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Intersociety Literacy Comparisons

Satya R. Chakravarty [1] and Amita Majumder [2]

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Abstract

Basu and Foster (1998) characterized a sophisticated literacy measure using five axioms. In this paper we argue that if a measure satisfies three of their five axioms, namely, anonymity, monotonicity and externality, then also it becomes suitable in some applications. We, therefore, introduce two classes of measures whose members will satisfy at least these three axioms. Two population principles for intersociety literacy comparisons are also suggested and their relationships with the Basu-Foster axioms are established. Finally, we illustrate our results numerically using Indian data and draw out some policy implications.

Keywords: literates, isolated illiterates, proximate illiterates, literacy measures, illustration, policy implication

JEL classification: I21

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[1] Corresponding author: Economic Research Unit, Indian Statistical Institute; email: satya@isical.ac.in

[2] Economic Research Unit, Indian Statistical Institute

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UNU World Institute for Development Economics Research (UNU-WIDER)
Katajanokanlaituri 6 B, 00160 Helsinki, Finland

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1 Introduction

Literacy is an individual's first step in knowledge-building. Therefore, literacy figures are essential in any quantification of human development. For instance, in the construction of the human development index, UNDP (1990-2003) used literacy as one of the key indicators of human development.

The most well-known measure of literacy (MOL) is the literacy rate, the proportion of adult population that is literate. However, as pointed out by Basu and Foster (1998), this measure ignores the positive impact of the presence of a literate person in a household on the illiterate persons of the household. More precisely, this measure does not take into account the fact that the illiterate persons of a household can benefit from the knowledge of a literate person in the household. The essential idea underlying this notion of benefit is that a literate person confers positive externality identically on all illiterate persons of the household to which he belongs. In other words, within a household literacy can be regarded as something like a pure public good, which is characterized by nonrivalry and nonexclusiveness. By nonrivalry we mean here that one illiterate person's benefit from the knowledge of a literate person in the household does not reduce the amount of benefit that another illiterate person in the same household can derive. On the other hand, nonexclusiveness means that the benefit an illiterate member of a household receives from having a literate person in the household does not exclude another illiterate person of the household from enjoying the same benefit. This kind of intra-household externality can arise in many ways. For instance, for filling in an official form an illiterate person can take the help of a literate person in the household.

Now, an illiterate person will belong either to (i) a household that has one or more literate persons or (ii) a household that has no literate person. Basu and Foster (1998: BF hereafter) referred to the first type of illiterate as a proximate illiterate (since he has proximity to literacy because of the presence of a literate person in the household) and the second type of illiterate as an isolated illiterate. In order to distinguish between these two types of illiterates, BF assumed that each proximate illiterate person counts for α literate persons, where α is a number lying between zero and one and an isolated illiterate is regarded as a 'zero literate' person. Thus, in 'literacy-equivalent' terms every proximate illiterate person has a status that lies somewhere in between that of complete illiteracy and that of complete literacy. They also suggested a measure, the 'effective literacy rate' that takes this into account. This measure is the usual literacy rate plus α times the fraction of proximate illiterates in the population.

BF provided a set of axioms that exactly characterizes the effective literacy rate. These axioms are externality, anonymity, monotonicity, normalization and decomposability. Externality requires, under ceteris paribus assumption, literacy of a population to decrease or remain unaltered according as a split of a household in the population is externality-reducing or externality-neutral. A household split is called externality-reducing (externality-neutral) if it creates (does not create) isolated illiterates. Anonymity demands that any characteristic other than literacy status of individuals or household is irrelevant to the measurement of literacy. Monotonicity means that the level of literacy of a population rises if, given other things, an illiterate person becomes literate. According to normalization, the literacy measure should take on the values zero and one in extreme cases of complete illiteracy and complete literacy, respectively. Finally, decomposability says that for any partitioning of the population into subgroups with respect to characteristics like race, region, religion etc., the overall literacy of the population is the weighted average of subgroup literacy levels, where the weights are the population shares of the subgroups.

While anonymity, monotonicity and externality are quite appealing, the remaining two axioms appear to be debatable to some extent. Any bounded measure of literacy can be made to satisfy the normalization axiom under suitable transformations. Sen (1992: 106) questioned the appropriateness of a poverty separability condition that parallels the decomposability axiom. He believed that one subgroup's poverty may be affected by what happens to other subgroups.[1] Clearly, if the literate persons of a subgroup can positively affect the literacy status of another subgroup, then the appropriateness of the literacy decomposability axiom also becomes questionable. To understand this more explicitly, note that one implication of subgroup decomposability is that for any society the overall literacy level can be expressed as the weighted average of literacy levels of individual households, where the weights are the adult population proportions of the respective households. Now, in India, particularly in rural areas, a person belonging to an illiterate family usually takes literal help from a literate neighbour in many respects, for example, for filling in a bank withdrawal form, reading and writing letters and so on. Therefore, in such a situation subgroup decomposability does not appear to be a suitable property. While we do not wish to claim that normalization and decomposability are always undesirable postulates, in view of the above discussion we can probably maintain the view that if a literacy index fails to meet these two properties but satisfies the remaining three, it cannot be discarded outright and may be suitable in some appropriate situations where normalization and decomposability do not hold.

Therefore, in this paper we first suggest a class of literacy measures whose members may or may not satisfy decomposability but satisfies the remaining four axioms. Evidently if some member of this class meets decomposability as well, then it must be the BF measure. This class can then be extended to a larger class of measures whose members will fulfil anonymity, monotonicity and externality but not necessarily the other two BF postulates.

To analyse the BF axioms further, we establish their independence, where independence means that none of these axioms implies or is implied by one or more of the other four. This shows that none of the BF axioms is redundant for deriving their measure. Next, as an extension of the BF exercise, we also propose two population principles for comparing literacy across societies. The first principle demands that if each household in a society is replicated any number of times, then the literacy levels of the original and replicated societies are the same. According to the second principle, literacy remains unchanged if replication occurs within all households not affecting their number. However, it turns out that decomposability along with anonymity implies the first principle. Thus, decomposability subsumes a weaker property, the first population principle, within an anonymous framework. Analogously, the second principle drops out as an implication of decomposability, anonymity and neutral part of externality. However, the reverse implications are not true. Note that since the BF index is decomposable, anonymous and invariant to

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1 For detailed discussion on the poverty separability axiom, see Anand (1983); Chakravarty (1983, 1990); Foster, Greer and Thorbecke (1984); Cowell (1988) and Foster and Shorrocks (1991).

externality-neutral splits, it is capable of making intersociety literacy comparison by either of the two population principles. The two classes of measures that we suggest in this paper also satisfy the two population principles.

We also illustrate different measures numerically using the national sample survey household level data for the rural sectors of seven states in India for the year 1993-93 and derive some policy implications of intra-household positive externality from literacy. It emerges that the literacy ranking of the states by a measure is sensitive to the values of α as well as to the functional forms of measures for the same value of α.

The paper is organized is follows. The following section starts with a discussion and demonstration of the BF axioms. Then we discuss the two population principles and their analytical relationship with decomposability. Next, we introduce the new classes of literacy measures. Section 3 provides the numerical illustration. Finally, section 4 concludes.

2 Properties for a measure of literacy, their implications and several new measures

Consider a society consisting of k households that contain n adults. The literacy profile of household h (h=1,…k) is given by the vector x^h, where x_j^h, the j-th coordinate of x^h, takes on the value 1 or 0 according as the jth member of household h is literate or illiterate. When we say that a person is literate, we are assuming that he/she has fulfilled some unambiguous criterion for literacy. For instance, according to Census of India (1991) a person is regarded as literate if he/she ‘can read or write with understanding in any language. However, a person who can merely read but cannot write is not literate’. The term ‘society’ is used to refer to the vector of household literacy profiles $x=(x^1,\ldots,x^k)$. Evidently, x represents the literacy levels of the society as well as a household structure. The literacy profile x^0 associated with x is obtained by concatenating the household vectors in x. For instance, consider a society of three households with two, three and four adult members respectively. Assuming that there is no literate in the first household and that only the first two members of the second household and the first member of the third household are literates, we have x=((0,0), (1,1,0), (1,0,0,0)) and x^0=(0,0,1,1,0,1,0,0,0).

Let D^n be the set of all societies with adult population size n. Note that two arbitrary societies in D^n may or may not have the same number of households. The set of all possible societies with arbitrary adult population size and number of households is $D=U_{n\in M} D^n$, where M is the set of positive integers. For any function f: $D \to R^1$, the restriction of f on D^n is denoted by f^n, where $n \in M$ is arbitrary and R^1 is the real line. For any $n \in M$, $x \in D^n$ we write $n_l(x)$ for the number of literate persons in the society x. For any k-household society $(x^1 \ldots. x^k) \in D^n, n \in M,$ the number of adult persons in x^h, h=1,…,k, is denoted by a_h. Evidently, $\sum_{h=1}^{k} a_h = n$.

Now, as stated in the introduction, an illiterate person is a member of a household that contains either at least one literate person or no literate person. Clearly, in the former case the illiterate person can derive literacy benefit from having a literate person in the household. We can, therefore, call this person proximate illiterate and regard him/her as an $\alpha(0 < \alpha < 1)$ literate person. In contrast, in the latter case the illiterate person can be called isolated illiterate because he/she does not get any literacy benefit from a member of the household. Evidently, α determines the extent to which a literate person's knowledge is able to help the illiterates literally. As BF argued, the value of α is to be determined from empirical estimation (see also Basu, Foster and Subramanian 2000). For any $x \in D^n$, where $n \in N$ is arbitrary, the number of proximate literates in *x* is denoted by $n_p(x)$.

The effective literacy profile of household *h*, $\hat{x}^h$, can be defined as

$$\hat{x}_j^h = \begin{cases} 1 & \text{if} \quad x_j^h = 1, \\ \alpha & \text{if} \quad x_j^h = 0 \quad \text{and} \quad x_q^h = 1 \text{ for some } q \neq j, \\ 0 & \text{if} \quad x_q^h = 0 \quad \text{for every } q. \end{cases}$$

The society effective literacy profile is given by $x^* = (\hat{x}^1, \hat{x}^2, ..., \hat{x}^k)^0$. The number of effective literates in household *h* in the vector x is $\sum_{j \in h} \hat{x}_j^h = e_h(x)$. For the purpose at hand we need some more preliminaries. For all $n \in N$, $x \in D^n$, we say that $y \in D^n$ is obtained from x by a 'simple increment' if y_j^h=1 and x_j^h=0, while $y_i^r = x_i^r$ for all $(r,i) \neq (h,j)$ and we write *yCx* to indicate this. That is, the societies x and y are identical except that one illiterate person (*j*) in x^h is becoming literate in y^h. We say that the (*k*+1)-household society $y \in D^n$ is obtained from the *k*-household society $x \in D^n$ through a household split if some household in *x*, say *r*, is broken down into two households, which are households *r* and (*r*+1) in *y*, while all the households numbered from 1 to (*r*-1) are the same in *x* and *y*, and household *h* in *x* is same as household (*h*+1) in *y*, where *h*=*r*+1,…,*k*. Equivalently, *x* is obtained from *y* as follows: the households numbered 1 to (*r*-1) in *x* are the corresponding households in *y*, household *r* in *x* is a concatenation of households *r* and (*r*+1) in *y*, and the *h*-th household in *x* is the (*h*+1) th household in *y*, where *h*=*r*+1,…,*k*. Thus,

$$\begin{aligned} x^h &= y^h, \quad 1 \le h < r, \\ &= y^h \, T y^{h+1}, \quad h = r, \\ &= y^{h+1}, \quad r < h \le k, \end{aligned}$$

where $y^h T y^{h+1}$ denotes the concatenation of the vectors y^h and y^{h+1}. For the given example of *x*, let *y*=((0,0), (1,1), (0), (1,0,0,0)). Then we have $x^1=y^1$, $x^3=y^4$ and $x^2=y^2 Ty^3$.

The household split will be called 'externality-neutral' if either (i) both y^r and y^{r+1} contain a literate person, or (ii) neither of y^r or y^{r+1} contains a literate person. We denote this relationship between x and y by yNx. The split is called 'externality-reducing' if exactly one of y^r and y^{r+1} contains a literate person and this is denoted by yEx. In the example considered above, the relationship yEx holds. A society $x \in D^n$ is called 'completely literate' if x_j^h =1 for all j and for all h ; x is 'completely illiterate' if x_j^h =0 for all j and h.

A measure of literacy H is a real valued function defined on D, that is, $H: D \to R^1$. Thus, for any $x \in D^n$, where $n \in M$ is arbitrary, $H^n(x)$ denotes the extent of literacy of the society x.

BF laid down the following desiderata for a measure of literacy (MOL).

Anonymity (ANY)

For all $n \in M, x \in D^n$, if $y \in D^n$ is obtained from x by either a reordering of households or a reordering of members of a household, then $H^n(y)=H^n(x)$.

Monotonicity (MON)

If $x,y \in D^n$, where $n \in M$ is arbitrary, are related as yCx, then $H^n(y)>H^n(x)$.

Externality (EXT)

For all $n \in M$, $x \in D^n$, suppose that $y \in D^n$ is obtained from x through a household split. Then (a) $H^n(y)=H^n(x)$ if yNx holds, and (b) $H^n(y)<H^n(x)$ if yEx holds.

Normalization (NOM)

For all $n \in M$, $x \in D^n$, $H^n(x)=1$ if x is completely literate and $H^n(x)=0$ if x is completely illiterate.

Subgroup decomposability (SUD)

For $x^{(n_i)} \in D^{n_i}$, i=1,2,…m, where $x^{(n_i)}$ is an n_i-member adult person society,

$H^n(x)=\sum_{i=1}^{m} \frac{n_i}{n} H^{n_i}(x^{(n_i)})$, where $x= x^{(n_1)}, x^{(n_2)},...x^{(n_m)}$ and $n=\sum_{i=1}^{m} n_i$.

Since we have already mentioned these properties in the introduction, and they have also been discussed earlier by BF, we skip any further discussion on them here.

In addition to the above axioms considered by BF, we also suggest the following as a postulate for an MOL.

Principle of population (POP)

For all $n \in M, x \in D^n$, $H^n(x)=H^{mn}(y)$, where y is the society obtained by replicating m times each household in x.

According to POP, for an m-fold replication of each household of the society, the degrees of literacy of the replicated and the original populations are the same, where $m \geq 2$ is arbitrary. Thus, POP leads us to view literacy in proportional terms. Evidently, POP is helpful for cross population comparisons of literacy.

An alternative to POP can be the following:

Alternative population principle (APP)

For all $n \in M, x \in D^n$, $H^n(x)=H^{mn}(z)$, where z is the society obtained from x by replicating the individuals within each household m times.

Given $x \in D^n$, y in POP and z in APP have the same population size mn. But the essential difference between y and z is that y has a higher number of households than z. In fact, the number of households in y is mk, where k is the common number of households in x and z.

The most commonly used MOL is the literacy rate, the proportion of adult population that is literate. Formally, the literacy rate $A: D \to R^1$ is defined as

$$A^n(x) = \sum_{i=1}^{n} \frac{x_i^0}{n} = \frac{n_\ell(x)}{n}, \quad (1)$$

where $n \in M$ and $x \in D^n$ are arbitrary. As observed by BF, A^n satisfies all their axioms except part (b) of EXT. It satisfies POP and APP also.

BF suggested a more sophisticated MOL, the effective literacy rate $B: D \to R^1$, where for all $n \in M, x \in D^n$,

$$B^n(x) = \sum_{i=1}^{n} \frac{x_i^*}{n} = A^n(x)+\alpha\, P^n(x), \quad (2)$$

where $P^n(x)=n_p(x)/n$ is the proportion of proximate illiterates in x. B^n can be rewritten as

$$B^n(x) = \alpha(1-C^n(x))+(1-\alpha)\, A^n(x), \quad (3)$$

where $C^n(x)$ is the fraction of isolated illiterates in x. The number $1-C^n(x)$ gives the proportion of population in x with one or more literate persons in the household and is equal to $A^n(x)+P^n(x)$. This has also been used as an indicator of literacy (see Rogers and Herzog 1966, and Sharma and Retherford 1993). Thus, B^n is a convex mix of the two indicators of literacy $1-C^n$ and A^n, with weights α and $1-\alpha$, respectively. If $\alpha=0$, $B^n(x)=A^n(x)$. On the other hand if $\alpha=1, B^n$ becomes $1-C^n$. Subramanian (2000) suggested a measure S^n, which is given by the product of A^n and $1-C^n$. In the particular case $\alpha = A^n$, B^n and S^n are related by $B^n(x) - S^n(x) = A^n(x)(1-A^n(x))$.

BF demonstrated that B^n is the only MOL that verifies their five axioms. It meets POP and APP also.

We now show that the five BF axioms are independent. The demonstration involves construction of an indicator that will satisfy any four of these axioms but not the remaining one. Thus, independence means that if we drop anyone of these five axioms then the resulting MOL will not be the BF index.

Proposition 1: Axioms ANY, MON, EXT, NOM and SUD are independent.

Proof:

i) The measure (1-($C^n(x)$)) may not satisfy MON but satisfies others.

ii) For a *k*-household society with *n* adult persons, the measure $\sum_{h=1}^{k} a_h \frac{e_h}{n}$ will not remain unchanged under an externality neutral split, thus violating part (a) of EXT. It, however, satisfies the other postulates.

As observed by BF, $A^n(x)$ violates part (b) of EXT but meets others.

iii) The MOL $\delta B^n(x)$, where $\delta > 0$, $\delta \neq 1$, does not fulfil the part of NOM that corresponds to complete literacy, but fulfils others. The MOL $\frac{1+B^n(x)}{2}$ meets all properties except the part of NOM which corresponds to complete illiteracy.

iv) The indicator $\alpha(1\text{-}C^n(x))^{\theta}+(1\text{-}\alpha)(A^n(x))^{\theta}$, where $\theta > 0, \theta \neq 1$, is a violator of SUD but not of others.

v) The measure $(1\text{-}\alpha)\sum_{i=1}^{n}\frac{(x_i^*)^{\eta_i}}{n}+\alpha(1-C^n(x))$ where $\eta_i > 0$, $\eta_i \neq \eta_j$ if $i \neq j$, fulfils all the five axioms except ANY.

The next proposition shows that SUD combined with one or more of the remaining BF axioms implies POP and APP.

Proposition 2: (a) If a literacy measure $H : D \rightarrow R^1$ satisfies ANY and SUD, then it satisfies POP. (b) If a literacy measure $H : D \rightarrow R^1$ fulfils ANY, SUD and part (a) of EXT, then it fulfils APP.

Proof:

(a) Suppose society *y* is obtained by replicating *t* times all households in $x \in D^n$ that consists of *k* households, where household *h* with literacy profile x^h includes a_h individuals giving a total of *n* individuals for *x*. Society *y*, which has a population size of *tn*, is composed of *tk* households. Let $H^{tn}(y)$ be the MOL for *y*. By SUD, we have $H^{tn}(y)=\sum_{h=1}^{tk}\frac{b_h}{tn}H^{b_h}(y^h)$, where y^h is the literacy profile of household *h* in *y*, b_h is the number of persons in y^h and $tn=\sum_{h=1}^{tk}b_h$. Applying anonymity all the MOLs associated to similar replicated households give the same value. Therefore,

$$H^{tn}(y)=\sum_{h=1}^{k}\frac{ta_h}{tn}H^{a_h}(x^h)=\sum_{h=1}^{k}\frac{a_h}{n}H^{a_h}(x^h)=H^n(x),$$

where the last equality is obtained by applying SUD. $H^{tn}(y)=H^n(x)$ is precisely the requirement of POP.

(b) Suppose society *z* is obtained by replicating *t* times all the individuals in every household in society $x\in D^n$. Society *x* is composed of *k* households where household *h* with literacy profile x^h includes a_h individuals, giving a total of *n* individuals for entire *x*.

Similarly, society *z*, by definition, consists of *tk* households, where household *h* includes ta_h individuals giving a total of *tn* individuals for entire z. Let $H^{tn}(z)$ be the MOL for *z*. We can now split each household of ta_h individuals in *z* into *t* identical households of a_h individuals to obtain society *y*. Since the operation is externality neutral, in view of part (a) of EXT, we have $H^{tn}(z)=H^{tn}(y)$. Applying the result in part (a) of the proposition, by SUD and ANY, we get $H^{tn}(y)=H^n(x)$, which gives $H^{tn}(z)=H^n(x)$, the requirement of APP.

It may be important to note that the converse of this proposition is not true. That is, POP and APP do not imply the postulates from which they are derived in Proposition 2. To see this, note that the MOL in (iv) in the proof of Proposition 1 verifies POP and APP, but not SUD. Similarly, the MOL in (v) in the same proof meets POP and APP but not ANY. Finally, we use the first MOL in (ii) in that proof to demonstrate the remaining part of the claim. It is also easy to construct examples which will satisfy POP (APP) but not ANY and SUD (ANY, SUD and EXT(a)) simultaneously.

Since the BF index B^n is a convex combination of the MOLs A^n and (1-C^n), a natural generalization of B^n can be the same convex combination of a transformation of A^n and (1-C^n). More precisely, as a generalization of *B* we may suggest the use of the MOL $G: D\to R^1$ where for all $n\in M, x\in D^n$,

$$G^n(x)=\alpha f(1-C^n(x))+(1-\alpha)f(A^n(x)), \tag{4}$$

where $f: Q\to R^1$, with *Q* being the set of rational numbers in [0,1].

The following proposition identifies the class of all real valued functions defined on *Q* for which *G* verifies NOM and part (b) of EXT.

Proposition 3: The general MOL G defined in (4) satisfies normalization and part (b) of externality if and only if $f(0)=0$, $f(1)=1$ and f is increasing.

Proof:

Suppose that $A^n(x)=0$. This in turn implies that $C^n(x)=1$. Then

$$G^n(x)=\alpha f(0)+(1-\alpha)f(0)=f(0). \quad (5)$$

But in this extreme case by normalization $G^n(x)=0$. This along with (5) gives $f(0)=0$. Next, suppose that $A^n(x)=1$ which gives $C^n(x)=0$. Then

$$G^n(x)=\alpha \text{ f}(1)+(1-\alpha)f(1)=f(1) \quad (6)$$

But normalization says that $G^n(x)=1$ in this case. Using this information in (6) we get $f(1)=1$.

Now, suppose that y has been obtained from x by a household split and the split is externality reducing. Then the split decreases the number of proximate illiterates. Part (b) of EXT then demands

$$G^n(y)=\alpha f(1-C^n(y))+(1-\alpha)f(A^n(y))<\alpha f(1-C^n(x))+(1-\alpha)f(A^n(x))=G^n(x) \quad (7)$$

Since the split does not reduce the number of literates, we have $A^n(y)=A^n(x)$. Therefore, $G^n(y)<G^n(x)$ means that $f(1-C^n(y))<f(1-C^n(x))$, that is, $f(A^n(y)+P^n(y))<f(A^n(x)+P^n(x))$. Since $P^n(y)<P^n(x)$ and $A^n(y)=A^n(x)$, we need increasingness of f for the inequality in (7) to hold. This establishes the necessity part of the proposition. The sufficiency is easy to verify.[2]

Let F be the class of all real valued increasing functions defined on Q that take on the values 0 and 1 at 0 and 1, respectively. More precisely, $f: Q \rightarrow R^1$ is a member of F if f is increasing, and $f(0)=0$ and $f(1)=1$. It is clear that to every $f\in F$ there corresponds a different index of the form (4). These indices will differ only in the manner how we specify f. For any $f\in F$, the underlying MOL G^n, in addition to being affirmatively responsive to part (b) of EXT and NOM, is anonymous, population replication invariant (since A^n and C^n are so), monotonic (since f is increasing) and invariat to externality neutral splits. However, G^n may not fulfil SUD.

Examples of functions which are members of F are

i) $f_1(t)=t^{\theta}, \theta>0$,

ii) $f_2(t)=(e^t-1)/(e-1)$,

iii) $f_3(t)=2t/(1+t)$.

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[2] Note that if the household split is externality neutral, then increasingness of f does not follow as a necessary condition for EXT to hold. However, if f is increasing then the underlying $G^n(x)$ satisfies both parts (a) and (b) of EXT.

Now, the MOL of the form (4) associated with f_1 in (i) becomes $G_\theta^n(x) = \alpha(1-C^n(x))^\theta + (1-\alpha)(A^n(x))^\theta, \theta > 0$. Clearly for $\theta = 1$, which gives $f_1(t)=t$, G_θ^n becomes the BF MOL B^n in (2).Note that for a given society x, which is neither completely literate, nor completely illiterate, $G_\theta^n(x)$ is decreasing in θ. As $\theta \to 0$, $G_\theta^n(x) \to 1$.

We will consider the explicit forms of MOLs corresponding to the functions f_2 and f_3 specified above in section 3. Clearly, we can have infinitely many functions to choose from F. The choice gets widened if in (4) we give up the assumptions f (0)=0 and f (1)=1 but retain increasingness of f. In such a case G^n will fulfil ANY, MON, EXT, POP and APP, but not NOM and also not necessarily SUD. Let $\overline{F}$ be the class of all real valued increasing functions defined on Q. Then $F \subset \overline{F}$. Examples of functions that belong to $\overline{F}$ -F can simply be constructed by adding a non-zero constant term to the examples given above. That is, if we define $g_i(t)=f_i(t)+\delta_i$, where $\delta_i \neq 0$ is a constant, i=1,2,3; then $g_i^{'}$ s are members of $\overline{F}$ -F.

3 Literacy in rural India: an illustration

The purpose of this section is to numerically illustrate the measures suggested in section 2 of the paper using statewise household level literacy data thrown up by the National Sample Survey Organisation (NSSO) for rural India for the period 1993-94. The states considered are Andhra Pradesh (AP), Arunachal Pradesh (AR), Haryana (HA), Karnataka (KA), Manipur (MA), Rajasthan (RA), and Sikkim (SI). The literacy measures chosen for the purpose are the generalized BF index G_θ, and the measures H_2 and H_3, whose restrictions on D^n are given by

$$H_2^n(x) = \alpha \frac{e^{1-C^n(x)} - 1}{e-1} + (1-\alpha)\frac{e^{A^n(x)} - 1}{e-1} \tag{8}$$

and

$$H_3^n(x) = \frac{2\alpha(1-C^n(x))}{2-C^n(x)} + \frac{(1-\alpha)A^n(x)}{1+A^n(x)}, \tag{9}$$

where $n \in M$ and $x \in D^n$ are arbitrary. It may be noted that H_2 and H_3 are special cases of G in (4), corresponding respectively to the functional forms $f_2, f_3 \in F$ considered in section 2.

Numerical estimates of literacy for rural India for the period considered are presented in Table 1. The first column of the table gives the names of the states for which the calculations are done. In columns 2 and 3 we show, for each state, the literacy and proximate illiteracy rates A and P. (Since interstate comparisons involve different population sizes, we drop superscript n from all indices.) Assuming that α=0.25, statewise generalized BF index for three different values of θ (θ=0.5,1.0, and 1.5) are presented in columns 4-6. It may be recalled that for θ=1, G_θ becomes the BF index B.

In columns 7 and 8 we show values of H_2 and H_3 for the same value of α. Columns 9 to 13 present the values of G_θ (θ=0.5,1.0, and 1.5), H_2 and H_3 corresponding to α=0.75. Thus, while in the former case a proximate illiterate is equivalent to one fourth of a literate, in the latter case we have three-fourth equivalence.

Several interesting features emerge from the table. For all ${}^7C_2$=21 pairwise comparable situations we have uniform ranking by all the three measures for α=0.25. The same ranking of the states is obtained for H_3 and $G_{0.5}$ for α=0.75 also. For this latter value of α, uniform ranking by all measures is generated in 20 cases, a disagreement arises for the pair (AP, RA). More precisely, while H_3 and $G_{0.5}$ make AP more literate than RA for both α=0.25 and 0.75, G_θ (θ=1.0 and 1.5) and H_2 agree (disagree) with this ordering for α=0.25 (0.75). From this observation we can conclude two features on literacy ranking. First, alternative measures may generate different orderings of societies for the same value of α (as in the case of α=0.75). Second, a change in the value of α may give rise to a change in ranking by the same measures (as G_θ for θ=1.0,1.5 and H_2 demonstrate this for the pair AP and RA, when α increases from 0.25 to 0.75). This second feature was noted by BF as well. Note that for the pair (AP, RA), we have A(AP)>A(RA), but P(AP)<P(RA). The dominance of the component αP in the calculation of the MOLs (G_θ for θ=1.0, 1.5 and H_2) for high values of α may be the underlying factor for the reverse ordering generated. However, for the remaining MOLs ($G_{0.5}$ and H_3) αP does not become a dominant factor. This clearly shows that the ranking of the societies is sensitive to the value of α.

Table 1
Literacy in rural India, 1993-94

State	Proportion of: Literates (A)	Proximate illiterates (P)	α=0.25 G_θ θ=0.5	θ=1.0	θ=1.5	H_2	H_3	α=0.75 G_θ θ=0.5	θ=1.0	θ=1.5	H_2	H_3
1	2	3	4	5	6	7	8	9	10	11	12	13
Andhra Pradesh (AP)	0.3988	0.4595	0.7052	0.5137	0.3877	0.4116	0.6586	0.8527	0.7434	0.6593	0.6645	0.8354
Arunachal Pradesh (AR)	0.3133	0.5357	0.6501	0.4472	0.3271	0.3552	0.5874	0.8310	0.7151	0.6306	0.6373	0.8080
Haryana (HA)	0.4874	0.4528	0.7660	0.6006	0.4831	0.5012	0.7338	0.9018	0.8270	0.7688	0.7725	0.8907
Karnataka (KA)	0.4799	0.4320	0.7583	0.5879	0.4671	0.4855	0.7249	0.8894	0.8039	0.7362	0.7396	0.8776
Manipur (MA)	0.6496	0.3212	0.8508	0.7299	0.6318	0.6379	0.8370	0.9405	0.8906	0.8484	0.8490	0.9358
Rajasthan (RA)	0.3431	0.5409	0.6744	0.4783	0.3685	0.3853	0.6178	0.8516	0.7488	0.6736	0.6796	0.8316
Sikkim (SI)	0.6321	0.3030	0.8381	0.7079	0.6030	0.6100	0.8226	0.9240	0.8594	0.8039	0.8038	0.9185

A comparison between AR and RA shows that RA has higher values of *A* and *P* than AR and since the three indices combine increasing transformations of *A* and *P* in a positive way, RA becomes more literate than AR by all the measures. The same phenomenon holds for the pair (HA, KA). For no other pair of states presented in the table this characteristic has been found. In such cases we have to consider specific indices for literacy ranking of states.

We conclude this section with some additional observations on the figures presented in the table. By all the indices considered, MA turns out to be the most literate state, whereas AR has the minimum literacy level. The situations for the remaining states are in between these two extremes. It might be of interest to note that although MA has the highest proportion of literates, its proportion of proximate illiterates is rather low. The converse is true for AR. For many states, the proportion of proximate illiterates is significantly greater than the proportion of isolated illiterates. Thus a substantial proportion of population in these states has immediate access to literacy because of intra-household externality. For each state the excess of an index over the literacy rate *A* shows the quantitative impact of the intra-household externality on assessment of literacy. These observations correspond closely to an important policy implication. Consider a cost-constrained literacy campaign programme in a region. Under the programme one person from each illiterate household can be made literate so that other members in the household can take advantage of intra-household positive externality from literacy. Since higher number of households can now be covered by the programme, the society becomes effective literates to a larger extent, which in turn demonstrates a greater success of the programme.

4 Conclusions

The results developed in the paper are based on the assumption that within a household, literacy can be regarded as a pure public good characterized by nonrivalry and nonexclusiveness. That is, all illiterate persons in a household derive literacy benefit from the presence of one or more literate person in the household under the conditions that no illiterate person in the household can be excluded from getting this benefit and one person's benefit does not reduce the level of benefit for another person. As a result, it has been assumed that each illiterate person in a household with one or more literates can be regarded as an $\alpha(0<\alpha<1)$ literate person. Thus, irrespective of the number of literate persons in the household, an illiterate person becomes an α literate person. Further, intrinsic to this beneficial connection between literates and illiterates is the assumption that there is no constraint on the time that a literate person can spend helping illiterates.

Now, it is quite likely that a higher fraction of literate persons may ensure a greater access to literacy skills of illiterates. The time constraint of a literate person is also likely to influence this access. Another issue is gender sensitivity to literacy analysis. The positive externality generated by literacy is likely to be higher if the source of literacy is a female instead of a male. (BF suggested a modification of their measure *B* along this line.) Thus, characteristics other than the sheer existence of a literate person in a household may be quite important in determining the value of α.

The next point concerns the domain of positive impact that a literate person can have on illiterates. In addition to conferring literal benefits to their own household members who are illiterate, a literate person may be able to affect positively the literacy status of their own society, caste, etc. This will depend on the individual's social connection in the community/caste. Extensions of our analysis along these directions will be quite worthwhile.

Since alternative literacy measures may rank two societies in different directions, another line of investigation can be the development of a quasi-ordering such that the ranking of the societies by a set of measures satisfying certain postulates will coincide with that generated by the ordering. This is left as a future research programme.

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