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Abstract. We study regulation of a bureaucratic provider of a public good in the presence of moral hazard and adverse selection. By bureaucratic we mean that it values output in itself, and not only profit. Three different financing systems are studied: cost reimbursement, prospective payment, and the optimal contract. In all cases, the output level increases with the bureaucratic bias. We find that the optimal contract is linear in cost (fixed payment plus partial cost-reimbursement). A stronger preference for high output reduces the tendency of the firm to announce a high cost (adverse selection), allowing a more powered incentive scheme (a lower fraction of the costs is reimbursed), which alleviates the problem of moral hazard.

Keywords: Procurement, Regulation, Adverse selection, Moral hazard, Bureaucracy.

JEL Classification Numbers: D82, H41, H51, I11.

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1 Introduction

The regulation of a public firm under asymmetric cost information has been the subject of intensive research since the pioneering papers of Baron and Myerson (1982), Baron and Besanko (1984) and Laffont and Tirole (1986). In this literature, it is assumed that the managers of the firms maximize profit net of the disutility of effort. This is a quite restrictive assumption, since managers are known to be interested not only in monetary rewards but also in managing a large firm. This preference may reflect the concern of the managers with their reputation and career.

The possibility of bureaucratic behavior should be taken into account when designing a financing system. The public bureau or public firm is characterized by weak external control on efficiency and weak internal incentives (Mueller, 2003). The goals of the bureaucrat are "*salary, perquisites of the office, public reputation, power, patronage, output of the bureau, ease of making changes, and ease in managing the bureau*" (Niskanen, 1971).[1] This motivates us to study procurement contracts between the government and a bureaucratic manager, and to examine whether this changes the results previously obtained in the literature.

In the theory of regulation and procurement, it is usual to assume that the firm (agent) is better informed about its cost than the regulator (principal). This is common to the contributions of Baron and Myerson (1982), Baron and Besanko (1984) and Laffont and Tirole (1986, 1993), which were important milestones. In the work of Baron and Myerson (1982), cost is unobservable. Therefore, the gross payment to the firm can only be a function of the cost function announced by the firm (prospective payment). In this context, the firm tends to announce a high marginal cost in order to receive a high payment while incurring in a low cost. The procurement contract should provide incentives for the firm to announce its true cost (rewarding the firm for announcing a low cost). In the setup of Baron and Besanko (1984), the regulator can, *ex post*, pay an auditing cost to observe (imperfectly) the firm's cost. The optimal scheme is to audit the firm when the reported cost is above a particular level and impose a penalty when the observed cost is low.

To this context of regulation under adverse selection, Laffont and Tirole (1986) add the problem of moral hazard. While in the models of Baron and Myerson (1982) and

[1] For a discussion of the motivation of bureaucrats in a public organization, see Wilson (1989, chapter 9).

Baron and Besanko (1984) the firm's only decision variable is the announcement of its marginal cost (adverse selection), in the model of Laffont and Tirole (1986), the firm's cost-reducing effort (unobserved by the government) is also a decision variable (adverse selection and moral hazard). Under moral hazard, the planner cannot penalize low observed costs as in the work of Baron and Besanko (1984). The firm would simply reduce its effort to increase cost. The optimal contract is shown to be linear in observed cost, being composed by a fixed payment plus a partial cost reimbursement. The fraction of realized cost that is reimbursed to the firm increases with the firm's announced cost (while the output decreases with the firm's announced cost).[2]

In this paper, we allow the manager of the firm to have a preference for higher output, deriving utility from the difference between the output level of the firm and a "reference output level". We study three different kinds of procurement contracts between a bureaucratic firm and a government: a cost reimbursement system, which consists of compensating the firm for the costs in which it incurs, a prospective payment system, which grants a fixed financing, independently of the costs that the firm comes to incur, and the optimal incentive scheme.

In all cases, we show that the output level, as one could expect, is increasing with the bureaucratic bias (strength of the preference for higher output). Since the cost savings associated with effort are proportional to the output, the effort level is increasing with the output level, and, therefore, with the bureaucratic bias (except in the case of the cost reimbursement system, in which cost-reducing efforts are not available). Generalizing the analysis of Laffont and Tirole (1986) to allow for bureaucratic behavior, we find that the optimal contract depends on the strength of the preference of the firm for higher output, but remains linear in cost (is still composed by a fixed prospective transfer plus a partial reimbursement). A stronger preference for high output reduces the tendency of the firm to announce a high cost (adverse selection), allowing a more powered incentive scheme (a lower fraction of the costs is reimbursed), alleviating the problem of moral hazard. In all the cases under study, the expected social welfare increases (decreases) with the bureaucratic bias whenever the expected

[2]Laffont and Tirole (1986) compare their setting with the case in which the regulator is unable to observe cost (as in Baron and Myerson, 1982). If the cost is unobservable, the optimal regulatory policy is a gross transfer that depends on the firm's cost report (prospective payment) in such a way that the firm has no incentive to misrepresent its costs. The prospective payment implies no effort distortion for a given output level contrary to the optimal incentive contract with cost observability, in which the effort is lower than optimal.

output is larger (lower) than the reference output level. This suggests that it is better (from the regulator's point of view) to hire a more bureaucratic manager in order to run a large firm and a less bureaucratic manager in order to run a small firm.

The paper is organized as follows. Section 2 describes the model and section 3 analyzes the benchmark case of complete information. In sections 4, 5 and 6, we derive the different procurement contracts: optimal incentive scheme, cost reimbursement and prospective payment, respectively. Finally, Section 7 offers some concluding remarks.

2 The model

We consider a model of procurement in which the government (principal) offers a contract to the firm (agent) for the provision of a public good. The firm produces an observable output, q, incurring in an observable total cost:

$$C = (\hat{\beta} - e)q + \epsilon.$$

The intrinsic cost parameter, $\hat{\beta}$, is drawn from a uniform distribution $\left[\underline{\beta}, \overline{\beta}\right]$ and is firm's private information (adverse selection). The effort level chosen by the firm after the contract is signed, $e \geq 0$, is also unobservable (moral hazard). Cost observation is subject to an error, ϵ, a random variable with zero mean.[3]

The government observes the total cost, C, incurred by the firm and pays in addition a net monetary transfer t.

The social value of output is $S(q)$, with marginal social value being strictly positive and decreasing, $S'(q) > 0$ and $S''(q) < 0$, for any $q \in [0, \overline{q})$. We also set $S(0) = 0$ and $S'(\overline{q}) = 0$ (where $\overline{q}$ can be interpreted as full coverage of the needs of the population).

The *ex ante* utility level of the firm is

$$U = \alpha E(t) + \delta(q - q_{ref}) - \psi(e), \tag{1}$$

where $E(t)$ is the expected value of the net monetary transfer, q_{ref} is the output reference level of the bureaucrat, and α and $\delta \geq 0$ are weight factors that measure the

[3] This random variable may also be interpreted as a cost disturbance that is unknown to the firm when it chooses its effort level.

biases of the bureaucrat toward profit and output, respectively. Finally, $\psi(e)$, stands for the disutility of effort, with $\psi'(e) > 0$, $\psi''(e) > 0$ and $\psi'''(e) \geq 0$.

The government finances the public good provision using a distortionary mechanism (taxes, for example) so that the social cost of raising one unit is $1+\lambda$. The welfare of consumers is the social value of the public good less the cost of providing it, $S(q) - (1+\lambda)E(t+C)$. The government seeks to maximize the sum of the consumer's welfare with the utility of the firm.

3 The case of complete information

As a benchmark case, we start by considering that the government is able to observe the marginal cost parameter, $\hat{\beta}$, as well as the level of effort, e. The problem of the government is:

$$\max_{q,e,t} \{S(q) - (1+\lambda)E(t+C) + U\} \tag{2}$$

subject to

$$U \geq 0.$$

Using (1), it can be written as:

$$\max_{q,e,U} \left\{ S(q) - \frac{1+\lambda}{\alpha} \left[U - \delta(q - q_{ref}) + \psi(e) + \alpha(\hat{\beta} - e)q \right] + U \right\} \tag{3}$$

subject to

$$U \geq 0.$$

It is necessary that $\lambda > \alpha - 1$ for the participation constraint to be binding ($U = 0$). Otherwise, we could always improve welfare by increasing the taxes and the payment to the firm.[4]

The first order conditions of problem (3) are:

$$S'(q) = (1+\lambda) \left(\hat{\beta} - e - \frac{\delta}{\alpha} \right), \tag{4}$$

$$\psi'(e) = \alpha q. \tag{5}$$

[4]This is in the same spirit as the usual assumption of $\lambda > 0$ (Laffont and Tirole, 1986).

The second order conditions of problem (3) are:[5]

$$
\begin{aligned}
&S''(q) < 0, \\
&\psi''(e) > 0, \\
&S''(q)\psi''(e) + \alpha(1+\lambda) < 0.
\end{aligned}
$$

We make the following assumptions for the problem to be well-behaved.

Assumption 1.

(i) $1+\lambda > \alpha$*;*

(ii) $\forall q \in [0,1],\ S''(q)\psi''(0) < -\alpha(1+\lambda)$*;*

(iii) $S'(0) > (1+\lambda)(\overline{\beta} - \delta/\alpha)$*;*

(iv) $\psi'(\underline{\beta} - \delta/\alpha) > \alpha\overline{q}$*.*

Together with $\psi''' \geq 0$, assumption (ii) guarantees that the second order conditions are always satisfied. Assumption (iii) ensures a positive output level. Assumption (iv) ensures that the marginal cost is positive.

For illustrative purposes, we shall often refer to the case where $S(q) = 2q - q^2$, $\psi(e) = e^2/2$, $\lambda = 0.1$, $\alpha = 1$, $\delta = 0.05$ or $\delta = 0$, $\underline{\beta} = 1.1$, $\overline{\beta} = 1.3$ and $q_{ref} = 1$.

From (4) and (5), we obtain (see Appendix A):

$$
\begin{aligned}
q_c^* &= \frac{1}{2-\alpha(1+\lambda)}\left[2-(1+\lambda)\left(\hat{\beta}-\frac{\delta}{\alpha}\right)\right], \\
e_c^* &= \alpha q_c^*, \\
t_c^* &= \frac{\delta}{\alpha}(q_{ref} - q_c^*) + \frac{e_c^* 2}{2\alpha}.
\end{aligned}
$$

The optimal output, q_c^*, and the level of effort, e_c^*, are decreasing functions of the intrinsic marginal cost, $\hat{\beta}$, and increasing functions of the bureaucratic bias toward higher output, δ. The bureaucratic behaviour reduces social welfare. This occurs because we have chosen a reference output that is higher than the expected output, aggravating the participation constraint.

[5]Corresponding to $\frac{\partial^2 f}{\partial q^2} < 0$, $\frac{\partial^2 f}{\partial e^2} < 0$ and $\frac{\partial^2 f}{\partial q^2}\frac{\partial^2 f}{\partial e^2} < \left(\frac{\partial^2 f}{\partial q \partial e}\right)^2$.

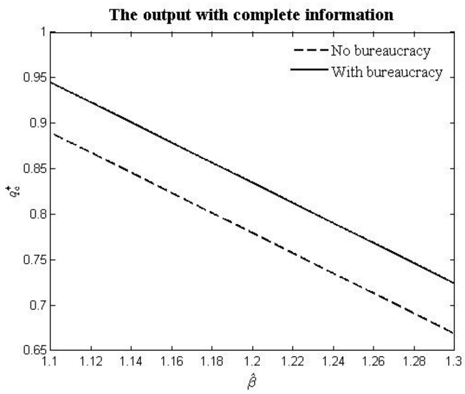

Figure 1: Output with ($\delta = 0.05$) and without bureaucracy ($\delta = 0$).

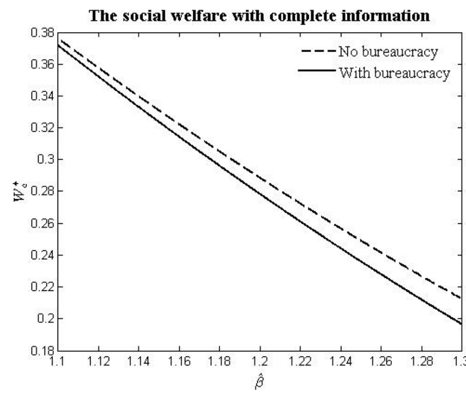

Figure 2: Social welfare with ($\delta = 0.05$) and without bureaucracy ($\delta = 0$).

4 The optimal incentive scheme

In this section, we consider the case in which the government is able to observe the output level, q, and the total cost, C, but not the marginal cost of the firm, $\hat{\beta}$, nor the effort made by the firm, e.[6]

Thanks to the Revelation Principle[7], we can restrict (without loss of generality) our attention to incentive compatible direct mechanisms.

The government offers a contract to the firm, specifying an output, $q(\beta)$, and a compensation scheme, $t(\beta, C)$, which depend on the intrinsic marginal cost announced by the firm, β. Given the compensation scheme, the bureaucrat chooses the level of effort, e, that maximizes its utility. The contract should be incentive compatible (induce truthful revelation):

$$\hat{\beta} \in arg \max_{\beta \in \left[\underline{\beta}, \overline{\beta}\right]} U(\beta, \hat{\beta}),$$

where $U(\beta, \hat{\beta})$ denotes the utility attainable by a firm with cost $\hat{\beta}$ that announces a

[6]Before the contract is signed, the government knows the objective function of the firm, and the prior probability distribution of the efficiency parameter, $\hat{\beta}$ (uniform on the interval $\left[\underline{\beta}, \overline{\beta}\right]$).

[7]By the revelation principle (Myerson, 1979), given a Bayesian Nash equilibrium of a game of incomplete information, there exists a direct mechanism that has an equivalent equilibrium where the players truthfully report their types. A direct-revelation mechanism is said to be "incentive compatible" if, when each individual is expecting the others to be truthful, then he/she has interest in being truthful.

cost β.

The net monetary transfer depends on the announced efficiency and on the observed cost. The expected net monetary transfer (with respect to the disturbance term ϵ) is:

$$s(\beta) = E\left\{t\left[\beta, C(\hat{\beta}, e, q, \epsilon)\right]\right\} = E\left\{t\left[\beta, (\hat{\beta} - e)q + \epsilon\right]\right\}.$$

4.1 The firm's optimization problem

We start by analyzing the case in which there is no cost disturbance ($\epsilon = 0$).

The firm announces its marginal cost, β, and the government recommends a level of effort, $e(\beta)$. Truthful behavior implies that a cost (observed by the government) given by $C(\beta) = [\beta - e(\beta)]q(\beta)$.

With perfect cost observation ($\epsilon = 0$), the observed cost must be exactly equal to $C(\beta)$. Otherwise, the government would impose an extreme penalty to the firm:

$$C \neq C(\beta) \;\Rightarrow\; t(\beta, C) = -\infty.$$

Still, a firm with cost $\hat{\beta}$ can claim to have a higher cost, $\beta > \hat{\beta}$, and choose a lower level of effort, $e(\beta, \hat{\beta})$, incurring in a cost $C = [\hat{\beta} - e(\beta, \hat{\beta})]q(\beta)$. The firm's deviation is concealed if and only if the firm makes an effort, $e(\beta, \hat{\beta})$, that is such that:

$$C = C(\beta) \;\Leftrightarrow\; e(\beta, \hat{\beta}) = e(\beta) + \hat{\beta} - \beta.$$

For any true efficiency parameter of the firm, $\hat{\beta}$, truth-telling must maximize the utility of the firm (incentive compatibility condition):

$$\hat{\beta} \in arg \max_{\beta \in \left[\underline{\beta}, \overline{\beta}\right]} \left\{\alpha s(\beta) + \delta\left[q(\beta) - q_{ref}\right] - \psi\left[e(\beta) + \hat{\beta} - \beta\right]\right\}. \tag{6}$$

Let $V(\hat{\beta})$ be the value function of the maximization problem:

$$V(\hat{\beta}) = \max_{\beta \in \left[\underline{\beta}, \overline{\beta}\right]} U(\beta, \hat{\beta}) = \max_{\beta \in \left[\underline{\beta}, \overline{\beta}\right]} \left\{\alpha s(\beta) + \delta\left[q(\beta) - q_{ref}\right] - \psi\left[e(\beta) + \hat{\beta} - \beta\right]\right\}.$$

Since the incentive compatibility condition (6) must be satisfied, the value function becomes:

$$V(\hat{\beta}) = \alpha s(\hat{\beta}) + \delta\left[q(\hat{\beta}) - q_{ref}\right] - \psi\left[e(\hat{\beta})\right]. \tag{7}$$

From the Envelope Theorem[8], we obtain the first order incentive compatibility constraint:

$$V'(\hat{\beta}) = -\psi' \left[e(\hat{\beta}) \right]. \tag{8}$$

Incentive compatibility implies equation (8), which tells us that the derivative of the value function is equal to the symmetric of the marginal disutility of equilibrium effort. More efficient firms obtain higher equilibrium utility.

Integrating, we obtain:

$$V(\hat{\beta}) = V(\overline{\beta}) + \int_{\hat{\beta}}^{\overline{\beta}} \psi' \left[e(\gamma) \right] d\gamma. \tag{9}$$

The local second order condition, $\frac{\partial^2 U(\beta,\hat{\beta})}{\partial \beta^2} \leq 0$, can be written using the first order condition, $\frac{\partial U(\beta,\hat{\beta})}{\partial \beta} = 0$, and $\psi'' > 0$. It becomes (see Appendix B.1):

$$e'(\hat{\beta}) \leq 1. \tag{10}$$

Which means that the actual marginal cost, $\hat{\beta} - e$, is increasing with the intrinsic marginal cost, $\hat{\beta}$.

Proposition 1 (Firm's optimization problem).

If deviations in the firm's concealment set are not profitable, then:

(i) the effort function and the utility function are differentiable almost everywhere;

(ii) the first order incentive compatibility constraint is given by (8);

(iii) this necessary condition is also sufficient if the effort function satisfies (10).

Proof. See Appendixes B.2 and B.3. □

[8] See, for example, Chiang and Wainwright (2005).

4.2 The government's optimization problem

The objective of the government is to maximize expected social welfare:

$$\max_{q(\hat\beta),e(\hat\beta),t(\hat\beta)} E\int_{\underline{\beta}}^{\overline{\beta}} S\left[q(\hat\beta)\right]-(1+\lambda)\left[t(\hat\beta)+C(\hat\beta)\right]+V(\hat\beta)\,d\hat\beta \tag{11}$$

subject to, for all $\hat\beta$,

$$V(\hat\beta)\geq 0, \tag{12}$$

$$V'(\hat\beta)=-\psi'\left[e(\hat\beta)\right]. \tag{8}$$

$$e'(\hat\beta)\leq 1, \tag{10}$$

From equation (8), V is a decreasing function of $\hat\beta$, so (12) is satisfied if and only if $V(\overline{\beta})\geq 0$. Therefore, we can replace (12) by $V(\overline{\beta})=0$.

We start by studying a relaxed problem in which the second order incentive compatibility condition (10) is ignored. We shall check later that the solution of this relaxed problem is the solution of the general problem.

The relaxed problem of the government is the following:

$$\max_{q(\hat\beta),e(\hat\beta),V(\hat\beta)} \int_{\underline{\beta}}^{\overline{\beta}} S\left[q(\hat\beta)\right]-\frac{1+\lambda}{\alpha}\left\{V(\hat\beta)-\delta\left[q(\hat\beta)-q_{ref}\right]+\psi\left[e(\hat\beta)\right]\right\}-$$
$$-(1+\lambda)\left[\hat\beta-e(\hat\beta)\right]q(\hat\beta)+V(\hat\beta)\,d\hat\beta \tag{13}$$

subject to

$$V(\overline{\beta})=0, \tag{14}$$

$$V'(\hat\beta)=-\psi'\left[e(\hat\beta)\right]. \tag{8}$$

This is an optimal control problem with state variable $V(\hat\beta)$ and control variables $e(\hat\beta)$ and $q(\hat\beta)$. The first order conditions, written below, are obtained in Appendix B.4.

Proposition 2 (Government's optimization problem - necessary conditions).

The following are necessary conditions for an interior optimum of problem (13):

$$V(\overline{\beta})=0, \tag{14}$$

$$V'(\hat\beta)=-\psi'\left[e(\hat\beta)\right], \tag{8}$$

$$S'\left[q(\hat{\beta})\right] = (1+\lambda)\left[\hat{\beta} - e(\hat{\beta}) - \frac{\delta}{\alpha}\right], \tag{15}$$

$$\psi'\left[e(\hat{\beta})\right] = \alpha q(\hat{\beta}) - \left(1 - \frac{\alpha}{1+\lambda}\right)(\hat{\beta} - \underline{\beta})\psi''\left[e(\hat{\beta})\right]. \tag{16}$$

This problem (13) has a unique interior optimum (see Appendix B.4).

We need to check that the condition which was omitted in the relaxed problem ($e'(\hat{\beta}) \leq 1$) is satisfied. Differentiating equations (15) and (16), we obtain:

$$\begin{cases} S''q^{*'} = (1+\lambda)(1-e^{*'}) \\ \psi''e^{*'} = \alpha q^{*'} - \left(1 - \frac{\alpha}{1+\lambda}\right)\left[\psi'' + (\hat{\beta} - \underline{\beta})\psi'''e^{*'}\right] \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} q^{*'} = \frac{1+\lambda}{S''}(1-e^{*'}) \\ e^{*'} = \frac{\alpha(1+\lambda) - \left(1-\frac{\alpha}{1+\lambda}\right)S''\psi''}{S''\psi'' + \alpha(1+\lambda) + \left(1-\frac{\alpha}{1+\lambda}\right)(\hat{\beta}-\underline{\beta})S''\psi'''} \end{cases}$$

Using Assumption 1 (i) and (ii), we find that $e^{*'} < 0$ and $q^{*'} < 0$, which implies that (10) is verified. The solution of (13) is also the solution of (11).

Observe that the more efficient is the firm, the higher are the output and the effort.

Proposition 3.

Under Assumption 1, the firm's effort increases with its efficiency. Therefore, the firm's second order condition is satisfied, and the solution of the relaxed problem (13) is the solution of the general problem (11).

The equilibrium transfer is such that:

$$\alpha t^*(\hat{\beta}) = V(\hat{\beta}) - \delta\left[q^*(\hat{\beta}) - q_{ref}\right] + \psi\left[e^*(\hat{\beta})\right].$$

Using (9), we obtain:

$$t^*(\hat{\beta}) = \frac{1}{\alpha}\left\{\int_{\hat{\beta}}^{\overline{\beta}} \psi'\left[e^*(\gamma)\right]d\gamma - \delta\left[q^*(\hat{\beta}) - q_{ref}\right] + \psi\left[e^*(\hat{\beta})\right]\right\}.$$

We find that the effort and the output levels are increasing on the manager's marginal utility of output, δ (see Appendix B.6, Lemma 7). The intuition behind these results is that an increase in the δ gives more weight to the output level in the firm's objective function and, hence, also in the social welfare function. This translates into higher output and higher effort levels, the later because the cost savings associated with the effort are proportional to the output level.

With the social value given by $S(q) = 2q - q^2$ and the disutility of effort given by $\psi(e) = e^2/2$ (with parameter values that satisfy Assumption 1), equations (15) and (16) allow us to determine the levels of output, effort and transfer (see Appendix B.4):

$$\begin{aligned}
q^*(\hat{\beta}) &= \frac{2}{2-\alpha(1+\lambda)} - \frac{1+\lambda}{2-\alpha(1+\lambda)}\left[\hat{\beta} - \frac{\delta}{\alpha} + \left(1 - \frac{\alpha}{1+\lambda}\right)(\hat{\beta} - \underline{\beta})\right], \\
e^*(\hat{\beta}) &= \alpha q^*(\hat{\beta}) - \left(1 - \frac{\alpha}{1+\lambda}\right)(\hat{\beta} - \underline{\beta}), \\
t^*(\hat{\beta}) &= \frac{1}{\alpha}\left\{\int_{\hat{\beta}}^{\overline{\beta}} e^*(\gamma)d\gamma - \delta\left[q^*(\hat{\beta}) - q_{ref}\right] + \frac{e^*(\hat{\beta})^2}{2}\right\}.
\end{aligned}$$

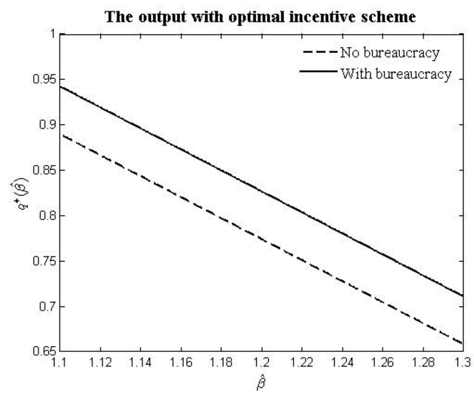

Figure 3: Output with ($\delta = 0.05$) and without bureaucracy ($\delta = 0$).

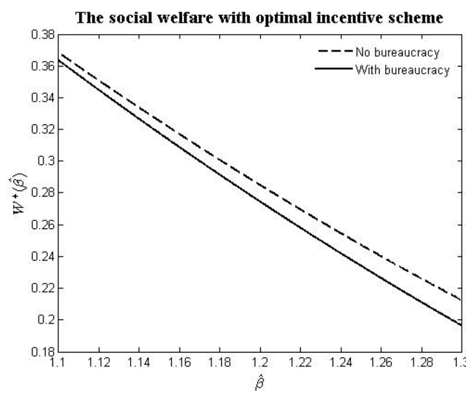

Figure 4: Social welfare with ($\delta = 0.05$) and without bureaucracy ($\delta = 0$).

In this example, the bureaucratic bias reduces social welfare. This occurs because the reference output is higher than the expected output, implying an increase of the monetary transfer from the government to the firm (for the participation constraint to be satisfied).

In general, we find that an increase in the value of δ increases (reduces) expected social welfare if the expected output level is larger (lower) than the reference output (see Appendix B.6, Lemma 8). This means that, when the reference output is lower than expected output, a manager who is more "bureaucratic" is less costly to society as a whole, because the manager receives in a non-monetary form a larger part of the informational rent V. The opposite occurs when the reference output is higher than expected output.

4.3 Implementation

We now consider the implementation problem. Let $\left\{e^*(\hat{\beta}), q^*(\hat{\beta}), V(\hat{\beta})\right\}$ denote the solution to (13), and let $t^*(\hat{\beta})$, $C^*(\hat{\beta})$ and $C(\hat{\beta}, e, q, \epsilon)$ denote the corresponding expected transfer, expected cost and observed cost.

When there is no disturbance, $\epsilon = 0$, to implement the solution, it suffices for the government to: (i) ask the firm to announce its marginal cost, β; (ii) choose output $q^*(\beta)$; and (iii) give transfer $t^*(\beta)$ if $C = C^*(\beta)$, and $-\infty$ otherwise. Laffont and Tirole (1986) defined this contract as the "knife-edge" mechanism.

However, the "knife-edge" mechanism does not work if the cost is not perfectly observed. If there is any noise, the probability of incurring in an extreme penalty becomes positive and makes the firm unwilling to participate.

To implement the optimal solution in the more general case of cost disturbance, we must find a transfer function $t(\beta, C)$ that is such that:

$$\left[\hat{\beta}, e^*(\hat{\beta})\right] \in arg \max \left\{\alpha E\left[t\left(\beta, (\hat{\beta} - e)q(\beta) + \epsilon\right)\right] + \delta\left[q(\beta) - q_{ref}\right] - \psi(e)\right\}$$

and

$$E\left\{t\left[\hat{\beta}, \left[\hat{\beta} - e^*(\hat{\beta})\right] q^*(\hat{\beta}) + \epsilon\right]\right\} = t^*(\hat{\beta}).$$

Consider the following transfer function (linear in observed cost):

$$t(\beta, C) = t^*(\beta) + K^*(\beta)\left[C^*(\beta) - C\right],$$

where

$$K^*(\beta) = \frac{\psi'\left[e^*(\beta)\right]}{\alpha q^*(\beta)}. \tag{17}$$

If the observed cost is higher than expected cost, $C > C^*(\beta)$, the government reimburses a fraction of the difference, $1 - K^*(\beta)$, while the firm supports the remaining fraction, $K^*(\beta)$.

A firm with intrinsic cost $\hat{\beta}$ solves:

$$\max_{\beta, e} \ \alpha t^*(\beta) + \alpha E\left\{K^*(\beta)\left[C^*(\beta) - C\right]\right\} + \delta\left[q^*(\beta) - q_{ref}\right] - \psi(e). \tag{18}$$

Notice that:

$$K^*(\beta)\left[C^*(\beta) - C\right] = \frac{\psi'[e^*(\beta)]}{\alpha}\left[e - e^*(\beta) + \beta - \hat{\beta} - \frac{\epsilon}{q^*(\beta)}\right].$$

Substituting in (18), we obtain:

$$\max_{\beta,e} \ \alpha t^*(\beta) + \psi'[e^*(\beta)] \left[e - e^*(\beta) + \beta - \hat{\beta}\right] + \delta \left[q^*(\beta) - q_{ref}\right] - \psi(e). \tag{19}$$

Optimization with respect to e yields:

$$\psi' \left[e^*(\beta)\right] = \psi'(e) \Leftrightarrow e = e^*(\beta).$$

Substituting again, problem (19) simplifies:

$$\max_{\beta} \ \alpha t^*(\beta) + \psi'[e^*(\beta)] \left(\beta - \hat{\beta}\right) + \delta \left[q^*(\beta) - q_{ref}\right] - \psi \left[e^*(\beta)\right].$$

Optimizing with respect to β:

$$\alpha t^{*'}(\beta) + \psi'' \left[e^*(\beta)\right] e^{*'}(\beta)(\beta - \hat{\beta}) + \psi' \left[e^*(\beta)\right] + \delta q^{*'}(\beta) - \psi' \left[e^*(\beta)\right] e^{*'}(\beta) = 0.$$

Substituting $\beta = \hat{\beta}$ we obtain:

$$\alpha t^{*'}(\hat{\beta}) + \psi' \left[e^*(\hat{\beta})\right] \left[1 - e^{*'}(\hat{\beta})\right] + \delta q^{*'}(\hat{\beta}) = 0.$$

Which we know that is true as it coincides with the first order incentive compatibility condition (8).

Notice that the firm's second order condition for (19) is satisfied, as it boils down to:

$$e^{*'}(\beta) \leq 0. \tag{20}$$

We draw the following conclusion.

Proposition 4.

Under Assumption 1, the optimal incentive compatible allocation, $[q^*(\hat{\beta}), e^*(\hat{\beta}), t^*(\hat{\beta})]$, *can be implemented by a contract that is linear in observed cost:*

$$t(\beta, C) = t^*(\beta) + K^*(\beta) \left[C^*(\beta) - C\right].$$

The second order condition (20) is stronger than (10). It is necessary for this way of implementing the optimal solution, which requires the transfer to be linear in cost. If (20) is satisfied (as is the case under our assumptions), then the linear scheme implements the optimal solution.

The linear scheme implements the optimal allocation. Furthermore, it has a very appealing property. Notice that the optimal allocation is independent of the distribution of cost uncertainty. The linear scheme is the only scheme that implements the optimal allocation for any probability distribution of the cost disturbance (see Appendix B.5).

Let us now turn to the effect of a variation of δ on the "power" of the incentive scheme, that is, on the fraction of the cost that is supported by the firm, K^*. We find that K^* is increasing in δ for any value of $\hat{\beta}$ (if ψ''' is small enough). A more bureaucratic firm leads to more powered incentive schemes (this proposition is made precise in Appendix B.6, Lemma 9).

5 The reimbursement payment system

In this section, we consider the case in which there is no cost-reducing effort (the cost function is $C = \hat{\beta}q + \epsilon$), and study the financing system known as cost-reimbursement, which consists in: (i) compensating the firm for all the costs which it incurs; plus (ii) a net payment in advance, $t(\hat{\beta})$, which can be negative because a bureaucratic firm enjoys producing a high output.

The bureaucratic bias, δ, leads to a significant difference with respect to the usual reimbursement payment (Laffont and Tirole, 1993): the equilibrium transfer to the firm depends on the intrinsic marginal cost.

The utility of the firm is $U = \alpha t(\beta) + \delta\,[q(\beta) - q_{ref}]$. It is independent of $\hat{\beta}$, therefore, it must be constant across β for the firm to be truth-telling. Therefore, the government will choose $t(\beta)$ and $q(\beta)$ such that $U(\beta) = 0$, for any announcement β (notice that the participation constraint is binding for $\alpha < 1 + \lambda$, which is the economically interesting case).

To produce an output lower than q_{ref}, the firm requires a positive net transfer to participate, $t(\hat{\beta}) > 0$. If the output is higher than q_{ref}, the firm accepts a negative transfer, $t(\hat{\beta}) < 0$.[9]

$$U(\hat{\beta}) = 0 \Leftrightarrow t(\hat{\beta}) = -\frac{\delta}{\alpha}\left[q(\hat{\beta}) - q_{ref}\right]. \tag{21}$$

[9]This has the flavor of the typical agency problem in which the managers value output while the owners value profits.

The government's problem is:

$$\max_{q(\hat{\beta}),t(\hat{\beta})} \int_{\underline{\beta}}^{\overline{\beta}} S\left[q(\hat{\beta})\right] - (1+\lambda)\left[C(\hat{\beta}) + t(\hat{\beta})\right] + \alpha t(\hat{\beta}) + \delta\left[q(\hat{\beta}) - q_{ref}\right]\, d\hat{\beta} \tag{22}$$

subject to

$$t(\hat{\beta}) = -\frac{\delta}{\alpha}\left[q(\hat{\beta}) - q_{ref}\right]. \tag{21}$$

Equivalently:

$$\max_{q(\hat{\beta})} \int_{\underline{\beta}}^{\overline{\beta}} S\left[q(\hat{\beta})\right] - (1+\lambda)\left\{\hat{\beta}q(\hat{\beta}) - \frac{\delta}{\alpha}\left[q(\hat{\beta}) - q_{ref}\right]\right\}\, d\hat{\beta}. \tag{23}$$

The first order condition of problem (23) is:

$$S'\left[q(\hat{\beta})\right] = (1+\lambda)\left(\hat{\beta} - \frac{\delta}{\alpha}\right).$$

The second order condition is verified because $S''(q) < 0$.

Appendix C studies the problem (23) with $S(q) = 2q(\hat{\beta}) - q(\hat{\beta})^2$. The solution is:

$$\begin{aligned} q_r^*(\hat{\beta}) &= 1 - \frac{1+\lambda}{2}\left(\hat{\beta} - \frac{\delta}{\alpha}\right), \\ t_r^*(\hat{\beta}) &= -\frac{\delta}{\alpha}\left[1 - \frac{1+\lambda}{2}\left(\hat{\beta} - \frac{\delta}{\alpha}\right) - q_{ref}\right]. \end{aligned}$$

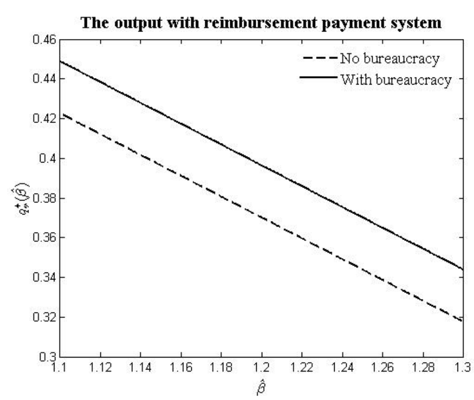

Figure 5: Output with ($\delta = 0.05$) and without bureaucracy ($\delta = 0$).

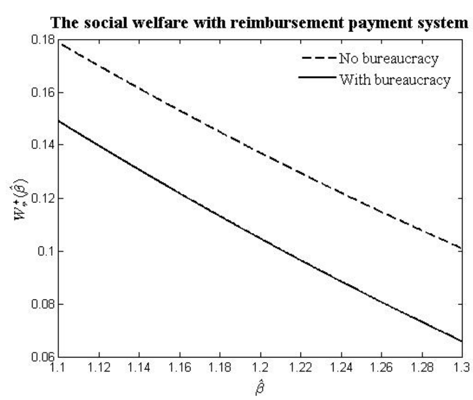

Figure 6: Social welfare with ($\delta = 0.05$) and without bureaucracy ($\delta = 0$).

With bureaucratic behavior, the cost reimbursement system yields a net transfer that is different from zero, a higher output and a decrease in social welfare. When compared with the optimum incentive scheme, we observe a lower level of output. It is easy to see that, as in the case of the optimal incentive scheme, expected social welfare increases (decreases) with the bureaucratic bias whenever the expected output level is higher (lower) than the reference output.

6 The prospective payment system

The prospective payment system consists of a fixed payment, $g(\beta)$, independent of the observed cost.[10]

The net monetary transfer is $t\left[\beta, C(\hat{\beta}, e, q, \epsilon)\right] = g(\beta) - C(\hat{\beta}, e, q, \epsilon)$.

6.1 The firm's optimization problem

The firm chooses the values of β and e that maximize expected utility:

$$U(\beta, \hat{\beta}) = \max_{e} \alpha g(\beta) - \alpha(\hat{\beta} - e)q(\beta) + \delta\left[q(\beta) - q_{ref}\right] - \psi(e).$$

The first order condition with respect to e determines the level of effort:

$$\psi'\left[e(\beta)\right] = \alpha q(\beta). \tag{24}$$

Since the government's transfer does not depend on the observed cost, the relationship between the effort level and the output level is the same as in the case of complete information.

The firm truthfully announces its efficiency ($\beta = \hat{\beta}$) if and only if:

$$\hat{\beta} \in arg \max_{\beta \in \left[\underline{\beta}, \overline{\beta}\right]} \left\{\alpha g(\beta) - \alpha\left[\hat{\beta} - e(\beta)\right] q(\beta) + \delta\left[q(\beta) - q_{ref}\right] - \psi\left[e(\beta)\right]\right\}.$$

[10] The prospective payment system is used, in some countries, in contracts between governments and hospitals for the provision of health care services. A fixed financing is attributed, based on the Diagnosis-Related Group (DRG) of an hospital's admission record, independently of the costs that the hospital comes to incur in.

The first order incentive compatibility constraint is:

$$V'(\hat{\beta}) = -\alpha q(\hat{\beta}). \tag{25}$$

Integrating, we obtain:

$$V(\hat{\beta}) = V(\overline{\beta}) + \int_{\hat{\beta}}^{\overline{\beta}} \alpha q(\xi)\, d\xi. \tag{26}$$

And the second order incentive constraint is:

$$q'(\hat{\beta}) \leq 0. \tag{27}$$

For the firm to participate, we must have $V(\hat{\beta}) \geq 0$. Given (25), we can substitute it for $V(\overline{\beta}) = 0$.

6.2 The government's optimization problem

The objective of the government is to maximize expected social welfare:

$$\begin{aligned} \max_{q(\hat{\beta}),e(\hat{\beta})} \int_{\underline{\beta}}^{\overline{\beta}} S\left[q(\hat{\beta})\right] - (1+\lambda)g(\hat{\beta}) + \alpha\left\{g(\hat{\beta}) - \left[\hat{\beta} - e(\beta)\right] q(\beta)\right\} + \\ +\delta\left[q(\hat{\beta}) - q_{ref}\right] - \psi\left[e(\hat{\beta})\right] d\hat{\beta} \end{aligned} \tag{28}$$

subject to

$$V(\overline{\beta}) = 0, \tag{29}$$
$$V'(\hat{\beta}) = -\alpha q(\hat{\beta}), \tag{25}$$
$$q'(\hat{\beta}) \leq 0. \tag{27}$$

We shall solve the following relaxed problem obtained by dropping (27) and then check that the solution satisfies this constraint.

$$\begin{aligned} \max_{q(\hat{\beta}),e(\hat{\beta}),V(\hat{\beta})} \int_{\underline{\beta}}^{\overline{\beta}} S\left[q(\hat{\beta})\right] - \frac{1+\lambda}{\alpha}\left\{V(\hat{\beta}) - \delta\left[q(\hat{\beta}) - q_{ref}\right] + \psi\left[e(\hat{\beta})\right]\right\} - \\ -(1+\lambda)\left[\hat{\beta} - e(\hat{\beta})\right] q(\hat{\beta}) + V(\hat{\beta})\, d\hat{\beta} \end{aligned} \tag{30}$$

subject to

$$V(\overline{\beta}) = 0, \tag{29}$$
$$V'(\hat{\beta}) = -\alpha q(\hat{\beta}), \tag{25}$$

This is an optimal control problem with state variable $V(\hat{\beta})$ and control variables $e(\hat{\beta})$ and $q(\hat{\beta})$. The first order conditions, written below, are obtained in Appendix D.1.

Proposition 5 (Government's optimization problem - necessary conditions).

The following are necessary conditions for an interior optimum of problem (30):

$$V(\overline{\beta}) = 0, \tag{29}$$

$$V'(\hat{\beta}) = -\alpha q(\hat{\beta}), \tag{25}$$

$$\psi'\left[e(\hat{\beta})\right] = \alpha q(\hat{\beta}), \tag{24}$$

$$S'\left[q(\hat{\beta})\right] = (1+\lambda)\left[2\hat{\beta} - \underline{\beta} - e(\hat{\beta}) - \frac{\delta}{\alpha}\right] - \alpha(\hat{\beta} - \underline{\beta}). \tag{31}$$

The first order conditions are sufficient under Assumption 1 (ii) (see Appendix D.1).

We can verify that the more efficient is the firm, the higher are the output and the effort, by differentiating equations (31) and (24):

$$\begin{cases} S'' q_p^{*'} = (1+\lambda)(2 - e_p^{*'}) - \alpha \\ \psi'' e_p^{*'} = \alpha q_p^{*'} \end{cases} \Leftrightarrow \begin{cases} q_p^{*'} = \psi'' \frac{2+2\lambda-\alpha}{\psi'' S'' + \alpha(1+\lambda)} \\ e_p^{*'} = \frac{\alpha(2+2\lambda-\alpha)}{\psi'' S'' + \alpha(1+\lambda)}. \end{cases}$$

We find that $e^{*'} < 0$ and $q^{*'} < 0$ which implies that (27) is verified. The solution of the relaxed problem (30) is the solution of the fully constrained problem (28).

Proposition 6. *Under Assumption 1, the firm's output increases with its efficiency. Therefore, the firm's second order condition is satisfied, and the solution of the relaxed problem (30) is the solution of the fully constrained (28) problem.*

The equilibrium transfer is such that $\alpha t_p^*(\hat{\beta}) = V_p^*(\hat{\beta}) - \delta\left[q_p^*(\beta) - q_{ref}\right] + \psi\left[e_p^*(\beta)\right]$ and, given (26), we obtain:

$$t_p^*(\hat{\beta}) = \frac{1}{\alpha}\left\{\int_{\hat{\beta}}^{\overline{\beta}} \alpha q_p^*(\xi)\, d\xi - \delta\left[q_p^*(\hat{\beta}) - q_{ref}\right] + \psi\left[e_p^*(\hat{\beta})\right]\right\}.$$

As in the previous sections, for the numerical illustration, we assume that the social value is $S(q) = 2q - q^2$ and that the disutility of effort is $\psi(e) = e^2/2$. Then (see

Appendix D.1):

$$
\begin{aligned}
q_p^*(\hat{\beta}) &= \frac{1}{2-\alpha(1+\lambda)}\left[2+\alpha(\hat{\beta}-\underline{\beta})\right] - \frac{1+\lambda}{2-\alpha(1+\lambda)}\left(2\hat{\beta}-\underline{\beta}-\frac{\delta}{\alpha}\right),\\
e_p^*(\hat{\beta}) &= \frac{\alpha}{2-\alpha(1+\lambda)}\left[2+\alpha(\hat{\beta}-\underline{\beta})\right] - \frac{\alpha(1+\lambda)}{2-\alpha(1+\lambda)}\left(2\hat{\beta}-\underline{\beta}-\frac{\delta}{\alpha}\right),\\
t_p^*(\hat{\beta}) &= \frac{1}{\alpha}\left\{\int_{\hat{\beta}}^{\overline{\beta}} \alpha q_p^*(\xi)\,d\xi - \delta\left[q_p^*(\hat{\beta})-q_{ref}\right] + \frac{e_p^*(\hat{\beta})^2}{2}\right\}.
\end{aligned}
$$

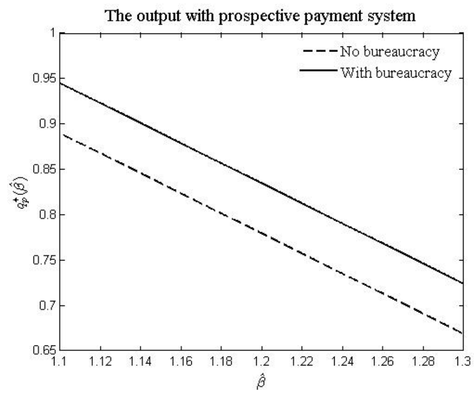

Figure 7: Output with ($\delta = 0.05$) and without bureaucracy ($\delta = 0$).

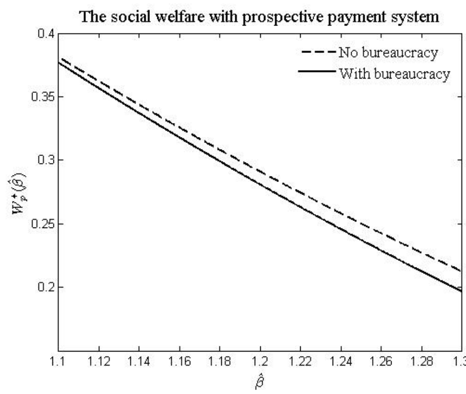

Figure 8: Social welfare with ($\delta = 0.05$) and without bureaucracy ($\delta = 0$).

The firm tends to announce a low efficiency for the government to transfer a high prospective payment. Bureaucratic behaviour counterbalances this tendency, alleviating the problem of adverse selection.

In the general case, we find that the output and effort levels are increasing in δ (see Appendix D.2, Lemma 10), and that the expected social welfare increases (decreases) with the bureaucratic bias whenever the expected output level is larger (lower) than the reference output, as in the previous cases (see Appendix D.2, Lemma 11).

7 Concluding remarks

We have studied procurement contracts between the government (principal) and a bureaucratic firm (agent), in the presence of moral hazard and adverse selection. Three different payment systems were considered: the optimal incentive scheme, cost reimbursement and prospective payment. In any case, with a bureaucratic provider, we observe a higher level of public good provision. Under the prospective payment and the optimal incentive schemes, the effort level is also higher. The optimal incentive scheme is shown to remain linear in observed cost but to become more powered (the firm supports a higher fraction of the costs) when the manager is more bureaucratic. Finally we suggest that it is more interesting for the regulator to have large public firms run by more bureaucratic managers and small ones run by less bureaucratic managers.

The value of the manager's marginal utility of output is, in the present model, known by the regulator. It would be interesting to account for the fact that it is more likely to be his/her private information and to analyze the resulting equilibrium in such a framework. This will be the subject of further research.

A Appendix: Complete information

For the numerical illustration, we replace the social value by $S(q) = 2q - q^2$ and the disutility of effort by $\psi(e) = e^2/2$ in problem (3):

$$\max_{q,e,U} \left\{ 2q - q^2 - \frac{1+\lambda}{\alpha} \left[U - \delta(q - q_{ref}) + \frac{e^2}{2} + \alpha(\hat{\beta} - e)q \right] + U \right\} \tag{32}$$

subject to

$$U \geq 0.$$

With the participation condition being always binding, problem (32) becomes:

$$\max_{q,e} \left\{ 2q - q^2 - (1+\lambda) \left[-\frac{\delta}{\alpha}(q - q_{ref}) + \frac{e^2}{2\alpha} + (\hat{\beta} - e)q \right] \right\}. \tag{33}$$

The first order conditions of problem (33) are:

$$\frac{\partial f}{\partial q} = 0 \Leftrightarrow 2 - 2q - (1+\lambda) \left(-\frac{\delta}{\alpha} + \hat{\beta} - e \right) = 0, \tag{34}$$

$$\frac{\partial f}{\partial e} = 0 \Leftrightarrow -(1+\lambda) \left(\frac{e}{\alpha} - q \right) = 0. \tag{35}$$

From (34) and (35), we obtain:

$$
\begin{aligned}
q_c^* &= \frac{1}{2-\alpha(1+\lambda)}\left[2-(1+\lambda)\left(\hat{\beta}-\frac{\delta}{\alpha}\right)\right], \\
e_c^* &= \alpha q_c^*, \\
t_c^* &= \frac{\delta}{\alpha}(q_{ref}-q_c^*)+\frac{e_c^{*2}}{2\alpha}.
\end{aligned}
$$

Assumption 1 guarantees that:

(i) the participation condition is always binding;

(ii) the second order conditions are satisfied.

(iii) the optimal output level is greater than zero.

(iv) the marginal cost is positive.

Assumption 1 is satisfied if we choose, for example, $\lambda = 0.1$, $\alpha = 1$, $\delta = 0$ or $\delta = 0.05$, $\underline{\beta} = 1.1$, $\overline{\beta} = 1.3$ and $q_{ref} = 1$.

B Appendix: The optimal incentive scheme

B.1 Problem of the firm - second order condition

The derivative of the value function with respect to $\hat{\beta}$ is:

$$
\frac{dV(\hat{\beta})}{d\hat{\beta}} = \frac{\partial U(\beta,\hat{\beta})}{\partial \beta}\frac{d\beta}{d\hat{\beta}} + \frac{\partial U(\beta,\hat{\beta})}{\partial \hat{\beta}} = \frac{\partial U(\beta,\hat{\beta})}{\partial \beta} + \frac{\partial U(\beta,\hat{\beta})}{\partial \hat{\beta}}. \tag{36}
$$

Differentiating (36) with respect to β we obtain:

$$
0 = \frac{\partial^2 U(\beta,\hat{\beta})}{\partial \beta^2} + \frac{\partial^2 U(\beta,\hat{\beta})}{\partial \beta \partial \hat{\beta}} \Leftrightarrow \frac{\partial^2 U(\beta,\hat{\beta})}{\partial \beta^2} = -\frac{\partial^2 U(\beta,\hat{\beta})}{\partial \beta \partial \hat{\beta}}.
$$

The local second order condition can, therefore, be written as:

$$
\frac{\partial^2 U(\beta,\hat{\beta})}{\partial \beta \partial \hat{\beta}}\Big|_{\beta=\hat{\beta}} \geq 0 \quad \text{for any } \hat{\beta}.
$$

Evaluating this second order derivative:

$$
\frac{\partial^2 U(\beta,\hat{\beta})}{\partial \beta \partial \hat{\beta}} = -\psi''[e(\beta)+\hat{\beta}-\beta][e'(\beta)-1].
$$

At the optimum:

$$\frac{\partial^2 U(\beta,\hat{\beta})}{\partial\beta\partial\hat{\beta}}\Big|_{\beta=\hat{\beta}} = -\psi''[e(\hat{\beta})][e'(\hat{\beta})-1].$$

Since $\psi'' > 0$, the local second order condition becomes:

$$e'(\hat{\beta}) \leq 1.$$

B.2 Differentiability of effort, transfer, and utility functions

The objective of this section is to prove **Proposition 1 (i)**.

Lemma 1. $\beta < \hat{\beta} \Rightarrow e(\beta,\hat{\beta}) \geq e(\hat{\beta},\hat{\beta})$.

Proof.

From the incentive compatibility constraints, we know that:

$$\alpha s(\hat{\beta}) + \delta\left[q(\hat{\beta}) - q_{ref}\right] - \psi\left[e(\hat{\beta},\hat{\beta})\right] \;\; \geq \;\; \alpha s(\beta) + \delta\left[q(\beta) - q_{ref}\right] - \psi\left[e(\beta,\hat{\beta})\right]$$

and

$$\alpha s(\beta) + \delta\left[q(\beta) - q_{ref}\right] - \psi\left[e(\beta,\beta)\right] \;\; \geq \;\; \alpha s(\hat{\beta}) + \delta\left[q(\hat{\beta}) - q_{ref}\right] - \psi\left[e(\hat{\beta},\beta)\right].$$

Adding the two inequalities, we obtain:

$$\psi\left[e(\beta,\hat{\beta})\right] - \psi\left[e(\beta,\beta)\right] \geq \psi\left[e(\hat{\beta},\hat{\beta})\right] - \psi\left[e(\hat{\beta},\beta)\right]. \tag{37}$$

Notice that, by definition:

$$e(\beta,\hat{\beta}) - e(\beta,\beta) \;\; = \;\; \hat{\beta} - \beta > 0$$

and

$$e(\hat{\beta},\hat{\beta}) - e(\hat{\beta},\beta) \;\; = \;\; \hat{\beta} - \beta > 0.$$

Since these differences coincide, strict convexity of ψ together with (37) implies that:

$$e(\beta,\hat{\beta}) \geq e(\hat{\beta},\hat{\beta}).$$

Q.E.D.

Lemma 2. $e(\beta, \hat{\beta})$ is nonincreasing in β.

Proof.

Let $\beta > \beta'$ and define $\Delta(\hat{\beta}) \equiv e(\beta', \hat{\beta}) - e(\beta, \hat{\beta})$.

We want to prove that $\Delta(\hat{\beta}) \geq 0$.

Notice that: $\Delta(\hat{\beta}) = e(\beta') - \beta' - e(\beta) + \beta$. Thus $\Delta(\hat{\beta})$ does not depend on $\hat{\beta}$.

Then, $\Delta(\hat{\beta}) = \Delta(\beta) = e(\beta', \beta) - e(\beta, \beta)$.

By Lemma 1, $\Delta(\beta) \geq 0$.

Q.E.D.

Since the effort level is bounded, Lemma 2 implies that $e(\beta, \hat{\beta})$ is almost everywhere differentiable in β. Therefore, $e(\beta) = e(\beta, \hat{\beta}) + \beta - \hat{\beta}$ is also almost everywhere differentiable in β.

Lemma 3. $U(\beta, \hat{\beta})$, as a function of β, is nondecreasing on $\left[\underline{\beta}, \hat{\beta}\right]$ and nonincreasing on $\left[\hat{\beta}, \overline{\beta}\right]$.

Proof.

Let us first show monotonicity on $\left[\underline{\beta}, \hat{\beta}\right]$. Assume that $\beta < \beta' < \hat{\beta}$ and, by way of contradiction, $U(\beta, \hat{\beta}) > U(\beta', \hat{\beta})$. Thus:

$$\alpha s(\beta) + \delta \left[q(\beta) - q_{ref}\right] - \psi\left[e(\beta, \hat{\beta})\right] > \alpha s(\beta') + \delta\left[q(\beta') - q_{ref}\right] - \psi\left[e(\beta', \hat{\beta})\right].$$

On the other hand, we know that a firm with cost β' prefers to announce β' rather than announce β. Thus:

$$\alpha s(\beta') + \delta\left[q(\beta') - q_{ref}\right] - \psi\left[e(\beta', \beta')\right] \geq \alpha s(\beta) + \delta\left[q(\beta) - q_{ref}\right] - \psi\left[e(\beta, \beta')\right].$$

Adding the last two equations, we get:

$$\psi\left[e(\beta, \beta')\right] - \psi\left[e(\beta', \beta')\right] > \psi\left[e(\beta, \hat{\beta})\right] - \psi\left[e(\beta', \hat{\beta})\right].$$

By definition:

$$e(\beta, \beta') - e(\beta', \beta') = e(\beta, \hat{\beta}) - e(\beta', \hat{\beta}).$$

From Lemma 1, $e(\beta, \beta') \geq e(\beta', \beta')$. Thus:

$$e(\beta, \beta') - e(\beta', \beta') = e(\beta, \hat{\beta}) - e(\beta', \hat{\beta}) > 0.$$

Again, by definition:

$$e(\beta, \beta') < e(\beta, \hat{\beta}).$$

The last two equations, together with convexity of ψ, imply:

$$\psi\left[e(\beta, \beta')\right] - \psi\left[e(\beta', \beta')\right] > \psi\left[e(\beta, \hat{\beta})\right] - \psi\left[e(\beta', \hat{\beta})\right].$$

Which is a contradiction.

Monotonicity on $\left[\hat{\beta}, \bar{\beta}\right]$ can be proved in the same way.

Q.E.D.

Lemma 4. $\alpha s(\beta) + \delta q(\beta)$ is nonincreasing.

Proof.

By definition:

$$U(\beta, \underline{\beta}) = \alpha s(\beta) + \delta\left[q(\beta) - q_{ref}\right] - \psi\left[e(\beta, \underline{\beta})\right] \Leftrightarrow$$

$$\Leftrightarrow \alpha s(\beta) + \delta\left[q(\beta) - q_{ref}\right] = U(\beta, \underline{\beta}) + \psi\left[e(\beta, \underline{\beta})\right].$$

From Lemma 2: $\psi\left[e(\beta, \underline{\beta})\right]$ is nonincreasing with β.

From Lemma 3: $U(\beta, \underline{\beta})$ is nonincreasing with β.

Therefore, $\alpha s(\beta) + \delta q(\beta)$ must also be nonincreasing with β.

Q.E.D.

Lemmas 2 and 4 imply that the functions $e(\beta, \hat{\beta})$, $e(\beta)$ and $\alpha s(\beta) + \delta q(\beta)$ are almost everywhere differentiable. Hence $U(\beta, \hat{\beta}) = \alpha s(\beta) + \delta\left[q(\beta) - q_{ref}\right] - \psi\left[e(\beta, \hat{\beta})\right]$ and $V(\hat{\beta}) = \alpha s(\hat{\beta}) + \delta\left[q(\hat{\beta}) - q_{ref}\right] - \psi\left[e(\hat{\beta})\right]$ are also almost everywhere differentiable.

Q.E.D.

B.3 The local second order condition implies the global one

Below, we prove **Proposition 1 (iii)**.

Lemma 5. If $\partial U/\partial \beta$ is (strictly) monotonic in $\hat{\beta}$, then the local second order condition implies the global one.

Proof.

The local second order condition implies that announcing the truth $\beta = \hat{\beta}$ gives a local maximum for the firm of type $\hat{\beta}$. Is there another announcement, $\beta \neq \hat{\beta}$, that satisfies the first order condition? That is, does there exist $\beta \neq \hat{\beta}$ such that:

$$\frac{\partial U(\beta, \hat{\beta})}{\partial \beta} = \frac{\partial U(\hat{\beta}, \hat{\beta})}{\partial \beta} = 0?$$

This would imply that

$$\frac{\partial U}{\partial \beta}(\beta, \hat{\beta}) = \frac{\partial U}{\partial \beta}(\beta, \beta) = 0.$$

But this is inconsistent with the strict monotonicity of $\partial U/\partial \beta$ with respect to its second argument.

Q.E.D.

Lemma 6. If the local second order condition is (strictly) satisfied, then $\partial^2 U/\partial \beta \partial \hat{\beta}$ is (strictly) positive.

Proof.

Recall that:

$$U(\beta, \underline{\beta}) = \alpha s(\beta) + \delta \left[q(\beta) - q_{ref}\right] - \psi \left[e(\beta, \underline{\beta})\right].$$

The partial derivative with respect to β yields an expression that we, then, differentiate with respect to $\hat{\beta}$, to obtain:

$$\frac{\partial^2 U}{\partial \beta \partial \hat{\beta}} = -\psi'' \left[e(\beta) + \hat{\beta} - \beta\right] \left[e'(\beta) - 1\right].$$

Using the strict version of (10) and the strict convexity of ψ, we obtain $\frac{\partial^2 U}{\partial \beta \partial \hat{\beta}} > 0$.

Q.E.D.

B.4 The study of the Hamiltonian

Government's optimization problem - necessary conditions

Consider problem (13). The Hamiltonian is:

$$H = S\left[q(\hat{\beta})\right] - \tfrac{1+\lambda}{\alpha}\left\{V(\hat{\beta}) - \delta\left[q(\hat{\beta}) - q_{ref}\right] + \psi\left[e(\beta)\right] + \alpha\left[\hat{\beta} - e(\hat{\beta})\right]q(\hat{\beta})\right\}$$
$$+V(\hat{\beta}) + \mu\left\{-\psi'\left[e(\hat{\beta})\right]\right\}, \tag{38}$$

where μ is the multiplier associated with (8). The Pontryagin principle[11] yields:

$$\frac{\partial H}{\partial q} = S'\left[q(\hat{\beta})\right] - (1+\lambda)\left[\hat{\beta} - e(\hat{\beta}) - \frac{\delta}{\alpha}\right] = 0,$$

$$\frac{\partial H}{\partial e} = -\frac{1+\lambda}{\alpha}\left\{\psi'\left[e(\hat{\beta})\right] - \alpha q(\hat{\beta})\right\} - \mu\psi''\left[e(\hat{\beta})\right] = 0, \tag{39}$$

$$\mu'(\hat{\beta}) = -\frac{\partial H}{\partial V} = \frac{1+\lambda}{\alpha} - 1. \tag{40}$$

Furthermore, $\underline{\beta}$ is a free boundary so that

$$\mu(\underline{\beta}) = 0. \tag{41}$$

Integrating (40) and using (41), we obtain

$$\mu(\hat{\beta}) = \left(\frac{1+\lambda}{\alpha} - 1\right)(\hat{\beta} - \underline{\beta}). \tag{42}$$

Substituting in (39) above:

$$\psi'\left[e(\hat{\beta})\right] = \alpha q(\hat{\beta}) - \left(1 - \frac{\alpha}{1+\lambda}\right)(\hat{\beta} - \underline{\beta})\psi''\left[e(\hat{\beta})\right].$$

Government's optimization problem - sufficient conditions

The second order derivatives of the Hamiltonian (38) are:

$$\begin{aligned}
\frac{\partial^2 H}{\partial q^2} &= S''\left[q(\hat{\beta})\right] < 0,\\
\frac{\partial^2 H}{\partial e^2} &= -\frac{1+\lambda}{\alpha}\psi''\left[e(\hat{\beta})\right] + \left(1 - \frac{\alpha}{1+\lambda}\right)\left(\hat{\beta} - \underline{\beta}\right)\psi'''\left[e(\hat{\beta})\right] < 0,\\
\frac{\partial^2 H}{\partial q \partial e} &= \frac{\partial^2 H}{\partial e \partial q} = 1+\lambda.
\end{aligned}$$

[11]See, for example, Chiang and Wainwright (2005).

The determinant of the Hessian is:

$$|H| = S'' \left[q(\hat{\beta})\right] \left\{-\tfrac{1+\lambda}{\alpha}\psi'' \left[e(\hat{\beta})\right] + \left(1 - \tfrac{\alpha}{1+\lambda}\right) \left(\hat{\beta} - \underline{\beta}\right) \psi''' \left[e(\hat{\beta})\right]\right\} -$$
$$-(1+\lambda)^2 \geq -S'' \left[q(\hat{\beta})\right] \tfrac{1+\lambda}{\alpha}\psi'' \left[e(\hat{\beta})\right] - (1+\lambda)^2.$$

Using Assumption 1 (ii), we find that $|H| > 0$. Assumption 1 implies that the first order conditions have a unique interior solution.

Finally, observe that the argument in Laffont and Tirole (1986, p.639) applies. Pontryagin's Principle requires V to be piecewise differentiable with a finite number of pieces, while we only know that V is a.e. differentiable and decreasing. The space of a.e. differentiable decreasing functions in $[\underline{\beta}, \overline{\beta}]$ is a closed and convex subset of the Banach space $L^\infty([\underline{\beta}, \overline{\beta}], \mathbb{R})$. Any decreasing function in $[\underline{\beta}, \overline{\beta}]$ that is a.e. differentiable can be approximated as closely as desired by a piecewise-continuous function. Therefore, the maximum in the subspace of piecewise-continuous functions (the solution that we found above) is the maximum in the general space of a.e. differentiable functions (the solution of the general problem).

Optimal incentive scheme - numerical example

Consider problem (13), and replace the social value by $S(q) = 2q - q^2$ and the disutility of effort by $\psi(e) = e^2/2$. The Hamiltonian becomes:

$$H = 2q(\hat{\beta}) - q(\hat{\beta})^2 - \frac{1+\lambda}{\alpha}\left\{V(\hat{\beta}) - \delta\left[q(\hat{\beta}) - q_{ref}\right] + \frac{e(\hat{\beta})^2}{2} + \frac{1}{\alpha}\left[\hat{\beta} - e(\hat{\beta})\right] q(\hat{\beta})\right\} +$$
$$+ V(\hat{\beta}) - \mu e(\hat{\beta}).$$

The Pontryagin principle yields:

$$\frac{\partial H}{\partial q} = 2 - 2q(\hat{\beta}) - (1+\lambda)\left[\hat{\beta} - e(\hat{\beta}) - \frac{\delta}{\alpha}\right] = 0, \quad (43)$$

$$\frac{\partial H}{\partial e} = -(1+\lambda)\left[\frac{e(\hat{\beta})}{\alpha} - q(\hat{\beta})\right] - \mu = 0, \quad (44)$$

$$\mu'(\beta) = -\frac{\partial H}{\partial V} = \frac{1+\lambda}{\alpha} - 1. \quad (45)$$

Furthermore, $\underline{\beta}$ is a free boundary so that

$$\mu(\underline{\beta}) = 0. \quad (46)$$

Integrating (45) and using (46), we obtain

$$\mu(\hat{\beta}) = \left(\frac{1+\lambda}{\alpha} - 1 \right) (\hat{\beta} - \underline{\beta}). \tag{47}$$

Replacing equation (47) into (44), we obtain:

$$e(\hat{\beta}) = \alpha q(\hat{\beta}) - \left(1 - \frac{\alpha}{1+\lambda} \right) (\hat{\beta} - \underline{\beta}). \tag{48}$$

Replacing equation (48) into (43), we find the level of output:

$$q^*(\hat{\beta}) = \frac{2}{2-\alpha(1+\lambda)} - \frac{1+\lambda}{2-\alpha(1+\lambda)} \left[\hat{\beta} - \frac{\delta}{\alpha} + \left(1 - \frac{\alpha}{1+\lambda} \right) (\hat{\beta} - \underline{\beta}) \right]. \tag{49}$$

Replacing equation (49) into (48), we obtain the level of effort:

$$e^*(\hat{\beta}) = \frac{2\alpha}{2-\alpha(1+\lambda)} - \frac{\alpha(1+\lambda)}{2-\alpha(1+\lambda)} \left[\hat{\beta} - \frac{\delta}{\alpha} + \left(1 - \frac{\alpha}{1+\lambda} \right) (\hat{\beta} - \underline{\beta}) \right] - \\ - \left[1 - \frac{\alpha}{1+\lambda} \right] (\hat{\beta} - \underline{\beta}).$$

The net transfer can be calculated from:

$$t^*(\hat{\beta}) = \frac{1}{\alpha} \left\{ \int_{\hat{\beta}}^{\overline{\beta}} e^*(\gamma) d\gamma - \delta \left[q^*(\hat{\beta}) - q_{ref} \right] + \frac{e^*(\hat{\beta})^2}{2} \right\}.$$

B.5 Nonlinearity and cost disturbances

Let us show that a scheme that is not linear in cost cannot implement the optimal solution for all probability distributions of the cost disturbance.

We know that $t(\beta, C)$ must satisfy:

$$s^*(\beta) = Et \left\{ \beta, [\beta - e^*(\beta)] q^*(\beta) + \epsilon \right\}.$$

If t is not linear in cost, there exist β, C_1, C_2 and C_3 such that

$$\frac{t(\beta, C_1) - t(\beta, C_2)}{C_1 - C_2} \neq \frac{t(\beta, C_1) - t(\beta, C_3)}{C_1 - C_3}.$$

Define $\epsilon_i \equiv C_i - [\beta - e^*(\beta)]q^*(\beta)$, and consider the family of discrete distributions with three atoms at e_1, e_2 and e_3 and no weight elsewhere (since these distributions can be approximated by continuous distributions, we could actually restrict ourselves to continuous distributions). It is clear that by varying the weights on the three disturbance levels and given the last equation, the first equation cannot always be satisfied.

B.6 Effect of the bureaucratic bias

Lemma 7. $q^*(\hat{\beta})$ and $e^*(\hat{\beta})$ are increasing in δ.

Proof.

Differentiating equations (15) and (16), in order to δ we obtain:

$$\begin{cases} S'' \frac{dq^*(\hat{\beta})}{d\delta} = -(1+\lambda)\left(\frac{de^*(\hat{\beta})}{d\delta} + \frac{1}{\alpha}\right) \\ \psi'' \frac{de^*(\hat{\beta})}{d\delta} = \alpha \frac{dq^*(\hat{\beta})}{d\delta} - \left(1 - \frac{\alpha}{1+\lambda}\right)(\hat{\beta} - \underline{\beta})\psi''' \frac{de^*(\hat{\beta})}{d\delta} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \frac{dq^*(\hat{\beta})}{d\delta} = -\frac{1+\lambda}{S''}\left(\frac{de^*(\hat{\beta})}{d\delta} + \frac{1}{\alpha}\right) \\ \frac{de^*(\hat{\beta})}{d\delta} = \frac{-\frac{1+\lambda}{S''}}{\psi'' + \frac{\alpha(1+\lambda)}{S''} + \left(1 - \frac{\alpha}{1+\lambda}\right)(\hat{\beta} - \underline{\beta})\psi'''} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \frac{dq^*(\hat{\beta})}{d\delta} = \frac{1+\lambda}{S''}\left[\frac{1+\lambda}{\psi'' S'' + \alpha(1+\lambda) + S''\left(1 - \frac{\alpha}{1+\lambda}\right)(\hat{\beta} - \underline{\beta})\psi'''} - \frac{1}{\alpha}\right] \\ \frac{de^*(\hat{\beta})}{d\delta} = -\frac{1+\lambda}{\psi'' S'' + \alpha(1+\lambda) + S''\left(1 - \frac{\alpha}{1+\lambda}\right)(\hat{\beta} - \underline{\beta})\psi'''} \end{cases}.$$

Using Assumption 1 (i) and (ii), we find that $\frac{de^*(\hat{\beta})}{d\delta} > 0$ and $\frac{dq^*(\hat{\beta})}{d\delta} > 0$.

Q.E.D.

Lemma 8. The expected social welfare, W^*, increases (decreases) with the bureaucratic bias, δ, whenever the expected output level is larger (lower) than the reference output, q_{ref}.

Proof.

The expected social welfare function is given by:

$$W^* = \int_{\underline{\beta}}^{\overline{\beta}} S\left[q^*(\hat{\beta})\right] - \frac{1+\lambda}{\alpha}\left\{-\delta\left[q^*(\hat{\beta}) - q_{ref}\right] + \psi\left[e^*(\hat{\beta})\right] + \alpha\left[\hat{\beta} - e^*(\hat{\beta})\right]q^*(\hat{\beta})\right\} + \\ + \left(1 - \frac{1+\lambda}{\alpha}\right)V^*(\hat{\beta})\, d\hat{\beta}.$$

It can be written as:

$$W^* = \int_{\underline{\beta}}^{\overline{\beta}} S\left[q^*(\hat{\beta})\right] - \frac{1+\lambda}{\alpha}\left\{-\delta\left[q^*(\hat{\beta}) - q_{ref}\right] + \psi\left[e^*(\hat{\beta})\right] + \alpha\left[\hat{\beta} - e^*(\hat{\beta})\right] q^*(\hat{\beta})\right\} +$$

$$+\left(1 - \frac{1+\lambda}{\alpha}\right)\left\{V^*(\underline{\beta}) - \int_{\underline{\beta}}^{\hat{\beta}} \psi'\left[e^*(\gamma)\right] d\gamma\right\} \, d\hat{\beta}.$$

Using the Envelope Theorem we find:

$$\frac{dW^*}{d\delta} = \frac{1+\lambda}{\alpha}\left\{\int_{\underline{\beta}}^{\overline{\beta}} q^*(\hat{\beta})d\hat{\beta} - q_{ref}(\overline{\beta} - \underline{\beta})\right\} = \frac{1+\lambda}{\alpha}\left\{E_{\hat{\beta}}\left[q^*(\hat{\beta})\right] - q_{ref}\right\}.$$

Q.E.D.

Lemma 9. For small enough ψ''', the fraction of cost that is supported by the firm, K^*, is increasing with δ. That is: $\frac{dK^*(\hat{\beta})}{d\delta} > 0$.

Proof.

We know that both $e^*(\hat{\beta})$ and $q^*(\hat{\beta})$ are increasing in δ. Using equation (16) we can rewrite (17) as:

$$K^*(\hat{\beta}) = 1 - \frac{\left(1 - \frac{\alpha}{1+\lambda}\right)(\hat{\beta} - \underline{\beta})\psi''\left[e^*(\hat{\beta})\right]}{\alpha q^*(\hat{\beta})}.$$

Therefore:

$$\frac{dK^*(\hat{\beta})}{d\delta} = -\frac{\left(1 - \frac{\alpha}{1+\lambda}\right)}{\alpha}(\hat{\beta} - \underline{\beta})\frac{\psi'''\left[e^*(\hat{\beta})\right]\frac{de^*(\hat{\beta})}{d\delta}q^*(\hat{\beta}) - \frac{dq^*(\hat{\beta})}{d\delta}\psi''\left[e^*(\hat{\beta})\right]}{q^*(\hat{\beta})^2}.$$

Thus, $\frac{dK^*(\hat{\beta})}{d\delta} > 0$ if and only if:

$$\psi'''\left[e^*(\hat{\beta})\right]\frac{de^*(\hat{\beta})}{d\delta}q^*(\hat{\beta}) - \frac{dq^*(\hat{\beta})}{d\delta}\psi''\left[e^*(\hat{\beta})\right] < 0 \Leftrightarrow \frac{\frac{dq^*(\hat{\beta})}{d\delta}}{\frac{de^*(\hat{\beta})}{d\delta}} > \frac{\psi'''\left[e^*(\hat{\beta})\right]q^*(\hat{\beta})}{\psi''\left[e^*(\hat{\beta})\right]}.$$

From the expressions for $\frac{dq^*(\hat{\beta})}{d\delta}$ and $\frac{de^*(\hat{\beta})}{d\delta}$ in Lemma 7, the condition above is always true when ψ''' is null. Therefore, we may conclude that, in this case, $K^*(\hat{\beta})$ is increasing in δ for any value of $\hat{\beta}$.

Q.E.D.

C Appendix: The reimbursement payment system

Replacing $S\left[q(\hat{\beta})\right] = 2q(\hat{\beta}) - q(\hat{\beta})^2$ and $t(\hat{\beta}) = -\frac{\delta}{\alpha}\left[q(\hat{\beta}) - q_{ref}\right]$ in problem (23):

$$\max_{q(\hat{\beta}),t(\hat{\beta})} \int_{\underline{\beta}}^{\overline{\beta}} 2q(\hat{\beta}) - q(\hat{\beta})^2 - (1+\lambda)\left\{\hat{\beta}q(\hat{\beta}) - \frac{\delta}{\alpha}\left[q(\hat{\beta}) - q_{ref}\right]\right\} d\hat{\beta}. \tag{50}$$

The first order condition of problem (50) is:

$$2 - 2q(\hat{\beta}) = (1+\lambda)\left(\hat{\beta} - \frac{\delta}{\alpha}\right).$$

Simplifying, we get the output:

$$q_r^*(\hat{\beta}) = 1 - \frac{1+\lambda}{2}\left(\hat{\beta} - \frac{\delta}{\alpha}\right).$$

The net transfer is:

$$t_r^*(\hat{\beta}) = -\frac{\delta}{\alpha}\left[1 - \frac{1+\lambda}{2}\left(\hat{\beta} - \frac{\delta}{\alpha}\right) - q_{ref}\right].$$

The second order condition of problem (50) is satisfied:

$$\frac{\partial^2 f}{\partial q^2} < 0 \Leftrightarrow -2 < 0.$$

D Appendix: The prospective payment system

D.1 The study of the Hamiltonian

Government's optimization problem - necessary conditions

Consider problem (30). The Hamiltonian is:

$$\begin{aligned} H = S\left[q(\hat{\beta})\right] - \tfrac{1+\lambda}{\alpha}\left\{V(\hat{\beta}) - \delta\left[q(\hat{\beta}) - q_{ref}\right] + \psi\left[e(\beta)\right] + \alpha\left[\hat{\beta} - e(\hat{\beta})\right] q(\hat{\beta})\right\} + \\ + V(\hat{\beta}) + \nu\left[-\alpha q(\hat{\beta})\right], \end{aligned} \tag{51}$$

where ν is the multiplier associated with (25). The Pontryagin principle yields:

$$\frac{\partial H}{\partial q} = S'\left[q(\hat{\beta})\right] - (1+\lambda)\left[\hat{\beta} - e(\hat{\beta}) - \frac{\delta}{\alpha}\right] - \nu\alpha = 0, \tag{52}$$

$$\frac{\partial H}{\partial e} = -\frac{1+\lambda}{\alpha}\left\{\psi'\left[e(\hat{\beta})\right] - \alpha q(\hat{\beta})\right\} = 0,$$

$$\nu'(\hat{\beta}) = -\frac{\partial H}{\partial V} = \frac{1+\lambda}{\alpha} - 1. \tag{53}$$

Furthermore, $\underline{\beta}$ is a free boundary so that

$$\nu(\underline{\beta}) = 0. \tag{54}$$

Integrating (53) and using (54), we obtain

$$\nu(\hat{\beta}) = \left(\frac{1+\lambda}{\alpha} - 1\right)(\hat{\beta} - \underline{\beta}). \tag{55}$$

Substituting in (52) above:

$$S'\left[q(\hat{\beta})\right] = (1+\lambda)\left[2\hat{\beta} - \underline{\beta} - e(\hat{\beta}) - \frac{\delta}{\alpha}\right] - \alpha(\hat{\beta} - \underline{\beta}).$$

Government's optimization problem - sufficiency conditions

The second order derivatives of the Hamiltonian (51) are:

$$\begin{aligned} \frac{\partial^2 H}{\partial q^2} &= S''\left[q(\hat{\beta})\right] < 0, \\ \frac{\partial^2 H}{\partial e^2} &= -\frac{1+\lambda}{\alpha}\psi''\left[e(\hat{\beta})\right] < 0, \\ \frac{\partial^2 H}{\partial q \partial e} &= \frac{\partial^2 H}{\partial e \partial q} = 1+\lambda. \end{aligned}$$

The determinant of the Hessian is:

$$\begin{aligned} |H| &= -\frac{1+\lambda}{\alpha}S''\left[q(\hat{\beta})\right]\psi''\left[e(\hat{\beta})\right] - (1+\lambda)^2. \\ |H| &> 0 \Leftrightarrow S''\left[q(\hat{\beta})\right]\psi''\left[e(\hat{\beta})\right] > \alpha(1+\lambda). \end{aligned}$$

We find that $|H| > 0$, by Assumption 1 (ii).

Prospective payment system - numerical example

In problem (30), replace the social value by $S(q) = 2q - q^2$ and the disutility of effort by $\psi(e) = e^2/2$. The Hamiltonian becomes:

$$H = 2q(\hat{\beta}) - q(\hat{\beta})^2 - (1+\lambda)\left\{\frac{V(\hat{\beta})}{\alpha} - \frac{\delta}{\alpha}\left[q(\hat{\beta}) - q_{ref}\right] + \frac{e(\hat{\beta})^2}{2\alpha} + \left[\hat{\beta} - e(\hat{\beta})\right]q(\hat{\beta})\right\} + V(\hat{\beta}) - \nu\alpha q(\hat{\beta}),$$

where ν is the multiplier associated with (25). The Pontryagin Principle yields:

$$\frac{\partial H}{\partial q} = 2 - 2q(\hat{\beta}) - (1+\lambda)\left[\hat{\beta} - e(\hat{\beta}) - \frac{\delta}{\alpha}\right] - \nu\alpha = 0, \tag{56}$$

$$\frac{\partial H}{\partial e} = -(1+\lambda)\left[\frac{e(\hat{\beta})}{\alpha} - q(\hat{\beta})\right] = 0 \Leftrightarrow e(\hat{\beta}) = \alpha q(\hat{\beta}), \tag{57}$$

$$\nu'(\beta) = -\frac{\partial H}{\partial V} = \frac{1+\lambda}{\alpha} - 1. \tag{58}$$

Furthermore, $\underline{\beta}$ is a free boundary so that

$$\nu(\underline{\beta}) = 0. \tag{59}$$

Integrating (58) and using (59), we obtain:

$$\nu(\hat{\beta}) = \left(\frac{1+\lambda}{\alpha} - 1\right)(\hat{\beta} - \underline{\beta}). \tag{60}$$

Replacing equation (60) in equation (56) we obtain:

$$2 - 2q(\hat{\beta}) = (1+\lambda)\left[\hat{\beta} - e(\hat{\beta}) - \frac{\delta}{\alpha}\right] + \alpha\left(\frac{1+\lambda}{\alpha} - 1\right)(\hat{\beta} - \underline{\beta}). \tag{61}$$

Replacing the equation (57) into the equation (61) we obtain the level of output:

$$q_p^*(\hat{\beta}) = \frac{1}{2 - \alpha(1+\lambda)}\left[2 + \alpha(\hat{\beta} - \underline{\beta})\right] - \frac{1+\lambda}{2 - \alpha(1+\lambda)}\left(2\hat{\beta} - \underline{\beta} - \frac{\delta}{\alpha}\right). \tag{62}$$

Replacing the equation (62) into the equation (57) we obtain the level of effort:

$$e_p^*(\hat{\beta}) = \frac{\alpha}{2 - \alpha(1+\lambda)}\left[2 + \alpha(\hat{\beta} - \underline{\beta})\right] - \frac{\alpha(1+\lambda)}{2 - \alpha(1+\lambda)}\left(2\hat{\beta} - \underline{\beta} - \frac{\delta}{\alpha}\right).$$

The net transfer is:

$$t_p^*(\hat{\beta}) = \frac{1}{\alpha}\left\{\int_{\hat{\beta}}^{\overline{\beta}} \alpha q_p^*(\xi)\, d\xi - \delta\left[q_p^*(\hat{\beta}) - q_{ref}\right] + \frac{e_p^{*2}(\hat{\beta})}{2}\right\}.$$

D.2 Effect of the bureaucratic bias

Lemma 10. $q_p^*(\hat{\beta})$ and $e_p^*(\hat{\beta})$ are increasing in δ.

Proof.

Differentiating equations (31) and (24), in order to δ we obtain:

$$\begin{cases} S'' \frac{dq_p^*(\hat{\beta})}{d\delta} = -(1+\lambda)\left(\frac{de_p^*(\hat{\beta})}{d\delta} + \frac{1}{\alpha}\right) \\ \psi'' \frac{de_p^*(\hat{\beta})}{d\delta} = \alpha \frac{dq_p^*(\hat{\beta})}{d\delta} \end{cases} \Leftrightarrow \begin{cases} \frac{de_p^*(\hat{\beta})}{d\delta} = -\frac{1+\lambda}{\psi'' S'' + \alpha(1+\lambda)}, \\ \frac{dq_p^*(\hat{\beta})}{d\delta} = -\frac{1+\lambda}{S''}\left(\frac{de_p^*(\hat{\beta})}{d\delta} + \frac{1}{\alpha}\right). \end{cases}$$

Using Assumption 1 (i) and (ii), we find that $\frac{dq_p^*(\hat{\beta})}{d\delta} > 0$ and $\frac{de_p^*(\hat{\beta})}{d\delta} > 0$.

Q.E.D.

Lemma 11. The expected social welfare, W_p^*, increases (decreases) with the bureaucratic bias, δ, whenever the expected output level is larger (lower) than the reference output, q_{ref}.

Proof.

The expected social welfare function can be written as:

$$W_p^* = \int_{\underline{\beta}}^{\overline{\beta}} S\left[q_p^*(\hat{\beta})\right] - \frac{1+\lambda}{\alpha}\left\{\psi\left[e_p^*(\hat{\beta})\right] - \delta\left[q_p^*(\hat{\beta}) - q_{ref}\right]\right\} -$$
$$-(1+\lambda)\left[\hat{\beta} - e_p^*(\hat{\beta})\right] q_p^*(\hat{\beta}) + \left(1 - \frac{1+\lambda}{\alpha}\right)\left[V_p^*(\underline{\beta}) - \int_{\underline{\beta}}^{\hat{\beta}} \alpha q_p^*(\hat{\beta})\right] d\hat{\beta}.$$

Using the Envelope Theorem we find:

$$\frac{dW_p^*}{d\delta} = \frac{1+\lambda}{\alpha}\left\{\int_{\underline{\beta}}^{\overline{\beta}} q_p^*(\hat{\beta}) d\hat{\beta} - q_{ref}(\overline{\beta} - \underline{\beta})\right\} = \frac{1+\lambda}{\alpha}\left\{E_{\hat{\beta}}\left[q_p^*(\hat{\beta})\right] - q_{ref}\right\}.$$

Q.E.D.

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