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Subjective Expectations Equilibrium in Economies with Uncertain Delivery*

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Abstract. In economies with uncertain delivery, agents trade their endowments for lists instead of bundles. A list specifies a set of bundles such that the agent has the right to receive one of them. In this paper, with continuity conditions on private beliefs about the bundle that will be delivered, we establish existence of a *subjective expectations equilibrium.*

Keywords: Private information, Uncertain delivery, Subjective expectations equilibrium, General equilibrium, Incomplete information, Real options.

JEL Classification Numbers: C62, D51, D82.

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1 Introduction

One of the fundamental steps in the development of general equilibrium theory was the formulation of objects of choice as contingent consumption claims (Arrow, 1953). Under this formulation, besides being defined by physical properties and location in space and time, a commodity is also defined by the state of nature in which it is made available.[1] With this extension of the commodity space, the model of Arrow and Debreu (1954) could cover the case of uncertainty.

But, in a context of uncertainty, agents usually have different information. To cover this more general situation, Radner (1968) introduced asymmetric information in the general equilibrium model, now known as a differential information economy. In this setting, agents have private information that determines which states of nature they can distinguish.[2] The competitive equilibrium of this model, known as a Walrasian Expectations Equilibrium, is based on the assumption that agents only make trades contingent on events that they can observe. As a result, agents consume the same in states of nature that they do not distinguish.[3] In practice, the sole impact of an agent's private information is this restriction on the consumption space.

In a recent paper, we suggested that this restriction on the consumption space should be relaxed. From the fact that an agent does not observe a difference between two states should not follow that her consumption should be the same in these states. It is true that this introduces some difficulties. Suppose that an agent buys the right to receive x_1 in state ω_1 and x_2 in state ω_2, but is not able to distinguish between the two states. If the actual state of nature is ω_1 and the

[1]For example, instead of talking about good A in state 1, or good B in state 2, we should talk about good $A1$ or good $B2$.

[2]With a finite number of states, the private information of an agent is modeled as a partition such that the agent can distinguish states that belong to different sets of the partition.

[3]For this to be true, it is also assumed that agents observe their endowments, that is, each agent has the same initial endowments in states of nature that she cannot distinguish.

bundle x_2 is delivered, the agent has to accept this, since she is ignorant about which is the actual state of nature. What the agent actually bought was the right to receive x_1 or x_2 if one of the states ω_1 or ω_2 occurs. We designated these uncertain bundles as lists.

What we have found, essentially, is that even if the agent expects to receive the worst bundle among the alternatives, she is better off buying the right to receive different bundles for delivery in states of nature that she does not distinguish, that is, buying lists. Under this assumption of extreme pessimism, the solution, which we designated as a *prudent expectations equilibrium*, is characterized by the fact that the agents consume bundles with the same utility in states of nature that they do not distinguish.

In this model, which may be designated as an economy with uncertain delivery, the trades which are allowed are extended from the classical structure of complete contingent markets to include also contingent lists of goods. A list is a set of bundles such that the market has the obligation to deliver one of the bundles specified in the list. Equivalently, it is a set of bundles such that the agent has the right to receive one of the bundles specified in the list. This formulation covers financial derivatives known as options.

For example, consider that East Timor has 10 units of oil and 1 unit of medicines. With the first coordinate representing oil and the second representing medicines: $e_E = (10, 1)$. If some trade is made giving Australia an option on East Timor to get 8 units of oil in exchange for 4 units of medicines, then, in our formulation, East Timor will have the list $lx_E = [(10, 1) \vee (2, 5)]$. This means that (depending on the preferences of Australia) East Timor will end up with 10 units of oil and 1 unit of medicines, or with 2 units of oil and 5 units of medicines.

We expand the market structure, but do not expect markets for each of the lists to clear. On the contrary, a particular list may only be traded by one of the agents. For example, let the initial endowments of East Timor and Australia be

$e_E = (10, 1)$ and $e_A = (42, 64)$. East Timor can get the list $lx_E = [(10, 1) \vee (2, 5)]$ with Australia getting the list $lx_A = [(50, 60) \vee (250, 700)]$. The trade of 8 units of oil in exchange for 4 units of medicines leads to $x_E = (2, 5)$ and $x_A = (50, 60)$, which is a feasible allocation that satisfies the requirements set by the lists. The lists are feasible if there is some combination of alternatives that constitutes a feasible allocation.

Each agent can get a different list, so to talk about clearing the market for a list is meaningless. Market clearing should be verified in the markets for the primitive commodities: oil and medicines. The derivatives markets are not expected to clear.

A possible interpretation of this model is the following. Each agent deals with a broker, who offers contingent lists in exchange for the agent's endowments. Among themselves, the brokers trade state-contingent commodities (primitives). That is, internally, the market works as an Arrow-Debreu economy only with the primitive commodities. To guarantee feasibility of the allocation in terms of lists, we restrict brokers to the sale of lists that they can deliver with 100% probability, and assume that they make no profits. These assumptions imply that the price charged for a list is the price (in the internal market) of the cheapest bundle which satisfies the requirements of the list.

A representative offering $lx = [(a \vee b \vee c), (a \vee b \vee c), (d \vee e)]$ has different possibilities of guaranteeing delivery of this list. One is to buy (in the internal markets for primitives) the state-contingent bundle (a, b, d), another is to buy (c, c, e), or (a, c, e), etc. In any case, the first and second coordinates are a, b or c, and the third is d or e. The representative will choose to buy the cheapest of the bundles that guarantees delivery of lx. This cheapest bundle has two fundamental characteristics: its price is the price of the list; and this is actually the bundle which will be delivered.

In this way, the prices of lists are uniquely determined by the prices of the contingent commodities in the internal market. Selection of bundles to be delivered is also determined internally - the bundle that is the cheapest is also the one that is delivered. In sum, the internal market mechanism is responsible for price-setting and for the selection of the bundles to be delivered to each agent in each state of nature, among the possibilities specified in the lists.

In the previous paper, we have shown that allowing this kind of trades leads to welfare improvements in the sense of Pareto. In that paper we assumed that agents were extremely prudent in their assessment of the value of lists: agents expected the worst possible bundle (in terms of utility) to be delivered. Now we seek to study the case in which agents are not extremely pessimistic. Facing a list, and observing prices, agents construct subjective expectations on the probabilities of receiving each of the different bundles in the list.

The main result in this paper is the existence of equilibrium independently of the expectations of the agents, in general conditions. We remark that in this model the preferences of the agents may also be a function of prices.

It makes some sense that these subjective expectations also depend on prices.[4] Price constitutes an indication of the economic difficulty to deliver the good. Having the right to receive a car or a bicycle, an agent should expect to receive a bicycle, which is a cheaper good.

The paper is organized as follows: in section 2, the model is presented; section 3 discusses preferences and makes some assumptions on expectations; and in section 4 existence of equilibrium is established.

[4]This dependence has been recognized before by Veblen (1899) and Pollak (1977). With price dependent preferences, it is known that equilibrium exists (Arrow and Hahn, 1971). Economies with price-dependent preferences were recently studied by Balasko (2003).

2 The economy with uncertain delivery

An *economy with uncertain delivery* is similar to a *differential information economy* in which agents are allowed to select *consumption lists* instead of bundles. Remember that a *list* is a set of bundles such that the market is obliged to deliver one of the bundles in the list. Another interpretation is that an agent is allowed to buy different bundles for delivery in states of nature that she does not distinguish, but has to accept any of the bundles that corresponds to a state that she is unable to distinguish from the actual state of nature.

We restrict our analysis to a finite set of possible states of nature: $\Omega = \{\omega_1, \omega_2, ..., \omega_\Omega\}$. Agents have private information that allows them to distinguish between some of these states, represented by a partition of Ω, P_i, such that agent i is able to distinguish states that belong to different sets of the partition. The set of states that agent i does not distinguish from ω_s is denoted by $P_i(\omega_s)$.

We also restrict the number of commodities in the economy to be finite. The consumption of agent i in the state of nature ω_s, can thus be written as a vector $x_i^s \in \mathbb{R}_+^l$. Since the number of contingent goods is also finite, the complete contingent consumption of agent i can be written as a vector $x_i \in \mathbb{R}_+^{\Omega l}$. With lists having K elements, we can write a list that specifies the alternative bundles that agent i may receive in state ω_s as a vector $lx_i^s \in \mathbb{R}_+^{Kl}$. Similarly, a complete vector of contingent lists can be written as $lx_i \in \mathbb{R}_+^{\Omega Kl}$. We also denote by lx^{sk} the k^{th} alternative in the list lx^s, which, in turn, specifies the bundles that the market may deliver in state ω_s.

The economy extends over two time periods. In the first, the agents $(i = 1, ..., n)$ trade their state-contingent endowments for a vector of state-contingent lists, $lx_i = (lx_i^1, lx_i^2, ..., lx_i^\Omega)$, specifying the bundles that the market may deliver in each state of nature. This vector is constant across states that the agents cannot distinguish, that is, if $\omega_t \in P_i(\omega_s)$, then $lx_i^t = lx_i^s$. This is the common measurability restriction of Radner (1968) and Yannelis (1991). In the second period, agents receive (and consume) one of the bundles in the list that corresponds to the state of nature that occurs. For example, suppose that the actual state of nature is ω_s. In this case, agent i has the right to receive one of the bundles lx_i^{sk} in the list lx_i^s.

For concreteness, let the set of possible states of nature be $\Omega = \{\omega_1, \omega_2, \omega_3\}$, the private information of agent i be $P_i = \{\{\omega_1, \omega_2\}, \{\omega_3\}\}$, and the measurable vector of consumption lists be $lx_i = (lx_i^1, lx_i^2, lx_i^3) = [(a \vee b \vee c), (a \vee b \vee c), (d \vee e)]$. In the second period, if the state of nature is ω_1 or ω_2, agent i receives one of the bundles a, b or c; while if it is ω_3 that occurs, then the agent receives d or e.

The economy with uncertain delivery is defined by $\mathcal{E} \equiv (e_i, u_i, P_i, q_i, E_i)_{i=1}^n$, where, for each agent i:

- A partition of Ω, P_i, represents private information. The set of states of nature that agent i does not distinguish from ω_s is denoted by $P_i(\omega_s)$.
- Subjective probabilities are attributed to the different possible states of nature. To each state $\omega_s \in \Omega$, corresponds a the subjective probability $q_i^s \geq 0$, with $\sum_{s=1}^{\Omega} q_i^s = 1$.
- Preferences are represented by Von Neumann-Morgenstern (1944) utility functions, $u_i^s : \mathbb{R}_+^{\Omega l} \to \mathbb{R}$, assumed to be continuous, weakly monotone and concave.
- For each state of nature, $\omega_s \in \Omega$, a continuous subjective expectations function, $E_i^s(lx_i, p) : \;LX \times \Delta_+^{\Omega l} \to \Delta_+^K$, gives the subjective probabilities of delivery of each of the bundles $lx_i^{sk} \in lx_i^s$.
- The initial endowments are constant across undistinguished states, and strictly positive: $e_i(\omega_s) \gg 0$ for all $\omega_s \in \Omega$.

It is clear that a bundle of primitives, x, allows a broker to offer various lists. Examples of deliverable lists are $(x \vee y)$ and $(0.5x \vee z)$, but certainly not $(2x \vee x+y)$.

Having x, the broker can keep the contract for the delivery of $(x \vee y)$ (by delivering x), and also the contract for the delivery of $(0.5x \vee z)$ (by delivering $0.5x$). But the broker cannot possibly deliver $2x$ nor $x + b$, which are strictly greater than the bundle x.

Precisely, with the physical feasibility restriction being satisfied, a bundle $x = (x^1, x^2, ..., x^\Omega) \in \mathbb{R}^{\Omega l}$ allows the delivery of the following set of lists:

$$LX(x) = \{lx = (lx^1, ..., lx^\Omega) \in LX : \forall \omega_s \in \Omega, \exists lx^{sk} \in lx^s \ s.t. \ lx^{sk} \leq x^s)\}.$$

Given a list, lx, and prices p, agents have prior beliefs on the probability of delivery of each of the bundles in the list. These are assumed to depend continuously on prices. A further technical assumption is that given a list with an infinite number of possible bundles for delivery, an agent only attributes strictly positive probabilities of delivery to a maximum of K bundles, and is indifferent between the original list and the truncated one, which has the relevant K elements. In this setting, the space of relevant lists can be reduced to $LX = \mathbb{R}_+^{\Omega K l}$. Note that a list with less than K alternatives can still be represented in $\mathbb{R}_+^{\Omega K l}$ by completing the remaining coordinates with repetitions of the alternatives already included.

A vector of primitive assets, $x \in \mathbb{R}_+^{\Omega l}$, allows a broker to offer a set of lists, $LX(x) = LX^1(x^1) \times LX^2(x^2) \times ... \times LX^\Omega(x^\Omega)$. Note that the possible alternatives for delivery in state ω_s only depend on the primitive assets deliverable in this state, that is, on x^s. With lists having a maximum of K alternatives, in state ω_s, the contingent list lx_i^s must belong to $LX^s(x^s) = \cup_{k=1}^K \{(\mathbb{R}_+^l)^{k-1} \times [0, x^s] \times (\mathbb{R}_+^l)^{K-k}\}$, with $[0, x^s]$ denoting the set of bundles y^s such that $y^s \leq x^s$. For example, with a single state of nature and a single commodity: $x = 1$ implies $LX(x) = \{[0, 1] \times \mathbb{R}_+\} \cup \{\mathbb{R}_+ \times [0, 1]\}$. Observe that the correspondence LX, from bundles to allowed lists is continuous.

The price that is charged to the agents for the list lx (a vector of "*derivatives*") is assumed to be equal to the price of the bundle x (a vector of "*primitives*") that the broker buys in the internal market to guarantee delivery of lx. This bundle x is the cheapest among those that allow the broker to comply with the contract.

Price functions can thus be defined over the vectors of state-contingent bundles (primitives), and not over lists (derivatives). We normalize the price functions to the simplex of $\mathbb{R}_+^{\Omega l}$:

$$p \in \Delta_+^{\Omega l} = \left\{ p \in \mathbb{R}_+^{\Omega l} : \sum_{s=1}^{\Omega} \sum_{t=1}^{l} p^{st} = 1 \right\}.$$

The "budget set" can also defined over the vectors of state-contingent bundles. For agent i, it is given by:

$$B_i(e_i, p) = \left\{ x_i \in \mathbb{R}_+^{\Omega l}, \text{ such that } \sum_{s=1}^{\Omega} p^s x_i^s \leq \sum_{s=1}^{\Omega} p^s e_i^s \right\}.$$

The preferences of the agents are given by an expected utility function. Recall that we denoting the k^{th} element of a list that is delivered when a state ω_s occurs by lx^{sk}, and the corresponding subjective probability by E_i^{sk}. With this notation, the objective function can be written as:

$$U_i(lx_i, p) = \sum_{s=1}^{\Omega} q_s \sum_{k=1}^{K} E_i^{sk}(lx_i, p) \; u_i^s(lx_i^{sk}).$$

The hypothesis of continuity of the expectation functions and of the state-dependent utility functions imply that this subjective expected utility function is also continuous.

It is useful to define an indirect expected utility function, $V_i(x_i, p)$, as the maximum expected utility that a broker with a bundle x_i can promise to deliver to agent i. That is, as the expected utility of the list, lx_i, that can be offered with a bundle x_i and that is such that the agent prefers none of the other lists which are also deliverable with x_i.

$$V_i(x_i, p) = \max_{lx_i \in LX(x_i)} U_i(lx_i, p).$$

Since any list can be truncated to one of K elements, the maximum is attainable in the compact $LX(x_i) \cap \mathbb{R}_+^{\Omega K l}$, so Berge's Maximum Theorem can be applied.

A triple (p^*, x^*, lx^*) is a *competitive equilibrium with subjective expectations* if p^* is a price system and $x^* = (x_1^*, ..., x_n^*)$ is a feasible allocation such that, for every i, $x_i^* \in \mathbb{R}_+^{\Omega l}$ maximizes indirect expected utility, $V_i(x_i^*, p^*)$, on $B_i(p^*, e_i)$. The triple is completed by a vector $lx^* = (lx_1^*, ..., lx_n^*)$ such that $U_i(lx_i^*, p^*) = V_i(x_i^*, p^*)$.

This means that the vector x_i^* is sold to the agent as the list lx_i^*, and that (given prices p^*) no list that is preferred by the agent can be delivered using resources x_i' such that $p^* \cdot x_i' \leq p^* \cdot x_i^*$.

3 Preferences over lists and resources

The utility attributed to a list depends on the list itself, but also on prices. Recall that the subjective expectations functions are assumed to be continuous. In the restricted space of lists with K bundles, these functions are defined as follows:

$$E_i^s(lx_i, p): \; LX_K \times \Delta_+^{\Omega l} \to \Delta_+^K.$$

To a pair composed by a list with K elements, $lx_i \in LX_K$, and a price vector $p \in \Delta_+^{\Omega l}$, the subjective expectations function of agent i for state ω_s, denoted $E_i^s(lx_i, p)$, gives the subjective probability beliefs (a vector in Δ_+^K) of receiving each of the elements of the list, if the state of nature that occurs is ω_s. Each agent has a vector of these functions, one for each state of nature:

$$E_i(lx_i, p) = [E_i^1(lx_i, p), E_i^2(lx_i, p), ..., E_i^\Omega(lx_i, p)].$$

After receiving information $P_i(\omega_s)$, agent i knows that the state of nature belongs to this set. In this *interim* stage, the market is sure of being obliged to deliver one of the bundles of the list lx_i^s. The correspondent coordinate from the vector of contingent commodities, x_i^s, is what the market will deliver, but the agent does not know that.

The agent only sees the list lx_i^s, and has subjective expectations regarding the probabilities of delivery of the different elements of the list. Remember that these beliefs are assumed to vary continuously with prices and that subjective expected utility is given by:

$$U_i(lx_i, p) = \sum_{s=1}^{\Omega} q_i^s \sum_k E_i^{sk}(lx_i, p) \; u_i^s(lx_i^{sk}).$$

In some sense, these preferences are not convex. To see this, consider two commodities and linear utility: $u(x, y) = x + y$. Thus, we have $u(1, 0) = u(0, 1) = 1$ and $u(a, 0) = u(0, a) = a$. Let also $a > 1$. How much is $u((1, 0) \vee (0, a))$? And $u((0, 1) \vee (a, 0))$? Both lists give an *ex post* utility of either 1 or a. Suppose that preferences are not *prudent*[5], that is, that agents look beyond the worst outcome: $u((1,0) \vee (0,a)) > 1$ and $u((0,1) \vee (a,0)) > 1$. Nevertheless, a realistic agent should be pessimistic regarding the average allocation: $\frac{(1,0)+(0,1)}{2} \vee \frac{(0,a)+(a,0)}{2} = (1/2, 1/2) \vee (a/2, a/2)$. Observe that the market will not deliver $(a/2, a/2)$ when there is a possibility of delivering $(1/2, 1/2) << (a/2, a/2)$. Therefore, the agent should consider $u((1/2, 1/2) \vee (a/2, a/2)) = u((1/2, 1/2) = 1$. Since the average allocation has an utility of 1, lower than the utility of the extremes, convexity is violated.

How do we get around this problem?

We make an assumption that may be interpreted as meaning that the agents take probabilities of delivery as given, neglecting the impact of their choices. This assumption, which we make precise in this section, implies that if the bundle $x \in \mathbb{R}_+^{\Omega l}$ allows the offer of a list lx, and $y \in \mathbb{R}_+^{\Omega l}$ allows the offer of a list ly, then, any convex combination $z = \lambda x + (1-\lambda)y$, with $\lambda \in [0,1]$, allows the offer of a list lz such that $U(lz,p) \geq \min\{U(lx,p), U(ly,p)\}$. Precisely, given $z = \lambda x + (1-\lambda)y$:

$$lx \in LX(x) \wedge ly \in LX(y) \Rightarrow \exists lz \in LX(z) : U(lz,p) \geq \min\{U(lx,p), U(ly,p)\}.$$

Given prices p, consider the lists that can be offered using resources x and y, and let the two following, lx and ly, be among the optimal for the agent:

$$lx = (lx^1, ..., lx^\Omega) = [(lx^{11} \vee lx^{12} \vee ... \vee lx^{1K}), ..., (lx^{\Omega 1} \vee lx^{\Omega 2} \vee ... \vee lx^{\Omega K})];$$

$$ly = (ly^1, ..., ly^\Omega) = [(ly^{11} \vee ly^{12} \vee ... \vee ly^{1K}), ..., (ly^{\Omega 1} \vee ly^{\Omega 2} \vee ... \vee ly^{\Omega K})].$$

Precisely (we omit subscripts i for clearness):

$$V(x,p) = U(lx,p) = \sum_{s=1}^{\Omega} q^s \sum_{k=1}^{K} E^{sk}(lx,p)\ u^s(lx^{sk}) =$$

$$= V(y,p) = U(ly,p) = \sum_{s=1}^{\Omega} q^s \sum_{k=1}^{K} E^{sk}(ly,p) u^s(ly^{sk}).$$

Observe that λx allows the offer of the list:

[5]See Correia-da-Silva and Hervés-Beloso (2005).

$\lambda lx = (\lambda lx^1, ..., \lambda lx^\Omega) = [(\lambda lx^{11} \vee ... \vee \lambda lx^{1K}), ..., (\lambda lx^{\Omega 1} \vee ... \vee \lambda lx^{\Omega K})]$.

And, similarly, that $(1-\lambda)y$ allows the offer of $(1-\lambda)ly$:

$(1-\lambda)ly = [(1-\lambda)ly^1, ..., (1-\lambda)ly^\Omega)]$.

As a result, a convex combination of the resources, $z = \lambda x + (1-\lambda)y$, allows delivery of a sort of convex combination of the two lists, lz. This list is defined, for every ω_s, by:

$lz^s = \bigvee_i \bigvee_j [\lambda lx^{si} + (1-\lambda)ly^{sj}]$.

Denote the expected probability of lx^{si} being selected from the list lx^s by r_x^{si}, that of ly^{sj} being selected from list ly^s by r_y^{sj}, and that of $[\lambda lx_i + (1-\lambda)ly_j]$ being selected from lz^s by r_z^{sij}.

By showing that the list $lz = (lz^1, ..., lz^\Omega)$ is not worse than lx and ly, we will arrive a convexity of preferences in a sense that is crucial to establish existence of equilibrium. For it, we assume that agents assume their influences on the delivery choices of the market (among the elements of the list) to be negligible. Precisely, the subjective expected probabilities associated with $\lambda lx^{si} + (1-\lambda)ly^{sj}$ are such that:

$\sum_j r_z^{sij} = r_x^{si}$, for each i;

$\sum_i r_z^{sij} = r_y^{sj}$, for each j.

Reformulating:

$U(lx,p) = \sum_{s=1}^{\Omega} q^s U^s(lx,p) = \sum_{s=1}^{\Omega} q^s \sum_i \sum_j r_z^{sij} u^s(lx^{si})$;

$U(ly,p) = \sum_{s=1}^{\Omega} q^s U^s(ly,p) = \sum_{s=1}^{\Omega} q^s \sum_j \sum_i r_z^{sij} u(ly^{sj})$.

Combining:

$\lambda U^s(lx,p) + (1-\lambda)U^s(ly,p) = \sum_i \sum_j r_z^{sij} \lambda u(lx^{si}) + \sum_j \sum_i r_z^{sij}(1-\lambda)u(ly^{sj}) =$

$= \sum_i \sum_j r_z^{sij} \lambda u(x_i) + (1-\lambda)u(y_j) \leq \sum_i \sum_j r_z^{sij} u(\lambda lx^{si} + (1-\lambda)u(ly^{sj})) =$

$$= U^s(lz, p).$$

Thus, for given prices, preferences over resources are convex. The indirect utility function is quasi-concave and the optimal demand correspondence (defined over primitives) is convex-valued:

$$V(\lambda x + (1 - \lambda)y, p) \geq \min\{V(x, p), V(y, p)\}.$$

4 Existence of equilibrium

Assume for now that the lists proposed to the agents, besides having a maximum of K alternatives, must also have bounded coordinates, that is, $LX_{KT} = [0, T]^{\Omega K l}$ and $X_T = [0, T]^{\Omega l}$. We already know that the subjective expected utility function, $U(lx_i, p)$, is continuous. Thus, it follows that, in this "compact" domain, the indirect utility, $V_i(x_i, p) = \max\limits_{lx_i \in LX_{KT}(x_i)} U_i(lx_i, p)$, is well defined (the maximum always exists).

Furthermore, the correspondence from bundles to sets of lists, $LX(x)$, is continuous with non-empty compact values. Applying Berge's Maximum Theorem, we find that the indirect utility function, $V_i(x_i, p) = \max\limits_{lx_i \in LX_{KT}(x_i)} U_i(lx_i, p)$, is continuous, and that the *argmax* correspondence is upper hemicontinuous.

Recall that in the previous section the indirect utility function, $V_i(x_i, p)$, has been shown to be quasi-concave in the first variable. Thus, to establish existence of equilibrium (in the internal market) we have the classical conditions, except for the fact that preferences also depend on prices. We assume that this dependence is continuous for the subjective utility function to be continuous, thus allowing application of Berge's Maximum Theorem. As usually, the auctioneer takes preferences as given, and sets prices that maximize the value of excess demand, which is a linear function.

Proof.

Consider a correspondence, ψ, which assigns to given prices, p, a vector of bundles, $x'_i \in X_T$, maximizers of $V_i(x_i, p)$, and to the total demand, $\sum_i x_i$, the prices, $p' \in \Delta$, which maximize the value of excess demand:

$\psi : \mathbb{R}_+^{\Omega l n} \times \Delta_+^{\Omega l} \to \mathbb{R}_+^{\Omega l n} \times \Delta_+^{\Omega l};$

$(x', p') \in \psi(x, p) \Leftrightarrow$

i) $x'_i \in argmax_{x_i \in B_i(e_i, p)} \{V_i(x_i, p)\}$, $\forall i$;

ii) $p' \in argmax_{p \in \Delta_+^{\Omega l}} \{p \cdot \sum_i (x_i - e_i)\}$.

The argument is well known. Applying Berge's Maximum Theorem, we find that this correspondence is upper semicontinuous with respect to every x_i and p. It also has non-empty, closed and convex values (from quasi-concaveness of V_i).

The consumption and price spaces, X_T and Δ, are compact, therefore existence of equilibrium can be established using the Theorem of Kakutani.

QED

A difficulty that appears when extending the proof to $LX = \mathbb{R}_+^{K\Omega l}$ and $X = \mathbb{R}_+^{\Omega l}$, is to guarantee existence of $V_i(x_i, p) = \max_{lx_i \in LX(x_i)} U_i(lx_i, p)$.

A straightforward way contour this difficulty is to grant the agents with some information about the total resources in the economy. Using this information, they would regard the alternatives that imply delivery of greater quantities than those that exist in the whole economy as impossible, that is:

$E_i^{sk}(lx, p) \geq 0 \Rightarrow lx^{skl} \leq e^{sl} \leq \sum_i e^{sl}$.[6]

Note that a list can be interpreted as a bundle together with real options of the market on the agent. A list $a \vee b \vee c$ can be seen as the bundle a together with two options given to the market: to trade a for b, and to trade a for c. So the agent may end up consuming a, b or c, depending on the preferences of the market.

Going back to our problem, a restriction to lx_i^s is that one of the alternatives lx_i^{sk} cannot exceed x_i^s. Suppose that it is actually equal to x_i^s. It may be seen, thus, as the combination between x_i^s and a maximum of K real options of the market to trade it for each of the lx_i^{sk}. Notice that it is the market that has the option, not the agent.

[6]The function lx^{skl} represents the quantity of commodity l in the alternative bundle k to be delivered in state ω_s, and the function $E_i^{sk}(lx, p)$ is the k^{th} coordinate of $E_i^s(lx, p)$, that is, the subjective probability of the market selecting alternative k for delivery in state ω_s.

Under this assumption on expectations, the argument for existence of equilibrium in the compact economy extends to $LX = \mathbb{R}_+^{K\Omega l}$ and $X = \mathbb{R}_+^{\Omega l}$. It is enough to work with $\bar{LX} = [0, \sum_i e_i]^K \subset \mathbb{R}_+^{K\Omega l}$ and $\bar{X} = [0, \sum_i e_i] \subset \mathbb{R}_+^{\Omega l}$.

The existence of equilibrium result is relative to the internal market, that is, to (p^*, x^*). To find the lists that are offered to the agents in order to complete the triple of equilibrium (p^*, x^*, lx^*) simply choose lists lx_i^* which maximize utility $U_i(lx_i^*, p^*)$ among those that can be offered through resources x_i^*. Again recurring to Berge's Maximum Theorem, we know that these *argmax* exist.

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