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Past expectations as a determinant of present prices - hysteresis in a simple economy*

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Abstract. We give an illustration of hysteresis (path-dependence) in a simple economy. In the presence of multiple possible equilibrium prices, we find that past expectations determine present prices. This phenomenon of path-dependence is robust under perturbations of the economy.

Keywords: Hysteresis, Path-dependence, Tatônnement, Equilibrium selection.

JEL classification: C62, D50.

1 Introduction

It is known that expectations play a key role in economics. Not only as deriving from an objective reality, but also as primary causes. This idea underlies what Keynes (1936) designated by "animal spirits", as well as the distinction that financial analysts find useful between the "bull market" and the "bear market". It is a consensus that expectations about the future are a determinant of present prices - good examples are the interest rate and the inflation rate. It is, however, not as widely accepted that past expectations may also be a determinant of present prices.

Hysteresis is associated to the permanence of effects from a temporary stimulus and to the idea of path dependence. Such phenomena are at the heart of evolutionary models, but do not appear frequently in mainstream economics. Some exceptions are the studies of international trade by Amable (1985) and Ljungqvist (1994), of investment by Dixit (1991), and of unemployment by Roed (1997).[2]

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[2]See Göcke (2002) and Katzner (1999) for surveys.

In most models, the concern for *uniqueness* and *determinacy* leaves no room for path-dependence. For example, the results of Arrow, Block and Hurwicz (1959) on stability of equilibrium use conditions that are strong enough to guarantee uniqueness. Nevertheless, in the presence of multiple equilibrium prices, the problem of selection arises and path-dependence may become an issue.

In order to study equilibrium selection, we need to make assumptions on the way market prices are formed. A famous endogenous process of reaching equilibrium is "the law of demand and supply": if demand exceeds supply, prices rise; and if supply exceeds demand, prices fall. In the lines of Arrow and Hurwicz (1958), we restrict our analysis to this kind of price formation processes.[3]

Our object of study is an economy with two agents and a single consumption good. There is subjective uncertainty about which of the two possible states of nature will occur, and a problem of risk sharing arises. Agents are subjective expected utility maximizers, having the same state-dependent utility functions but different beliefs. When agents have very asymmetric beliefs, the economy exhibits a multiplicity of equilibria, and this opens the way for the study of path-dependence.

We assume that the beliefs of the agents change during the bargaining process, and that prices adjust continuously. In this setting, we find that the equilibrium price that results from the tatônnement depends on past (transitory) beliefs. This is an example of hysteresis. Past expectations are a determinant of present prices, and this path-dependence is robust under perturbations of the economy under study.

The rest of the paper is organized as follows: in section 2 we present the model of the economy; in section 3 we find hysteresis in a concrete example; and in section 4 we make a qualitative study of price trajectories and equilibrium dynamics.

2 The model

Consider a simple economy with two agents, a single good, and two states of nature. The agents have the same state-dependent utility functions, but different beliefs, $q^i = (q^i_1, q^i_2)$, concerning the probabilities of occurrence of the two states of nature. Denote

[3] We focus on what Arrow and Hurwicz call the "instantaneous adjustment process", but the same conclusions would be reached if the "lagged adjustment process" was assumed instead.

consumption of agent 1 in the two states of nature by (x_1, x_2) and consumption of agent 2 by (y_1, y_2).[4] The objective of the agents is to maximize subjective expected utility:

$$U^1(x_1, x_2; a - \epsilon, \mu) = -x_1^{\frac{\mu}{\mu-1}} - (a - \epsilon)x_2^{\frac{\mu}{\mu-1}}$$
$$U^2(y_1, y_2; a, \mu) = -y_2^{\frac{\mu}{\mu-1}} - ay_1^{\frac{\mu}{\mu-1}},$$

where the expectations of the agents are:

$$q^1 = (q_1^1, q_2^1) = (\tfrac{1}{1+a-\epsilon}, \tfrac{a-\epsilon}{1+a-\epsilon}), \text{ and } q^2 = (q_1^2, q_2^2) = (\tfrac{a}{1+a}, \tfrac{1}{1+a}).$$

The initial endowments of the agents in terms of contingent goods are $e^1 = (1, \theta)$, and $e^2 = (0, 1)$, so the economic problem is of risk sharing.

Normalizing prices to $p_1 = p$ and $p_2 = 1$, and maximizing each agent's utility subject to the budget constraint, we solve for the demand of good 1 to find:

$$x_1(p; a - \epsilon, \mu, \theta) = \tfrac{p+\theta}{p+(a-\epsilon)^{1-\mu}p^{1-\mu}}, \text{ and } y_1(p; a, \mu) = \tfrac{p^{\mu-1}}{a^{\mu-1}+p^{\mu}}.$$

Hence, excess demand in the market for good 1 is:

$$z_1(p; a, \epsilon, \mu, \theta) = (a - \epsilon)^{\mu-1}p^{\mu-1}\tfrac{p+\theta}{1+(a-\epsilon)^{\mu-1}p^{\mu}} + \tfrac{p^{\mu-1}}{a^{\mu-1}+p^{\mu}} - 1.$$

The budget restrictions are obviously active. As suggested by Walras' Law, equilibrium in one market implies equilibrium in the two markets.[5]

We consider the classical tatônnement process, designated in Arrow and Hurwicz (1958) as the "instantaneous adjustment process". It is described by the differential equation: $\dot{p} = z_1(p; a, \epsilon, \mu, \theta)$.

The case considered by Bala (1997), which results in a pitchfork, is similar to the perfectly symmetric economy with $\epsilon = 0$ and $\theta = 0$. This yields:

$$z_1(p; a, \mu) = a^{\mu-1}p^{\mu-1}\tfrac{p}{1+a^{\mu-1}p^{\mu}} + \tfrac{p^{\mu-1}}{a^{\mu-1}+p^{\mu}} - 1.$$

Observe that $p = 1$ is an equilibrium price for all values of the parameters a and μ since $z_1(1; a, \mu) = 0$. Equilibrium price $p = 1$ undergoes a bifurcation when the bifurcation parameter a is such that: $\frac{\partial z_1}{\partial p}(1; a, \mu) = 0$ which occurs for $a^* = (2\mu - 1)^{\frac{1}{1-\mu}}$.

We also have $\frac{\partial^2 z_1}{\partial a \partial p}(1, a^*) > 0$, $\frac{\partial^2 z_1}{\partial p^2}(1, a^*) = 0$, and $\frac{\partial^3 z_1}{\partial p^3}(1, a^*) > 0$, for all $\mu \in [0, 1]$.

[4] Notice that everything applies to economies with two goods. The consumption of a single good in two states of nature can also be seen as consumption of two goods (Arrow, 1953).

[5] It is enough to consider the market for good 1, excess demand for good 2 is $z_2 = -pz_1$.

Hence, from Golubitsky and Schaffer (1985, Prop. II, 9.2) we know that, near $p = 1$ and $a = a^*$, the dynamics of the tatônnement are qualitatively equivalent to those of a subcritical pitchfork.[6]

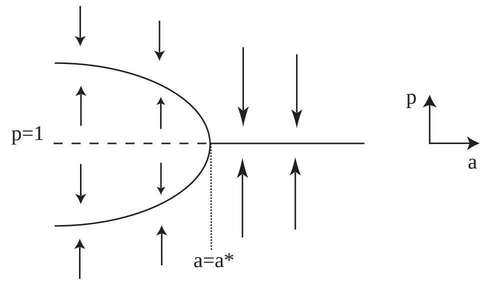

Figure 1: The pitchfork occurs for $a^* = (2\mu - 1)^{1/(1-\mu)}$. Stability of branches is indicated both by solid lines and by arrows representing price movements.

However, the pitchfork occurs only in the fully symmetric economy, and is not robust to all perturbations of the original problem. It may be confirmed that our economy is an *universal unfolding* of the symmetric problem, that is, it includes all the perturbations using as few parameters as possible.[7]

It is known that small perturbations of a problem described by a pitchfork bifurcation may originate different kinds of dynamics. In section 4 we present a qualitative study, but before we provide a numerical example of hysteresis.

3 An Illustrative Example

For concreteness, consider $\mu = 0.8$ and $\theta = 0.6$. Let expectations about the future state of nature be extremely different: use $a = 0.03$ and $\epsilon = -0.04$, which is equivalent to $q_1 = (93.5\%, 6.5\%)$ and $q_2 = (2.9\%, 97.1\%)$.

Solving for equilibrium prices, we obtain three solutions: $\check{p} = 0.353$, $\bar{p} = 1.150$ and $\hat{p} = 4.334$. The intermediate solution ($\bar{p} = 1.150$) is unstable, that is, the law of demand is not satisfied, and therefore, a small perturbation of demand is amplified. Prices adjust either to $\check{p} = 0.353$ or to $\hat{p} = 4.334$.

[6]There is a coordinate system in which these dynamics are described by $\dot{x} = x^3 + \lambda x$.

[7]To see this, construct a matrix of partial derivatives as required to apply Golubitsky and Schaffer (1985, Prop. III, 4.4).

To determine prices and equilibrium allocations, we assume that the adjustment process follows continuous trajectories, according to the "instantaneous adjustment process". This may have multiple interpretations. For example, the trade under study may correspond to one period of the true economy, with endowments and preferences varying each period. The essential assumption is that beliefs are changing and prices adjust in a continuous way.

Suppose that preliminary expectations were such that $a = 0.02$ rather than $a = 0.03$, corresponding to $q_1 = (94.3\%, 5.7\%)$ and $q_2 = (2.0\%, 98.0\%)$. In this economy, equilibrium is unique: $p = 0.141$. If the expectations change continuously to reach $a = 0.03$, the price that results is $\check{p} = 0.353$.

On the other hand, past expectations such that $a = 0.04$, that is, $q_1 = (92.6\%, 7.4\%)$ and $q_2 = (3.8\%, 96.2\%)$, produce a different outcome. Equilibrium is also unique, but $p = 4.043$. Now if expectations changed continuously to $a = 0.03$, the resulting price would be $\hat{p} = 4.334$.

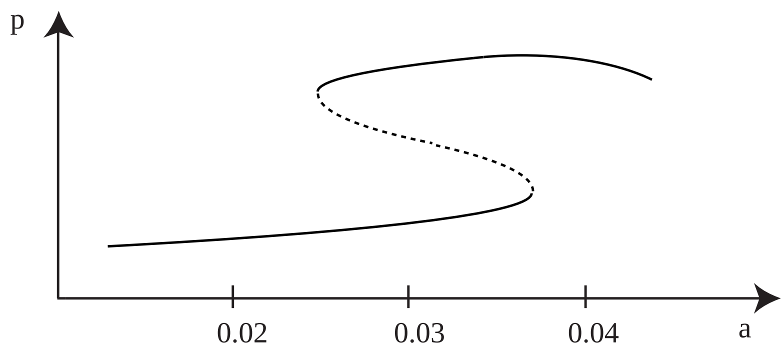

Figure 2: Equilibrium prices for $\mu = 0.8$, $\theta = 0.6$ and $\varepsilon = -0.04$. For $a = 0.02$ and $a = 0.04$ there is only one equilibrium price, $p = 0.141$ and $p = 4.043$, respectively. For $a = 0.03$ there are two stable equilibrium prices given by $\check{p} = 0.353$ and $\hat{p} = 4.334$ and one unstable equilibrium price given by $\bar{p} = 1.150$.

The economy exhibits hysteresis (path-dependence). Past expectations are a determinant of equilibrium prices.

4 Qualitative Bifurcation Diagrams

In this section, we do the local bifurcation analysis to describe of the qualitative behavior of the generic problem in terms of the unfolding parameters ε and θ. Since we are interested uniquely in small perturbations of the original (symmetric) problem around the bifurcation point, we can, without loss of generality, consider the Taylor polynomial of z_1 at $(p^*, a^*) = (1, (2\mu - 1)^{\frac{1}{1-\mu}})$. Furthermore, since a pitchfork is 3-determined (that is, equivalent to its Taylor polynomial of degree 3), it is sufficient to consider Taylor expansion up to order 3. This can be further simplified if we consider the weighted homogeneous setting.[8]

The weighted homogenous Taylor polynomial of weighted degree 3, with respect to weights $(1, 2)$, at (p^*, a^*) of z_1 is given by:

$$G(x, \lambda, \varepsilon, \theta) = a_1 + a_2 x + a_3 \lambda + a_4 x\lambda + a_5 x^2 + a_6 x^3,$$

where $x = p - 1$, $\lambda = a - a^*$, and the coefficients are functions of the unfolding parameters ε, θ, and of the parameter governing risk aversion μ.

Locally, the tatônnement problem is qualitatively equivalent to $\dot{x} = G(x, \lambda, \varepsilon, \theta)$, and the equilibrium price depends on the values of all three parameters: a (through λ), ε and θ. The different dynamics that result from perturbations of the symmetric economy are depicted in the bifurcation diagrams of figure 3.[9] The three regions are separated by curves known as bifurcation set, $\mathcal{B}$ and hysteresis set, $\mathcal{H}$, defined by:

$$\mathcal{B} = \{(\varepsilon, \theta) \in \mathbb{R}^2 : \exists (x, \lambda) : G = G_x = G_\lambda = 0\};$$

$$\mathcal{H} = \{(\varepsilon, \theta) \in \mathbb{R}^2 : \exists (x, \lambda) : G = G_x = G_{xx} = 0\}.$$

Again see Golubitsky and Schaeffer (1985, chapter III, 5.1) for more detail.

The bifurcation diagram between the curves $\mathcal{B}$ and $\mathcal{H}$ exhibits the hysteresic behaviour observed in our example.

[8] A function $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is weighted homogeneous of weight r with respect to a set of weights (α_1, α_2) if $f(t^{\alpha_1} x_1, t^{\alpha_2} x_2) = t^r f(x_1, x_2)$, for all $t \in \mathbb{R}$.

[9] A *bifurcation diagram* is a low-dimension qualitative representation of all equilibria as the parameter varies. Here, bifurcation diagrams are drawn in the (a, p)-plane and branches of equilibria represented by solid lines correspond to stable equilibria whereas dotted lines correspond to unstable values of the equilibrium price.

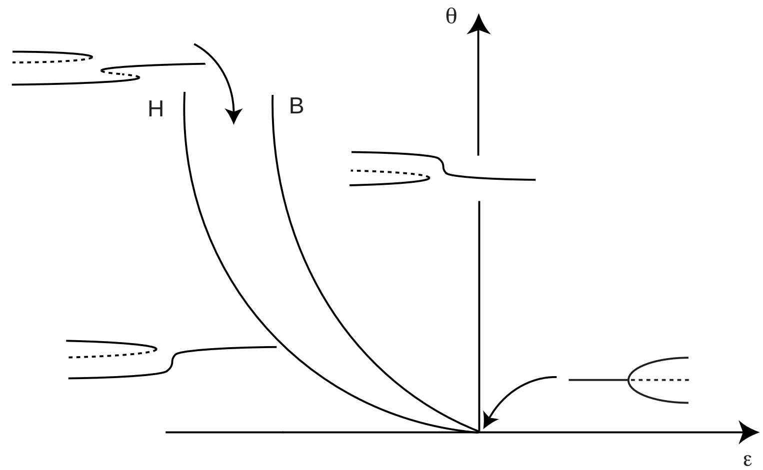

Figure 3: Bifurcation and hysteresis sets (in bold) in the upper half-plane. Notice that θ represents the endowment of agent 1 in state 2. The bifurcation diagrams drawn (in the (a, p)-plane) remain qualitatively equivalent in each connected component. A solid line in a bifurcation diagram represents stability while a dotted line means the branch consists of unstable equilibria.

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