View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by Research Papers in Economics

THE UNIVERSITY OF ADELAIDE AUSTRALIA

DOING WITHOUT MONEY: A CRITICAL ASSESSMENT OF WOOLFORD'S ANALYSIS OF MONETARY POLICY IN A POST-MONETARY WORLD

Colin Rogers

Working Paper 03-1

SCHOOL OF ECONOMICS
University of Adelaide
SA 5005 AUSTRALIA

ISSN 1444 8866

DOING WITHOUT MONEY: A CRITICAL ASSESSMENT OF WOOLFORD'S ANALYSIS OF MONETARY POLICY IN A POST-MONETARY WORLD

Colin Rogers

ABSTRACT

Woodford (1997, 1998) employs an inter-temporal general equilibrium model to examine the properties of the monetary system as it evolves form the use of a physical medium of exchange to an electronic medium. He presents a structure in which cash (money) as a means of payment can be made to vanish at the limit but where he claims the price level remains determinate. It is shown that Woodford's analysis involves fundamental but still widespread conceptual and methodological errors. His general equilibrium model does not map into the world of electronic money and his analysis has no implications for the art of central banking.

JEL Classifications: B40, E40, E42, E50

Keywords: Electronic Money, Walrasian General Equilibrium

Contact author:
Colin Rogers
School of Economics
University of Adelaide
SA 5005 AUSTRALIA
Tel: +61 8 8303 5528
Fax: +61 8 8223 1460
colin.rogers@adelaide.edu.au

I Introduction

The electronic revolution in banking has thrown up challenges for regulators and at the same time raised some interesting questions about the fundamentals of monetary theory. Of particular interest is Woodford's (1997, 1998)[1] model that is intended to shed light on the properties of a monetary economy as it evolves from the use of a physical medium of exchange (cash) to an electronic medium. This is an important area of research and has obvious implications for the implementation of monetary policy in the face of the electronic revolution in the payments mechanism. Monetary theorists have recognised that we have to get this right if errors in policy and regulation are to be avoided (Green, 1999). Furthermore, the issue is of more than purely theoretical interest. Woodford's work has been cited with approval by Henckel, Ize and Kovanen (HIK) (1999, p. 31, fn 35). They claim that "…the path-breaking analysis in Woodford (1997), [….] shows that in a model where money vanishes a stable local equilibrium continues to exist for the rate of inflation even when money balances are zero". It is this sort of claim with which I wish to take issue.

In my view the HIK description of Woodford's results is incorrect. Woodford's analysis involves well-documented, if not well known, conceptual errors. Specifically, Woodford's analysis 'works' by sleight of hand[2] because it confounds the meaning of terms such as the price level (as measured by the CPI or GDP deflator) and nominal rates of interest, by attaching to them theoretical concepts that have fundamentally different properties. Consequently, nothing in Woodford's model maps to identifiable institutions or entities that would be recognisable to policy makers.

[1] All page references are to Woodford (1997)
[2] Apparently Woodford is not the first to commit this error. See the discussion of Fama (1980) in Hoover (1988).

In this note I focus on the conceptual and theoretical confusions that result from Woodford's analysis. Essentially, the difficulties for Woodford arise because he does not specify what a monetary economy is. A necessary requirement in this regard is that money exist and that it serve *all three functions*- as a unit of account (*numeraire*), store of value and medium of exchange. In Woodford's post-monetary world, by contrast, only the *numeraire* function exists. A survey of the issue by McCallum (1985) concluded that the three functions of money are non-separable in any workable monetary economy[3]. The absence of all three functions of money in any model reveals a deeper conceptual problem first identified by Frank Hahn (1965). Hahn pointed out that adding something called 'money' to any real general equilibrium system is an *inessential* addition in the sense that it is easy to show that a non-monetary equilibrium exists. Woodford's model has this property.

The remainder of the note is arranged as follows. Section II outlines Woodford's analysis. Section III outlines why 'money' in Woodford's model is an inessential addition in the sense of Hahn (1965). Section IV then briefly outlines the consequent conceptual errors that arise when Woodford's model is used to assess policy issues. Section V concludes.

II Woodford's post-monetary world

[3] Attempts to separate the functions of money have led to confusion elsewhere in the literature. See Rogers and Rymes (1998).

The model is based on an infinitely lived representative household-producer, a self-employed artisan, for example, and each artisan seeks to maximize lifetime utility. To introduce a role for money Woodford draws his inspiration from Lucas (1980) for the use of Clower's cash-in-advance constraint and from Lucas and Stokey (1987) for the variation on this theme based on the distinction between what he calls "cash" and "credit" goods. As is well known the cash-in-advance feature gives money a role in the model because in this sort of set-up artisans must hold cash even though it is dominated in rate of return by bonds (or capital).

But Woodford moves beyond the traditional cash-in-advance specification because his intention is to set up a payments technology that will allow him to shrink to zero the number of payments requiring cash settlement. If the price level remains determinate in such a situation then he will have achieved his goal of "..determining money prices of goods and services without any reference either to the money supply or to a money demand equation" Woodford (1997, p. 3). If successful this would be a truly remarkable achievement – the determination of money prices in a world in which money does not exist! It is my contention the Woodford has slipped into confusion at this point. Specifically he has confused the existence of a *numeraire* with the broader concept of money, particularly the existence of money as a means of payment or settlement. The latter accounts for the existence of the 'network externality' that attaches to money and explains why prices are quoted in the means of settlement. As we will see, in Woodford's set-up the *numeraire* can be selected arbitrarily because no means of payment is required.

To make the point I consider Woodford's payments system and then examine its implications under the cashless limit in two settings: (a) when something called 'money' is treated as exogenous and a determinant of the price level (one version of the traditional quantity theoretic view of monetary theory and policy), and, (b) a "Wicksellian" world, without 'money' where Woodford attempts to analyse how the interest rate is manipulated to achieve a price level or inflation target.

The payments system

Although there is an infinite planning horizon, time is divided into discrete periods. Each period, artisans must purchase two types of good each associated with it's own payment technology. One category of goods can be purchased on credit during the period and settled at the end of the period without the need for the buyer to hold cash balances. Woodford calls this "informal credit". The other category of goods is purchased with the second technology and requires the buyer to hold cash balances at the end of the period sufficient to cover all charges against his account (for the second category of goods only) during the period. Cash must then be used to discharge any outstanding debt at the beginning of the next period.

Note that only the second type of payments technology requires the artisan to hold cash balances during the period. As Woodford (1997, p. 10) explains, "Thus the second type of transactions, but not the first, require the buyer to maintain cash balances". In addition, the second technology is specified so that it contains a parameter that can be adjusted to zero. Setting the parameter to zero then eliminates both the second category of goods and the second payments technology. It also

obviously eliminates the cash-in-advance constraint and Woodford describes this case as the cashless limit. Thus in the cashless limit the only payments system which remains is "informal credit". Hence, it is essential to have a clear understanding of what "informal credit" is because it is the only element of the payments technology that remains when Woodford eliminates money from his model in what he calls the cashless limit.

But on this crucial issue Woodford is ambiguous to say the least. We know that in a monetary economy transactions can be settled in spot cash or deferred cash. In the latter case we say that purchases are made on credit and the associated debt must be serviced and/or settled using cash (money) at some future date. By contrast, in Woodford's model both types of transaction are said to be a form of credit but only one requires cash settlement. Woodford (1997, pp. 10) notes simply that "informal credit" must be settled by the end of the period but he does not specify how this is to be achieved. But how is informal credit settled if not by cash?[4] I return to this question below.

A quantity-theoretic world in a real general equilibrium setting

Woodford's quantity-theoretic version of the model - the version, in which *M* is treated as exogenous- can be described in terms of the following system of equations. (I follow Woodford's numbering for ease of comparison)

[4] In this regard, is the Lucas and Stokey distinction between credit and cash goods any assistance? Lucas and Stokey (1987) define 'credit goods' (goods which do not need to be paid for in cash) as 'non-market goods', such as leisure. This does not seem to be what Woodford has in mind. When he sets up the model we are left with the distinct impression that both types of goods are market goods that are distinguished only by the form of payments technology that must be used for their payment. But without further explanation we are left to surmise that these goods are exchanged somehow without the need for money (cash) or credit (as those two terms are usually understood).

$$R_t \geq 1 \tag{2.16}$$

$$\frac{M_t}{p_t} \geq \mu(R_t) y_t \tag{2.23}$$

$$\frac{u'(\omega(R_t) y_t)}{p_t} \theta_1 (R_t)^{\gamma-1} = \beta R_t \frac{u'(\omega(R_{t+1}) y_{t+1})}{p_{t+1}} \theta_1 (R_{t+1})^{\gamma-1} \tag{2.24}$$

$$\sum_{T=t}^{\infty} \beta^T u'(\omega(R_T) y_T) \omega(R_T) y_T < \infty \tag{2.26}$$

$$\lim_{T \to \infty} \beta^T u'(\omega(R_T) y_T) \theta_1 (R_T)^{\gamma-1} \frac{M_{T-1}}{p_T} = 0 \tag{2.27}$$

Woodford presents this system of equations as a complete set of equilibrium conditions that can be used to determine the sequence $\{p_t, R_t\}$. He requires, in addition that at least one of (2.16) and (2.23) holds with equality. A brief explanation of each equation follows.

Expression (2.16) states that the endogenous nominal *gross* interest rate, $R_t \geq 1$ and imposes the condition that the nominal *net* interest rate is always non-negative and is required to ensure that artisans' debt is bounded (Woodford, 1997, 16). Expression (2.23) is the cash-in-advance constraint where M_t and y_t are the exogenous money supply and known output in each period *t*. Expression (2.24) is the Euler equation which determines the optimal choice path for the representative artisan. Expression

(2.24) includes the term $\omega(R_t)$, which reflects the use of the payments system that requires cash settlement. Woodford introduces the parameter, α as the fraction of transactions requiring cash settlement. He then argues that as the cost of each 'informal credit" good is p_t the cost of each good subject to cash settlement is $R_t p_t$. Those goods requiring cash settlement involve an additional interest charge because of the credit extended.

Artisans purchase the same quantities, c_{1t}, of each "informal credit" good and, c_{2t}, of each cash settlement good. By definition $c_t = (1-\alpha)c_{1t} + \alpha c_{2t}$, and under market clearing conditions (assuming no government purchases) $(1-\alpha)c_{1t} = y_{1t}$ and $\alpha c_{2t} = y_{2t}$. Expressions (2.26) and (2.27) are boundary conditions where $0 < \beta < 1$ is a discount factor. Also, expression (2.26) requires that artisans' do not borrow more than they can possibly repay- based on their potential capacity to produce and maintain a minimum positive level of consumption (Woodford 1997, 11-12). Expression (2.27) is a transversality condition of a form required for an infinite planning horizon problem.

To model the post-monetary world Woodford generates the cashless limit by setting $\alpha = 0$ and examining whether it is still possible to determine the price level p_t. Accepting the analysis of the necessary mappings between infinite sequences, and that the required continuity conditions are met, it is possible to show that the first order non-linear difference equation which describes the equilibrium state of the model can determine p_t when $\alpha > 0$ (subject to the two boundary conditions). That is, writing the sequence of equilibrium conditions in general form as

$$\Phi(p; M, y; \alpha) = 0. \qquad (3.2)$$

Woodford shows that it is possible to find a function,

$$p = f(\alpha)\ , \qquad (3.4)$$

such that (3.2) holds so long as $\alpha > 0$. However, it is not possible to express the perfect foresight equilibrium price sequence as a continuous function, $p = f(\alpha)$ on a domain that includes $\alpha = 0$. Woodford (1997, p. 25) explains the intuition behind this result as follows: "This result makes intuitive sense, of course, from the point of view of the quantity-theoretic approach to price level determination; as the fraction of transactions that require cash falls to zero, desired real money balances fall to zero as well, and so one would expect the equilibrium price level to cease to be defined. The failure of the solution to be continuous near $\alpha = 0$ simply means that one cannot [derive] a solution for the limiting case as an approximation to the small- α case; instead the use of money in transactions is intrinsic to the model's ability to determine the price level".

All this is no doubt familiar to anyone acquainted with Patinkin (1965). The problems arise in Woodford's model when he attempts to circumvent this result. In that endeavour, Woodford is, quite rightly, not concerned with a world in which *M* is exogenous. He regards that case as of no practical importance because today central bankers make no attempt to render some *M* exogenous. Today's world of electronic banking has more in common with Wicksell than Friedman.

A "Wicksellian" world in a real general equilibrium setting

Woodford (1997, p. 27) suggests the following relationships as a complete set of necessary and sufficient conditions for sequences $\{p_t, R_t\}$ to constitute equilibrium in a model which is a proxy for a world in which central bankers manipulate a nominal interest rate to achieve a price level or inflation target.

$$R_t = \phi(p_t) \tag{3.7}$$

$$\frac{u'(\omega(R_t)y_t)}{p_t}\theta_1(R_t)^{\gamma-1} = \beta R_t \frac{u'(\omega(R_{t+1})y_{t+1})}{p_{t+1}}\theta_1(R_{t+1})^{\gamma-1} \tag{2.24}$$

$$\sum_{T=t}^{\infty} \beta^T u'(\omega(R_T)y_T)\omega(R_T)y_T < \infty \tag{2.26}$$

$$\lim_{T\to\infty} \beta^T u'(\omega(R_T)y_T)\theta_1(R_T)^{\gamma-1}\frac{W}{p_T} = 0 \tag{3.9}$$

Expression (3.7) is simply the monetary policy rule that links interest rates to prices (or rates of change in prices in the case of an inflation target). There is no change to expressions (2.24) or (2.26) but (3.9) does involve a re-specification of fiscal policy to incorporate a target level for the total nominal value of government liabilities, $W > 0$. This accounts for the appearance of W in expression (3.9). Expression (2.23), which

embodied the cash-in-advance feature of the previous model, is still implicit in the model in the sense that it could be used to determine the equilibrium path of the now endogenous money supply once the equilibrium sequence $\{p_t, R_t\}$ has been determined. (Presumably this is true only for those cases where $\alpha > 0$ because *M* disappears in the cashless limit as all payment is by "informal credit").

This model is essentially identical to the model with exogenous money (Woodford, 1997, pp. 28-29) but Woodford argues that it is *qualitatively* different in one important respect: in this version of the model the equilibrium equations remain well-behaved in the cashless limit. In other words, the function *f* in expression (3.4) is continuous in the cashless limit; ie., when $\alpha = 0$. Hence, the price sequence $p_t = \hat{p}(y_t, y_{t+1,}.....)$ defined by $\hat{p} = f(y,0)$ is a solution to the equilibrium conditions in the cashless limit ($\alpha = 0$). It also means that this solution approximates a perfect foresight equilibrium in the case of any parameter sequence in which α_t is small enough for all *t*.

In the cashless limit the equilibrium conditions in the "Wicksellian" version become:

$$\frac{u'(y_t)}{p_t \phi(p_t)} = \beta \frac{u'(y_{t+1})}{p_{t+1}} \tag{3.13}$$

$$\sum_{T=t}^{\infty} \beta^T u'(y_T) y_T < \infty \tag{3.14}$$

$$\underset{T\to\infty}{Lim} \beta^T u'(y_T) p_t^{-1} = 0 \tag{3.15}$$

Clearly, the Euler equation (expression 3.13) makes no reference to any money or transactions technology. This is the crucial step in Woodford's (1997, p. 34) analysis because he takes this as evidence ".. that there is in fact no need to describe the way in which cash facilitates transactions, ..., in order to derive these equations.... One might instead have simply written down the equilibrium conditions

$$u'(y_t) = \beta \frac{R_t p_t}{p_{t+1}} u'(y_{t+1}) \qquad (3.19)$$

and

$$\underset{T\to\infty}{Lim} \beta^T u'(y_T) \frac{W_T}{p_T} = 0 . \qquad (3.20)$$

Combining these with the monetary and fiscal policy rules produces (3.13) and (3.15) directly[5].

III Critique of Woodford's analysis

What is wrong with Woodford's analysis? The answer lies initially with an observation by Frank Hahn (1965). Hahn pointed out that it was not possible, in general, to prove the existence of a monetary equilibrium by introducing 'money' as one of the *n* commodities in what is otherwise a real general equilibrium model. This

[5] The fiscal policy rule is simply that the conventional budget deficit is zero in each period (Woodford, 1997, p. 27). The monetary policy rule is expression (3.7) given above.

is clearly true in the Arrow-Debreu world of complete contingent futures markets but it applies also to Patinkin-type models and overlapping generations models. The issues involved have been admirably surveyed by Hoover (1988, Chs. 5-6).

There are two fundamental problems with treating 'money' as the *nth* commodity in a real general equilibrium system. First, the model does not require all three functions of money. This leads to the second difficulty: the absence or redundancy of a transactions technology. Transactions technologies are redundant in models that are designed to address questions of resource allocation. For example, the OLG tradition includes only the store of value function that is easily dominated by any durable asset with a non-zero rate of return. To avoid this problem many monetary theorists have taken to imposing a cash-in-advance constraint. This forces agents to hold 'money' even though 'money' is dominated by other durable assets in rate of return. But that does not solve the deeper and more fundamental problem: the irrelevance of a transactions technology in real general equilibrium models in which 'money' is treated as the *nth* commodity. As Hahn (1965) stressed, 'money' in these models is an *inessential* addition in the sense that, although it may be incorporated, it is usually easy to show that a non-monetary equilibrium exists.

To see that 'money' is an inessential addition in Woodford's model, consider the following features of his analysis. As noted above, 'money' is inessential if it is easy to show that a non-monetary equilibrium exists. This is straightforward to illustrate because Woodford repeatedly draws attention to this property of his model.

To begin with, in the version with exogenous *M*, Woodford derives the non-monetary version of the Euler equation (expression (2.24) by applying the condition $\alpha = 0$ and deriving the consequent results; $\theta_1(R) = 1$, $\theta_2(R) = R^{-1/1-\gamma}$, $\omega(R) = 1$ and $\mu(R) = 0$ for all $R \geq 1$. [check these; theta 1 and theta 2 seem to be the wrong way round given the defs on page 17] In that case the Euler equation (2.24) reduces to:

$$\frac{u'(y_t)}{p_t} = \beta \frac{u'(y_{t+1})}{p_{t+1}} \qquad (3.5)$$

which, as Woodford (1997, p. 24) notes, is the "..familiar asset-pricing equation for a non-monetary economy, applied to an asset which pays no dividend and has an exchange value (in terms of the consumption good) of $1/p_t$".

In terms of Hahn's criterion, money is obviously an inessential addition to this version of Woodford's model. [As it was in Patinkin (1965)]

The same conclusion also applies to Woodford's Wicksellian world in the cashless limit as he himself points out when he notes that there is no need to take account of any transactions technology when deriving the equilibrium solution. One might more conveniently simply write down the equilibrium conditions from the non-monetary version of the model. In this regard expression (3.13) is particularly interesting because it is apparent that it differs from expression (3.5) only by the inclusion of the term $\phi(p_t)$. But in the cashless limit there is no 'money' so by definition there is no nominal rate of interest either. Hence it must be the case that $R = 1$ so that expressions (3.5) and (3.13) or (3.19) are identical in the cashless limit. The same

equilibrium solution can be derived for the model even when $\alpha > 0$ if we consider the case of $R = 1$. Because expression (2.16) may hold with equality in this model (which Woodford allows) this case also generates the non-monetary Euler equation (3.5). My point here is that these results are simply examples of Hahn's (1965) conclusion that it is easy to show that a non-monetary equilibrium exists when a non-essential addition of money is made to a real general equilibrium system.

Woodford's strategy of appending a cash-in-advance transactions technology to a real general equilibrium model then appears for what it really is; an inessential addition that in no way compromises the integrity of the underlying real system. Consequently 'money' can be made to vanish without undermining the existence of a real equilibrium. We should not be surprised that appending a redundant transactions technology to the model causes nary a ripple when it is removed. It does, however, throw up some interesting conceptual issues.

IV Conceptual problems raised by Woodford's analysis

Woodford clearly intends his analysis to have policy relevance in view of Rotemburg and Woodford (1997) and the use to which he puts his own model. To be of relevance to policy makers the variables in Woodford's model must be proxies for their real world counterparts. For example, a short-term nominal interest rate is the instrument of choice in modern monetary systems, and policy is directed to achieve price level stability defined in terms of the CPI (or nominal GDP). In Woodford's cashless limit, however, neither of these concepts can be given any meaningful interpretation. That leaves no role for central bankers.

Consider first the nominal rate of interest. Whenever $\alpha > 0$ the gross and net nominal rates of interest are defined. In each case there exists some *M* by virtue of the cash-in-advance constraint which is imposed on artisans. But in the cashless limit, where $\alpha = 0$ there is no 'money' and hence there *are no nominal values*. There is no money so the nominal rate of interest, the rate charged for borrowing money, is conceptually undefined. In any event, no artisans use the second technology in the cashless limit and any arbitrarily selected commodity can be labelled the *numeraire*. But even if the $ was selected as the *numeraire,* and all prices quoted in $s, how is actual payment to be made in this world? The model has nothing to say about this question.

What then, are we to make of the role of central bankers? If central bankers use a nominal interest rate as the instrument of policy, albeit that they have an inflation-adjusted rate in mind, how are they to implement monetary policy in Woodford's post-monetary world where no nominal instrument exists? In Woodford's post-monetary world only real interest rates exist. Could central bankers not use a real rate directly? It seems not. Real interest rates in Woodford's post monetary world are implicit in the sequence of real inter-temporal relative prices. The set of real interest rates is simply another way of presenting the set of relative prices. The set of real relative prices is generated given the tastes, endowment and technology as a solution to the dynamic optimising behaviour of the artisans. There is simply no way in which someone called a central banker can interfere with this process. If the model has no essential role for money it certainly has no essential role for central bankers. What purpose would central bankers serve in this world? It seems to me that there is a form of cognitive dissonance at work here. Consequently, because the model is one in

which any transactions technology is redundant it really makes no sense to use it to examine the properties of a world in which the transactions technology is evolving[6].

For similar reasons I find the description of the model as "Wicksellian" in the cashless limit to be problematic. The essence of Wicksell's analysis is the distinction between the market and natural rates of interest. The market rate is the rate on money loans and the natural rate represents the return on real capital investment. Assuming full employment, Wicksell argued that inflationary (deflationary) pressure resulted whenever the market rate lay below (above) the natural rate. The discrepancy between the two rates could occur as a result of monetary and/or productivity shocks. Furthermore, Wicksell also had a model that bears some relationship to the cashless world Woodford has in mind- the pure credit model.

Wicksell's pure credit model is a world in which no cash exists and all transactions are conducted by book entry. Today he would no doubt say by electronic transfer. Nevertheless, Wicksell's theory was clearly intended to apply to a monetary economy in which all three functions attached to the book entries (or in a modern setting the electronic medium of exchange). For example, even though the $ bill does not exist (there is no cash) in Wicksell's pure credit economy all prices are expressed in $'s and electronic entries expressed in $ units are the medium of exchange and store of value. Woodford's model does not have either of the last two functions in the cashless limit and the numeraire can be selected arbitrarily from the list of commodities[7]. In a

[6] Buiter (1999, p.36, fn 30) concludes, correctly in my view, that:
" Searching for limiting behaviour of the equilibrium price of money sequence in a demonetising monetary economy, in the hope of finding one which converges to an equilibrium price of money sequence in a non-monetary economy, is therefore bound to be a will-o'-the wisp".
This conclusion also applies to the concept of the interest rate in Woodford's model.
[7] Buiter (1999) stresses this point.

monetary economy the numeraire is not arbitrary. Finally, because there is no nominal rate of interest in Woodford's cashless limit there is nothing in the model equivalent to Wicksell's market rate. For these reasons I don't see any connection between Wicksell's pure credit economy and Woodford's cashless limit.

Additional conceptual difficulties arise when we begin to inquire into the meaning of the term "price level" in Woodford's post-monetary world. In the cashless limit there is no *M*, so there is no sense in describing the price sequence, $p_t = \hat{p}(y_t, y_{t+1,}.....)$ as a price level. The price sequence is simply a set of real relative inter-temporal prices and bears no resemblance to the CPI or the GDP deflator. In that case Woodford's model cannot provide the basis for sensible advice about the process of inflation or price level targeting undertaken by central bankers. As a matter of interest, Hoover pointed out some time ago that a model by Fama (1980) involved just this conceptual error[8]. Hoover (1988, p. 99) alerted us to the confusion when he observed that: "Fama's use of the term 'price level' is somewhat misleading. The barrell-of-oil price of each good is a *relative* price while the price level is usually taken to refer to *absolute* or *nominal* prices." The same conceptual error is found in Woodford's analysis when he suggests that inflation of the price level can meaningfully be analysed by a model in which only an inter-temporal real relative price vector exists. Essentially the same point has been made by Buiter (1999, p. 36, fn 30) when he concludes that:

[8] Fischer Black (1974) makes a similar conceptual error. It seems that these authors confuse the existence of an auctioneer (with amazing computing ability) with the use of electronic means of payment and the widespread adoption of computers in the banking sector. There is an obvious conceptual distinction.

"In an economy without money the price of money is not just indeterminate. It is conceptually undefined."

All this suggests that the conceptual errors inherent in Woodford's analysis are nothing new. In addition to Hahn (1965), whom, it might be thought, was making a rather esoteric point, monetary theorists from across the spectrum have reached essentially the same conclusion. In particular, Laidler (1990) and Clower and Howitt (1993) have drawn attention in the literature to this form of conceptual error: real general equilibrium models are exactly that- real general equilibrium models that have nothing to say about how transactions are actually carried out. The existence of the Walrasian auctioneer is usually identified as the assumption that precludes the need to model any transactions technology. Clower and Howitt (1993, p. 2) make the point rather forcefully:

"Although in every case some attempt is made to motivate the holding of money by invoking some kind of transactions cost, the very existence of such costs is ruled out by the assumption (explicit or implicit) that 'the auctioneer' establishes the terms of all planned trades without cost to any 'agent'. In all such theories, transactions cost are ruled out by the assumption that trading plans can be executed whenever for each good the sum of everyone's planned purchases equals the sum of their planned sales, without regard to the question of who trades what for which or with whom".

Laidler (1990) raises the same issue when he stresses that, in any model, the Walrasian auction and money are substitutes not complements. The existence of a

Walrasian auction obviates the need for money. Simply put, a model with an explicit, or implicit, Walrasian auction precludes the need for a transactions technology; the model is simply not designed to address the question of how transactions will be executed. Using it to address such questions leads to the sort of confusions exhibited by Woodford's analysis.

The most recent example of how the conceptual errors embedded in Woodford's analysis come home to roost has been provided by Buiter's (1999) critique of the fiscal theory of the price level. What, one may ask, is the connection between the fiscal theory of the price level and monetary theory? At first sight there appears to be no connection between these two issues. However, it turns out that the connection is immediate. If we have an analysis in which money is not required to determine the price level what other mechanism might do the job? Woodford's answer, it turns out, is to introduce a government and use the government budget constraint to determine the price level in a world without 'money'. Buiter (1999) regards this claim as just too fantastic for words. How is it possible, he asks, to determine the price of money in a world where money does not exist either as a physical object or a financial claim? The answer is simple once the sleight of hand is exposed: this is Woodford's so-called post-monetary world where a real relative price vector is parading as the price level.

Buiter (1999, p. 34) accurately outlines the properties of Woodford's cashless limit in the following terms and raises some (by now) familiar conceptual questions:

"Now consider the case of $\alpha = 0$. The demand for real money balances will be zero in equilibrium. Households don't demand money and the government sector does not

issue money. Money is dropped from the asset menu. It is no longer the store of value, medium of exchange or means of payment. Something called 'money' could still be numeraire, but it has no existence other than as a numeraire or unit of account. In addition there is a government debt instrument that has a coupon payment, Γ, specified in terms of money. ….. Note that the payment of the nominal debt instrument, Γ, cannot be made in money, since there is no monetary asset in the economy. The nominal bond can only promise to make a payment worth Γ units of money. The payment would have to be made in some good or financial claim that actually exists."

Here, Buiter is describing the property of a real general equilibrium model, outlined above, in which the *nth* commodity can be selected arbitrarily and treated as *numeraire,* and in which the other two functions of money can be imposed via the cash-in advance constraint. But the introduction of nominal magnitudes in this fashion amounts to nothing more that and *inessential* addition to the underlying real structure. Nothing of analytical substance is altered. Hence it is easy for Woodford to eliminate the cash-in-advance feature of his payments technology because it is in any event inessential. Removing 'money' in the cashless limit reveals the real core of the model by stripping away all nominal values (values quoted in the money unit). Only real relative prices remain and confusion results when theorists attempt to relate these real relative price concepts, which measure equivalent quantities of commodities, to the nominal magnitudes observed by policy makers (or even their inflation adjusted real equivalents).

VI Conclusion

Woodford presents an analysis that purports to model the evolution of the monetary system from a world of cash to a world of electronic money. The analysis is unconvincing because it proceeds by a well documented, but apparently not well-known, sleight of hand that obscures fundamental conceptual errors. In short, there is nothing in Woodford's model that corresponds with anything that would be recognizable to a central banker. It cannot then provide a basis for sensible advice to central bankers or provide a basis for empirical analysis of real economies.

References

Black, F. (1970) 'Banking and interest rates in a world without money' *Journal of Bank Research*, Autumn: 9-20.

Buiter, W. H. (1998) "The Young Person's Guide to Neutrality, Price Level Indeterminacy, Interest rate Pegs, and Fiscal Theories of the Price Level, *National Bureau of Economic Research*, Working Paper 6396.

Buiter, W. H. (1999) "The Fallacy of the Fiscal Theory of the Price Level", *National Bureau of Economic Research*, Working Paper 7302.

Clower, R. and Howitt, P. (1993a) "Money, Markets and Coase", mimeo.

Fama, E. (1980) 'Banking in the theory of finance, *Journal of Monetary Economics*, 6(1), 39-57.

Green, E. (1999) "We need to Think Straight about Electronic Payments", *Journal of Money Credit and Banking*, August, part 2, 669-670.

Hahn, F. H. (1965) " On Some Problems of Proving the Existence of Equilibrium in a Monetary Economy", in *The Theory of Interest Rates*, ed. Hahn and Brechling, London: Macmillan.

Henckel, T., Ize, A., and Kovanen, A. (1999) "Central Banking Without Central Bank Money", *IMF Working Paper*, WP/99/92.

Hoover, K. D. (1988) *The New Classical Macroeconomics*: *A skeptical inquiry*, Oxford: Basil Blackwell.

Laidler, D. (1990) *Taking money seriously and other essays*, Cambridge, Mass: M I T Press.

Lucas, R. E. Jr. (1980) "Equilibrium in a Pure Currency Economy", in J.H. Kareken and N. Wallace, eds, *Models of Monetary Economies*, Minneapolis: Federal reserve Bank of Minneapolis.

Lucas, R. E. Jr. and Stokey, N. L (1987), Money and Interest in a Cash-in Advance Economy", *Econometrica* 55, 491-514.

McCallum, B. T. (1985) 'Bank deregulation, accounting systems of exchange, and the unit of account: a critical review", in Karl Brunner and Allan Meltzer (eds) *The 'New Monetary Economics", Fiscal Issues and Unemployment*, Carnegie-Rochester Conference Series on Public Policy, 23, Amsterdam: North Holland, 13-46.

Patinkin, D. (1965) *Money Interest and Prices*, (second edition), New York: Harper and Row.

Rogers, C. and Rymes, T.K. (1998) "Indirect convertibility and quasi-futures contracts: Two non-operational schemes for stabilisation of the price level? School of Economics, University of Adelaide, WP 98-17.

Rotemburg, J. J. and Woodford, M. (1997) "Am Optimization Framework for the Evaluation of Monetary Policy", *NBER Macroeconomics Annual*, ???.

Woodford, M. (1997) 'Doing without money: controlling inflation in a post-monetary world", *National Bureau of Economic Research,* Working paper 6188.

Woodford, M. (1998) 'Doing without money: controlling inflation in a post-monetary world", *Review of Economic Dynamics,* 1, 173-219, 1998.