

# Aggregation in activity-based costing and the short run activity cost function

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# **Aggregation in activity-based costing and the short run activity cost function**

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**Abstract:** This paper first identifies the conditions that support the construction of an aggregate output, i.e. the conditions under which a single measure of output can be used to accurately determine cost object incremental costs within a cost pool. This is a significant issue which has not been fully explored in the management accounting literature. Two conditions are jointly necessary and sufficient. The first one is the linear homogeneity property associated with each cost object production function. This condition ensures that costs are linear with output, which is essential if the cost reported by an activity-based costing (ABC) system is also to be a relevant cost for decision-making. The second is that all (cost object) cost driver rates for a given cost pool are equal. This condition guarantees that the cost function at a given activity depends on only one cost driver. The short run structure of ABC is also introduced. It is shown that the fundamental ABC property of linearity between costs and output does not generally hold in the short run, even assuming that technologies are linearly homogeneous. Only under very particular conditions, such as when inputs are combined in completely fixed proportions, are short run costs linear with output.

**Keywords:** activity-based costing; aggregate output; cost driver; multi-output technologies; short run activity cost function.

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## 1. Introduction

Activity-based costing has received considerable attention since its emergence in the late eighties, as evidenced by the significant number of articles published in professionally oriented journals and, to a lesser extent, in academic accounting journals (Lukka and Granlund, 2002; Bjornenak and Mitchell, 2002).

Only a few, however, have focused on the theoretical foundations of ABC. On this theme, Noreen (1991) constitutes the first significant example. He has focused on the theoretical foundations of ABC. These constitute conditions relating to cost functions and he has derived three necessary and sufficient conditions for ABC systems to measure relevant costs for decision-making<sup>1</sup>. These are that (i) total costs can be divided into independent cost pools, each of which depends only on one activity, (ii) the cost in each cost pool is strictly proportional to the level of activity in that cost pool and (iii) the volume of an activity is simply the sum of activity measures utilised by the individual products. Christensen and Demski (1995) and Bromwich and Hong (1999) have supplemented this work by developing a more fundamental analysis of the theoretical foundations of ABC, in the sense that they consider technology, apart from input prices, as the primary determinant of cost functions. This constitutes a perspective which, although well established in the production economics literature (e.g. Chambers, 1988), had been systematically absent in the management accounting literature. In an ABC context, Christensen and Demski (1995) have interpreted concepts already familiar in the production economics literature, namely cost function separability and linearity of the cost function. Bromwich and Hong (1999) have investigated the technological conditions that support ABC systems capable of measuring incremental costs. Their analysis has investigated the conditions related to technology that more generally satisfy the three conditions derived by Noreen for ABC systems to measure decision relevant output costs. Specifically, they have derived the following conditions: (i) non-jointness, to rule out the existence of economies or diseconomies of scope, (ii) local homotheticity and (iii) linear homogeneity between a cost driver and the inputs

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<sup>1</sup>In this paper, decision relevant is taken to mean relevant to decision making on final output variation, e.g. the expansion or reduction of output of existing products, the introduction of a new product, make or buy (outsourcing) decisions or special orders. This is consistent with Noreen (1991).

within a cost pool, in order to represent the inputs used in a cost pool by a cost driver, (iv) technological separability between cost pools, to ensure the independence between cost pools.

The first purpose of this paper is to extend the analysis of ABC decision relevant costs to include consideration of output characteristics. This is done by deriving the necessary and sufficient conditions that support the construction of an aggregate output, in a context where the cost in each cost pool is strictly proportional to that output. The aggregate output is a single measure of output or single cost driver that fully captures the cost of the resources used by the various cost objects within a cost pool. Moreover, the aggregate output in each cost pool is the sum of the activity measures utilised by the individual cost objects. The point that must be emphasised here is that the ABC literature above has taken the aggregate output for granted, without deriving the necessary and sufficient conditions that support its construction. It remains therefore unclear under what conditions (i) the output in each cost pool is the sum of the activity measures utilised by the individual cost objects and (ii) activity costs are strictly proportional to that output, two fundamental properties of an ABC system (Noreen, 1991). As demonstrated below, this analysis is necessary to complete a specification of the conditions characterising an ABC system which will generate relevant costs.

The previous analysis of ABC is undertaken in the long run, where all inputs are variable with output. The second purpose of this paper is to introduce the short run structure of ABC. One of the major recognised innovations of ABC systems is the introduction of the distinction between the cost of resources used and the cost of resources supplied, where the difference between the two is given by the cost of resources not used (Cooper and Kaplan, 1992). It is in the short run, where some inputs are fixed, that this analysis has to be carried out. Another limitation of the existing literature is that it has only considered the long run. The significance of the restrictions on cost variability in the short run and the implications for ABC are therefore investigated.

This paper is organised into three further sections. The first investigates the theoretical foundations of an aggregate output. It assumes a long run perspective, where all inputs are variable with output, i.e. there are no fixed costs. The second concentrates

on the structure of the short run activity cost function while the final section presents the conclusions.

## 2. Theoretical foundations of an aggregate output

This section is organised as follows. Sub-section one describes the general production and cost models. Sub-section two characterises the accounting model. Sub-section three discusses some implications of the results derived in sub-section two. Finally, sub-section four relates the results of this section with the results of Bromwich and Hong (1999).

### 2.1. Technology and costs

To begin the analysis, suppose that  $p$  inputs are aggregated at cost pool  $t$ , which are used in the production of  $m$  cost object outputs. In other words, I have a multi-output technology at cost pool  $t$ .

I assume that the technology is non-joint, a necessary condition for an ABC system to generate incremental costs (Noreen, 1991; Bromwich and Hong, 1999). Basically, a multi-output technology is non-joint if the cost of producing the  $m$  outputs jointly equals the cost of producing them separately (see Hall, 1973). Therefore, the total cost at cost pool  $t$  can be obtained by, first, calculating the cost of producing separately each output and, second, summing the costs of producing the  $m$  outputs<sup>2</sup>.

In order to calculate the cost of producing separately each output, let  $x^{tj} = (x_{1,j}^t, \dots, x_{p,j}^t)$  represent a  $p$ -dimensional vector of inputs used by cost object  $j$  at cost pool  $t$ . Moreover, under the assumption that the technology is non-joint, the vector of inputs used at cost pool  $t$  when the  $m$  outputs are produced jointly is  $x^t = (x_1^t, \dots, x_p^t)$ , where  $x_i^t = \sum_{j=1}^m x_{i,j}^t$ . Let also  $y^{tj}$  denote the output associated with cost object  $j$ . Next sub-section derives the conditions under which the various cost object outputs can be added together in order to construct an aggregate output. For now, simply visualise  $y^{tj}$  as the output associated with an individual cost object. For example,  $y^{tj}$  can be the number

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<sup>2</sup> Furthermore, non-jointness applies not only within each activity but also between activities (see Bromwich and Hong, 1999). Given my purposes, I concentrate my analysis on a given activity.

of set-ups of product  $j$ , the number of deliveries of product  $j$  or the total machine hours required to produce the same product.

The production function for cost object  $j$  at cost pool  $t$  is given by:

$$f_j^t(x^{t,j}) = y^{t,j} \quad (1)$$

I assume that  $f_j^t(x^{t,j})$  is smooth, increasing and (strictly) quasi-concave. I also consider that all inputs are essential to the production, in the sense that a positive amount of output cannot be produced without a strictly positive utilisation of all inputs.

In general, the  $m$  cost object technologies will be different. For example, the technologies supporting say products  $j$  and  $k$ ,  $j \neq k$ , might be such that one and another use different input mixes within a cost pool. As will be later shown, the possibility that different cost objects use different technologies has several important implications in relation to the conditions that allow the construction of an aggregate output.

The cost of producing output  $y^{t,j}$  is given by:

$$\begin{aligned} c_j^t(w^t, y^{t,j}) &\equiv & (2) \\ \text{Min } \sum_{i=1}^p x_{i,j}^t w_i^t & \\ \text{Subject to } f_j^t(x^{t,j}) &\geq y^{t,j} \end{aligned}$$

Where  $w^t = (w_1^t, \dots, w_p^t)$  denotes a  $p$ -dimensional vector of strictly positive input prices. Problem (2) identifies the input vector that minimises the cost of producing output  $y^{t,j}$ , when the input price set is  $w^t$ . For later reference, it is useful to represent problem (2) in terms of the *Lagrangean function*:

$$L(x^{t,j}, \mu) = \sum_{i=1}^p x_{i,j}^t w_i^t + \mu (y^{t,j} - f_j^t(x^{t,j})) \quad (3)$$

Where  $\mu$  is a *Lagrange multiplier*. The first-order conditions for the existence of a minimum imply the following<sup>3</sup>:

$$\frac{\partial f_j^t(x^{tj})/\partial x_{i,j}^t}{\partial f_j^t(x^{tj})/\partial x_{u,j}^t} = \frac{w_i^t}{w_u^t} \quad (4)$$

Expression (4) is the well know result that at an optimum the marginal rate of technical substitution of input  $i$  for input  $u$  ( $MRTS_{i,u}^{tj}$ ) equals the ratio of the corresponding input prices<sup>4</sup>.

Finally, the cost of producing the  $m$  cost object outputs or the  $m$ -dimensional output vector  $y^t = (y^{t,1}, \dots, y^{t,m})$  at cost pool  $t$  is given by:

$$c^t(w^t, y^t) = \sum_{j=1}^m c_j^t(w^t, y^{tj}) \quad (5)$$

While the left-hand side of expression (5) represents the total cost of producing the  $m$  outputs jointly, the right-hand side denotes the total cost of producing them separately. As observed above, this equality takes place if and only if the multi-output technology is non-joint.

## 2.2. Accounting system

A fundamental property underlying the architecture of both conventional and ABC systems is the application of average cost driver rates to cost outputs. This procedure can only be justified when cost functions are linear with output. Otherwise, average and marginal costs will differ. If this is the case, the cost reported by an ABC system does not measure incremental costs.

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<sup>3</sup> More precisely, the first-order conditions are  $\frac{\partial L}{\partial x_{ij}^t} = w_i^t - \mu \frac{\partial f_j^t(x^{tj})}{\partial x_{ij}^t} = 0$  and  $\frac{\partial L}{\partial \mu} = f_j^t(x^{tj}) - y^{tj} = 0$ .

Dividing  $\frac{\partial L}{\partial x_{ij}^t}$  by  $\frac{\partial L}{\partial x_{uj}^t}$ ,  $i \neq u$ , establishes the result.

<sup>4</sup> The assumption that  $f_j^t(x^{tj})$  is smooth, increasing and (strictly) quasi-concave ensures that the first-order conditions are not only necessary but also sufficient for the existence of a (unique) minimum (see Chiang, 1984, 387-404).

If the cost function for cost object  $j$  is linear with output, the cost of producing output  $y^{tj}$  is given by:

$$c_j^t(w^t, y^{tj}) = y^{tj} \phi_j^t(w^t) \quad (6)$$

Where  $\phi_j^t(w^t)$  is the average and marginal cost for cost object  $j$  at cost pool  $t$ . Moreover, by the envelope theorem,  $\mu = \phi_j^t(w^t)$ , i.e. the *Lagrange multiplier* in (3) is the marginal cost. Hereafter, I will refer to  $\phi_j^t(w^t)$  as the cost driver rate for cost object  $j$  at cost pool  $t$ . For example,  $\phi_j^t(w^t)$  can represent the cost per set-up of product  $j$  or the cost per machine hour used in the production of the same product. The following Lemma is fundamental. It is based on the duality between costs and technology and shows that only linearly homogeneous technologies give rise to cost functions linear with output. As will be shown in sub-section four, this result strongly contrasts with existing ABC literature.

**Lemma 1.** *The cost function  $c_j^t(w^t, y^{tj})$  is linear with output if and only if the production function  $f_j^t(x^{tj})$  is linearly homogeneous.*

**Proof.** *By the Lemma of Shephard, the derived demand for input  $i$ ,  $x_{i,j}^t$ , equals the derivative of the cost function with respect to the price of the same input,  $w_i^t$ . If I apply it to (6), I obtain:*

$$\frac{\partial c_j^t(w^t, y^{tj})}{\partial w_i^t} = y^{tj} \frac{\partial \phi_j^t(w^t)}{\partial w_i^t} = x_{i,j}^t(w^t, y^{tj}), \text{ for all } i \quad (A.1)$$

Condition (A.1) shows that if the cost function for cost object  $j$  is linear with output, the optimal input-output relationships are also linear. For later reference, denote the rate at which the quantity of input  $i$  changes with output as:

$$\frac{\partial x_{i,j}^t(w^t, y^{tj})}{\partial y^{tj}} = \frac{\partial \phi_j^t(w^t)}{\partial w_i^t} = \frac{1}{\alpha_{i,j}^t(w^t)}, \text{ for all } i \quad (A.2)$$



And the cost driver rate for cost object  $j$  as:

$$\phi_j^t(w^t) = \sum_{i=1}^p \frac{\partial x_{i,j}^t(w^t, y^{t,j})}{\partial y^{t,j}} w_i^t = \sum_{i=1}^p \frac{w_i^t}{\alpha_{i,j}^t(w^t)} \quad (\text{A.3})$$

Condition (A.1) implies that, given the input price set, if the input combination  $x^{t,j}$  is associated with the production of output  $y^{t,j}$ , the input combination  $\lambda x^{t,j}$  is associated with the production of output  $\lambda y^{t,j}$ .

What technologies give rise to such (optimal) input-output relationships? This requires that  $f_j^t(x^{t,j})$  is linearly homogeneous (since  $f_j^t(\lambda x^{t,j}) = \lambda f_j^t(x^{t,j})$ ). Basically, all I need to show is that, given the input price set,  $x^{t,j}$  and  $\lambda x^{t,j}$  are the optimal input vectors when the outputs are  $y^{t,j}$  and  $\lambda y^{t,j}$ , respectively.

If  $f_j^t(x^{t,j})$  is homogeneous of degree one,  $\partial f_j^t(x^{t,j})/\partial x_{i,j}^t$  is homogeneous of degree zero ( $\partial f_j^t(\lambda x^{t,j})/\partial x_{i,j}^t = \partial f_j^t(x^{t,j})/\partial x_{i,j}^t$ ). Taking into account (4), this implies that an increase of  $\lambda$  in all the  $p$  inputs does not change the MRTS $_{i,u}^{t,j}$ . Therefore, given the input price set, if the first-order conditions are fulfilled by the input combination  $x^{t,j}$  they must also be fulfilled by the combination  $\lambda x^{t,j}$ . Moreover, if the input combination  $x^{t,j}$  produces output  $y^{t,j}$  the input combination  $\lambda x^{t,j}$  produces output  $\lambda y^{t,j}$  (since  $f_j^t(\lambda x^{t,j}) = \lambda f_j^t(x^{t,j})$ ). ■

It is well known that a linearly homogeneous technology is a special case of a more general class of technologies, called homothetic technologies. Specifically, a technology is homothetic if it can be written as  $h_j^t(f_j^t(x^{t,j})) = y^{t,j}$ , where  $f_j^t(x^{t,j})$  is linearly homogeneous and  $dh_j^t(f_j^t(x^{t,j}))/df_j^t(x^{t,j}) > 0$ . The cost function dual to a homothetic technology takes the following form  $c_j^t(w^t, y^{t,j}) = \varphi^t(y^{t,j}) \phi_j^t(w^t)$ , where  $\varphi^t(y^{t,j}) = h_j^{t-1}(y^{t,j})/h_j^{t-1}(1)$  (see Jehle, 1991, 233).

This cost function is not linear with output and thus average and marginal costs are not, in general, constant<sup>5</sup>. There is an exception, however, when  $h_j^t(f_j^t(x^{tj})) = f_j^t(x^{tj})$ , i.e when the technology is linearly homogeneous<sup>6</sup>. This, of course, is the essence of the Lemma.

The condition that cost object technologies are linearly homogeneous is necessary for the construction of an aggregate output. It is not sufficient, however. The construction of an aggregate output presupposes that a second condition is also verified. Basically, this second condition ensures that each cost pool depends on only one cost driver.

At this point, the question that should be asked is to know whether the various cost object measures of output are or are not various volume levels of the same cost driver. They will represent various volume levels of the same cost driver if they affect activity costs in the same way, more precisely, if the various (cost object) cost driver rates are equal. In this case, the various cost object outputs can simply be added together. For example, if the outputs of cost objects  $j$  and  $k$ ,  $j \neq k$ , are the same cost driver,  $y^{tj} + y^{tk}$  constitute the aggregate output associated with both cost objects  $j$  and  $k$ . If, however, they are not the same cost driver,  $y^{tj}$  and  $y^{tk}$  affect costs differently and so cannot be added together. Otherwise, and as will be shown, some cost distortion is introduced. The following definition can now be introduced.

**Definition 1.**  $y^{tj}$  and  $y^{tk}$  are the same cost driver at cost pool  $t$  if  $\phi_j^t(w^t) = \phi_k^t(w^t)$ ,  $j \neq k$ . Similarly, they are not the same cost driver if  $\phi_j^t(w^t) \neq \phi_k^t(w^t)$ .

For example, the number of set-ups of products  $j$  and  $k$  represent two volume levels of the same cost driver if the cost per set-up of product  $j$  equals the cost per set-up of product  $k$ . Proposition 1 shows that in order to construct an aggregated output all cost

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<sup>5</sup> For example, consider the following homothetic production function:

$h_j^t(f_j^t(x^{tj})) = \frac{1}{2} \text{Log} [f_j^t(x^{tj})]^2$ , where  $f_j^t(x^{tj}) = \sqrt{x_{1j}^t x_{2j}^t}$ . The *Lagrangian function* can be written as  $L = w_1^t x_{1j}^t + w_2^t x_{2j}^t - \mu \left( \frac{1}{2} \text{Log} x_{1j}^t x_{2j}^t - y^{tj} \right)$  and the first-order conditions as  $\frac{\partial L}{\partial x_{ij}^t} = w_i^t - \mu \frac{1}{2 x_{ij}^t} = 0$ ,  $i = 1, 2$  and  $\frac{\partial L}{\partial \mu} = \frac{1}{2} \text{Log} x_{1j}^t x_{2j}^t - y^{tj} = 0$ . After simplification I obtain  $c_j^t(w^t, y^{tj}) = 2 \sqrt{w_1^t w_2^t} e^{y^{tj}}$ . This cost function is not compatible with ABC (average and marginal costs are not constant).

<sup>6</sup> Formally,  $c_j^t(w^t, y^{tj}) = y^{tj} \phi_j^t(w^t) \Leftrightarrow h_j^t(f_j^t(x^{tj})) = f_j^t(x^{tj})$ .

object outputs, at a given cost pool, have to represent various volume levels of the same cost driver.

**Proposition 1.**  $g^t(y^t) = \sum_{j=1}^m y^{t,j}$  is an aggregate output that accurately measures cost object incremental costs at cost pool  $t$  if and only if  $y^{t,j}$  and  $y^{t,k}$  are the same cost driver, for all  $j \neq k$ .

**Proof.**

**Sufficiency**

The Lemma implies that  $c_h^t(w^t, y^{t,h}) = y^{t,h} \phi_h^t(w^t) = \sum_{i=1}^p x_{i,h}^t(w^t, y^{t,h}) w_i^t$ . Now, if  $y^{t,j}$  and  $y^{t,k}$  are the same cost driver, then  $\phi_j^t(w^t) = \phi_k^t(w^t) = \phi^t(w^t)$ , for all  $j \neq k$ . The cost allocated to cost object  $h$  under ABC is equal to:

$$y^{t,h} \frac{\sum_{j=1}^m \sum_{i=1}^p x_{i,j}^t(w^t, y^{t,j}) w_i^t}{\sum_{j=1}^m y^{t,j}} = y^{t,h} \frac{\sum_{j=1}^m y^{t,j} \phi_j^t(w^t)}{\sum_{j=1}^m y^{t,j}} = y^{t,h} \frac{\phi^t(w^t) \sum_{j=1}^m y^{t,j}}{\sum_{j=1}^m y^{t,j}} =$$

$$y^{t,h} \phi_h^t(w^t) = \sum_{i=1}^p x_{i,h}^t(w^t, y^{t,h}) w_i^t. \text{ That is, the cost allocated to cost object } h \text{ equals}$$

its incremental cost. ■

**Necessity**

Suppose that  $\phi_j^t(w^t) < \phi_k^t(w^t)$  and  $\phi_k^t(w^t) = \phi^t(w^t)$ , for all  $k \neq j$ . Let also  $y^{t,h} > 0$ , for all  $h$ . The cost allocated to cost object  $h$  under ABC is now equal to:

$$y^{t,h} \frac{\sum_{i=1}^p x_{i,j}^t(w^t, y^{t,j}) w_i^t + \sum_{k \neq j} \sum_{i=1}^p x_{i,k}^t(w^t, y^{t,k}) w_i^t}{\sum_{j=1}^m y^{t,j}} = y^{t,h} \phi^t(w^t, y^{t,j}, g^t(y^{t_{k \neq j}}))$$

$$\text{Where } \phi^t(w^t, y^{t,j}, g^t(y^{t_{k \neq j}})) = \frac{y^{t,j} \phi_j^t(w^t) + g^t(y^{t_{k \neq j}}) \phi^t(w^t)}{g^t(y^t)} \text{ and } g^t(y^{t_{k \neq j}}) = \sum_{k \neq j} y^{t,k}$$

Since  $y^{t,h} > 0$  and  $\phi_j^t(w^t) < \phi^t(w^t)$ , then  $\phi_j^t(w^t) < \phi^t(w^t, y^{t,j}, g^t(y^{t_{k \neq j}})) < \phi^t(w^t)$ . This implies that  $y^{t,h} \phi^t(w^t, y^{t,j}, g^t(y^{t_{k \neq j}})) \neq y^{t,h} \phi_h^t(w^t) = \sum_{i=1}^p x_{i,h}^t(w^t, y^{t,h}) w_i^t$ . That is, the cost allocated to cost object  $h$  distorts its incremental cost.

To sum up, the condition that  $y^{t,j}$  and  $y^{t,k}$  are the same cost driver, for all  $j \neq k$ , is a necessary and sufficient condition for the construction of an aggregate output that accurately measures cost object incremental costs at cost pool  $t$ . ■

Remarks 1 and 2 follow directly from Proposition 1. While Remark 1 derives the fundamental relationship between cost object technologies, the aggregate output and activity costs, Remark 2 shows the consequences of using the output measure  $g^t(y^t)$  to allocate costs when cost pool  $t$  depends on more than one cost driver.

**Remark 1.** *The cost function at cost pool  $t$  can be written as:*  $c^t(w^t, y^t) = \sum_{j=1}^m c_j^t(w^t, y^{t,j})$   
 $= \sum_{j=1}^m y^{t,j} \phi_j^t(w^t) = g^t(y^t) \phi^t(w^t)$ .

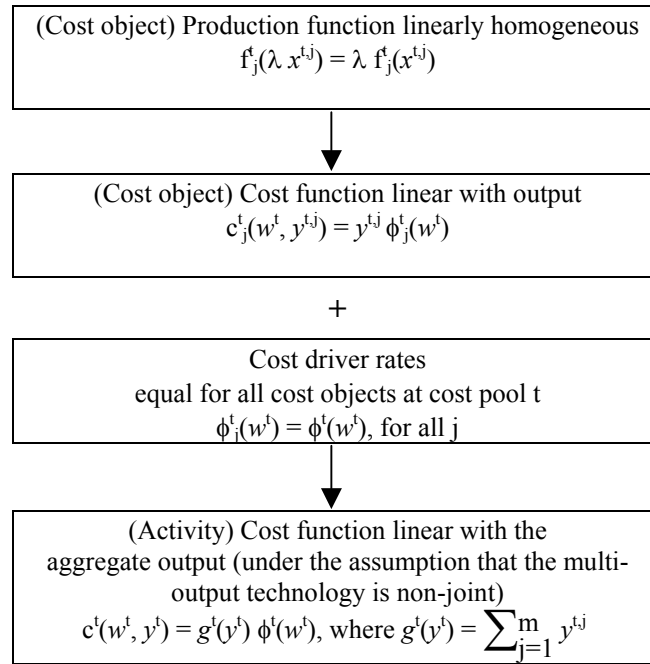
**Proof.** *The first equality results from the fact that the multi-output technology is non-joint, the second from the fact that cost objects technologies are linearly homogenous and the third from the fact that  $y^{t,j}$  and  $y^{t,k}$  are the same cost driver, for all  $j \neq k$ . ■*

**Remark 2.** *If  $\exists j \neq k: \phi_j^t(w^t) \neq \phi_k^t(w^t)$ , the cost function at cost pool  $t$  depends on more than one cost driver. In this case, allocate the cost at cost pool  $t$  based on the output measure  $g^t(y^t)$  distorts cost object incremental costs.*

**Proof.** *This follows directly from the demonstration of the necessity of Proposition 1. ■*

The following table summarises the theoretical foundations of an aggregate output.

**Table I – Theoretical foundations of an aggregate output**



### 2.3. Discussion

It was demonstrated that in order to create an aggregate output both cost objects technologies have to be linearly homogeneous and all cost object outputs have to represent various volume levels of the same cost driver. As will be demonstrated, if cost objects technologies are not only linearly homogenous but also identical, all cost object outputs represent various volume levels of the same cost driver. The contrary, however, is not true, as also will be shown. In other words, it might be the case that two cost object technologies are not identical while their outputs are still the same cost driver (at least for some input price set).

To pursue the analysis let first assume that all cost objects technologies are identical.

**Assumption 1.**  $f_j^t(x^{tj})$  and  $f_k^t(x^{tk})$  are identical, for all  $j \neq k$ .

It is obvious that when cost object technologies are identical the various cost object outputs represent different volume levels of the same output. In other words, I have a single output technology, as the various cost object technologies are indistinguishable.

In this case, the various cost object outputs also represent different volume levels of the same cost driver (Remark 3).

**Remark 3.** *If  $f_j^t(x^{t,j})$  and  $f_k^t(x^{t,k})$  are identical,  $y^{t,j}$  and  $y^{t,k}$  are the same cost driver at cost pool  $t$ , for all  $j \neq k$ ,  $w^t$ . In this case, any input aggregated at cost pool  $t$  can be used as an aggregate measure of output.*

**Proof.** *It follows directly from (A.1) and (A.2) that the optimal input-output relationships for cost object  $j$  at cost pool  $t$  are  $y^{t,j} = \alpha_{i,j}^t(w^t) x_{i,j}^t$ , for all  $i$ . Additionally, if all cost object technologies are identical, then:*

$$\alpha_{i,j}^t(w^t) = \alpha_{i,k}^t(w^t) = \alpha_i^t(w^t), \text{ for all } i, k \neq j, w^t \quad (\text{A.4})$$

Using (A.3) and (A.4), I obtain  $\phi_j^t(w^t) = \phi_k^t(w^t) = \phi^t(w^t)$ , for all  $j \neq k$ ,  $w^t$ . That is,  $y^{t,j}$  and  $y^{t,k}$  are the same cost driver at cost pool  $t$ . Moreover, any input aggregated at cost pool  $t$  can be used as an aggregate measure of output. More formally, using (A.1), (A.2) and (A.4), the cost driver rate at cost pool  $t$  when input  $u$  is used as a measure of output,  $\phi^t(w^t)_u$ , is:

$$\phi^t(w^t)_u \equiv \phi_j^t(w^t)_u = \sum_{i=1}^p \frac{\partial x_{i,j}^t}{\partial x_{u,j}^t} w^t = \alpha_u^t(w^t) \sum_{i=1}^p \frac{w_i^t}{\alpha_i^t(w^t)} \quad \text{for all } j \quad (\text{A.5})$$

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It remains to show the possibility that two cost object outputs are the same cost driver even though their technologies are not identical. An example will be sufficient to illustrate this point.

**Example.** Consider that cost pool  $t$  aggregates three inputs. Assume also that there are two products ( $P_1$  and  $P_2$ ). The production function for  $P_j$  can be represented as  $f_j^t(x_{1,j}^t, x_{2,j}^t, x_{3,j}^t) = y^{t,j}$ , where  $y^{t,j}$  is the number of set-ups of  $P_j$ . Additionally,

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<sup>7</sup> Thus Remark 1 can be rewritten as  $c^t(w^t, x_{u,1}^t, \dots, x_{u,m}^t) = \sum_{j=1}^m c_j^t(w^t, x_{u,j}^t) = \sum_{j=1}^m x_{u,j}^t \phi_j^t(w^t)_u = x_u^t \phi^t(w^t)_u$ .

$f_j^t(x_{1,j}^t, x_{2,j}^t, x_{3,j}^t) = \min(\alpha_{1,j}^t x_{1,j}^t, \alpha_{2,j}^t x_{2,j}^t, \alpha_{3,j}^t x_{3,j}^t)$ , that is, the technology supporting  $P_j$  is Leontief, a special case of a linearly homogeneous technology that does not allow any substitution between inputs.

The cost per set-up of  $P_j$  is  $\phi_j^t(w^t) = \sum_{i=1}^3 \frac{w_i^t}{\alpha_{i,j}^t}$ . It is obvious that  $\phi_1^t(w^t) = \phi_2^t(w^t)$  when  $\alpha_{i,1}^t = \alpha_{i,2}^t$ , for all  $i$ . However, it is no less obvious that even when  $\alpha_{i,1}^t \neq \alpha_{i,2}^t$ , for all  $i$ , I might have  $\phi_1^t(w^t) = \phi_2^t(w^t)$ , at least for some input price set. That is to say, it is possible that  $P_1$  and  $P_2$  use different input mixes while the cost per set-up of  $P_1$  is still equal to the cost per set-up of  $P_2$ . If this is the case, the total number of set-ups is in fact an aggregate measure of output that accurately measures incremental product costs (i.e.  $y^{t,1}$  and  $y^{t,2}$  are the same cost driver).

Suppose now that I consider using as an allocation base either the total number of set-ups or the total quantity of input  $u$ . It might be the case that  $y^{t,1}$  and  $y^{t,2}$  are not the same cost driver ( $\phi_1^t(w^t) \neq \phi_2^t(w^t)$ ) while  $x_{u,1}^t$  and  $x_{u,2}^t$  are the same cost driver ( $\phi_1^t(w^t)_u = \phi_2^t(w^t)_u$ )<sup>8</sup>. Table II presents a numerical example to illustrate this point.

**Table II – Product cost driver rates**

<b>Panel A: Parameters</b>	
$P_1$ :	$1/\alpha_{1,1}^t = 20; 1/\alpha_{2,1}^t = 60; 1/\alpha_{3,1}^t = 40$
$P_2$ :	$1/\alpha_{1,2}^t = 65; 1/\alpha_{2,2}^t = 70; 1/\alpha_{3,2}^t = 5$
$w^t$ :	$(w_1^t, w_2^t, w_3^t) = (£1, £1, £1)$
<b>Panel B: Product cost driver rates (Allocation base: number of set-ups)</b>	
$P_1$ :	$\phi_1^t(w^t) = \sum_{i=1}^3 \frac{w_i^t}{\alpha_{i,1}^t} = 20 \times £1 + 60 \times £1 + 40 \times £1 = £120$
$P_2$ :	$\phi_2^t(w^t) = \sum_{i=1}^3 \frac{w_i^t}{\alpha_{i,2}^t} = 65 \times £1 + 70 \times £1 + 5 \times £1 = £140$
<b>Panel C: Product cost driver rates (Allocation base: input 2)</b>	
$P_1$ :	$\phi_1^t(w^t)_2 = \alpha_{2,1}^t \sum_{i=1}^3 \frac{w_i^t}{\alpha_{i,1}^t} = \frac{20 \times £1 + 60 \times £1 + 40 \times £1}{60} = £2$
$P_2$ :	$\phi_2^t(w^t)_2 = \alpha_{2,2}^t \sum_{i=1}^3 \frac{w_i^t}{\alpha_{i,2}^t} = \frac{65 \times £1 + 70 \times £1 + 5 \times £1}{70} = £2$

<sup>8</sup>  $\phi_j^t(w^t)_u = \alpha_{u,j}^t \sum_{i=1}^3 \frac{w_i^t}{\alpha_{i,j}^t}$  (see (A.5)).

As Table II shows, if the total number of set-ups is the allocation base activity costs are incorrectly distributed between  $P_1$  and  $P_2$  (since  $\phi_1^t(w^t) \neq \phi_2^t(w^t)$  – see Panel B). However, if input 2 is the allocation base activity costs are accurately distributed between  $P_1$  and  $P_2$  (since  $\phi_1^t(w^t)_2 = \phi_2^t(w^t)_2$  – see Panel C)<sup>9</sup>.

Finally, observe that, given the specific (heterogeneous) product technologies, the conclusion that  $y^{t,1}$  and  $y^{t,2}$  (or  $x_{u,1}^t$  and  $x_{u,2}^t$ ) are or not the same cost driver depends on the input price set. In other words, it might be the case (or not) that for some input price set  $y^{t,1}$  and  $y^{t,2}$  (or  $x_{u,1}^t$  and  $x_{u,2}^t$ ) are the same cost driver. Only when product technologies are identical  $y^{t,1}$  and  $y^{t,2}$  (and  $x_{u,1}^t$  and  $x_{u,2}^t$ ) are the same cost driver for all the input price sets (see Remark 3).

The previous example implies that even when product technologies are heterogeneous, i.e. even when the various products use different input mixes, product cost distortions might be small. Hwang *et al* (1993) observe that product cost distortions, due to the use of an allocation base to distribute the cost of the inputs aggregated in an activity cost pool among the various products, increase when product technologies are significantly different. Although this can be accepted as a general observation, the preceding analysis shows that high product technology heterogeneity does not necessarily lead to high product cost distortions. I thus establish the following Remark.

**Remark 4.** *Even when  $f_j^t(x^{t,j})$  and  $f_k^t(x^{t,k})$  are not identical,  $y^{t,j}$  and  $y^{t,k}$  might still be the same cost driver (for some input price set).*

Overall, what Remarks 3 and 4 show is that even though it is true that when cost object technologies are identical (and linearly homogeneous) their outputs are the same cost driver, the contrary is not true. In other words, cost driver rates might be equal for two cost objects even though their technologies are not identical (for some input price set). Nevertheless, it should be recognised that it is a very strong assumption that two

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<sup>9</sup> The same cannot be concluded when input 1 or input 3 is the allocation base.



cost object outputs are the same cost driver when their (linearly homogeneous) technologies are not identical.

#### **2.4. Some related results**

This sub-section is devoted to a discussion of a paper that also addresses the conditions under which ABC generates incremental costs: Bromwich and Hong (1999).

The condition that the multi-output technology is non-joint is fundamental in ABC. This property has been emphasised by Noreen (1991) and Bromwich and Hong (1999) and, as was observed, supports all the analysis undertaken in the previous sub-sections.

The specific contribution of this section is to highlight the additional significance of the two necessary and sufficient conditions that support the construction of an aggregate output, compatible with cost being strictly proportional to that output. This point has not been fully developed by Bromwich and Hong (1999), although they do refer to three conditions, two related to the technology and one related to the accounting system. With respect to the technology, they emphasise both the fact that a constant input mix has to be common to all products in a cost pool irrespective of volume, if aggregation of elementary inputs is to be allowed, and the fact that the technology has to be homothetic. With respect to the accounting system, they claim that a cost driver should be linearly homogeneous with respect to the elementary inputs making up a cost pool (Bromwich and Hong, 1999, 48-53).

However, the results derived in the previous sub-sections show that only linearly homogenous technologies, a special case of homothetic technologies, give rise to cost functions compatible with ABC (see the demonstration of the Lemma). Further, a cost driver will be linearly homogeneous with respect to the elementary inputs making up a cost pool if and only if cost object technologies are both linearly homogeneous and identical. Thus, homothetic production functions do not, in general, allow such representation. Also, the possibility that cost object technologies are not identical, i.e. they use different input mixes, does not necessarily invalidate the possibility of constructing an aggregate output or cost driver. That is, the condition that all cost objects use the same input mix in a cost pool is not a necessary condition for the

construction of an aggregate output or cost driver (see the example of the previous subsection).

The point that should be emphasised here, which constitutes the essence of the above results, is that a cost driver fully reflects both the primitive assumptions of the technology and the interaction between technology and input prices.

To sum up, assuming that activity technologies are non-joint, the two necessary and sufficient conditions supporting the construction of an aggregate output or cost driver are cost object production functions linearly homogeneous and cost object cost driver rates equal for a given cost pool (see Table I).

### 3. Short run activity cost function

The analysis undertaken in the last section has assumed a long run perspective, where all inputs are variable with output. In the short run, however, some inputs are fixed. This section investigates the structure of the short run activity cost function.

As was previously demonstrated, when cost object technologies are both linearly homogeneous and identical, cost pools depend on only one cost driver. As was observed previously, the assumption that cost object technologies are identical is equivalent to imposing that, at a given activity, there is a single output technology. In other words, all cost object outputs represent various volume levels of the same output. This permits the visualisation of the activity output as an intermediate input that is used by the various cost objects. The analysis undertaken in this section explicitly assumes this (see Assumption 1)<sup>10</sup>.

To incorporate the distinction between variable and fixed inputs, let represent the vector of inputs supplied at cost pool  $t$  as  $x^t_{(\text{supplied})} = (x^t_{f1}, \dots, x^t_{fu}, x^t_{u+1}, \dots, x^t_p)$ , where  $u$  inputs ( $i = 1, \dots, u$ ) are fixed and  $(p - u)$  inputs ( $i = u+1, \dots, p$ ) are variable in the period in consideration. Let us also denote the short run vector of inputs used at cost pool  $t$  as  $x^{t(\text{SR})} = (x^{t(\text{SR})}_1, \dots, x^{t(\text{SR})}_p)$ . Note that  $x^{t(\text{SR})}_i \leq x^t_{fi}$ ,  $i = 1, \dots, u$ , i.e. in the case of the fixed inputs usage is lower than, or equal to, supply. In the case of the variable inputs,

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<sup>10</sup> As was shown, although the assumption that cost object technologies are identical, together with the necessary condition that they are linearly homogenous, is sufficient to construct an aggregate output, it is not necessary. Only for the sake of analytical simplicity in the analysis of the short run cost minimisation problem it is assumed here that cost object technologies are identical.

$x^{t(\text{SR})}_i = x^t_i$ ,  $i = u+1, \dots, p$ , i.e. usage equals supply. The short run cost minimisation problem is given by:

$$\begin{aligned} c^{t(\text{SR})}(w^t, g^t(y^t), x^t_{f1}, \dots, x^t_{fu}) &\equiv & (7) \\ \sum_{i=1}^u x^t_{fi} w^t_i + \text{Min}_{x^{t(\text{SR})}_i} \sum_{i=u+1}^p x^{t(\text{SR})}_i w^t_i \\ \text{Subject to } f^t(x^{t(\text{SR})}) &\geq g^t(y^t) \text{ and } x^t_{fi} \geq x^{t(\text{SR})}_i, i = 1, \dots, u \end{aligned}$$

Where  $f^t(x^{t(\text{SR})})$  is the production function at cost pool  $t$  and  $x^{t(\text{SR})}$  the input vector that minimises the cost of producing output  $g^t(y^t)$ . In order to characterise the optimal solution to problem (7) it is useful to represent its long run counterpart:

$$\begin{aligned} c^{t(\text{LR})}(w^t, g^t(y^t)) &\equiv & (8) \\ \text{Min}_{x^{t(\text{LR})}_i} \sum_{i=1}^p x^{t(\text{LR})}_i w^t_i \\ \text{Subject to } f^t(x^{t(\text{LR})}) &= g^t(y^t) \end{aligned}$$

Where  $x^{t(\text{LR})} = (x^{t(\text{LR})}_1, \dots, x^{t(\text{LR})}_p)$  denotes the input vector that resolves problem (8). In contrast with problem (7), all the  $p$  inputs are variable in problem (8). When  $f^t(x^{t(\text{LR})})$  is linearly homogeneous, the long run cost function can simply be written as (see Lemma 1):

$$c^{t(\text{LR})}(w^t, g^t(y^t)) \equiv \phi^{t(\text{LR})}(w^t) g^t(y^t) \quad (9)$$

Where  $\phi^{t(\text{LR})}(w^t)$  is the long run marginal cost. Let finally introduce the following assumption.

**Assumption 2.** *There is an output level, say  $g^t(y^t)^\bullet$ , such that  $x^{t(\text{supplied})} \equiv x^{t(\text{LR})}$  and thus  $c^{t(\text{LR})}(w^t, g^t(y^t)^\bullet) = c^{t(\text{SR})}(w^t, g^t(y^t)^\bullet, x^t_{f1}, \dots, x^t_{fu})$ .*

The output  $g^t(y^t)^*$  is usually interpreted as the capacity of activity  $t$ . It corresponds to the output level for which long run costs are minimised, i.e. long and short run costs coincide (see Chambers, 1988, p. 104). Proposition 2 constitutes the main result of this section. It characterises the optimal solution to problem (7).

**Proposition 2.** *Assuming that  $x^{t(SR)}_i > 0$  (input essentiality) and  $\partial f^t(x^{t(SR)})/\partial x^{t(SR)}_i > 0$ , for all  $i$ , the optimal solution to problem (7) exhibits the following properties:*

P2.(i) *All resources supplied in the short run are used.*

P2.(ii) *The short run cost function is non-linear in output.*

**Proof.**

*Property P2.(i)*

*The Lagrangean function for problem (7) is:*

$$\begin{aligned} L(x^{tj}, \lambda, \mu_1, \dots, \mu_u) = \\ \sum_{i=1}^u x^t_{fi} w^t_i + \sum_{i=u+1}^p x^{t(SR)}_i w^t_i + \lambda (g^t(y^t) - f^t(x^{t(SR)})) + \\ \sum_{i=1}^u \mu_i (-x^t_{fi} + x^{t(SR)}_i) \end{aligned} \quad (A.6)$$

*Where  $\lambda$  and  $\mu_i$  are Lagrange multipliers. Since  $x^{t(SR)}_i > 0$  and  $\partial f^t(x^{t(SR)})/\partial x^{t(SR)}_i > 0$ , the complementary slackness condition implies:*

$$\begin{aligned} \frac{\partial L(\cdot)}{\partial x^{t(SR)}_i} = -\lambda \frac{\partial f^t(x^{t(SR)})}{\partial x^{t(SR)}_i} + \mu_i = 0, \quad i = 1, \dots, u \\ \frac{\partial L(\cdot)}{\partial x^{t(SR)}_j} = w^t_j - \lambda \frac{\partial f^t(x^{t(SR)})}{\partial x^{t(SR)}_j} = 0, \quad j = u+1, \dots, p \end{aligned}$$

*Or,*

$$\mu_i = w^t_j \frac{\partial f^t(x^{t(SR)})/\partial x^{t(SR)}_i}{\partial f^t(x^{t(SR)})/\partial x^{t(SR)}_j} > 0 \quad (A.7)$$

*Given that the  $\mu_i$ 's are positive the restrictions  $x^t_{fi} \geq x^{t(SR)}_i$ ,  $i = 1, \dots, u$ , bind. Therefore, all resources supplied in the short run are used.*

Property P2.(ii)

Note first that the solution to the restricted problem (7) cannot be better than the solution to the unrestricted problem (8), that is  $c^{t(\text{SR})}(w^t, g^t(y^t), x_{f1}^t, \dots, x_{fu}^t) \geq c^{t(\text{LR})}(w^t, g^t(y^t))$ . Additionally, since  $c^{t(\text{LR})}(w^t, 0) = 0$ ,  $c^{t(\text{SR})}(w^t, 0, x_{f1}^t, \dots, x_{fu}^t) > 0$  and  $c^{t(\text{LR})}(w^t, g^t(y^t)^\bullet) = c^{t(\text{SR})}(w^t, g^t(y^t)^\bullet, x_{f1}^t, \dots, x_{fu}^t)$ , short run costs are non-linear in output (observe that  $c^{t(\text{LR})}(w^t, g^t(y^t))$  is linear in output). Also, as the fixed inputs are fully used, the input mix changes with output<sup>11</sup>. This implies that short run costs are non-linear when  $0 < g^t(y^t) < g^t(y^t)^\bullet$  as well as when  $g^t(y^t) > g^t(y^t)^\bullet$ . ■

Two points should be noted here. The first is that the possibility of substitution between inputs plays a crucial role in Proposition 2<sup>12</sup>. In fact, one way of ensuring that short run costs are linear with output is to restrict the possibility of substitution between inputs. For example, suppose that inputs are combined in completely fixed proportions, such a Leontief technology (a special case of a linearly homogeneous technology). It is not hard to see that, in this case, short run costs are linear with output<sup>13</sup>. The second point is that the above results hold whether the technology is linearly homogeneous or not. Thus, and in contrast with the long run, imposing that technologies are linearly homogeneous no longer guarantee that short run costs are linear with output, a fundamental property of an ABC system.

#### 4. Conclusion

The contribution of this paper is, primarily, to identify the necessary and sufficient conditions that support the construction of an aggregate output, compatible with costs

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<sup>11</sup> By contrast, in the long run, the input mix is constant for a given input price set. Using both the *Lemma of Shephard* and (9), I obtain  $\frac{x_{i}^{t(\text{LR})}}{x_{k}^{t(\text{LR})}} = \frac{\partial \phi^{t(\text{LR})}(w^t) / \partial w_i^t}{\partial \phi^{t(\text{LR})}(w^t) / \partial w_k^t}$ , which is constant for a given input price set.

<sup>12</sup> This is a direct implication of the assumption that the marginal productivity of each input is positive,  $\frac{\partial f^t(x^{t(\text{SR})})}{\partial x_i^{t(\text{SR})}} > 0$ .

<sup>13</sup> In particular, the rate at which short run costs change is constant and equal to  $\sum_{i=u+1}^p \frac{w_i^t}{\alpha_i}$ . Moreover, the (short run) maximum output is given by  $\text{Min}(\alpha_1 x_{f1}^t, \dots, \alpha_u x_{fu}^t) = g^t(y^t)^\bullet$ .

being directly proportional to that output. This is an important issue which has not been fully developed in the existing management accounting literature. Two conditions were derived. The first is that (i) cost object production functions are linearly homogeneous. This condition ensures that marginal costs are constant, which is essential if the product cost reported by an ABC system is also to be a relevant cost for decision-making. The second condition is that (ii) all cost object cost driver rates at a given cost pool are equal. This condition ensures that the cost function, at a given cost pool, depends on only one cost driver. In other words, this condition ensures that the output in each cost pool is the sum of the activity measures utilised by the individual cost objects, a fundamental property of an ABC system (Noreen, 1991). These two conditions are jointly necessary and sufficient for the construction of an aggregate output in ABC. As was also observed, a fundamental assumption underlying this analysis is that multi-output technologies are non-joint.

It was shown than when all (linearly homogeneous) cost object technologies are identical condition (ii) is automatically ensured. However, and at least in theory, condition (ii) might still occur when cost object technologies are heterogeneous (for some input price set).

These results contrast with Bromwich and Hong (1999). They claim both that homothetic technologies give rise in general to cost functions compatible with ABC and that a constant input mix has to be common to all products in a cost pool if aggregation of elementary inputs is to be allowed. As was shown, only linearly homogeneous technologies, a special case of homothetic technologies, ensure that costs are strictly proportional with output, a fundamental property of an ABC system. Additionally, the fact that not all products use the same input mix within a cost pool does not necessarily preclude the possibility of constructing an aggregate output.

The second major contribution of this paper is the analysis of the short run activity cost function. In the short run, imposing that technologies are linearly homogeneous no longer guarantee that short costs are linear with output, a fundamental property of an ABC system. If the technology is such that inputs can be substituted for each other short run costs are non-linear with output.

The analysis developed in this paper provides guidance for further research that empirically tests the basic conditions supporting the construction of an aggregate output for an activity.

The investigation of cost distortions arising in situations where those conditions do not apply would be of particular relevance to those who wish to assess the utility of an existing costing system or who wish to design a new system. Christensen and Demski (1997, 2003), Gupta and Datar (1994), Gupta (1993) and Hwang et al (1993) have addressed some of the questions that the aggregation of inputs and the selection of cost drivers pose, but more research is necessary in this area. In particular, the analysis of situations where cost object production functions are not linearly homogenous (and are not identical), and do not therefore permit the creation of a single measure of output for an activity, might provide new insights.

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