clarida, Gali and Gertler (2002) and Corsetti and Pesenti (2005) assume a unit elasticity of international trade. This implies that financial market structure is irrelevant. Obstfeld and Rogoff (1995) and Kollmann (2002) assume that international financial trade is confined to non-contingent bonds.

²Note that asset market timing is only relevant for welfare evaluation from the point of view of national policymakers. When the global welfare effects of policy are analysed from the point of view of a global policymaker, the spillover generated by asset trade is fully internalised by the policymaker regardless of the timing of asset trade. The timing of asset trade therefore has no impact on the incentives faced by a global policymaker.

are, however, much less obvious. For instance, in a dynamic model where asset trade takes place period-by-period it appears that asset trade must inevitably take place after a policy change has been announced. Does this imply that asset markets do not provide insurance against policy changes? This paper provides a systematic analysis of the links between policy decisions, consumption and welfare and shows explicitly how the timing of asset trade affects the incentives of the policymaker.

We show that policy decisions affect consumption (and therefore welfare) via two asset market transmission channels. One is a flow effect which arises in periods subsequent to the policy change, while the other is a one-off capital gain effect which potentially arises in the period in which a policy change is announced.

The flow income effect is generated by a difference between home and foreign income. If, for instance, the home monetary authority follows a policy rule which tends to depress the expected level of home output, the flow income effect will, other things being equal, imply a reduction in home consumption. The capital gain effect, on the other hand, is the change in the value of the home country portfolio which occurs at the time monetary policy is announced (i.e. in the initial period). A fall in expected home income causes a fall in the value of home equity, which leads to a one-off capital gain for the home population in the initial period (because home households optimally hold a negative external position in home equity).

It is shown that the timing of asset trade in the *initial* period is critical in determining whether the capital gain valuation effect is present or not. If asset trade in the initial period takes place *before* policy is determined then the capital gain valuation effect is present. But if asset trade in the initial period takes place *after* policy is determined, the capital gain valuation effect is absent. It is shown below that the presence or absence of this capital gain valuation effect has an important impact on the incentives faced by the monetary policymaker.

While there is now an extensive literature examining optimal monetary policy in open economy models, there has been no previous detailed analysis of the implications of asset-trade timing for the welfare effects of policy in a dynamic multi-period setting. Indeed in much of the current literature (see e.g. Galí and Monacelli (2005), Benigno and Benigno (2006), Pappa (2004) and Faia and Monacelli (2008)) there is an implicit assumption that policy decisions are made after asset trade takes place in the initial period. There is rarely any justification for this assumption nor is there any recognition that it can have important implications for the welfare effects of monetary policy.

Senay and Sutherland (2007) do provide a basic analysis of asset market timing in

a very simple static single-period model and merely state and demonstrate the general principle that asset market timing can affect optimal policy choices.³ However, asset market trade, and all matters related to the sequencing of events, obviously raise questions which can only adequately be addressed in a dynamic multi-period setting. In addition, the single-period model used in Senay and Sutherland (2007) lacks the complex dynamics that are integral to the standard workhorse models used in the international macro literature. As we show below, the extension of the basic single-period result to a dynamic setting is far from straightforward and the main contribution of the current paper is therefore to show exactly how the timing of asset trade and the welfare effects of policy interact in a multi-period model of the type which is standard in the international macro literature.

In dynamic multi-period models with Arrow-Debreu asset trade, one can think about asset trade, and policy being set, in the initial time period. In such a framework, one way to approach the dynamic analogue of the Senay and Sutherland (2007) analysis would be to consider the timing of trade in Arrow-Debreu securities relative to the timing of policy within that initial period. However, in a more realistic dynamic setting, trade in realistic assets (such as bonds and equities) takes place period by period, rather than exclusively in the initial period. In this more realistic setting, it seems that asset trade will inevitably be taking place *after* policy decisions have been made. This paper focuses on this more realistic setting and analyses the welfare effects of policy in a model with period-by-period trade in equity shares. Our analysis shows clearly that the question of the timing of asset trade relative to policy arises even when asset trade takes place period by period. The timing issue relates to the selection of an initial portfolio of asset holdings. It is this initial portfolio which determines the capital gain valuation effect in the initial period. The fact that asset trade also takes place in all periods subsequent to the policy decision does not undermine the importance of the timing of asset trade in the initial period.⁴

³In an analysis of optimal capital taxation in a small open economy Schmitt-Grohe and Uribe (2003) also discuss an issue related to the timing of asset trade. Rather than focusing on the timing of asset trade *per se*, they frame the problem in terms of the presence or absence of Arrow-Debreu securities which are specifically contingent on the decisions of the policymaker. The assumption that trade in Arrow-Debreu securities takes place after policy decisions are made is effectively equivalent to assuming there are no Arrow-Debreu securities which are contingent on policy decisions.

⁴We assume that policy is represented by a credible once-and-for-all decision about a policy rule. An alternative approach would be to assume that policy is re-optimised period by period. This creates a dynamic game between the policymaker and traders in asset markets. The equilibrium of this dynamic game will depend on the timing of asset trade relative to the policy decision within each period. We focus on the case where policy is a once-and-for-all decision because this corresponds more closely to the standard assumption in the existing literature on monetary policy in open economies. The alternative assumption (where policy is re-optimised period by period) is likely to be an interesting topic for further

Before describing our analysis in detail, it is important to emphasise that we are not arguing that the modelling of asset market timing (relative to policy decisions) in itself represents a way to analyse market imperfections in international financial markets. What we are showing is that asset market timing has important logical implications for the interaction between financial markets and policy which have not been addressed in the existing literature. This issue arises in both complete markets and incomplete markets settings and can co-exist with many forms of asset market imperfection.

The analysis presented below is based on a simple two-country new-Keynesian model. Policy is represented in terms of a choice of a feedback parameter in a monetary policy targeting rule. This choice is made in the initial period. This simple framework provides clear analytical solutions and thus helps to establish the main underlying principles. The issues highlighted here are, however, applicable to a wide range of open economy macro models.⁵

The paper proceeds as follows: Section 2 presents the model structure; Section 3 discusses the general approach to solving the model; Section 4 analyses the impact of the timing of asset trade on the determination of consumption; Section 5 demonstrates the implications for the welfare effects of policy; and Section 6 concludes.

2 Model structure

The model is a standard open economy DSGE model of the type which has been widely used to analyse monetary policy in open economies (see for instance, Gali and Monacelli (2005) and Benigno and Benigno (2006)). The details of the specific model presented below are chosen for illustrative purposes only. The results emphasised in this paper, however, apply to a wide range of models.

The model consists of two countries, home and foreign, inhabited by a continuum of infinitely lived individual households which are both consumers and producers. House-

⁵In the model used below, equity trade is sufficient to support full risk sharing (for a given setting of monetary policy). Our asset-trade-before-policy case corresponds precisely to the Arrow-Debreu case which is the standard assumption in the literature. But the same issues (about the timing of asset trade) also arise when there are not sufficient assets to support full risk sharing. In cases such as this there is not full insurance, but the timing of asset trade affects the *degree* to which there is insurance against policy changes, and this insurance will work through a capital gain valuation effect in the initial period. See Devereux and Sutherland (2008b) for an example of how the setting of monetary policy can affect portfolio allocation in a model where markets are incomplete. Devereux and Sutherland (2008b) do not explicitly analyse the welfare effects of policy, nor do they consider issues related to the timing of asset trade.

holds consume a basket of differentiated, perishable goods of total measure unity. Home country households produce fraction n of goods while foreign households produce the remaining $n^* = 1 - n$. Each individual household uses labour effort to produce a single good and is the monopoly supplier of that good. Nominal price inertia is modelled in the form of Calvo (1983) price setting.

Asset trade takes the form of trade in equity claims on home and foreign income. There is assumed to be only one source of random disturbances in the model, so trade in two equity claims allows full sharing of consumption risk arising from this single source of shocks.

We focus on monetary policy and welfare from the point of view of home country households. The foreign country's monetary stance is taken as given. The structure of the foreign economy is otherwise identical to the home country, so the model description focuses on the home country equations. Where foreign variables do arise, they are indicated with an asterisk.

2.1 Households and the goods market

Representative household h in the home country has a utility function of the form:

$$U_t(h) = E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{C_s^{1-\rho}(h)}{1-\rho} - \frac{K}{\mu} y_s^\mu(h) \right) \right] \quad (1)$$

where C is a consumption index defined across all home and foreign goods, P is the consumer price index, $y(h)$ is the output of good h and E_t is the expectations operator conditional on time- t information. K , ρ and μ are positive constants and $0 < \beta < 1$.

The consumption index C for home agents is given by:

$$C = \left[\left(\frac{1}{2} \right)^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + \left(\frac{1}{2} \right)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \quad (2)$$

where C_H and C_F are indices of individual home and foreign produced goods with an elasticity of substitution between individual goods ϕ , where $\phi > 1$. The parameter θ is the elasticity of substitution between home and foreign goods. Home and foreign goods are assumed to have equal weight in the consumption basket. Combined with an assumption of producer currency pricing, this ensures that purchasing power parity holds in all states

of the world. The aggregate consumer price index for home agents is:

$$P = \left[\frac{1}{2} P_H^{1-\theta} + \frac{1}{2} P_F^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (3)$$

where P_H and P_F are the aggregate price indices for home and foreign goods.

Goods prices are assumed to be set in the currency of the producer and are subject to Calvo (1983) price contracts. The probability that a given producer changes its price in any particular period is taken to be a constant, $(1 - \gamma)$. The first-order condition for the choice of prices implies the following

$$E_t \left\{ \sum_{s=t}^{\infty} (\beta\gamma)^{s-t} \left[(\phi - 1) \frac{p_{H,t} y_{t,s}}{C_s^p P_s} - A_s \phi K y_{t,s}^\mu \right] \right\} = 0 \quad (4)$$

where $y_{t,s}$ is the period- s output of a home producer whose price was last set in period t . Prices are assumed to be subject to ‘‘cost-push’’ disturbances, A , where

$$\log A_t = \zeta \log A_{t-1} + \varepsilon_{A,t} \quad (5)$$

where ε_A is symmetrically distributed over the interval $[-\varepsilon, \varepsilon]$ with $E[\varepsilon_A] = 0$ and $Var[\varepsilon_A] = \sigma_A^2$. Cost push disturbances are assumed to affect only home country pricing and are the only source of shocks in the model. Foreign producers are not subject to cost push disturbances.

2.2 Asset markets

International financial trade takes the form of trade in equity claims on the value of home and foreign aggregate output. Thus the home equity is a claim on $Y_t = y_t P_{H,t} / P_t$, while the foreign equity is a claim on $Y_t^* = y_t^* P_{F,t}^* / P_t^*$, where y_t and y_t^* are aggregate outputs of home and foreign goods. Equity trade takes place period by period. At the end of period t , home and foreign households allocate their net asset position across portfolios of the two equity assets. In period $t + 1$, shocks are realised and output, goods prices, equity prices and equity pay-offs are determined. At the end of period $t + 1$ equity trade is repeated and portfolios are reallocated and held into period $t + 2$, and so on for each future period.

The real pay-off to a unit of the home equity purchased in period t is defined to be $Y_{t+1} + Z_{t+1}$, where Z_{t+1} is the real price of home equity in period $t + 1$. Thus the gross

real rate of return on the home equity is $r_{1,t+1} = (Y_{t+1} + Z_{t+1})/Z_t$, and the gross real return on foreign equity is $r_{2,t+1} = (Y_{t+1}^* + Z_{t+1}^*)/Z_t^*$. The aggregate budget constraint of the home country can then be defined as follows

$$\alpha_{1,t} + \alpha_{2,t} = \alpha_{1,t-1}r_{1t} + \alpha_{2,t-1}r_{2t} + Y_t - C_t \quad (6)$$

where $\alpha_{1,t-1}$ and $\alpha_{2,t-1}$ represent the real external holdings of home and foreign equity, brought into period t from the end of period $t - 1$.⁶

It is useful to define $W_t = \alpha_{1,t} + \alpha_{2,t}$ to be the total net claims of home agents on the foreign country at the end of period t (i.e. the net foreign assets, or *NFA*, of home agents). The budget constraint can then be re-written as

$$W_t = r_{2,t}W_{t-1} + Y_t - C_t + \alpha_{1,t-1}r_{x,t} \quad (7)$$

where

$$r_{x,t} = r_{1,t} - r_{2,t} \quad (8)$$

Here the foreign equity is used as a numeraire and $r_{x,t}$ measures the "excess return" on the home equity. Because $\alpha_{1,t}$ and $\alpha_{2,t}$ measure the external position of the home country in home and foreign equity, market clearing in asset markets implies

$$n\alpha_{1,t} + n^*\alpha_{1,t}^* = 0, \quad n\alpha_{2,t} + n^*\alpha_{2,t}^* = 0$$

To simplify notation, in what follows we will drop the subscript from $\alpha_{1,t}$ and simply refer to α_t . It should be understood, therefore, that $\alpha_t = \alpha_{1,t} = -\frac{n^*}{n}\alpha_{1,t}^*$, $\alpha_{2,t} = W_t - \alpha_t$ and $\alpha_{2,t}^* = W_t^* + \frac{n}{n^*}\alpha_t$.

Preferences and the structure of asset markets imply that optimal consumption choices satisfy the following Euler equation

$$C_t^{-\rho} = \beta E_t C_{t+1}^{-\rho} r_{2,t+1} \quad (9)$$

⁶We adopt the notational convention that α_1 and α_2 represent *external* holdings of equities. That is, α_1 is the value of claims on home output sold by home households to foreign households, and α_2 is the value of claims on foreign output sold by foreign households to home households. An alternative notational convention is to measure portfolio positions in terms of the total (internal plus external) holdings of assets. Our choice of notational convention involves no loss of generality and proves to be particularly convenient for deriving our results. See the Appendix for an explanation of the link between the two notational conventions.

while optimal portfolio choices imply

$$E_t C_{t+1}^{-\rho} r_{x,t+1} = 0 \quad (10)$$

2.3 Monetary policy and asset trade in period 0

Monetary policy is modelled in the form of a targeting rule. The monetary authority in the home country is assumed to choose the monetary instrument (which is not modelled explicitly) in order to ensure that the following targeting relationship holds

$$\log \frac{P_{H,t}}{P_{H,t-1}} + \delta \log A_t = 0 \quad (11)$$

Thus the monetary authority follows a state-contingent inflation targeting policy where δ measures the degree to which producer-price inflation is allowed to vary in response to cost push shocks. The analysis below focuses on the welfare implications of the choice of δ . For the purposes of explaining and illustrating the main points of this paper, the precise functional form of the policy rule is not a central issue. The key comparison which we analyse below is the difference in the optimal choice of δ between the asset-trade-before-policy case and the asset-trade-after-policy case.⁷

The foreign monetary authority is assumed to follow a similar targeting rule. In the foreign case, however, δ is assumed to be zero (i.e. the foreign monetary authority completely stabilises the foreign PPI inflation rate). The foreign rule is taken as exogenous and fixed and our analysis is focused on the policy problem of the home country.

In the initial period (i.e. period 0), it is assumed that the only events that occur are:

- A once-and-for-all decision by the home policymaker about δ , which is immediately announced publicly.
- International trade in equities to establish portfolio allocations to be carried into period 1.

The aim of the paper is to investigate the implication of the timing of asset trade in period 0 relative to the timing of the policymaker's decision about δ . There are two alternative assumptions: (1) asset trade before policy; or (2) asset trade after policy.

⁷For the purposes of this paper, a rule of the form given in (11) is nevertheless a reasonable choice as a benchmark example because it is known that optimal policy takes this form in the context of a closed economy model analogous to the model outlined above (see for instance Woodford, 2003).

We assume that households enter period 0 with a common prior belief about the value of δ which is held with subjective certainty. The value of δ actually chosen by the policymaker may, however, be different from households' prior belief.

We further assume that households enter period 0 with zero net foreign assets and zero gross asset positions, so prior to trading assets in period 0, households have zero net and gross portfolio positions. Asset trade allows households to establish optimal gross portfolio positions which hedge against future shocks to A .

The crucial difference between the asset-trade-before-policy case and the asset-trade-after-policy case is that households' knowledge of δ at the time of asset trade differs between the two cases. This is illustrated in the time-lines shown in Figures 1 and 2.

Figure 1 shows that, in the case where asset trade in period 0 takes place *before* the policymaker determines δ , households will trade assets in period 0 at prices which are determined before the true value of δ is known. Asset prices at the time of asset trade in period 0 will incorporate household expectations of δ , i.e. asset prices will be determined by the prior belief about δ . But the value of the portfolio at the start of period 1 (i.e. the portfolio payoff in period 1) depends on asset prices determined *after* δ is actually determined (i.e. asset prices at the start of period 1 will incorporate information about the true value of δ). In other words, asset payoffs in period 1 will reflect any difference between the prior belief about δ and its true value, so households will receive a capital gain (or loss) as a result of the policy announcement. This capital gain affects the *NFA* position of home households at the start of period 1 and therefore affects consumption plans and welfare from period 1 onwards.

Figure 2 shows that, in the case where asset trade in period 0 takes place *after* the policymaker determines δ , households will trade assets in period 0 at prices which are determined after δ is chosen. These prices will also determine the value of the portfolio at the start of period 1, so, in this case, the policy announcement will not create a capital gain or loss at the start of period 1. The absence of the capital gain implies that policy in this case will have a different effect on consumption and welfare compared to the asset-trade-before-policy case.⁸

One way to interpret events in period 0 is in terms of a Stackelberg leader-follower game. In the asset-trade-before-policy case the asset market acts as the Stackelberg leader and the policymaker is the follower. In the asset-trade-after-policy case the roles

⁸Note that in the asset-trade-after-policy case it is important that households do not hold any gross portfolio positions at the time of the policy announcement. "Asset prices" implicitly respond to the policy announcement, but, in the absence of any gross positions, this has no impact on *NFA*.

are reversed, the policymaker is the leader and the asset market is the follower. In each case the follower is able to make optimal decisions in the light of the decisions made by the leader.

3 Model solution

The aim of this paper is to investigate the implications of the timing of asset trade for the evaluation of home welfare in period 0, i.e. welfare at the time that the policymaker makes a decision about policy, i.e. δ .

In models of the form outlined above, welfare analysis is typically based on a second-order approximate solution for aggregate utility. Aggregate (per capita) home welfare in period 0 is given by

$$\Omega = \frac{1}{n} E_0 \sum_{s=0}^{\infty} \beta^s \left\{ \int_0^n \left(\frac{C_s^{1-\rho}(h)}{1-\rho} - \frac{K}{\mu} y_s^\mu(h) \right) dh \right\} \quad (12)$$

A second-order approximation of Ω can be written as follows

$$\begin{aligned} \hat{\Omega} = (1-\beta) E_0 \sum_{s=0}^{\infty} \beta^s \left\{ \hat{C}_s + \frac{1}{2} (1-\rho) \hat{C}_s^2 \right. \\ \left. - \frac{\phi-1}{\phi} \left[\hat{y}_s + \frac{1}{2} \mu \hat{y}_s^2 + \frac{1}{2} \frac{\phi\gamma(1-\phi+\phi\mu)}{(1-\gamma)(1-\beta\gamma)} \pi_s^2 \right] \right\} + O(\varepsilon^3) \end{aligned} \quad (13)$$

where $O(\varepsilon^3)$ contains terms of order higher than two in the variables of the model,⁹ and $\pi_s = \hat{P}_{H,s} - \hat{P}_{H,s-1}$.

Equation (13) shows that home welfare depends on the first and second moments of consumption, output and the rate of producer price inflation (as measured by π). In the literature on optimal monetary policy there has been much discussion and analysis of the properties of welfare functions of this form. This analysis is now very standard and need not be repeated here. For the purposes of the current paper the main point that should be noted is that home welfare depends positively on the first moment of home consumption and negatively on the first moment of home output, i.e. positively on $E_0[\hat{C}_s]$ and negatively on $E_0[\hat{y}_s]$ for $s = 1.. \infty$.

In general, the policy parameter, δ , affects the way the monetary instrument responds to shocks and therefore affects the second moments of the endogenous variables of the

⁹Note that $\hat{\Omega} \equiv (1-\beta)(\Omega - \bar{\Omega})\bar{C}^{\rho-1}$. By writing welfare in this form, $\hat{\Omega}$ can be interpreted in terms of “steady-state consumption units”.

model. In turn, second moments affect first moments of variables at the level of a second-order approximation. So, for instance, δ affects the risk premium on home equity (i.e. the expected return differential between home and foreign equity) and also, via the impact of risk on labour supply, δ affects the expected level of output.¹⁰ The precise nature of the links between δ and these variables is not the central issue in our analysis. It is sufficient to note that the policy parameter affects the expected level of output. The main question of concern in our analysis is how the timing of asset trade affects the link between the expected level of output and the expected level of consumption.

Equation (13) shows that a reduction in the expected level of home output directly increases home welfare (because it represents a reduction in work effort for home households). The overall welfare impact of a fall in home output, however, depends on how home consumption is affected by the consequent fall in home income. Asset markets impinge on the linkage between income and consumption via the impact of asset trade on the cross-country sharing of consumption risk. The first step in our analysis of asset market timing is therefore to derive a second-order approximation of the relationship between income, portfolio returns and consumption. This is based on a second-order approximation of aggregate budget constraints.

A second-order approximation for the aggregate home budget constraint is given by

$$\hat{W}_t = \frac{1}{\beta} \hat{W}_{t-1} + \hat{Y}_t - \hat{C}_t + \tilde{\alpha} \hat{r}_{x,t} + \lambda_{B,t} + O(\varepsilon^3) \quad (14)$$

where for convenience the second-order terms have been gathered together in the following expression

$$\lambda_{B,t} = \frac{1}{2} \hat{Y}_t^2 - \frac{1}{2} \hat{C}_t^2 + \hat{\alpha}_{t-1} \hat{r}_{x,t} + \frac{1}{\beta} \hat{W}_{t-1} \hat{r}_{2,t} \quad (15)$$

In general, a bar over a variable indicates its value in the non-stochastic steady state and a hat indicates the log-deviation from the non-stochastic steady state, except for $\hat{W}_t = (W_t - \bar{W})/\bar{C}$, $\tilde{\alpha} = \bar{\alpha}/(\beta\bar{Y})$, $\hat{\alpha}_{t-1} = (\alpha_{t-1} - \bar{\alpha})/(\beta\bar{Y})$, $\hat{r}_{1,t} = \beta(r_{1,t} - \bar{r}_1)$, $\hat{r}_{2,t} = \beta(r_{2,t} - \bar{r}_2)$ and $\hat{r}_{x,t} = \hat{r}_{1,t} - \hat{r}_{2,t}$.

Notice that it is necessary to derive a solution for the gross portfolio position in the non-stochastic steady state, $\tilde{\alpha}$. For the above model, the solution method described by

¹⁰The policy parameter δ affects the equilibrium variance of output and thus the variance of the realised return on home equities. In equilibrium this affects the risk premium on home equities. Likewise, the variance of output affects optimal labour supply because households are risk averse (i.e. a higher variance of output reduces equilibrium work effort).

Devereux and Sutherland (2008a) yields the following result¹¹

$$\tilde{\alpha} = -\frac{n^*}{1-\beta} \quad (16)$$

This implies that the home country holds a negative external position in home equity and a positive external position in foreign equity. The intuition for this is obvious - optimal risk sharing is achieved by holding a diversified portfolio of claims on home income and foreign income. This is achieved by holding a negative external position in home equity and a corresponding positive external position in foreign equity. Foreign households hold the mirror-image portfolio.¹²

Note that, in principle, the portfolio in period 0 may depend on the timing of asset trade, i.e. $\tilde{\alpha}$ in period 0 may differ from $\tilde{\alpha}$ in all subsequent periods. However, in this model the steady state portfolio is given by (16) in all periods (including period 0) regardless of the timing of asset trade.

We are interested in the evaluation of welfare at the time policy is determined in period 0, so the focus of the analysis is on the expectation of \hat{C} at the time of the policy decision, i.e. $E_0[\hat{C}_t]$, where E_0 denotes expectation conditional on information at the time of the policy decision. It is useful to decompose \hat{C}_t as follows

$$\hat{C}_t = n\hat{C}_t + n^*\hat{C}_t^* + n^*\hat{C}_t^D$$

where $\hat{C}_t^D = \hat{C}_t - \hat{C}_t^*$. By definition total world real income equals total world real consumption, so it follows that (to a second order approximation)

$$n\hat{C}_t + n^*\hat{C}_t^* = n\hat{Y}_t + n^*\hat{Y}_t^* + \lambda_{Y,t} + O(\varepsilon^3)$$

where

$$\lambda_{Y,t} = \frac{1}{2}nn^* \left(\hat{Y}_t - \hat{Y}_t^* \right)^2$$

¹¹Strictly speaking, it is not necessary to use the Devereux and Sutherland method to derive solutions for $\tilde{\alpha}$ and $\hat{\alpha}_{t-1}$. The financial structure in the above model implies that full consumption risk sharing between home and foreign households is possible. Equilibrium gross portfolio positions can therefore simply be backed-out from a solution of the model where income pooling is imposed. This approach yields the same result as the Devereux and Sutherland method.

¹²Equation (15) also contains a term in the first-order deviation of gross portfolios from the non-stochastic steady state, $\hat{\alpha}_{t-1}$. Devereux and Sutherland (2010) explain how to derive a solution for $\hat{\alpha}_{t-1}$. This term, however, drops out of the analysis once the conditional expectations operator is applied, so an explicit solution is not required for the results reported below.

so¹³

$$E_0[\hat{C}_t] = E_0[n\hat{Y}_t + n^*\hat{Y}_t^* + \lambda_{Y,t} + n^*\hat{C}_t^D] + O(\varepsilon^3) \quad (17)$$

This provides part of the relationship between home-country consumption and home-country income. The next step is to derive an expression for $E_0[\hat{C}_t^D]$.

The consumption Euler equation (9), and its foreign counterpart, imply that $E_0[\hat{C}_t^D]$ is equal to a constant for period 1 onwards i.e.

$$E_0[\hat{C}_t^D] = E_0[\hat{C}_1^D] \text{ for all } t > 1 \quad (18)$$

so the analysis proceeds by deriving an expression for $E_0[\hat{C}_1^D]$. Using the home budget constraint (14), its foreign counterpart, the usual transversality condition, equation (18) and $\hat{W}_0 = 0$, the following is derived

$$E_0[\hat{C}_1^D] = (1 - \beta) \sum_{t=1}^{\infty} \beta^{t-1} E_0 \left[\hat{Y}_t - \hat{Y}_t^* + \frac{1}{n^*} \tilde{\alpha} \hat{r}_{x,t} + \lambda_{A,t} \right] + O(\varepsilon^3) \quad (19)$$

where

$$\lambda_{A,t} = \frac{1}{2}(\hat{Y}_t^2 - \hat{Y}_t^{*2}) + \frac{1}{n^*} \hat{\alpha}_{t-1} \hat{r}_{x,t} + \frac{1}{\beta n^*} \hat{W}_{t-1} \hat{r}_{2,t}$$

This expression shows that $E_0[\hat{C}_1^D]$ is equal to the sum of the discounted value of expected future income differences, $\hat{Y}_t - \hat{Y}_t^*$, and portfolio returns (and a number of second-order terms captured by $\lambda_{A,t}$).

It is convenient to rewrite (19) as follows

$$\begin{aligned} E_0[\hat{C}_1^D] &= (1 - \beta) E_0 \left[\hat{Y}_1 - \hat{Y}_1^* + \frac{1}{n^*} \tilde{\alpha} \hat{r}_{x,1} + \lambda_{A,1} \right] \\ &\quad + (1 - \beta) \sum_{t=2}^{\infty} \beta^{t-1} E_0 \left[\hat{Y}_t - \hat{Y}_t^* + \frac{1}{n^*} \tilde{\alpha} \hat{r}_{x,t} + \lambda_{A,t} \right] + O(\varepsilon^3) \end{aligned} \quad (20)$$

This expression separates out the impact of income and portfolio returns in period 1 from the impact of the same variables in future periods. The portfolio return in period 1, $\tilde{\alpha} \hat{r}_{x,1}$, depends on the timing of asset trade in period 0, whereas the portfolio return from period 2 onwards, $\tilde{\alpha} \hat{r}_{x,t}$ for $t > 1$, does not depend on the timing of asset trade. It is therefore useful to treat these two terms separately.

¹³The derivation of $\lambda_{Y,t}$ (and a number of other expressions used below) is considerably simplified by noting that, at the level of a first-order approximation, equity trade ensures that $\hat{C}_t = \hat{C}_t^* + O(\varepsilon^2)$ for all t and in all states of the world, regardless of the timing of asset trade in period 0.

The combination of equations (17) and (20) can now be used to analyse the impact of asset market timing on the determination of home-country consumption.

4 The timing of asset trade, portfolio returns and consumption

This section describes in detail the determination of portfolio returns. First, the expected portfolio return for period 2 onwards is derived and substituted into the equation for $E_0[\hat{C}_1^D]$ (i.e. equation (20)). We then analyse portfolio returns in period 1. Because this component of portfolio returns depends on the timing of asset trade, we consider separately the asset-trade-after-policy and asset-trade-before-policy cases.

4.1 Portfolio return for period 2 onwards

In each period from period 2 onwards, optimal portfolio allocation and asset market clearing imply that the expected excess return is given by

$$E_t[\hat{r}_{x,t+1}] = \frac{\rho}{2} E_t \left[(\hat{C}_{t+1} + \hat{C}_{t+1}^*) \hat{r}_{x,t+1} \right] + O(\varepsilon^3) \quad (21)$$

This is the risk premium on home equity.¹⁴ This expression shows that the risk premium, up to a second order approximation, depends on one-period ahead conditional second moments. These are constant by assumption, so $E_t[\hat{r}_{x,t+1}]$ is a constant for $t > 1$. It is useful to define $R = E_t[\hat{r}_{x,t+1}]$. The law of iterated expectations implies

$$E_0[\hat{r}_{x,t+1}] = E_t[\hat{r}_{x,t+1}] = R \quad (22)$$

So equation (20) becomes

$$\begin{aligned} E_0[\hat{C}_1^D] &= (1 - \beta) E_0 \left[\hat{Y}_1 - \hat{Y}_1^* + \frac{1}{n^*} \tilde{\alpha} \hat{r}_{x,1} + \lambda_{A,1} \right] \\ &\quad + (1 - \beta) \sum_{t=2}^{\infty} \beta^{t-1} \left(E_0[\hat{Y}_t - \hat{Y}_t^*] + \frac{1}{n^*} \tilde{\alpha} R + E_0[\lambda_{A,t}] \right) + O(\varepsilon^3) \end{aligned} \quad (23)$$

¹⁴See Devereux and Sutherland (2008a) for a more detailed derivation of this expression.

4.2 Asset trade after policy

The expected excess return in period 1 depends on whether asset trade in period 0 takes place before or after policy is determined. If asset trade takes place *after* policy is determined, asset prices in period zero are determined with full knowledge of the true value of δ . The policy announcement therefore does not give rise to any capital gain or loss, so the expected excess return in period 1 is again simply given by equation (21).

Equation (22) can be substituted into (23), which, after much simplification, yields

$$E_0[\hat{C}_1^D] = V + O(\varepsilon^3) \quad (24)$$

where

$$V = (1 - \beta) \sum_{t=1}^{\infty} \beta^{t-1} \left(E_0[\hat{Y}_t - \hat{Y}_t^*] + \lambda_{V,t} \right) \quad (25)$$

and

$$\lambda_{V,t} = \frac{1}{2} E_0 \left[(1 - 2n\rho) \hat{Y}_t^2 - (1 - 2n^*\rho) \hat{Y}_t^{*2} - 2(n^* - n) \rho \hat{Y}_t \hat{Y}_t^* \right]$$

The Appendix provides a more detailed derivation of these expressions.

Equations (24) and (25) show how policy affects $E_0[\hat{C}_1^D]$ in the asset-trade-after-policy case. The value of δ can affect V via its impact on the expected level of home income (relative to foreign income), $(\hat{Y}_t - \hat{Y}_t^*)$, or its impact on the second moments of home and foreign income. These effects can be thought of as capturing the on-going “flow income effect” of policy on the sustainable level of home consumption.

To see the impact on home welfare it is necessary to consider the impact on the discounted present value of home consumption. Equation (17) can be used to derive the following

$$\sum_{t=1}^{\infty} \beta^{t-1} E_0[\hat{C}_t] = \sum_{t=1}^{\infty} \beta^{t-1} E_0[\hat{Y}_t + \lambda_{C,t}] + O(\varepsilon^3) \quad (26)$$

$$\lambda_{C,t} = \frac{1}{2} n^{*2} (\hat{Y}_t - \hat{Y}_t^*)^2 + n^*(1 - \rho) (\hat{Y}_t - \hat{Y}_t^*) (n\hat{Y}_t + n^*\hat{Y}_t^*)$$

Equation (26) shows the link between home income and home consumption in the asset-trade-after-policy case. In order to analyse the implications of asset market timing it is now necessary to derive a relationship corresponding to equation (26) for the asset-trade-before-policy case.

4.3 Asset trade before policy

If asset trade takes place *before* policy is determined, there is a potential capital gain or loss which must be added to expected asset returns in period 1. At the time asset trade takes place in period 0, market clearing in the asset market will imply that

$$E_0^T[\hat{r}_{x,1}] = R$$

where E_0^T denotes expectations conditional on information at the time of asset trade. These expectations will be based on the prior belief about δ . Define \hat{Z}_0^{*T} and \hat{Z}_0^T to be asset prices at the time of asset trade in period 0 (i.e. \hat{Z}_0^T and \hat{Z}_0^{*T} incorporate the prior belief about the policy parameter δ). If \hat{Z}_0 and \hat{Z}_0^* are equity prices after the announcement of policy (i.e. asset prices which incorporate knowledge of the true value of δ), the capital gain caused by the policy announcement is given by

$$CG = (\hat{Z}_0 - \hat{Z}_0^T) - (\hat{Z}_0^* - \hat{Z}_0^{*T}) \quad (27)$$

The total expected excess return at the time of the policy announcement is thus

$$E_0[\hat{r}_{x,1}] = R + CG \quad (28)$$

Substitution of (28) into (23) and simplification yields

$$E_0[\hat{C}_1^D] = \frac{1}{n^*}(1 - \beta)\tilde{\alpha}CG + V + O(\varepsilon^3) \quad (29)$$

This can be contrasted with (24), which is the corresponding equation in the asset-trade-after-policy case. The comparison between these two equations shows clearly the different ways policy will affect $E_0[\hat{C}_1^D]$ in the two cases. Equation (24) shows that, in the asset-trade-after-policy case, policy will only affect $E_0[\hat{C}_1^D]$ via the impact of policy on V . Equation (29) shows that these effects will also arise in the asset trade before policy case, but in this case policy will have an additional effect on $E_0[\hat{C}_1^D]$ via the impact of policy on the capital gain term, CG .

In order to understand the impact of policy in the asset-trade-before-policy case it is obviously necessary to consider the capital gain term in more detail. The capital gain term is effectively a one-off valuation effect created by the policy change. It will now be shown that the capital gain term exactly offsets the flow income effect of policy operating

via V .

It is shown in the Appendix that the difference between the home equity price and the foreign equity price can be written in the form

$$\hat{Z}_t - \hat{Z}_t^* = \sum_{i=1}^{\infty} \beta^{i-1} \left\{ E_t \left[(1 - \beta)(\hat{Y}_{t+i} - \hat{Y}_{t+i}^*) + \lambda_{E,t+i} \right] - R \right\} + O(\varepsilon^3)$$

where $\lambda_{E,t}$ is a collection of second-order terms which is defined in the Appendix. The capital gain term, $CG = (\hat{Z}_0 - \hat{Z}_0^*) - (\hat{Z}_0^T - \hat{Z}_0^{*T})$, is thus

$$\begin{aligned} CG &= \sum_{t=1}^{\infty} \beta^{t-1} \{ (1 - \beta)(E_0[\hat{Y}_t - \hat{Y}_t^*] - E_0^T[\hat{Y}_t - \hat{Y}_t^*]) \\ &\quad + (E_0[\lambda_{E,t}] - E_0^T[\lambda_{E,t}]) - (R - R^T) \} + O(\varepsilon^3) \end{aligned} \quad (30)$$

where R^T and E_0^T are evaluated using households' prior belief about the value of δ .

This expression can be substituted into (29) and, after much further algebra, it can be shown that

$$E_0[\hat{C}_1^D] = V^T + O(\varepsilon^3) \quad (31)$$

where

$$V^T = (1 - \beta) \sum_{t=1}^{\infty} \beta^{t-1} \left(E_0^T[\hat{Y}_t - \hat{Y}_t^*] + \lambda_{V,t}^T \right) \quad (32)$$

and

$$\lambda_{V,t}^T = \frac{1}{2} E_0^T \left[(1 - 2n\rho)\hat{Y}_t^2 - (1 - 2n^*\rho)\hat{Y}_t^{*2} - 2(n^* - n)\rho\hat{Y}_t\hat{Y}_t^* \right]$$

The Appendix provides a more detailed derivation of these expressions.

Note that E_0^T terms are determined by the prior belief about δ . These terms are therefore exogenous and independent from the true value of δ . Equation (31) therefore shows that policy (in terms of the true value of δ) has no effect on $E_0[\hat{C}_1^D]$ in the case where asset trade takes place before policy is determined. In other words the one-off valuation effect generated by the capital gain term exactly offsets the flow income effect operating through the future impact of policy on home income.

Equation (31) can now be used to derive the following expression for the discounted value of home consumption

$$\sum_{t=1}^{\infty} \beta^{t-1} E_0[\hat{C}_t] = \sum_{t=1}^{\infty} \beta^{t-1} E_0[n\hat{Y}_t + n^*\hat{Y}_t^* + \lambda_{Y,t}] + \frac{n^*}{1 - \beta} V^T + O(\varepsilon^3) \quad (33)$$

Notice that the expectational terms in V^T are formed before policy is determined, while the expectational terms in other parts of (33) are formed after policy is determined. So, unlike the asset-trade-after-policy case, no further simplification of this expression is possible. This expression can be used to show the impact of policy on home welfare. In particular, it can be compared to (26), which is the corresponding equation in the asset-trade-after-policy case.

5 The welfare effects of policy

Before examining the welfare implications of the above results in more detail, it is useful to summarise how the timing of asset trade affects the link between policy and consumption. Equation (29) shows that the policy choice creates two distinct effects that impact on expected consumption. The first is a flow effect, which operates directly via the term, V , in (29). The second is a valuation effect which arises via the capital gain term, CG , in (29). If asset trade in period 0 takes place *after* policy is decided, the capital gain term is not present. In this case the choice of δ affects consumption only via the V term. On the other hand, when asset trade takes place *before* policy is decided, equation (31) shows that policy has no effect on $E_0[\hat{C}_1^D]$. This is because policy has an indirect effect on consumption via the capital gain term, CG . The capital gain valuation effect exactly offsets the flow income effect. The CG is, in effect, the payoff to a portfolio which precisely hedges against policy changes.¹⁵

The implications for the welfare effects of policy can now be assessed by comparing equations (26) and (33). These two equations show the relationship between the discounted value of home income and the discounted value of home consumption. The main difference between the two equations is in the size of the coefficient on the first moment of home income. Equation (26) shows that this coefficient is unity in the asset-trade-after-policy case. This contrasts with equation (33) where the coefficient is n , which is less than unity. In other words, a monetary policy rule which depresses the expected level of home income will have a one-for-one negative impact on home consumption in the asset-

¹⁵In the existing literature it is typically noted that complete international risk sharing implies a relationship of the form $U_{C^*}/U_C = k(SP^*/P)$, where U_C and U_{C^*} are home and foreign marginal utilities of consumption and k is an exogenous constant. In the context of the model of this paper, $U_C = C^{-\rho}$, $U_{C^*} = C^{*-\rho}$ and $SP^*/P = 1$, so equation (31), which shows that $E_0[\hat{C}_1^D]$ is exogenous and constant in the asset-trade-before-policy case, is consistent with the assumption that k is exogenous and constant. On the other hand, in the asset-trade-after-policy case, equation (29) shows that $E_0[\hat{C}_1^D]$ depends on the policymaker's choice of policy parameter. The asset-trade-after-policy case therefore implies that k is endogenous and depends on policy choices.

trade-after-policy case, but will have a less than one-for-one impact on consumption in the asset-trade-before-policy case. The underlying explanation for the contrast between the two cases relates directly to the hedging effect of the CG term.

The differing impact of policy on consumption in the two cases obviously has implications for the incentives facing the policymaker. In particular it has implications for the trade-off between output (i.e. work effort) and consumption. If the output of home goods (y) and home income (Y) are positively related (as will be the case when $\theta > 1$) then the policymaker faces a less favourable trade-off between output and consumption in the asset-trade-after-policy case than in the asset-trade-before-policy case. In the asset-trade-after-policy case any policy which leads to a reduction in the expected level of home output and income will be penalised by a one-for-one reduction in home consumption. This tends to discourage the choice of a policy rule which depresses the expected level of home output. In the asset-trade-before-policy case the impact on consumption is less than one-for-one so that policymaker faces an incentive to move the policy parameter in a direction which reduces the expected level of home output and income. In effect, part of the consumption impact of the policy choice is shifted on to foreign consumers.¹⁶

5.1 Numerical example

The results derived above are now illustrated using a calibrated version of the model. For the purposes of this exercise the following parameter values are used:

$$\beta = 0.99, \gamma = 0.75, \theta = 4, \phi = 8, \rho = 1, \mu = 2, \zeta = 0.95, \sigma_A = 0.01$$

We report results for two values of n : a large country case where $n = 0.5$; and a small country case where $n = 0.01$. The large country results are shown in Figure 3 and the small country results are shown in Figure 4. In the figures, welfare (in steady-state consumption units), consumption and output are measured in terms of the percentage deviation from a baseline stochastic solution where $\delta = 0$.¹⁷

As discussed above, the main implications of the timing of asset trade operate via the impact of the policy parameter on the discounted value of expected home output and expected home consumption. Figure 3 plots these quantities, together with home

¹⁶The fact that international risk sharing can distort the incentive faced by national policymakers has previously been noted by Wagner (2007). However, Wagner does not analyse the implications of the timing of asset trade.

¹⁷In the asset-trade-before-policy case, the model is solved while imposing equation (31). The solution in the asset-trade-after-policy case requires that equation (24) is imposed.

welfare, for a range of values of δ . The upper panel of Figure 3 shows these plots for the asset-trade-before policy case, while the lower panel shows the asset-trade-after-policy case.

Figure 3 shows that the welfare maximising value of δ differs between the two cases. Optimal δ is 0.016 in the asset-trade-before-policy case and 0.006 in the asset-trade-after-policy case. The underlying explanation for this difference is clear from the plots of consumption and output. The upper panel shows that, as δ increases, the discounted expected value of home output declines. This tends to raise home welfare (because it represents a fall in work effort). The discounted expected value of home consumption also declines as δ rises, but the decline in consumption is less than the decline in output. This reflects the cushioning effect of the capital gain in the asset-trade-before-policy case. The decline in consumption tends to reduce home welfare, but (initially at least) this is not sufficient to offset the welfare benefit of lower work effort. The optimal value of δ is therefore relatively high.

These effects can be contrasted with the asset-trade-after-policy case shown in the lower panel of Figure 3. There it can be seen that the expected discounted value of home output also declines as δ is increased. But in this case the declining level of home output is closely matched by the decline in the expected discounted value of home consumption. The welfare benefit of lower work effort is almost exactly offset by the welfare cost of lower consumption. The optimal value of δ is therefore relatively low.

Figure 4 shows the same set of comparisons for the small country example, where $n = 0.01$. The lower panel of Figure 4 shows that the general shapes of the welfare, output and consumption relationships are similar to the large country example. The main difference between the small country and large country examples occurs in the asset-trade-before-policy case. The upper panel shows that, in this case, welfare is now convex in δ rather than concave (at least within the range of values of δ shown here). The explanation for this is evident from equation (33) and from the plot of consumption in the upper panel of Figure 4. Equation (33) shows that, when n is very small, home country consumption becomes almost entirely insulated from the level of home country income. In terms of the upper panel of Figure 4, the relationship between consumption and δ is close to a horizontal straight line. This implies that, as δ increases, the positive welfare effect caused by the reduction of home output is not offset by any reduction in consumption. Welfare is therefore monotonically increasing in δ (within the range of δ shown here).

6 Conclusion

Using a standard dynamic general equilibrium model of an open economy, this paper has shown how the timing of asset trade relative to policy decisions can affect the welfare evaluation of policy. It is shown that, if asset trade in the initial period takes place before the announcement of the monetary policy rule, home consumers are effectively insured against the choice of policy rule. This allows the home country policymaker to choose a policy rule which reduces home country work effort in the knowledge that the impact on home country consumption is cushioned by portfolio returns at the time of the policy announcement. If, on the other hand, asset trade in the initial period takes place after the announcement of the policy rule, this insurance is not present and home consumers have to bear the full consumption consequences of a reduction in home output. The welfare incentives faced by the policymaker are significantly different between the two cases. In the asset-trade-before-policy case a welfare maximising policymaker has an incentive to choose a policy rule which depresses the expected level of home output. This incentive does not arise in the asset-trade-after-policy case. Numerical examples confirm that this can have a significant impact on the welfare maximising policy rule (particularly in the small economy example).

The current literature on monetary policy in open economies tends (implicitly) to focus on the asset-trade-before-policy case. The analysis reported above shows that this is not an innocuous assumption and demonstrates in detail how and why the timing of asset market trade matters. In itself, however, the analysis provides no definitive guidance on which assumption about asset market timing is most appropriate, either from an empirical or theoretical point of view. From an empirical perspective, it is arguable that policy decisions are always made against a background where agents hold diversified portfolios. Policy announcements frequently give rise to asset price movements and therefore capital gains and losses. The asset-trade-before-policy case therefore has some claims to empirical relevance. However, regardless of empirical considerations, from a theoretical perspective it seems unsatisfactory to judge the welfare effects of policy in a framework where the population of the country in question is implicitly insured against the potential adverse effects of policy. This paper shows that, when analysing the welfare effects of policy in open economies, it is important to acknowledge and understand the welfare incentives created by international financial markets. This is an issue which has, hitherto, received little attention in the related literature.

Appendix

Asset holdings and the budget constraint

In the text, we focus on a case where assets are defined to be in zero net supply. Here we show that a more conventional model with trade in equities which are in positive net supply may easily be transformed into the algebra of zero net supply used in the text. To see this in the context of the example model, assume that, instead of receiving income from production, all income comes in the form of returns on holdings of home and foreign equity. Let $\omega_{1,t}$ and $\omega_{1,t}^*$ represent the home and foreign share of home equity, and normalize so that the total supply of home equity is unity. Likewise, let $\omega_{2,t}$ and $\omega_{2,t}^*$ represent the home and foreign holdings of foreign equity, with total supply again normalized to unity. Then in an economy where there is asset trade only in the two equities, the home country faces a budget constraint given by:

$$Z_t \omega_{1,t} + Z_t^* \omega_{2,t} = (Z_t + Y_t) \omega_{1,t-1} + (Z_t^* + Y_t^*) \omega_{2,t-1} - C_t \quad (34)$$

where, as before, Z_t and Z_t^* are equity prices. The equity market clearing conditions are now

$$\omega_{1,t} + \omega_{1,t}^* = 1, \quad \omega_{2,t} + \omega_{2,t}^* = 1$$

It is easy to show that (34) may be transformed into the budget constraint used in the text, where assets are defined to be in zero net supply. To see this, re-write (34) in the form

$$Z_t(\omega_{1,t} - 1) + Z_t^* \omega_{2,t} = \frac{Z_t + Y_t}{Z_{t-1}} Z_{t-1}(\omega_{1,t-1} - 1) + \frac{Z_t^* + Y_t^*}{Z_{t-1}^*} Z_{t-1}^* \omega_{2,t-1} + Y_t - C_t \quad (35)$$

Now redefining $\alpha_{1,t} = Z_t(\omega_{1,t-1} - 1)$ as external holdings of the home asset and $\alpha_{2,t} = Z_t^* \omega_{2,t}$ as external holdings of the foreign asset, we arrive at

$$\alpha_{1,t} + \alpha_{2,t} = \frac{Z_t + Y_t}{Z_{t-1}} \alpha_{1,t-1} + \frac{Z_t^* + Y_t^*}{Z_{t-1}^*} \alpha_{2,t-1} + Y_t - C_t \quad (36)$$

which, given the definitions of $r_{1,t}$ and $r_{2,t}$, is identical to (6). Thus, the model where assets are in positive net supply is transformed into a model where the defined assets are in zero net supply.

Derivation of equation (25)

Equation (22) can be substituted into (23) to yield

$$\begin{aligned} E_0[\hat{C}_1^D] &= (1 - \beta) \left(E_0[\hat{Y}_1 - \hat{Y}_1^*] + \frac{1}{n^*} \tilde{\alpha} R + E_0[\lambda_{A,1}] \right) \\ &\quad + (1 - \beta) \sum_{t=2}^{\infty} \beta^{t-1} \left(E_0[\hat{Y}_t - \hat{Y}_t^*] + \frac{1}{n^*} \tilde{\alpha} R + E_0[\lambda_{A,t}] \right) + O(\varepsilon^3) \end{aligned}$$

which can be simplified to yield

$$E_0[\hat{C}_1^D] = (1 - \beta) \sum_{t=1}^{\infty} \beta^{t-1} \left(E_0[\hat{Y}_t - \hat{Y}_t^*] + \frac{1}{n^*} \tilde{\alpha} R + E_0[\lambda_{A,t}] \right) + O(\varepsilon^3)$$

Note that R and λ_A contain only second-order terms and can thus be evaluated using first-order accurate expressions for \hat{C} , \hat{C}^* , \hat{r}_1 , \hat{r}_2 and \hat{W} . Furthermore, it is possible to obtain expressions for \hat{C} , \hat{C}^* , \hat{r}_1 , \hat{r}_2 and \hat{W} in terms of \hat{Y} and \hat{Y}^* and thus express R and λ_A in terms of the second moments of \hat{Y} and \hat{Y}^* . After much further algebra it is possible to show that

$$\begin{aligned} \sum_{t=1}^{\infty} \beta^{t-1} \left(\frac{1}{n^*} \tilde{\alpha} R + E_0[\lambda_{A,t}] \right) &= \\ \sum_{t=1}^{\infty} \beta^{t-1} \frac{1}{2} E_0 \left[(1 - 2n\rho) \hat{Y}_t^2 - (1 - 2n^*\rho) \hat{Y}_t^{*2} - 2(n^* - n)\rho \hat{Y}_t \hat{Y}_t^* \right] &+ O(\varepsilon^3) \end{aligned}$$

so

$$E_0[\hat{C}_1^D] = (1 - \beta) \sum_{t=1}^{\infty} \beta^{t-1} \left(E_0[\hat{Y}_t - \hat{Y}_t^*] + \lambda_{V,t} \right) + O(\varepsilon^3)$$

where

$$\lambda_{V,t} = \frac{1}{2} E_0 \left[(1 - 2n\rho) \hat{Y}_t^2 - (1 - 2n^*\rho) \hat{Y}_t^{*2} - 2(n^* - n)\rho \hat{Y}_t \hat{Y}_t^* \right]$$

Derivation of equation (32)

Equation (30) can be substituted into (29) and simplified (using $\tilde{\alpha} = -n^*/(1 - \beta)$) to yield

$$\begin{aligned} E_0[\hat{C}_1^D] &= (1 - \beta) \sum_{t=1}^{\infty} \beta^{t-1} \left\{ E_0^T[\hat{Y}_t - \hat{Y}_t^*] + \frac{1}{n^*} \tilde{\alpha} (E_0[\lambda_{E,t}] - E_0^T[\lambda_{E,t}]) + \frac{1}{n^*} \tilde{\alpha} R^T \right\} \\ &\quad + (1 - \beta) \sum_{t=1}^{\infty} \beta^{t-1} E_0[\lambda_{A,t}] + O(\varepsilon^3) \end{aligned}$$

Note that λ_E and λ_A contain only second-order terms and can thus be evaluated using first-order accurate expressions for \hat{C} , \hat{C}^* , \hat{r}_1 , \hat{r}_2 and \hat{W} . As above, it is possible to obtain expressions for \hat{C} , \hat{C}^* , \hat{r}_1 , \hat{r}_2 and \hat{W} in terms of \hat{Y} and \hat{Y}^* and thus express λ_E and λ_A in terms of the second moments of \hat{Y} and \hat{Y}^* . After much further algebra it can be shown that

$$\sum_{t=1}^{\infty} \beta^{t-1} E_0 \left[\frac{1}{n^*} \tilde{\alpha} \lambda_{E,t} + \lambda_{A,t} \right] = 0 + O(\varepsilon^3)$$

so

$$E_0[\hat{C}_1^D] = (1 - \beta) \sum_{t=1}^{\infty} \beta^{t-1} \left(E_0^T[\hat{Y}_t - \hat{Y}_t^*] + \lambda_{V,t}^T \right) + O(\varepsilon^3)$$

where

$$\lambda_{V,t}^T = \frac{1}{2} E_0^T \left[(1 - 2n\rho) \hat{Y}_t^2 - (1 - 2n^*\rho) \hat{Y}_t^{*2} - 2(n^* - n)\rho \hat{Y}_t \hat{Y}_t^* \right]$$

Equity prices

The returns on home and foreign equities are given by $r_{1,t} = X_t/Z_{t-1}$ and $r_{2,t} = X_t^*/Z_{t-1}^*$, where, for convenience, equity payoffs are defined as $X_t = Y_t + Z_t$ and $X_t^* = Y_t^* + Z_t^*$. Second-order approximation of these relationships imply

$$\begin{aligned} \hat{r}_{1,t} &= \hat{X}_t - \hat{Z}_{t-1} + \frac{1}{2} \hat{r}_{1,t}^2 + O(\varepsilon^3) \\ \hat{r}_{2,t} &= \hat{X}_t^* - \hat{Z}_{t-1}^* + \frac{1}{2} \hat{r}_{2,t}^2 + O(\varepsilon^3) \end{aligned} \quad (37)$$

and

$$\begin{aligned} \hat{X}_t &= (1 - \beta) \hat{Y}_t + \beta \hat{Z}_t + \lambda_{X,t} + O(\varepsilon^3) \\ \hat{X}_t^* &= (1 - \beta) \hat{Y}_t^* + \beta \hat{Z}_t^* + \lambda_{X^*,t} + O(\varepsilon^3) \end{aligned} \quad (38)$$

where

$$\begin{aligned} \lambda_{X,t} &= \frac{1}{2} (1 - \beta) \hat{Y}_t^2 + \frac{1}{2} \beta \hat{Z}_t^2 - \frac{1}{2} \hat{X}_t^2 \\ \lambda_{X^*,t} &= \frac{1}{2} (1 - \beta) \hat{Y}_t^{*2} + \frac{1}{2} \beta \hat{Z}_t^{*2} - \frac{1}{2} \hat{X}_t^{*2} \end{aligned} \quad (39)$$

Using (37), (38) and the fact that $R = E_t[\hat{r}_{1,t+1} - \hat{r}_{2,t+1}]$, it follows that

$$\begin{aligned} \hat{Z}_t - \hat{Z}_t^* &= \beta E_t(\hat{Z}_{t+1} - \hat{Z}_{t+1}^*) \\ &+ E_t \left[(1 - \beta)(\hat{Y}_{t+1} - \hat{Y}_{t+1}^*) + \frac{1}{2}(\hat{r}_{1,t+1}^2 - \hat{r}_{2,t+1}^2) + (\lambda_{X,t+1} - \lambda_{X^*,t+1}) \right] \\ &- R + O(\varepsilon^3) \end{aligned}$$

and thus the difference between home and foreign equity prices can be written as follows

$$\hat{Z}_t - \hat{Z}_t^* = \sum_{i=1}^{\infty} \beta^{i-1} \left\{ E_t \left[(1 - \beta)(\hat{Y}_{t+i} - \hat{Y}_{t+i}^*) + \lambda_{E,t+i} \right] - R \right\} + O(\varepsilon^3) \quad (40)$$

where

$$\lambda_{E,t} = \frac{1}{2}(\hat{r}_{1,t}^2 - \hat{r}_{2,t}^2) + \lambda_{X,t} - \lambda_{X^*,t}$$

After some rearrangement $\lambda_{E,t}$ can be written as follows

$$\begin{aligned} \lambda_{E,t} = & \frac{1}{2}(1 - \beta)(\hat{Y}_t^2 - \hat{Y}_t^{*2}) + \frac{1}{2}\beta(\hat{Z}_t^2 - \hat{Z}_t^{*2}) \\ & - \frac{1}{2}(\hat{Z}_{t-1}^2 - \hat{Z}_{t-1}^{*2}) - (\hat{r}_{1,t}\hat{Z}_{t-1} - \hat{r}_{2,t}\hat{Z}_{t-1}^*) \end{aligned}$$

Expression (40) is used in the main text to derive an expression for the capital gain at the time of the policy announcement.

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Figure 1: Time line - asset trade before policy

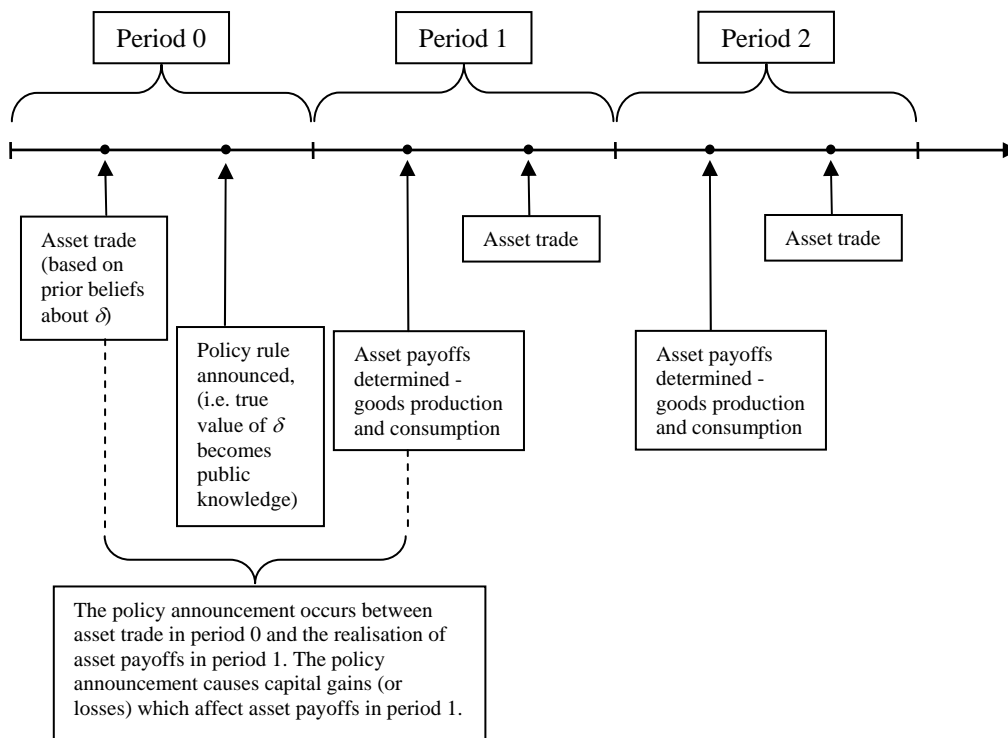


Figure 2: Time line - asset trade after policy

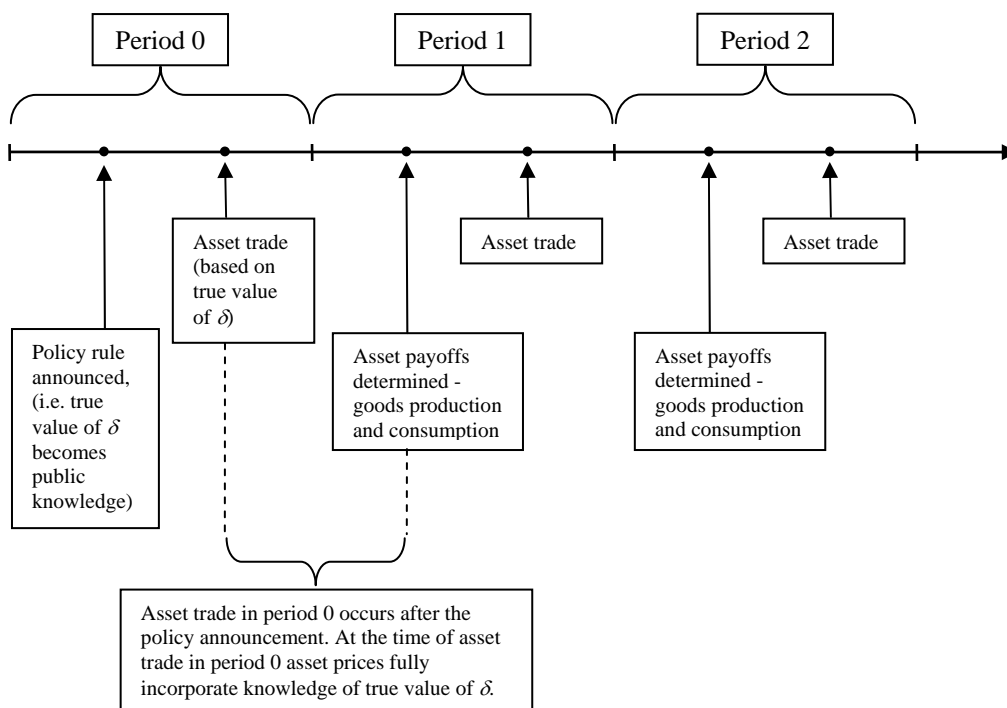


Figure 3: Large Country Example

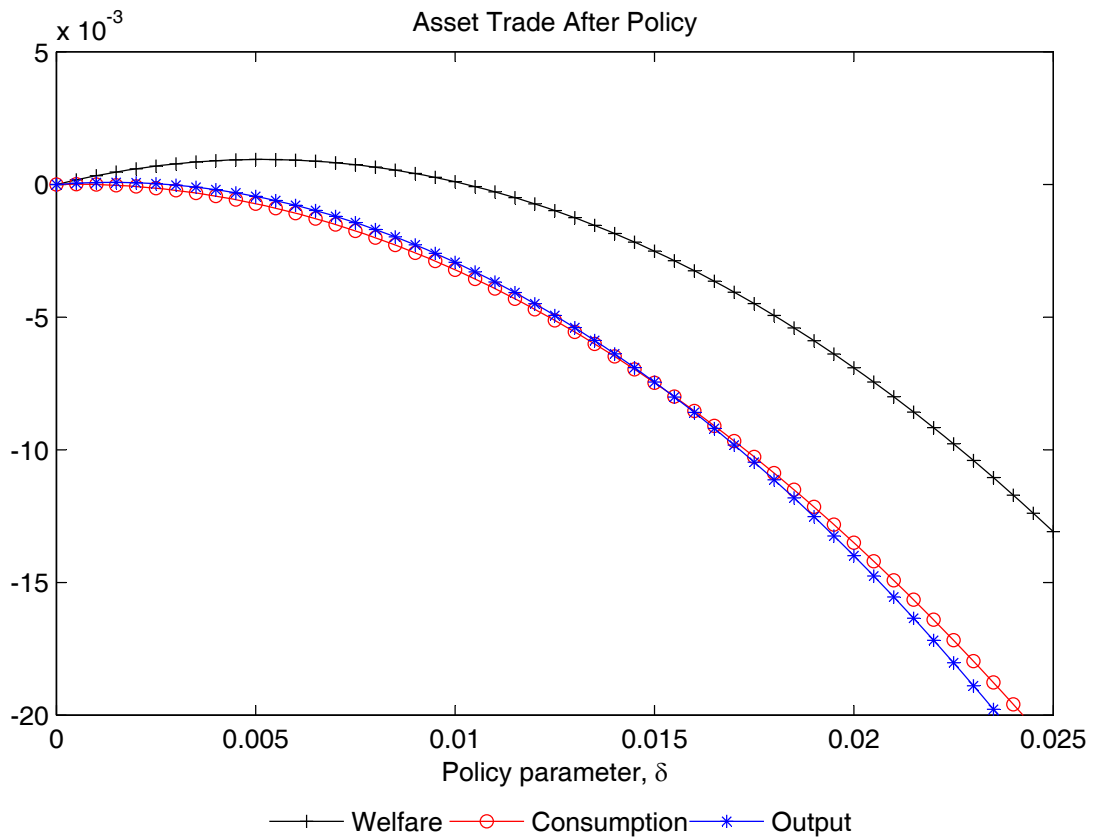
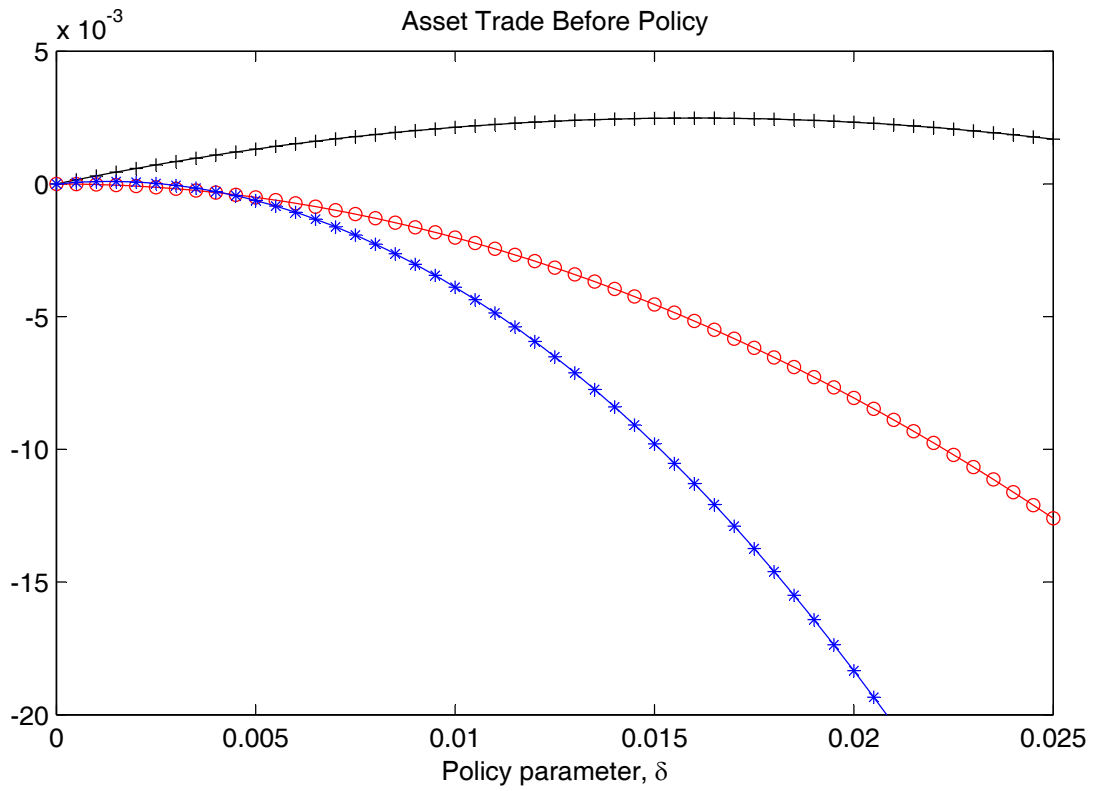
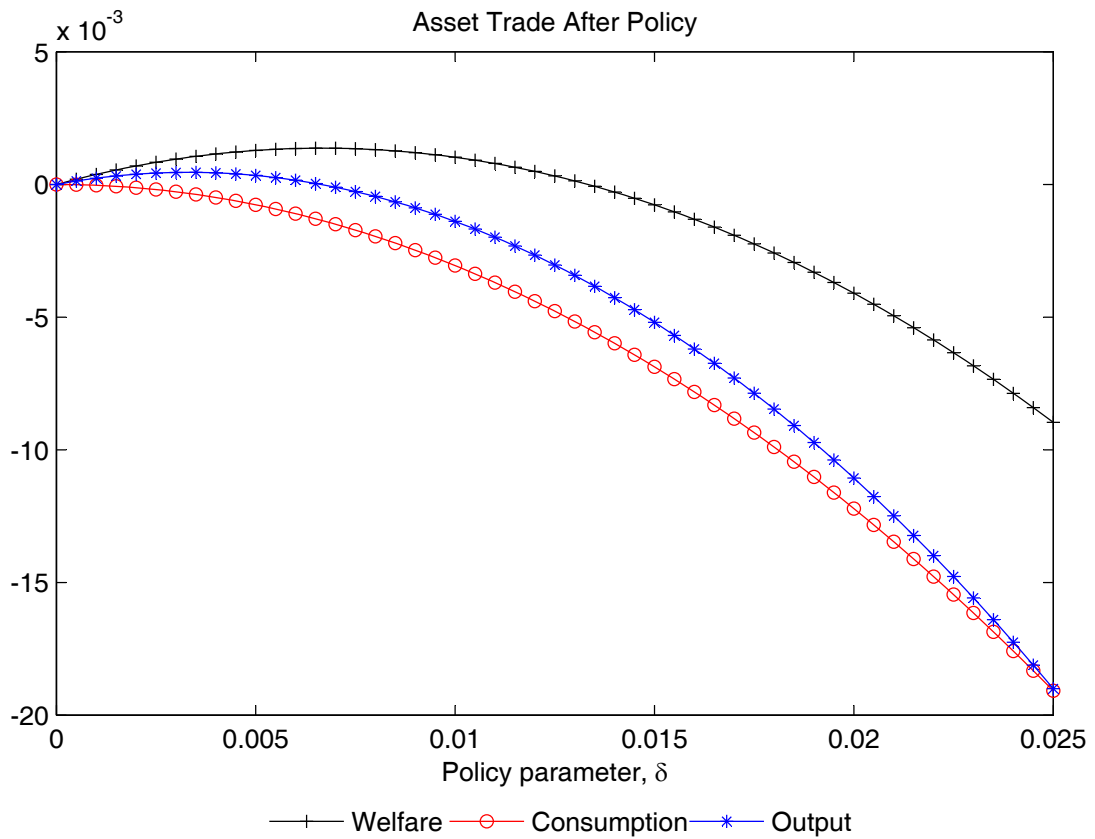
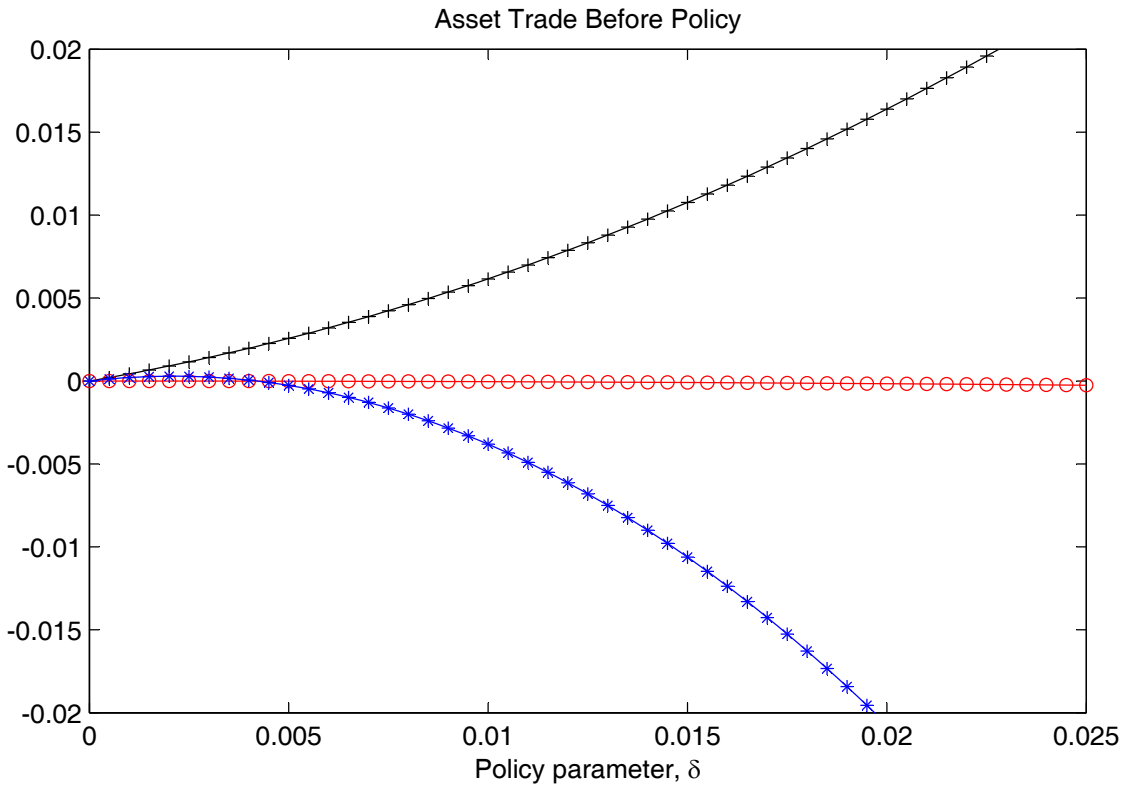


Figure 4: Small Country Example



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Jinyu Chen
Castlecliffe, School of Economics and Finance
University of St Andrews
Fife, UK, KY16 9AL
Email: jc736@at-andrews.ac.uk; Phone: +44 (0)1334 462445; Fax: +44 (0)1334 462444.