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A CONTEST WITH THE TAXMAN – THE IMPACT OF TAX RATES ON TAX EVASION AND WASTEFULLY INVESTED RESOURCES

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A contest with the taxman - The impact of tax rates on tax evasion and wastefully invested resources*

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Abstract

We develop a moral hazard model with auditing where both the principal and the agent can influence the probability that the true state of nature is verified. This setting is widely applicable for situations where fraudulent reporting with costly state verification takes place. However, we use the framework to investigate tax evasion. We model tax evasion as a concealment-detection contest between the taxpayer and the authority. We show that higher tax rates cause more evasion and increase the resources wasted in the contest. Additionally, we find conditions under which a government should enforce incentive compatible auditing in order to reduce wasted resources.

JEL-Classification: H26, D82, K42

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1 Introduction

The purpose of this paper is to develop a moral hazard model with auditing where both the principal and the agent can influence the probability that the true state of nature is verified. We do not allow the principal to commit to an audit strategy before observing the signal from the agent. Such a setting is widely applicable to situations of fraud. Fraudulent claims for benefits, insurance payments, or loans are examples. It even could be applied to the broad range of situations where bilateral trade of goods takes place. Whenever it is hard and expensive to verify the value of a good for a potential buyer (antiques, paintings), while the seller has private information about this value, such a moral-hazard situation may arise. However, the application we choose is the case of tax evasion. This will enable us to draw conclusions about the impact of tax rates on tax evasion and the resources wasted by the agents' attempts to influence the detection probability.

The early neoclassical approach to income tax evasion (e.g. Allingham and Sandmo, 1972; Yitzhaki, 1974) treats the detection probability as an exogenous parameter.[1] In later contributions the audit probability was endogenized in two different ways. Reinganum and Wilde (1985) derive an optimal audit rule under the assumption that the authority has to invest in the audit probability.[2] In a neoclassical optimal taxation framework Cremer and Gahvari (1994) allow for the taxpayer to influence the audit probability by spending some resources on covering actions. In this paper, we explicitly model both, the tax authority investing in detection and the taxpayer spending some income to cover his evasion activity. The detection probability is determined by the effort exerted by both parties. We believe that for many countries the relationship between taxpayer and tax authority is quite competitive, and accordingly is accurately described by such a contest.

Furthermore, in the real world we observe that different sources of income lead to different evasion and concealment opportunities.[3] We include this fact in our model by just focusing on single components of income with different marginal coverage and fixed evasion costs. So we end up with separate evasion, coverage and detection decisions for different possible income components. The sum of all these decisions determines the over all income after tax - including possible fines.[4] We think that this approach, that allows for income structures with distinct income parts, is more realistic than the widely used framework where the aggregate income is considered to be homogenous and evasion decisions are modelled as continuous choices. This approach is related to Macho-Stadler and Perez-Castrillo (1997). There taxpayers are heterogeneous in income and income sources are heterogeneous in the (exogenous) probability of verification if an audit takes place. We endogenize the verification probability by introducing a contest. Furthermore, we abstain from the assumption that the tax authority can commit to an audit strategy. This reflects our aim, rather to analyse the interaction between tax authority and taxpayer positively, than to characterize an optimal committable audit, penalty, and tax

[1] For a detailed survey and many extensions to the basic neoclassical model see Cowell (1990).

[2] For a more general characterisation of optimal enforcement schemes see Chander and Wilde (1998).

[3] The Taxpayer Compliance Measurement Program of the U.S. (IRS, 1983) e.g. estimates for 1981 that tax compliance for wages and salaries was 93.9%, 59.4% for capital gains, and only 37.2% for rents.

[4] By restricting our analysis to uncorrelated earnings probabilities, a linear tax system, and a penalty that does not dependend on the over-all income, we can treat these decisions as independent.

structure. We think that the normative approaches in the latter tradition suffer the problem that maximizing social welfare only with respect to tax evasion does not take into account that tax rates and fines may have more influence on welfare through other channels. The results of those models may be misleading for this reason.

We examine the equilibrium predictions of the model and find conditions the parameters have to satisfy in order that certain equilibria are obtained (such as e.g. "contest" or "honest taxpayer"). Our main finding is that in the tax evasion setting with incomplete and imperfect information no credible strategy for the tax authority exists that prevents tax evasion with certainty if the taxpayer has the opportunity of evading.[5] This finding differs from the standard literature (see e.g. Reinganum and Wilde (1985), Border and Sobel (1987), Mookherjee and Png (1989), Mookherjee and Png (1990) or Chander and Wilde (1998)), where optimal incentive-compatible enforcement schemes are derived. There the possibility of committing to a certain strategy is the key for the nonexistence of evasion. We follow the non-commitment assumption, which was introduced into the tax evasion literature by Reinganum and Wilde (1986) and Graetz et al. (1986). Our model shares some characteristics with Khalil (1997), who uses the price-regulation setup of Baron and Myerson (1982) and combines it with production-cost auditing. We think that the result in our model - i.e. the taxpayer always evades at least with a very small probability if he has the opportunity to do so - is empirically more realistic. In addition, our model predicts more tax evasion if tax rates rise. This empirically established fact is hardly explainable with the traditional neoclassical models.[6] In the normative part of this paper we analyse the impact of tax rates on resources wastefully invested in detection and concealment. Additionally, we examine the conditions to be satisfied in order that a government directive, which commits the authority to an audit rule that always prevents the taxpayer from cheating, reduces the resources wasted in the enforcement process.

The remainder of the chapter is organized as follows: In the next section we will discuss the timing of the game and our main assumptions. Then we develop the basic setup and analyse the impact of tax rates on evasion and wasted resources. In following section we examine the conditions an external commitment device such as a law or government directive that commits the authority to an effort leading to truthful revelation has to fulfil in order to reduce the waste. In section 5 we introduce some extensions to the basic moral hazard framework. There, we at first relax the assumption of a dichotomous income distribution, to finally allow for continuously distributed evasion costs privately known to the taxpayer. We conclude with some remarks on the policy implications of the presented model.

2 Timing and basic assumptions

In this section we develop the structure of the model, and briefly discuss the underlying assumptions. We begin with the timing.

[5] We define a positive evasion opportunity as a situation where the fixed evasion costs are not prohibitive.

[6] For a discussion see chapter one. A concise overview over the logic of different generations of tax-evasion models can be found in Franzoni (1999).

2.1 Timing

Before we comment on the reasons for choosing the present structure, we introduce the timing of our model and some notation. The sequence of events is as follows:

1. Nature determines the actual income y_i^a for every possible income source i.
2. The taxpayer observes y_i^a.
3. The taxpayer declares his income $d_i \in \{0, y_i^a\}$, and chooses the effort $e_i \in [0, \infty)$ to cover a possible evasion for every possible income source i.
4. The authority observes the declared income d_i for every source i. I tdoes not observe the true income y_i^a and the concealment effort e_i exerted by the taxpayer.
5. The authority chooses a certain verification effort $a_i \in [0, \infty)$ for every possible income source.
6. Nature decides whether a possible evasion or avoidance is verifiable or not. The probability of verifiability is given by $p_i(a_{i,}e_i)$ for the different possible income components.
7. Taxpayer and authority receive their pay-offs U_i and R_i, respectively.

Since we are not primarily interested in the effects of taxes on the income generation decision, we treat income as endogenously determined by nature. Furthermore, the induction of the income-generating mechanism does not lead to additional strategic effects. For reasons of clarity and simplicity we prefer not to model them.

2.2 The basic assumptions

Here we will explain the basic assumptions to be used in the main part of this chapter.

A1 Declaration is a binary decision for the different income sources, i.e. $d_i \in \{0, y_i^a\}$.

This assumption closely corresponds with our usage of the term "income" sources. Income sources in our sense are specific components of possible income, which are not divisible in terms of certification. Usually, the tax authorities - at least in systems with developed tax collection - ask for documents proving the value of declared income components. These are e.g. payment certificates issued by the employer, bank certificates for interest payments or copies of bills for deductions of expenses. So we assume that it is only possible to declare and certify a certain income component or not to declare it at all. However, this assumption is not crucial at all. In the linear framework we use, an interior declaration level is never optimal. To exclude the possibility of interior declaration levels from the beginning makes the notation easier and proves convenient for expositional reasons.[7]

[7] We are aware that there are income sources that are poorly described by this assumption (e.g. tips in restaurants); but we think that our assumption in general is more appropriate than to assume declarations to be continuous in general. The model naturally extends to continuous income declaration if we assume common knowledge about the fact that a source can be declared continuously.

A2 Assume that the distribution of the realizations for different income sources y_i^a is dichotomous:

$$y_i^a = \begin{cases} y_i & \text{with probability} \quad \lambda_i \\ 0 & \text{with probability} \quad 1-\lambda_i \end{cases}$$

A2 considerably simplifies the analysis and could indeed be regarded as an oversimplification. However, the main results still hold if we relax this assumption. This is shown in section 5.1.

A3 Both taxpayer and tax authority are risk neutral. They maximize expected net income and net revenue, respectively.

We showed in a related paper (, Bayer, 2002) that it might be an appropriate approximation to assume a risk neutral taxpayer if there are tax-evasion costs. To assume a risk-neutral tax authority is a standard assumption in tax evasion games (e.g. Reinganum and Wilde, 1985). However it is less obvious what the objective function for the tax authority should be. There are alternative formulations that seem reasonable. Assuming that the authority maximises net penalties instead of net revenue does not have any qualitative influence on our results. If the tax authority were assumed to maximize net recovered revenue - an assumption indicating that bureaucrats care about their percepted performance - we would obtain qualitatively equivalent results.

A4 The tax system is linear (i.e. $T(d) = t\sum_{i=1}^{n} d_i$) and the penalty is proportional to the amount of taxes evaded or avoided (i.e. $F(d, y^a) = f \cdot t\sum_{i=1}^{n}(y_i^a - d_i)$ with $f > 1$).

This assumption serves two purposes. Firstly, it makes the results of this chapter comparable to most of the existing work on tax evasion, since such tax and penalty systems are widely used in the literature. Secondly, this assumption makes sure that we can treat the overall tax liability and possible penalties as a simple sum of outcomes for the single income components. In this setting the choices of declared income, concealment and detection effort are independent for the different income sources. We are aware that in real life the decision whether to evade the income from e.g. letting a house might depend on the decision over the declaration of other income components. But in our opinion, it is worth neglecting these side effects in order to simplify the model in such a way that the main effects can be identified.

A5 The verification probability p_i increases with detection effort a_i and decreases with concealment effort e_i. The marginal cost of influencing the verification probability in the favourable direction increases with the effort.

To achieve this we use a formulation for the verification probability that is commonly used in the contest literature:

$$p_i(a_i, e_i) = \begin{cases} 0 & \text{if} \quad a_i, e_i = 0 \\ \frac{a_i}{a_i+e_i} & \text{else} \end{cases} \tag{1}$$

A6 The concealment costs C_i and the detection costs A_i are linear in effort.

The marginal concealment cost may depend on some parameters that describe the specific environment for the concealment of that income component. Banking system, laws to prevent money laundering, and the degree of transparency in capital markets are examples. Realistically, the marginal detection cost could depend on the amount of income concealed, since it is harder to conceal large amounts of money.[8]

$$C_i(e_i, \cdot) = c_i(\cdot) \cdot e_i. \tag{2}$$

Without loss of generality we can normalize the marginal detection cost to unity.

$$A_i(a_i) = a_i \tag{3}$$

This assumption reflects the observation that it is more costly for the tax evader to hide his evasion more effectively. He will take the cheaper measures to conceal before using the more expensive ones. On the other hand, it seems reasonable to assume that it is getting more and more expensive for the authority to achieve an extra percent of detection or verification probability, because tax inspectors should begin seeking where it is easiest to find evidence. The only property we need for our main results, is that the marginal costs of influencing the probability in its favoured direction are increasing. We do not allow the concealment cost to depend directly on the tax rate. We think, this is a realistic restriction that considerably simplifies the algebra.

A7 Tax evasion causes fixed evasion cost K_i to the evader.

Finally, we allow for some evasion costs K_i, which are incurred whenever the taxpayer tries to evade an income component. This reflects the observation that not only concealment, but also evasion may be costly. Sometimes an evasion opportunity has to be created in order to have the possibility to evade. There are expenses that do not vary with the level of concealment. Another part of K_i are the often cited moral costs of evasion. We use these moral cost as - admittedly, a somewhat crude - black box variable that describes psychological differences of taxpayers (like ethics, attitudes etc.) leading to different evasion behaviour in identical situations.[9]

2.3 The pay-offs and some notation

Let us now specify the payoff functions for the two players. It follows from our assumptions that the expected interim payoff - after declaration and efforts are determined - for the taxpayer can be written as

$$\begin{aligned} EU(\mathbf{d}, \mathbf{e}, \mathbf{a}) &= (1-t)\sum_{i \in H} y_i^a + \sum_{i \notin H} y_i^a \left[1 - f \cdot t \cdot p_i(e_i, a_i)\right] \\ &\quad - \sum_{i \notin H} (c_i \cdot e_i + K_i) \end{aligned} \tag{4}$$

where H is the set of all i with $d_i = y_i^a$, which is the set of truthfully declared income sources. The first sum gives the certain after-tax income for all income components that are declared. The second represents the

[8] To simplify the notation in what follows we will drop the possible arguments of the marginal cost function.

[9] For some experimental evidence on psychological differences as predictors for evasion behaviour see Bayer and Reichl (1997) or Anderhub et al. (2001).

income for the undeclared income parts - expected penalties included. The final, negative sum contains the concealment and evasion cost.

Following our assumption about the objective function of the tax authority (A3), we can write the expected payoff of the authority as:

$$ER(\mathbf{d}, \mathbf{e}, \mathbf{a}) = (1-t) \sum_{i \in H} y_i^a + \sum_{i \notin H} [y_i^a \cdot f \cdot t \cdot p_i(e_i, a_i)] - \sum_{i=1}^{n} a_i \tag{5}$$

Because of the linear system and the risk neutrality assumption we immediately see that the maximization for both objective functions is piecewise.[10] This leads to the following lemma:

Lemma 1 *The decisions (declaration d_i, concealment effort e_i and detection effort a_i) for income source i are independent of the decisions for all other income sources d_j, e_j, and a_j with $j \neq i$.*

Proof. Obvious. ■

Lemma 1 tells us that we can restrict ourselves to examining the decisions for a single income component. To simplify our notation we can drop the indices for the potential income sources.

In order to be able to interpret the results we will derive, it might be helpful to define the ratio of marginal detection cost to marginal concealment cost as the relative *concealment opportunity* η for the income component:[11]

$$\eta = \frac{1}{c} \tag{6}$$

A further definition that will help the intuition is to use the ratio of fixed evasion cost to the possible tax bill reduction in the following way to define the *evasion opportunity* ω:

$$\omega = 1 - \frac{K}{ty} \tag{7}$$

An evasion opportunity ω of 0 means that a taxpayer has to invest the same amount of money (or time, nerves, and moral tension, respectively) to be able to evade as the possible tax bill reduction would be. More precisely, the evasion opportunity here is a percentage measure of the possible tax bill reduction net of evasion cost. Note that the concealment costs are not included here - they are measured by η.

It remains to define the amount of resources invested wastefully in the process of declaration and auditing. The waste is defined as the sum of the costs for evasion, covering action, and detection activity:

$$W = ce + a + \phi K, \tag{8}$$

where ϕ is an indicator variable, which is equal to 1 if the income component is earned and evaded and 0 otherwise.

[10] For the moral hazard case where the expected pay-off function depends on beliefs we certainly need as well the assumption that the different sources are uncorrelated.

[11] Note, that η depends on the same arguments as c.

3 Signaling with hidden action

We concentrate on the genuine tax evasion situation, where the authority can neither observe the true income, nor the concealment effort. This might be the most frequent situation the tax authority faces when receiving tax return.[12] The only information the tax authority has is the probability distribution over the distinct income parts. As a simplifying assumption (see A2) we assumed a dichotomous distribution.[13] Consequentially, the tax inspector knows that the taxpayer has the income component (worth an amount of y) with probability λ. With a probability of $1-\lambda$ the income from this income source is 0. This is common knowledge.

The solution concept that will be applied is that of a Perfect Bayesian Equilibrium (PBE). The game the actors face can be classified as a signaling game with hidden action. In our case an equilibrium consists of three elements: a strategy for the taxpayer, a strategy for the authority, and the authority's beliefs about the true income of the taxpayer. The strategy for the taxpayer specifies a declaration (signal) and a concealment effort (hidden action) conditioned on whether he earned the income or not. The strategy for the authority is a detection effort depending on the observed declaration. The authority's beliefs assign probabilities to the income of the taxpayer. They depend on the observed declaration and are updated by using Bayes' Rule. In equilibrium the strategies maximise the actors' pay-offs given the beliefs. The beliefs have to be consistent with the equilibrium strategies.

Unlike other models (e.g. Chander and Wilde, 1998) in our case the revelation principle does not hold. The reason is twofold. Firstly, we do not allow the authority to commit beforehand to a certain action. But even if we allowed for that, the revelation principle would fail, since secondly the nature of the contest restricts the set of feasible contracts. We will see the difference of outcomes when we compare an externally enforced incentive compatible effort scheme to the equilibrium in our original game.

3.1 Equilibria for different parameter settings

Let us begin with an obvious statement about the taxpayer's behaviour. Declaring any non-existent income never pays. So the strategy reporting zero if no income is earned is part of any equilibrium. The corresponding concealment effort is also zero.

$$d^*(y^a|y^a = 0) = 0 \tag{9}$$

$$e^*(y^a|y^a = 0) = 0 \tag{10}$$

When the tax authority has to decide how much to invest in detection, its only information is the declaration of the taxpayer. It may face a declaration of $d = 0$ or $d = y$. If an income declaration of y is observed it is optimal for the authority to do nothing. This is also part of any equilibrium:

$$a^*(d|d = y) = 0 \tag{11}$$

[12] A treatment of different informational settings, which describe the case of tax avoidance, is contained in a longer version of this paper and can be obtained on request from the author.

[13] We will relax this assumption in a later section.

But if the inspector representing the authority finds that the taxpayer declared no income, he might not be sure whether he faces a tax evader or just a person who really received no income from the source in question. He has to form some beliefs. Denote the belief that he faces a tax evader, which is the subjective probability that the true type of the taxpayer is y if he reports 0, as $\mu(y^a = y|d = 0)$. Applying Bayes' Rule this belief should be

$$\mu(y^a = y|d = 0) = \frac{\alpha \cdot \lambda}{\alpha \cdot \lambda + 1 - \lambda}, \tag{12}$$

where α is the probability that a taxpayer with positive income does not declare it.[14] We allow the taxpayer to play a mixed strategy.[15] Now we can express the objective function of the authority if it faces a declaration of 0 as the expected fine collected net of detection costs:

$$ER(a, e, \mu, d|d = 0) = \mu \cdot f \cdot t \cdot y \cdot p(a, e) - a,$$

where μ is the abbreviated form of *lhs* in (12).

Investing valuable resources in concealment if there is nothing to conceal is a strictly dominated strategy for the taxpayer. So we have

$$e^*(y^a|y^a = y\ ,\ d|d = y) = 0.$$

Taking this into account and allowing for mixing we can state the relevant ex ante objective function for the taxpayer under the condition that he earned the income component:

$$EU(e, a, y^a|y^a = y) = \alpha(y - p(e, a) \cdot f \cdot t \cdot y - c \cdot e) + (1 - \alpha)(1 - t)y$$

3.2 Pure strategy equilibrium

Let us now look for pure strategy equilibria. Note that equations (9) to (11) are part of any equilibrium. To find a pure strategy equilibrium we let $\alpha = 1$ (the taxpayer always evades) or $\alpha = 0$ (the taxpayer never evades). For the case of a pure evasion equilibrium we plug $\alpha = 1$ into both of the objective functions and find the optimal values for a and e. This is to make sure that the beliefs of the authority are consistent with the strategy of the tax evader. Later on, we have to check whether - given the outcome in the simultaneous effort stage - it is really optimal for the taxpayer to declare no income if he earned it. For $\alpha = 1$ - and $\mu = \lambda$ consequently - the two first-order conditions are:[16]

$$\frac{\partial}{\partial a} ER(a, e, \alpha, 0) = \frac{\lambda \cdot e \cdot f \cdot t \cdot y}{(a + e)^2} - 1 \leq 0 \tag{13}$$

$$\frac{\partial}{\partial e} EU(e, a, \alpha) = \frac{a \cdot f \cdot t \cdot y}{(a + e)^2} - c \leq 0 \tag{14}$$

[14] Implicitly we already apply the consistency requirement that the authority puts a zero probability on the taxpayer declaring some income if he has not got it (see equation 9).

[15] To abbreviate the notation we will refer to this belief as μ.

[16] The second order conditions are obviously fulfilled.

The resulting efforts are:

$$a_p^* = \frac{\lambda^2 \cdot f \cdot t \cdot \eta \cdot y}{(\eta + \lambda)^2} \quad (15)$$

$$e_p^* = \frac{\lambda \cdot f \cdot t \cdot \eta^2 \cdot y}{(\eta + \lambda)^2} \quad (16)$$

This is an equilibrium whenever:

$$ER(e_p^*, a_p^*, d|d = 0) \geq 0$$

$$EU(e_p^*, a_p^*, \phi) \geq (1-t)y$$

The first condition is always fulfilled.[17] The second only holds for certain parameter configurations.[18] The requirements on the parameters for a pure strategy evasion equilibrium to exist is:

$$K < t \cdot y \left[1 - f + f(\frac{\eta}{\eta + \lambda})^2\right] \text{, which simplifies to}$$

$$\omega \geq \frac{\lambda \cdot f(2\eta + \lambda)}{(\eta + \lambda)^2} \quad (17)$$

The question is now what the equilibrium looks like if the evasion opportunity ω is not high enough to ensure an evasion equilibrium. A natural candidate seems to be a pure non-evasion equilibrium. But in fact, pure strategy non evasion is not necessarily an equilibrium in this case. The argument goes as follows. Being honest dominates evasion if the authority exerts the best-response level of effort. It is a best response for the authority to exert no effort if it believes the taxpayer to be honest with certainty ($\alpha = 0$). But if the authority is exerting no effort it is not a best response for the taxpayer to be honest if the fixed evasion costs are not prohibitive. Then the beliefs off the equilibrium path required for this equilibrium would not be consistent. The equilibrium would require $\mu > 0$, but since α should be equal to zero by using Bayes' rule, μ should be equal to zero as well. This is an obvious contradiction.

The condition for a pure strategy non-evasion equilibrium to exist is the trivial case where evasion is a dominated strategy. This is the case whenever the fixed evasion costs K are higher than the maximal gain from evasion ty. In terms of the evasion opportunity we have a pure strategy non-evasion equilibrium, whenever

$$\omega \leq 0. \quad (18)$$

3.3 Hybrid equilibrium

For all the cases where the evasion opportunity is too low for a pure strategy evasion equilibrium, but too high for pure strategy non evasion, we can find a hybrid equilibrium. This is an equilibrium where one type (in our case $y^a = 0$) plays a pure strategy, while the other type ($y^a = y$) randomises. To find this equilibrium we have to find the evasion probability α that yields the same payoff in equilibrium as reporting truthfully. To do so we

[17] ER is equal to $fty\lambda^3/(\eta + \lambda)^2 > 0$.

[18] Note that ϕ again is the abbreviation for $y^a = y$, $d = 0$.

use the first-order conditions for optimal efforts as functions of α. Then we solve for the α that guarantees the taxpayer the honesty payoff $y(1-t)$. The first-order conditions for the efforts are:

$$\begin{aligned}
\frac{\partial}{\partial a}ER(a,e,\alpha,0) &= \mu(\alpha)\frac{e\cdot f\cdot t\cdot y}{(a+e)^2}-1\leq 0\\
\frac{\partial}{\partial e}EU(e,a,\alpha,y) &= \alpha(\frac{f\cdot t\cdot y\cdot a}{(a+e)^2}-c)\leq 0
\end{aligned}$$

Solving simultaneously for the optimal effort depending on α leads to

$$\begin{aligned}
a^*(\alpha) &= \frac{\mu(\alpha)^2\cdot f\cdot t\cdot\eta}{(\eta+\mu(\alpha))^2}\\
e^*(\alpha) &= \frac{\mu(\alpha)\cdot f\cdot t\cdot\eta^2}{(\eta+\mu(\alpha))^2}
\end{aligned}$$

Equating the resulting expected payoff $EU\left(e^*(\alpha),a^*(\alpha),y\right)$ to the honesty payoff $(1-t)\,y$ gives the equilibrium belief of facing a tax evader the authority has to have, whenever it observes a zero income declaration:

$$\mu(\alpha^*)=\eta\left(\sqrt{\frac{f}{f-\omega}}-1\right) \tag{19}$$

The requirement that this belief has to be consistent with behaviour leads us to the equilibrium probability of evasion that the taxpayer will use in mixing:

$$\alpha^*=\frac{\eta\,(1-\lambda)\left(\sqrt{f}-\sqrt{f-\omega}\right)}{\lambda\left[(1+\eta)\sqrt{f-\omega}-\eta\sqrt{f}\right]} \tag{20}$$

Substituting α^* back into the optimal effort function gives the equilibrium efforts for the hybrid equilibrium:

$$a_h^*=\begin{cases} 0 & \text{if}\quad d=y\\ t\cdot y\cdot\eta\left(\sqrt{f}-\sqrt{f-\omega}\right)^2 & \text{if}\quad d=0\end{cases} \tag{21}$$

$$e_h^*=\begin{cases} t\cdot y\cdot\eta\left(\sqrt{f(f-\omega)}-f+\omega\right) & \text{if}\quad y^a=y\wedge d(\alpha^*)=0\\ 0 & \text{else}\end{cases} \tag{22}$$

Insert figure one about here

Figure 1 shows the dependence of the equilibrium type on the income source parameters. We see that for a lower earning probability λ the concealment opportunity η for every fine level f has to be lower to deter the taxpayer from always evading. The intuition is the following: A lower earning probability reduces the expected recoverable income (including fines) for the authority. The tax authority reduces its detection effort. Knowing this the taxpayer realizes that evading with certainty pays. Note that the auditing office knows (according to its equilibrium beliefs) that the taxpayer will evade, whenever he got the income: but it just does not pay to step up the effort, because the probability of facing a honest taxpayer, who did not earn the income, is too high. Also intuitive is the result that a higher fixed evasion cost K (= lower evasion opportunity to evade ω) ceteribus paribus requires a higher concealment opportunity for a taxpayer to cheat with certainty.

Less intuitive, however, is our result that the detection effort is deterministic. In models with commitment and perfect auditing it may be optimal for the authority to mix between auditing and doing nothing (e.g. Mookherjee and Png, 1989). On the first sight, this feature seems to be very appealing, because we observe in reality that similar tax declarations may trigger different auditing behaviour. However, the superiority of a random audit rule is driven by the restrictive assumptions that audits are perfect, that the authority can commit to an audit strategy, and that taxpayers are risk-averse. The authority commits beforehand to an audit probability that is just high enough to deter every single taxpayer from evasion. Perfect auditing without commitment, does not cause random audits to be optimal.

In our model, where the detection probability is determined by the efforts in a contest, not even allowing for commitment would cause the authority to mix over different detection efforts (see section 4). We do not believe that the observed randomness in auditing stems from a situation where the authority can commit to perfect audits, because that would mean that the authority would knowingly audit honest taxpayers.[19] We believe rather that the tax authority conditions its audits on the belief of facing a tax evader after having received a tax declaration. If we enrich our model by assuming that the authority has limited auditing resources it might become optimal to concentrate resources randomly on single taxpayers. In our view this reason for random audits is the more plausible.

Returning to our main purpose, to examine the effect of tax rates on tax evasion and waste, we can state the following propositions.

Proposition

Proposition 1 *In the imperfect and incomplete information scenario a higher tax rate ceteribus paribus weakly increases tax evasion for a specific income component if there are fixed evasion costs.*

Proof. See appendix. ■

Proposition 2 *In the imperfect and incomplete information scenario a higher tax rate ceteribus paribus weakly increases the wastefully invested resources for a specific income component if there are fixed evasion costs.*[20]

Proof. See appendix. ■

The intuition behind these results is straightforward. A higher tax rate provides stronger incentives for the taxpayer to evade by increasing the possible gain from tax evasion. This effect is strengthened by the fact that the evasion opportunity increases with the tax rate, since the ratio between possible gains and fixed evasion cost becomes more favourable. The tax authority, anticipating the stronger evasion incentives, has an incentive to exert more effort, because the potential revenue to recover rises with higher incentives for evasion. The nature of the contest forces the taxpayer to raise his concealment effort, as well, to keep track with the higher detection effort of the authority. These effects over all lead to more tax evasion, higher efforts, and consequentially to more wastefully invested resources.

[19] In these models in equilibrium all taxpayers are honest.

[20] Here the presence of evasion cost are not necessary. We abstain from strengthening the proposition to avoid the very messy proof.

Insert figure 2 about here

Figure 2 shows how the evasion probability for a certain income component depends on the tax rate (dashed line).[21] For low tax rates the income is reported because the fixed evasion cost are prohibitive $\omega < 0$. As the tax rate rises, the taxpayer (in the hybrid equilibrium) evades with increasing probability, until it pays to evade with certainty if he earned the income component (pure evasion equilibrium). The solid line depicts the expected waste in percent of the expected earned income for the same parameter configuration.

It is also interesting to investigate the role of λ,which is the prior probability that a specific income component is earned. The influence of the earnings probability comes from its relevance for the beliefs the tax authority might have, whenever it observes a zero declaration. A very low earnings probability tells the authority that it is very unlikely to face a tax evader after a zero declaration - even if it believes that the taxpayer evades with certainty if he earns the income. Knowing this, the tax man will not exert a big detection effort. In return it is likely that for the taxpayer evasion will pay: so he evades with certainty. With an increasing earnings probability the expected recoverable income for the tax authority increases. Consequentially, it increases the detection effort. This makes the taxpayer - still evading with certainty - try harder to conceal his evasion. The over-all waste increases with the earnings probability. At a certain level of the earnings probability the detection effort of the authority is becoming so massive that for the taxpayer evasion with certainty no longer pays. The equilibrium switches from pure evasion to the hybrid case. The higher the earnings probability becomes the larger the expected revenue from detection effort for the authority becomes if it believes that it is facing a tax evader. To reduce the tax inspector's belief that he is facing an evader the taxpayer reduces the evasion probability α. In return the authority reduces the effort. The expected waste now falls with an increasing probability that the income source generates the income.

Figure 3 illustrates the intuition above. It shows how the amount of waste depends on the earnings probability λ.[22] The two graphs correspond to two different tax rates (dashed line .4, solid line .25). On the left of the spike the taxpayer is evading with certainty (pure evasion equilibrium), while we have a hybrid equilibrium to the right where the taxpayer mixes between evasion and reporting truthfully.

Insert figure 3 about here

4 Externally enforced incentive compatibility

In this section we consider a situation, where the government externally forces the tax authority to exert as much effort as necessary to deter tax evasion with certainty. We examine the resources required under this regime and compare them to the expected waste under the discretionary audit rule without such an external commitment device.

[21] The parameter settings were $y = 1$, $\eta = 2/3$, $\lambda = .1$, $K = .2$, and $f = 3$.

[22] The parameter settings are the same as for the previous figure.

The reason that in our model the revelation principle does not hold is - besides the restriction of the set of feasible contracts - the fact that we do not allow the authority to commit beforehand to a certain effort level if it observes a declaration of zero. Assume that the government externally enforces an incentive compatible effort level upon the authority. That means the government puts a law or a directive in place that forces the authority to exert an effort level for any zero declaration that makes sure that the taxpayer always truthfully reports his income. Here, it is necessary that this law is common knowledge. Our aim is to examine whether such an external commitment device is suitable for reducing the wastefully invested resources.

Let Ψ denote the set of all possible parameter configurations. Let $\psi \in \Psi$ be a specific parameter configuration. Such a configuration contains values for the earnings probability λ, the tax rate t, the evasion opportunity ω, the concealment opportunity η, and the potential income y.

Then the government wants the authority to exert an effort $a^*(\psi, d)$ such that the incentive constraint (IC) for the taxpayer holds for every possible parameter configuration:

$$EU\left(d = y, y^a = y, a^*(\psi, y), e^*\right) \geq EU\left(d = 0, y = y^a, a^*(\psi, 0), e\right) \quad \forall e \geq 0 \;\; \forall \psi \in \Psi \tag{IC}$$

This just means that the expected payoff of the taxpayer if he earned an income component and declared it is at least as high as if he evaded it. We do not have to bother with the IC for the case the taxpayer did not earn the income component, since it is a dominant strategy to truthfully report zero, no matter what the effort of the tax authority will be. Since we are interested in the minimal a^* we already know the optimal effort, in the case that the taxpayer reports truthfully if he got the income, which has to be zero.

$$a^*(\psi, y) = 0 \;\; \forall \psi \in \Psi \tag{23}$$

The best the taxpayer can do if he is forced to report his earned income is to exert no effort ($e^* = 0$). Therefore, the left hand side of (IC) reduces to $EU(y, y, 0, 0) = y(1-t)$. We also know from our previous analysis that the expected ex post income after evasion decreases with the authorities effort. In order to minimize a^* the authority will choose to make (IC) binding. Then the best a tax evader can do is to choose a concealment effort that maximizes his payoff, given the commitment effort of the authority. (IC) becomes:

$$y(1-t) = EU\left(0, y, a^*(\psi, 0), e^*(a^*)\right) \quad \forall \psi \in \Psi. \tag{24}$$

To solve this problem for $a^*(\psi, 0)$ is straightforward. Using the envelope theorem we maximise $EU(0, y, \cdot)$ with respect to a given a, and choose a such that the equality holds. This leads to the optimal incentive-compatible detection effort for the authority, whenever it observes $d = 0$:[23]

$$a^*(\psi, 0) = \begin{cases} t \cdot y \cdot \eta(2f - 2\sqrt{f(f-\omega)} - \omega & \text{if} \quad \omega > 0 \\ 0 & \text{else} \end{cases} \tag{25}$$

The question is whether this external commitment that deters the taxpayer from cheating is generally resource saving. That this is not the case is easily seen if we express the waste in terms of a percentage of the

[23] Here the taxpayer is indifferent between evading or not evading. With an infinitesimal higher effort deterrence would be certain.

expected income. Then the expected waste W_c for positive evasion probability ω is given by

$$W_{c,\%} = \frac{1-\lambda}{\lambda \cdot y}[t \cdot y \cdot \eta(2f - 2\sqrt{f(f-\omega)} - \omega],$$

which is the effort that is pointlessly exerted in the case of a true income declaration of 0 times the probability that the income is not earned, divided by the expected income. We see that W_c tends to infinity whenever λ approaches zero. Recall the expected waste in our non-commitment scenario, when for small probabilities a pure evasion equilibrium is played:

$$W_{nc,\%} = \frac{(2t \cdot f \cdot \eta \cdot \lambda)}{(\eta + \lambda)^2} + (1-\omega)\, t.$$

In this case the expected percentage waste tends to $t(1-\omega)$ when λ approaches zero. This suggests that the external commitment might be not a good solution for income sources where the earning probability is small. For high earning probabilities this policy might reduce wasted resources. This is documented in the simulation in figure 4, where the solid (dashed) line represents the waste in the non-commitment (commitment) case.

Insert figure 4 about here

This result yields some important policy implications. The presence of a tax evasion contest causes an extra welfare loss. This loss consists of the resources that are unproductively spent on concealment and detection. The analysis above provides some guidelines how a government should organize tax enforcement activities in order to keep this loss as small as possible. The regime appropriate for income sources that are common to most citizens should be different from the regime for sources that generate income for only few people. For likely income sources such as income from dependent employment or interest payments on savings a resource saving enforcement policy has to guarantee that the costs for concealment are prohibitive. A policy that deters evasion of such income components may reduce wasted resources even if it is expensive to set up such a policy. The intuition is straightforward. The resources saved by deterring the many people that earn such income components from investing in concealment may outweigh the costs for conducting the policy. This fact may be a reason why in most countries taxes on income from dependent work and taxes on interest payments are deducted at source. This regime causes considerable costs for firms, banks, and authorities, but makes evasion almost impossible.

The enforcement of taxes paid on income from unlikely sources should consist of audits conducted by an authority with certain discretionary powers. In this case a regime that eliminates all evasion incentives may cost more than it saves concealment costs by deterring the few people that earn such income components from entering a contest.

5 Extensions for the signaling setting

In this section we provide some extentions in order to make the underlying assumptions more realistic. First, we relax the assumption that the income distribution is dichotomous. We consider an arbitrary continuous income distribution and show that a higher tax rate still induces more tax evasion. Finally, we allow for the case where the evasion costs are private information. Then the tax authority does not know what type of taxpayer it faces: a law-obeying citizen or a crook. We show that our results concerning the influence of tax rate on evasion behaviour and wasted resources still holds.

5.1 Non dichotomous probability distributions

Assumption A2 - the taxpayer may have earned an income component or not, and the potential value of the income component is commonly known - is not very realistic. We already argued that it is possible to relax this assumption. However, the analysis gets more complicated if we do so. For this reason we just look for an equilibrium where the taxpayer evades whenever he has an actual income above some cutoff income level. Suppose there exists a commonly known probability distribution over possible values of incomes with density $f(y)$. Let the support of the distribution be bounded between 0 and $\bar{y}$. To make things interesting we need a positive probability that the taxpayer has no income to declare.[24] For convenience we assume a continuous distribution function. So we also have to assume that below a certain threshold y_0 the income de jure is treated as being zero.[25]

The situation the tax authority faces if it observes a declaration of zero is the following: It has to form a belief about the probability that the taxpayer is cheating as well as an expectation of the true income of the taxpayer if he is cheating. Now suppose the authority believes that the taxpayer will cheat whenever he has got an income higher than a certain cutoff income level $\hat{y}$. Then the probability μ_e of facing an evader if a declaration of 0 is observed is given by:

$$\mu_e = \frac{1 - F(\hat{y})}{1 - F(\hat{y}) + F(y_0)}. \tag{26}$$

Consequently, the expected detectable income $y^e(\hat{y})$ can be written as:

$$y_e(\hat{y}) = \mu_e \cdot E(y|y \geq \hat{y}) = \frac{\int_{\hat{y}}^{\bar{y}} y \cdot f(y)dy}{1 - F(\hat{y}) + F(y_0)}. \tag{27}$$

The objective function for the tax man after observing a zero income declaration becomes:

$$ER(a, e, y_e, d|d = 0) = f \cdot t \cdot y_e \cdot p(a, e) - a.$$

The objective function for a taxpayer after earning some income above $\hat{y}$ and declaring $d = 0$ can be written as:

$$EU(e, a, y|y \geq \hat{y}) = y - p(e, a) \cdot f \cdot t \cdot y - c \cdot e - K$$

Maximizing simultaneously with respect to the corresponding efforts leads to:

$$a^* = \frac{f \cdot t \cdot c \cdot y_e^2}{(1 + c \cdot y_e)^2} \tag{28}$$

$$e^* = \frac{f \cdot t \cdot y_e}{(1 + c \cdot y_e)^2} \tag{29}$$

[24] Otherwise there would be no uncertainty for the authority observing a declaration of 0 whether it faces a tax evader or not .

[25] The other possibility to ensure a positive probability of a zero tax liability would be to use a distribution with an atom at $y = 0$.

The resulting expected payoff for the authority has to be positive to justify the effort to be optimal. And in fact, this is the case:

$$ER(e^*, a^*, \hat{y}, d|d = 0) = \frac{f \cdot t \cdot c^2 \cdot y_e^3}{(1 + c \cdot y_e)^2} > 0.$$

The expected payoff after evading for the taxpayer is given by:

$$EU(e^*, a^*, \hat{y}, d|d = 0) = y - f \cdot t \cdot y \cdot \left(1 - \frac{1}{(1 + c \cdot y_e)^2}\right) - K \tag{30}$$

To find the cutoff income level that leads to an equilibrium, we have to find the value of y for that the expected payoff is equal to the payoff from reporting truthfully. Furthermore, we have to check that given this cutoff value evading pays for higher income levels than $\hat{y}$. The level(s) of income where the pay-offs for evading and reporting truthfully are equal is implicitly defined by:

$$y_e = \frac{\sqrt{\frac{f \cdot t \cdot \hat{y}}{K + (f-1) t \cdot \hat{y}}} - 1}{c} \tag{31}$$

Let us assume that there exists at least one $\hat{y}$ that satisfies this equation. To show that evading pays if and only if $y > \hat{y}$ it is sufficient that for the given cutoff income the payoff from evading increases with the actual income y. Differentiating equation 30 and substituting in equation 31 shows that this is true:

$$\frac{\partial EU(\cdot)}{\partial y} = 1 - t + K/\hat{y} > 0. \tag{32}$$

Further investigation shows that there exists at least one $\hat{y}$ satisfying equation 31 if the income distribution is such that the taxpayer earns with positive probability an income higher than the prohibitive fixed evasion cost, i.e. $\bar{y} > K/t$. This is captured by the following lemma.

Lemma 2 *There exists at least one PBE. For $\bar{y} < K/t$ we have a non-evasion, non-effort equilibrium with $d^* = y$. For $\bar{y} \geq K/t$ we have at least one cutoff equilibrium with $d^*(y|y < \hat{y}) = y$ and $d^*(y|y \geq \hat{y}) = 0$.*

Proof. The first part is obvious. If the fixed cost of evasion is weakly higher than the possible gain of evasion - $K \geq ty \; \forall y \in [0, \bar{y}]$ - then, with both players knowing this, the only equilibrium is the non-evasion, non-effort equilibrium.

To proof the existence of a cutoff equilibrium for $K < ty$ we have to show that there exists at least one $y \in [K/t, \bar{y}]$ that satisfies equation 31. To do this we use a simple fixed point argument. Let us denote the right hand side of equation 31 as $g(y)$. $y^e(y)$ and $g(y)$ are continuous. If we can find two income levels y' and y'' $\in [K/t, \bar{y}]$ such that $g(y') < y^e(y')$ and $g(y'') \geq y^e(y'')$ the two functions have at least one intersection and at least one $\hat{y}$ exists.

Let $y' = K/t$, then $g(y\prime) = 0 < \left(\int_{K/t}^{\bar{y}} y \cdot f(y)dy\right) / (1 - F(K/t) + F(y_0)) = y^e(y')$. Also let $y'' = \bar{y}$. Then $g(y'') = [(t\bar{y} - K)/(K + (f-1)t\bar{y})]^{1/2} / c \geq 0$, since $\bar{y} \geq K/t$ and $f > 1$.

We know that $y^e(y'') = \left(\int_{\bar{y}}^{\bar{y}} y \cdot f(y)dy\right) / (F(y_0)) = 0$. It follows that $g(y'') \geq y^e(y'')$. This concludes the proof. ■

Having established the existence of at least one cutoff equilibrium in the case of $\bar{y} \geq K/t$ we have to deal with the possibility of multiple cutoff equilibria. And in fact, if we apply the Intuitive Criterion of Cho and Kreps (1987), we can rule out all equilibria, but the one with the lowest $\hat{y}$.

Lemma 3 *The only cutoff PBE satisfying the Intuitive Criterion for signaling games is the one with the lowest* $\hat{y}$.

Proof. The Intuitive Criterion requires that the receiver (the authority in our case) puts a zero probability on the sender (taxpayer) being type y if the sent message (d) is equilibrium dominated for that type. So for the message $d = y$ equilibrium domination is given whenever the equilibrium payoff from sending $d = 0$ is bigger than the maximum payoff the taxpayer can achieve from sending $d = y$. In our notation:

$$EU^*(d|d = 0, y) > \max EU(d|d = y, y).$$

Now suppose we have two PBE with cutoff values $\hat{y}_l$ and $\hat{y}_h$, where $\hat{y}_l < \hat{y}_h$. All taxpayers who have an actual income (are of type) $y \in (\hat{y}_l, \hat{y}_h)$ and who report truthfully ($d = y$) earn their compliance net income. So we have $\max EU(d|d = y, y) = (1 - t)y$. But in the equilibrium with cutoff $\hat{y}_l$ the taxpayer reports $d = 0$ for all $y \in (\hat{y}_l, \hat{y}_h)$ and realizes an expected payoff that is strictly higher than $(1-t)y$. This has to be the case, because we know that $EU^*(y|y = \hat{y}_l) = (1 - t)y$ (due to equilibrium condition 31) and that $dEU^*/dy > 0$ (see equation 32). This means that the report $d = y$ is equilibrium dominated for types $y \in (\hat{y}_l, \hat{y}_h)$. According to the Intuitive Criterion the authority should put a positive probability on the taxpayer being of type $y \in (\hat{y}_l, \hat{y}_h)$ if he observes $d = 0$. But for beliefs like that, $\hat{y}_h$ is not a solution to equilibrium condition (31). The equilibrium with the higher cutoff value $\hat{y}_h$ is ruled out. Since this is true for any two PBE cutoff values, only the equilibrium with the lowest cutoff value satisfies the Intuitive Criterion. ∎

Examining this remaining equilibrium a bit more closely we can find out that our main result - a higher tax rate leads to more tax evasion - still holds. This leads to the following proposition.

Proposition 3 *If the income is drawn from a continuous distribution where the probability that the taxpayer has no income that is liable to tax is positive the unique cutoff equilibrium satisfying the Intuitive criterion is such that a higher tax rate leads to more expected tax evasion.*

Proof. Given our equilibrium strategy of the taxpayer (evade whenever the actual income is higher than the critical income) ex ante more tax evasion is expected when the critical income level sinks. Hence we have to determine the sign of $d\hat{y}/dt$. We can use the implicit definition of the cutoff income from equation 31. Again, denote the right-hand side as $g(\hat{y}, t)$. Rearranging leads to

$$y_e(\hat{y}) - g(\hat{y}, t) = 0.$$

Implicitly differentiating gives

$$\frac{d\hat{y}}{dt} = -\frac{dg(\hat{y}, t)/dt}{dg(\hat{y}, t)/dy - dy_e(\hat{y})/dy}.$$

The sign for $d\hat{y}/dt$ is negative whenever $dg(\hat{y},t)/dy - dy_e(\hat{y})/dy > 0$, since the numerator is positive - i.e.

$$\frac{d}{dt}g(\hat{y},t) = \frac{k\sqrt{f \cdot \hat{y}}}{2c\sqrt{t}\,[k+(f-1)\hat{y}\cdot t]^{\frac{3}{2}}} > 0.$$

Concentrating on the minimum cutoff value (to ensure the equilibrium satisfying the Intuitive Criterion) we know that $\hat{y}$ is equal to the minimum y solving $y^e(y) - g(y,t) = 0$. If we find a $y' < \hat{y}$ where $y^e(y') > g(y',t)$, then we know that $y^e(y)$ is crossing $g(y,t)$ from above at $\hat{y}$. This would mean that $dg(\hat{y},t)/dy - dy_e(\hat{y})/dy > 0$ has to be true. To show that this is the case we compare $y_e(0)$ and $g(0,t)$. We find:

$$y_e(0) = \frac{E(y)}{1+F(y_0)} > -\frac{1}{c} = g(0,t).$$

This concludes the proof. ■

5.2 Privately known moral cost

In this chapter so far we maintained the rather restrictive assumption that the fixed evasion costs (including the moral cost) are common knowledge. In this section we allow for the more realistic situation where the authority does not know what kind of taxpayer it faces. The taxpayer may be a crook with a high criminal energy (i.e. low moral evasion cost) or a law-abiding citizen with scruples about cheating the government (high moral evasion cost). Tax evasion experiments have shown (e.g. Anderhub et al., 2001) that under the same circumstances some people evade and others do not. In Bayer and Reichl (1997) the evasion behaviour is correlated with personal dispositions (such as egoism etc.) and with attitudes towards government and fiscal system.

For this section we return to our initial assumption that the actual income is dichotomously distributed ($y^a \in \{0, y\}$). The probability that the income y is earned is denoted by λ once again. Now assume here that the fixed evasion cost - including the moral cost - are continuously distributed. The prior distribution is common knowledge. Let the cumulative density function be $G(K)$ with $K \in [\underline{K}, \overline{K}]$

Denote the tax authority's beliefs about facing an evader after having observed a declaration of zero as μ. Then the expected interim payoff functions for the taxpayer (EU) and for the authority (ER) are:[26]

$$EU(e,a,y^a|y^a=y) = \begin{cases} y - p(e,a)\cdot f\cdot t\cdot y - e/\eta - K & \text{for} \quad d=0 \\ (1-t)\cdot y - e/\eta & \text{for} \quad d=y \end{cases}$$

$$ER(a,e,\mu,d) = \begin{cases} \mu\cdot f\cdot t\cdot y\cdot p(a,e) - a & \text{for} \quad d=0 \\ t\cdot y - a & \text{for} \quad d=y \end{cases}$$

Suppose there exists a cutoff value for the fixed evasion cost $\hat{K}$. A taxpayer is honest whenever his realized opportunity is lower ($K > \hat{K}$) and evades for lower values of K. Then the equilibrium strategies are as follows:[27]

[26] We omit EU for $y^a = 0$, since this part of he equilibrium stays the same.

[27] We directly state the equilibrium strategies, since apart from the belief formation the derivation is the same as in section 3.

$$d^* = \begin{cases} 0 & for \quad y^a = 0 \\ 0 & for \quad K \le \hat{K} \wedge y^a = y \\ y & for \quad K > \hat{K} \wedge y^a = y \end{cases}$$

The taxpayer declares no income if he hasn't earned it or if his scruples are so small that evasion pays. He truthfully reports y if the moral costs are too high. The corresponding efforts depending on the beliefs of the tax authority, which in equilibrium have to be consistent with the taxpayer's perception, are given by:[28]

$$e^* = \begin{cases} 0 & for \quad y^a = 0 \\ \frac{\mu(\hat{K})^2 \cdot f \cdot t \cdot \eta}{(\eta + \mu(\hat{K}))^2} & for \quad K \le \hat{K} \wedge y^a = y \\ 0 & for \quad K > \hat{K} \wedge y^a = y \end{cases} . \tag{33}$$

The tax officer conditions his detection effort only on the observed declaration. In equilibrium the optimal detection effort is:

$$a^* = \begin{cases} 0 & for \quad d = y \\ \frac{\mu(\hat{K}) \cdot f \cdot t \cdot \eta^2}{(\eta + \mu(\hat{K}))^2} & for \quad d = 0 \end{cases} . \tag{34}$$

After describing the possible equilibrium declaration and efforts we have to investigate if there are consistent beliefs that support such an equilibrium. We use the equilibrium concept of a PBE. This imposes the requirement that the authority uses Bayes' Rule after observing the declaration. Then the belief μ (of facing an evader after observing a declaration of zero) has to be

$$\mu(\hat{K}) = \frac{\lambda \cdot G(\hat{K})}{\lambda \cdot G(\hat{K}) + 1 - \lambda},$$

which is the probability that the taxpayer earned the income, and has evasion costs not higher than the cutoff value $\hat{K}$, normalized by the probability that the taxpayer declares zero. And in fact, we can establish that we obtain a unique equilibrium for these beliefs.

Proposition 4 *For appropriately bounded evasion costs $K \in (\underline{K}, \overline{K})$ a unique interior cutoff PBE exists when moral costs are private information.*

Proof. For a given cutoff value the expected evasion payoff is obviously decreasing with the moral cost. This means that we have an equilibrium whenever we find a $\hat{K}$ that makes the taxpayer indifferent between reporting and not reporting (given the consistent beliefs) in equilibrium. The cutoff value is now implicitly defined by

$$EU\left(d = 0, e^*(\hat{K}), a^*(\hat{K})\right) = EU\left(d = y, 0, 0\right).$$

Substituting and solving for $\hat{K}$ gives:

$$\hat{K} = t \cdot y \left[1 - f \cdot \left(1 - \left(\frac{\eta}{\eta + \mu(\hat{K})}\right)^2\right)\right]. \tag{35}$$

[28] Note, that the beliefs now also depend on the cutoff value for K.

Inspection shows that the right-hand side is continuous and decreasing in $\hat{K}$, since

$$\frac{\partial rhs}{\partial \hat{K}} = -\frac{2 \cdot f \cdot t \cdot y \cdot \eta^2 \cdot \mu'(\hat{K})}{(\eta + \mu(\hat{K}))^3} < 0$$

with

$$\mu'(\hat{K}) = \frac{(1-\lambda) \cdot \lambda \cdot g(\hat{K})}{(1-\lambda+\lambda \cdot G(\hat{K}))^2} > 0.$$

The left-hand side is continuous and increasing. This ensures that for appropriate $\underline{K}$ and $\overline{K}$ a single interior fixed point exists.[29] ■

Implicitly differentiating the implicit definition of the cutoff value from equation 35 tells us whether more or less tax evasion will take place when the tax rate rises. If the cutoff value increases, people with higher moral cost will begin to evade after a tax rise. And in fact, higher tax rates lead generally to more evasion.[30]

Proposition 5 *Higher tax rates in an interior cutoff PBE lead to more tax evasion when moral costs are private information.*

Proof. Implicit differentiation of condition 35 leads to

$$\frac{d\hat{K}}{dt} = \frac{y\left[1 - f \cdot \left(1 - \left(\frac{\eta}{\eta+\mu(\hat{K})}\right)^2\right)\right]}{1 + \frac{2 \cdot f \cdot t \cdot y \cdot \eta^2 \cdot \partial\mu(\hat{K})/\partial\hat{K}}{(\eta+\mu(\hat{K}))^3}}. \tag{36}$$

First we show that the numerator is positive. From our assumption $K \geq 0$ follows $\hat{K} \geq 0$. If the numerator were negative this would imply a negative $\hat{K}$ in equation 35, since there $\hat{K}$ is defined as t times the numerator in equation 36. This is a contradiction. The denominator is obviously positive, since $\partial\mu(\hat{K})/\partial\hat{K}$ and all the parameters are positive. ■

After having established that higher tax rates lead to more evasion, it is straightforward to show that higher tax rates lead to more resources expected to be wasted in the contest. Higher tax rates intensify the covering/detection contest, because more is at stake. Together with more sources being evaded, this leads to more wasted resources. The following proposition states this result.

Proposition 6 *Higher tax rates in an interior cutoff PBE lead to more resources wasted in the contest when moral cost are private information.*

Proof. See appendix. ■

[29] I turns out that $\underline{K} < ty$, $\overline{K} > ty(1 - f(1 - \eta^2/(\eta + \lambda)^2)$, and $\overline{K} > \underline{K}$ are sufficient for existence. The two conditions ensure that evasion pays at least for some K but nor for all.

[30] There is one interesting exception that we ruled out by assumption in this chapter. If we allow for negative moral cost (psychological gratification from evading) then for very high fines the effect of the tax rate on fines dominates, and higher taxes lead to less evasion.

6 Conclusion

Our main interest in this paper was to examine the impact of the tax rate on tax evasion and the resources spent on concealment and detection activities. Our finding is that higher tax rates lead to more tax evasion. This rather intuitive result does not lead per se to any policy implications based on welfare considerations. We do not want to enter the discussion about the relationship between welfare and tax evasion, since the widely used standard measures for welfare do not appear to be sufficient to make a sensible judgement if we look at tax evasion. The welfare effect of more or less tax evasion measured by some form of social welfare criterion is highly sensitive to the criterion used, to assumptions about the state the economy is in (distortions, public good provision etc.), and to assumptions about individual preferences.[31] Thus it seems to be reasonable to base judgements about the desirability of tax evasion on broader foundations than traditional welfare economics does. What we can say is that if one considers tax evasion as undesirable - which we implicitly do - then lower tax rates might be a good policy measure to reduce it.

More clear-cut are the consequences of our result that higher income tax rates imply more wasteful investment in income concealment and detection. Higher tax rates lead - beside a higher excess burden - to some extra cost. More scarce resources are unproductively absorbed by the contest between taxpayer and tax authority.

Furthermore, our model provides an additional insight into the effectiveness and desirability of measures to prevent tax evasion. We saw that an external commitment device, such as law or governmental directives, which forces the tax authority to make sure that no tax evasion takes place, might not be desirable for income sources which rarely generate income. This is due to the fact that the detection resources needed to induce truthful revelation then become excessive compared to the small small revenue collected.

Additionally, we can provide an explanation - although not explicitly pursued in this chapter - why increasing the fines is not necessarily a cheap and effective measure to deter tax evasion. Higher fines intensify the contest between taxpayer and tax authority and may consequentially lead to more wastefully invested resources. Unscrupulous cheats will not react to higher fines with tax compliance, but will instead step up their effort to conceal their tax fraud. To keep up with them the authority has to intensify its efforts, too. Raising the fines can backfire even more, if some formerly at least partly honest citizens perceive this as unfair. The dissatisfaction caused may reduce scruples and consequently moral evasion costs. For this group of taxpayers the anger effect can possibly compensate for the deterrence effect of higher fines. So it is not clear at all that increasing the fines will reduce tax evasion, while such a policy runs the risk to increase the wasted resources. This adds to the non-economic argument that the principle of proportionality between crime and fine should be maintained.

Although we have not formally modelled it, it is straightforward that reducing opportunities for evasion and concealment is a sensible strategy for reducing tax evasion and waste. There are numerous real world examples of governments trying to reduce these opportunities. Taxation at source reduces evasion opportunities, while the banks' duty to report high pay-ins in cash reduces concealment opportunities. We see in our model that lower concealment opportunities are more effective in reducing the waste, while small evasion opportunities

[31] A detailed discussion can be found in Cowell (1990), chapter 7.

control the extend of evasion more effectively.

Finally, note that all the influence factors on tax evasion known from economic psychology (subsumed under attitudes towards tax system, government and authority) play a role in our model. Such attitudes may be the main influence on the moral cost of evasion (contained in the fixed evasion cost). Dissatisfaction reduces the scruples (hence the moral cost) of evasion. So, beside the technical means, a tax system that is conceived as fair, efficient expenditure policy, and a good government performance may effectively deter tax evasion, as well as reduced opportunities or low tax rates.

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A Proofs of some propositions

This appendix contains proofs for propositions in the main text of the paper.

A.1 Proposition 1

Proof. It is sufficient to show that the probability of cheating α^* in the hybrid equilibrium increases with t and that some income sources that previously were reported with positive probability are evaded with a higher probability as t rises. Note: This also ensures that an income that was previously evaded with certainty will be evaded with certainty after the tax rise. Furthermore, an income source that has been evaded with positive probability will never be declared with certainty, since the condition for the pure non evasion equilibrium is $\omega(t) \leq 0$ and $\omega(t') > \omega(t)$ for $t' > t$.

To show that α^* is rising with t we use equation 20. Since $d\alpha^*/dt = d\alpha^*/d\omega \cdot dw/dt$ and $dw/dt = K/t^2y > 0 \; \forall \; K > 0$ it is sufficient to show that $d\alpha^*/d\omega > 0$ as well. Differentiation leads to:

$$d\alpha^*/d\omega = \frac{(1-\lambda)\eta\sqrt{f}}{2\lambda\sqrt{f-\omega}\left(\eta\sqrt{f}-(1+\eta)\sqrt{f-\omega}\right)^2} > 0$$

Since the fine parameter $f > 1$ and the evasion opportunity $\omega \leq 1$ the derivative is necessarily real. Since $0 < \lambda < 1$, the derivative is positive.

The condition for a taxpayer just to play a hybrid equilibrium was $\omega(t_0) = \lambda \cdot f(2\eta + \lambda)/(\eta + \lambda)^2 - \varepsilon$. A change of the tax rate does not effect the right hand side of this equation. The change on the left hand side is $d\omega/dt = K/ty^2$. It follows $\omega(t') > \lambda \cdot f(1 + 2\eta)/(1 + \eta)^2 - \varepsilon$ if $t' > t_0$ and $K > 0$. This means that for some income sources taxpayers change from the hybrid equilibrium to the pure evasion equilibrium as the tax rate rises. ■

A.2 Proposition 2

Proof. The proof consists of three steps: Firstly (a), we show that an increase in the tax rate increases the waste for parameter configurations that lead to a hybrid equilibrium. Then we do the same for the pure strategy equilibrium (b). To conclude the proof it will be sufficient to show that the waste function is continuous at the point where the hybrid equilibrium becomes a pure evasion equilibrium (c). Note: For an income component where before and after the tax rise a pure non-evasion equilibrium was played ($\omega(t), \omega(t') \leq 0$) the waste remains zero.

(a) We can write the expected waste in the hybrid equilibrium as $W = a^*(\lambda\alpha^* + 1 - \lambda) + (ce^* + K)\lambda\alpha^*$, where α^*, a^* and e^* depend on t.[32] Differentiation with respect to t leads to

$$\frac{dW}{dt} = \frac{da^*}{dt}(\lambda\alpha^* + 1 - \lambda) + \frac{d\alpha^*}{dt}a^* + \lambda\frac{d\alpha^*}{dt}(ce^* + K) + \lambda c\frac{de^*}{dt}\alpha^*.$$

If $d\alpha^*/dt$, da^*/dt, and de^*/dt are positive then dW/dt is positive, too. In the previous proof we showed that $d\alpha^*/dt > 0$ for $K > 0$. Taking the detection effort from equation 21 and differentiating with respect to t leads to:

$$\frac{da^*}{dt} = \eta y\left[(2f - 1 - \frac{2\sqrt{f(f-1+\frac{K}{ty})}(K + 2(f-1)ty)}{K + (f-1)ty}\right]$$

Since we cannot determine the sign globally, we have to look on the values for t that are relevant for the hybrid equilibrium. The lower bound condition is $t > K/y$. Since

$$\frac{da^*}{dt}\Big|_{t=K/y} = \eta y\left(2f - 1 - \frac{K + 2K(f-1)}{K}\right) = 0,$$

$da^*/dt > 0\ \forall t > K/y$ if a^* is convex for $t \geq K/y$. And indeed, a^* is globally convex in t - i.e.

$$\frac{d^2a^*}{dt^2} = \frac{K^2\eta\sqrt{f}}{2t^2\sqrt{(f-1+\frac{K}{ty})}(K + (f-1)ty)} > 0 \text{ for } K > 0,$$

since $f > 1$ by construction.

To finish part (a) we just have to make sure that $de^*/dt > 0$. Here, it is convenient to use equation 22, and again to express ω in terms of t. By differentiation we obtain:

$$\frac{de^*}{dt} = \frac{\eta y}{2}\left[(2 - 2f + \frac{\sqrt{f}(K + 2(f-1)ty)}{ty\sqrt{f - 1 + \frac{K}{ty}}}\right]$$

some manipulation and replacing K/ty by the equivalent $1 - \omega$ leads to:

$$\frac{de^*}{dt} = \frac{\eta y}{2}\left[-(2f-2) + (2f - 2 + 1 - \omega)\frac{\sqrt{f}}{\sqrt{(f-\omega)}}\right].$$

[32] e^* and a^* are to be understood as the equilibrium efforts in the case that the income is earned and evaded. To simplify the notation we drop the subscript h.

Since $2f-2+1-\omega > 2f-2$ and $\sqrt{f}/\sqrt{(f-\omega)} > 1$ for $\omega \in (0,1)$ the term in brackets, and also $de^*/dt > 0$ for the relevant range of the evasion opportunity ω.

(b) The expected waste in the pure evasion equilibrium is $W = a_p^* + \lambda e_p^*/\eta + \lambda K = (2tf\eta y\lambda^2)/(\eta+\lambda)^2 + \lambda K$ (from equation 15 and 16), which obviously increases with t.

(c) Suppose there exists (obtained from equation 17) a

$$\hat{t} = \frac{K}{y}\frac{(\eta+\lambda)^2}{\eta^2 - 2(f-1)\eta\lambda - (f-1)\lambda^2} > \frac{K}{y},$$

which is the maximum t for that a hybrid evasion equilibrium is obtained. If it does not exist then the parameters do not allow for a hybrid equilibrium, and we do not have to check for continuity. To show that the waste function is continuous at $\hat{t}$, where the hybrid equilibrium becomes a pure evasion equilibrium, we have to show that

$$W_p(\hat{t}) = \lim_{t->\hat{t}+} W_h(t).$$

This is equivalent to

$$a_p^*(\hat{t}) + \frac{\lambda}{\eta}e_p^*(\hat{t}) = (\lambda \cdot \lim_{t->\hat{t}+} \alpha^*(t) + 1 - \lambda) \cdot \lim_{t->\hat{t}+} a_h^*(t) + \frac{\lambda}{\eta} \cdot \lim_{t->\hat{t}+} \alpha^*(t) \cdot \lim_{t->\hat{t}+} e_h^*(t).$$

The condition above is obviously fulfilled if

$$\lim_{t->\hat{t}+} \alpha^*(t) = 1, \ \lim_{t->\hat{t}+} a_h^*(t) = a_p^*(\hat{t}), \text{ and } \lim_{t->\hat{t}+} e_h^*(t) = e_p^*(\hat{t}).$$

Using the definition of α^* from equation 20, replacing ω by $1 - K/ty$, and taking the right-hand limit at $\hat{t}$ leads to

$$\lim_{t->\hat{t}+} \alpha(t) = \frac{\eta(1-\lambda)\left(\sqrt{f} - \frac{\sqrt{f}\eta}{\eta+\lambda}\right)}{\lambda\left[(1+\eta)\frac{\sqrt{f}\eta}{\eta+\lambda} - \eta\sqrt{f}\right]} = 1.$$

Using the definitions of the pure evasion equilibrium efforts from equations 15 and 16, expressing c as $1/\eta$, and substituting $\hat{t}$ gives:

$$\begin{aligned} a_p^*(\hat{t}) &= \frac{f \cdot \eta \cdot \lambda^2 \cdot K}{\eta^2 + 2 \cdot \eta \cdot \lambda(f-1) + \lambda^2(f-1)} \\ e_p^*(\hat{t}) &= \frac{f \cdot \eta^2 \cdot \lambda \cdot K}{\eta^2 + 2 \cdot \eta \cdot \lambda(f-1) + \lambda^2(f-1)}. \end{aligned}$$

Taking the limits of $a_h^*(t)$ and $e_h^*(t)$ at $\hat{t}$ in equations 21 and 22 (knowing that $\lim_{t->\hat{t}+} \mu = \lambda$) yields

$$\begin{aligned} \lim_{t->\hat{t}+} a_h^*(t) &= \frac{f \cdot \hat{t} \cdot y \cdot \eta^2 \cdot \lambda}{(\eta+\lambda)^2} = a_p^*(\hat{t}) \\ \lim_{t->\hat{t}+} e_h^*(t) &= \frac{f \cdot \hat{t} \cdot y \cdot \eta \cdot \lambda^2}{(\eta+\lambda)^2} = e_p^*(\hat{t}), \end{aligned}$$

if $\hat{t}$ from equation 17 is plugged in. This concludes the proof. ■

A.3 Proposition 6

Proof. The expected waste can be written as

$$EW = \left(\lambda \cdot G\left(\hat{K}(t)\right) + 1 - \lambda\right) \cdot a^* + \lambda \cdot G\left(\hat{K}(t)\right) \cdot \frac{e^*}{\eta},$$

which is the probability that the tax authority observes a declaration of 0 times the corresponding effort plus the probability that the taxpayer has the income and evades, multiplied by the covering effort in that case. ■

Proof. Observing from the optimal efforts in equations 33 and 34 that $e^*/\eta = a^* \cdot \mu[t]$ we can rewrite EW as

$$EW = a^* \left(\lambda \cdot G\left(\hat{K}(t)\right)(1 + \mu(t)) + 1 - \lambda\right).$$

Differentiating with respect to t leads to

$$\begin{aligned}\frac{d}{dt}EW &= \frac{\partial}{\partial t}a^* \left(1 - \lambda + \lambda \cdot G\left(\hat{K}(t)\right) \cdot (1 + \mu(t))\right) + \\ &+ \lambda \cdot a^* \left((1 + \mu(t)) \cdot \frac{\partial}{\partial t}G\left(\hat{K}(t)\right) + G\left(\hat{K}(t)\right) \cdot \frac{\partial}{\partial t}\mu(t)\right).\end{aligned}$$

Obviously, since $0 \leq \lambda \leq 1$, $G\left(\hat{K}(t)\right) \geq 0$, and $\mu(t) \geq 0$, this expression is positive, whenever the partial derivatives of $G\left(\hat{K}(t)\right)$, $\mu(t)$, and a^* are non-negative.

$$\frac{\partial}{\partial t}G\left(\hat{K}(t)\right) = \frac{\partial}{\partial t}\hat{K}(t) \cdot \frac{\partial}{\partial \hat{K}(t)}G\left(\hat{K}(t)\right) = \frac{\partial}{\partial t}\hat{K}(t) \cdot g\left(\hat{K}(t)\right) > 0$$

This is true, because $g\left(\hat{K}(t)\right)$ is positive as a density function, while we established $\partial \hat{K}(t)/\partial t > 0$ in proposition 5. Differentiating the equilibrium beliefs gives:

$$\frac{\partial}{\partial t}\mu(t) = \frac{(1 - \lambda) \cdot \lambda \cdot \frac{\partial}{\partial t}G\left(\hat{K}(t)\right)}{\left[1 - \lambda + \lambda \cdot G\left(\hat{K}(t)\right)\right]^2} > 0.$$

This follows from knowing that $G\left(\hat{K}(t)\right)$ increases with t.

It remains to be checked whether the effort of the authority increases with t:

$$\frac{\partial}{\partial t}a^* = f \cdot y \cdot \mu(t) \cdot \frac{(\eta + \mu(t)) \cdot \mu(t) + 2t \cdot \eta \cdot \frac{\partial}{\partial t}\mu(t)}{(\eta + \mu(t))^3} > 0$$

Note: That the strict inequalities are induced by an interior cutoff equilibrium - i.e. $\underline{K} < \hat{K} < \overline{K}$. ■

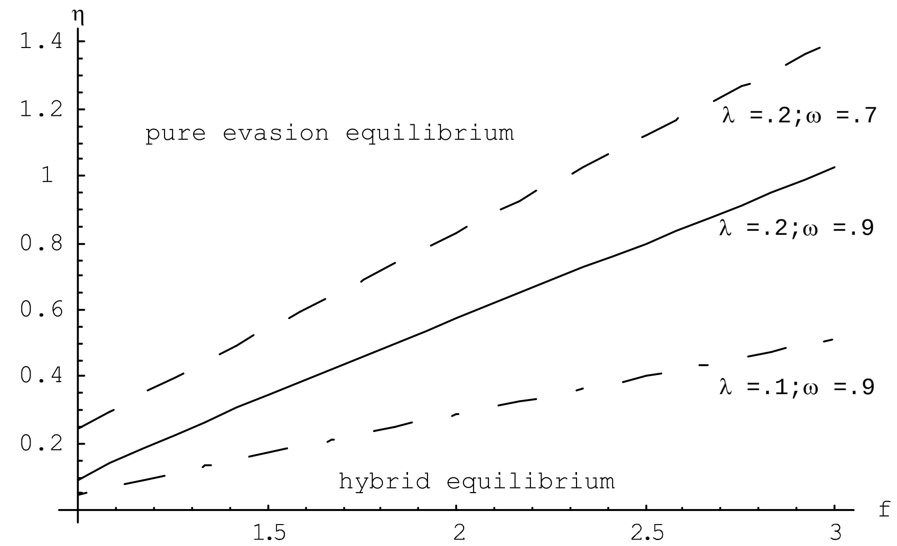

Figure 1: Parameter configurations for pure evasion and hybrid equilibria

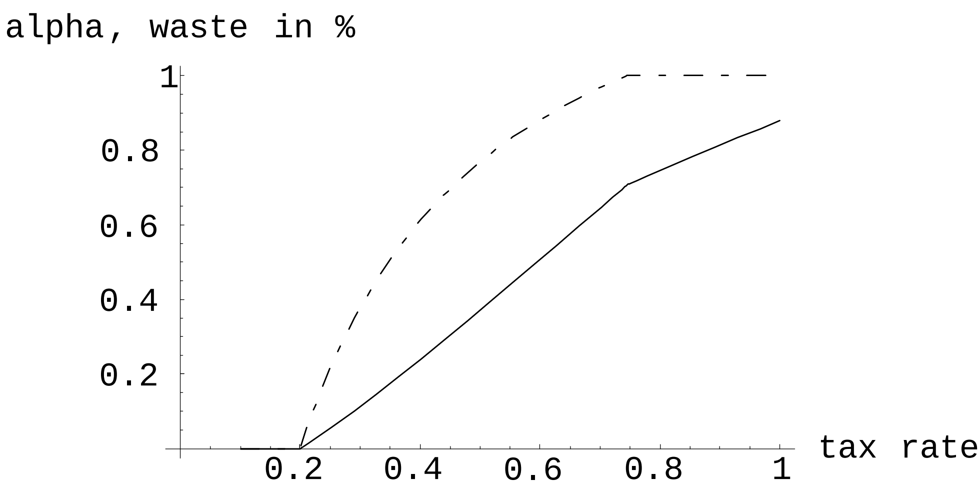

Figure 2: Evasion probability and waste pecentage for different tax rates

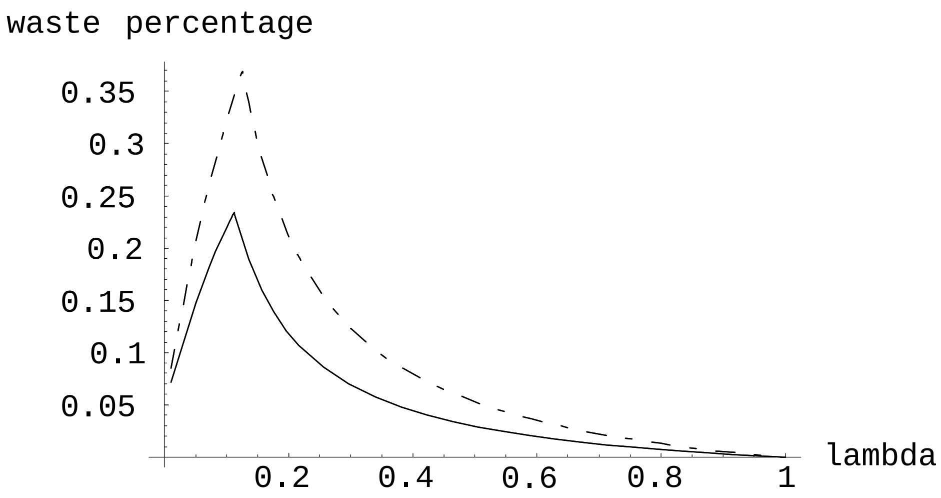

Figure 3: Waste percentage for different earning probabilities

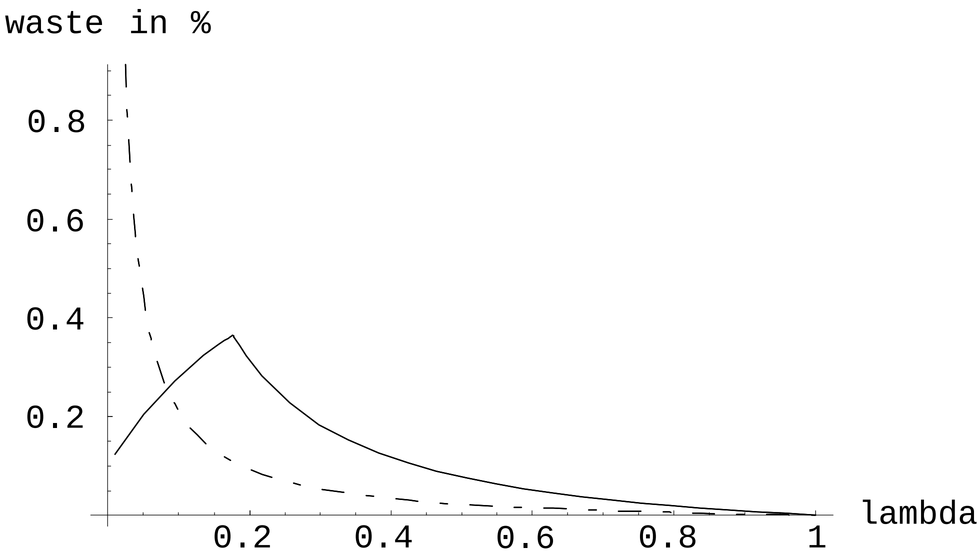

Figure 4: Waste percentages for the commitment and non-commitment case