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BY

Diwakar Poudel, Leif K. Sandal,
Sturla F. Kvamsdal, AND **Stein I. Steinshamn**

Norges Handelshøyskole

NORWEGIAN SCHOOL OF ECONOMICS.

FISHERIES MANAGEMENT UNDER IRREVERSIBLE INVESTMENT: DOES STOCHASTICITY MATTER?

Diwakar Poudel[a]*, Leif K Sandal[a], Sturla F Kvamsdal[a] and Stein I Steinshamn[b]

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Abstract

We present a continuous, nonlinear, stochastic and dynamic model for capital investment in the exploitation of a renewable resource. Both the resource stock and capital stock are treated as state variables. The resource owner controls fishing effort and the investment rate in an optimal way. Biological stock growth and capital depreciation rate are stochastic in the model. We find that the stochastic resource should be managed conservatively. The capital utilization rate is found to be a non-increasing function of stochasticity. Investment could be either higher or lower depending on the interaction between the capital and the resource stocks. In general a stochastic capital depreciation rate has only weak influence on optimal management. In the long run, the steady state harvest for a stochastic resource becomes lower than the deterministic level.

Keywords:

Physical capital, irreversible investment, stochastic growth, long-term sustainable optimal

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[a] NHH Norwegian School of Economics, Department of Finance and Management Science

[b] Institute for Research in Economics and Business Administration (SNF)

* Corresponding author, E-mail: Diwakar.poudel@nhh.no

1. Introduction

The exploitation of a natural resource requires capital investments. Investments, accumulated over time, may lead to excessive exploitation capacities causing an overexploitation of natural resources (Sandal *et al.*, 2007). Management models that do not include capital dynamics to suggest optimal exploitation of natural resources could be biased toward over-exploitation. Over-exploitation as a result of capital accumulation is one of the serious problems in natural resource management except if the capital invested in the resource exploitation is perfectly malleable, where the capital resource can be put into alternative use and the optimization model can simply be a single state - control optimization (Clark *et al.*, 1979). Some studies have employed capital dynamics on the optimal management of natural resources taking into account that capital invested in resource management can often be non-malleable. Although the capital accumulation in natural resource management was considered by Smith (1968), the irreversibility in capital investment was first considered by Clark *et al.* (1979). The model analyzed effect of irreversible capital investment on optimal policy of renewable natural resource. Their model is deterministic, continuous and linear in control variables. They concluded that the short-term optimal policies are significantly affected by the assumption of irreversibility in capital investment although the long-term optimal policy at steady state remains unaffected, particularly in an unexploited fishery. McKelvey (1985) also studied similar problems with convexities in price, cost and production function in an open access fisheries and found to hold the optimal capital accumulation paths suggested by Clark *et al.* (1979). Boyce (1995) investigated a non-linear version of Clark *et al.* (1979) model. He introduced non-linearities in the objective function both employing a non-linear cost of investment and a non-linear harvest profit function. He concluded that it is never optimal to stop gross investment once it starts in the nonlinear model and claimed that the difference in the optimal capital accumulation paths is due to the linearity assumptions made by Clark *et al.* (1979) with regards to the harvest and the investment cost functions.

After Boyce (1995), the study was extended by Sandal *et al.* (2007). They included the resource stock in the objective function and removed the non-negativity constraint on investment. Furthermore, they considered a cost associated to the capital management irrespective of its utilization. They showed that both the steady state and paths leading to steady states are affected by the new assumptions. Also they concluded that the convexity in cost of capital and cost of investment call for more conservative exploitation of the natural resources.

The seminal work by Clark *et al.* (1979) and later are all the deterministic models. In reality, most of the decisions are required to take place in an uncertain environment (Charles & Munro, 1985). Species interactions, growing environment and different external shocks make the stock growth uncertain and more complex. The stochastic process is central in explaining the uncertainty in growth and development of natural resources. However, in the bioeconomic literature, stochastic models are much fewer than deterministic models (Agnarsson *et al.*, 2008; Nøstbakken *et al.*, 2011), particularly in stock-capital interaction models due to the complexity of the stochastic elements in the model (Nøstbakken, *et al.*, 2011).

Charles (1983) and Charles and Munro (1985) were among the first few authors to study the effect of stochasticity in natural resources management with irreversible investment. They extended Clark *et al.* (1979) model. This is discrete time, linear model with stochasticity in stock growth. They found optimal harvest more conservative. Charles (1985) included the stochasticity in his model. The model is continuous in time and has nonlinearity in the cost function. Thereafter, very few studies considered the stochastic analysis of optimal management of natural resources with irreversible investment. Researchers have been unable to incorporate both stochasticity in stock and capital dynamics in the analysis of the fisheries management due to the computational difficulties (Nøstbakken, *et al.*, 2011; Singh *et al.*, 2006). A study by Singh et al (2006) that included uncertainty in the stock dynamics and linear cost of capital adjustment suggest lower optimal exploitation level.

In our analysis, we have extended and modified previous studies to analyze the effect of stochasticity in optimal exploitation of natural resources in a multi-dimensional state space. This study differs from Clark *et al.* (1979) due to the nonlinearity in control variables and cost functions and the stochasticity in the model. Furthermore, this study is different from Boyce (1995) due to addition of capital management cost and removal of non-negativity constraints on investment and inclusion of stocks in the objective function. It differs from Charles and Munro (1985) because this is a continuous time, non-linear model with stochasticity in both of the state variables. This study is an extended version of the deterministic multi-dimensional model by Sandal *et al*. (2007).

In addition to the commonly considered stochasticity in the growth of biological stock, we also include stochasticity in the capital depreciation rate in order to understand the effect of uncertainties in optimal management of natural resources with irreversible investment. The intuition of having a stochastic depreciation rate is that the actual depreciation rates of assets are hard to observe as they are inextricably intertwined with many other environmental factors (Yiu, 2007) rather than the most common approach of treating the depreciation rate as an exogenous variable (Dueker *et al.*, 2007). Furthermore, an increase in the capital utilization may induce an overuse of the capital and therefore an unexpected increase in its depreciation rate (Bourguignon, 1974). Moreover consideration of stochastic depreciation allows depreciation shocks to serve as an additional driving force behind macroeconomic fluctuations along with technology shocks (Dueker, *et al.*, 2007). Furthermore, the uncertainty may potentially have a great impact on the investment decisions of the fishing firms and the capacity development of their capital resources (Nøstbakken, *et al.*, 2011). Therefore, we find it appropriate to include stochasticity in the capital management in our model.

Apart from inclusion of stochasticity, the model is non-linear in both the cost and harvest functions, and both the resource stocks and capital appear in the objective function. The non-malleability of capital investments is incorporated through an asymmetric cost function of investment with a positive depreciation rate. Finally we analyzed the sensitivity in optimal decision due to change in parameters. These include different degrees of stochasticity, discounting rate, depreciation rate, and the disinvestment boundaries.

This study employs a feedback approach (Sandal & Steinshamn, 1997a, 1997c, 2001), where the control variable is the direct function of the state variables. In contrast to the commonly used time paths approach, the feedback approach is of better use when faced with stochasticity and uncertainty (Agnarsson, *et al.*, 2008). Feedback models take prevailing stock (states) as an input and therefore, these models automatically respond to the unexpected changes in the stock and adapt to the new situations (Sandal & Steinshamn, 1997a, 1997c).

2. Model

The model is a dynamic optimization model in continuous time with two state variables and two control variables. Let the biomass of natural resource such as fish stock be denoted by x and the harvest rate of the stock as h. We assume that the extraction of these resources requires a capital k, which can in this case be interpreted as the fleet. The instrument used to control the state variable capital is investment, denoted by $I \in (-\infty, \infty)$, where $I > 0$ is the investment while $I < 0$ is disinvestment. The instrument to control the extraction of the natural resource is the capital utilization rate, denoted by $\phi \in [0,1]$. Here $\phi = 0$ represents zero harvest and $\phi = 1$ represents full capacity utilization. The cases with $0 < \phi < 1$ represent reduced capital utilization rate. Following Schaefer (1957), the harvest function $h(x,k,\phi)$ is given by:

(1) $$h(x,k,\phi) = q\,x\,\phi\,k$$

In this equation, q is an exogenous coefficient. Notice that k is the maximum capital capacity available whereas ϕ is the utilization rate of this capacity. The term ϕk in the Schaefer production function (1) can be interpreted as fishing effort.

The variables x and k are the state variables while ϕ and I are the control variables. The dynamic equation for the biological stock with stochastic growth can be obtained by adding a stochastic term in the deterministic growth function of Clark (1990) and Sandal *et al.*(2007):

(2) $$dx = (f(x) - h)\,dt + \sigma_x(x)dB_x$$

Here $f(x)$ is the natural growth function of the biological stock x, where $f(x)$ is positive on $0 < x < K$ with $f(0) = f(K) = 0$. The term $\sigma_x(x)dB_x$ represents the stochastic part of the stock growth relationship. The term $dB_{(.)}$ represents the incremental Brownian motion which are iid with mean zero and dt is the time increment.

A wildly used deterministic equation for the capital change over time is:

(3) $$dk = (I - \beta k)\,dt$$

In equation (3), βk is the capital depreciation with depreciation rate $\beta \in (0,1)$. Uncertainty in the capital dynamics due to uncertain depreciation rate can be introduced into the model by decomposing the change in capital into two parts (Fousekis & Shortle, 1995). The first part is the expected change due to the investment and the observed rate of depreciation as given in equation (3). The second part is the stochastic deviation from the expected change due to stochastic variation in the depreciation rate. To obtain the stochastic component, let β evolve stochastically (Fousekis & Shortle, 1995) as:

(4) $$d\beta = \sigma_k\, dB, \text{ with } \quad \sigma_k > 0$$

Now the dynamic equation for capital change can be obtained by adding the stochastic deviation equation (4) into the expected change equation (3) as:

(5) $$dk = (I - \beta k)\,dt - k * d\beta = (I - \beta k)\,dt + \sigma_k k\, dB_k$$

Equation (5) provides the actual change in capital as the drift $(I - \beta k)$ captures the expected change conditional on the observed depreciation rate (β) and the unexpected change in the capital is captured by $k\ d\beta = \sigma_k k dB_k$. The term $\sigma_k k dB_k$ represents the stochastic volatility part of the capital evolution in time. In absence of investments the capital is reduced through a Geometric Brownian process with standard deviation $\sigma_k k$. We make the natural assumption that B_k and B_x are not correlated.

The objective is to maximize the expected net present value from the fishery over an infinite time horizon subject to the dynamic constraints (2) and (5). The dynamic optimization problem can now be formulated by maximizing the expected net present value given as:

(6) $$J = E\left[\int_0^{\infty} e^{-\delta t} \prod(x,k,\phi,I)\,dt\right]$$

The function $\prod(x,k,\phi,I)$ is defined as the profit from the fishery that depends on the stock, capital, capital utilization rate and investment rate. E is the expectation operator and the non-negative parameter δ is the discount rate. The profit function can be given as:

$$\prod(x,k,\phi,I) = \pi(x,h) - \xi(k) - C(I),$$

with following assumptions

$\pi_x \geq 0,$

$\xi'(k) \geq 0,$

$C'(I) \geq 0$ and $C'(I) > 0$ for $I > I_{\min}$

$C''(I) \geq 0$ and $C''(I) > 0$ for $I > I_{\min}$

$\pi(x,h)$ is the net revenue associated with the exploitation activity of the resources, with $\pi_x > 0$, meaning that a more abundant resource base reduces harvesting cost.

$\xi(k)$ is the cost associated with the capital whether it is used or not.

$C(I)$ is the cost (or revenue) associated with the investment (or disinvestment) of capital. We assume that $I \cdot C(I) \geq 0$, meaning that there is cost associated with positive investment (buying) and a revenue (negative costs) associated with negative investment (selling). The conditions listed on $C'(I)$ and $C''(I)$ imply that marginal cost of buying capital is typically increasing and marginal revenue on selling is typically decreasing while not excluding a situation where some excess capital may have zero market value. The cost of investment is zero when investment is zero. There is an asymmetric relationship between selling price and buying price because of $C''(I) > 0$.

The convexity of the investment cost function controls the degree of malleability. By adjusting $C(I)$ we can have anything from perfect malleable to almost completely non-malleable capital.

Now we can formulate the Hamilton Jacobi-Bellman (HJB) equation for the stated problem as follows.

(7) $$\delta V = \max_{\substack{I\in R \\ \phi\in[0,1]}} \{ \prod(x,k,\phi,I) + [f(x) - h(x,k,\phi)]V_x + (I - \beta k)V_k + \frac{1}{2}\sigma_x{}^2 x^2 V_{xx} + \frac{1}{2}\sigma_k{}^2 k^2 V_{kk} \}$$

The subscripts of V denote partial derivatives with respect to the index $i = x, k$. Optimal solution can be found by solving the Hamilton-Jacobi-Bellman (HJB) equation (7). The inner optimums with respect to control are given in appendix A. Since it is difficult to solve analytically the Hamilton-Jacobi-Bellman (HJB) equation with boundary conditions, we approach the problem by using numerical approximation methods.

3. Numerical approximation approach

Our problem is a two-dimensional and strongly non-linear in control problem. Explicit solutions to such problems are extremely rare and it is difficult to solve the Hamilton-Jacobi-Bellman (HJB) equation together with nonlinearity and given boundary conditions. Numerical approximation methods are the only viable alternatives. The Markov chain approximation approach is one of the most effective methods (Song, 2008), which is based on probability theory. Numerical algorithms for optimal stochastic control problems of this kind can be found in Kushner and Dupis (2001). When dealing with the convergence of numerical methods, it is shown that the value functions to which our approximations converge are the optimal value functions.

The numerical technique entails discretizing the state space for the HJB control problem (7), constructing transition probabilities for the controlled Markov chain by applying finite difference techniques and then iterating on the HJB equation with initial guess V_0 for the value function. The combined approximation in policy space and value space is more powerful and faster as the value function is updated with the new policy at each step. The iteration is carried out until the value function converges to the optimal value functions (for details of the approximation refer to Kushner & Dupuis, 2001).

4. Application of the model

We apply the model for optimal fisheries management by adopting the functional forms of biological and economic components from the Norwegian cod fishery. The selected numerical example only generalizes the application of the model for the optimal management of stochastic biological stocks and capital but not for a thorough empirical analysis of the Norwegian cod fishery.

4.1 Specification of the functional forms and the parameters

The biological growth function of the stock is given as $f(x) = x(r - sx)$ where r and s are biological growth parameters. The profit function from the harvest is specified as $\pi(x,h) = p(h)h - c(x,h)$, where, $p(h)$ is the inverse demand function for harvest and is given by $p(h) = p - \gamma h$ while $c(x,h) = \frac{c_x h}{x}$

is the cost of per unit harvest.

The cost of capital management is given by $\xi(k) = c_k k$, where, c_k is the fixed unit cost associated with the total capital k, whether it is utilized or not.

The cost of investment or the revenue from the sale of the capital is modeled by the function

$$C(I) = \begin{cases} \rho(-\mathrm{n}^2 + (\mathrm{I}+\mathrm{n})^2) & \text{if } I \geq -n \\ -\rho \mathrm{n}^2 & \text{otherwise} \end{cases}$$

where ρ and n are positive coefficients. Notice that this cost function is convex and gives no extra revenue if one puts more than n units of capital on the market, i.e. it allows for free dumping of the capital exceeding n.

Parameters for this numerical example are adopted from the deterministic version of the paper by Sandal *et al.* (2007), which are specified as follows:

$r = 0.54 \text{ year}^{-1}$

$s = 1.22\times10^{-4}\left(\text{year} \times 10^{6}\,\text{kg}\right)^{-1}$

$q = 0.04 \text{ year}^{-1}$

$p = 13.5 \left(10^{6}\,\text{NOK}\right)\times\left(10^{6}\,\text{kg}\right)^{-1}$

$c_x = 7{,}500 \left(10^{6}\,\text{NOK}\right)$

$c_k = 7 \left(10^{6}\,\text{NOK}\right) \left(\text{year} \times \text{vessel}\right)^{-1}$

$\gamma = 0.01 \left(10^{6}\,\text{NOK}\right) \text{ year } \left(10\,\text{kg}\right)^{-1}$

$\beta = 0.05 \text{ year}^{-1}$

$n = 1 \text{ vessel year}^{-1}$

$\rho = 6.67 \left(10^{6}\,\text{NOK}\right) \text{ year vessel}^{-2}$

$\delta = 0.05 \text{ year}^{-1}$

$\sigma_x = 0,\ 0.3\ \&\ 0.7$

$\sigma_k = 0\ \&\ 0.01$

The detailed results described in the subsequent sections are based on the above specified parameters.

4.2 Results for deterministic model

This section provides a glimpse of the control policy in a deterministic setting of the model. We present the policy behavior in three dimensional space. The deterministic result is considered as the base case for the comparison of the stochastic optimal policy behavior in the study.

4.2.1 Optimal capital utilization rate: Since harvest (h) is the function of physical capital (k), biomass stock (x) and capital utilization rate (ϕ), an increase in capital or stock or both would decrease the capital utilization rate for a given harvest and vice-versa. In this example, it can be seen that capital is utilized in full capacity until the capital size $(k) < 47.98$ units (figure 1). The capital utilization rate decreases with an increase in capital or biomass due to the functional relationships $h(x,k,\phi) = q\,x\,\phi\,k$.

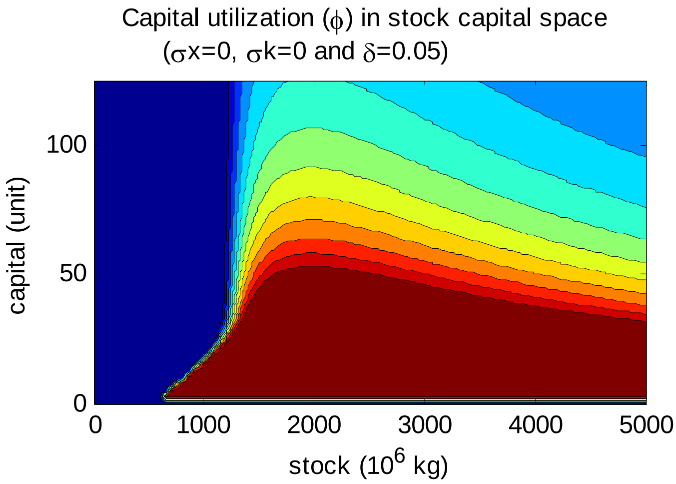

Figure 1: Capital utilization in stock capital state space in the deterministic feedback model

4.2.2 Optimal exploitation of the natural resource stock: The optimal exploitation rate of natural resources in stock-capital state space is different from the single state optimization model. Exploitation is not possible for any biomass stock with zero capital (figure 2). Even with increased capital, any exploitation of resource is suboptimal for a biomass level below 0.7 million ton. In other words, the harvest moratorium is at 0.7 million ton for small capital level. When capital increases, the harvest moratorium moves further towards higher biological stock (about 1.1 million ton). The economic gain of saving biological stock to this level is higher compared to earlier exploitation. Furthermore, the exploitation rate can be increased either by employing more capital or by allowing an increase in biological stock i.e. a higher capital and higher stock size means higher exploitation rate. However, if the capital $(k) > 49.24$ units, it is not optimal to increase the exploitation due to the stock dependent cost of harvest. An increase in exploitation increases the cost and reduces the revenue. Similarly, an increase in biological stock has only a small marginal gain in optimal harvest due to the price-quantity relationship.

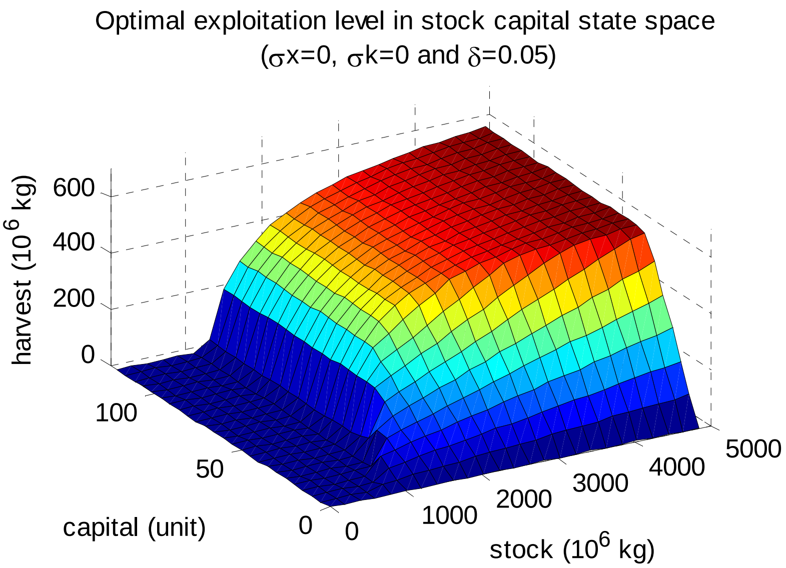

Figure 2: Optimal exploitation level in stock capital state space in a deterministic feedback model

4.2.3 Optimal investment or disinvestment in the capital resource: It is optimal to invest when the capital size is very low and the stock is positive. The investment rate should be increased with biological stock size for low capital but it should be decreased faster as the capital size increases. The economic intuition is that at a low capital, one should invest to increase the capital. Large biological stock further needs more capital and therefore needs higher investment. On the other hand, when capital accumulates above the optimal level, the excess capital needs disinvestment (figure 3).

4.2.2 Free disposal (dumping) of excess capital: Due to the cost of capital whether used or not and its depreciation, it may be optimal to sell the excess of the accumulated capital. However due the non-malleability nature of the capital and the market saturation, the marginal gain from the sale of the excess capital becomes zero. At this point, free disposal or dumping of the fraction of excess capital is optimal because the cost of the remaining extra capital reduces the discounted value in the long-run. If the cost of capital is high, the dumping rate becomes further up. In our study we have employed small cost of capital management and the free disposal is only a fraction of the capital even in low or zero stock level.

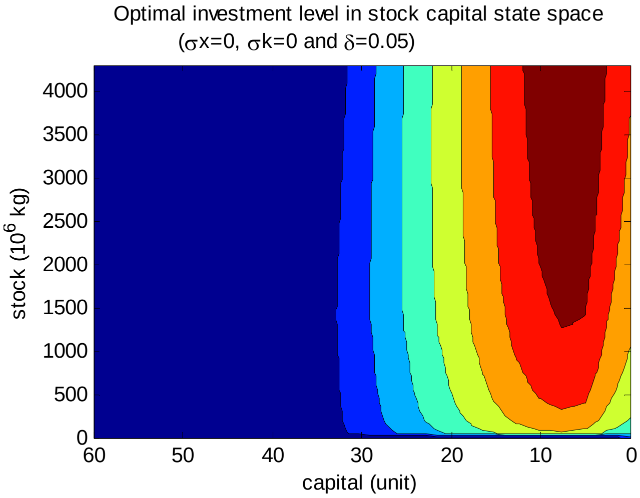

Figure 3: Optimal capital investment or disinvestment level in stock capital state space in feedback model

4.3 Results for the stochastic model

In this section, we describe the effect of stochasticity on the optimal exploitation and investment policy. Stochasticity is considered both on the biological stock growth as well as on the depreciation of capital. We examine how stochastic dynamic affects the fisheries management. The stochastic parameters are σ_x and σ_k are exogenous. We examine the effect of stochasticity in the capital utilization rate, exploitation of the resources and capital investment rate.

4.3.1 Effect of stochasticity on capital utilization rate: Although small stochasticity in stock growth does not influence the capital utilization rate significantly, the inclusion of stochasticity ($\sigma_x > 0.1$) resulted in lower capital utilization compared to the deterministic model. In figures 4a-d, we have shown the capital utilization rate at different level of stochasticity. At high level of stochasticty (for example $\sigma_x = 0.7$), the maximum capital size that can be utilized in full capacity decreased to 32.83 units compared to 47.98 units in deterministic model ($\sigma_x = 0$). A further increase in stochasticity results in large decrease in capital utilization. The decrease in capital requirement is relatively less than the decrease in the stock and its harvest under stochastic management, leading to a lower capital utilization under stochastic model.

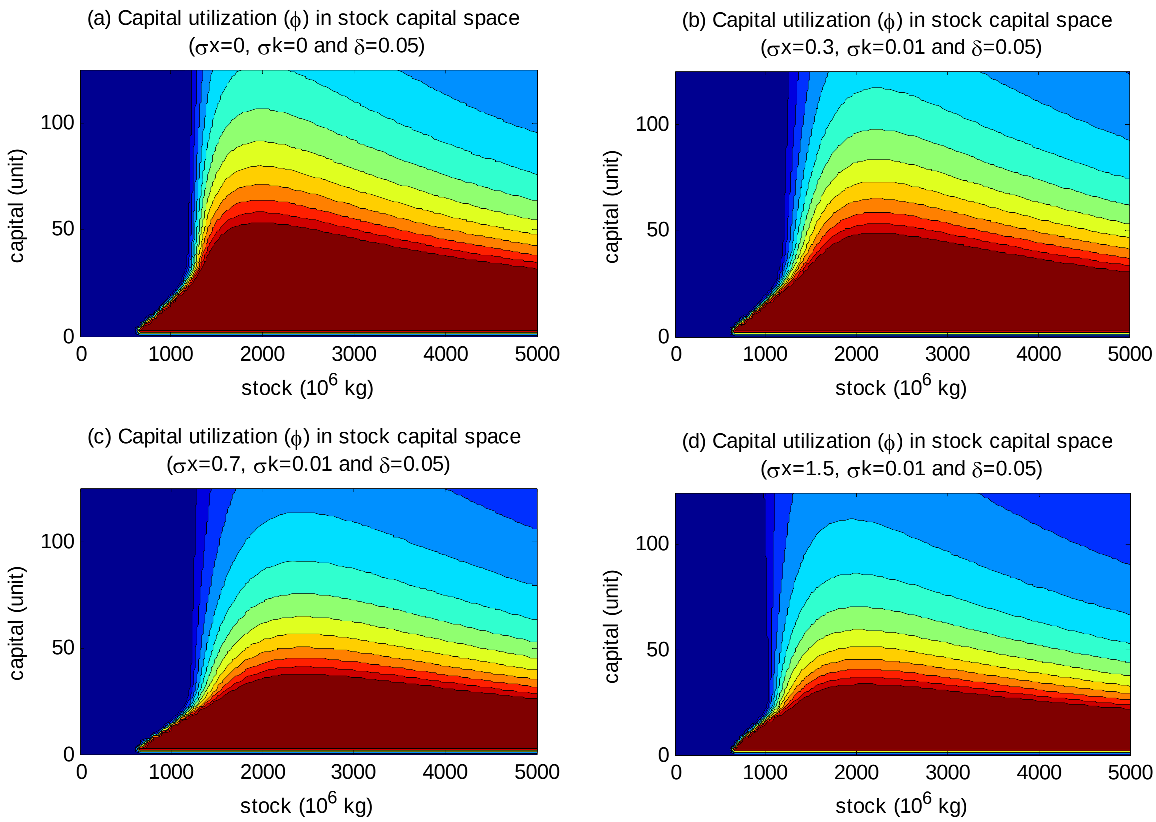

Figure 4: The capital utilization rates in exploitation for biological stock in stock capital state space in a feedback model (a) Deterministic growth model with σ_x=0 and σ_k=0 (b) Stochastic growth model with σ_x=0.3 and σ_k=0.01 (c) Stochastic growth model with σ_x=0.7 and σ_k=0.01 and (d) stochastic growth model with σ_x=1.5 and σ_k=0.01

4.3.2 Effect of stochasticity on optimal exploitation rate: The inclusion of stochasticity in the model resulted in a more conservative exploitation of the renewable resource. Although a very small level of stochasticity does not make any big difference compared to the deterministic case, an increased level of stochasticity (for example σ_x =0.3) implies that the exploitation of the resources should be much more conservative at a smaller biomass stock level compared to the large biomass stock level. At small biomass levels the chance of extinction is high compared to large biomass levels, therefore conservative exploitation is necessary when the stock is small. At higher levels of stochasticity (for example σ_x >1), the harvesting strategy should be closer to myopic at smaller biological stock because the chance of extinction is high due to stochastic critical depensation, when the stock is small even not harvested. Therefore it is optimal to harvest myopically. Figures 5a-d shows the effect of stochasticity level on optimal exploitation rate. Further increase in the stochasticty ($\sigma_x \rightarrow \infty$), exploitation should become myopic for any biomass stock size due to the higher possibility of extinction.

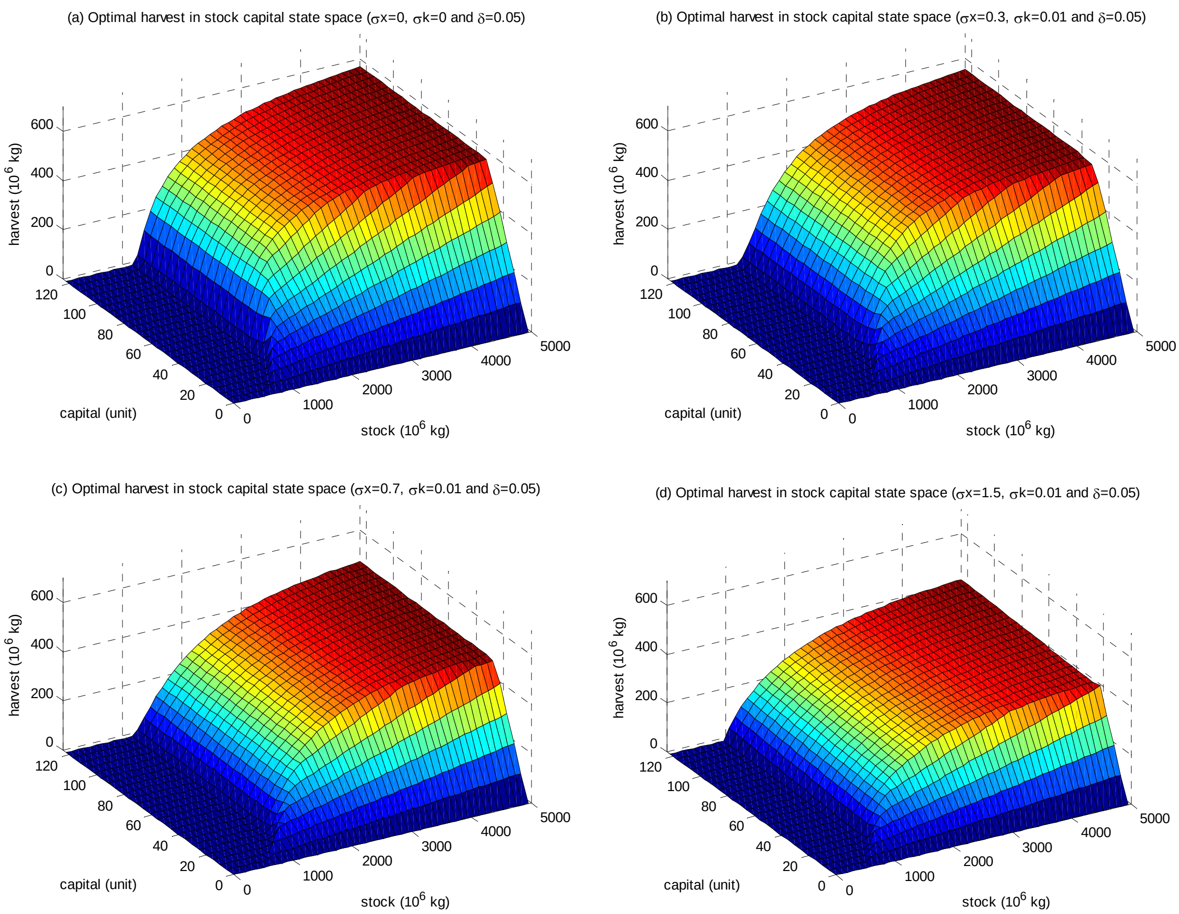

Figure 5: Optimal exploitation paths for biological stock in stock capital state space in feedback model (a) Deterministic growth model with σ_x=0 and σ_k=0 (b) Stochastic growth model with σ_x=0.3 and σ_k=0.01 (c) Stochastic growth model with σ_x=0.7 and σ_k=0.01 and (d) stochastic growth model with σ_x=1.5 and σ_k=0.01

4.3.3 Effect of stochasticity on the capital investment rate: A small level of stochasticity in stock growth does not have strong influence on the capital investment rate. But at higher levels of stochasticity (σ_x >0.3), the capital investment rate goes either way depending on the stock-capital interaction. With low capital and low biomass, the optimal investment rate is lower compared to the deterministic because smaller exploitation needs lower investment. But at large capital and large biomass need higher investment than in the deterministic model as the stochasticity has more influence even though there is high capital capacity. However, with increasing capital, the investment rate decreases faster than in deterministic setting. If the stochasticity (σ_x) >0.3, the capital disinvestment (sale) should be done earlier as compared to the deterministic case because it is optimal to get rid of the excess capital due to the cost (figure 6). Similarly the free disposal should also be done earlier under the stochastic condition compared to the deterministic model.

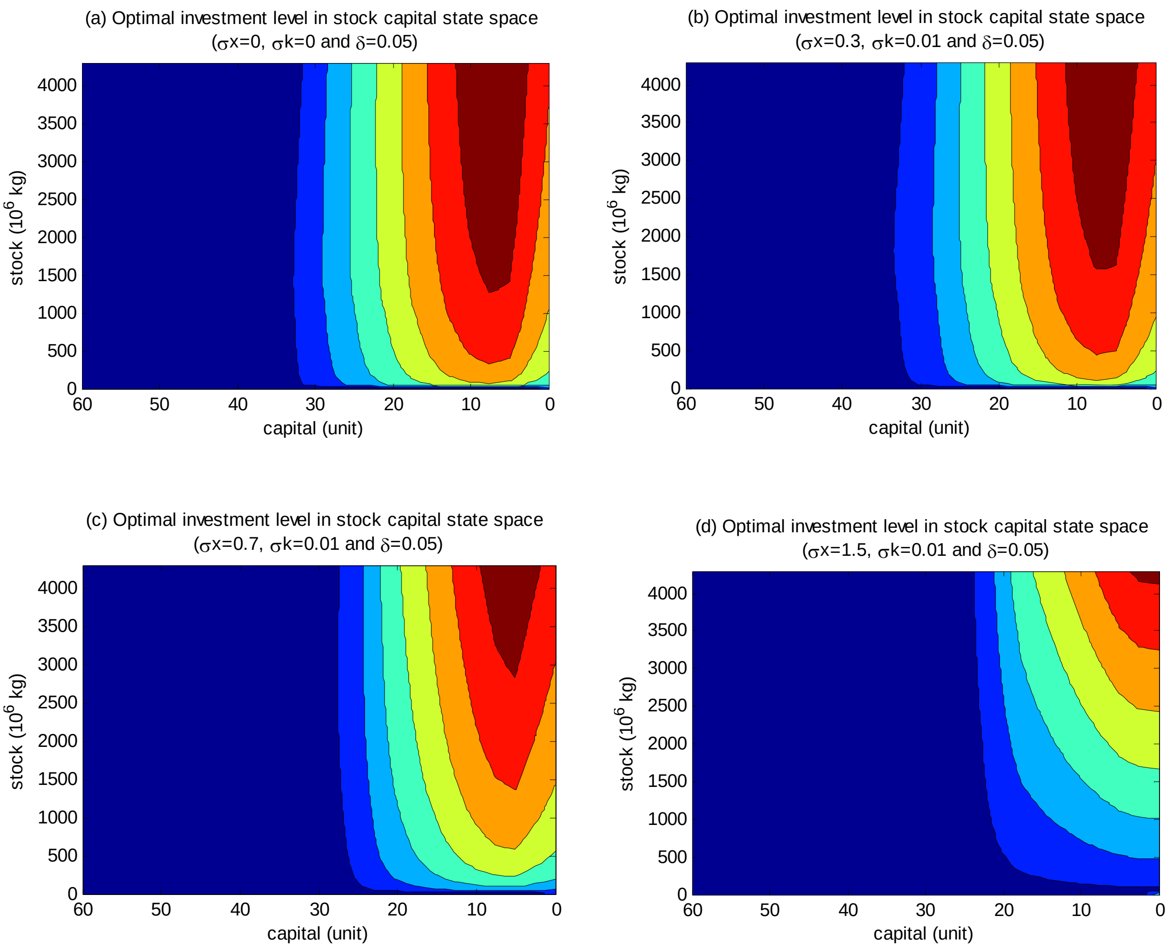

Figure 6: Optimal capital investment paths in stock capital state space in a feedback model (a) Deterministic growth model with σ_x=0 and σ_k=0 (b) Stochastic growth model with σ_x=0.3 and σ_k=0.01 (c) Stochastic growth model with σ_x=0.7 and σ_k=0.01 and (d) stochastic growth model with σ_x=1.5 and σ_k=0.01

4.4 The long-term sustainable optimal (LSO) condition

With the optimal solutions available, we simulate the system forward in time from a range of initial levels for biological stocks and capital. Overtime, the simulated paths approach to a level for stock, capital, exploitation level and investment. The level is defined as the long term sustainable optimal (LSO) level. These long term sustainable levels for deterministic and the stochastic models are defined as follows.

4.4.1 The long-term sustainable optimal (LSO) level in the deterministic model: The long-term sustainable optimal (LSO) level is defined as the steady state that can be achieved after a certain period of time if the capital and stock is managed properly. The LSO level in deterministic model can be characterized as follows:

$$x^* = 3.4882\mathrm{e}^{+003} \times 10^6\, kg$$
$$k^* = 28.61\ unit$$
$$h^*(x^*, k^*, \phi^*) = 399.1 \times 10^6\, kg\ year^{-1}$$
$$I^* = 1.1432\, unit\ year^{-1}$$

4.4.2 The long-term sustainable optimal (LSO) level in the stochastic model: The long-term sustainable optimal (LSO) level in the stochastic setting can be characterized as the mean of the stochastic oscillation that relatively stabilizes after a certain period of time. In other words, it can be defined as optimal stochastic steady state (Smith, 1986).

The stochasticity in the biological stock growth affects the LSO levels. Increased stochasticity leads to a lower LSO stock, lower capital and subsequently lower exploitation. The LSO stock, capital, exploitation rate and investment rate are presented in the table 1. There is about 6.7 percent lower LSO stock and only about 1.7 percent lower capital while about 11.2 percent lower LSO exploitation rate under the stochasticity in biological stock growth. This shows that the stochasticity leads to a very conservative exploitation. It is interesting to note that with increased stochasticity, the LSO investment rate does not change compared to the deterministic steady state. This seems to be counterintuitive. But the reason is that the capital intensive exploitation in the stochastic case requires high investment to increase the capital at the LSO level. However, a small stochasticity level in capital (σ_k=0.01) does not have any strong influence on LSO biomass, capital, exploitation rate or investment rate.

Table 1: Effect of stochasticity in optimal capital investment and exploitation rate in long-term sustainable optimal (LSO) levels at δ=0.05 and β=0.04.

State and control	Deterministic model†	Stochastic model		
	σ_x=0,σ_k=0	σ_x=0.3,σ_k=0	σ_x=0,σ_k=0.01	σ_x=0.3,σ_k=0.01
x^* (10^6 kg)	3488.2 (0)	3153.0 (1096.7)	3487.53 (8.78)	3153.9 (1105.2)
k^* (unit year-^{1})	28.6034 (0)	28.0862 (0.0679)	28.6087 (0.3508)	28.1054 (0.3902)
I^* (unit year-^{1})	1.1432	1.1396	1.1418	1.1394
h^* (10^6 kg year-^{1})	399.0953	354.2269	399.0947	354.5695

†figures in the parentheses represent standard deviation.

4.5 Evolution of biological stock and capital toward LSO level

We have illustrated how the stock and capital evolve over time to approach the long term sustainable (LSO) level. Twenty one different optimal paths for different combinations of initial biological stock and capital are shown in figure 7-8. We observe that there is overshooting and undershooting in the stock and capital evolution. The overshooting occurs when a large stock remains unexploited and an undershooting occurs due to the exploitation of a small stock otherwise not optimal. Both overshooting and undershooting may occur at an early stage which can be explained by the combination of the nonlinearities in the model such as the downward sloping demand function and the convexity in investment costs.

4.5.1 The biological stock evolution over time: Figure (7) shows the stock development for various starting points for both biomass and capital. Both deterministic and stochastic evolutions follow similar paths although these LSO levels differ significantly. There are many paths starting from the same biomass level but capital levels are different. These paths split up immediately or after a while representing several different paths but it can be seen that all the paths approach the same level in the long run. This represents the steady state in the deterministic case and LSO state in the stochastic case.

Overshooting occurs when there is a large initial biological stock and small capital. In this case it is suboptimal to harvest more either because it is too costly to invest more capital or because more harvest will reduce the price. Hence the stock will increase for a while. With high initial capital however, a large stock approaches LSO level very fast because the high capital will be utilized. Even though the price of the catch could go down due to higher landing, the lower cost of harvesting makes it optimal to harvest more aggressively.

Undershooting occurs when there is *very high initial capital* and a reasonably large initial biomass. As discussed earlier, it is optimal to reduce the high capital through disinvestment (sale). But it is not optimal to sell too much because it reduces the price due to market saturation. On the other hand, it is not optimal to let the capital be idle because there is a cost of capital. Therefore, to cover the cost of capital management, it is optimal to harvest the stock initially before it is allowed to increase to the LSO level. There could also be time tradeoff such as it is costly to wait due to discounting. Similarly, the smaller stock takes longer to reach the LSO level either because takes time to grow or because with a high initial capital it is optimal to harvest slightly to cover the costs before it approaches the LSO level.

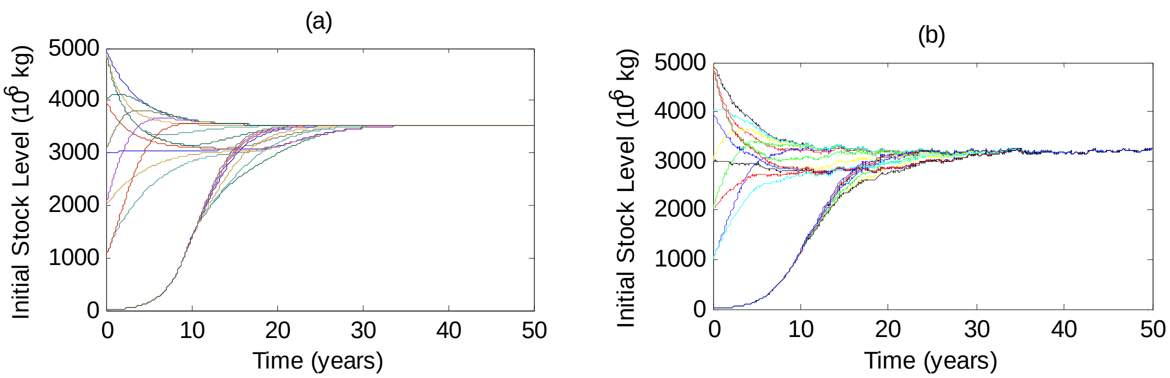

Figure 7: The optimal evolution of biological stock over time for different initial capital levels (a) deterministic growth model (σ_x=0), (b) stochastic growth model (σ_x=0.3)

4.5.2 The capital evolution over time: Figure (8) shows the capital development for various starting points for initial biomass and capital levels. When initial capital is high, it takes longer time to get rid of it because if we put more capital in the market its price decreases and therefore it is optimal to reduce capital slowly, while the smaller the initial capital the faster it approaches to the LSO level.

No overshooting occurs in the capital but undershooting can occur when the initial biomass is too low and the initial capital is slightly below or around LSO level. The reason is that it is costly to keep the capital until stock grows to the LSO level but optimal to sell due to an unsaturated capital market. Therefore, the undershooting implies that it is optimal to sell the capital at early stage and invest later to increase capital when the stock approaches to its LSO level.

The initial stock level does not affect the evolution of very high initial capital towards LSO level because these can not be utilized at any stock level. Similarly the evolution of such high capital towards LSO takes longer because disinvestment should be done at a slow rate due to market saturation. If both biomass and capital are very low, the evolution of capital to LSO should be slow because it should only be increased when the biomass increases to LSO.

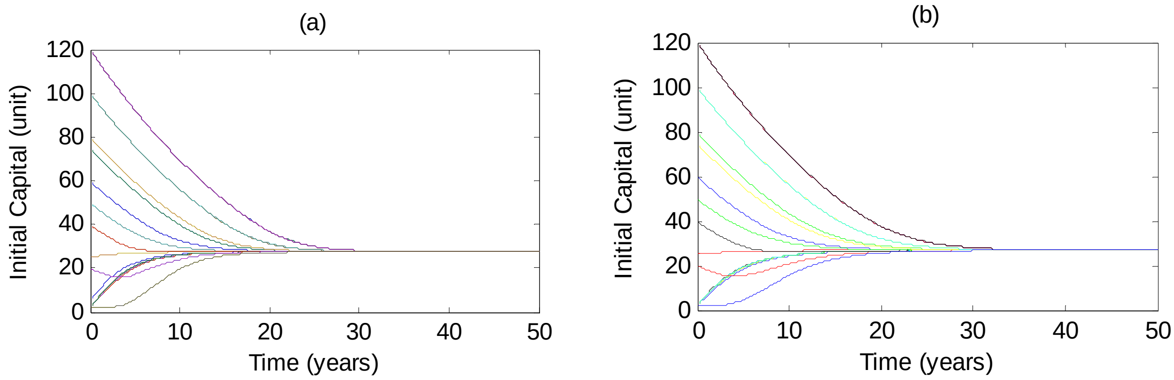

Figure 8: The optimal evolution of physical capital over time for different stock levels (a) deterministic growth model (σ_k=0), (b) stochastic growth model (σ_k=0.01)

4.5.3 The optimal paths in stock-capital state space: Twenty one different paths were drawn from various combinations of initial biomass and capital levels. The development over time is shown in figure (9). Both the deterministic and stochastic dynamic paths follow a similar trend but the long-term equilibrium occurs at lower biomass level and capital level in the stochastic case. Both overshooting and undershooting can be observed for the stock as expected, which is also reported in the previous studies (Sandal, *et al.*, 2007).

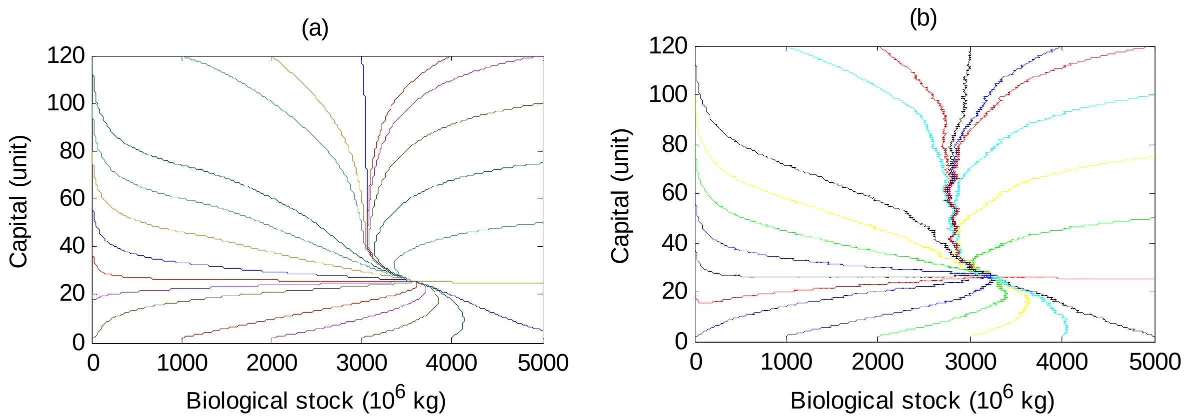

Figure 9: Optimal dynamic paths in stock-capital state space (a) deterministic growth model (σ_x=0, σ_k=0), (b) stochastic growth model (σ_x=0.3, σ_k=0.01)

4.6 Effect of parameters in long-term sustainable optimal (LSO) condition

4.6.1 Effect of discount rate (δ) *in the LSO exploitation rate:* The discount rate is an important parameter for deciding the optimality in natural resource exploitation. Hannesson(1987) and Sandal and Steinshamn (1997b) have shown that discount rate affects the long-term steady state management of biological stock and capital. In our example, three different discount rates are used to illustrate the sensitivity in the long-term sustainable optimal management of the fisheries.

An increased discount rate resulted in higher standing stocks for biological resources at the LSO level, while the capital level, investment rate and exploitation rate decreased. Although this result seems contrary to the general findings in single species management model in which higher discount rate leads to a myopic exploitation with lower standing stocks. Our finding holds due to the addition of the capital dynamics in the model.

In principle, it is optimal to increase the present harvest for higher net return considering the higher future discount. But refer the harvest function $h(x,k,\phi) = q\,x\,\phi\,k$ and note that at the steady state, we utilize the capital resources in full capacity ($\phi = 1$) and q is constant at all time. In this case, the exploitation can only be increased either by increasing the capital level or by increasing the biological stock. If we increase the capital size the cost of capital goes up along with investment to increase the capital size. Also note that the unit harvest cost goes up. The net return will be increased only if return from harvest is higher than these costs, otherwise it is not optimal to increase the harvest. Secondly, the harvest can be increased by increasing the stock size but it needs to wait which is not optimal due to higher discounting.

Therefore only decreasing harvest and increasing stock is feasible and the net return could be higher because per unit cost goes down due to increased stock and capital cost goes down due to decreased capital and investment. Therefore, it is optimal to maintain large biological stocks and lower capital, investment and exploitation rate at higher discount rate in contrary to the single species models that do not include capital dynamics. The effect of discount rate in stochastic resource management is higher than in the deterministic case. An increase in four percent discount rate results in sixteen percent lower optimal exploitation in the deterministic case but nineteen percent lower exploitation in the stochastic case (table 2).

Table 2 Effect of discount rate (δ) in long-term sustainable optimal (LSO) stock, capital, investment rate, and exploitation rate in deterministic and stochastic model.

State & control	Growth models and discount rates†					
	Deterministic (σ_x=0,σ_k=0)			Stochastic (σ_x=0.3,σ_k=0.01)		
	δ=0.05	δ=0.07	δ=0.09	δ=0.05	δ=0.07	δ=0.09
$x^*(10^6$ kg)	3488.2	3595.6	3678.2	3153.9	3260.1	3343.0
	(0)	(0)	(0)	(1105.2)	(1116.0)	(1128.7)
k^* (unit year-^{1})	28.6034	25.2716	22.6805	28.1054	24.4037	21.5571
	(0)	(0)	(0)	(0.3902)	(0.4022)	(0.4248)
I^* (unit year-^{1})	1.1432	0.9978	0.8726	1.1394	0.9709	0.8256
$h^*(10^6$ kg year-^{1})	399.095	363.466	333.697	354.5695	318.2304	288.2655

†figures in the parentheses represent standard deviation.

4.6.2 Effect of depreciation rate (β) in LSO condition: The capital depreciation rate is an important parameter that may influence optimal investment in capital and also the optimal exploitation of natural resources. We employed three different depreciation rates to study the effect on the long-term sustainable optimal (LSO) exploitation of natural resources and the optimal capital investment rate. We found that with an increase in the depreciation rate, the capital size decreases at the LSO. Due to the lower capital at the LSO level, the exploitation rate decreases and the biological stock level increases which is reasonable and convincing. In contrast to the decrease in capital at LSO state, the capital investment rate increases with increased depreciation rate to compensate the loss of capital at the LSO state (table 3).

The increased depreciation rate *(β)* caused a decrease in the LSO state with increased stochasticity in the model for all variables except investment, which has further increased compared to the deterministic result. This is convincing in the sense that exploitation of stochastic resources with stochastically depreciating capital needs higher investment.

Table 3 Effect of depreciation rate *(β)* in long-term sustainable optimal (LSO) biomass, capital, investment rate, and exploitation rate in deterministic and stochastic model.

State & control	Growth models and depreciation rates†					
	Deterministic (σ_x=0,σ_k=0)			Stochastic (σ_x=0.3,σ_k=0.01)		
	β=0	β=0.04	β=0.05	β=0	β=0.04	β=0.05
x^*(10^6 kg)	3362.1 (0)	3488.2 (0)	3514.7 (0)	3013.3 (1077.5)	3153.9 (1105.2)	3183.6 (1102.2)
k^* (unit year-^{1})	32.7820 (0)	28.6034 (0)	27.7935 (0)	32.5492 (0.6302)	28.1054 (0.3902)	27.0739 (0.3710)
I^* (unit year-^{1})	0.0	1.1432	1.3886	0.0167	1.1394	1.3714
h^*(10^6 kg year-^{1})	436.5619	399.0953	390.7432	404.0411	354.5695	344.7724

†figures in the parentheses represent standard deviation.

4.6.3 Effect of depreciation (β) in free disposal (dumping) of capital: The free disposal or dumping of capital is required if the capital level is very high and there is no marginal gain from its sale. This should be done to reduce the cost of capital management. The amount of capital dumping is affected by the depreciation rate of the capital. We studied the effect of β on 'maximum' dumping of excessive capital unit. Although it is not very significant, figure (10) shows that the increase in depreciation rate decreases the free disposal of the capital. The result seems to be very intuitive. Because capital that depreciates faster need more of it in future to compensate the loss, therefore dumping should be smaller while slowly depreciating capital need less to compensate its depreciation loss therefore it is wise to dump larger fraction of the capital. But we note that the stochasticity has no effect on the optimal disposal. The 'maximum' free disposal rate (dumping out) is not affected by the stochasticity because the capital unit at stochastic LSO state is always lower than the deterministic state.

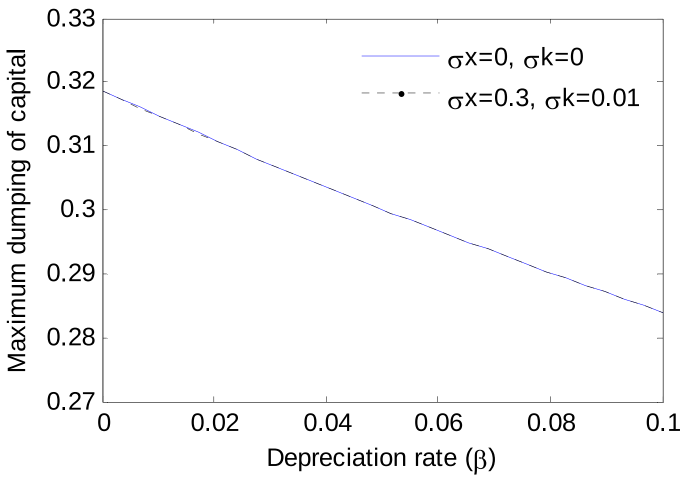

Figure 10: Effect of depreciation rate (β) in free disposal (dumping) of capital due to excessive accumulation

4.6.4 Effect of depreciation rate (β) on initial capital investment: The decision regarding investment during the low capital stage can be altered by the capital depreciation rate (β). If the depreciation rate is high, the capital investment rate should be lower as compared to a lower depreciation rate. Although it seems to be somewhat strange at first thought, this is intuitively acceptable. At the long-term sustainable biological stock level, the capital requirement is high. The capital investment today is made to approach a level of long-term sustainable capital level (k^*) in the future. If the depreciation rate is high, the capital invested today will depreciate faster before reaching the optimal steady state (k^*), whereas, lower depreciation means that capital lasts longer. Therefore, the investment rate should be altered based on the depreciation rate of physical capital. The investment rate in the stochastic case is lower compared to the deterministic case (figure 11).

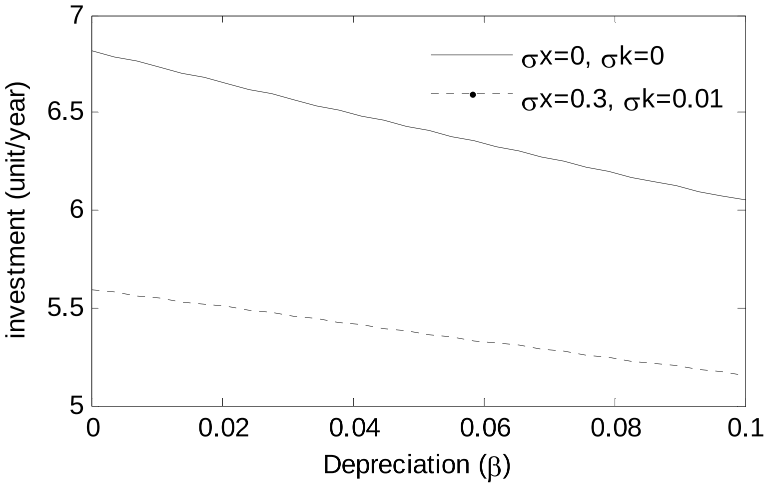

Figure 11: Effect of depreciation rate (βI in capital investment during low initial capital level

5. Summary and conclusion

This article studies optimal management of a fishery subject to stochastic biological growth and stochastic capital depreciation. We solve a stochastic dynamic programing problem and find optimal feedback policies. Our results demonstrate the importance of jointly determining resource exploitation and capital investment rates in stock capital state space.

This article has attempted to address three principle issues.

(a) Suggest optimal feedback policy in deterministic and stochastic setting under the assumption of irreversible investment.
(b) Studies and analyze the long-term sustainable optimal (LSO) behavior of the stock and capital with optimal exploitation of resource.
(c) Analyze the effect of exogenous parameters, in particular the discount and depreciation rates.

Our study shows that the optimal solutions from a single-species model are suboptimal when capital dynamics is included. During a phase with low capital availability it is optimal to harvest the stock earlier as compared to the optimal single species model without considering capital dynamics. This leads to overshooting in the fisheries management although in the long-run this is managed through appropriate investment decisions. The addition of stochasticity in the model affects the decisions compared to the deterministic case. We find that the exploitation of the resource is more conservative in the stochastic case. This result confirmed the findings of previous studies (Agnarsson, *et al.*, 2008; Sandal & Steinshamn, 1997c) in fisheries management.

Viewed from the capital investment perspective, the results suggest that the investment rate should be increased with stock at lower capital. But at high capital the investment should be reduced faster in high stock compared to low stock. The stochasticity suggests for earlier disinvest and free disposal of capitals compared to deterministic case. However, the stochasticity in capital depreciation doesn’t have strong influence on optimal management strategy.

Due to the stochasticity in biological and capital resources, the long term sustainable optimal (LSO) or the stochastic equilibrium becomes lower than the deterministic equilibrium level in both the state and control variables. The LSO exploitation rate is more affected by the stochasticity than LSO stock level and capital level. It has also been shown that the paths approach to LSO state levels may differ depending on the initial levels of stock and capital suggesting overshooting and undershooting scenarios.

Furthermore, an increase in discount rate results in a lower LSO exploitation rate and capital investment rate, whereas, an increase in stochasticity results in much lower exploitation rate and investment rate at higher discount rate. Similarly, the increase in depreciation rate results in a decrease LSO state with increased stochasticity in the model. This does not apply to the capital investment rate: it further increases as compared to the deterministic result.

References

Agnarsson, S., Arnason, R., Johannsdottir, K., Ravn-Jonsen, L., Sandal, L. K., Steinshamn, S. I., & Vestergaard, N. (2008). Comparative evaluation of the fisheries policies in Denmark, Iceland and Norway: multispecies and stochastic issues: SNF, Bergen, Norway.

Bourguignon, F. (1974). A particular class of continuous-time stochastic growth models. *Journal of Economic Theory, 9*(2), 141-158.

Boyce, J. R. (1995). Optimal capital accumulation in a fishery: A nonlinear irreversible investment model. *Journal of Environmental Economics and Management, 28*(3), 324-339.

Charles, A. T. (1983). Optimal fisheries investment under uncertainty. *Canadian Journal of Fisheries and Aquatic Sciences, 40*(12), 2080-2091.

Charles, A. T. (1985). Nonlinear costs and optimal fleet capacity in deterministic and stochastic fisheries. *Mathematical biosciences, 73*(2), 271-299.

Charles, A. T., & Munro, G. R. (1985). Irreversible investment and optimal fisheries management: a stochastic analysis. *Marine Resource Economics, 1*(3), 247-264.

Clark, C. W. (1990). *Mathematical bioeconomics: the optimal management of renewable resources*: John Willey and Sons, Inc.

Clark, C. W., Clarke, F. H., & Munro, G. R. (1979). The optimal exploitation of renewable resource stocks: problems of irreversible investment. *Econometrica, 47*(1), 25-47.

Dueker, M., Fischer, A., & Dittmar, R. (2007). Stochastic capital depreciation and the co-movement of hours and productivity. *The BE Journal of Macroeconomics, 6*(3), 6.

Fousekis, P., & Shortle, J. S. (1995). Investment demand when economic depreciation is stochastic. *American Journal of Agricultural Economics, 77*(4), 990-1000.

Hannesson, R. (1987). The Effect of the Discount Rate on the Optimal Exploitation of Renewable Resources. *Marine Resource Economics, 3*(4).

Kushner, H. J., & Dupuis, P. (2001). *Numerical methods for stochastic control problems in continuous time* (Vol. 24): Springer Verlag.

McKelvey, R. (1985). Decentralized regulation of a common property renewable resource industry with irreversible investment. *Journal of Environmental Economics and Management, 12*(4), 287-307.

Nøstbakken, L., Thébaud , O., & Sørensen, L. (2011). Investment Behaviour and Capacity Adjustment in Fisheries: A survey of the Literature. *Marine Resource Economics, 26*(2), 95-117.

Sandal, L. K., & Steinshamn, S. I. (1997a). A feedback model for the optimal management of renewable natural capital stocks. *Canadian Journal of Fisheries and Aquatic Sciences, 54*(11), 2475-2482.

Sandal, L. K., & Steinshamn, S. I. (1997b). Optimal steady states and the effects of discounting. *Marine Resource Economics, 12*(2), 95.

Sandal, L. K., & Steinshamn, S. I. (1997c). A stochastic feedback model for optimal management of renewable resources. *Natural Resource Modeling, 10*(1), 31-52.

Sandal, L. K., & Steinshamn, S. I. (2001). A simplified feedback approach to optimal resource management. *Natural Resource Modeling, 14*(3), 419-432.

Sandal, L. K., Steinshamn, S. I., & Hoff, A. (2007). Irreversible investments revisited. *Marine Resource Economics, 22*(3), 255.

Schaefer, M. B. (1957). Some considerations of population dynamics and economics in relation to the management of the commercial marine fisheries. *Journal of the Fisheries Research Board of Canada, 14*, 669-681.

Singh, R., Weninger, Q., & Doyle, M. (2006). Fisheries management with stock growth uncertainty and costly capital adjustment. *Journal of Environmental Economics and Management, 52*(2), 582-599.

Smith, J. B. (1986). Stochastic steady-state replenishable resource management policies. *Marine Resource Economics, 3*(2), 155-168.

Smith, V. L. (1968). Economics of production from natural resources. *The American Economic Review, 58*(3), 409-431.

Song, Q. S. (2008). Convergence of Markov chain approximation on generalized HJB equation and its applications. *Automatica, 44*(3), 761-766.

Yiu, C. Y. (2007). Building depreciation and sustainable development. *Journal of Building Appraisal, 3*(2), 97-103.

Appendix A: The inner optimum with respect to harvest policy

The inner optimum with respect to control (harvest) is obtained from the HJB equation as follows.

Note the specification, $\pi(x,h) = ph - \gamma h^2 - \frac{c_x}{x}$, $h(x,k,\phi) = q\,x\,\phi\,k$,

$$C(I) = \begin{cases} \rho(-\mathrm{n}^2 + (\mathrm{I+n})^2) & if \;\; I \geq -n \\ -\rho\,\mathrm{n}^2 & otherwise \end{cases},$$

$K(k) = c_k k$, $f(x) = x(r - sx)$

$\prod(x,k,\phi,I) = p\,q\,x\,\phi\,k - \gamma(qx\phi k)^2 - c_x q\phi k - c_k k - \rho(-\mathrm{n}^2 + (\mathrm{I+n})^2)$ and the HJB can be rewritten as:

$$\delta V = \max_{h\geq 0}\{\, pqx\phi k - \gamma(qx\phi k)^2 - \frac{c_x qx\phi k}{x} - c_k k - \rho(-\mathrm{n}^2 + (\mathrm{I+n})^2)$$
$$+(x(r-sx) - qx\phi k)V_x + (I - bk)V_k + \frac{1}{2}\sigma_x{}^2 x^2\, V_{xx} + \frac{1}{2}\sigma_k{}^2 k^2\, V_{kk}\} \qquad (A1)$$

The FOC for (A1) with respect to u is

$$0 = pqxk - 2\gamma(qxk)^2\phi - c_x qk - qxkV_x$$

$$2\gamma(qxk)^2\phi = pqxk - c_x qk - qxkV_x$$

$$\phi = 0.5\frac{(pqxk - c_x qk - qxkV_x)}{\gamma(qxk)^2}$$

$$\phi = 0.5\frac{(px - c_x - xV_x)}{\gamma q(x)^2 k} \qquad (A2)$$

The FOC for (A1) with respect to I is

$$0 = -\rho 2(\mathrm{I+n}) + \mathrm{V}_k$$

$$2\rho I = -\rho 2\mathrm{n} + \mathrm{V}_k$$

$$I = \begin{cases} \frac{\mathrm{V}_k}{2\rho} - n & if \;\; I \geq -n \\ 0 & otherwise \end{cases} \qquad (A3)$$

The subscripts of V denote partial derivatives with respect to the index $i = x,\ y$.