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Modelling and forecasting the global financial crisis: Initial findings using heterosckedastic log-periodic models

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Abstract

The financial crisis of 2007-2009 has begun in July 2007 when a loss of confidence by investors in the value of securitized mortgages in the United States resulted in a liquidity crisis. World stock markets peaked in October 2007 and then entered a period of high volatility which culminated with the market crashes in September and October 2008. Since March 2009, the world stock markets have rebounded, but strong uncertainties still remain. In order to get more insights into the current world markets operation, we consider log-periodic models of price movements, which has been largely used in the past to forecast financial crashes and "anti-bubbles". Both the original and an extended model which accounts for heteroskedasticity and autocorrelation are fitted to the American S&P500 index. The empirical analysis reveal three interesting points: i) the log-periodic models outperform standard financial models when long-term out-of-sample forecasting is of concern. ii) the log-periodic-AR(1)- GARCH(1,1) model has residuals with better statistical properties than the original model and iii) the current market rebound should peak at the beginning of 2010.

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1 Introduction

Sornette et al. (1996), Sornette and Johansen (1997), Johansen et al. (2000) and Sornette (2003) suggested that, prior to crashes, the mean function of a index price time series is characterized by a power law acceleration decorated with log-periodic oscillations, leading to a finite-time singularity that describes the onset of the market crash. Within this model, this behavior would hold for months and years in advance, allowing the anticipation of the crash from the log-periodic oscillations exhibited by the prices. The underlying hypothesis of this model is the existence of a growing cooperative action of the market traders due to an imitative behavior among them. In the pre-crash regime, clusters of correlated trades with arbitrary sizes would drive the financial system, and therefore the observed financial variables would exhibit scaling invariant properties. Particularly, Johansen et al. (2000) put forward the idea that stock market crashes are caused by the slow buildup of long-range correlations leading to a collapse of the stock market in one critical instant: in mathematical terms, complex dynamical systems can go through the so-called "critical" points, defined as the explosion to infinity of a normally well-behaved quantity. It is then natural to think of the stock market in terms of complex systems with analogies to dynamically driven out-of-equilibrium systems such as earthquakes, avalanches, crack propagation etc. However, there is a non-negligible and extremely important difference: the "microscopic" building blocks, i.e. the traders, are conscious of their action. This has been masterly captured by Keynes under the parable of the beauty contest. As a consequence, Johansen et al. (2000) develop an interesting model based on the interplay between economic theory and statistical physics. Since then, several authors have reported a large number of empirical results for a variety of unrelated crashes in worldwide stock markets indices. We refer to Sornette (2003) for a recent review of the theoretical framework of the log-periodic model and a compilation of empirical evidences. Johansen and Sornette (1999) and Zhou and Sornette (2005) examined the problem of whether the cooperative herding behavior of traders might also produce market evolutions that are symmetric to the accelerating speculative bubbles often ending in crashes. More specifically, they show that there seems to exist critical times t_c at which the market peaks with, either a power law increase with accelerating log-periodic oscillations ending with a crash (i.e. a bubble), or *a power law decrease after the critical time t_c with decelerating log-periodic oscillations*: in the latter case we have a so called "*anti-bubble*". Johansen and Sornette (1999) and Zhou and Sornette (2005) show that the traders' herding behavior can progressively occur and strengthen itself also in "bearish" decreasing market phases, thus forming anti-bubbles with decelerating market devaluations following market peaks.

What we do in this paper is to use log-periodic models to model and forecast the global financial crisis of 2007-2009 as a special case of "anti-bubble". By observing that world stock markets peaked in October 2007, which thus represents our critical time t_c, we present an econometric investigation of the log-periodic models, which looks at the residuals properties and tackles potential inferential problems arising from autocorrelation and heteroskedasticity. Particularly, we consider the AR(1)-GARCH(1,1) log-periodic model recently proposed by Gazola et al. (2008), which is intended to aggregate latent dynamical features and mechanisms of the normal phase of the market onto the critical long-range dynamics of price fluctuations encompassed by the original log-periodic model. We then compare the previous set of log-periodic models with standard times series models in terms of long-term out-of-sample forecasting performances. In this perspective, we perform forecasting exercises with the American SP500 stock index, considering 120-step ahead and 180-step ahead forecasts, showing that the AR(1)-GARCH(1,1) log-periodic model clearly outperforms standard financial models. Interestingly, we find out that the current rebound should peak at the beginning of 2010.

The rest of the paper is structured as follows. In Section 2 we review the log-periodic models for "anti-bubble" modelling, whereas in Section 3 we show an empirical application with the American S&P500 index. We perform extensive forecasting exercises in Section 4, while Section 5 briefly concludes.

2 Methodology

We consider the following log-periodic model for the price evolution trajectory of an anti-bubble:

$$p(t) = A + B(t - t_c)^{\beta} + C(t - t_c)^{\beta} \cos[w \ln(t - t_c) + \phi] + u_t \quad (1)$$

where $p(t)$ is the stock index price, $B < 0$ for a (bearish) anti-bubble to exist, $C \neq 0$ guarantees the significance of the log-periodic oscillations, β should be positive to ensure a finite price at the critical initiation time t_c of the anti-bubble and quantifies the primary power law acceleration of prices, w is the angular log-frequency which measures the frequency of oscillations of the correction term in logarithmic time units, ϕ is a phase which can be absorbed in a re-definition of the unit of the time, while $t - t_c$ is the distance to the critical time t_c which is the starting point of the anti-bubble.

The previous deterministic component describes decreasing oscillations whose period increases as the time departs from the critical time t_c: as discussed in Sornette (2003 a,b) and Zhou and Sornette (2006), the power law acceleration $B(t - t_c)^\beta$ and the log-periodicity $\cos[w \ln(t - t_c) + \phi]$ are both intimately linked to the herding behavior of agents whose investments involve a competition between positive and negative feedbacks leading to a critical point. Furthermore, previous empirical studies as well as theoretical fundamentals discussed in Johansen et al. (2000) and Sornette (2003) suggest that the parameter governing the bubble growth should satisfy $0 < \beta < 1$, whereas the log-frequency parameter should satisfy $2 < w < 15$, meaning that there must be some oscillations embedded in the fit to give weight to the model. Particularly, for w close to zero, we would not observe any oscillations in the whole sample, so that the present modelling would not be very representative of the data under analysis. On the other hand, very high values for w would imply a lot of oscillations, thus resulting in an over-fitting of the noise. Besides, being ϕ a phase parameter, Gazola et al. (2008) proposed $0 < \phi < 2\pi$, which has also the advantage of ruling out some obvious non-identifiability problems. The time index t is converted in units of one year, so that 1 day = 1/365=0.0027397260 of the year.

Under normality assumption for the error term u_t, the maximum likelihood estimator for the parameter vector $\mathbf{\Pi} = [A, B, C, t_c, \beta, w, \phi]$ is obtained through the minimization of the squared residuals. Therefore, the estimates $\hat{\mathbf{\Pi}}$ for the log-periodic specification are estimated by nonlinear least squares (NLS).

In order to improve the optimization procedure, each parameter of the log-periodic model (1) denoted by θ and defined in a restricted interval denoted by $[a, b]$, can be re-parameterized according to the following monotonic transformation:

$$\theta = b \frac{\exp(\tilde{\theta})}{1 + \exp(\tilde{\theta})} + a \left(1 - \frac{\exp(\tilde{\theta})}{1 + \exp(\tilde{\theta})} \right) \tag{2}$$

This monotonic transformation turns the original estimation problem over a restricted space of solutions into an unrestricted problem, which eases estimation particularly when poor starting values are chosen. In this situation, the standard errors of the estimates can be computed by using the delta method. We remind that the delta method is used to compute an estimator for the variance of functions of estimators and the corresponding confidence bands. Let $\hat{V}[\tilde{\theta}]$ be an estimated variance-covariance matrix of $\tilde{\theta}$, then by the delta method, a variance-covariance matrix for a general nonlinear transformation $g(\tilde{\theta})$ is given by (see Greene (2002) or Hayashi (2000) for more details):

$$\hat{V}[g(\tilde{\theta})] = \frac{\partial g(\tilde{\theta})}{\partial \tilde{\theta}'} \hat{V}[\tilde{\theta}] \frac{\partial g(\tilde{\theta})}{\partial \tilde{\theta}}'$$

However, the delta method is only a first order approximation and may work very poorly in small samples like the ones used in our empirical analysis. A viable alternative is to maximize the log-likelihood with respect to $\tilde{\theta}$ in a first step, and then use the estimated transformed parameters $\hat{\theta}$ as starting values in a second maximization using only the original parameters θ.

While the previous log-periodic model (1) can model long-range dynamics of price fluctuations, nevertheless it is unable to consider the short-term market dynamics, thus showing residual terms $\hat{u}_t$ which are strongly autocorrelated and heteroskedastic. As a consequence, Gazola et al. (2008) proposed the following AR(1)-GARCH(1,1) log-periodic model:

$$\begin{aligned} p_t &= A + B(t - t_c)^\beta + C(t - t_c)^\beta \cos[w \ln(t - t_c) + \phi] + u_t \qquad (3) \\ u_t &= \rho u_{t-1} + \eta_t \\ \eta_t &= \sigma_t \varepsilon_t, \quad \varepsilon_t \sim N(0, 1) \\ \sigma_t^2 &= \alpha_0 + \alpha_1 \eta_{t-1}^2 + \alpha_2 \sigma_{t-1}^2 \end{aligned}$$

where ε_t is a standard white noise term satisfying $\mathrm{E}[\varepsilon_t] = 0$ and $\mathrm{E}[\varepsilon_t^2] = 1$, whereas the conditional variance σ_t^2 follows a GARCH(1,1) process. We propose here a slightly different formulation for the autoregressive structure in the conditional mean that we found much easier to estimate compared to (3):

$$\begin{aligned} p_t &= A + B(t-t_c)^{\beta} + C(t-t_c)^{\beta}\cos[w\ln(t-t_c)+\phi] + \rho p_{t-1} + \eta_t \\ \eta_t &= \sigma_t\varepsilon_t, \quad \varepsilon_t \sim N(0,1) \\ \sigma_t^2 &= \alpha_0 + \alpha_1\eta_{t-1}^2 + \alpha_2\sigma_{t-1}^2 \end{aligned} \tag{4}$$

The two formulations are very close and can be shown to differ only for the initial conditions. However, model (4) reached numerical convergence when applied to the times series of the SP500 in the years 2007-2009, while model (3) did not, thus confirming similar problems in Gazola et al. (2008). Moreover, the chosen lag orders in the mean and variance specifications in (4) minimized a set of information criteria: AIC, BIC and their modified versions proposed by Ng and Perron (2001). Besides, model (4) passed a large set of specification tests, as we will discuss in more details in Section 3.2 .
We remark that in case of processes with a deterministic linear trend and white noise shocks, the usual t and F-statistics have the same asymptotic distributions as those for the stationary processes, but with different convergence rates. Unfortunately, similar results for processes with deterministic non-linear trends (such as the ones exhibited by the log-periodic models) are not standard as in the linear case, and some care must be exercised when looking at the inferential results in the next section. However, an unreported preliminary small-scale simulation study seems to confirm that the t and F-statistics in case of nonlinear deterministic trends have the same asymptotic distributions as in the case of linear trends.

3 Empirical Analysis

We explore the possibility that log-periodic models provide a better empirical description than standard time series models of the long-run price dynamics of the American SP&500 market index, during the global financial crisis in the years 2007-2009. To answer this question, we estimate models (1) and (4) using data ranging between 10/10/2007 and 13/04/2009 for a total of 379 observations.

3.1 Estimation Results

When estimating the log-periodic models, we found out (not surprisingly) that the critical time t_c was very close to the market peak on the 09/10/2007, when the SP500 reached the value of 1565.15. Therefore, we fixed $t_c = 7.773$, i.e. October the 9th 2007, so to increase the degrees of freedom and improve the estimation efficiency. Besides, the numerical maximization of the log likelihood resulted in being much less cumbersome. The estimated parameters for the log-periodic model (1) and the AR(1)-GARCH(1,1) log-periodic model (4) are reported below in Tables I-II.

Table I: Estimation Results: Log-periodic model (1)

θ	$\hat{\theta}$	Std.err(*)	T-stat	P-value
A	1525.27	10.94	139.43	0.00
B	-388.71	16.50	-23.56	0.00
β	0.99	0.06	16.33	0.00
C	168.06	11.86	14.17	0.00
w	4.88	0.14	33.71	0.00
ϕ	2.11	0.08	25.34	0.00
Log lik.	-2022.90	Schwarz criterion		10.77

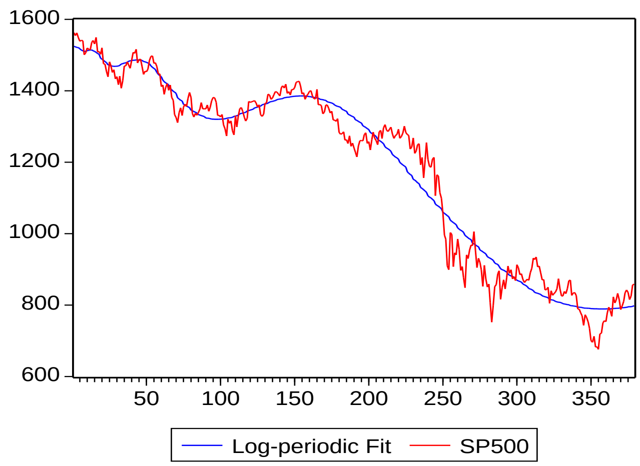
(*) Newey-West HAC Standard Errors

Table II: Estimation Results: AR(1)-GARCH(1,1) Log-periodic model (4)

θ	$\hat{\theta}$	Std.err	T-stat	P-value
A	184.78	35.14	5.26	0.00
B	-45.21	10.06	-4.49	0.00
β	0.96	0.15	6.60	0.00
C	21.20	4.18	5.08	0.00
w	4.52	0.30	15.29	0.00
ϕ	2.23	0.13	17.45	0.00
ρ	0.88	0.02	38.64	0.00
α_0	17.94	10.21	1.76	0.08
α_1	0.09	0.04	2.52	0.01
α_2	0.87	0.05	18.06	0.00
Log lik.	-1695.26	Schwarz criterion		9.13

Comparing Tables (I) and (II), it is possible to observe that all the log-periodic parameters are strongly significant and with the correct sign. Furthermore, the fundamental parameters β, w and ϕ are rather robust against the incorporation of the residual structure. On the other hand, the autocorrelation parameter ρ is strongly significant and smaller than 1 (we will see in the next section that the residuals are stationary), while the GARCH parameters take values well known in the financial literature, that is a small α_1 for the short-term shocks and a large α_2 for the long-term volatility persistence (see Tsay (2005) for a review of GARCH models). Figure 1 displays the plot of the SP500 stock index in the years 2007-2009 and the fitted curve according to the log-periodic specification given by model (1).

Figure 1: SP500 (10/10/2007 - 13/04/2009) and the log-periodic fit.



3.2 Specification Tests

The estimation of statistical models for non-stationary series like financial assets can be problematic, due to the possibility of a spurious regression. In this case, the residuals would be non-stationary and the parameter estimates would lack any statistical meaning. We test for unit roots in the residual series of the log-periodic specification (1) and (4) by using the Augmented Dickey-Fuller (ADF), the Dickey-Fuller test with GLS Detrending (DF-GLS) by Elliott et al. (1996) and the test by Kwiatkowski, Phillips, Schmidt and Shin (KPSS, 1992), which is based on the null of covariance stationarity rather than integratedness.

Table III: Tests statistics for ADF, DF-GLS and KPSS unit root tests applied to the residuals of the log-periodic models (1) and (4)

Residuals	**ADF**	**DF-GLS**	**KPSS**
Log-periodic model (1)	-4.823 (**)	-2.878 (**)	0.037
AR(1)-GARCH(1,1) Log-periodic model (4)	-21.236 (**)	-21.193 (**)	0.054

(**) Significant at the 1% level.

A careful analysis of the tests results reported in Table III shows that stationarity is the main feature of the residuals under scrutiny, similarly to the results found in Gazola et al. (2008). We then tested the goodness-of-fit of the log-periodic models employed for the conditional marginal distributions by using Ljung-Box tests on the residuals in levels and squares to test the null of no autocorrelation in the mean and in the variance, together with the specification tests discussed in Granger et al. (2006): we used standard empirical distribution tests for density specification (see D'Agostino and Stephens (1986)), together with the "Hit" test in order to test jointly for the adequacy of the dynamics and the density specification in the marginal distribution models, where the null hypothesis is that the density model is well specified. The latter test divides the support of the density into five regions and then applies interval forecast evaluation techniques to each region separately, and then to all regions jointly (see Granger et al. (2006) for more details). For the sake of space and interest, we report in Table IV only the p-values of each test.

The AR(1)-GARCH(1,1) log-periodic model (4) passes all the tests without any problem, thus highlighting that it is correctly specified, whereas the original log-periodic model (1) clearly shows some forms of

Table IV: P-values for the specification tests.

Model	**Ljung-Box (levels) (25)**	**Ljung-Box (squares) (25)**	**Jarque-Bera Test (H_0: Normal)**	**Lilliefors (D) (H_0: Normal)**
(1)	0.000	0.000	0.006	0.040
(4)	0.254	0.423	0.225	> 0.1
Model	**Cramer-von Mises (W2) (H_0: Normal)**	**Watson (U2) (H_0: Normal)**	**Anderson-Darling (A2) (H_0: Normal)**	**Joint Hit Test**
(1)	0.000	0.000	0.000	0.000
(4)	0.241	0.314	0.207	0.581

misspecifications. This result was expected, given that model (1) considers only the long-range dynamics of price movements, without taking the short-term dynamics into account.

4 Forecasting Exercises

In this section we perform some forecasting exercises with the American SP500 stock index, considering 120-step ahead and 180-step ahead forecasts in order to assess the accuracy of our approach with regard to a long-term forecasting. The competing models are the following ones:

1. The original log-periodic model (1);
2. The AR(1)-GARCH(1,1) log-periodic model (4);
3. An AR(p)-GARCH(1,1) model computed with log-returns;
4. An ARMA(1,1)-GARCH(1,1) model computed with log-returns.
5. A simple Random Walk.

The third and fourth models are standard models in the financial literature, while the random walk is the most classical benchmark in forecasting. We use the observations ranging between 10/10/2007 and 26/06/2008 as the first initialization sample, while those from 27/06/2008 till 13/04/2009 are used to perform 120-step ahead and 180-step ahead forecasts. A summary of the forecasting performances is reported in Table V.

Table V: Root Mean Squared Error and Mean Absolute Error for the 120 and 180-step ahead forecasts

	RMSE		**MAE**	
	120-step ahead	*180-step ahead*	*120-step ahead*	*180-step ahead*
LOG-PERIODIC	269.20	244.84	236.41	241.34
AR(1)-GARCH(1,1) -LOG-PERIODIC	250.48	170.17	223.50	158.22
AR(P)-GARCH(1,1)	266.16	220.95	244.81	214.03
ARMA(1,1)-GARCH(1,1)	266.95	227.05	244.78	221.52
RANDOM WALK	274.23	217.60	254.25	213.99

The previous table shows that the AR(1)-GARCH(1,1) log-periodic model yields better forecasting statistics than the competing models for both the 120-step ahead and the 180-step ahead forecasts, and the improvement increases with the forecasting distance. This result was expected since the main advantage of log-periodic models is to take the long range dynamics of price fluctuations into account, which is missing in standard financial models like the random walk. Instead, the original log-periodic model (1) shows mixed results, probably due to the misspecifications arising from not considering the short-term dynamics and which could have caused biased estimates and poor forecasts.

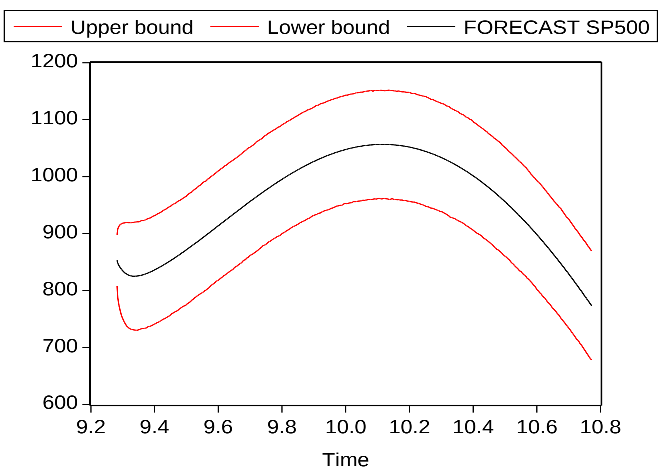
In order to compare the predictive accuracy of our models, we perform the Hansen and Lunde's (2005) and Hansen's (2005) Superior Predictive Ability (SPA) test, which compares the performances of two or more forecasting models. The forecasts are evaluated using a loss function like the MAE and the RMSE. The best forecasting model is the model that produces the smallest expected loss. The SPA test compares for the best standardized forecasting performance relative to a benchmark model, and the null hypothesis is that none of the competing models is better than the benchmark, see Hansen (2005) for

Table VI: Hansen's SPA test. P-values smaller than 0.05 are reported in bold fonts.

	RMSE		MAE	
Benchmark	*120-step ahead*	*180-step ahead*	*120-step ahead*	*180-step ahead*
LOG-PERIODIC	**0.01**	**0.00**	0.11	**0.00**
AR(1)-GARCH(1,1) -LOG-PERIODIC	0.52	0.51	0.61	0.51
AR(P)-GARCH(1,1)	0.31	**0.00**	0.34	**0.00**
ARMA(1,1)-GARCH(1,1)	0.27	**0.00**	0.32	**0.00**
RANDOM WALK	**0.00**	**0.00**	**0.00**	**0.00**

more details. The p-values produced by the SPA test with the RMSE and the MAE as loss functions are presented in Table VI.
The reported Hansen's SPA-consistent p-values show whether there is evidence against the hypothesis that the benchmark model is the best forecasting one. A low p-value (less that 0.05) means that the benchmark model is inferior to one or more of the competing models: the empirical results highlight that the sample being analyzed does yield strong evidence that all the competing models are outperformed by the AR(1)-GARCH(1,1) log-periodic model in case of the 180 step-ahead forecasts. As for the 120 step-ahead forecasts, the random walk and the original log-periodic model (1) can be outperformed by some other competing model, but this is not true for the AR(p)-GARCH(1,1), the ARMA(1,1)-GARCH(1,1) and the AR(1)-GARCH(1,1)-log-periodic models. This confirms our previous discussion with forecasting performances reported in Table V.
Finally, we report in Figure 2 the long-term out-of-sample forecast produced by the AR(1)-GARCH(1,1) log-periodic model from the 14/04/2009 till the 09/10/2010, together with 95% bootstrap confidence bands.

Figure 2: SP500 forecast: 14/04/2009 - 09/10/2010. Time t converted in units of one year.

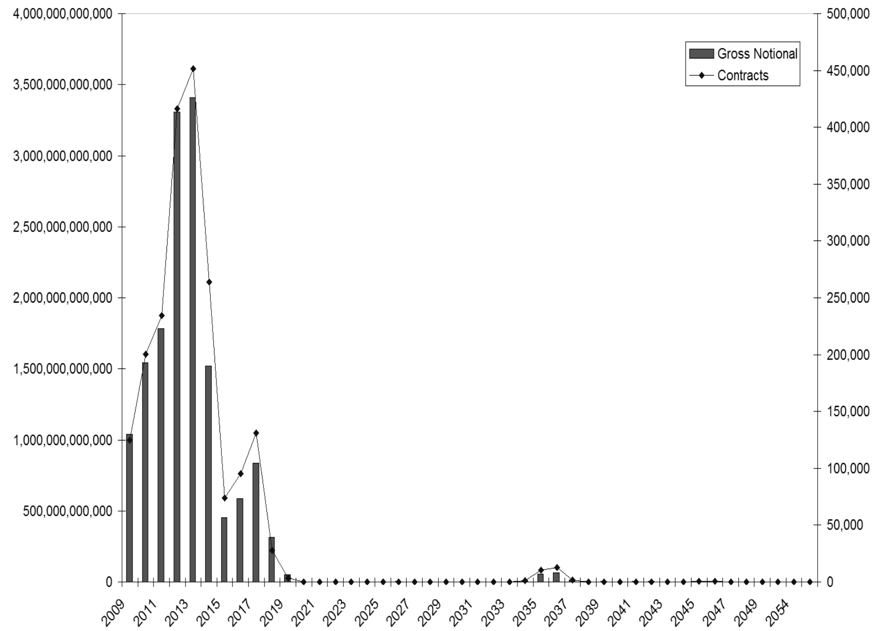


We considered here a forecast reaching a maximum distance of 3 years from the critical point t_c, i.e. the market peak on the 09/10/2007. We do so because the log-periodic models considered here are first order approximations valid only in the neighborhood of the critical point. Sornette and Johansen (1997) proposed a more general formula with additional degrees of freedom to better capture behavior away from the critical point up to 10 years. Moreover, Zhou and Sornette (2005) highlighted that, as time flows, the cumulative effect of exogenous news may detune progressively the anti-bubble pattern. This phenomenon may be accelerated in the presence of strong exogenous shocks such as the Federal Reserve interest rate and monetary policies.
The previous forecast of a renewed future market downturn in 2010 seems to be indirectly confirmed by the data on credit default swaps (CDS) reported online by the Depository Trust & Clearing Corporation's (DTCC's) Trade Information Warehouse[1]. These figures include the aggregate CDS gross notional

[1]DTCC is the world's largest post-trade financial services company and in 2007 it settled the vast majority of securities transactions in the United States, more than $ 1.86 quadrillion in value.

positions for Warehouse records as well as the aggregate number of contracts (see Figure 3 below).

Figure 3: CDS Gross notional positions and number of contracts (data retrieved on the 27/03/2009)



These data seem to suggest that the peak of the current crisis will be reached in the years 2012-2013, after which the situation should start normalizing. It will be very interesting to see whether this rather simple but extremely powerful forecasting tool will be confirmed by reality.

5 Conclusions

This paper proposed the use of log-periodic models to model and forecast the global financial crisis of 2007-2009 as a special case of "anti-bubble". By observing that world stock markets peaked in October 2007, we presented an econometric investigation of the log-periodic models, which tackled potential inferential problems arising from autocorrelation and heteroskedasticity. We considered the AR(1)-GARCH(1,1) log-periodic model recently proposed by Gazola et al. (2008), which aggregates latent dynamical features and mechanisms of the normal phase of the market, with the critical long-range dynamics of price fluctuations encompassed by the original log-periodic model. We then compared the previous set of log-periodic models with standard times series models in terms of long-term out-of-sample forecasting performances. We performed forecasting exercises with the American SP500 stock index, considering 120-step ahead and 180-step ahead forecasts. We showed that the AR(1)-GARCH(1,1) log-periodic model yielded better forecasting statistics than the competing models for both the 120-step ahead and the 180-step ahead forecasts, and the improvements increased with the forecasting distance. Instead, the original log-periodic model (1) showed mixed results, probably due to the misspecifications resulting from not considering the short-term dynamics: such misspecifications may have determined biased parameter estimates and poor forecasts. Interestingly, we found out that the current market rebound should peak at the beginning of 2010.

The main implication that we draw here is that the traders' herding behavior during the "Second Great Contraction" (using the definition by Reinhart and Rogoff (2009)) has progressively strengthened itself in bearish market phases, thus forming an anti-bubble. It remains to be seen whether the latter will last only 3 years, similarly to what happened with the SP500 in the years 2000-2003 and discussed by Zhou and Sornette (2005), or much longer like the Japanese anti-bubble in the 90's which was discussed in Johansen and Sornette (1999).

The future work will be directed to studying the small sample properties and the computational aspects of the ML estimators in case of models with nonlinear trends, similarly to the Monte Carlo studies recently performed in Fantazzini (2009, 2010).

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