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ABSTRACT

Four economists, Mauro Gallegati, Steven Keen, Thomas Lux, and Paul Ormerod, published a paper after the 2005 Econophysics Colloquium criticizing conservative particle systems as models of income and wealth distribution. Their critique made science news: coverage in a feature article in ***Nature***. A particle system model of income distribution is a hypothesized universal statistical law of income distribution. Gallegati et al. (2006) claim that the Kuznets Curve, well known to economists, shows that a universal statistical law of income distribution is unlikely and that a conservative particle system is inadequate to account for income distribution dynamics. The Kuznets Curve is the graph of income inequality (ordinate variable) against the movement of workers from rural subsistence agriculture into more modern sectors of the economy (abscissa). The Gini concentration ratio is the preferred measure of income inequality in economics. The Kuznets Curve has an initial uptick from the Gini concentration ratio of the earned income of a poorly educated agrarian labor force. Then the curve falls in near linear fashion toward the Gini concentration ratio of the earned incomes of a modern, educated labor force as the modern labor force grows. The Kuznets Curve is concave down and skewed to the right. This paper shows that the iconic Kuznets Curve can be derived from the Inequality Process (IP), a conservative particle system, presenting a counter-example to Gallegati et al.'s claim. The IP reproduces the Kuznets Curve as the Gini ratio of a mixture of two IP stationary distributions, one characteristic of the wage income distribution of poorly educated workers in rural areas, the other of workers with an education adequate for industrial work, as the mixing weight of the latter increases and that of the former decreases. The greater purchasing power of money in rural areas is taken into account.

THE KUZNETS CURVE AND THE INEQUALITY PROCESS

1. INTRODUCTION

Four economists, Mauro Gallegati, Steven Keen, Thomas Lux, and Paul Ormerod attended the 2005 Econophysics Colloquium and published a paper in its proceedings criticizing conservative particle systems as models of income distribution (Gallegati et al. 2006). Their critique made science news: a feature news article in an issue of ***Nature***. Their paper did a service to research on conservative particle systems as models of income distribution by raising its visibility and encouraging discussion. We agree with Gallegati et al. that a conservative particle system model of income distribution is a hypothesized universal statistical law. Gallegati et al. assert that the economics literature on the Kuznets Curve shows the unlikelihood that such a law exists. They write that, in economics, relationships between phenomena can change. They claim a conservative particle system cannot account for change. They give the Kuznets Curve as an example of change a conservative particle system cannot explain (Kuznets 1955, 1965). Simon Kuznets won the third Nobel Prize in economics for, inter alia, finding the Kuznets Curve. There is a literature in economics on the Kuznets Curve which continues today. Neither we nor Gallegati et al. (2006) have seen in this literature a conservative particle system used to explain the Kuznets Curve. See Nielsen (1994) for a review of Kuznets Curve studies in economics and sociology. Gallegati et al. see explaining the Kuznets Curve as an open problem. Kuznets (1955, 1965) observed that, during the industrialization of an agrarian economy, income inequality first rises and then falls. Gallegati et al. (2006) write that there are "good reasons" for the Kuznets Curve. One reason they cite is the rising proportion of human capital in the labor force. Another is the shift of the labor force out of subsistence agriculture into the modern sector of manufacturing and services. We provide a counter-example to Gallegati et al.'s (2006) claim that a conservative particle system cannot account for the Kuznets Curve. Gallegati et al. have no mathematical model behind the assertion of "good reasons" for the curve. They cite none.

Figure 1 about here

2. THE KUZNETS CURVE

The oldest and best known statistical law of income distribution is the Pareto Law, a broad statement of which is that all size distributions of personal income (in large populations defined geographically) are right skewed with gently tapering right tails, power series tails. In 1954, Simon Kuznets (1955) announced a statistical law of personal earned income, now called the Kuznets Curve. By volume of literature generated, the Kuznets Curve approaches the fame of the Pareto Law.

We examine the Kuznets Curve as the graph of the Gini concentration ratio of personal earned income (or a related income concept such as household income) against the movement of workers from low-skilled, poorly paid work in subsistence agriculture requiring little education into more productive modern sectors of the economy, requiring at least a secondary education and offering higher pay. Social scientists use the word 'inequality' casually to name any of several statistics of income when they find the values of these statistics disagreeable. Besides the Gini concentration ratio and the Lorenz Curve of which it is a summary statistic, measures such as %poor, %poor and % rich (with various income cut points for these categories), and dispersion (e.g., variance, interquartile range) have been used as indicators of inequality. These statistics do not necessarily covary (Wolfson, 1994), who terms the Gini concentration ratio the "gold standard" of income inequality statistics (Wolfson, 1994:353). See Kleiber and Kotz (2003: 20-29, 164) for a discussion of the Gini concentration and the Lorenz Curve.

The iconic shape of the Kuznets Curve is an initial uptick in the Gini concentration ratio from that of the earned income of a poorly educated 100% agrarian labor force, a Gini higher than that characteristic of a modern economy, followed by a long, nearly linear decline to a modern Gini as the labor force shifts into the modern sector. The iconic Kuznets Curve is concave down, often called an "inverted U" although skewed to the right, with its right endpoint lower than its left endpoint. See, for an empirical example, Nielsen (1994: 667), a graph of the Gini concentration of income as a function of the percent of a birth cohort that eventually enrolls in secondary school in 56 countries circa 1970. Figure 1 is a stylized iconic Kuznets Curve.

The present paper shows how a particular conservative particle system model of income distribution gives rise to the iconic Kuznets Curve as the Gini concentration ratio of the mixture of a model agrarian distribution of earned income and a model modern distribution as the mixing weight goes from 100% agrarian to 100% modern. The particle system generating the model agrarian and model modern earned income distributions is the Inequality Process (Angle, 1983, 1986, 2002, 2006), a conservative particle system. We use Gallegati et al.'s measure of the transition of a labor force from agrarian to modern: the acquisition of human capital as workers move from subsistence agriculture in rural areas to employment in the modern sector in cities. We take into account the greater purchasing power of a unit of currency in rural than in urban areas.

Kuznets (1965) argued that the shift of the labor force from the agrarian sector with low average income to the modern sector with higher average income produces a trajectory of the inequality of income in both labor forces combined that rises, levels off, and declines during the transition. Using point estimates of the agrarian and modern wage, the result follows for the Gini concentration ratio from its definition in the case of discrete observations (Kleiber and Kotz, 2003: 164):

$$\Delta_n \equiv \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} |x_i - x_j|}{n(n-1)} \tag{1}$$

where Δ_n is Gini's mean difference, x_i is the income of the i^{th} recipient in a population of n recipients. The Gini concentration ratio, G_n:

$$G_n \equiv \frac{\Delta_n}{2\mu} \tag{2}$$

where mean income of the population is μ. If all agrarian workers earn an income of x_a and all modern workers earn an income of x_m, and $x_m > x_a$, then the number of nonzero terms contributing to Δ_n and G_n, i.e.,$|\, x_a - x_m \,|$ and $|\, x_m - x_a |$, is proportional to pq where p is the proportion of agrarian workers and q the proportion of modern workers, and p + q = 1. Hence the concave down curve of G_n plotted against q. Since μ increases as the proportion, q, of modern workers rises, the concave down graph of the Gini concentration ratio, G, against q, is skewed to the right.

However, this result is not a satisfactory account of the empirical Kuznets Curve since at the start point and end point of the transition, i.e., p or q equals 0.0 or 1.0, the Gini concentration ratio, (1), equals 0.0, a value of the Gini concentration ratio never seen or approached empirically. At least two more considerations have to be taken into account to generate an empirically relevant Kuznets Curve.

3. Explaining The Empirical Kuznets Curve

The two considerations needed to account for the empirical Kuznets Curve are: a) the difference in the purchasing power of money, or, equivalently, the difference in the cost of living, between the agrarian and modern sectors, and b) the difference between the earned income distribution of the poorly educated agrarian labor force and that of more educated workers in the modern sector.

a) The Kuznets Curve and the Metro-Nonmetro Gap in the Cost of Living in the U.S.

Gallegati et al. (2006) define the transition from agrarian to modern sectors of employment in terms of the education level of the labor force and the migration of labor from rural to urban areas. Nord (2000) estimated the difference in the cost of living between the metro and nonmetro {footnote 1} U.S. in the 1990's. Joliffe (2006) also estimated this difference. Nord estimated that the cost of living in the nonmetro U.S. was about 84% that of the metro. Joliffe estimated the cost of living in the nonmetro U.S. at 79% that of the metro. Taking the mean of these two estimates at 81.5% implies that $1 of earned income in the nonmetro U.S. has the purchasing power of approximately $1.23 in the metro U.S. A similar difference in the purchasing power of currency exists between urban and rural areas worldwide. This difference is likely much greater in economies whose labor forces are transitioning out of subsistence agriculture to employment in the modern sector. This transition was made in the U.S. in the 19^{th} and early 20^{th} centuries. The U.S. metro and nonmetro labor forces are similar, although nonmetro wages are lower partly due to a lower cost of living in the nonmetro U.S. and partly due to the somewhat lower level of education of the nonmetro labor force.

Figures 2 and 3 about here

b) Modern and Agrarian Income Distributions and The Kuznets Curve

The 'metro' and 'nonmetro' concepts are the nearest approximation to the concepts 'modern' and 'agrarian' within the U.S. national statistical system. Besides the effect of the cost of living difference between metro and nonmetro areas on wage incomes, there is also the effect of the difference in the distribution of education in the metro and nonmetro labor forces. The distributions of annual wage income conditioned on education in the metro and nonmetro U.S. are similar. See figures 2 and 3. The two parameter gamma pdf offers a good fit to the distribution of annual income in the U.S. conditioned on education in the period 1961-2003 (Angle,1996, 2006). The mixture of the partial distributions of this conditional distribution (each partial distribution weighted by its share of the labor force) has a right tail heavy enough to account for the National Income and Product Account estimates of aggregate wage income in the U.S., an approximately Pareto right tail (Angle 2001,2003). The shape parameters of the gamma pdfs fitted to partial distributions of the distribution of annual wage income conditioned on education scale from low to high with worker education in the whole U.S. (Angle, 1996, 2006; and Table 1).

The two parameter gamma pdf is:

$$f(x) \equiv \frac{\lambda^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x} \qquad (3)$$

where, $x > 0$, x is interpreted as earned income, α is the shape parameter, λ is the scale parameter, and (3) is referred to as GAM(α,λ). In terms of a gamma pdf model of earned income distribution of the whole U.S. labor force, a mixture of the metro (m) and nonmetro (nm) distributions, the Kuznets Curve is the graph of G, the Gini concentration ratio of h(x) plotted

1 The term 'rural' has a specific meaning in the U.S. Federal statistical system, a meaning farther from what the expression 'rural' in, for example, 'rural America' means than does the term 'nonmetropolitan' ('nonmetro'). A nonmetro county is a county not in a Metropolitan Statistical Area (MSA) as defined by the Office of Management and Budget (OMB), the regulator of the U.S. Federal statistical system. MSA's include core counties containing a city of 50,000 or more people or having an urbanized area of 50,000 or more and total area population of at least 100,000. Additional contiguous counties are included in the MSA if they are economically integrated with the core county or counties. The metropolitan status of every county in the U.S. is re-evaluated following the Decennial Census. While there has been a net decline in counties classified as nonmetro since 1961, the definition of nonmetro has remained roughly constant. A nonmetro wage income is defined here as the annual wage and salary income of an earner whose principal place of residence is in a nonmetro county. The percentage of the U.S. labor force thus classified has declined in the data on which figures 2 and 3 are based from about 31 to 18 percent from 1961 to 2003.

against q, the proportion metro, where h(x) is:

$$h(x) \equiv p\left(\frac{\lambda_{nm}^{\alpha_{nm}}}{\Gamma(\alpha_{nm})} x^{\alpha_{nm}-1} e^{-\lambda_{nm}}\right) + q\left(\frac{\lambda_{m}^{\alpha_{m}}}{\Gamma(\alpha_{m})} x^{\alpha_{m}-1} e^{-\lambda_{m}}\right)$$

$$\equiv p\, GAM(\alpha_{nm}, \lambda_{nm}) + q\, GAM(\alpha_m, \lambda_m) \tag{4}$$

and,

p + q =1

α_{nm} = shape parameter of the gamma pdf model of the nonmetro wage income distribution
λ_m = scale parameter of the gamma pdf model of the metro wage income distribution.

The two parameter gamma pdf is not in general closed under mixture, i.e., h(x) is not itself a two parameter gamma pdf unless either p or q = 0.

Table 1. Gamma shape parameters of partial distributions of the distribution of annual wage income conditioned on education in the U.S., 1961-2003. Standard errors of estimate are negligibly small. Source: Angle (2006)

Highest Level of Education	Estimate of shape parameter, α_i , of i^{th} partial distribution
Eighth Grade or Less	1.2194
Some High School	1.4972
High School Graduate	1.8134
Some College	2.0718
College Graduate	2.8771
Post Graduate Education	3.7329

Most of the workers in the lowest level of education in table 1 were close to the upper limit of that category. A fully agrarian labor force, in the sense of a labor force uninvolved with an industrial economy, would be largely illiterate and, extrapolating from table 1, would have a shape parameter fitted to their earned income distribution distinctly smaller than 1.2 . To extrapolate conservatively, we specify the shape parameter of the gamma pdf of an agrarian distribution of earned income as 1.0. For much of the 20^{th} century in the U.S. a high school diploma (completion of secondary education) was the standard qualification for industrial, “blue collar” labor. We take the gamma shape parameter of U.S. high school graduates, 1.8, as the model of earned income distribution of the modern sector of an economy.

The Gini Concentration Ratio of a Gamma PDF and a Mixture of Two Gamma PDF’s

McDonald and Jensen (1979) give the Gini concentration ratio, G_Γ, of a two parameter gamma pdf (3) as:

$$G_\Gamma = \frac{\Gamma\left(\alpha + \frac{1}{2}\right)}{\sqrt{\pi}\ \Gamma(\alpha+1)} \tag{5}$$

G_Γ is a monotonically decreasing function of α. $G_\Gamma = .5$ when $\alpha = 1.0$. The G of a mixture of gamma pdfs cannot be expressed, in general, as a linear function of the G_Γ's of the gamma pdf summands. The G_Γ of a gamma pdf is a function of its shape parameter alone. The G of a mixture of two gamma pdf's is, in general, a function of all four gamma parameters. There is no simple expression for the Gini concentration ratio of a mixture of two gamma pdf's with distinct shape and scale parameters. However, the G of h(x), (4), can be found by numerically integrating the Lorenz Curve of h(x) and subtracting that integral from the integral of the Lorenz Curve of perfect equality. The Gini concentration ratio of h(x) is twice that difference. See Kleiber and Kotz (2003) for a discussion of the Gini concentration ratio as a summary statistic of the Lorenz Curve.

Does the Greater Purchasing Power of Money in the Agrarian Sector Account for the Kuznets Curve?

If a unit of currency has greater purchasing power for the agrarian labor force than the modern labor force, an agrarian wage income with purchasing power equal to that in the modern sector is smaller. Assuming that education levels in both the rural and urban labor force were equal, gamma models of the wage income in both sectors will differ only in their scale parameters, i.e., $GAM(\alpha_M,\lambda_M)$ is the model of the distribution of the modern sector, $GAM(\alpha_A,\lambda_A)$ the model of the agrarian sector, $\alpha_M = \alpha_A$ and $\lambda_M < \lambda_A$. Suppose the purchasing power of a unit of currency in the agrarian sector is twice that of the modern sector, i.e., $\lambda_A = 2.0\ \lambda_M$. Since the mean of the two parameter gamma pdf model is α/λ, mean wage income in the modern sector is twice that of the agrarian sector. Figure 4 graphs the Gini concentration ratio of the mixture of the two gamma pdfs, $h(x) = p\ GAM(\alpha_A = 1.0, \lambda_A = 2.0) + q\ GAM(\alpha_M = 1.0, \lambda_M = 1.0)$, as q, the proportion in the modern sector, goes from 0.0 to 1.0. Figure 4 shows that when the purchasing power of a unit of currency in the agrarian sector is twice that in the modern sector, that difference alone cannot produce the iconic Kuznets Curve of figure 1. Figure 4 shows 1) the Gini concentration ratios of the 100% agrarian and the 100% modern labor forces as equal, and 2) the Kuznets Curve as nearly symmetric. Thus, figure 4's hypothesis is not empirically relevant.

Figure 4 about here

Does the Rise of the Educational Level of the Labor Force during the Agrarian -> Modern Transition Account for the Kuznets Curve?

Suppose that there is no difference in the purchasing power of a unit of currency received by a worker in the agrarian sector and a worker in the modern sector ($\lambda_A = \lambda_M = 1.0$), but rather there is a substantial difference in education and a concomitant difference in the shape parameters of the gamma pdfs fitting the distributions of earned income in each sector. Let the shape parameter of the gamma pdf model of wage income distribution in the agrarian sector be $\alpha_A = 1.0$, i.e., somewhat smaller than the shape parameter of the gamma pdf fitted to the wage income distribution of U.S. workers with eight years or less of elementary schooling. Let the shape parameter of the gamma pdf model of wage income distribution in the modern sector be $\alpha_A = 1.8$, i.e., the estimate of the shape parameter of the gamma pdf fitted to the wage income distribution of U.S. workers who completed high school (secondary education). Figure 5 shows the Gini concentration ratio of the mixture, $h(x) = p\ GAM(\alpha_A = 1.0, \lambda_A = 1.0) + q\ GAM(\alpha_M = 1.8, \lambda_M = 1.0)$, as the mixing weight, q, the proportion in the modern sector, goes from 0.0 to 1.0. Figure 5 demonstrates that a rise in the education level of the labor force in its transition from the agrarian to the modern sectors accounts for the decrease in the Gini concentration ratio of earned income but not for the initial uptick of the curve.

Figure 5 about here

The Joint Effect of Greater Purchasing Power in the Agrarian Sector and A Rise in Education Level in the Agrarian -> Modern Transition Accounts for the Kuznets Curve

The greater purchasing power of a unit of currency accounts for the upward movement of the Kuznets Curve over its left side, i.e., as the fraction of the labor force in the modern sector moves up from 0. The rise in education level of the labor force accounts for the fall in the Kuznets Curve. Suppose the cost of living in the agrarian sector is 81.5% of that of the modern sector. The greater purchasing power of a unit of currency in the agrarian sector would be 1.23 that of the modern sector, using estimates of the greater purchasing power of a U.S. dollar in the nonmetro U.S. than the metro U.S. in the 1990's. Suppose the education level of the modern sector results in a wage income distribution that is fitted by a gamma pdf with the same shape parameter as that fitted to the wage income distribution of high school graduates (secondary school completion) in the U.S., a gamma shape parameter of 1.8 . The graph of the Gini concentration ratio of $h(x) = p\ GAM(\alpha_A = 1.0, \lambda_A = 1.23) + q\ GAM(\alpha_M = 1.8, \lambda_M = 1.0)$ is shown in figure 6. Figure 6 contains both defining features of the iconic Kuznets Curve, the initial uptick in the Gini concentration ratio over a small proportion of the labor force in the modern sector followed by a long, nearly linear decline to the lower Gini of the modern sector as the proportion of the labor force in the modern sector rises. If the cost of living in the agrarian sector is somewhat lower than 81.5% that of the modern sector – say 2/3, and if there is the difference in education levels of figures 5 and 6, then figure 1 results. Figure 1 is the iconic Kuznets Curve of the introduction to this paper. So, if a conservative particle system can account for a) the approximately gamma distribution of wage income, and b) the shape of this distribution by level of education, we have a counter-example to Gallegati et al.'s proposition that a conservative particle system cannot account for the Kuznets Curve. The difference in cost of living by sector is an adjustment that is easily made.

Figure 6 about here

4. The Inequality Process and The Kuznets Curve

The earliest article we have found that develops a statistical mechanical theory of income distribution is Harro Bernadelli's 1943 article in **Sankhyā**, "The Stability of the Income Distribution", a paper that recognizes that stable features of this distribution indicate its generation by a statistical law. The Inequality Process is a candidate model of that law similar to the Kinetic Theory of Gases particle system model of statistical mechanics (Angle, 1990). The Inequality Process (Angle, 1983, 1986, 2002, 2006) randomly matches pairs of particles for competition for each other's "wealth", a positive quantity that is neither created nor destroyed in the particle encounter. The Inequality Process is thus a conservative particle system model, i.e., in the class of model criticized by Gallegati et al. The transition equations of the Inequality Process are:

$$
\begin{aligned}
x_{it} &= x_{i(t-1)} + d_t\, \omega_{\theta j}\, x_{j(t-1)} - (1-d_t)\, \omega_{\psi i}\, x_{i(t-1)} \\
x_{jt} &= x_{j(t-1)} - d_t\, \omega_{\theta j}\, x_{j(t-1)} + (1-d_t)\, \omega_{\psi i}\, x_{i(t-1)}
\end{aligned}
\tag{6}
$$

where x_{it} is the wealth of particle i at time step t; $\omega_{\theta j} \in (0,1)$ is the fraction lost in loss by particle j; $\omega_{\psi i} \in (0,1)$ is the fraction lost in loss by particle i; and d_t is a sequence of dichotomous independent random variables equal to 1 with probability 1/2 and to 0 with probability 1/2.

The provenance of the Inequality Process is a verbal theory of social science (Angle, 1986, 2006) that identifies competition as the generator of income distributions. In particular, the source of the Inequality Process asserts that more skilled and productive workers are more sheltered in this competition, i.e., a particle with smaller ω_ψ represents a more productive worker. Consequently, the Inequality Process must show that particles more sheltered from competition have a distribution of wealth that fits the empirical distribution of earned income of more productive workers. We agree with Gallegati et al. that worker education is a measure of worker productivity. The Inequality Process must account for the distribution of earned income conditioned on education. The test of whether it does so is performed by equating an ω_ψ equivalence class of particles with observations on workers who report a given level of education and then by fitting the stationary distribution of particle wealth in the ω_ψ equivalence class to the income distribution of workers at that level of education. The Inequality Process passes this test (Angle, 2006). Gallegati et al. are concerned about testing a model of the stock form of wealth against data on its flow form, income. Capitalizing aggregate earned income shows that most of the stock of wealth of an industrial economy is in human capital, largely the educations, of its workers. Earned income is the annuitization of human capital. Earned income is closely correlated with human capital. The substitution of one variable for another one that is closely correlated is well established in economics. See Friedman (1970 [1953]).

The Inequality Process' stationary distribution of wealth in the ω_ψ equivalence class is approximately a gamma pdf (Angle, 1983, 1986, 2002, 2006) for ω_ψ's estimated from earned income distributions conditioned on education:

$$f(x) \equiv \frac{\lambda_{\psi t}^{\alpha_\psi}}{\Gamma(\alpha_\psi)} x^{\alpha_\psi - 1} e^{-\lambda_{\psi t} x} \quad (7)$$

where X > 0 represents wealth (income) in the ω_ψ equivalence class; the shape parameter is $\alpha_\psi \approx \frac{1-\omega_\psi}{\omega_\psi}$; the scale parameter is $\lambda_{\psi t} \approx \frac{1-\omega_\psi}{\tilde{\omega}_t \mu_t}$, with $\tilde{\omega}_t$ the harmonic mean of the ω_ψ's, and μ_t the unconditional mean of x at time t. $\mu_{\psi t}$ is the mean of x in the ω_ψ equivalence class; $\mu_{\psi t} \approx \alpha_\psi / \lambda_{\psi t} = (\tilde{\omega}_t \mu_t)/\omega_\psi$.

The Macro Model of the Inequality Process, (7), (Angle, 2007) represents the agrarian distribution of earned income as a gamma pdf with a larger ω_ψ (smaller α_ψ) and larger $\lambda_{\psi t}$, than those of the modern distribution, i.e., able to reproduce figure 1, the iconic Kuznets Curve, as the Gini concentration ratio of the mixture of the two pdf's, h(x):

$h(x) = p \; GAM(\alpha_A, \lambda_A) + q \; GAM(\alpha_M, \lambda_M)$,

where the subscript A indicates the agrarian distribution and the subscript M the modern distribution. Reproduction of the iconic Kuznets Curve requires both a lower cost of living in the agrarian sector and a higher education level of the labor force in the modern sector. {footnote 2}

5. Conclusions

The Inequality Process, a conservative particle system, implies its macro model, a model of its stationary distribution in each equivalence class of its particle parameter. The macro model of the Inequality Process (7) presents a counter-example to Gallegati et al.'s claim that the iconic Kuznets Curve is a dynamic empirical income phenomenon a conservative particle system cannot

2 We have not excluded the possibility that other conservative particle system models of income distribution might do the same. Nor do we assert that we have identified all factors that might give rise to an iconic Kuznets Curve. Indeed we think it likely that the left censoring of the income distribution (exclusion of small incomes from tabulation) in countries with small GDP per capita is also involved.

explain. Kuznets (1965) thought the curve named after him resulted from the transition of a labor force from employment in the agrarian sector to employment in the modern sector. The macro model of the Inequality Process explains the Kuznets Curve the same way. Its explanation is more satisfactory because more relevant information is included and more of the features of the iconic Kuznets Curve are reproduced. We thank Gallegati et al. (2006) for stimulating discussion and research into particle system models of economic phenomena, indeed for encouraging the present paper. Their paper succeeded in directing attention to the subject of particle system models of income distributions more effectively than publications on the wide empirical relevance of such models. Economists need not fear this class of model. We expect that this line of research will put many of the verbal tenets and basic insights of the paradigm of economics on a firm scientific footing for the first time. We welcome Gallegati et al. as collaborators in this enterprise.

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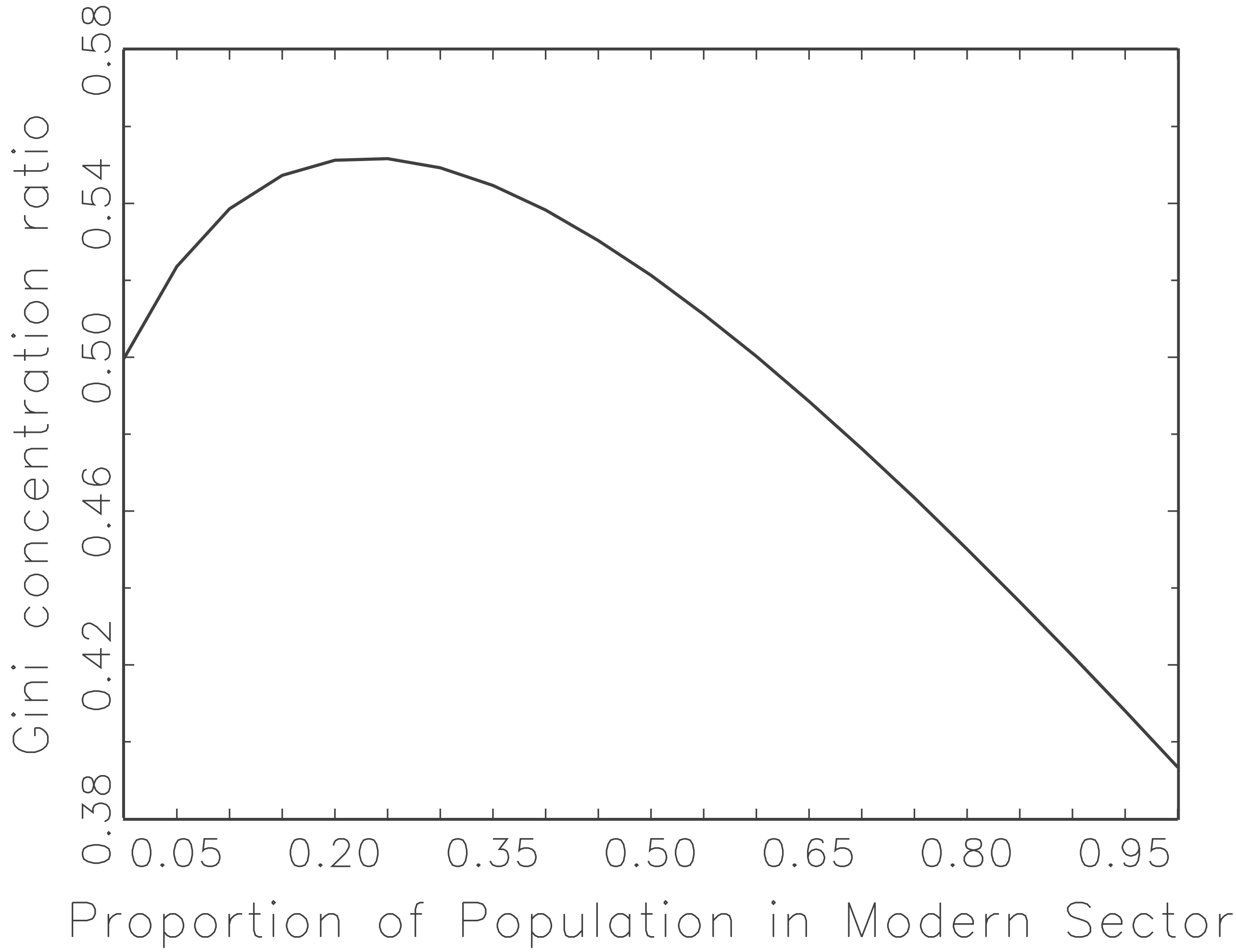

Figure 1: An Iconic Kuznets Curve

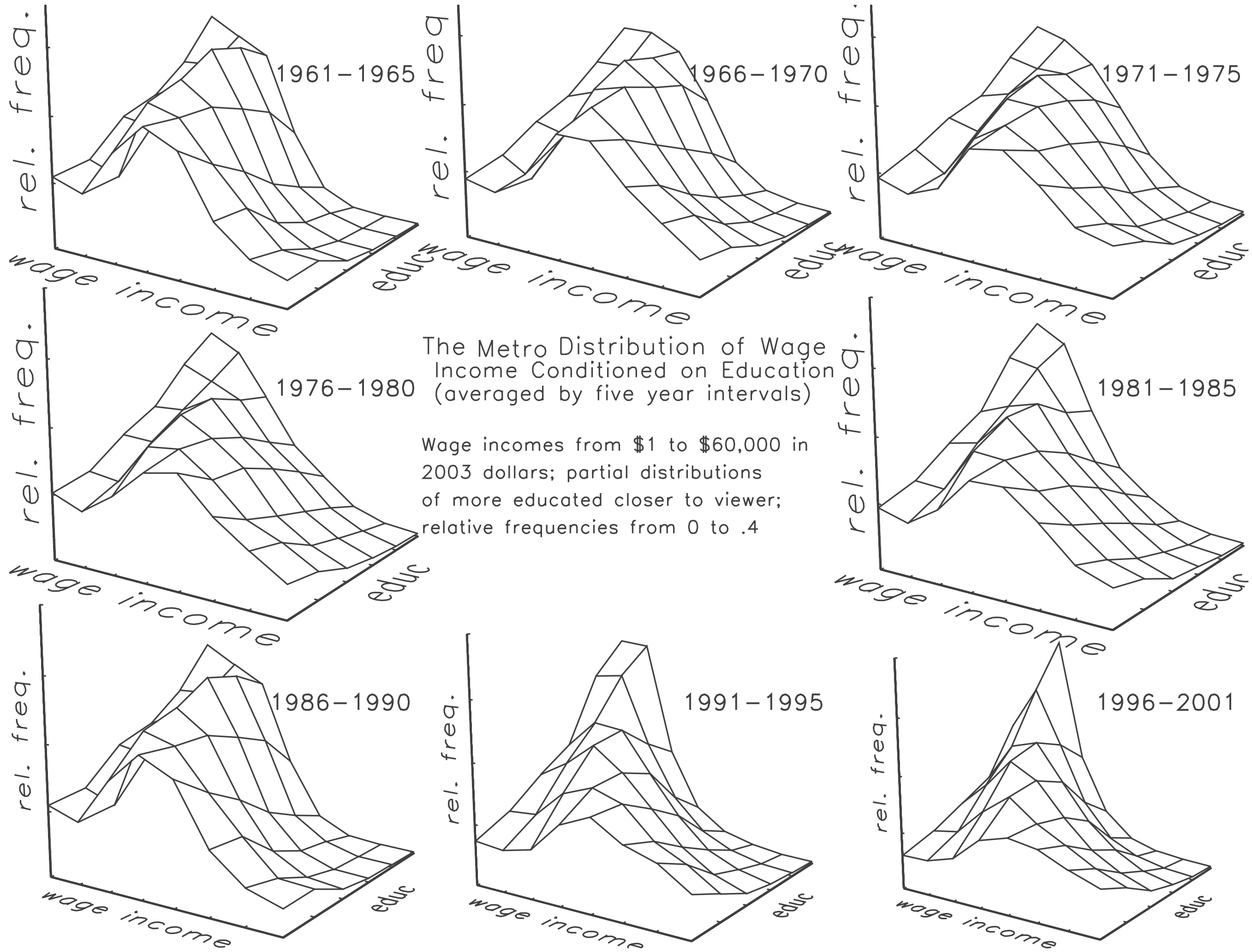

Figure 2: The U.S. Metro Distribution of Annual Wage and Salary Income Conditioned on Education

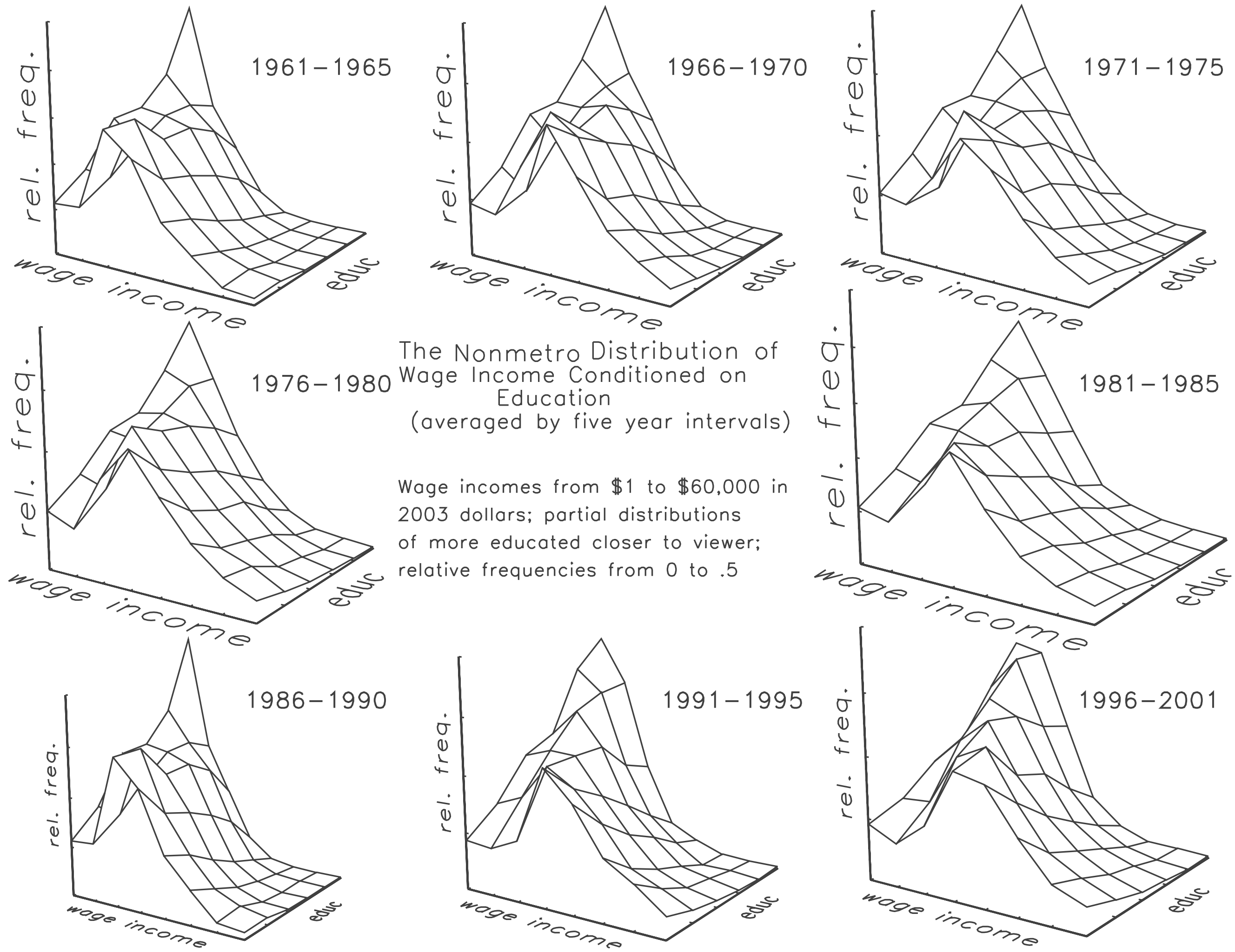

Figure 3: The U.S. Nonmetro Distribution of Annual Wage and Salary Income Conditioned on Education

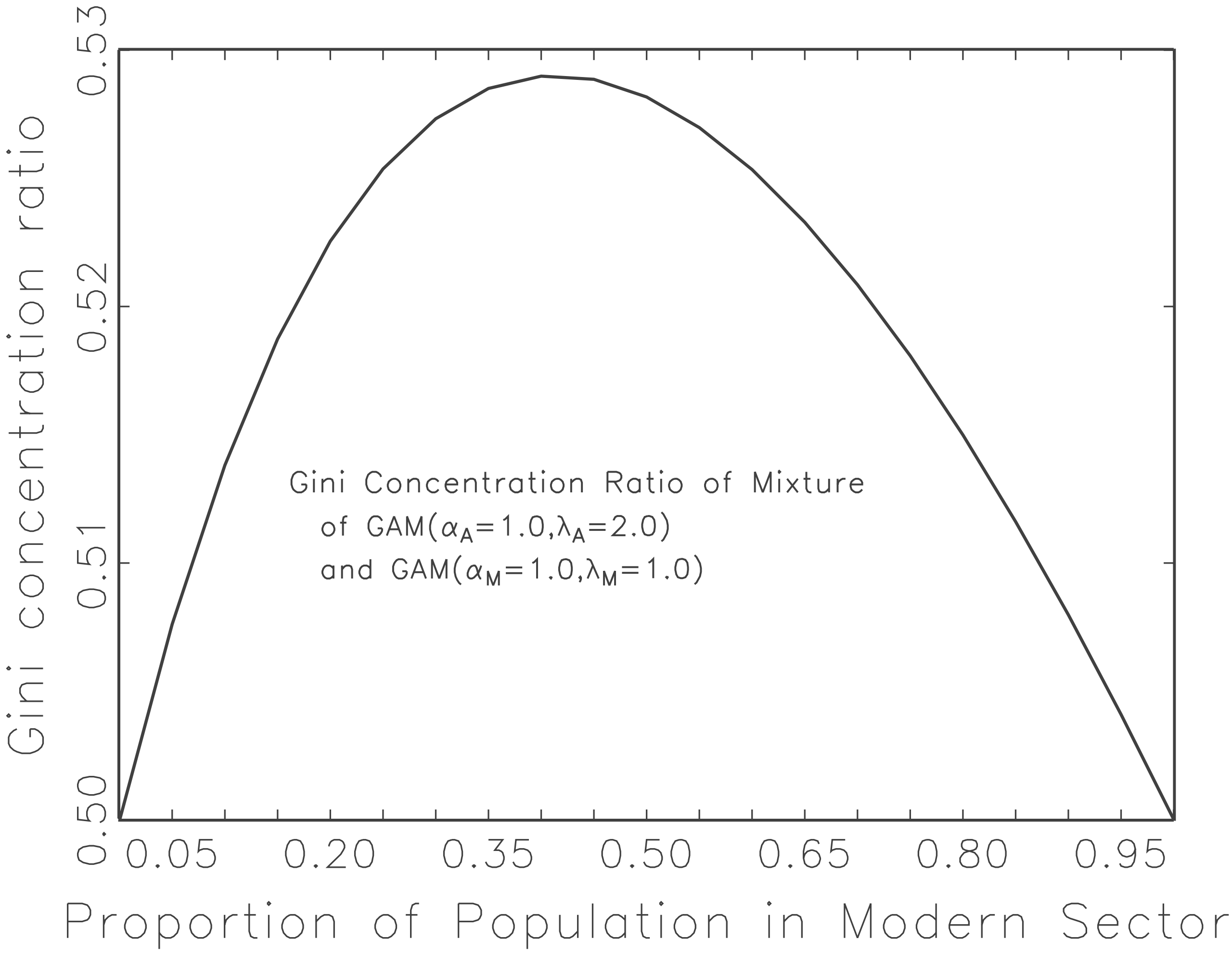
Gini concentration ratio
0.53
0.52
0.51
0.50
0.05
0.20
0.35
0.50
0.65
0.80
0.95
Gini Concentration Ratio of Mixture
of GAM(α_A=1.0,λ_A=2.0)
and GAM(α_M=1.0,λ_M=1.0)
Proportion of Population in Modern Sector

Figure 4

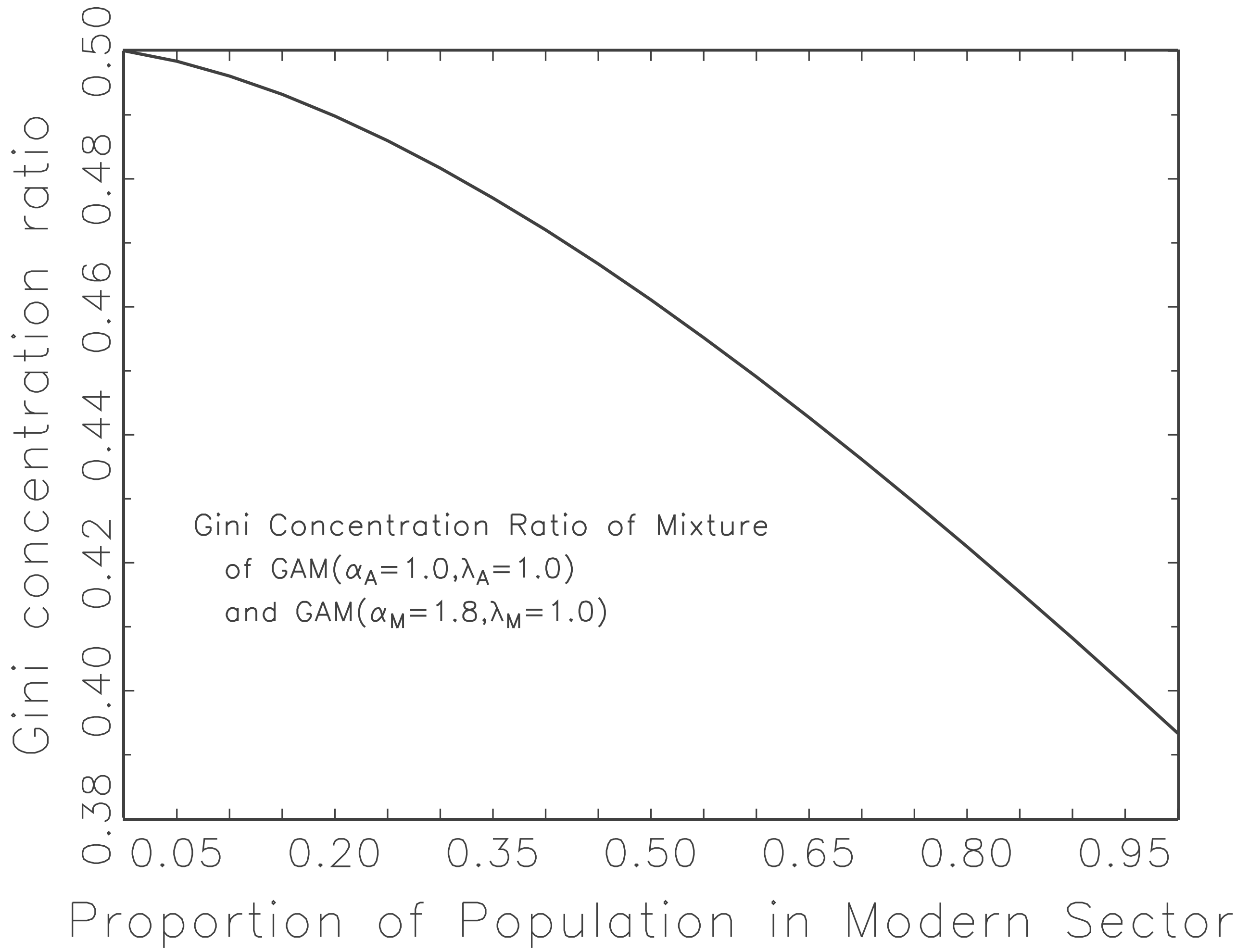
Gini concentration ratio
0.38
0.40
0.42
0.44
0.46
0.48
0.50
0.05
0.20
0.35
0.50
0.65
0.80
0.95
Proportion of Population in Modern Sector
Gini Concentration Ratio of Mixture
of GAM(α_A=1.0,λ_A=1.0)
and GAM(α_M=1.8,λ_M=1.0)

Figure 5

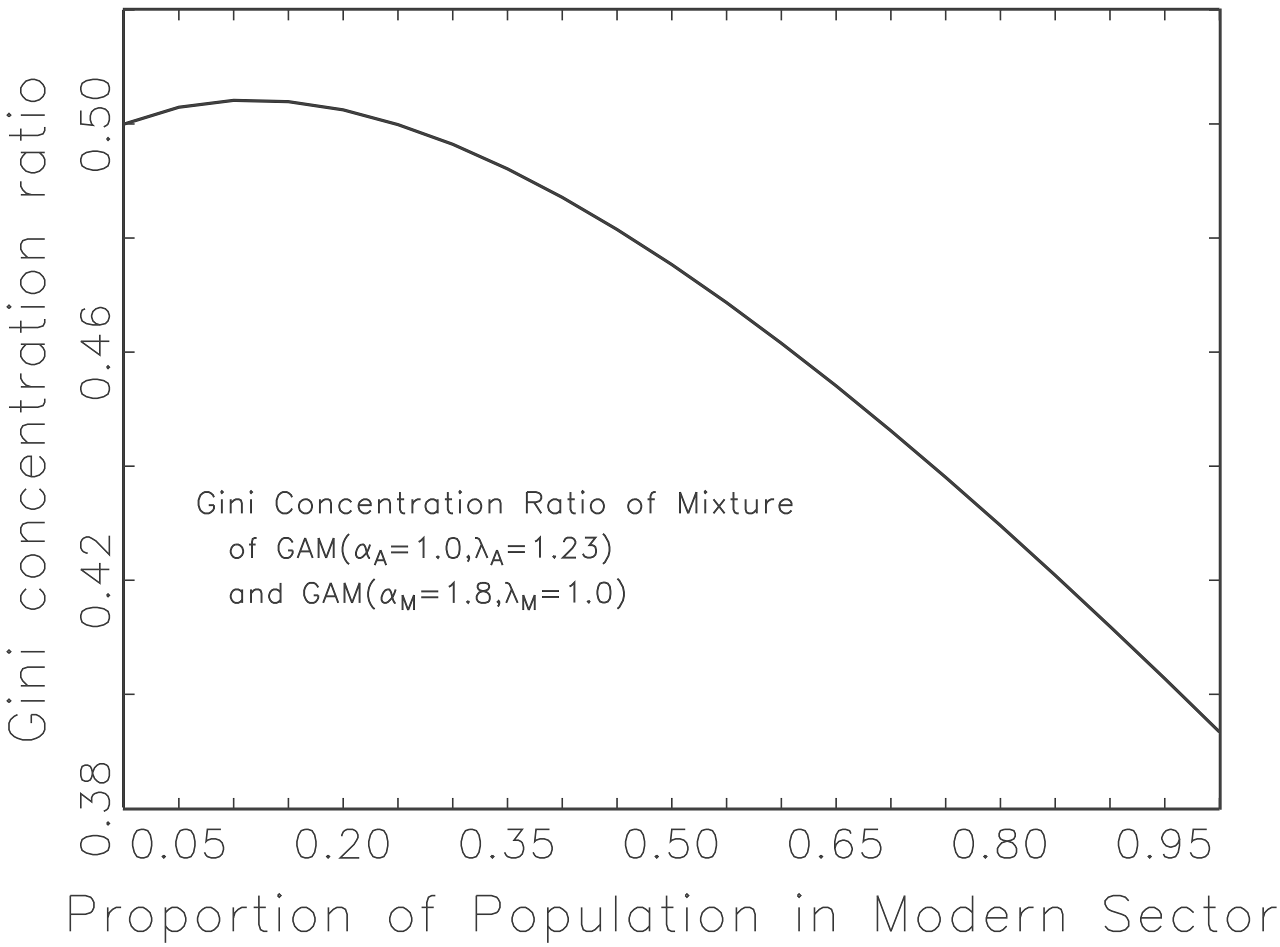
Gini concentration ratio
0.38
0.42
0.46
0.50
0.05
0.20
0.35
0.50
0.65
0.80
0.95
Proportion of Population in Modern Sector
Gini Concentration Ratio of Mixture
of GAM(α_A=1.0,λ_A=1.23)
and GAM(α_M=1.8,λ_M=1.0)

Figure 6