The Economics of Collective Brands

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Abstract

We consider the consequences of a shared brand name such as geographical names used to identify high quality products, for the incentives of otherwise autonomous firms to invest in quality. We contend that such collective brand labels improve communication between sellers and consumers, when the scale of production is too small for individual firms to establish reputations on a stand alone basis. This has two opposing effects on member firms' incentives to invest in quality. On the one hand, it increases investment incentives by increasing the visibility and transparency of individual member firms, which increases the return from investment in quality. On the other hand, it creates an incentive to free ride on the group's reputation, which can lead to less investment in quality. We identify parmater values under which collective branding delivers higher quality than is achievable by stand alone firms.

1 Introduction

Geographical names have been used to identify high quality products since ancient times. Corinthian wines, almonds from Naxos and Sicilian honey have been renowned for their quality since the 4th century BC (Bertozi, 1995). Other examples include Parma ham, California fruits, Jaffa oranges, and Washington apples. Some of these regional brands, such as spirits from Burgundy, Champagne, Chianti and Cognac are marketed individually by individual producers while others are marketed collectively, either by producer owned firms or by State Trading Enterprises (STEs). Two central features characterize these brands. First, their brand label are perceived as badges of superior quality by consumers who are willing to pay premium prices for them (e.g. Landon and Smith (1998) and Loureiro and McCluskey (2000, 2003)). Second, individual member - producers are generally autonomous firms, which make independent business decisions and retain their own profits, and only share a brand name.

The fact that collective brand labels are associated with superior quality suggests that firms which are members of these brands invest more to maintain brand quality than they would as stand alone firms (or at least are perceived to do so by consumers). This seems surprising. If consumers' perception of the collective brand label's quality is jointly determined by their experience with the qualities provided by different individual members, and if the provision of high quality requires costly investment, it would seem that each member has an incentive to free ride on the investments of fellow members. If so, why are these brand labels perceived as badges of quality?

It is true that in some cases, the perception of superior quality may be partly attributable to exogenous advantages such as climate, soil quality, access to superior inputs, technology and so on. However, even when such natural advantages are present the achievement of superior quality presumably also requires the requisite investment of effort and other resources. The free riding problem might also be mitigated to some extent by monitoring the efforts and investments of individual members to maintain quality standards. However, monitoring is costly and imperfect and is therefore unlikely to eliminate free riding altogether. Thus it would seem that producers have less of an incentive to invest in quality as members of a collective brand than they would as stand alone firms.

The purpose of this paper is to show that, despite the incentive to free ride, members of collective brands may nevertheless have a greater incentive to invest in quality than stand alone firms. Thus institutions like STE's, may increase welfare by providing consumers with better quality.

This has important implications for the current public debate concerning antitrust policy in the agricultural sector. For example, demands to ban STE's and limit regional branding have been voiced during recent rounds of the World Trade Organization negotiations on the grounds that these institutions reduce welfare and market efficiency by endowing their members with market power. Our analysis suggests that these institutions, by facilitating collective branding, can have positive welfare effects by promoting more efficient investment in quality.[1]

The idea is the following. When product quality is difficult to observe before purchase and is revealed to consumers only after consuming the product ('experience goods'), their perception of quality and the amount they are willing to pay for the product is based on past experience with the product - its reputation. Thus the extent to which a firm is able to receive a good return on its investment in quality depends on how well consumers are informed about its past performance. When information about past performance disseminates imperfectly, such as through word of mouth communication, the extent to which consumers are informed is likely to depend on the number of consumers who have experienced the product in the past. In particular, the smaller the firm the less informed its customers are about its past quality. Small firms may therefore be unable to effectively establish individual reputations on their own and consequently will have little incentive to invest in quality. Here collective branding may come to the

[1] An alternative position expressed in defense of STE's is that they provide economies of scale in production and promotion.

rescue and serve as a vehicle for reputation formation by facilitating the transmission of information about quality to consumers.

Specifically, suppose individual firms which are too small to establish good reputations on their own market their products under a collective brand name, sharing a collective reputation, while otherwise retaining full autonomy. Since their collective brand name covers a larger segment of the market than that of any individual member firm, the customers of each individual member are more likely to have previously experienced or interacted with past customers of the same brand, albeit not with that particular seller. This 'reputation effect' of collective branding increases the value of a good reputation and hence may increase members' incentive to invest in quality even if brand membership has no effect on individual members' market share.

But as noted above, branding may also have an opposing effect on investment incentives. Unless the brand is able to effectively monitor individual investment, sharing a collective reputation may encourage individual members to free ride on the efforts of other members. Therefore the full effect of collective branding on investment in quality is determined by the interaction of these two opposing factors - the fact that, on the one hand, a good collective reputation is more valuable than a stand alone reputation, against the incentive to free ride, on the other.

Accordingly, we analyze the effects of branding in two polar cases. First, as a benchmark, we consider the case in which the brand can deter free riding by perfectly monitoring members' investments and expelling members which don't invest. We show that in that case, since only the reputation effect is operative, a brand member's incentive to invest is always greater than that of a stand alone firm. Moreover, the incentive increases with brand size (the number of firms which are members of the brand) - the larger the brand, the greater the incentive of each member to invest and therefore the more profitable membership is.

We find that this pro - investment effect of collective branding also extends to the case in which the brand is unable to monitor individual members' investment. In particular, we show that collective branding can always enable at least some high ability firms to invest in quality when no stand alone firms will. Thus collective branding can always lead to higher expected quality in the market. Moreover, if either (i) investment is a sufficiently important ingredient for the attainment of high quality - that is, if the difference between the expected product quality of a firm which invests in quality and one which doesn't is sufficiently large - and/or (ii) firms with the ability to produce high quality through investment are sufficiently scarce, then collective branding enables most or all high ability firms to invest in quality when stand alone firms will not.

However, in contrast to perfect monitoring case, this is true only if brand size is not too large. If the brand is too large, the marginal contribution of an individual member's investment to the brand's reputation becomes too small to override free riding, reducing the brand's incentive to invest relative to stand alone firms.

In an econometric study of the determinants of reputation in the Italian wine industry, Castriota and Delmastro (2008) show that brand reputation is increasing in the number of bottles produced by the brand and decreasing in the number of individual producers in the brand. This is consistent with our analysis. Keeping output fixed, an increase in the number of individual producers has no reputation effect since the number of units whose quality consumers observe is unchanged. However, it does increase the incentive for free riding (which increases with the number of members), and hence lowers investment incentives and reduces the brand's reputation. Conversely, keeping the number of individual producers fixed, increasing output does not increase the incentive to free ride but does increase the reputation effect and hence increases incentives to invest.

An experimental study by Huck and Lűncer (2009) is also consistent with our analysis. They find that more sellers invest in quality when buyers are informed about the average past quality of all sellers - which corresponds to a collective brand in our model - than when they only know the record of the seller from whom they actually buy. However, as in our model, when the number of sellers increases, the average quality declines.

Finally, our analysis suggests that a regional brand may be viewed as an institution to regulate the collective brand size, keeping the number of producers large enough to enable successful reputation building but small enough to discourage individual free riding on the brand name. Thus one might speculate that a regional brand like Champagne wines owes its success not only to unique soil and climate but also to fortuitous natural boundaries which encompass "just the right" number of producers under its brand label.

1.1 Relationship to the Literature

Our emphasis on the centrality of a firms' reputation for quality for its success connects with the large and growing literature on firm reputation with a similar emphasis, e.g., Klein and Leffler (1981) Shapiro (1982), Kreps (1990), Tadelis (1999), Mailath and Samuelson (2001), Horner (2002). Banerjee and Duflo (2000) and Gorton (1996) provide empirical evidence that firms with reputation behave differently from firms without it. All of those papers are concerned with the reputation of individual firms. By contrast, our focus is on the collective reputation of otherwise autonomous firms.

More directly related are papers which explore the relationship between firm size and incentives for reputation formation. These include Andersson (2002), Cabral (2000, 2007), Choi (1997), Dana and Speir (2006), who analyze the effect of expanding the firm's product line (umbrella branding) on its reputation and profit, Cai and Obara (2009), who analyze the effect of horizontal integration on the integrated firms' cost of maintaining its reputation, Rob and Fishman (2005), who show that a firm's investment in quality increases with size, Yacouel (2005) and Guttman and Yacouel (2006), who show that larger firms benefit more from a good reputation. All those papers are concerned with size effects on incentives of an individual firm governed by a centralized decision maker. By contrast, our concern is with the effect of changing the number of member firms of fixed size, each governed by a distinct and autonomous decision maker.

The most closely related literature is the literature on collective reputation, beginning with Tirole (1996). He analyzes how group behavior affects individual incentive to invest (behave honestly). Evans and Guinnane (2007) analyze the conditions under which groups of heterogenous producers can create a common reputation and show that a common reputation can be created only if the members are not too different from each other and if marginal costs are declining. In these papers the group's size is fixed exogenously. By contrast, our focus is precisely on the role of the group size on individual incentives and, in particular, to compare a firm's incentives when standing alone with its incentives as a brand member.

There are also similarities between our approach and the common trait literature (for example Benabou and Gertner, 1993, Fishman 1996), in which observation of one agent's behavior reveals information about a common trait which she shares with other agents in the group. While in that literature the size of the group and the common trait itself are exogenously given, our focus is to understand how inferences about the common trait affect incentives to join the group, and motivate reputation formation.

2 Model - Stand Alone Firms

We consider a market for an experience good - consumers observe quality only after buying, but not at the time of purchase. There are two periods,[2] N risk neutral firms and we normalize the number of consumers per firm to be 1. There are two possible product quality levels, low (l) and high (h). Firms are of two types, H and L, which are distinguished by their technological ability to produce high quality. The probability with which a firm is able to produce high quality depends on its type and whether or not it invests in quality. An L firm produces high quality with probability b whether or not it invests. An H firm produces high quality with probability b if it does not invest but if it invests, it produces high quality with probability $g > b$. All units produced by a

[2] It will be apparent that the qualitative properties of the model extend straightforwardly to any horizon length, an extension which would complicate algebra and notation without adding insight. The intuition of our analysis should also apply to a repeated game setting in which current quality depends only on current investment.

firm at any period are of the same quality. We denote by N_H and N_L the total number of H and L firms respectively, $N_H + N_L = N$, and by n the proportion of H firms (that is $n = \frac{N_H}{N}$).

Investment is "once and for all": Prior to period 1, each firm decides whether or not to invest and that investment determines the quality of the products it sells at every future period[3]. The cost of investment is fixed at e.[4] We assume that $g - b \geq e$, so that investment is efficient.

Each consumer is in the market for one period, has a demand for one (discrete) unit at the most and exits the market at the end of the period. Her utility from a unit of low quality is zero and her utility from a unit of high quality is 1. Consumers cannot directly observe a firm's type (whether it is H or L) and are also unable to observe if a firm has invested or not. At period 1, consumers' only information about firms is their prior belief. We assume that at the beginning of period 2, each period 2 consumer is informed (through interaction with consumers of the previous generation) about the realized quality of each firm at the preceding period.[5] This information is used to update her expectations about firm quality at the second period.

In order to focus on the reputational effects of collective branding on investment incentives in the most direct possible way, it is convenient (though completely inessential for our main results) to assume that firms have monopolistic market power; That is, if consumers' expected utility from a unit of firm i is v_i, the price of firm i is v_i.[6] Thus

[3] This captures the idea that investment decisions are relatively inflexible in comparison to prices, which are easy to change.

[4] One could consider a richer model in which the probability of high quality is an increasing function of the amount invested. In that case, the intuition of our analysis suggests that whenever collective branding increases investment incentives, the brand invests more than stand alone firms and delivers higher expected quality.

[5] A more general formulation is that a consumer is informed about the first period quality of $\alpha \leq N$ randomly selected firms.

[6] This could be because consumers have high transportation costs which effectively endows firms with local monopoly pricing power. Alternatively, consider a standard consumer sequential search market setup: A consumer knows only the price distribution but not which firm charges what price, is randomly and costlessly matched with one firm and can either buy from that firm or sequentially search for other

branding cannot affect firms' pricing power or market share, and can only affect firms' investment incentives via reputational considerations.

An equilibrium specifies for each firm whether or not it invests and its price at each period, possibly as a function of previous quality realizations. Trivially, there always exists an equilibrium in which no firm invests[7].

The more interesting possibility is the existence of an 'investment equilibrium' (IE) in which H firms invest (L firms obviously don't invest in any equilibrium since investment has no effect on their quality). Suppose there is such an equilibrium. At the first period no firm has any history. Therefore, given that H firms invest, the expected utility from the product of any firm is $ng + (1 - n)b$ which is therefore the equilibrium price of each firm. At the second period consumers update their beliefs on the basis of the firms' realized quality at the preceding period (recall that consumers observe the first period quality realization of each firm).

Let $\Pr(H \mid h)$ be the posterior belief at period 2 that a firm is type H given that it produced high quality at period 1 and let $\Pr(H \mid l)$ be the posterior belief at period 2 that a firm is type H given that it produced low quality at period 1. Then by Bayes' Rule,

$$\Pr(H \mid h) = \frac{g\, n}{gn + b(1 - n)}.$$

$$\Pr(H \mid l) = \frac{(1-g)\, n}{(1-g)\, n + (1-b)(1 - n)}.$$

Thus the second period price of a firm which produced high quality at the first period, p_h, is

$$\begin{aligned} p_h &= g\Pr(H \mid h) + b\Pr(L \mid h) = g\Pr(H \mid h) + b(1 - \Pr(H \mid h)) \qquad (1) \\ &= b + (g-b)\Pr(H \mid h) = b + \frac{(g-b)gn}{gn + b(1-n)}. \end{aligned}$$

firms, incurring a positive search cost at each search. As is well known, these assumptions imply that firms have monopoly pricing power (Diamond, 1971).

[7] In this equilibrium consumers believe that no firm invests, which makes it optimal for firms not to invest.

The second period price of a firm which produced l at the first period, p_l, is

$$\begin{aligned} p_l &= g\Pr(H \mid l) + b\Pr(L \mid l) = g\Pr(H \mid l) + b(1 - \Pr(H \mid l)) \\ &= b + (g-b)\Pr(H \mid l) = b + \frac{(g-b)(1-g)n}{(1-g)n + (1-b)(1-n)}. \end{aligned} \tag{2}$$

Let R be the expected second period revenues of an H firm that invests and R_{-1} the expected second period revenues of an H firm that doesn't invest. Then, since its probability of producing high quality is g if it invests and b if it doesn't,

$$R = gp_h + (1-g)p_l$$

and

$$R_{-1} = bp_h + (1-b)p_l.$$

An H firm's expected gain from investment is $e_1 \equiv R - R_{-1}$. By (1) and (2):

$$e_1 = (g-b)^2 \left[\frac{gn}{gn + b(1-n)} - \frac{(1-g)n}{(1-g)n + (1-b)(1-n)} \right]. \tag{3}$$

Thus, when firms stand alone an *IE* exists only if $e \leq e_1$.

A stand - alone firm has only a limited opportunity to establish a reputation for quality, since at period 2 consumers have only limited information about its past quality - one observation in our formulation. Hence if $e > e_1$ an *IE* does not exist because the cost of investment exceeds the individual firm's expected return from maintaining a good reputation.

We proceed to show below that collective branding can lead to higher quality by increasing the incentive to invest and thus expanding the range of investment costs for which an *IE* exists. That is, if two or more firms sell their product under a common brand name, an *IE* may exist even when it does not exist if firms stand alone.

3 Collective Branding

We define a *brand* as two or more firms which market their products under a common brand name, while retaining full autonomy with respect to all business decisions and retaining their own profits. In particular, members of the brand decide individually whether or not to invest in quality. We denote by m the brand size (the number of firms which are members of the brand).

We are not committed to any specific process of brand formation but a convenient way to think of brand formation is that firms mutually agree to share a brand name, much as members of a cooperative mutually agree about its membership. We do assume that firms discern each other's type - i.e., can discern if a firm is type H or L. Accordingly, we define a pure H -brand as a brand all of whose members are type H. Similarly a pure $L-$brand is a brand all of whose members are type L. As is explained below, our focus is on pure brands.

Consumers evaluate a firm's quality on the basis of their experience with the brand label which it carries and not only on the basis of its own past performance. That is, we assume that a consumer who observes the qualities of j units from the same brand label, whether from the same firm or from different firms, forms her belief about the expected quality of any firm associated with that brand on the basis of all j observations.

Specifically, suppose that all brands are pure and of size m, that consumers believe that all members of H brands invest and the proportion of H brands (out of the total number of brands of both types) is f. Then a consumer who observes that k members of a brand produced high quality at the first period and $m-k$ produced low quality, believes that the brand is type H with the probability

$$\Pr(H \mid k, m) = \frac{g^k(1-g)^{m-k}f}{g^k(1-g)^{m-k}f + b^k(1-b)^{m-k}(1-f)}. \tag{4}$$

Thus the second period price of this brand, denoted p_k^m, is,

$$\begin{aligned} p_k^m &= g\Pr(H \mid k, m) + b\Pr(L \mid k, m) = g\Pr(H \mid k, m) + b(1 - \Pr(H \mid k, m)) \\ &= b + (g-b)\Pr(H \mid k, m). \end{aligned} \tag{5}$$

Collective branding provides consumers with more information about individual brand members' expected quality than if those firms stood alone (i.e., owned distinct individual brand names) because instead of getting only one observation of quality per *firm*, a consumer now effectively gets $m > 1$ observations per *brand*. Therefore the reputation of the brand affects the profit of its members more than their stand alone reputation would. Specifically, as is established by Lemma 1 below, if all members of a pure H brand invest, consumers are willing, on average, to pay each brand member a higher price than if it invested as a stand-alone firm. Formally, let R^m be the expected second period revenues of a member of a pure $H-$brand of size m *when every member invests and consumers believe that each member of such a brand invests.* Let R^m_{-1} be the same for a member which alone does not invest (but all other members do invest). And let $\Pr(k \mid H, m)$ be the probability that a pure H brand of size m produces k high quality goods when all the firms invest. Then:

$$R^m = \sum_{k=0}^{m} \Pr(k \mid H, m) p_k^m \tag{6}$$

and

$$R^m_{-1} = (1-b) \sum_{k=0}^{m-1} \Pr(k \mid H, m-1) p_k^m + b \sum_{k=0}^{m-1} \Pr(k \mid H, m-1) p_{k+1}^m.$$

Lemma 1 *(reputation effect): R^m and R^m_{-1} are increasing in m* for $m \geq 1$

Proof: In the appendix

Thus, in the case of a pure $H-$ brand, *provided all members invest and consumers believe that each member invests*, the larger the brand, the more profitable it is. In the case of a pure $L-$brand the reverse is true; the greater the number of outcomes consumers observe, the more likely they are to conclude that the brand is of type L, and the less they are willing to pay for it. And yet, as we show below, in equilibrium, L firms form L-brands to conceal their type.

Even in the case of a $H-$ brand, however, brand membership may also have a countervailing effect by motivating firms to free ride on other members' investment. The effect of collective branding on investment thus depends on the interaction of these opposing effects. Branding increases incentives to invest and leads to higher quality if the reputation effect dominates but can lead to lower quality if the free riding effect dominates. We wish to determine the conditions under which brands have a greater incentive to invest in quality than stand alone firms, and the effect of the brand size on these incentives.

We focus on equilibria in which all brands are pure. That is because this configuration provides H firms with the greatest incentive to invest because the expected number of high quality units, and therefore the price the brand commands, is greater if all members are type H than if one or more members are type L (see equations (4) and (5)). Hence, collective branding has the greatest potential to lead to higher quality if brands are pure. Moreover, for the same reason, it is natural to expect that H firms would only admit other H firms into their ranks. Also, we consider *symmetric* equilibria, in which all brands are of the same size. Henceforth, the terms '$H-$brand' and '$L-$brand' refer to pure brands.

In analogy to the stand alone setting, we define a *Brand IE (BIE)* as an equilibrium in which all brands are pure, are the same size, and *each* member of every $H-$ brand invests. In a BIE it must be individually optimal for each member of an $H-$brand to invest and it must be more profitable for a firm to be a member of the brand (and invest) than to stand alone. The revenue of a stand alone H firm is $max\{R_{-1}, R-e\}$.Thus in a BIE:

$$R^m - e \geq max\{R^m_{-1}, R_{-1}, R-e\}.$$

By Lemma 1, for $m \geq 1$, $R^m \geq R$ and therefore the preceding inequality is equivalent to:

$$R^m - e \geq max\{R^m_{-1}, R_{-1}\}. \tag{7}$$

Recall that at the second period consumers observe the first period realized quality of each firm. Thus, under collective branding, by counting the number of observations per brand consumers can deduce the brand size. Hence, in a *BIE,* it must be the case that consumers cannot perfectly infer the brand type (whether H or L) by brand size alone; otherwise H firms would have no incentive to invest.

Our primary aim is to determine circumstances under which a *BIE* exists when stand alone firms would not invest; that is, to determine when a *BIE* exists for $e > e_1$.

We shall consider two different regimes. In the first, called '*perfect monitoring*', an H brand is able to monitor individual members' investment and prevent firms which do not invest from using the brand label. In that case, free riding is not an option and so incentives to invest are driven only by the reputation effect. In the second case, '*no monitoring*', the brand is unable to monitor its members' investments.

3.1 Perfect Monitoring

Consider first the '*perfect monitoring*' regime. In that case, a member of an H brand *must* invest. Thus, for a brand of size m, condition (7) boils down to:

$$R^m - e \geq R_{-1}.$$

Let e_m solve the preceding inequality with equality; that is:

$$e_m \equiv R^m - R_{-1}. \tag{8}$$

Thus a necessary condition for the existence of a BIE in which all brands are of size m is that $e \leq e_m$. The following proposition states that the larger the brand size, the greater the value of e for which a BIE exists.

Proposition 1 *(i)* e_m *is strictly increasing in* m *for* $m \geq 1$.

(ii) For all $m > 1$, *a* BIE *exists for* $e_1 < e \leq e_m$.[8]

Proof: (i) By Lemma 1, R^m is increasing in m. Since R_{-1} is independent of m, $e_m = R^m - R_{-1}$ is increasing in m. In particular, for any $m > 1$, $e_m > e_1$.

(ii) For each $m > 1$ we construct a BIE for e, $e_1 < e \leq e_m$. Suppose that $N_H < N_L$ (recall that N_H and N_L are the number of H and L firms respectively) [9] Let s be the largest integer $\leq \frac{N_H}{m}$ and r the largest integer $\leq \frac{N_L}{m}$.

[8] We restrict attention to $e > e_1$ since for $e \leq e_1$ even stand alone firms invest.

[9] If $N_H \geq N_L$, then the proof holds for $m \leq N_L$.

Let there be s pure $H-$brands, and r pure $L-$brands, each of size m. Let the remaining $N_H - ms$ $H-$firms and the remaining $N_L - mr$ L -firms stand alone. Suppose that the $H-$brands invest and the stand alone $H-$firms do not invest and let consumers' beliefs be that H- brands of size m invest, that stand alone H firms do not invest, and that any brand of size $\neq m$ is an L brand. We now show that, given these beliefs, it is optimal for each member of an H brand to invest and for stand alone $H-$firms not to invest. An H firm's profit as a member of an H- brand is $R^m - e$ while if it stands alone its profit is R_{-1}. Since,

$$R^m - e - R_{-1} = e_m - e$$

it's optimal for an H -firm to be a brand member and invest if and only if $e \leq e_m$.

Next consider L firms. As a member of the L brand, an L firm's profit is greater than b. [10] Given that the H -brands are of size m, any stand alone firm (or member of brand of size $\neq m$) is identified by consumers as either an L -firm or a stand alone $H-$firm that does not invest and hence cannot charge a price higher than b. Thus an L -firm's profit as a stand alone firm is less than its profit as a $L-$brand member. Hence, L firms are also optimizing[11]. Hence, the preceding is a BIE for $e_1 < e \leq e_m$. That completes the proof of the proposition.

Thus corresponding to any brand size, a BIE exists for $e > e_1$. Moreover, the larger the brand size, the wider the range of investment costs which are compatible with investment and the greater the profit of each brand member. In particular, the highest range of investment costs compatible with investment in quality and the highest profit per member

[10] By (5)

$$p_k^m = b + (g - b)\Pr(H \mid k, m) > b.$$

[11] Note that if the stand alone $L-$firms were to form a separate brand, since its size would be smaller than m, it would be perceived as type L and hence its members' profit would not increase. By the same token, if any of those firms was admitted into one of the L - brands, its size would be greater than m, hence would be perceived as an $L-$ brand, decreasing existing members' profits. The same applies to stand alone $H-$ brands. However, if the sum of stand alone H and L firms $\geq m$, there exists an alternative equilibrium in which these firms form a 'hybrid' brand of size m, in addition to the pure H and L brands.

corresponds to the *BIE* in which all the H -firms form a single H -brand.

This is very intuitive. Under perfect monitoring there is no opportunity to free ride so only the reputation effect is operative. But that effect increases with brand size because the larger the brand, the more observations consumers receive about the brand's type. Therefore, given that the brand is type H and that all members invest, the larger the brand the more positive signals consumers receive on average and therefore the higher the price they are willing to pay for that brand label.

Although there are multiple equilibria, the equilibrium in which all H firms are in the same brand (of size N_H) seems more stable than the others because it maximizes the profit of individual H firms. Thus if there is more than one H brand, one might expect the brands to merge into a larger brand to increase the profit of both brands' members. As we show in the next subsection, this is no longer true if the brand cannot monitor its members.

3.2 No Monitoring

The more realistic and interesting case is the one in which the brand is unable to perfectly enforce investment by individual members. Therefore, we now consider the other extreme case, in which each brand member decides on its own whether or not to invest and that decision is undetectable by other brand members.

By Lemma 1, R^m and R^m_{-1} are increasing in m, which implies that $R^m \geq R$ and $R^m_{-1} \geq R_{-1}$; Thus, condition (7) becomes:

$$R^m - e \geq R^m_{-1}$$

or

$$e \leq R^m - R^m_{-1}$$

Define $\widetilde{e}_m \equiv R^m - R^m_{-1}$; $\widetilde{e}_m$ is the analog, for the no monitoring regime, of e_m for the perfect monitoring regime, and represents the highest value of the investment cost for which a *BIE* in which brands are of size m can exist under the no monitoring regime ($\widetilde{e}_m > 0$ exists for any m because it is easy to see that $R^m > R^m_{-1}$).

For any $e \leq \widetilde{e}_m$ a BIE in which all pure brands are of size m can be constructed in exactly the same way that it was constructed for the perfect monitoring regime in the proof of Proposition 1. Our concern is to determine when $\widetilde{e}_m > e_1$. In the perfect monitoring regime it was seen that e_m is increasing and therefore the larger the brand size, the greater the range of investment costs corresponding to which a BIE exists. Because of free riding, this is no longer the case in the no monitoring regime, as is established by the following proposition (proved in the appendix).

Proposition 2 *Under the no monitoring regime: (i) if $g = 1$, $\widetilde{e}_m$ is increasing in m for $m \geq 1$. (ii) If $g < 1$ and m is sufficiently large, then $\widetilde{e}_m < e_1$.*

The key to understanding this Proposition is that $R^m - R^m_{-1}$ reflects the adverse effect of a single low quality observation on the brand's reputation. Consider first the case $g = 1$; that is, if an H firm invests, it produces high quality with certainty. In that case, if the brand produces even a single low quality unit, consumers will believe that the brand is type L (with probability 1) regardless of how many high quality units it produces and be unwilling to pay it more than b. Therefore there is no free riding, regardless of how large the brand is. On the other hand, if all members invest, the consumers' posterior probability that the brand is type H, and hence the price they are willing to pay, increases with brand size, just as under perfect monitoring. Thus when $g = 1$, an individual brand member's loss from not investing, and hence its incentive to invest, increases with m.

By contrast, if $g < 1$, even when all brand members invest, some of the brand's units are low quality with positive probability, and this probability rises with the brand size. Therefore, when $g < 1$, the negative effect of another low quality observation declines with the brand's size and therefore the free riding effect eventually dominates the reputation effect if the brand is sufficiently large. Thus, if $g < 1$, for sufficiently large m, collective branding leads to lower quality than stand alone firms.

The preceding implies that without monitoring, if $g < 1$,branding does not necessarily enable more investment than stand alone firms. The following proposition (proved in the appendix) characterizes the conditions under which, nevertheless, $\widetilde{e}_m > e_1$ if m is not too large. In particular, $\widetilde{e}_m > e_1$ if the difference between the expected quality of a firm which invests and one which doesn't is sufficiently large and/or if the proportion of H firms in the firm population is sufficiently small.

Proposition 3 *(i) Given b, there is $g^* < 1$ such that if $1 \geq g \geq g^*$, then there is $m' > 1$ such that $\widetilde{e}_{m'} > e_1$.*

(ii) Given b and g, there is $n^* > 0$ such that if $0 < n \leq n^*$ then there is $m' > 1$ such that $\widetilde{e}_{m'} > e_1$.

In words, if g is relatively large or n is relatively small, there is a brand size for which each member has a greater incentive to invest than a stand alone firm.

The intuition for part (i) of the proposition is that even if $g < 1$, but still close to 1, a low quality observation is expected with low probability if the brand is not too large. Therefore, a low quality observation still has a large effect on consumers' posterior if the brand is relatively small, which discourages free riding. By contrast, if g is small, consumers expect a low quality unit with relatively high probability even if the brand is small, hence the marginal negative effect of free riding on consumers' posterior, and consequently the incentive to invest, is small.

The intuition behind part (ii) is that the smaller the proportion of H firms, the smaller is the proportion of H brands (cf proof of Proposition 1), hence the smaller is the prior probability that a brand is type H, and hence the greater the impact of each quality realization on the posterior probability that a brand is H. Put differently, when the proportion of H brands is small, a greater number of successes is required to convince consumers that the brand is indeed H and hence the greater the impact of each high quality realization on the brands's price. Hence the smaller is n, the greater the expected damage to the brand's reputation from unilateral failure to invest and thus the smaller the incentive to free ride. Therefore for sufficiently small n the reputation effect of branding dominates the free riding effect.

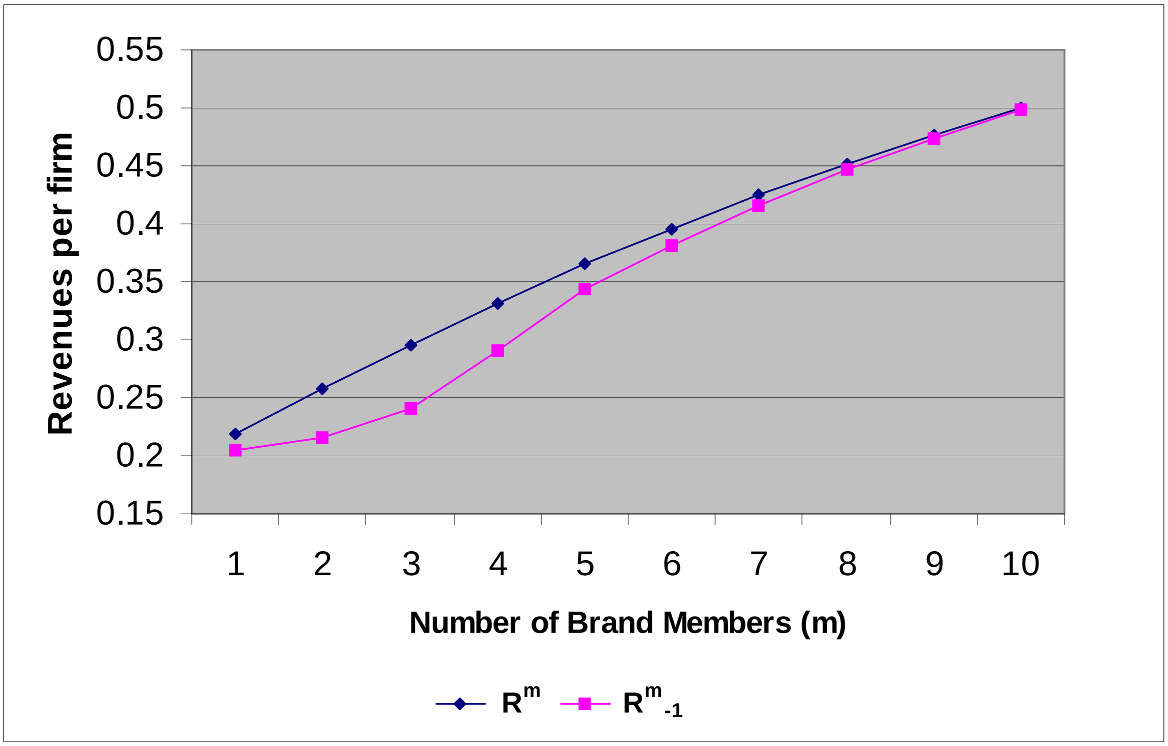

Figure 1:

The proposition is illustrated in the figure, where R^m and R^m_{-1} are sketched for the parameters $b = 0.2$, $g = 0.95$ and $f = 0.2$. For these parameters, $e_1 = 0.015$, which in the figure is represented by the vertical distance between R and R_{-1}, at $m = 1$. For $m = 2, 3, 4, 5$ and 6, $\widetilde{e}_m > e_1$ (where e_m is represented by the distance between R^m and R^m_{-1}). Thus, for example if $e = 0.02$, a stand alone firm would not invest but all $H-$ brands invest if $2 \leq m \leq 5$ and do not invest if $m \geq 6$. Note that in this case the brand size which maximizes H- brand's profits is $m = 5$. That is because as long as the brand size is small enough to deter free riding, the brand member's profit increases with the brand size.

4 The Optimal Brand Size

What is the 'optimal' brand size? For a given value of the investment cost, say e_0, all values of m for which $\widetilde{e}_m > e_0$ are equally efficient in the sense that each of these brand sizes lead to the efficient outcome of investment by H firms. Thus in the example sketched in Figure 1, if $e = 0.015$, brands of size $2, 3, 4$ and 5 are equally efficient. The same is not true with respect to firms' profits, however. By lemma 1, for a given value of e, the larger the brand size, the more information consumers have and therefore the higher the price they are willing to pay to an H brand, on average. Therefore, given e_0, the brand size which maximizes firms' profits[12] is the largest m satisfying the requirement that $\widetilde{e}_m \geq e_0$. Thus in our example, the brand size which maximizes firms' expected profit is $m = 5$.

An alternative notion is that the socially optimal brand size is the one under which the efficient outcome is realized in the widest range of circumstances; that is, that the larger is $\tilde{e}_m$, the better. Proposition 1 implies that in this sense there is indeed a socially optimal brand size which maximizes $\widetilde{e}_m$. In the example, the brand size which is optimal in this sense is 3 where $\tilde{e}_3 = 0.055$.

The preceding is subject to the caveat that we have assumed away any effects of branding on pricing by considering a model with unit demand and monopoly power. In a more general setting, brand size may also affect welfare adversely by facilitating collusion. In that case the socially optimal brand size would have to balance the positive effect of branding on reputation building against its negative effect on prices.

5 Concluding Remarks

Collective Brands and State Trading Enterprises are often perceived as a means of fostering collusion and therefore as an obstacle to efficient markets. On these grounds, they have been targeted by free market advocates in recent WTO rounds. Here we take a contrasting view, arguing that by affecting reputational incentives, these institutions can increase efficiency and welfare by enabling higher product quality than would be attainable in their absence.

[12] The reverse is true for L firms - the larger the brand size and the more information consumers have, the lower their profit. Nevertheless in equilibrium, these firms must form brands of the same size as H firms to keep from being detected as L firms on the basis of brand size alone.

In our model, collective branding can lead to greater investment, and higher quality, in two senses. First, the institution of collective branding enables investment in quality under conditions when stand alone firms would not invest - specifically when $e_1 < e < \widetilde{e}_m$. Second, while we have assumed that all firms are able to form brands, in reality institutional, financial and other exogenous factors may prevent some firms from forming brands. For example, in the case of regional brands (Jaffa oranges, Champagne wines, Washington apples and so on) a geographical region may be too small to support the critical number of firms required to support investment. In such cases, only firms which succeed in forming brands invest and command commensurately high prices while firms which are prevented from forming or joining brands stand alone and don't invest.

Because the equilibrium we derive is pooled - both L and H firms form brands - branding *per se* is not a signal of quality in our model. Rather, all brand members command higher prices than stand alone firms and high quality H brands, by investing, acquire a good reputation and command higher prices than L brands, which fail to acquire such a reputation.

It would nevertheless be interesting to formulate a model which has a separating equilibrium, in which only H firms form brands and L firms don't. Recall that in our model such an equilibrium doesn't exist because consumers observe the first period records of all firms, and can therefore tell which firms are brand members and which stand alone. Thus the key to achieving a separating equilibrium would be to formulate the model so that consumers cannot perfectly tell if a firm is a brand member. For example, suppose that at the second period a consumer observes a random number of observations of the first period quality of the firm from which it buys as well as a random number of observations of some other firms. Then a consumer who buys from a firm about whose label she has only one observation, she cannot tell for sure if that firm is stand alone or a brand member.

There are interesting parallels between our characterization of collective brands as vehicles of reputation formation and franchisees of chain stores and restaurants. Like a member of a collective brand, a franchisee is affected by and affects the reputation of the entire chain. Therefore it may be motivated to free ride on the chain's reputation and for this reason is closely regulated by chain ownership. Indeed, Jin and Leslie (2008) present evidence, that chain-affiliation is indeed a source of reputational incentives which may drive chain restaurants to have better hygiene than independent restaurants. They also find that franchisees invest less in hygiene than company owned restaurants of the same chain, which they interpret as evidence of free riding by franchisees on the chain reputation.

In contrast to franchises, the collective brand has no centralized ownership or control and each member is an autonomous firm. What we have shown is that despite incentives for free riding, collective branding can create greater reputational incentives than is possible with stand alone firms, even in the absence of a centralized ownership and even in the complete absence of any regulation or monitoring.

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6 Appendix

The following lemma presents a useful decomposition of R^m for the analysis below.

Lemma 2 *(i)*

$$R^m = \sum_{h=0}^{m-1} \Pr(h \mid H, m-1) \left[g p_{h+1}^m + (1-g) p_h^m \right] \tag{9}$$

(ii)

$$R_{-1}^m = \sum_{h=0}^{m-1} \Pr(h \mid H, m-1) \left[b p_{h+1}^m + (1-b) p_h^m \right]$$

Proof of (*i*): Let us divide the m members into two separate groups, the first of which includes $m-1$ members and the second includes the remaining member. Suppose that the first group produces h, $0 \leq h \leq m-1$ high quality and $m-1-h$ low quality units. Since the mth firm is also investing, its success probability is g and therefore the expected revenue of each brand member is,

$$R^m = g p_{h+1}^m + (1-g) p_h^m.$$

Since the probability that the first group produces h high quality units is $\Pr(h \mid H, m-1)$,

$$R^m = \sum_{h=0}^{m-1} \Pr(h \mid H, m-1) \left[g p_{h+1}^m + (1-g) p_h^m \right].$$

which completes the proof of (i)

Proof of (*ii*): Let us divide the m members into two separate groups, the first of which includes $m-1$ members which invest and the second of which includes the remaining single member which doesn't invest. Suppose that the first group produces h high quality and $m-1-h$ low quality units. Since the mth firm does not invest, its success probability is b and therefore the expected revenue of each brand member is,

$$R_{-1}^m = bp_{h+1}^m + (1-b)p_h^m.$$

Since the probability that the first group produces h high quality units is $\Pr(h \mid H, m-1)$,

$$R_{-1}^m = \sum_{h=0}^{m-1} \Pr(h \mid H, m-1) \left[bp_{h+1}^m + (1-b)p_h^m\right].$$

which completes the proof of (ii).

Substituting directly from the lemma, we have-

$$R^m - R_{-1}^m = (g-b) \sum_{h=0}^{m-1} \Pr(h \mid H, m-1) \left[p_{h+1}^m - p_h^m\right] \tag{10}$$

which is always positive.

6.1 Proof of Lemma 1

Substituting (9) for R^m, (6) for R^{m-1}, and (5) for p_h^m and rearranging:

$$\begin{aligned}
& R^m - R^{m-1} \\
= \; & \sum_{h=0}^{m-1} \Pr(h \mid H, m-1) \left[gp_{h+1}^m + (1-g)p_h^m\right] - \sum_{h=0}^{m-1} \Pr(h \mid H, m-1) p_h^{m-1} \\
= \; & \sum_{h=0}^{m-1} \Pr(h \mid H, m-1) \left[gp_{h+1}^m + (1-g)p_h^m - p_h^{m-1}\right] \\
= \; & (g-b) \sum_{h=0}^{m-1} \Pr(h \mid H, m-1) \left\{g \Pr(H \mid h+1, m) + (1-g) \Pr(H \mid h, m) - \Pr(H \mid h, m-1)\right\}.
\end{aligned}$$

Using equation (4)

$$
\begin{aligned}
& R^m - R^{m-1} \\
= \; & (g-b)\sum_{h=0}^{m-1} \Pr(h \mid H, m-1)\left\{ g\frac{g^{h+1}(1-g)^{m-h-1}f}{g^{h+1}(1-g)^{m-h-1}f + b^{h+1}(1-b)^{m-h-1}(1-f)} \right. \\
& +(1-g)\frac{g^h(1-g)^{m-h}f}{g^h(1-g)^{m-h}f + b^h(1-b)^{m-h}(1-f)} \\
& \left. -\frac{g^h(1-g)^{m-h-1}f}{g^h(1-g)^{m-h-1}f + b^h(1-b)^{m-h-1}(1-f)} \right\}.
\end{aligned}
$$

Define $X^h \equiv \frac{1-f}{f}\left(\frac{b}{g}\right)^h (\frac{1-b}{1-g})^{m-h-1}$. Then,

$$
\begin{aligned}
& R^m - R^{m-1} \\
= \; & (g-b)\sum_{h=0}^{m-1} \Pr(h \mid H, m-1)\left(\frac{g}{1+\frac{b}{g}X^h} + \frac{1-g}{1+\frac{1-b}{1-g}X^h} - \frac{1}{1+X^h} \right) \\
= \; & (g-b)\sum_{h=0}^{m-1} \Pr(h \mid H, m-1) \cdot \\
& \frac{g\left(1+\frac{1-b}{1-g}X^h\right)\left(1+X^h\right) + (1-g)\left(1+\frac{b}{g}X^h\right)\left(1+X^h\right) - \left(1+\frac{b}{g}X^h\right)\left(1+\frac{1-b}{1-g}X^h\right)}{\left(1+\frac{b}{g}X^h\right)\left(1+\frac{1-b}{1-g}X^h\right)\left(1+X^h\right)}.
\end{aligned}
$$

Define N^h as

$$
\begin{aligned}
N^h \;\equiv\; & g\left(1+\frac{1-b}{1-g}X^h\right)\left(1+X^h\right) + (1-g)\left(1+\frac{b}{g}X^h\right)\left(1+X^h\right) \\
& -\left(1+\frac{b}{g}X^h\right)\left(1+\frac{1-b}{1-g}X^h\right).
\end{aligned}
$$

Since $(g-b)\Pr(h \mid H, m-1) > 0$ and

$$
\left(1+\frac{b}{g}X^h\right)\left(1+\frac{1-b}{1-g}X^h\right)\left(1+X^h\right) > 0,
$$

$R^m > R^{m-1} > 0$ if $N^h > 0$ for all h. Manipulating N^h,

$$
\begin{aligned}
N^h &= g\left(1+\frac{2-g-b}{1-g}X^h+\frac{1-b}{1-g}X^{h2}\right)+(1-g)\left(1+\frac{g+b}{g}X^h+\frac{b}{g}X^{h2}\right) \\
&\quad -\left(1+\frac{b+g-2bg}{g(1-g)}X^h+\frac{b(1-b)}{g(1-g)}X^{h2}\right) \\
&= g^2\left[2-g-b+(1-b)X^h\right]+(1-g)^2\left(g+b+bX^h\right) \\
&\quad -\left[b+g-2bg+b(1-b)X^h\right] \\
&= g^2(2-g-b)+(1-g)^2(g+b)-(b+g-2bg) \\
&\quad +\left[g^2(1-b)+(1-g)^2b-b(1-b)\right]X^h \\
&= \frac{X^h}{g(1-g)}\cdot \\
&\quad \left[g^2(2-g-b+g+b-2)+g\left(1-2b-1+2b\right)+b-b\right. \\
&\quad \left.+(g^2-bg^2+b-2gb+g^2b-b+b^2)X^h\right] \\
&= \frac{X^h}{g(1-g)}(g-b)^2>0. \qquad (11)
\end{aligned}
$$

Thus, $R^m - R^{m-1} > 0$ for every $m > 1$ and it follows that R^m is increasing with m. The proof for R^m_{-1} is analogous. This completes the proof of Lemma 1.

6.2 Proof of Proposition 2

By Lemma 2, when $g = 1$,

$$R^m - R^m_{-1} = (1-b)\sum_{h=0}^{m-1}\Pr(h \mid H, m-1)(p^m_{h+1} - p^m_h) > 0$$

where the last inequality follows immediately from equations (4) and (5). Hence, and since by Lemma 1 $R^m_{-1} > R_{-1}$, it follows from equation (7) that firms belonging H brands invest. Hence, $\widetilde{e}_m = e_m$ which by Proposition 1 is increasing with m.That proves part (i) of the proposition.

To prove part (ii), substitute equation (4) in equation (10):

$$\begin{aligned} R^m - R^m_{-1} &= (g-b)^2 \sum_{h=0}^{m-1} \Pr(h \mid H, m-1) \cdot \\ &\quad \left[\tfrac{g^{h+1}(1-g)^{m-h-1} f}{g^{h+1}(1-g)^{m-h-1} f + b^{h+1}(1-b)^{m-h-1}(1-f)} - \tfrac{g^{h}(1-g)^{m-h} f}{g^{h}(1-g)^{m-h} f + b^{h}(1-b)^{m-h}(1-f)} \right] \\ &= (g-b)^2 \sum_{h=0}^{m-1} \Pr(h \mid H, m-1) A^m_h \end{aligned} \tag{12}$$

where

$$A^m_h = \frac{1}{1 + \frac{b(1-g)}{g(1-b)} c q^m} - \frac{1}{1 + c q^m} \qquad \text{and} \qquad q = \left(\frac{b}{g}\right)^{\frac{h}{m}} \left(\frac{1-b}{1-g}\right)^{\frac{m-h}{m}}.$$

Note that $q > 0$ and is strictly decreasing with h. Hence, for any given m, there is at most one value of h, denoted h^*, for which $q = 1$. If $h < h^*$, then $q > 1$, implying $\lim_{m\to\infty} q^m = \infty$ and it follows that $\lim_{m\to\infty} A^m_h = 0$. If $h > h^*$, then $q < 1$, implying $\lim_{m\to\infty} q^m = 0$ and it follows that $\lim_{m\to\infty} A^m_h = 0$. If $h = h^*$, then $q = 1$, in which case $A^m_{h^*}$ is some finite number.

Hence, from equation (12)

$$\begin{aligned} & \lim_{m\to\infty} \frac{\left(R^m - R^m_{-1}\right)}{(g-b)^2} \\ = \; & \lim_{m\to\infty} \left\{ \sum_{h=0}^{h^*-1} \Pr(h \mid H, m-1) A^m_h + \Pr(h^* \mid H, m-1) A^m_{h^*} + \sum_{h=h^*+1}^{m-1} \Pr(h \mid H, m-1) A^m_h \right\} \\ = \; & 0 \end{aligned}$$

since $\lim_{m\to\infty} \Pr(h^* \mid H, m-1) = 0$. This completes the proof of part (iii), which completes the proof of the proposition.

6.3 Proof of Proposition 3

We prove parts (i) and (ii) by showing that (i) and (ii) hold for $m' = 2$. That is, we will show that: (i) Given b, there is $g^* < 1$ such that if $g > g^*$, then $R^2 - R^2_{-1} > e_1$ and (ii) : Given b and g, there is $n^* > 0$ such that if $0 < n \le n^*$ then $R^2 - R^2_{-1} > e_1$.

Substituting $m = 2$ in equation (10) yields

$$R^2 - R_{-1}^2 = (g - b) \left\{ \Pr(0 \mid H, 1) \left[p_1^2 - p_0^2 \right] + \Pr(1 \mid H, 1) \left[p_2^2 - p_1^2 \right] \right\}.$$

Substituting equation (5) and equation (4) yields

$$\begin{aligned} \frac{R^2 - R_{-1}^2}{(g-b)^2} &= (1-g) \left[\frac{g(1-g)f}{g(1-g)f + b(1-b)(1-f)} - \frac{(1-g)^2 f}{(1-g)^2 f + (1-b)^2(1-f)} \right] \\ &\quad + g \left[\frac{g^2 f}{g^2 f + b^2(1-f)} - \frac{g(1-g)f}{g(1-g)f + b(1-b)(1-f)} \right]. \end{aligned}$$

Defining

$$z \equiv \frac{b}{g}, \qquad s \equiv \frac{1-b}{1-g}, \qquad c = \frac{1-f}{f}$$

we get

$$\frac{R^2 - R_{-1}^2}{(g-b)^2} = \frac{1-g}{1+zsc} - \frac{1-g}{1+s^2 c} + \frac{g}{1+z^2 c} - \frac{g}{1+szc}.$$

Recalling that $e_1 = R - R_{-1}$ and substituting (3) for $R - R_{-1}$ it follows that

$$\frac{R^2 - R_{-1}^2 - e_1}{(g-b)^2} = \frac{1-g}{1+zsc} - \frac{1-g}{1+s^2 c} + \frac{g}{1+z^2 c} - \frac{g}{1+szc} - \frac{1}{1+zc} + \frac{1}{1+sc}.$$

Rearranging,

$$\frac{R^2 - R_{-1}^2 - e_1}{(g-b)^2} = \frac{\frac{(g-b)^2 c^2}{(1-g)g}}{1 + (z+s)\,c + zsc^2} \cdot B$$

where

$$\begin{aligned} B &\equiv z \frac{(z+s+zsc) + z^2 sc}{1 + (z+s)\,zc + z^3 sc^2} - s \frac{(z+s+zsc) + zs^2 c}{1 + (z+s)\,sc + zs^3 c^2} \\ &= \frac{(s-z)(1+zsc)\left[(zsc)^2 - (z+s+zsc) \right]}{\left[1 + (z+s)\,zc + z^3 sc^2 \right] \left[1 + (z+s)\,sc + zs^3 c^2 \right]}. \end{aligned}$$

Note that $R^2 - R_{-1}^2 > e_1$ if and only if $B > 0$.

Since the denominator of the preceding expression is positive and $z < 1$ and $s > 1$, then $B > 0$ if and only if $Y \equiv (zsc)^2 - (z + s + zsc) > 0$.

Y is quadratic with a single positive root, c^*, and is positive for all $c \geq c^*$. Since f is strictly decreasing with c it follows that there exists $f^* = 1/(1 + c^*) > 0$ such that if $0 < f < f^*$ then $Y > 0$ and $B > 0$.

Define n^* such that

$$\frac{n^* N_H}{N_H - n^*} = f^*.$$

It is easy to see that if $n \leq n^*$, then

$$\frac{N_H}{N_H + N_L - 1} < f^*.$$

On the other hand, if N_H and N_L both divide by 2 $f = n$, if N_L divide by 2 and N_H does not, $f = \frac{N_H - 1}{N_H + N_L - 1}$ and if N_H divide by 2 and N_L does not, $f = \frac{N_H}{N_H + N_L - 1}$. In addition,

$$\frac{N_H - 1}{N_H + N_L - 1} < \frac{N_H}{N_H + N_L - 1} < \frac{N_H}{N_H + N_L - 1}.$$

Hence, whether N_H and N_L divide by 2, if $0 < n < n^*$ then $f \leq f^*$ and it follows that $Y > 0$ and $B > 0$ which completes the proof of (ii).

To prove (i), express Y in terms of g, b and c.as follows:

$$Y = \left(\frac{b}{g}\frac{1-b}{1-g}c\right)^2 - \frac{b}{g} - \frac{1-b}{1-g} - \frac{b}{g}\frac{1-b}{1-g}c. \tag{13}$$

It is easily seen that as $g \to 1$, $Y \to \infty$. Since Y is continuous in g,it follows that there exists $g^* < 1$ such that $Y > 0$ for all $g > g^*$, which proves (i).

Thus, by (i) and (ii), the proposition is satisfied for $m' = 2$,which completes the proof of the proposition.

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