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REAL OPTIONS USING MARKOV CHAINS: AN APPLICATION TO PRODUCTION CAPACITY DECISIONS

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ABSTRACT

In this work we address investment decisions using real options. A standard numerical approach for valuing real options is dynamic programming. The basic idea is to establish a discrete-valued lattice of possible future values of the underlying stochastic variable (demand in our case). For most approaches in the literature, the stochastic variable is assumed normally distributed and then approximated by a binomial distribution, resulting in a binomial lattice. In this work, we investigate the use of a sparse Markov chain to model such variable. The Markov approach is expected to perform better since it does not assume any type of distribution for the demand variation, the probability of a variation on the demand value is dependent on the current demand value and thus, no longer constant, and it generalizes the binomial lattice since the latter can be modelled as a Markov chain. We developed a stochastic dynamic programming model that has been implemented both on binomial and Markov models. A numerical example of a production capacity choice problem has been solved and the results obtained show that the investment decisions are different and, as expected the Markov chain approach leads to a better investment policy.

Keywords: Flexible Capacity Investments, Real Options, Markov Chains, Dynamic Programming.

J.E.L. Classification. C61, G31.

1 INTRODUCTION

Manufacturing flexibility has become a very important competitive aspect for production oriented companies. Many researchers, e.g. (Fine & Hax 1985) also rank manufacturing flexibility as a competitive priority together with e.g. cost and quality. Several types of flexibility can be evaluated. Here, we are concerned with the some times called “volume” flexibility, which can be defined as the ability to operate with profit at different output levels. A possible way of measuring flexibility is to estimate its value, which can be compared with the cost of acquiring it. Some authors advocate that its value can be given by the Net Present Value (NPV) calculations, while others believe that flexibility can be best seen as options, or Real Options (RO) to separate them from the more familiar financial options. The NPV analysis typically assumes that an immediate accept-or-reject decision must be made and thus, it ignores the flexibility to optimally time project initiation. NPV says that an investment should be made whenever the expected discounted future cash-flows match investment costs. RO on the other hand, require that expected discounted future cash-flows to be significantly above the investment costs. This happens, since making the investment implies the loss of the waiting option and hence, it represents an opportunity cost.

The term “real options” was coined by Stewart Myers in 1977. It referred to the application of option pricing theory to the valuation of non-financial or “real” investments with learning and flexibility, such as multi-stage R&D, manufacturing plant expansion, and the like (Myers 1977). This type of approach is based on three factors, namely: the existence of uncertainty about the project rewards, i.e. future cash-flows; the investment irreversibility, at least partially, i.e. money cannot be fully recovered once an investment decision has been done; and the investment timing, for which there is some flexibility that allows for the arrival of new information.

A large and rapidly growing literature on investment under uncertainty interprets the firm as consisting either wholly or in part of a portfolio of options, and uses options-based models and option pricing techniques to study the investment decision. This approach also recognizes the sequential nature of investment decisions as a key feature. The options studied include the option to temporarily shut down (McDonald & Siegel 1985, Brennan & Schwartz 1985), the option to continue or discontinue a planned series of investments (Majd & Pindyck 1987), the option to defer investment (McDonald & Siegel 1986), the option to abandon project earlier (Myers & Majd 1990), the option to increment capacity (Bollen 1999, Pindyck 1988, Kandel & Pearson 2002), amongst others.

Managerial flexibility has been valued by option pricing for almost two decades and during this time different kinds of real options have been treated. Kulatilaka (1988) uses a stochastic dynamic programming model to evaluate the options in a flexible production process and incorporates the effects of switching costs. Andreou (1990) evaluates process flexibility in different configurations of dedicated and flexible equipment when demand of two products is uncertain. Triantis & Hodder (1990) evaluate process flexibility, in a given fixed capacity equipment as a complex option. The profit margins of different products are assumed to be stochastic and dependent on quantity produced. The latter effect is a result of allowing for downward sloping demand curves. In their model there are no switching costs. Capacity constraints are considered and the model allows the firm to temporarily shut down and restart operation. He & Pindyck (1992) examine investments in flexible production capacity. Here, the capacity choice problem is considered, i.e. whether to buy flexible or non-flexible equipment and how much capacity with respect to the fact that investment is irreversible. As in (Tannous 1996) demand is uncertain but in this case differs, via a demand shift parameter depending on whether market is perfectly competitive or not. Kamrad & Ernst (1995) model multi-product manufacturing with stochastic input price, output yield uncertainty and capacity constraints to value multi-product production agreements. During one period, only one product type is produced with respect to the inventory available. Tannous (1996) carries out capital budgeting for volume flexible equipment and compares a non-flexible to a flexible system in a case based on a real company. In his model, demand is uncertain and dependent on price and a stochastic factor. The effect of having inventory available is also considered. Bollen (1999) evaluates the option to switch between production capacities. The demand stochastic process is governed by a stochastic product life cycle which is modelled by using a regime switching process. In his study a comparison between flexible and fixed capacity projects is also made.

In this work, we present a model to evaluate investment decisions based on real options. In the problem considered we incorporate partial reversibility into the model by letting the firm reverse its capital investment at a cost, both fully or partially. Therefore, three options are considered: temporarily shutdown, defer investment, and switch between production capacities. The standard RO approach considers the stochastic variable to be normally distributed and then approximated by a binomial distribution, resulting in a binomial lattice. In this work, we propose to discretize the stochastic variable by means of a sparse Markov chain, which is derived from the demand data previously collected. The main advantages of the proposed approach are: i) the Markov chain does not assume any type of distribution for the stochastic variable, ii) the probability of a variation is not constant, actually it depends on the current value, and iii) it generalizes current literature using binomial distributions since they can be modelled by a Markov chain. Nevertheless, the Markov approach is computationally more demanding. However, the sparsity of the Markov matrix, that naturally results from data streams like ours, can be exploited to improve the computational performance of the algorithm.

2 Problem Description and Formulation

Following the approach outlined in Dixit & Pindyck (1994) and Trigeorgis (1996), the opportunity of adopting a specific value of production capacity can be viewed as representing a real option to the firm. This type of investment decisions can be casted as a sequence of embedded decisions since the current capacity decision has implications on future decisions.

Our starting point is the irreversible investment model by Pindyck (1988), which is a flexible and tractable example of the options-based models, and can be readily generalized to allow for partial reversibility. In his model, a monopolist faces a demand function that shifts stochastically, towards and away from the origin, over time as given by

$$Q = \theta - \lambda P, \tag{1}$$

where Q is the industry output and θ models the dynamics of demand. Of course, for the case of monopoly (1) is also the demand curve faced by the firm. (In financial options it is standard to assume that the underlying security is traded in a perfectly competitive market. However, many real asset markets are monopolistic or oligopolistic, rather than perfectly competitive.)

The total variable production costs are assumed to be a quadratic function of quantity produced, which is a standard assumption, see for example (Pindyck 1988, Trigeorgis 1996, Bollen 1999).

Thus, the total production costs are

$$C(Q,m) = c_1 Q + \frac{c_2}{2m} Q^2 + c_3 m, \tag{2}$$

where the fixed and variable coefficients of the marginal cost function are c_1 and $\frac{c_2}{2m}Q$, m is the installed production capacity, and the fixed component $c_3 m$ represents the overhead costs.

The operating profit of period t, given the demand and production capacity installed, is then computed as

$$\pi\left(\theta_t, m_{t-1}\right) P(Q_t) - C(Q_t, m_{t-1}) = \left(\frac{\theta_t}{\lambda} - c_1\right) Q_t - \left(\frac{1}{\lambda} + \frac{c_2}{2m_{t-1}}\right) Q_t^2 - c_3 m_{t-1}. \tag{3}$$

(It should be noticed that we are assuming that the capacity in place at period t is the capacity chosen in the previous period. This is not a limitation since the reasoning and formulae given can be applied with any number of installation periods.)

The firm maximizes operating profit over produced quantity and hence, the optimum quantity Q^*, which is obtained by solving $\frac{\partial \pi}{\partial Q} = 0$, is given by

$$Q^*(\theta_t) = \frac{\theta_t - \lambda c_1}{2 + \lambda c_2 / m_{t-1}}. \tag{4}$$

Furthermore, the quantity to be produce in each period Q_t is bounded from above by the production capacity and from below by zero, thus it is given as

$$Q^{**}(\theta_t) = max\left(0, min\left(Q^*(\theta_t), m_{t-1}\right)\right) \tag{5}$$

and therefore, the optimal operating profit is

$$\pi^*\left(\theta_t, m_{t-1}\right) = P\left(Q^{**}(\theta_t)\right) - C\left(Q^{**}(\theta_t), m_{t-1}\right). \tag{6}$$

We also consider partial reversibility, which is incorporated into the model by letting the firm reverse its capital investment at a cost. The ability to partially reverse the capital investment is modelled through capacity sell out, which allows for recovering of a fraction of the purchase price for each unit sold. More specifically, following the work by Bollen (1999), we use $S\left(m_1, m_2\right)$ to represent additional investment or recovered investment associated with changing capacity level from m_1 to m_2:

$$\begin{aligned} S\left(m_1, m_2\right) &= s_1 c_4 \left(m_2 - m_1\right) + s_3, \quad \text{if } m_2 > m_1, \\ S\left(m_1, m_2\right) &= s_2 c_4 \left(m_1 - m_2\right) + s_3, \quad \text{otherwise.} \end{aligned} \tag{7}$$

In the cost function (7), s_1 and s_2 are percentages of the initial capacity cost c_4 and s_3 is a fixed switching capacity cost. It should be noticed, that although in the example solved in this work we set $s_1 = 1$ and $0 < s_2 < 1$, they could assume any positive or negative value. Moreover, the switching costs could also be time dependent.

We assume that markets are dynamically complete, implying that there exists a risk-neutral probability or equivalent martingale measure such that the value of the firm is given by the expected discounted value of its profits less the investments in capacity. The assumption that markets are dynamically complete amounts to assuming that stochastic changes in demand are spanned by existing assets, or that markets are sufficiently complete that the firms decision to invest does not change the opportunities available to investors.

3 Solution Methodology

To solve the problem it is necessary to find the optimal sequence of capacity choices, namely: invest in additional capacity, sell out excessive capacity, keep exactly the same capacity; and the optimal production in each period given the capacity decision previously made. These two types of decisions must be addressed simultaneously since the existence of switching costs implies that a capacity decision made in a period alters future switching costs and future profits and thus, future switching decisions. Therefore, the project value must be determined simultaneously with the optimal production capacity policy. The solution approach we use is to discretize the problem and set up a discrete-valued lattice or grid for which a dynamic programming model is derived and then solved by backward induction.

The standard RO approach to discretize the stochastic variable is through the use of a binomial lattice as explained in Section 3.1. Here, we propose another approach to discretize the problem that makes use of a Markov chain, a sparse one-step transition probabilities matrix. This is further explained in Section 3.2.

3.1 The Binomial Lattice

A standard assumption in the real options literature is that the underlying stochastic variable is governed by a geometric diffusion, which implies that at each period there is only one constant growth/decay rate. If this is assumed then a natural way of obtaining a valued-lattice for the stochastic variable (demand) is to discretized it through a standard binomial lattice, see Figure 1.

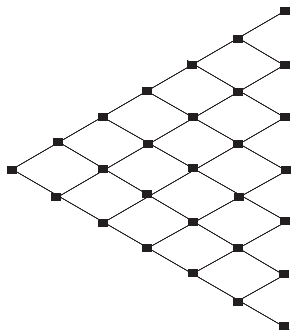

Figure 1: A lattice discretizing demand.

A node of value θ_t^i can lead to two nodes with their values being given by $\theta_{t+1}^j = u\theta_t^i$ and $\theta_{t+1}^k = d\theta_t^i$ with probability q_u and $q_d = 1 - q_u$, respectively. The probability of reaching each of these nodes is the usual equivalent martingale measure used in the binomial option pricing model of Cox, Ross & Rubinstein (1979):

$$q_u = \frac{(1 + r_f) - d}{u - d} \text{ and } q_d = 1 - q_u, \tag{8}$$

where r_f is the risk free rate over the interval Δt, $u = exp(\sigma\sqrt{\Delta t})$, and $d = exp(-\sigma\sqrt{\Delta t})$.

3.2 The Markov Grid

If the stochastic variable under consideration is the demand, it seems unrealistic to assume it to be governed by a geometric diffusion, since this implies that at each period demand grows/decays at a constant rate. The implication of the constant rate of growth assumption are twofold. On one hand, it undervalues the option to contract or abandon project, since it underestimates the probability of low demand in future. On the other hand, it overvalues the option to expand capacity as demand is able to increase forever.

We propose the use of a Markov chain. This way, not only we allow for demand variations dependent on demand current value, but we also allow for probability values dependent on demand current value. Moreover, the grid obtained by a Markov chain, generalizes the binomial lattice approach since it can be used to model a binomial lattice. We obtain the Markov chain from data previously collected.

A Markov chain is defined by a one step transition probability matrix. The levels of demand can be easily described by a square grid of all possible states $i = 1, 2, \ldots, n$, see Figure 2, where the planning horizon is discretized in Δt intervals and the state variable in $\Delta\theta$.

PSfrag replacements

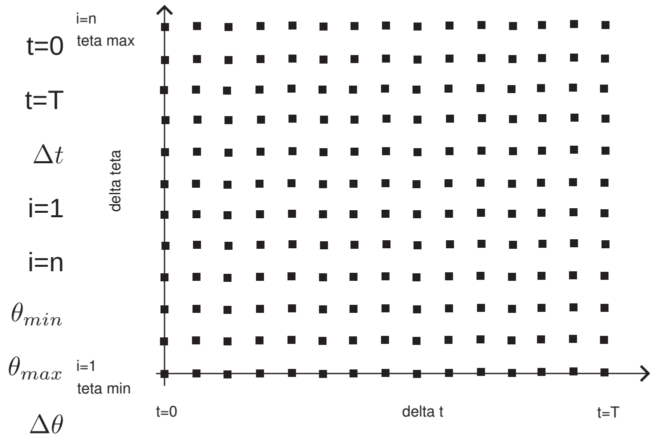

Figure 2: A grid for the Markov discretization of demand.

The Markov matrix of transition probabilities can be obtained from the data as follows. The probability of reaching state j at some period of time being in state i at the previous time period is given by the ratio between the number of transitions from demand value θ_i to demand value θ_j in consecutive periods and the total number of transitions out of demand value θ_i to all other possible demand values in consecutive periods, which can be written as

$$Prob\left(\theta_j, \theta_i\right) = \frac{\sum_{t=1}^{T}\left(\theta_i^{t-1}, \theta_j^{t}\right)}{\sum_{t=1}^{T}\sum_{k=1}^{n}\left(\theta_i^{t-1}, \theta_k^{t}\right)} \tag{9}$$

where $\left(\theta_i^{t-1}, \theta_j^{t}\right)$ denotes a transition from state i at period $t-1$ to state j at period t. Note that these probabilities are time invariant.

3.3 The Decision Process

Recall that, we have defined to have three options, namely temporarily shutdown, defer investment, and switching capacity. The temporarily shut down option is a reversible option that is only taken if we are better off not producing, which in our problem is done implicitly. Such a decision is reached whenever the optimal quantity to be produced is determined to be zero.

The defer investment option corresponds to a capacity switch to the same value and thus, it can be simultaneously addressed with the capacity switching option. This way, we only have to explicitly address the capacity switching option.

In each period the firm must make two decisions, one regarding the quantity to be produce Q_t and another regarding the production capacity m_t that is to be in place for the following period. The decision on the quantity to be produce is given by maximizing the operating profit and hence, the optimal quantity Q_t to be produced in period t is given as in equation (5). The decision about the production capacity is related to the future periods profits since the chosen capacity will be available from next period. Recall that an increase in installed capacity requires an investment, while a sell out of capacity leads to a partial recover of investment, as given in equation (7). Therefore, at each period the project value is dependent on the level of demand and production capacity and is obtained by maximizing the sum of the optimal current period's profit with the optimal continuation value for each possible capacity. The latter value is given by the discounted expected future profits net of switching costs.

Before proceeding let us review the notation:

- $\pi^*(\theta_t, m_{t-1})$ represents the optimal operating profit at state (demand value) θ_t when the installed production capacity is m_{t-1}. Recall that π^* is computed as in equation (6).
- $f(\theta_t, m_{t-1}, t)$ represents the optimal value of the project at state θ_t when the installed production capacity m_{t-1}, assuming optimal future switching decisions.
- m_t represents the optimal production capacity value to be installed and working at period $t+1$ given that at period t state θ_t is entered with production capacity m_{t-1}.
- $E[\,]$ is the risk neutral expectations operator.
- $S(m_1, m_2)$ represents the cost to be paid for to switch production capacity from m_1 to m_2.

The optimal project value at period t given the demand θ_t and available production capacity m_{t-1} is then given by

$$f(\theta_t, m_{t-1}, t) = \pi^*(\theta_t, m_{t-1}) + \max_m \left\{ \frac{E[f(\theta_{t+1}, m_t, t+1)]}{1+r_f} + S(m_{t-1}, m_t) \right\}. \tag{10}$$

As said before, and in order to allow for earlier exercise, the valuation procedure begins at the last stage and works backwards to initial time. At the final period $t = T$, for each demand value and capacity available, the project value is given by the final operating profit plus the capacity salvage value

$$f\left(\theta_T, m_{T-1}, T\right) = \pi^*\left(\theta_T, m_{T-1}\right) + S\left(m_{T-1}, 0\right). \tag{11}$$

The implementation of the Dynamic Programming recursion, given by equation (10), on a standard binomial lattice computes expected value of future profits as

$$E\left[f\left(\theta_t, m_{t-1}, t\right)\right] = q_u f\left(u\theta_t, m_t, t+1\right) + q_d f\left(d\theta_t, m_t, t+1\right), \tag{12}$$

where q_u, q_d, u, and d are as explained in Section 3.1, while the implementation on a Markov grid is computed as

$$E\left[f\left(\theta_t, m_{t-1}, t\right)\right] = \sum_{i=1}^{n} P_{\theta_t,\theta_i} f\left(\theta_i, m_t, t+1\right), \tag{13}$$

where P_{θ_i,θ_j} is the transition probability from demand value θ_i to demand value θ_j in consecutive periods.

4 Results

In order to test our methodology we have implemented, in MATLAB, the dynamic programming model on the binomial lattice and on the Markov grid, both derived from the collected data. These models differ only in the way of taking the expectation of future profits. The binomial lattice model takes the expectation by considering that from each node only two nodes can be reached at the following period, as given in equation (12), while the Markov model considers that regardless the current node, i.e. demand value, all possible demand values can be reached in the following period, as given in equation (13). As we have considered the initial production capacity also to be decided we have to solve

$$Project\ Value = \max_{m_0} \left\{f\left(\theta_1, m_0, 1\right)\right\}/\left(1 + r_f\right) - c_4 m_0 \tag{14}$$

using the dynamic programming recursion given in equation (10) and the terminal conditions of equation (11).

Both the Binomial and Markov models have been used to find out an optimal capacity investment policy, which we call a priori solution. The quality of these models is then tested by evaluating the policy performance on specific data realization sets, which we call a posteriori solution.

We have collected monthly sales data for a 48 months period, given in the Appendix. The first 24 months of data are used to set up both the Binomial and the Markov models. These models are then used to obtain the optimal capacity policy and the predicted project value.

It follows the description of the specific problem solved. The values for the parameters associated with selling price, production and switching costs, and production capacity have been taken from Bollen (1999). The demand data collected has been scaled in order to be of the same magnitude of demand values used in (Bollen 1999). The initial demand was set to the average demand over the first 24 months period.

Production costs: $c_1 = 0.1$, $c_2 = 0.5$, and $c_3 = 0.1$.

Capacity and capacity switching costs: $c_4 = 2$, $s_1 = -1$, $s_2 = 0.85$, and $s_3 = 0.05$.

Production capacity values: ranging from 0 up to 2.5 with capacity step values varying between 0.05 and 0.5.

Price parameter: $\lambda = 1$.

Annual risk free rate. $r_f = 10\%$.

The binomial lattice parameters have been computed by using the first 24 months of data, as given in Section 3.1: $u = 1.695$, $d = 1/u$, $q_u = 0.378$, and $q_d = 1/q_u$. The Markov grid was computed as in equation (9) also by using the first 24 months of data, see Appendix.

Several tests have been performed in order to compare the results obtained by each methodology. For these tests we have used both the collected data and randomly generated data.

4.1 Comparing the Accuracy of the Models

To evaluate project value accuracy, we compare the predicted project value (or model value) to the value obtained by applying the policy found to the data used to derive the model (months 1 to 24), see Table 1. For each possible value of capacity changing step, we report the model value, i.e. the predicted project value which is computed as given in equation (14), and the corresponding initial capacity. We also give the data value, which is the value obtained by applying the optimal capacity changing policy to the data set used to set up the model.

From the results reported it can be concluded that the strategies proposed by the two models are different since the initial capacity values are different. As expected, the better values for the

	Binomial Lattice				Markov Grid				Data
Cap.	Model	Init.	Data	Mod/Data	Model	Init.	Data	Mod/Data	Mark/Bin
Step	Value	Cap.	Value	Ratio (%)	Value	Cap.	Value	Ratio (%)	Ratio (%)
0.5	7.91	0.5	4.80	165.03	5.63	1.0	5.28	106.63	140.48
0.4	7.93	0.8	5.13	154.93	5.78	0.8	5.45	106.11	137.12
0.3	7.94	0.6	5.10	155.94	5.74	0.6	5.44	105.67	138.16
0.2	7.94	0.6	4.97	160.16	5.78	0.8	5.45	106.11	137.30
0.1	7.94	0.7	5.13	155.20	5.79	0.7	5.47	105.87	137.12
0.05	7.95	0.65	4.90	162.58	5.79	0.75	5.47	106.00	137.15

Table 1: Predicted project value for binomial and Markov models (months 1-24).

predicted project value are obtained for smaller capacity steps, in both models. Furthermore, the predicted project value is larger for the Binomial model, which although might seem to be an advantage is actually a drawback since in both cases the project value tends to be an overestimation. This can be observed in the columns giving the model to data project value ratio, where we have computed the ratio between the predicted project value and the project value obtained by applying the optimal capacity policy to the data which was used to set up the model. The project value obtained for the first 24 months period data, is better if the capacity changing policy used is the one provided by the Markov model. The Markov model provides values between 37% and 40% better than the Binomial model, as can be seen in the Mark/Bin Ratio column.

4.2 Comparing the Performance of the Models on *Real* Data

To test the efficiency of the models we used the capacity policies of each model on 7 different sets of data as given in Tables 2 and 3. Data sets 1 and 2 correspond to, respectively, the first and the last 24 months of the collected data. The remaining data sets were randomly generated between the minimum and maximum values of the data collected having demand averages of 1.4, 1.5, 1.6, 1.7, and 1.8.

As it can be seen, the *real* project values are higher when the Markov policies are used. Only in 1 out of the 42 values computed the Binomial model performs better. Furthermore, the project values obtained by using the Markov model vary from 99.99% to 114.95% of the project values obtained by using the Binomial model.

	Binomial - Project Value						
Step	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 7
0.5	4.804	6.729	5.573	5.835	7.357	7.852	9.078
0.4	5.130	7.304	6.075	6.168	8.296	8.600	10.353
0.3	5.101	7.075	5.895	6.011	7.881	8.416	9.744
0.2	4.969	7.075	5.895	6.011	7.881	8.416	9.744
0.1	5.131	7.246	6.052	6.147	8.182	8.734	10.153
0.05	4.900	7.178	5.992	6.097	8.058	8.602	9.978

Table 2: Project values for specific data realizations using the Binomial model policy.

	Markov - Project Value						
Step	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 7
0.5	5.280	7.230	5.939	6.037	8.271	8.866	10.435
0.4	5.449	7.304	6.075	6.168	8.296	8.862	10.353
0.3	5.435	7.075	5.895	6.011	7.881	8.416	9.744
0.2	5.449	7.304	6.075	6.168	8.296	8.862	10.353
0.1	5.473	7.246	6.052	6.147	8.181	8.734	10.153
0.05	5.467	7.287	6.077	6.168	8.257	8.816	10.275

Table 3: Project values for specific data realizations using the Markov model policy.

4.3 Comparing the Effect of Project Duration on the Models Performance

In order the analyze the impact of project life in the models performance, we have considered horizons of 12, 36, 48, and 60 months in addition to the 24 months horizon previously considered. In Tables 4 and 5, project values are reported for data sets 3 to 7 considering capacity steps of 0.5 and 0.1, respectively . Data sets 1 and 2 cannot be considered for horizons larger than 24 months since they consist of 24 demand values only.

Again, it can observed that the performance of the Markov model is better. For a capacity changing value of 0.5, the Binomial project value is never better than the Markov project value and the latter one can be up to 15.8% better.

When comparing the project values by considering 0.1 and 0.5 as the capacity changing steps, the Binomial model improves more than the Markov model. Nevertheless, project values given by using the Markov policy are still larger. Only in 2 out of the 25 values computed the Bi-

	Binomial Lattice					Markov Grid				
T	Set 3	Set 4	Set 5	Set 6	Set 7	Set 3	Set 4	Set 5	Set 6	Set 7
12	1.749	2.639	4.407	4.521	4.598	1.749	2.639	4.407	4.521	4.598
24	5.573	5.835	7.357	7.852	9.078	5.939	6.037	8.271	8.866	10.435
36	8.567	10.162	9.947	11.151	12.734	9.390	10.364	11.058	12.578	14.610
48	10.112	12.803	12.661	15.211	16.588	10.913	13.226	14.133	17.352	19.110
60	12.221	15.387	15.215	18.342	20.242	13.198	16.145	16.938	20.936	23.444

Table 4: Project value for varying time horizon using a 0.5 capacity changing step.

	Binomial Lattice					Markov Grid				
T	Set 3	Set 4	Set 5	Set 6	Set 7	Set 3	Set 4	Set 5	Set 6	Set 7
12	1.802	2.744	4.760	4.872	4.942	1.800	2.766	4.966	5.075	5.161
24	6.052	6.147	8.182	8.734	10.153	6.052	6.147	8.181	8.734	10.153
36	9.436	10.338	10.979	11.675	14.219	9.436	10.338	10.979	12.363	14.219
48	11.053	13.182	14.002	15.276	18.559	11.053	13.182	14.002	16.951	18.559
60	13.342	16.057	16.777	18.291	22.717	13.432	16.239	16.995	20.800	23.184

Table 5: Project value for varying time horizon using a 0.1 capacity changing step.

nomial model produced a better value. The Markov project values are now between 99.91% and 113.72% of the Binomial project values. It should be noticed that except for a capacity changing value of 0.4, the best performance of the Binomial model was observed when it was 0.1, see Table 2 above, which is not the case for the Markov model.

5 CONCLUSIONS

In this work, we address the problem of making investment decisions on a flexible production capacity firm. The problem involves deciding not only the optimal quantity to be produced at every single period but also the in place production capacity policy throughout the whole planning period. We consider the investments to be, at least, partially reversible since capacity sell out allows for partial investment recovering.

We propose to address this problem by using dynamic programming implemented on a Markov grid rather than on the standard binomial lattice. The Markovian approached was expected to perform better since it is a generalization of the binomial lattice in the sense that is can be used to model the latter one. Furthermore, the binomial approach assumes that demand varies according to normal distribution and the lattice is constructed based on demand constant growth rates at constant probabilities. Moreover, the Markov grid takes into account the current value of demand not only into the growth rate but also into the probabilities.

An example using *real* data for the stochastic variable (demand) has been solved, using both discretization approaches. It has been shown that the Markov approach is more reliable and leads to a better decision policy. The computational tests performed, also allowed for the conclusion that the Markov model is less sensitive to project time horizon and to capacity changing steps.

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Appendix

In this Appendix we give the demand data collected and also the one step transition probability Markov matrix.

	Monthly Demand			
Month	2000	2001	2002	2003
January	1117	1288	1342	1566
February	1520	1558	1767	1696
March	1758	1656	1724	1608
April	1099	1105	1630	1473
May	1669	1675	1851	1480
June	1345	1148	1717	1241
July	1456	1872	2322	1763
August	611	726	867	536
September	1363	1657	1951	1409
October	1664	1703	2152	1549
November	1811	1734	1981	1110
December	1033	1388	1465	998

Table 6: Collected data – monthly demand.

Collected data

Markov transition probability matrix

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1/4 & 0 & 1/2 & 0 & 1/4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2/7 & 1/7 & 1/7 & 0 & 0 & 2/7 & 1/7 & 0 \\ 0 & 0 & 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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