Pumped storage and energy saving

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Abstract

The pump storage technique allows to use cheap thermal electricity at periods of low demand to restore water resources that can be used to generate electricity at periods of peak demand. When the thermal plant and the hydro plant are managed by the same operator, the two plants are used in an efficient way to substitute low cost fuel for high cost fuel. The paper first analyzes the efficient use of the technology when the outputs at each period are given. We determine the frontier between the storage and no-storage solutions and its sensibility to cost variations. We then determine the optimal dispatch given the intertemporal preferences of electricity consumers. The model allows to emphasize the economic driver of the technology, that is the net social gain from transfering social surplus from off peak to peak period.

Keywords : water resource; pumping; hydroelectricity; energy saving

JEL classification: L12, Q25, Q32, Q42

1 Introduction

In all countries where global warming is a first-rank concern, governments have chosen various combinations of indirect and direct policies devoted to the abatement of polluting emissions, the promotion of renewable sources of energy and energy savings. For example, in 2007 and 2008 the European Commission put forward a package of measures to implement climate and renewable energy targets, in particular a strengthening and expansion from 2013 of the Emissions Trading System for cutting emissions, and a reinforcing of energy efficiency legislation on buildings and energy-using products.

In the electricity industry, much can be done both at the stages of production and consumption. But energy savings are difficult because electricity is not storable. Non storability implies that electricity must be produced at the very same moment of its consumption, including during the small number of short periods where demand is peak. The consequence is that the extra power necessary to supply peak periods is mainly produced by means of high operating cost technologies, and these technologies often are more polluting than base technologies. Every time it is possible to decrease consumption at peak periods, energy savings and emissions abatement are obvious. But it is also possible to save energy without consumption reduction by relying on pumped storage. Like in all industries, load smoothing consists in producing more than demand at off-peak periods, storing the extra production and injecting it at peak periods. The only difference is that, since electricity is not storable, the extra production of off-peak periods is used to produce primary fuel. Specifically pumped storage consists in using electricity to transform low-altitude water into high-altitude water at peak periods and then to produce electricity at peak periods with water turbines. Because of large energy losses in the transformation "electricity → water → electricity", this process can be profitable only if *i*) the difference in the costs of electricity production between peak and off-peak periods is high and *ii*) the losses implied by the double transformation are not too high.

Pumped storage is also an efficient contribution to energy saving when the installed technologies cannot be adapted to instantaneous demand. This is the case of coal plants and nuclear plants for which stop-and-go strategies would be non profitable so that a share of their off-peak output is available "for free". Similarly, solar and wind energies are not necessarily available when demand is high. Therefore, if controllable plants cannot be slowed down sufficiently at off peak periods, the extra production (thermal + intermittent - consumption) can be used to pump water into high altitude reservoirs. All increases in hydro capacity are valuable because hydroelectricity can be produced almost instantaneously. This flexibility makes it a pivotal contributor to system reliability (reactive energy, voltage control, operating reserves, back-up supply). The need to increase the availability of a flexible resource for peak periods explains the interest of governments for this technology and the momentum to develop it.[1]

The economics of pumped storage have not so far attracted many researchers, with two main exceptions: Jackson (1973) and Horsley and Wrobel (2002). Jackson (1973) insists on the advantages of pump storage for nuclear plants that cannot be turned off during periods of low demand. Pumped storage allows nuclear generators to operate almost permanently at their efficient level. Thanks to an illuminating graphical illustration, Jackson characterizes the first best of pumped storage and explains how energy should be priced to decentralize first best. Contrary to ours, his paper takes for granted that the pumping station will be used at first best. It does not characterize the technical and economic parameters separating the with-hydro and the without-hydro solutions.

Horsley and Wrobel (2002) rely on dynamic programming to extend results from a paper by Koopmans (1957) devoted to the evaluation of storage rents. Their analysis depart from ours on two grounds. First, they assume that the conversion of electricity into water involves no losses and is symmetrically reversible. In their model, the main technical constraint is the "converter's capacity" for charging or discharging the reservoir in a given time duration. The second difference is that they analyze a cost minimization problem assuming that demand is exogeneous. In our analysis, following on Jackson (1973) we determine the first best allocation given consumers'preferences.

Another strand of literature has focused on the necessity of time varying prices to implement first best and to provide unbiased signals to investors. This analysis in terms of "peak-load pricing" (see for instance Joskow (1976) and Williamson (1966)) has been recently revisited by Borenstein and Holland (2005) and by Joskow and Tirole (2007) to take into account the lack of reactivity of consumers who are not equipped to receive information on spot prices. As we will see, pumped storage alleviates but does not cancel the price variability required to implement first best.

The paper is organized as follows. In section 2 we present the hypotheses

[1] At present, about 30 to 35 GW of pumped hydro storage capacity is installed across the EU-27, to be added to 106 GW of hydro-power installed capacity. "The retrofitting of existing facilities into a storage scheme provides an important potential base for pumped hydro storage development". European Commission (2007).

on technologies and we determine the efficient dispatching given the total quantities of electricity to supply to consumers. In section 3, we specify the hypotheses on the demand side and we determine the quantities to be produced by the hydro and thermal plants in order to maximize social welfare. Section 4 presents an illustration for the case where both the preferences and the thermal technology are separable between periods. Section 5 proposes several developments of the basic model, in particular as regards the liberalization of energy markets. In section 6, we conclude.

2 The pumped-storage technology

Pumped-storage consists in a combination of the thermal and hydraulic technologies. The hydraulic side of the system requires two reservoirs. The upper reservoir is used like any water resource to produce electricity by turbinating stored water, except that the flow released is collected in a lower reservoir, until it is pumped up the mountain into the originating reservoir by means of electric power from thermal plants. This means that the hydro resource is not from exogeneous inflows but from controlled inflows.

We first explain our hypothesis on the thermal system, and then on the hydro system. In a second step, we determine the least-cost combination of technologies required to supply a given vector of electricity consumption at two periods. In effect, we assume that each time interval, say a day, is made of two periods, a night (or off-peak) period labeled $t = 1$ and a day (or peak) period labeled $t = 2$. The two periods are assumed to be of equal duration to facilitate the graphical presentation of the model.

2.1 Components

The hydro system is totally dependant on the water pumped and stored thanks to the thermal production. Therefore, the ground of the supply system is the thermal system. The hydraulic plants are nothing but an adjunct system permitting to store (a part of) the electricity produced by the thermal system in some period to make it available for the final users in the other period. Then let us examine first the thermal system and next how the hydraulic system adds flexibility within the supply apparatus, although at a cost.

2.1.1 The thermal system

Let us denote by (q_1^T, q_2^T) the production bundle of the thermal system and by $\bar{Q}^T$ the frontier of the set of bundles which are technically feasible given the available thermal capacities. A convenient and realistic assumption is that the maximal production levels are the same at both periods: (q_1^T, q_2^T) is technically feasible provided that $q_t^T \leq \bar{q}^T, t = 1, 2$. The highest rate at which the thermal system is exploited in some period does not reduce the highest rate at which the system can be used in the other period while preserving the production potential of the system.

The cost function $C(q_1^T, q_2^T)$ is assumed to be three times continuously differentiable, strictly increasing, strictly convex, and symmetrical.[2] Symmetry means that $C(q + \Delta q, q) = C(q, q + \Delta q), q \neq q + \Delta q$ and $0 \leq q + \Delta q \leq \bar{q}^T$. We denote by C_t, $C_{tt'}$ and $C_{tt't''}$ the first, second and third partial derivatives of C with respect to $q_t, q_{t'}$ and $q_{t''} (t, t', t'' = 1, 2)$.

The iso-cost curves of the thermal system in the space (q_1^T, q_2^T) are convex, symmetrical with respect the 45^o line, with slope -1 along this line, and truncated at $\bar{q}^T$.

Depending on the sign of $C_{tt'}, t \neq t'$, the two outputs can be either substitutes ($C_{tt'} > 0$) or complements ($C_{tt'} < 0$). The first possibility refers to the case where a higher intensity of use at one period hampers the efficiency of the plant at the other period. The second case is when there are some starting or warming requirements so that a positive output at one period reduces marginal cost at the other period.

A special case is the separable function:

$$C(q_1^T, q_2^T) = c(q_1^T) + c(q_2^T) \quad , \quad q_t^T \leq \bar{q}^T \quad , \quad t = 1, 2,$$

where c is a three times continuously differentiable function, strictly increasing and strictly convex.

2.1.2 The hydroelectric system

The hydroelectric system we have in mind is an Alpine mountain system in which the dams are located at some high altitude sites whereas the turbine generators are located at lower altitude sites so that the heights of the water stocks within the dams can be neglected.

[2]We therefore exclude that the non-hydro electricity could be produced from wind or solar energy. Its source is the energy from a reliable thermal plant.

We assume that there exist no natural water inflows entering the system. All the water available in the dams must have been pumped from some source using outside electric energy, that is energy generated by the thermal system.

Let us measure water in the dams in terms of energy units. Denote by $\bar{q}^H$ the capacity of the hydroelectric system and by (q_1^H, q_2^H) its production bundle: $q_t^H \leq \bar{q}^H$.

Pumping water necessitates more energy than the pumped water can generate.[3] Let $\alpha, \alpha > 1$, be the quantity of energy required to add one unit to the stock of energy available in the dams for use at the beginning of the next period. Thus if $q_{t'}^H$ is the quantity of hydroelectricity to be produced at period t', a production $\alpha q_{t'}^H$ has to be supplied by the thermal system at period t $(t, t' = 1, 2 \text{ and } t \neq t')$.

2.2 Efficient production schemes

In this section, we provide intuitions and geometrical illustrations of the solution to cost minimization using pumped storage. The formal proof is given in Appendix A.

Because the hydroelectrical system does not benefit from outside water flows and because filling up the dams with thermal energy converted into pumped water is implying some energy loss ($\alpha > 1$), then the set of consumption bundles (q_1^c, q_2^c) technically feasible is nothing but the set of feasible thermal production bundles $(q_1^T, q_2^T) : q_t^T \leq \bar{q}^T, t = 1, 2$. The thermal electricity system is the background of the supply system and the pumped storage system can just reduce the costs. It is not a system permitting to enlarge the set of technically feasible consumption bundles.

Given the symmetrical structure of the cost function of the thermal system and the fact that the iso-cost curves are convex, it may happen that, for supplying strongly asymmetrical consumption bundles, the least costly way to proceed is to use the hydroelectric system even if it is implying losses of the energy produced by the thermal plant.

2.2.1 Transfers of energy

We first analyse the feasible transfer of water between the two periods.

[3] "New state-of-the art hydro power plants have an efficiency well above 93%, and new pump storage plants may have an energy ratio of about 76 %" (European Commission, 2008).

Let us consider a consumption bundle (q_1^c, q_2^c) which would be feasible for the thermal system: $q_t^c \leq \bar{q}^T, t = 1, 2$. Without loss of generality we assume that $q_2^c > q_1^c$ since 2 is the peak period.

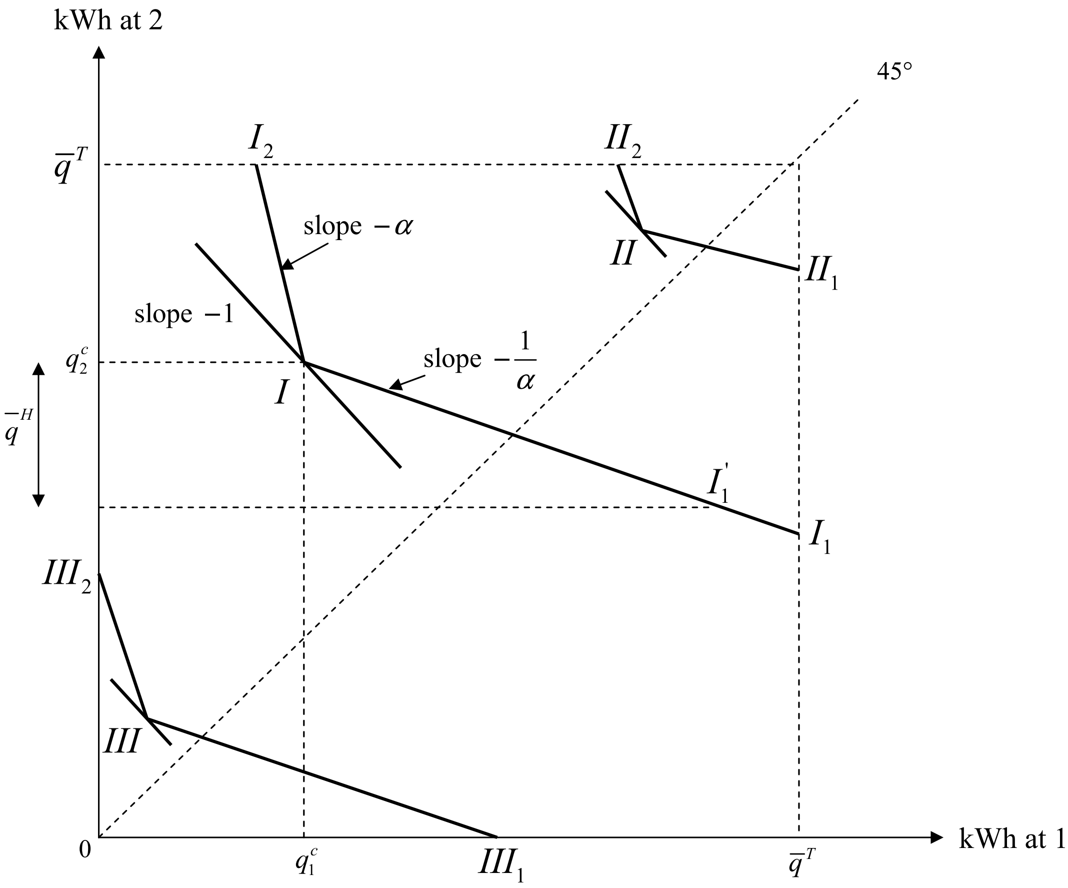

Figure 1: Technical transfers of energy

How can the electric system supply a bundle such as point I in Figure 1. It could be first supplied by using only the thermal system: $q_t^c = q_t^T, t = 1, 2$, that is point I itself. But it can also be supplied by exploiting both the thermal system and the hydro system, either by producing $q_1^T > q_1^c$ and keeping $q_1^T - q_1^c$ for stockpiling water into the dams to obtain an hydro production $q_2^H = \frac{1}{\alpha}(q_1^T - q_1^c)$ in period 2, or symmetrically by producing $q_2^T > q_2^c$ and stockpiling $q_2^T - q_2^c$ to obtain an hydro production $q_1^H = \frac{1}{\alpha}(q_2^T - q_2^c)$ in period 1. But because 2 is the peak period, the latter cannot be efficient and we will skip explanations on this technical possibility.

In Figure 1 the possibility of a thermal system producing more than q_1^c in period 1, is illustrated by the line between points I and I_1. The slope of this line is equal to $-1/\alpha$, less than 1 in absolute value. At I_1, on the vertical through $\bar{q}^T$, the capacity of the thermal system in period 1 is saturated because $q_1^T = \bar{q}^T$. This is implicitely assuming that the capacity of the dams, $\bar{q}^H$, is larger than the stockpiled energy $\frac{1}{\alpha}(\bar{q}^T - q_1^c)$. But it could happen that the capacity of the dams be first saturated when the thermal production of period 1 is increased. If it were the case, the transformation possibility line would be shorter, for example from I to I_1' instead of to I_1. At I_1' the difference between q_2^c and the thermal production in period 2 at I_1' is precisely equal to $\bar{q}^H$.

Clearly for (q_1^c, q_2^c) sufficiently high like point II in Figure 1, the capacity of the thermal system is the limiting factor to the use of the hydro system: indeed point II_1 is located on the vertical through $\bar{q}^T$. But for (q_1^c, q_2^c) sufficiently low like point III, the capacity of the thermal system is never the limiting factor: in effect given $\bar{q}^T$, III_1 is located on the horizontal axis.

2.2.2 Least-cost dispatch

For a given bundle (q_1^c, q_2^c) to supply, what is the efficient (leas-cost) way to proceed? The two possibly efficient ways are illustrated in Figures 2 and 3 where the straight lines depict the hydro possibilities like in Figure 1, whereas the convex decreasing curves stand for the thermal iso-cost curves.

In the case illustrated in Figure 2 the best to do is to produce exclusively with the thermal system the consumption to supply in each period: $q_t^T = q_t^c, t = 1, 2$. The reason is that at $I = (q_1^c, q_2^c)$, the iso-cost curve of the thermal system though I is in absolute value, higher than $1/\alpha$ and smaller than α. Hence the iso-cost curves through the points of the segment $]I, I_1]$ or the points of the segment $]I, I_2]$ correspond to higher costs, like the iso-cost curve through I'.

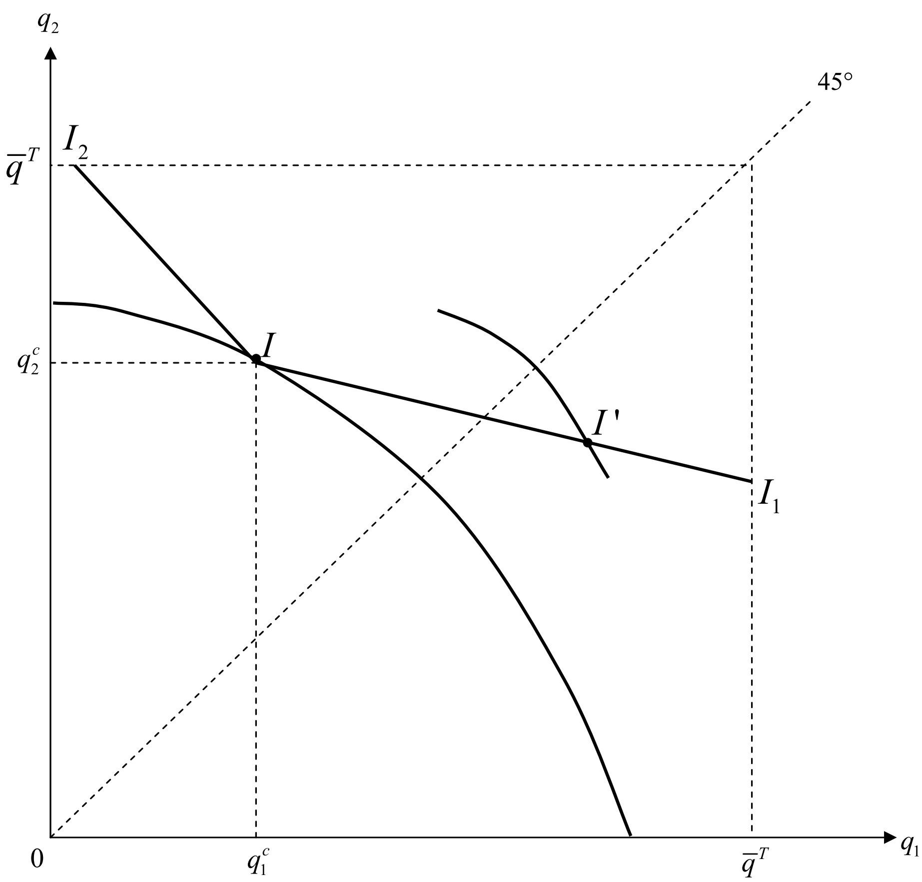

Figure 2: Efficient dispatch without pumping

Economically, this situation is such that producing (q_1^c, q_2^c) costs less than transfering some water from period 1 to period 2, i.e. $\Delta C \stackrel{def}{=} C(q_1^c + \Delta q_1, q_2^c + \Delta q_2) - C(q_1^c, q_2^c) > 0$ where $\Delta q_1 > 0$ and $\Delta q_2 = -\dfrac{\Delta q_1}{\alpha}$. Therefore $\lim\limits_{\Delta q_1 \to 0} \dfrac{\Delta C}{\Delta q_1} = \dfrac{\partial C}{\partial q_1} + \dfrac{\partial C}{\partial q_2}\dfrac{\Delta q_2}{\Delta q_1} = C_1 - \dfrac{C_2}{\alpha} > 0$, that is $\dfrac{C_1}{C_2} > \dfrac{1}{\alpha}$.

By contrast in Figure 3 the best solution is to use the hydroelectric system. The reason is that at $I = (q_1^c, q_2^c)$ the slope of the iso-cost curve of the thermal system through I is, in absolute value lower than the hydro transformation cost, namely $\dfrac{C_1}{C_2} < \dfrac{1}{\alpha}$. Then $C(q_1^c, q_2^c) > C(q_1^c + \Delta q, q_2^c - \dfrac{\Delta q}{\alpha})$ as long as $\Delta q > 0$ is not too large.

Let us denote by I'' the point at which the iso-cost curve through I intersects the segment $]I, I_1]$. Through any point of the interval $]I, I''[$ there exists an iso-cost lower than the cost corresponding to the iso-cost curve through I. The lowest cost is reached at the thermal production bundle corresponding to point I''' of $]I, I''[$ at which the cost to substitute 1 unit of energy of period 2 for 1 unit of energy of period 1 is the same with the hydro system and with the thermal system. Graphically an iso-cost curve is tangent to the segment $[I, I_1]$, provided that the required use of the hydro

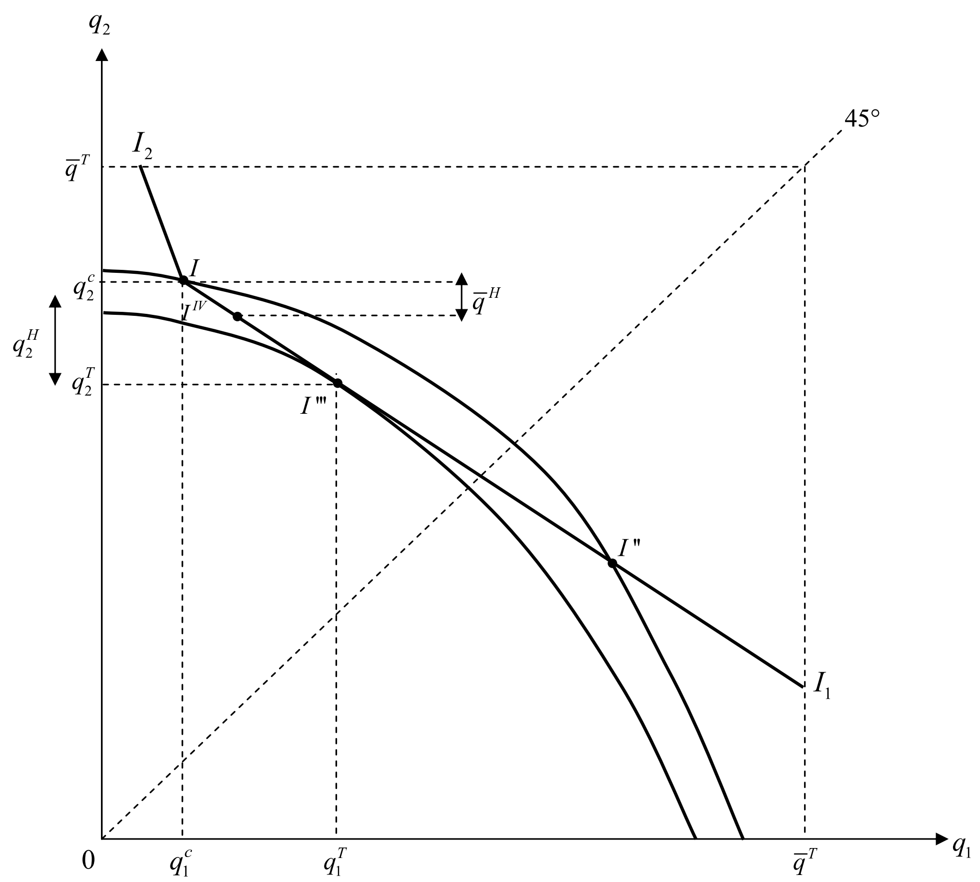

Figure 3: Efficient dispatch with pumping

system be not higher than its capacity: $\dfrac{1}{\alpha}(q_1^T - q_1^c) \leq \bar{q}^H$, where q_1^T is the thermal production of period 1 corresponding to I'''. If $q_2^H = \dfrac{1}{\alpha}(q_1^T - q_1^c)$ is higher than $\bar{q}^H$, the best is to choose the bundle of the interval $]I, II'''[$ at which the capacity constraint of the hydro system is saturated, point I^{IV} in Figure 3.

Another point to be noticed is that, when $q_2^c > q_1^c$, the efficient production bundle of the thermal system cannot be located on the segment $]I, I_2]$. The reverse holds when $q_2^c < q_1^c$ in which case the efficient thermal production bundle cannot lie on the segment $]I, I_1]$. We will not consider this case hereafter since it is similar to the one we analyze.

The last point to be noticed is that for consumption bundles (q_1^c, q_2^c) along the main diagonal, $q_1^c = q_2^c$, the slope of the thermal iso-curves is equal to -1, whatever $q_1^c = q_2^c \leq \bar{q}^T$. Thus any move either along $]I, I_1]$ (see I' in Figure 2) would induce higher costs so that the best solution is to use only the thermal system: $q_t^T = q_t^c, t = 1, 2$. A straightforward continuity argument suggests that the same holds for asymmetrical consumption bundles: $(q_1^c, q_2^c), q_1^c \neq q_2^c$, provided that the difference $|q_1^c - q_2^c|$ be not too high. In words, pumped storage is inefficient where peak demand and off-peak demand are not very different.

2.2.3 The efficient frontier

The locus of most asymmetrical consumption bundles for which efficiency requires to keep idle the hydro system is determined as follows. Let us consider consumption bundles $(q_1^c, q_2^c) : q_2^c > q_1^c$. Assume first that for any such bundle the slope of the iso-cost curve of the thermal system is, in absolute value, higher than $1/\alpha$. Then it is never efficient to use the hydro system according to the argument developped above.

Assume now that there is a point (q_1^c, q_2^c) located strictly within the set of the thermally feasible frontier like I' in Figure 4 at which the slope of the iso-cost curve is precisely equal to $-1/\alpha$. At any point on this iso-cost curve between I' and the vertical axis, the slope of the iso-cost curve is in absolute value lower than $1/\alpha$, thanks to the convexity of C. Thus for such consumption bundles the best is to use the hydro system as explained above. At any point on this iso-cost curve between I' and the 45^o line, the slope of the iso-cost curve is in absolute value higher than $1/\alpha$, thanks again to the convexity of C. Thus for such bundles the best is to keep idle the hydro system. This is the key argument of the zoning.

Proposition 1 *Let $q_2^E(q_1)$ be the locus of points such that $q_2 > q_1$ and $C_1(q_1, q_2)/C_2(q_1, q_2) = 1/\alpha$. Then all the thermally feasible bundles (q_1, q_2) located above this frontier $q_2^E(q_1)$ require the use of the hydro-system and all the points under this frontier require to keep idle the hydro system.*

Since the curve $q_2^E(q_1)$ is implicitely defined by:

$$\frac{C_1(q_1, q_2)}{C_2(q_1, q_2)} = \frac{1}{\alpha},$$

by differentiation, we get:

$$\frac{dq_2^E}{dq_1} = \frac{C_{11} - \frac{1}{\alpha} C_{21}}{\frac{1}{\alpha} C_{22} - C_{12}}$$

Thus, provided that the cross effects $C_{tt}, t, t' = 1, 2$ and $t \neq t'$, be sufficiently small with respect to the direct effects[4] $C_{tt}, t = 1, 2$, we obtain:

$$\frac{dq_2^E}{dq_1} > 0.$$

It is always the case for separable functions $C(q_1^T, q_2^T) = c(q_1^T) + c(q_2^T)$.

Note that, depending on the shape of the thermal cost function $C(.,.)$ the Hydro/No-Hydro frontier $q_2^E(q_1)$ can intersect the vertical axis like in Figure 4 or start from the origin. Also, note that there is another frontier $q_1^E(q_2)$ in the zone below the 45^o line, but we neglect it because, as we will see in the next section, the relevant zone is above the 45^o line since 2 is the peak period.

3 Optimal integrated management

In the former section we have supposed that demand at both periods were exogenous and we have depicted the combination of technologies minimizing the cost to produce exogeneous quantities. Here, we first set the hypotheses on the demand side. Then we characterize the optimal management of a pure thermal system. Finally we examine when and how hydro facilities can improve the net surplus of the industry.

A well known result in economic analysis is that any optimal state of a system must be an efficient one. Thus having determined what would be the optimal state of a purely thermal system, the question to ask is: Is this no-hydro state efficient according to the criteria determined in the preceding section? If the answer is yes this state is the true optimal state of the complete system because the hydro system has not to be used at the optimum. Otherwise we must consider the hydro option.

[4] Recall that $C_{tt'}$ is negative (resp. positive) when the thermal outputs are complements (resp. substitutes).

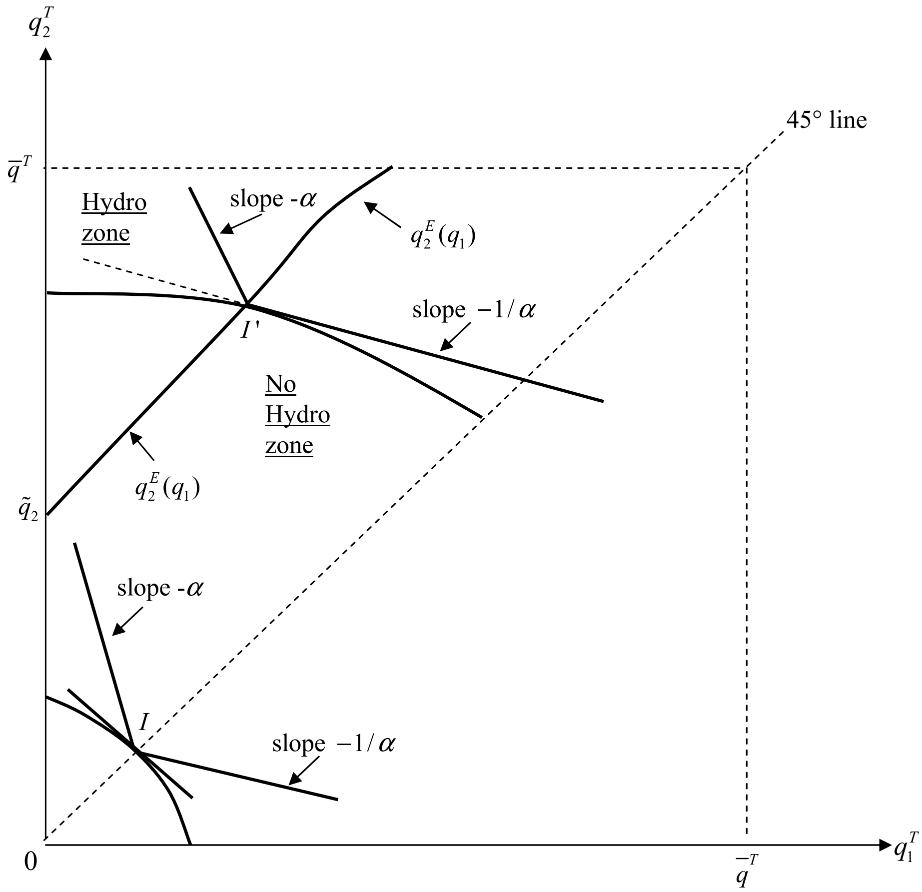

Figure 4: The pumped storage frontier

3.1 The demand side

Let $q_t^c, t = 1, 2$, be the electricity consumption in period t and $u(q_1^c, q_2^c)$ be the gross surplus measured in monetary units generated by a consumption bundle (q_1^c, q_2^c).

The gross surplus function is assumed to be three times continuously differentiable, strictly increasing and strictly concave. We denote by $u_t, u_{tt'}$ and $u_{tt't''}$ $(t, t', t'' = 1, 2)$ the partial first, second and third derivatives of u respectively.

The gross surplus function depends on the final users' activities and the easiness to transfer these activities or some parts of the activities from one period to the other. In turn this easiness is partly determined by the equipment endowment of the final users. The strict concavity assumption implies that $u(.,.)$ is strictly quasi-concave. Therefore along an iso-surplus curve transfers from one period to the other require an increasing total consumption of electricity $q_1^c + q_2^c$.

We characterize the fact that 2 is the peak period by

$$u(q + \Delta q, q) < u(q, q + \Delta q)$$
$$\text{for any} \quad q > 0 \quad \text{and any} \quad \Delta q > 0$$

so that:

$$u_1(q,q) = \lim_{\Delta q \downarrow 0} \frac{u(q+\Delta q, q) - u(q,q)}{\Delta q}$$
$$< \lim_{\Delta q \downarrow 0} \frac{u(q, q+\Delta) - u(q,q)}{\Delta q} = u_2(q,q)$$

Together with the concavity assumption, this condition implies that in the (q_1^c, q_2^c) space, when iso-surplus curves cross the 45^o line their slope is larger than -1.

Depending on the sign of $u_{tt'}, t \neq t'$, the consumptions can be gross substitutes ($u_{tt'} < 0$) or gross complements ($u_{tt'} > 0$). The first case refers to most usages of electricity by households who can switch on their electrical appliances such as washing machine or dish-washer at day or at night. Complementary rather concerns industrial users who need power at both periods.

A special case of gross surplus function is the separable function:

$$u(q_1^c, q_2^c) = v_1(q_1^c) + v_2(q_2^c)$$

where $v_t,\ t = 1,2$ are three times continuously differentiable functions, strictly increasing and strictly concave. Period 2 is the peak period provided that, for any $q > 0$ and $\Delta q > 0$

$$v_1(q+\Delta q) + v_2(q) < v_1(q) + v_2(q+\Delta q)$$

which implies that $v_1'(q) < v_2'(q)$.

3.2 The optimal dispatch without pumped storage

Absent the hydro system $q_t^T = q_t^c,\ t = 1,2$, hence the optimal dispatch is determined by solving the following problem (P.O.1).

$$\text{(P.O.1)} \qquad \max_{q_1^T, q_2^T} \quad u(q_1^T, q_2^T) - C(q_1^T, q_2^T)$$

$$s.t. \quad \bar{q}^T - q_t^T \geq 0 \quad \text{and} \quad q_t^T \geq 0 \quad , \quad t = 1,2$$

Assuming that the solution is interior, i.e. that the constraints are not binding, the first order condition are

$$u_t(q_1^T, q_2^T) = C_t(q_1^T, q_2^T) \quad , \quad t = 1, 2. \tag{3.1}$$

Then the marginal gross surplus u_t must be equal to the marginal cost C_t of the same period, and that for each period $t = 1, 2$. From this equality at $t = 1$ and $t = 2$, we can deduce

$$\frac{u_1(q_1^T, q_2^T)}{u_2(q_1^T, q_2^T)} = \frac{C_1(q_1^T, q_2^T)}{C_2(q_1^T, q_2^T)}.$$

Using this condition, we can check whether this dispatch is the true optimal state of the system or not. The test is the efficiency test. If the common slope of the iso-cost curve and the iso-surplus curve at the solution is, in absolute value, higher than or equal to $1/\alpha$ then it is not optimal to store water and (3.1) characterizes the true optimal state of the system. If this slope is lower than $1/\alpha$ then the hydro plants must be used and (3.1) does not determine the true optimal state.

Note that the test consists in checking the location of the point defined by (3.1) with respect to the curve $q_2^E(q_1)$ defined in the preceding section (see Figure 4). If the solution to (3.1) is located either on or under the $q_2^E(q_1)$ curve, then it is within the non-hydro zone and it is efficient. If the solution to (3.1) is located above the $q_2^E(q_1)$ curve, then it is in the hydro zone, thus it is not efficient.

3.3 The optimal dispatch with pumped storage

Here again we only propose intuitions and illustrations of the solution. The formal proof is given in Appendix B.

Now contrary to problem (P.O.1), we are not constrained by $q_t^c = q_t^T$ at $t = 1, 2$. On the contrary, the pumped-storage technology allows to disconnect consumption in one period from the thermal output of the same period.

The problem to solve is

$$\text{(P.O.2)} \quad \max_{\{(q_t^c, q_t^T)\ ,\ t=1,2\}} \quad u(q_1^c, q_2^c) - C(q_1^T, q_2^T)$$

$$s.t. \quad \begin{aligned} & q_1^T - q_1^c \geq 0 \\ & q_2^T + \frac{1}{\alpha}(q_1^T - q_1^c) - q_2^c \geq 0 \\ & \bar{q}^H - \frac{1}{\alpha}(q_1^T - q_1^c) \geq 0 \\ & q_t^c \geq 0 \ , \quad \bar{q}^T - q_t^T \geq 0 \quad \text{and} \quad q_t^T \geq 0 \ , \quad t = 1, 2. \end{aligned}$$

The solution to (P.O.2) combines the efficiency properties described in section 2 and the optimality properties of section 3.2. The solution[5] to P.0.2 is (q_1^{c*}, q_2^{c*}) and (q_1^{T*}, q_2^{T*}) such that

Proposition 2

$$u_1(q_1^{c*}, q_2^{c*}) = C_1(q_1^{T*}, q_2^{T*}) = \frac{1}{\alpha} u_2(q_1^{c*}, q_2^{c*}) = \frac{1}{\alpha} C_2(q_1^{T*}, q_2^{T*})$$

$$\textit{and} \quad q_2^{c*} - q_2^{T*} = \frac{1}{\alpha}(q_1^{T*} - q_1^{c*}).$$

The equality of marginal utility and marginal cost at each period is the standard condition for the maximization of instantaneous surplus (like in 3.1). Similarly, the equality between the marginal rates of transformation of one good (period 1 electricity) into another good (period 2 electricity) for all available technologies is the standard condition for efficiency, the only originality here is that we speak of intertemporal efficiency (like en section 2). Both must be satisfied at the solution of P.0.2 as it is illustrated in Figure 5.

Note that this first best dispatching can be implemented by prices reflecting the marginal value of the energy as long as consumers and producers are price-takers. These prices are such that $p_2^* = \alpha p_1^*$, which means that we still have peak-load pricing, but the difference in prices is smaller than if there is no pumped storage.

In the case where the hydro plant has a capacity too small to accommodate the optimal quality of water pumped at period 1, $q_1^{T*} - q_1^{c*}$, the inflow is constrained to be $\bar{q}^H$. It remains true that marginal utility is equal to

[5] We limit the analysis to the case where the capacity limits are not binding at the solution. Therefore the solution is internal. The formal proof of Proposition 2 is in Appendix B.

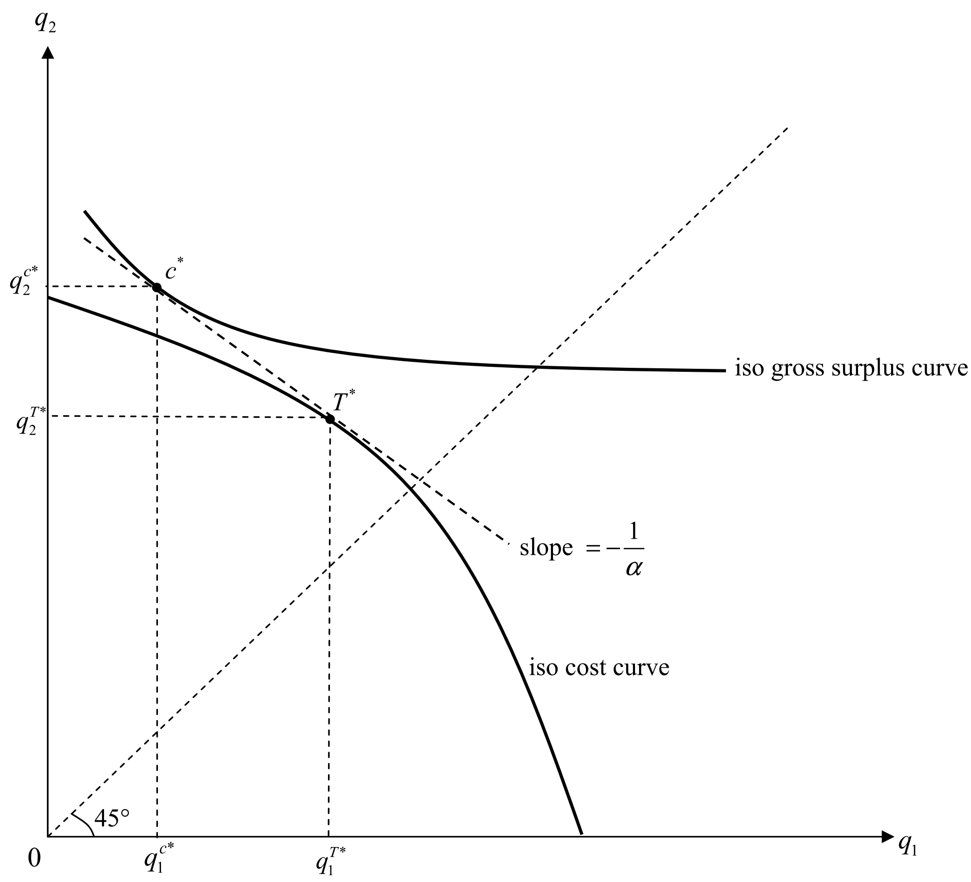

Figure 5: Optimal dispatch with pumped-storage

marginal cost at each period, but the equality of the marginal rates of transformation is lost. In terms of Figure 5 it means that the horizontal distance between the constrained pair of consumption $\hat{c}$ and the constrained pair of thermal production $\hat{T}$ is equal to $\bar{q}^H$, that the slope of the iso-surplus curve at $\hat{c}$ and the slope of the iso-cost curve at $\hat{T}$ are equal, but they are larger than $-\dfrac{1}{\alpha}$. The cost of this shortage in hydro capacity is that the constrained $\hat{c}$ is on an iso-surplus curve lower than c^* and the constrained $\hat{T}$ is on an iso-cost curve higher than T^*.

4 Separable utility and separable cost

Assume that $u(q_1^c, q_2^c) = v_1(q_1^c) + v_2(q_2^c)$ and $C(q_1^T, q_2^T) = c(q_1^T) + c(q_2^T)$. This simple case of separable functions combined with the hypothesis of stationary cost allows to better illustrate the social gain from pumped storage.

It is easy to check from Proposition 2 that the optimal solution must satisfy

- if $v_2'(q_{2i}) > \alpha c'(q_{1i})$, then:

$$q_2^H > 0 \ \text{ and } \ q_1^H = 0$$

$$v_1'(q_1^T + \alpha q_2^H) = c'(q_1^T + \alpha q_2^H)$$
$$v_2'(q_2^T + q_2^H) = c'(q_2^T) = \alpha c'(q_1^T + \alpha q_2^H)$$

- if $v_2'(q_{2i}) \leq \alpha c'(q_{1i})$, then:

$$q_1^H = q_2^H = 0 \text{ and } q_t^T = q_{ti}\ ,\ t = 1, 2$$

where $q_{ti}(t = 1, 2)$ is the output and the consumption of period t absent the hydro complement (see above, section 3.2).

In effect, as we saw in section 2, for a given pair of final consumptions, depending on the cost function and the loss coefficient α, the optimal solution is either without hydrogeneration or a mix of hydro and thermal generation. The higher α, the more likely the no-hydro solution.

Starting from the only-thermal solution (q_{1i}, q_{2i}), we can define the threshold value

$$\hat{\alpha} = \frac{c'(q_{2i})}{c'(q_{1i})}$$

such that if $\alpha \geq \hat{\alpha}$ the no-hydro dispatch is optimal and if $\alpha < \hat{\alpha}$, the best dispatch commands to mix the two technologies. Clearly if $\alpha c'(q_{1i}) < c'(q_{2i})$, consuming the same quantity at each period but producing some part q_2^H of q_{2i} in period 1, that is producing thermally $q_{1i} + \alpha q_2^H$ in period 1 and $q_{2i} - q_2^H$ in period 2 would reduce the total cost provided that q_2^H is not too large. This is the case illustrated in Figure 6. It corresponds to the first part of the optimal dispatch above. Symmetrically when $\alpha c'(q_{1i}) > c'(q_{2i})$ it is better not to use the hydro system. Equivalently for a given loss index α and a given cost function c, if the difference between v_1 and v_2, and v_1' and v_2' is not too large so that $q_{2i} - q_{1i}$ is small, then the hydro transfer system is useless; and for a given loss index α and a given utility function, if the marginal cost is slowly increasing, again the hydro system is not used. This case corresponds to the second part of the optimal dispatch above.

The first best social gain from pumped storage is also illustrated in Figure 6. The solution in a temporally isolated pure thermal system is given by points A_t $(t = 1, 2)$, for period t. Note that we have $q_1^T < q_{1i}$ that is the first best consumption in period 1 is lower than the consumption in period 1 in the temporally isolated pure thermal system, whereas the consumption in period 2, the peak period, is higher $q_2^T + q_2^H > q_{2i}$.

But the thermal production in period 1 is higher, and in period 2 is lower than in the pure thermal system. Compared to the pure thermal system there is a welfare loss in period 1, measured by the shaded area 1, more than compensated by a welfare gain in period 2, measured by the shaded area 2. If marginal utility is very inelastic, welfare gains from pumping are essentially technical. Otherwise, consumers incur a loss at night but their surplus is increased at day.

Nuclear plants provide another interesting polar case. In effect, they have very low flexibility, mainly for safety reasons which means that the iso-cost curves of figures 2 to 5 are just a point on the 45^o line. Starting from this initial point any transfer of electricity from one period to the other has an infinite cost whereas pumping water has a finite cost. Therefore, when nuclear plants are installed, pumped storage is always profitable on technical grounds. But efficiency does not mean optimality. We can use a simplified version of Figure 6 to identify a case where pumping is obviously efficient. Assume we have a nuclear plant with a constant production q_i. It means that in Figure 6 $q_{1i} = q_{2i} = q_i$ is exogeneous. Then, if $q_1^{max} = arg\{v_1'(q_1^c) = 0\} < q_i$, it is obviously optimal to transfer $\dfrac{q_i - q_1^{max}}{\alpha}$ for an extra consumption at period 2. More generally, when $v_1'(q_1^c)$ intersects the horizontal axis on the right of q_i, as long as $v_1'(q_i) < v_2'(q_i)$ some energy transfer is optimal because the surplus lost off-peak is lower than the surplus gained at peak hours.

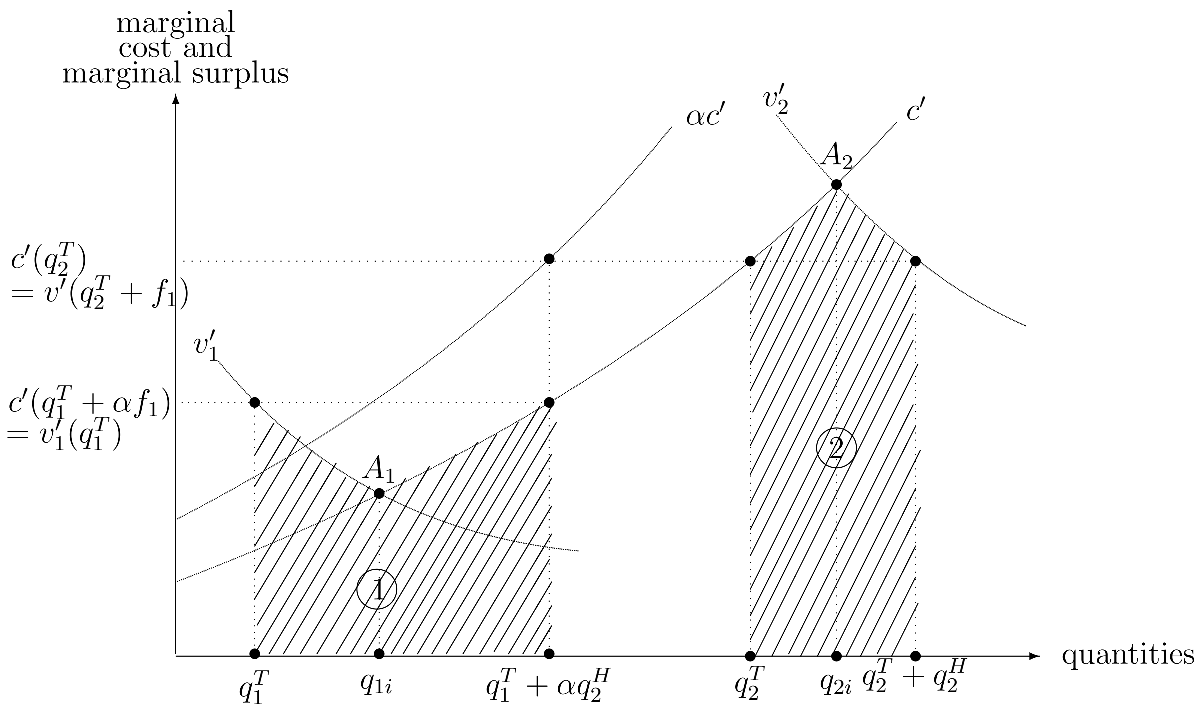

Figure 6: The welfare gains from pumping

5 Extensions

The basic model used in the former sections can be developed in several directions. Hereafter we give some hints on three of them: i) staircase thermal marginal cost and decreasing returns in hydrogeneration; ii) intermittent sources of energy and iii) market implementation.

5.1 Technologies

Instead of continuous differentiable cost functions, electricity-generating plants are characterized by piecewise functions where the unit variable cost is constant (equal to the cost of fuel) up to the production capacity of the plant and then becomes infinite as it is impossible to produce beyond capacity. Aggregating all the thermal production plants results in the so-called merit order, that is a staircase function made of the least unit-cost plant up to capacity, then the second least unit-cost plant up to capacity, and so on. This does not change drastically our former results as shown by the following example. Suppose that the thermal cost is separable and stationary and that marginal cost to produce q can be written $c'(q) = c_0 + cq$. Under this specification, it is easy to derive the functional form of the frontier between the hydro zone and the no-hydro zone: $q_2^E(q_1) = (\alpha - 1)\dfrac{c_0}{c} + \alpha q_1$. Therefore, the staircase can be obtained from $c \to 0$ for $q_2 \leq \bar{q}^T$. This means that, if there is one single thermal plant, the smooth increasing function $q_2^E(q_1)$ of figure 4 must be replaced by an horizontal line at $q_2 = \bar{q}^T$. If there are two thermal plants a and b where a is the one with the smaller variable unit cost, the frontier between hydro and no hydro is a staircase with stairs equal to $\bar{q}_a^T$ and $\bar{q}_a^T + \bar{q}_b^T$. And so on if there are n thermal plants with different unit generation cost and capacity. It remains true that if the quantity to supply at period 2 is large enough as compared to the quantity of period 1, that is if (q_1^c, q_2^c) is located above the frontier, efficiency commands to use the pumped storage system.

The second possibility to generalize the technical setting of the model is to assume that the conversion coefficient α is not constant. In effect, in a given electric system, we can have several pumping installations, each with its own physical characteristics such as the height and the slope between successive reservoirs. This means that there is also a merit order for hydro plants and the production function of pumping is a piecewise linear convex function instead of being made of two segments like in figure 1. The consequence is that the optimal dispatch is no longer a bang-bang solution (hydro vs. non hydro) since it must include the number and identity of the plants operated.

5.2 Intermittent sources of energy

Instead of considering the case where demand is changing between period 1 and period 2, we can use a similar model to address the case where demand remains the same in time but the availability of the non hydro plant is not constant, essentially because *i*) 1 and 2 are states of nature instead of time periods and *ii*) the no-hydro technology is state dependant like solar or wind plants are.

Here again, the pumped storage technology is a natural complement of the intermittent technology in so far as it allows to supply demand even when the primary fuel is not available, thanks to the water stored in the state of nature where the wind is blowing or the sun is shining. Actually, the strong development of renewable sources of energy to produce electricity encouraged by the European authorities (see the 2007 Roadmap) raises several problems of reliability to producers and transporters of electricity. Since demanders do not want to depend on the availability of intermittent sources, it is necessary to install back-up plants, called into operation when there is not enough wind or sun. If back-up plants burn coal or fuel, a part of the environmental benefits of green energy is lost. By contrast, the pumped storage technology only requires building costs. Except for the risk of damaging the landscape, the pumped storage technology should then be promoted together with the wind and solar technologies in the electricity industry.

5.3 Market implementation

Another natural extension of the model consists in deriving demand functions from the utility function and analyzing the behavior of profit maximizers supplying these demands. The structure of the industry will obviously influence the outcome and by how much it departs from first best. For example if all plants are under the control of the same profit maximizer, the total output will be decreased at all periods but, given these levels of output, the combination of thermal and hydro sources will be efficient since the monopolist earns more profits when it minimizes the cost of a given vector of outputs.

The outcome of a duopoly where the ownership of the thermal plant and the hydro plant are separated is less obvious. In effect, the thermal producer is successively a provider at period 1 and a competitor at period 2 of the hydro producer. Consequently, depending on the closed loop or the open loop structure of the model, the thermal producer will manage its plant differently, probably restricting its output at the off peak period when it internalizes the fact that it is supplying an input to its competitor for period 2. If it appears that the integration of the two technologies is welfare enhancing by preventing this type of opportunism, competition should be organized between integrated oligopolists. On practical grounds, the policy of encouraging intermittent producers without any obligation of investing in back-up equipments, in particular in pumped storage stations, should be revised.

6 Conclusion

A the time where global warming is a major concern in the public opinion, it would be desirable that governments design a coherent energy policy, including all possibilities of storing electricity at a competitive cost. Storage is feasible with pumping stations. Pumped storage is a potential efficient driver of energy saving when coupled with inflexible installations (such as thermal plants) or intermittent installations (such as windmills and solar batteries).

The model developped in this paper provides a simple framework to assess when pumped storage is efficient and when it should be optimally dispatched. It recalls that it is neither the electricity saved nor the cost saved at peak hours that should drive the decision on water pumping. The social profitability index should be the net increase in surplus, that is the difference between the larger gross surplus of an increased quantity consumed at peak period but partially produced at low cost at off peak periods less the welfare loss of the off peak period where more is produced than consumed. How this optimal use of pumped storage can be implemented in the competition framework sustained by governments in industrialized countries remain to be analyzed.

7 Appendix A: Efficient dispatch

The different cases illustrated in Figures 2-4 are the solution of the program (P.E) below. To simplify the proof we assume that $q_2^c > q_1^c$, which requires $q_1^T \geq q_1^c$.

$$
\begin{aligned}
\text{(P.E)} \quad \max_{q_1^T, q_2^T} \quad & -C(q_1^T, q_2^T) \\
s.t. \quad & q_1^T - q_1^c \geq 0 \\
& q_2^T + \frac{1}{\alpha}(q_1^T - q_1^c) - q_2^c \geq 0 \\
& \bar{q}^H - \frac{1}{\alpha}(q_1^T - q_1^c) \geq 0 \\
& \bar{q}^T - q_t^T \geq 0 \quad \text{and} \quad q_t^T \geq 0 \quad , \quad t = 1, 2.
\end{aligned}
$$

Let L^E be the Lagrangian of the problem :

$$
\begin{aligned}
L^E = & -C(q_1^T, q_2^T) + \gamma_1^c[q_1^T - q_1^c] + \gamma_2^c[q_2^T + \frac{1}{\alpha}(q_1^T - q_1^c) - q_2^c] \\
& + \gamma^H[\bar{q}^H - \frac{1}{\alpha}(q_1^T - q_1^c)] + \sum_{t=1,2} \{\bar{\gamma}_t^T[\bar{q}^T - q_t^T] + \underline{\gamma}_t^T q_t^T\}
\end{aligned}
$$

where the γ's are the Kuhn-Tucker multipliers corresponding to the inequality constraints.

The first order conditions are:

$$
\frac{\partial L^E}{\partial q_1^T} = 0 \quad \Rightarrow \quad C_1(q_1^T, q_2^T) = \gamma_1^c + \frac{1}{\alpha}[\gamma_2^c - \gamma^H] - \bar{\gamma}_1^T + \underline{\gamma}_1^T
$$

$$
\frac{\partial L^E}{\partial q_2^T} = 0 \quad \Rightarrow \quad C_2(q_1^T, q_2^T) = \gamma_2^c - \bar{\gamma}_2^T + \underline{\gamma}_2^T,
$$

together with the usual complementary slackness conditions.

Assume that the solution is such that $0 < q_t^T < \bar{q}^T$ so that $\bar{\gamma}_t^T = \underline{\gamma}_t^T = 0$ $(t = 1, 2)$. We may have three types of solution.

Type 1: The hydro system is used but not at its full capacity $\bar{q}^H$. It implies that $q_t^T - q_1^c > 0$, hence $\gamma_1^c = 0$ together with $\gamma^H = 0$, so that:

$$
\frac{C_1(q_1^T, q_2^T)}{C_2(q_1^T, q_2^T)} = \frac{1}{\alpha}.
$$

This is the internal solution illustrated by I''' In Figure 3.

Type 2: The hydro system is used at full capacity.

Now $\gamma_1^c = 0$ but γ^H is generically positive, hence:

$$\frac{C_1(q_1^T, q_2^T)}{C_2(q_1^T, q_2^T)} = \frac{1}{\alpha}\left[1 - \frac{\gamma^H}{\gamma_2^c}\right] \leq \frac{1}{\alpha}.$$

This is the solution illustrated by I^{IV} in Figure 3. The iso-cost curve through I^{IV} has not been drawn to simplify the figure. At this point the slope of the iso-cost curve is, in absolute value, lower than $1/\alpha$.

Type 3: The hydro system is not used so that $q_t^T - q_t^c = 0, t = 1, 2$.

Now γ_t^c is generically positive at $t = 1, 2$ and since the hydro system is not used $\gamma^H = 0$, so that:

$$\frac{C_1(q_1^T, q_2^T)}{C_2(q_1^T, q_2^T)} = \frac{1}{\alpha} + \frac{\gamma_1^c}{\gamma_2^c} \geq \frac{1}{\alpha}.$$

This is the solution illustrated by I in Figure 2. The slope of the iso-cost curve through I is, in absolute value, higher than $1/\alpha$.

Other corner cases can be listed (for example when $q_1^c = 0$ and/or $q_2^c = \bar{q}^T$) but they are of minor interest and therefore we will not analyze them.

8 Appendix B: Optimal dispatch

Assume now that the efficiency test has shown that the hydro plants must be used. Since the optimal program is efficient, given the asymmetrical structure of the surplus function and the symmetrical structure of the cost function, the optimal scheme must be located above the 45^o line in the (q_1, q_2) space with $q_1^T - q_1^c > 0$. Therefore, the problem to solve is now (P.O.2)

$$(\text{P.O.2}) \qquad \max_{\{(q_t^c, q_t^T),\ t=1,2\}} \quad u(q_1^c, q_2^c) - C(q_1^T, q_2^T)$$

$$s.t. \qquad q_1^T - q_1^c \geq 0$$

$$q_2^T + \frac{1}{\alpha}(q_1^T - q_1^c) - q_2^c \geq 0$$

$$\bar{q}^H - \frac{1}{\alpha}(q_1^T - q_1^c) \geq 0$$

$$q_t^c \geq 0 \quad , \quad \bar{q}^T - q_t^T \geq 0 \quad \text{and} \quad q_t^T \geq 0 \quad , \quad t = 1, 2.$$

Let L^0 be the Lagrangian of the problem:

$$\begin{aligned} L^0 = \; & u(q_1^c, q_2^c) - C(q_1^T, q_2^T) + \gamma_1^c[q_1^T - q_1^c] \\ & \gamma_2^c[q_2^T + \frac{1}{\alpha}(q_1^T - q_1^c) - q_2^c] + \gamma^H[\bar{q}^H - \frac{1}{\alpha}(q_1^T - q_1^c)] \\ & + \sum_{t=1,2} \underline{\gamma}_t^c q_t^c + \sum_{t=1,2} \left\{ \bar{\gamma}_t^T(\bar{q}^T - q_t^T) + \underline{\gamma}_t^T q_t^T \right\} \end{aligned}$$

where the $\gamma's$ are the Kuhn-Tucker multipliers associated to the inequality constraints.

The first order conditions are:

$$\frac{\partial L^0}{\partial q_1^c} = 0 \quad \Rightarrow \quad u_1(q_1^c, q_2^c) = \gamma_1^c + \frac{1}{\alpha}[\gamma_2^c - \gamma^H] - \underline{\gamma}_1^c$$

$$\frac{\partial L^0}{\partial q_2^c} = 0 \quad \Rightarrow \quad u_2(q_1^c, q_2^c) = \gamma_2^c - \underline{\gamma}_2^c$$

$$\frac{\partial L^0}{\partial q_1^T} = 0 \quad \Rightarrow \quad C_1(q_1^T, q_2^T) = \gamma_1^c + \frac{1}{\alpha}[\gamma_2^c - \gamma^H] - \bar{\gamma}_1^T + \underline{\gamma}_1^T$$

$$\frac{\partial L^0}{\partial q_2^T} = 0 \quad \Rightarrow \quad C_2(q_1^T, q_2^T) = \gamma_2^c - \bar{\gamma}_2^T + \underline{\gamma}_2^T$$

together with the usual complementary slackness conditions. Note that the two last conditions are nothing but the efficiency conditions examined in

Appendix A but here for the optimal consumption bundle rather than for an arbitrary bundle.

Assume that the optimal solution is an interior one: $q_t^c > 0, \bar{q}^T - q_t^T > 0$ and $q_t^T > 0, t = 1, 2$, so that the corresponding multipliers are nil. Because the hydro plants must be active, we have that $q_1^T - q_1^c > 0$, and $\gamma_1^c = 0$. Thus we are left with two cases depending upon the fact that the capacity of the hydro plant is saturated or not.

<u>The hydro plants capacity is not saturated</u>

In this case $\gamma^H = 0$, and given that all the other multipliers are nil, we must have:

$$u_1(q_1^c, q_2^c) = \frac{1}{\alpha}\gamma_2^c \quad \text{and} \quad u_2(q_1^c, q_2^c) = \gamma_2^c$$

$$C_1(q_1^T, q_2^T) = \frac{1}{\alpha}\gamma_2^c \quad \text{and} \quad C_2(q_1^T, q_2^T) = \gamma_2^c.$$

These four equations, joint with the energy balance $q_2^c - q_2^T = \frac{1}{\alpha}(q_1^T - q_1^c)$ determine the optimal bundles $(q_1^{c*}, q_2^{c*}), (q_1^{T*}, q_2^{T*})$, and the energy marginal value γ_2^{c*}.

<u>The hydro plants capacity is saturated</u>

In this case, γ^H is generically positive. The solution is given by solving:

$$u_1(q_1^c, q_2^c) = \frac{\gamma_2^c - \gamma^H}{\alpha} = c_1(q_1^T, q_2^T)$$

$$u_2(q_1^c, q_2^c) = \gamma_2^c = c_2(q_1^T, q_2^T)$$

$$q_1^c - q_1^T = \bar{q}_H \quad , \quad q_2^T - q_2^c + \frac{1}{\alpha}(q_1^T - q_1^c) = 0.$$

given the value of the hydroplant capacity $\bar{q}_H$.

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