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# INCOMPLETE REGULATION, Asymmetric Information and Collusion-Proofness

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# Incomplete Regulation, Asymmetric Information and Collusion-Proofness

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### Abstract

In an incomplete regulation framework the Regulator cannot replicate all the possible outcomes by himself since he has no influence on some firms present in the market. When facing asymmetric information regarding the regulated firm's costs, it may be better for the Regulator to allow the other competitors to extract a truthful report from her through side-payments in a collusion and therefore the "Collusion-Proofness Principle" may not hold. In fact, by introducing an exogenous number of unregulated competitors, Social Welfare differences seem to favour a Collusion-Allowing equilibrium. However, such result will strongly depend on the relative importance given by the Regulator to the Consumer Surplus.

**Keywords:** Incomplete Regulation, Asymmetric Information, Collusion, Market Competition

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### 1 – Introduction

The main goal of the present paper is to assess which is the optimal incomplete regulation when the marginal costs of the regulated firm are private information, not available to the regulator and neither to the other competitors, and when it is possible to form a coalition between these unregulated firms and the regulated firm. We will also evaluate how such results may be influenced by the relative importance given by the regulator to the Consumer Surplus in the Social Welfare function.

Most of the literature on economic regulation studied optimal contracts in the context of complete regulation, either between a regulator and a firm (monopoly) or between a regulator and all the firms in the market (usually a duopoly).

Baron and Myerson (1982), Laffont and Tirole (1986) and Lewis and Sappington (1988) studied a monopoly where the main problem was related to the design of an optimal contract under asymmetric information. On the first two papers the regulator was not aware of the monopolist's marginal costs, while in the last work that private information was related to the market's demand function. All of these models represent departures from the first-best solution due to the presence of incomplete information. The regulator has to pay a price above the marginal cost to avoid an untruthful report by the monopolist. However, Lewis and Sappington (1988) also concluded that under some conditions (nondecreasing marginal costs), in spite of having private information about the demand function, we would get the same first-best optimum equilibrium.

Caillaud (1990), Laffont and Martimort (1997, 2000) and Tangeras (2000) were departures from the previously mentioned models, by extending optimal complete regulation under asymmetric information to a competition framework. Caillaud (1990) focused on the informational effect of the existence of a competitive fringe for the regulation of a dominant firm under asymmetric information. If both the dominant firm's and the fringe's costs are unknown and positively correlated, the regulator could use the threat of entry of that fringe into the market as an endogenous incentive mechanism for the dominant firm to always reveal truthfully its costs. It was concluded that the presence of a fringe is always welfare enhancing but its magnitude depends on the degree of information correlation and on the characteristics of the demand function. Both Laffont and Martimort (1997, 2000) and Tangeras (2000) introduced the possibility of collusion between two firms in a context of complete regulation under asymmetric information. The first mainly stresses the role of correlated information between the firms as a determinant of the strength of the coalition. They also develop a new methodology to analyze collusion and have concluded that in the presence of correlated private information the regulator can create a regulation contract which can replicate exactly the collusion outcome: the Collusion-Proofness Principle. Tangeras (2000) studied the incentives for collusion when a market is regulated through yardstick competition. The regulator was able to design a contract for each firm separately. Since firms decided to collude before knowing their own productivity, the collusion would be costly to society only if firms could commit to the side payments agreed.

Biglaiser and Ma (1995) and Aubert and Pouyet (2006) are two very important contributions to the study of optimal regulation when the regulator is only able to make a contract with a dominant firm (incomplete regulation) under asymmetric information. Biglaiser and Ma (1995) focuses on optimal incomplete regulation when only the dominant firm has private information regarding the demand function and the unregulated competitor has some market power, acting as a Stackelberg follower. They proved that depending on the weight given by the regulator to consumer surplus on Social Welfare, the equilibrium outcome could both separating and pooling or just separating. Aubert and Pouyet's (2006) model is the closest to the one I propose. They have worked in a framework of incomplete regulation under asymmetric information and there is the possibility of collusion between the two firms operating in the market. Due to product imperfect substitutability, the unregulated competitor has incentives to bribe the regulated firm such that it overstates its costs and produce less. They have concluded that it is not optimal to design a Collusion-Proof contract for regulation. This kind of contract imposes both distortions at the bottom and at the top (inefficient and efficient regulated firms, respectively), while by allowing collusion the regulator may induce the non-regulated firm to indirectly tax its competitor. The Collusion-Proofness Principle will not hold in the incomplete regulation framework since the regulator is unable to contract with the unregulated firm and therefore it has limited possibilities for contracts when compared to what can be achieved within the coalition of the two firms. The difference between the Collusion-Proof contract and a contract that allows collusion is that in the first the regulator has to pay larger amounts to the regulated firm to ensure that it reveals its efficiency despite collusion. In the last, the regulator uses collusion to make the unregulated firm pay to ensure that the regulated firm reveals its efficiency.

Given the previously mentioned literature, the main contribution of the present paper is to discuss the role of collusion in the optimal design of incomplete regulation under asymmetric information when we introduce more than one unregulated firm into the market. In the present framework the market is composed by a dominant (regulated) firm and by a small number of unregulated firms, which produce the same homogeneous product (although different from the one produced by the regulated firm) and choose their quantities as Stackelberg followers. Similarly to Aubert and Pouyet (2006), I have assumed that the only private information on the market is the marginal cost of the dominant firm, which can take one of two possible values. Additionally, I have decided to introduce differentiation among unregulated firms by admitting different marginal costs, which are publicly known. During all the analysis we will also use a similar Social Welfare function to the one used by Aubert and Pouyet (2006) which allows us to compare the results directly. These assumptions may be shortcomings of the present paper and a reason for further research. In reality it is more likely the costs of the unregulated firms to be also private information and since economic regulation is usually decided in terms of price-cap, we should try to assess optimal regulation when firms compete using prices. As we will also see further ahead in the paper, by giving more weight to the Consumer Surplus in the Social Welfare function the conclusions will change dramatically, emphasising the importance of the regulator's priorities when ensuring competition in a market.

The present paper also has a wide range of empirical applicability. An example is the Portuguese fixed telecommunications market which is constituted by a dominant regulated firm (Portugal Telecom) and by a small number of unregulated competitors. Once again, consumers view telecommunications services of Portugal Telecom as different from the other competitors. We may also apply the model to other sectors as Health and Education in Portugal. In each region, we could look at the health-care market as being composed by one (or two) dominant public (regulated) hospitals and by a small number of private hospitals that compete in most of the type of health services. The same can be said about the regional market for Undergraduate Degrees. Usually the public university is the dominant firm, which is also regulated in terms of tuitions and there is also a small number of private universities that are free to impose the tuition they would like. However, we must stress that in these last two sectors the unregulated sector is not regulated in terms of prices, but it is regulated in terms of minimum level of quality of its services.

The paper is organized as follows: in the next section I will describe the model and its characteristics, also describing the timing to better understand the sequence of events. In later sections I will analyse the optimal regulation outcome (quantities, prices, profits, transfer from the regulator, side-payment from the unregulated firms to the regulated firm and welfare level) in different frameworks within incomplete regulation: complete information, incomplete information in the absence of collusion and incomplete information with the possibility of collusion. I will then compare the outcome from contracts that are Collusion-Proof with the outcome obtained with contracts that allow collusion and assess which one entails a higher level of Social Welfare. Finally, I will discuss the importance of the Social Welfare function to the robustness of such findings, by comparing the Consumer Surplus values in Collusion-Proofnes and Collusion-Allowing equilibria.

### 2 – The Model

### 2.1. – The Firms

The market is composed by a dominant regulated firm  $F^A$  and by *n* unregulated firms, each denoted by  $F_i^B$ , where i=1,...,n. All the firms compete in quantities. Firm  $F^A$  has a constant marginal cost  $\theta^a$  which can take two values,  $\underline{\theta}$  or  $\overline{\theta}$ ,  $\overline{\theta} - \underline{\theta} \equiv \Delta \theta > 0$ . The value of this marginal cost is private information for  $F^A$ , however its distribution is public knowledge: with probability  $\underline{p} = F^A$  is efficient ( $\theta^a = \underline{\theta}$ ) and it is inefficient ( $\theta^a = \overline{\theta}$ ) with probability  $\overline{p} = 1 - \underline{p}$ . The unregulated firms have different marginal costs ( $\theta_i^b$ ) which are publicly known. For simplification we have assumed that the unregulated firms are ordered from the most efficient to the least efficient, such that:  $\theta_1^b \le \theta_2^b \le ... \le \theta_n^b$ .

### 2.2. – The Consumers

The consumers can buy two differentiated products:  $q^a$  produced by the regulated firm in market A and  $q_i^b$  produced by each firm belonging to the unregulated market B, where  $Q^b = \sum_{i=1}^n q_i^b$  represents the total quantity produced in market B. The Gross Consumer Surplus when a quantity  $q^a$  is produced by the firm  $F^A$  and a quantity  $Q^b$  is produced by all the firms in market B, is given by:

$$GS(q^{a},Q^{b}) = d^{a}q^{a} + d^{b}Q^{b} - \frac{1}{2}(q^{a})^{2} - \frac{1}{2}(Q^{b})^{2} - sq^{a}Q^{b}$$

where the parameter  $s \in (0,1)$  measures the degree of substitutability between the two types of products, whereas the parameter  $d^{j}$  represents the size of market  $j = A, B^{3}$ . The inverse demand functions for both markets are given by:

$$P^{a}(q^{a},Q^{b}) = d^{a} - q^{a} - sQ^{b}$$
$$P^{b}(q^{a},Q^{b}) = d^{b} - Q^{b} - sq^{a}$$

We have also assumed that  $d^b - d^a = \theta_1^b - \underline{\theta} = \theta_n^b - \overline{\theta}$ , which means that the difference between the marginal costs of the most efficient unregulated firm  $(F_1^b)$  and the efficient type of  $F^A$  is exactly the same as the difference between the costs of the most inefficient firm in market B  $(F_n^b)$  and the inefficient type of the regulated firm and they are both equal the difference between the size of the two markets.<sup>4</sup>

### **2.3.** – The Regulator

Only firm  $F^{A}$  is regulated by the regulator R, while all other competitors are left unregulated. The regulation contract is composed by a quantity-transfer pair  $\left\{q^{a}\left(\theta_{m}^{a}\right),t^{a}\left(\theta_{m}^{a}\right)\right\}_{\theta_{m}^{a}\in\left\{\underline{\theta},\overline{\theta}\right\}}$ , which depends on the message  $\left(\theta_{m}^{a}\right)$  sent by the regulated firm about its cost to the regulator. For a given contract  $\{q^a, t^a\}$ , the total *ex post* profits are equal to:

$$\pi^{a} = \left[ P^{a}(q^{a}, Q^{b}) - \theta^{a} \right] q^{a} - t^{a}$$
  
$$\pi^{b}_{i} = \left[ P^{b}(q^{a}, Q^{b}) - \theta^{b}_{i} \right] q^{b}_{i} \qquad \forall i = 1, ..., n$$

Similarly to Aubert and Pouvet (2006), I have decided not to consider any relationship between the regulator and any other firm, but that could be considered by

 <sup>&</sup>lt;sup>3</sup> Such Consumer Surplus function results from the standard quadratic utility function proposed by Dixit.
 <sup>4</sup> We will see later on that the conclusions remain the same if we disregard such assumption.

extending the analysis to a lump-sum tax or taxes proportional either to profits or to output.

I will also assume that the firms' profits do not enter the objective function of the regulator, so that this objective is reduced to net consumer surplus plus the transfer paid by the regulated firm. Hence, the rents left to regulated firm are socially costly for the regulator since they represent the amount that the regulator has to pay for her to reveal truthfully its marginal costs. The objective of the regulator is to maximize Social Welfare, given by:

$$W = GS(q^a, Q^b) - \theta^a q^a - P^b(q^a, Q^b)Q^b - \pi^a$$

### 2.4. – The Timing

In the present model, the agents decide sequentially. The temporal sequence of events will be:

- 1) Nature draws one of the two possible values for  $\theta^a$ , which is only known to the regulated firm  $F^A$ .
- 2) The regulator *R* proposes a contract  $\left\{q^a\left(\theta_m^a\right), t^a\left(\theta_m^a\right)\right\}_{\theta_m^a \in \left\{\underline{\theta}, \overline{\theta}\right\}}$  to the regulated firm  $F^A$ .
- 3) The regulated firm  $F^A$  decides whether to accept or reject this contract. In case of refusal, it gets a reservation gain exogenously normalized to zero. If  $F^A$  accepts the contract the game continues as follows.
- 4) After accepting the contract and before choosing which signal to give to the regulator, the other firms may try to pay  $F^A$  the amount *b* such that she claims to be inefficient: **collusion**. The incentives for collusion reside on the substitutability between the different products. The smaller the quantity produced by the regulated firm, the greater will be the quantity produced and the profits of the firms in market B. The collusion will be made under

asymmetric information since the unregulated firms do not know the true cost of  $F^A$ . The outcome will be a pair of side-payment and report  $(b, \theta_m^a)$ .

- 5) After the coalition is made between the firms in the market,  $F^{A}$  sends a message  $\theta_{m}^{a}$  to the regulator, produces the corresponding quantity and receives the corresponding transfer.
- 6) All the unregulated firms act as Stackelberg followers, deciding their individual quantities simultaneously given the quantity produced by  $F^A$ .

### 3 – Optimal regulation with perfect information

For the purpose of this section, let us assume that the firms' efficiency parameters  $\theta^a$  and  $\theta_i^b$  for i = 1, ..., n are known to all economic agents. The best response function for each of the unregulated firms is given by:

$$q_{i}^{b} = \frac{d^{b} - \theta_{i}^{b} - sq^{a} - \sum_{j \neq i} q_{j}^{b}}{2}, \forall i = 1, ..., n$$

Knowing that all unregulated firms will decide simultaneously their quantities, we can get an aggregate best response function for market B, given by:

$$Q^{b} = \frac{1}{n+1} \left[ nd^{b} - \sum_{i=1}^{n} \theta_{i}^{b} - nsq^{a} \right]$$

### 3.1. - Complete Regulation and Complete Information: the Social Optimum

The socially optimal quantities  $(q_{opt}^a, q_{iopt}^b, Q_{opt}^b)$  would be the ones that the regulator would ideally choose if we had complete information and complete regulation. Although this framework is not considered, it becomes a reference benchmark to other possible situations. These optimal quantities are such that the prices equal the marginal costs<sup>5</sup> and are given by:

$$q_{opt}^{a} = \frac{d^{a} - \theta^{a} - s(d^{b} - \theta_{n}^{b})}{1 - s^{2}}$$
$$q_{iopt}^{b} = \theta_{n}^{b} - \theta_{i}^{b}$$
$$Q_{opt}^{b} = \frac{d^{b} - \theta_{n}^{b} - s(d^{a} - \theta^{a})}{1 - s^{2}}$$

Notice that the only two marginal costs relevant for the optimal quantities are the ones related to  $F^A$  and  $F_n^B$  since they are directly linked to the equilibrium prices for both markets. Such quantities also depend positively on their own market dimension and negatively on the competitor's market dimension. It is also worth to mention that none of the equilibrium prices of the first-best situation depend on the number of firms in market B (*n*). Another interesting feature is that the quantity produced by each of the unregulated firms is directly given by the absolute costs advantage, leading to a quantity equal to zero for the least efficient firm.

### 3.2. - Incomplete Regulation and Complete Information: the second-best

Let us now assume an incomplete regulation framework in which the regulator R can only regulate firm  $F^A$ . Since we have seen that the rent of  $F^A$  is socially costly, the regulator R in equilibrium decides to extract all the profits from the regulated firm. Incorporating the aggregate best response function from the unregulated firms into the

<sup>&</sup>lt;sup>5</sup> For the firms operating in market B, the optimal quantities are set by making the price of that product equal to the marginal cost of the least efficient firm (firm n). By doing this we are implicitly considering entry and exit of firms in the market as exogeneous.

Social Welfare function given previously, the solution yields the following output levels<sup>6</sup> for any  $\theta^a = \left\{\underline{\theta}, \overline{\theta}\right\}$ :  $q_*^a(\theta^a) = \frac{1}{(n+1)^2 - (ns)^2} \left[ (n+1)^2 (d^a - \theta^a) - ns \left( nd^b - \sum_{i=1}^n \theta_i^b \right) \right]$  $q_{i^*}^b(q_*^a(\theta^a)) = \frac{1}{(n+1)^2 - (ns)^2} \left[ (n+1) \left( d^b - s(d^a - \theta^a) \right) + (n+1-ns^2) \sum_{j \neq i} \theta_j^b + n \left( s^2(n-1) - (n+1) \right) \theta_i^b \right]$  $Q_*^b(q_*^a(\theta^a)) = \frac{n+1}{(n+1)^2 - (ns)^2} \left[ nd^b - \sum_{i=1}^n \theta_i^b - ns(d^a - \theta^a) \right]$ 

Notice that if s = 0 the two quantities  $q_*^a(\theta^a)$  and  $q_{opt}^a$  are the same. When the two products are independent, the unregulated firms do not compete with the regulated firm and therefore there is no way that the regulator can influence their behaviour in an incomplete regulation framework. However, when the two products are substitutes, since the profits of the unregulated firms are not included in the Social Welfare expression, the regulator will want to incentive firm  $F^A$  to produce more such that the Consumer Surplus is greater, even if that means smaller profits for the unregulated firms. Also notice that all optimal quantities do depend non linearly on the number of firms in market B (*n*).

# 4 – Optimal incomplete regulation under asymmetric information and in the absence of collusion

In this section we will assume that the regulator and all the unregulated firms do not know the firm  $F^A$ 's actual marginal costs, however its distribution is publicly known. The marginal costs of all unregulated firms are public knowledge and the regulator can only influence  $F^A$ , leaving all the other competitors unregulated. For the time being we will also disregard the possibility of collusion between all the firms.

<sup>&</sup>lt;sup>6</sup> See appendix A.1. for objective function, constraints and first order conditions.

Accordingly to the timing defined previously, after  $F^A$  deciding which quantity to produce the other competitors will simultaneously choose how much they want to produce by their respective best response functions. Therefore, even in the presence of incomplete regulation, we can assume that the Revelation Principle will still hold<sup>7</sup> and we can focus the attention of the regulator *R* to direct and truthful contracts. We will denote  $\pi^a(\theta^a, \theta^a_m)$  as the profits for firm  $F^A$  when the marginal cost is  $\theta^a$  and she reports to the regulator to have the marginal cost  $\theta^a_m$ ,  $\overline{q^a} \equiv q^a(\overline{\theta^a})$  as the quantity that should be produced by the regulated firm when inefficient and  $\underline{q}^a \equiv q^a(\underline{\theta}^a)$  the quantity produced when  $F^A$  is efficient. We will concentrate our analysis on the profits  $\underline{\pi}^a \equiv \pi^a(\underline{\theta}^a, \underline{\theta}^a)$  and  $\overline{\pi^a} \equiv \pi^a(\overline{\theta^a}, \overline{\theta^a})$  which represent the rents for the efficient and inefficient regulated firms at a truthful equilibrium, respectively. Let us also denote  $\overline{Q^b}$  and  $\underline{Q^b}$  as the total quantity produced in market B when firm  $F^A$  is inefficient and efficient, respectively.

The problem for the regulator will be to maximize Social Welfare, subject to incentive compatibility (ICC) and participation constraints (PC), which can be written as follows:

$$\max_{\left\{\underline{q^{a}},\overline{q^{a}}\right\}} E(W) = \underline{p} \left\{ d^{a} \underline{q^{a}} + d^{b} \underline{Q^{b}} - \frac{1}{2} \left(\underline{q^{a}}\right)^{2} - \frac{1}{2} \left(\underline{Q^{b}}\right)^{2} - s \underline{q^{a}} \underline{Q^{b}} - \underline{\theta} \underline{q^{a}} - \left(d^{b} - \underline{Q^{b}} - s \underline{q^{a}}\right) \underline{Q^{b}} - \underline{\pi^{a}} \right\} + \frac{1}{p} \left\{ d^{a} \overline{q^{a}} + d^{b} \overline{Q^{b}} - \frac{1}{2} \left(\overline{q^{a}}\right)^{2} - \frac{1}{2} \left(\overline{Q^{b}}\right)^{2} - s \overline{q^{a}} \overline{Q^{b}} - \overline{\theta} \overline{q^{a}} - \left(d^{b} - \overline{Q^{b}} - s \overline{q^{a}}\right) \underline{Q^{b}} - \overline{\pi^{a}} \right\}$$

*s.t*.

$\underline{\pi^a} \ge 0$	(PC for $\underline{\theta}$ )
$\overline{\pi^a} \ge 0$	(PC for $\overline{\theta}$ )
$\underline{\pi^{a}} \geq \overline{\pi^{a}} + \Delta \theta \overline{q^{a}}$	(ICC for $\underline{\theta}$ )
$\overline{\pi^a} \geq \underline{\pi^a} - \Delta \theta \underline{q}^a$	(ICC for $\overline{\theta}$ )

Usually in this kind of problems only the Participation Constraint for the inefficient  $F^A$  and the Incentive Compatibility Constraint for the efficient regulated firm are binding. Then, plugging these constraints into the objective function and using the first order conditions we get that:

<sup>&</sup>lt;sup>7</sup> See Green and Laffont (1977) or Myerson (1979), among others, on the Revelation Principle.

$$\frac{q^{a}}{q^{a}} = \frac{1}{(n+1)^{2} - (ns)^{2}} \left[ (n+1)^{2} (d^{a} - \underline{\theta}) - ns \left( nd^{b} - \sum_{i=1}^{n} \theta_{i}^{b} \right) \right] = q_{*}^{a} (\underline{\theta})$$

$$\overline{q^{a}} = \frac{1}{(n+1)^{2} - (ns)^{2}} \left[ (n+1)^{2} (d^{a} - \overline{\theta}) - ns \left( nd^{b} - \sum_{i=1}^{n} \theta_{i}^{b} \right) \right] - \frac{(n+1)^{2}}{(n+1)^{2} - (ns)^{2}} \frac{\underline{p}}{\underline{p}} \Delta \theta = q_{*}^{a} (\overline{\theta}) - \frac{(n+1)^{2}}{(n+1)^{2} - (ns)^{2}} \frac{\underline{p}}{\underline{p}} \Delta \theta$$

In the case where the firms cannot collude and under asymmetric information and incomplete regulation we can verify that the standard no distortion at the top equilibrium in an adverse selection model holds. An efficient firm  $F^{A}$  will produce the complete information output, but the regulator needs to leave an information rent of  $\Delta \theta \overline{q^a}$  to this firm in order to induce a truthful revelation of its efficiency. Since this rent increases with the quantity produced by the inefficient firm  $F^A$ , it will be distorted downward relatively to the respective complete information quantity. This informational rent that we take to  $\overline{q^a}$  will be greater the probability that the firm  $F^A$  will be efficient and the more substitutable the products of the two markets are. In these two cases, the consequences for the expected Social Welfare of decreasing the quantity for the inefficient regulated firm are minimized since the risk of having an inefficient  $F^{A}$  is lower and because the consumers can more easily compensate this decrease in  $\overline{q^a}$  by consuming more from the other unregulated firms. This informational rent will also increase with the number of unregulated firms n. Once again, a lower  $\overline{q^a}$  will bring smaller distortions to the expected Social Welfare since the ability of capture higher profits of the unregulated firms will be very low when they are many.

At this point we should also notice that due to product substitutability there is a stake for collusion<sup>8</sup>. We have seen that the Incentive Compatibility Constraint for the efficient  $F^A$  will lead to  $\underline{q}^a \ge \overline{q}^a$  leading to  $\overline{\pi_i^b} \ge \underline{\pi_i^b}, \forall i = 1, ..., n$  and therefore the unregulated firms have incentives to make the regulated firm always pretend to be inefficient. Since this increase in the profits for the unregulated firms will play an important role in the next section, we will denote  $\Delta \pi_i^b = \overline{\pi_i^b} - \underline{\pi_i^b}$  as the increase in the

<sup>&</sup>lt;sup>8</sup> Proof in the appendix A.2.

profits of firm *i* in the unregulated sector by having firm  $F^A$  producing  $\overline{q^a}$  instead of  $\underline{q^a}$ .

# 5 – Optimal regulation under asymmetric information and with the possibility of collusion

As it was previously mentioned, it is possible for the firms to collude such that the firm  $F^{A}$  always reports to be inefficient to the regulator. Such decision to collude takes place after the firm  $F^{A}$  knows its true costs but before deciding which costs to report to the regulator. To model collusion under asymmetric information we are going to use a similar methodology used by Laffont and Martimort (1999, 2000) and Aubert and Pouyet (2006). Since  $\theta^a$  is private information at the time that the collusion takes place, we can model the bargaining process within the coalition by considering a hypothetical benevolent mediator M whose objective is to maximize the aggregate expected profits of all the firms in the coalition, subject to participation and incentive compatibility constraints. This mediator M can be viewed as another Principal to whom the Revelation Principle applies: the mediator will offer a collusive agreement such that all firms are willing to participate and firm  $F^{A}$  truthfully reports its type to the coalition We should also notice that since the marginal costs of all the unregulated firms are public knowledge they cannot deviate from their best response function to a certain report from the firm  $F^{A}$ , leading to an immediate detection of the collusion from an antitrust authority, which we don't model explicitly but is present nevertheless. Conversely, this authority does not detect any type of collusion between the firms if they act accordingly to the report firm  $F^{A}$  made to the regulator.

The aim of this section is to study to which extent the Collusion-Proofness Principle stated by Laffont and Martimort (1999, 2000) holds in the present framework. With complete regulation, there is no loss of generality in only to concentrate on regulation contracts that will replicate the collusion outcomes since the Regulator is able to enforce any output to all firms in the market. However, as already pointed out by Aubert and Pouyet (2006), in an incomplete regulation setting the regulator is not able to replicate all possible outcomes using a regulation contract and therefore we should not restrict attention only to those contracts that avoid collusion. All situations that involve a side-payment from the unregulated firms to the regulated firm  $F^A$  are out of reach from the regulator's point of view since he cannot impose any tax on any firm  $F_i^B$  which could replicate those payments. By comparing the Social Welfare from Collusion-Proofness Contracts with the Welfare obtained through contracts that allow collusion we are able to assess about the robustness of the Collusion-Proof Principle.

### 5.1. – Collusion-Proof Contracts

We start by focusing on Collusion-Proof Contracts, which are contracts that will induce a passive response from the coalition. When designing such a contract, the mediator will ask firm  $F^A$  to truthfully report its marginal costs to the regulator and no side-payment will be made from the unregulated firms to  $F^A$ .

Let us denote  $\theta_m^a(\theta^a)$  as the report recommended by the mediator when the marginal cost of firm  $F^A$  is  $\theta^a$  and let  $\theta^{nc}(\theta^a)$  be the report that firm  $F^A$  would made to the regulator if no collusion occurs. In case of collusion, the regulated firm will produce  $q^a(\theta_m^a)$  and each of the unregulated firms would have to produce  $q_i^b(\theta_m^a)$  and pay firm  $F^A$  the amount  $b_i(\theta^a)$ , where  $B(\theta^a) = \sum_{i=1}^n b_i(\theta^a)$  represents the aggregate side-payment made to firm  $F^A$ .

The problem for the mediator is to maximize the coalition's aggregate profits subject to the Participation Constraints (PC) for all firms and to the Incentive Compatibility Constraints (ICC) for both types of the regulated firm such that it will always reveal truthfully its marginal cost to the coalition. It can be written as:

$$\begin{split} \max_{\left\{(\theta^a_m),b_i(\theta^a)\right\}_{i=1}^n} E_{\theta^a} \left[\sum_{i=1}^n \pi^b_i(\theta^a_m) + \pi^a(\theta^a_m, \theta^a)\right] \\ \text{s.t.} \\ \pi^a(\theta^a_m(\underline{\theta}),\underline{\theta}) + B(\underline{\theta}) \geq \pi^a(\theta^{nc}(\underline{\theta}),\underline{\theta}), \quad \text{PC for } \mathbf{F}^a \\ \pi^a(\theta^a_m(\overline{\theta}),\overline{\theta}) + B(\overline{\theta}) \geq \pi^a(\theta^{nc}(\overline{\theta}),\overline{\theta}), \quad \text{PC for } \mathbf{F}^a \\ \pi^a(\theta^a_m(\underline{\theta}),\underline{\theta}) + B(\underline{\theta}) \geq \pi^a(\theta^a_m(\overline{\theta}),\underline{\theta}) + B(\overline{\theta}), \quad \text{ICC for } \mathbf{F}^a \\ \pi^a(\theta^a_m(\overline{\theta}),\underline{\theta}) + B(\overline{\theta}) \geq \pi^a(\theta^a_m(\underline{\theta}),\underline{\theta}) + B(\underline{\theta}), \quad \text{ICC for } \mathbf{F}^a \\ \pi^a(\theta^a_m(\overline{\theta})) - b_i(\underline{\theta}) \geq \pi^b_i(\theta^{nc}(\underline{\theta})), \quad \forall i = 1,...,n, \text{ PC for } \mathbf{F}^b_i \\ \pi^b_i(\theta^a_m(\overline{\theta})) - b_i(\overline{\theta}) \geq \pi^b_i(\theta^{nc}(\overline{\theta})), \quad \forall i = 1,...,n, \text{ PC for } \mathbf{F}^b_i \end{split}$$

Using the same methodology as Aubert and Pouvet (2006) it is also possible to prove that in the case of a Collusion-Proof contract, the regulator is able to use the information asymmetry within the coalition to create a regulation contract which leads to "individual" truthful revelation of its type by firm  $F^{A}$ . Hence, the ICC for the inefficient type will not be binding and the collusive equilibrium would be the same as in complete information<sup>9</sup>. This result is opposite to the one found by Laffont and Martimort (2000) in a complete regulation context, where asymmetric information generated inefficiencies in the functioning of the coalition.

By solving the mediator's problem we get the Collusion-Proofness Constraint which the regulator needs to include in his design of the regulation contract problem in order to induce a passive response from the coalition. This constraint can be written as $^{10}$ :

$$\underline{\pi^{a}} \geq \overline{\pi^{a}} + \Delta \theta \overline{q^{a}} + \sum_{i=1}^{n} \Delta \pi_{i}^{b}$$

This result is consistent with the constraint reached by Aubert and Pouvet (2006) for the case of two firms. Such constraint can be understood intuitively: an efficient firm  $F^{A}$  is willing to report truthfully its marginal costs whenever its profits from telling the truth are greater than the ones she would get by pretending to be inefficient plus the

<sup>&</sup>lt;sup>9</sup> See proof in the appendix A.3. <sup>10</sup> See proof in the appendix A.3.

highest amount of bribe that the unregulated firms are willing to give to firm  $F^{A}$  (which is equal to the increase on their profits when the regulated firms misleads the regulator).

By solving the regulator's problem, including now the Collusion-Proofness constraint derived above, the best separating collusion-proof contract is characterized by the following rents and quantities<sup>11</sup>:

$$\begin{aligned} \underline{q}_{cp}^{a} &= \frac{1}{(n+1)^{2} - ns^{2}(2+n)} \left[ (n+1)^{2} \left( d^{a} - \underline{\theta} \right) - (2+n)s \left( nd^{b} - \sum_{i=1}^{n} \theta_{i}^{b} \right) \right] \\ \overline{q}_{cp}^{a} &= \frac{1}{(n+1)^{2} - ns^{2} \left( n - 2\frac{\underline{p}}{\underline{p}} \right)} \left[ (n+1)^{2} \left( d^{a} - \overline{\theta} - \frac{\underline{p}}{\underline{p}} \Delta \theta \right) - s \left( n - 2\frac{\underline{p}}{\underline{p}} \right) \left( nd^{b} - \sum_{i=1}^{n} \theta_{i}^{b} \right) \right] \\ \overline{\pi}_{cp}^{a} &= 0 \\ \underline{\pi}_{cp}^{a} &= \Delta \theta \overline{q}_{cp}^{a} + \sum_{i=1}^{n} \Delta \pi_{i}^{b} \end{aligned}$$

Notice that both quantities are distorted when compared with the complete information outcomes already mentioned in the previous section and that this best collusion-proof contract is separating only if  $q_{cp}^a \ge \overline{q_{cp}^a}$  or otherwise entails pooling for a quantity given  $by^{12}$ :

$$q_{pool}^{a} = \frac{1}{(n+1)^{2} - (ns)^{2}} \left[ (n+1)^{2} (d^{a} - \overline{\theta}) - ns \left( nd^{b} - \sum_{i=1}^{n} \theta_{i}^{b} \right) \right]$$

Which is the complete information outcome if  $F^A$  was inefficient.

With collusion-proof contracts, the regulator has to provide firm  $F^A$  with an extra rent in order to compensate for any side-payment that it might get through collusion. Since such amount is affected by the quantity produced by both an efficient and an inefficient firm  $F^{A}$ , they will be distorted downward to minimize the cost of inducing

<sup>&</sup>lt;sup>11</sup> See proof in the appendix A.3.<sup>12</sup> See proof in the appendix A.3.

truthful revelation, contrary to the standard result of *no distortion at the top*. In fact, to ensure Collusion-Proofness, the regulator has to reduce simultaneously the rent  $\Delta \theta \overline{q_{cp}^a}$ and the stake of collusion  $\sum_{i=1}^{n} \Delta \pi_i^b$ . The first demands a decrease in  $\overline{q_{cp}^a}$  while the second demands simultaneously a decrease in  $\underline{q_{cp}^a}$  and an increase of  $\overline{q_{cp}^a}$  to bring these two quantities closer to each other. We have thus two opposite effects on  $\overline{q_{cp}^a}$ . As in Aubert and Pouyet (2006) we can verify that such quantity will be smaller than in the case with no possibility of collusion, concluding that the first effect dominates the second mentioned.

The second objective of bringing both quantities closer together also conflicts with the condition of sustainability of a separating equilibrium for collusion-proof contracts:  $\underline{q}_{cp}^a \ge \overline{q}_{cp}^a$ . When taken to an extreme, a pooling equilibrium may emerge. Such equilibrium will be especially desirable when the stake of collusion  $\sum_{i=1}^{n} \Delta \pi_i^b$  is very large and very sensitive to regulated quantities. This happens when the profitability of the unregulated market is high  $\left(nd^b - \sum_{i=1}^{n} \theta_i^b\right)$  and when the scale of production  $\left(\underline{q}^a + \overline{q}^a\right)$  of firm  $F^A$  is low. Contrary to the findings of Aubert and Pouyet (2006), the stake of collusion does not depend monotonically on the substitutability between the products. It becomes larger for an increase in *s* when the products are almost independent (*s* close to zero) and it also becomes larger for a decrease in *s* for almost perfect substitutable products (*s* close to one). The stake of collusion is also larger for a small number of firms operating in the unregulated sector and decreases when *n* increases.

### 5.2. – Collusion-Allowing Contracts

As already mentioned, with incomplete regulation we cannot rule out contracts that allow collusion since there are some outcomes only attainable by taxing the unregulated firms and therefore unattainable by the regulator alone. A potential benefit of contracts inducing active collusion is precisely the possibility of extracting some rents from the unregulated firms and direct them towards the regulated firm through side-payments. Allowing collusion, the regulated firm  $F^A$  will have higher utility and therefore will be more willing to accept the regulation contract even if it yields higher taxes.

As in the Aubert and Pouyet (2006) model, it is possible to show that the Revelation Principle still applies to the present framework, where the "agent" to be considered is the coalition as a whole and no longer the regulated firm only<sup>13</sup>. Hence, we can restrict our attention to direct mechanisms which induce active collusion and truthful reports from the coalition. The regulator must now include in its problem the Incentive Compatibility Constraint and the Participation Constraint for the coalition as a whole as well as the Incentive Compatibility Constraints and Participation Constraints internal to the coalition, which will reflect the expected aggregate profits. Therefore, the regulator's problem may be written as:

$$\max_{\left\{\underline{q^{a}}, \overline{q^{a}}, b_{i}(\underline{\theta}), b_{i}(\overline{\theta})\right\}_{i=1}^{n}} E_{\theta^{a}}(W) = \underline{p} \left\{ d^{a} \underline{q^{a}} + d^{b} \underline{Q^{b}} - \frac{1}{2} \left(\underline{q^{a}}\right)^{2} - \frac{1}{2} \left(\underline{Q^{b}}\right)^{2} - s \underline{q^{a}} \underline{Q^{b}} - \underline{\theta} \underline{q^{a}} - \left(d^{b} - \underline{Q^{b}} - s \underline{q^{a}}\right) \underline{Q^{b}} - \underline{\pi^{a}} \right\} + \overline{p} \left\{ d^{a} \overline{q^{a}} + d^{b} \overline{Q^{b}} - \frac{1}{2} \left(\overline{q^{a}}\right)^{2} - \frac{1}{2} \left(\overline{Q^{b}}\right)^{2} - s \overline{q^{a}} \overline{Q^{b}} - \overline{\theta} \overline{q^{a}} - \left(d^{b} - \overline{Q^{b}} - s \overline{q^{a}}\right) \underline{Q^{b}} - \overline{\pi^{a}} \right\}$$

<sup>&</sup>lt;sup>13</sup> Proof in the appendix A.4.

$$\begin{split} s.t. \\ \underline{\pi^{a}} + \sum_{i=1}^{n} \underline{\pi^{b}_{i}} \geq \overline{\pi^{a}} + \Delta \theta \overline{q^{a}} + \sum_{i=1}^{n} \overline{\pi^{b}_{i}} & \text{ICC for coalition } (\underline{\theta}) \\ \overline{\pi^{a}} + \sum_{i=1}^{n} \overline{\pi^{b}_{i}} \geq \underline{\pi^{a}} - \Delta \theta \underline{q}^{a} + \sum_{i=1}^{n} \underline{\pi^{b}_{i}} & \text{ICC for coalition } (\overline{\theta}) \\ \underline{\pi^{a}} + \sum_{i=1}^{n} \overline{\pi^{b}_{i}} \geq 0 & \text{PC for coalition } (\underline{\theta}) \\ \overline{\pi^{a}} + \sum_{i=1}^{n} \overline{\pi^{b}_{i}} \geq 0 & \text{PC for coalition } (\underline{\theta}) \\ \overline{\pi^{a}} + \sum_{i=1}^{n} \overline{\pi^{b}_{i}} \geq 0 & \text{PC for coalition } (\underline{\theta}) \\ \underline{\pi^{a}} + B(\underline{\theta}) \geq \pi^{a} (\theta^{nc}(\underline{\theta}), \underline{\theta}), & \text{PC for F}^{a} \\ \overline{\pi^{a}} + B(\overline{\theta}) \geq \pi^{a} (\theta^{nc}(\overline{\theta}), \overline{\theta}), & \text{PC for F}^{a} \\ \underline{\pi^{a}}^{a} + B(\underline{\theta}) \geq \pi^{a} (\theta^{a}_{m}(\overline{\theta}), \underline{\theta}) + B(\overline{\theta}), & \text{ICC for F}^{a} \\ \overline{\pi^{a}}^{a} + B(\overline{\theta}) \geq \pi^{a} (\theta^{a}_{m}(\underline{\theta}), \overline{\theta}) + B(\underline{\theta}), & \text{ICC for F}^{a} \\ \overline{\pi^{a}}^{b} - b_{i}(\underline{\theta}) \geq \pi^{b}_{i} (\theta^{nc}(\overline{\theta})), & \forall i = 1, ..., n, & \text{PC for F}_{i}^{b} \\ \overline{\pi^{b}_{i}}^{b} - b_{i}(\overline{\theta}) \geq \pi^{b}_{i} (\theta^{nc}(\overline{\theta})), & \forall i = 1, ..., n, & \text{PC for F}_{i}^{b} \\ \end{array}$$

Given the monotonicity condition that  $\underline{q}_{ac}^a \ge \overline{q}_{ac}^a$ , the last six constraints will not be binding. Rearranging the other first four constraints it will yield the following two relevant conditions:

$$\underline{\underline{\pi}^{a}} = \Delta \theta \overline{q^{a}} - \sum_{i=1}^{n} \underline{\underline{\pi}^{b}}_{i}$$
$$\overline{\underline{\pi}^{a}} + \sum_{i=1}^{n} \overline{\underline{\pi}^{b}}_{i} = 0$$

Hence, the best contract with active collusion is then characterized by the following rents and quantities:

$$\begin{aligned} \underline{q}_{ac}^{a} &= \frac{1}{(1-s^{2})n(n+2)+1} \left[ (n+1)^{2} (d^{a}-\underline{\theta}) - s(n+2) \left( nd^{b} - \sum_{i=1}^{n} \theta_{i}^{b} \right) \right] \\ \overline{q}_{ac}^{a} &= \frac{1}{(1-s^{2})n(n+2)+1} \left[ (n+1)^{2} \left( d^{a} - \overline{\theta} - \frac{\underline{p}}{\underline{p}} \Delta \theta \right) - s(n+2) \left( nd^{b} - \sum_{i=1}^{n} \theta_{i}^{b} \right) \right] \\ \underline{\pi}_{ac}^{a} &= \Delta \theta \overline{q}_{ac}^{a} - \sum_{i=1}^{n} \underline{\pi}_{i}^{b} \\ \overline{\pi}_{ac}^{a} &= -\sum_{i=1}^{n} \overline{\pi}_{i}^{b} \end{aligned}$$

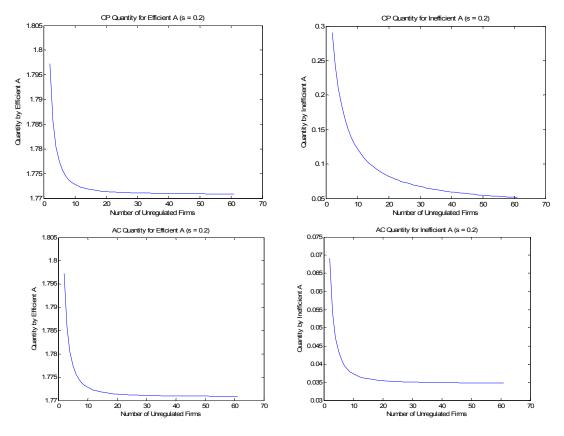
By comparing these quantities with the ones we have obtained when the contract was collusion-proof we can verify that  $\underline{q}_{ac}^{a} = \underline{q}_{cp}^{a}$  and  $\overline{q}_{ac}^{a} < \overline{q}_{cp}^{a}$ . This can be intuitively understood. By allowing collusion, the regulator is aware the firm  $F^{A}$  will have higher profits when reports to be inefficient through the side-payments made by the other firms in the unregulated sector. Hence, instead of rewarding the efficiency report the regulator penalizes even further firm  $F^{A}$  if she reports as being inefficient. Another way of understanding this result is that when allowing active collusion, the regulator is now internalizing not only the benefit that consumers derive from consumption of the good  $q^{b}$ but also the profits that this consumption generates, which will be forward to the regulated firm  $F^{A}$  through side-payments. Hence, imposing lower quantities for  $F^{A}$  will increase the profits of the unregulated firms and therefore smaller will be the share that the regulator needs to transfer himself to  $F^{A}$ .

### 5.3. – Equilibrium quantities Comparison

From this section onwards, due to algebraic complexity we have decided to compare quantities, Social Welfare and Consumer Surplus using graphical representations as a result of a computational simulation (using Matlab) using a set of parameters<sup>14</sup>. We will start by comparing the firm  $F^A$ 's and the unregulated firms' equilibrium quantities for the both types of contracts.

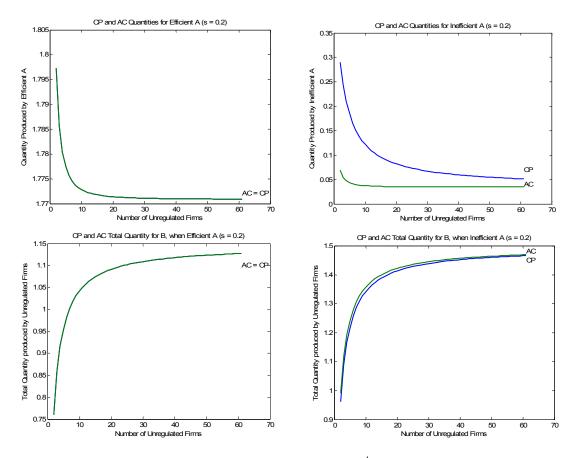
<sup>&</sup>lt;sup>14</sup> See appendix A.5. for parameter values used and detailed description of the simulation.

Firstly we will analyse how the equilibrium quantities produced by the firm  $F^{A}$  change with the number of competitors:



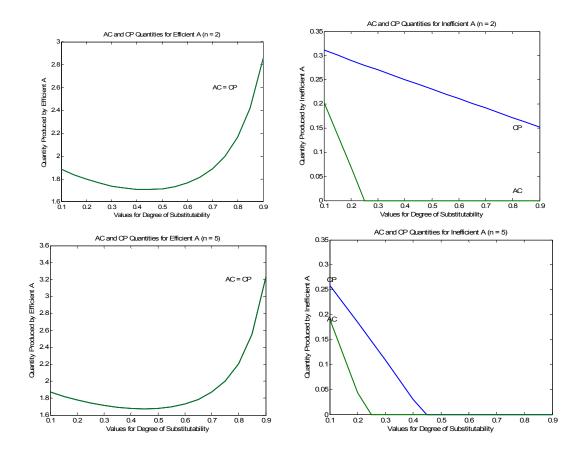
From the above diagrams we can see that there is a negative (and convex) relationship between the quantities produced by firm  $F^A$  and the amount of competitors it faces in the unregulated market. Although the graphs chosen as an example reflect a relatively low substitutability between the products of the two markets, we can see that when the number of competitors remains low but increases (between 1 and 10) this has a dramatic effect in the quantity produced by  $F^A$ . The same happens for other values of the substitutability degree (as we will see later on), where the quantity of  $F^A$  will tend to zero after a certain threshold in the number of competitors.

The following graphs allow us to better understand how the two types of equilibrium quantities for  $F^{A}$  and for all unregulated firms are related:



From above, we can see that an efficient  $F^A$  it will not matter if we allow collusion or not, as already expected. However, if firm  $F^A$  is inefficient then we have different levels of output across the two situations.  $F^A$ 's quantities tend to be lower when we allow for collusion, which is an evidence that the competitors are able to detect firm  $F^A$ 's inefficiency and that the "Truth-telling Constraint" holds. We should also notice the intuitive result that the total output produced in market B increases with the number of firms belonging to that market and that it is slightly higher in the situation where we allow for collusion.

The following diagrams compare firm  $F^A$ 's equilibrium outcomes both for the Collusion-Allowing and for Collusion-Proof situations for different levels of the substitutability degree.



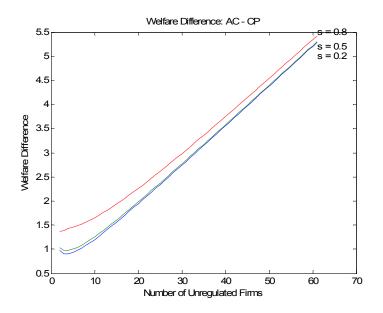
We can immediately notice that when firm  $F^A$  is efficient its equilibrium quantities are approximately the same across the two possible situations, similarly to the previously made analysis regarding the number of unregulated firms. However, when firm  $F^A$  is inefficient its Collusion-Proof quantities are greater than the equilibrium quantities when collusion is allowed. Once more, this is evidence that when designing the collusive agreement, the unregulated firms are able to extract from  $F^A$  some of the truthful information about its costs.

The results obtained for increasing values of the substitutability degree are also very intuitive. If firm  $F^A$  is efficient its equilibrium quantities will increase, as the consumers start to perceive the two products as close substitutes. The opposite happens when firm  $F^A$  is inefficient, where its quantities decrease with the degree of substitutability and becoming zero after a certain threshold. It is also worth mentioning that the level of equilibrium quantities for  $F^A$  also decrease when we increase the number unregulated firms from two to five, and such pattern continues throughout the rest of the levels considered.

### 5.4. – Social Welfare differences

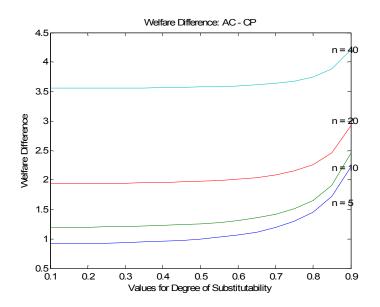
As previously mentioned, in an incomplete regulation framework the Collusion-Proofness Principle put forth by Laffont and Martimort (2000) may not hold and therefore we should also pay attention to regulation contracts that induce an active role by the coalition. The goal of this section is to compare the levels of Social Welfare obtained by a collusion-proof contract and by a contract that allows collusion.

Let us start by looking at the differences between the value of expected Welfare when we change the number of unregulated firms in market B.



From the above diagram we can clearly see that the Welfare Difference is always positive, meaning that the Collusion-Allowing Equilibrium always entails a higher expected Welfare than the one given by Collusion-Proof Contracts. Furthermore, this value is increasing with the number of unregulated firms and its level is slightly increasing with substitutability degrees. Such result can be justified in two ways. First, as we have argued before, the advantage of allowing collusion is that the unregulated firms in an effort of collude and decide side-payments will eventually lead firm  $F^A$  to reveal its real costs, eliminating part of the information asymmetry. Such result would entail a much higher cost for the Regulator if Collusion was not allowed. Secondly, as the number of unregulated firms increases the quantity and the relative weight of firm  $F^A$  on the total output produced by both markets decreases and therefore it will also decrease its importance in terms of Social Welfare.

Next, we will analyze the influence of the substitutability degree on Welfare differences, similarly to Aubert and Pouyet (2006). Additionally, such analysis will be taken across four different values for the number of firms in market B.



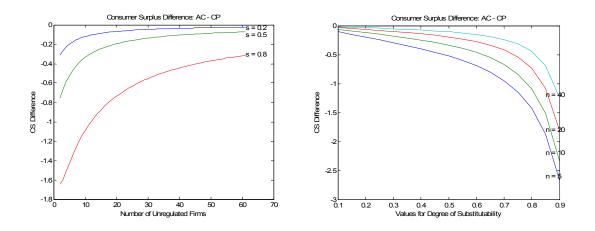
From the previous graph we can draw the same conclusions made previously for the number of unregulated firms. The Welfare differences are always positive, which means that the Collusion-Allowing equilibrium leads to greater Social Welfare. This Welfare difference increases with the degree of substitutability meaning that as the products are seen as close substitutes, the two markets are also seen as closer and the weight that firm  $F^A$  has in the total output and in Social Welfare is much smaller. Additionally, by allowing Collusion the unregulated firms are able to decrease the information asymmetry in the economy and therefore the bias it will have on the Welfare. Once again, we can also notice that the level of Welfare difference increases with the number of unregulated firms.

Finally it is also worth mentioning that such analysis still holds when the range of values for the unregulated firms' marginal costs increases, creating new firms more efficient than the efficient  $F^A$  and other firms that are more inefficient than the inefficient regulated firm<sup>15</sup>. The only difference lies in the level of values for the Welfare differences, they are still increasing and strictly positive but for even greater values. Such result may be intuitively explained by the fact that with increasing marginal costs differences across firms, the Regulator not only doesn't need to impose costs on society by creating incentives to extract marginal costs information from  $F^A$  but also the risk that an effective collusion might appear is much lower.

### 5.5. – The importance of Consumer Surplus differences

From previous analysis a somewhat paradoxal result was raised: it is always Welfare improving to allow collusion between the regulated firm  $F^A$  and the unregulated firms. In their paper, Aubert and Pouyet (2006) have already stated the failure of the Collusion-Proofness for some values of the substitutability degree and we were able to prove that if a new dimension is added to the model (an exogeneous fixed number of unregulated firms) this result is reinforced. The reasons behind such conclusion were also already stated. In an incomplete regulation framework the Regulator has no control over most of the firms and therefore he cannot replicate through contract design all the possible outcomes. Hence, it will be socially less costly to allow the unregulated firms, through side-payment in a collusion, to extract firm  $F^A$ 's information regarding her true marginal costs. Before such evidence, why do we still prohibit collusion? The answer may be given if we look only at Consumer Surplus instead of Social Welfare.

<sup>&</sup>lt;sup>15</sup> Results and diagrams are in the appendix A.5.



By observing the two previous graphs, the first and most important conclusion is that Active Collusion will always entail lower values of Consumer Surplus than the situation where Collusion-Proofness holds. By allowing collusion, the aggregate quantity produced by both the regulated and unregulated markets will be lower<sup>16</sup> and therefore the prices will be higher, which together will harm consumers. However, the number of unregulated firms and the substitutability degree will have opposite effects on the consumer surplus. As the number of unregulated firms increases the consumer surplus differences will decrease, while such differences will increase with the substitutability degree. Once again both results can be explained by the differences on the aggregate quantity produced. When the number of unregulated firms increases we have a convergence between the levels of quantities produced as for increases in the substitutability degree will lead to greater differences between the Collusion-Allowing and the Collusion-Proof outcomes.

Hence, a new and important conclusion rises from this analysis: Social Welfare differences will depend heavily on the relative weight that the regulator gives to Consumer Surplus on Social Welfare. The importance of this fact was also pointed out by Biglaiser and Ma (1995) and here lies the greatest argument against collusion even in a framework as the one presented in this paper.

<sup>&</sup>lt;sup>16</sup> See the graphs in section 5.3..

### 6 – Conclusion

The main purpose of this paper was to add a new dimension to the model put forth by Aubert and Pouyet (2006), by introducing an exogeneous number of unregulated firms in the market, however it was also able to produce a powerful argument against collusion even in situation when apparently it would be Welfare improving to allow it.

Auber and Pouyet (2006) proposed a model of incomplete regulation in a duopoly with asymmetric information regarding the costs of the regulated firm, in a context of product differentiation. The main conclusion of that paper was the failure of the "Collusion-Proofness Principle", which stated that higher Social Welfare would be attained if the regulator would design a contract with the regulated firm that prevented any form of collusion between the two of them. In fact, they have proven that for decreasing substitutability degrees the Social Welfare would be maximized in the situation where the regulator would allow collusion between firms and not when it is prevented. They argued that such result could be justified by the fact that through collusion the unregulated firm could extract a truthful cost information from the regulated firm and the regulated firm's costs information would be obtained at a much lower social cost than what it would have if such task would be left to the Regulator alone.

The present model extends such context to a case where the regulated firm faces competition from *n* unregulated firms and analyses how the conclusions drawn by Aubert and Pouyet (2006) would hold. Although this added dimension lead to a much more mathematically demanding model, it has reinforced the failure of the "Collusion-Proofness Principle". As the number of unregulated competitors increases, the Welfare difference between the Collusion-Allowing equilibrium and the Collusion-Proof equilibrium is always positive and increasing. Such result may be justified by the decreasing weight of the regulated firm in the aggregated market and on Social Welfare and by the ability that the unregulated firms have to extract the truthful information regarding the costs. A similar result holds when we study the impact of substitutability on the Welfare difference. Nevertheless, by analysing the Consumer Surplus differences we were able to find the most powerful argument why the regulator should still not allow

collusion. Results presented earlier depend heavily on the relative weight that the regulator gives to Consumer Surplus in the Social Welfare function. This new fact brings a new perspective on the results reached by Aubert and Pouyet (2006).

However some further research may still be made by adding further dimensions to the present model. First and foremost, we could start by analysing the role of the weight of the Consumer Surplus in the Social Welfare function in the simpler framework of Aubert and Pouyet (2006). Secondly, we have assumed up to this moment that the marginal costs for the unregulated sector were public knowledge. We can study what would happen if we had again only two firms (one regulated and the other unregulated) and both of them had marginal costs which were private information. Another way of improving the model is to introduce endogeneity in the number of unregulated firms operating in the market, by introducing entry and exit. Another idea would be to introduce the possibility of collusion between the regulated firm and only some of the unregulated firms in the market.

### Appendix

### A.1. Incomplete Regulation and Complete Information

The problem for the regulator when he cant influence any of the firms in market B but all the marginal costs are publicly known will be:

$$\max_{q^a} \quad W = GS(q^a, Q^b) - \theta^a q^a - P^b(q^a, Q^b)Q^b - \pi^a$$
  
s.t.  $\pi^a = 0$ 

and therefore, the first-order condition will be:

$$\frac{\partial W}{\partial q^{a}} = \frac{\partial W}{\partial q^{a}} + \frac{\partial W}{\partial Q^{b}} \times \frac{\partial Q^{b}}{\partial q^{a}} = 0$$

which entails the following solution:

$$q_*^{a}(\theta^{a}) = \frac{1}{(n+1)^2 - (ns)^2} \left[ (n+1)^2 (d^{a} - \theta^{a}) - ns \left( nd^{b} - \sum_{i=1}^{n} \theta_i^{b} \right) \right]$$

### A.2. Proof for the existence of a stake for collusion

There are incentives for collusion since the unregulated firms are better off, by product substitutability, if the firm  $F^A$  produces the quantity corresponding to her inefficient type.

Using the aggregate reaction function for the firms in market B, and by plugging the equilibrium quantities for  $F^A$ , we get that:

$$\overline{Q^{b}} - \underline{Q^{b}} = \frac{ns(n+1)}{(n+1)^{2} - (ns)^{2}} \left(1 + \frac{\underline{p}}{\underline{p}}\right) \Delta \theta > 0$$

which proves that the aggregate quantity produced by the firms in market B is greater when the firm  $F^A$  is inefficient. Substituting both  $Q^b$  and  $q^a$  we get the same conclusion for an individual firm  $F_j^b$ , for any j = 1, ..., n:

$$\overline{q_j^b} - \underline{q_j^b} = \frac{s(n+1)}{(n+1)^2 - (ns)^2} \left(1 + \frac{\underline{p}}{\underline{p}}\right) \Delta\theta > 0$$

and substituting this into the individual's profit function for any firm in the unregulated market, we have that:

$$\overline{\pi_{j}^{b}} - \underline{\pi_{j}^{b}} = \left(d^{b} - \overline{Q^{b}} - s\overline{q^{a}} - \theta_{j}^{b}\right)\overline{q_{j}^{b}} - \left(d^{b} - \underline{Q^{b}} - s\underline{q^{a}} - \theta_{j}^{b}\right)\underline{q_{j}^{b}} \\ = \left(d^{b} - \overline{Q^{b}} - s\overline{q^{a}} - \theta_{j}^{b}\right)\left(\overline{q_{j}^{b}} - q_{j}^{b}\right) + \left[\underline{Q^{b}} - \overline{Q^{b}} + s\left(\underline{q^{a}} - \overline{q^{a}}\right)\right]\underline{q_{j}^{b}} > 0$$

hence, each firm in market B will have higher profits if firm  $F^A$  is inefficient and therefore there is a stake for collusion.

### A.3. Collusion-Proof Contracts

### **Collusion-Proofness Constraints**

As stated before, the collusion-proof contracts are the ones yielding a passive response from the coalition. Since we have a sequence of decisions about the collusion and about the regulatory contract, the first step is to solve the mediator's problem presented earlier.

Let us denote by  $v^k(\theta^a)$  the multiplier of the coalition participation constraint for the firm k = a, b and by  $\delta^a(\theta^a)$  the multiplier of the coalition incentive compatibility constraint for each of the types of  $F^A$ . Solving that problem with respect to the bribes yields two first-order conditions that can be combined to obtain a relationship between those multipliers:

$$v^{a}(\underline{\theta}) - v^{b}_{i}(\underline{\theta}) = v^{b}_{i}(\overline{\theta}) - v^{a}(\overline{\theta}), \quad \forall i = 1, ..., n$$

By plugging the above expression into the mediator's problem, it is possible to separate the collusion problem with respect to reports into two parts:

$$\max_{\{\theta_{m}^{a}(\theta)\}} \left\{ \left[ \underline{p} + \delta^{a}\left(\overline{\theta}\right) + v^{b}\left(\underline{\theta}\right) \right] \pi^{a}\left(\theta_{m}^{a}(\underline{\theta}), \underline{\theta}\right) - \delta^{a}\left(\overline{\theta}\right) \pi^{a}\left(\theta_{m}^{a}(\underline{\theta}), \overline{\theta}\right) + \left[ \underline{p} + v^{b}\left(\underline{\theta}\right) \right] \sum_{i=1}^{n} \pi_{i}^{b}\left(\theta_{m}^{a}(\underline{\theta})\right) \right\}$$
$$\max_{\{\theta_{m}^{a}(\overline{\theta})\}} \left\{ \left[ \overline{p} + \delta^{a}\left(\overline{\theta}\right) + v^{b}\left(\overline{\theta}\right) \right] \pi^{a}\left(\theta_{m}^{a}(\overline{\theta}), \overline{\theta}\right) - \delta^{a}\left(\underline{\theta}\right) \pi^{a}\left(\theta_{m}^{a}(\overline{\theta}), \underline{\theta}\right) + \left[ \overline{p} + v^{b}\left(\overline{\theta}\right) \right] \sum_{i=1}^{n} \pi_{i}^{b}\left(\theta_{m}^{a}(\overline{\theta})\right) \right\}$$

To ensure collusion-proofness the regulator must offer a contract such that it always leads to a truth-telling decision from the collusion problem, meaning that  $\theta_m^a(\underline{\theta}) = \underline{\theta}$  and  $\theta_m^a(\overline{\theta}) = \overline{\theta}$  have to maximize the two objective functions stated above. Therefore, the two collusion-proofness constraints can be written as:

$$\pi^{a}(\underline{\theta},\underline{\theta}) \geq \pi^{a}(\overline{\theta},\overline{\theta}) + \Delta\theta \overline{q^{a}} + \sum_{i=1}^{n} \Delta\pi_{i}^{b} - \frac{\delta^{a}(\overline{\theta})}{\underline{p} + v^{b}(\underline{\theta})} \left(\underline{q^{a}} - \overline{q^{a}}\right) \Delta\theta$$
$$\pi^{a}(\overline{\theta},\overline{\theta}) \geq \pi^{a}(\underline{\theta},\underline{\theta}) - \Delta\theta \underline{q^{a}} - \sum_{i=1}^{n} \Delta\pi_{i}^{b} - \frac{\delta^{a}(\underline{\theta})}{\overline{p} + v^{b}(\overline{\theta})} \left(\underline{q^{a}} - \overline{q^{a}}\right) \Delta\theta$$

Let us consider for the time being only the first collusion-proofness constraint, related to the efficient  $F^A$  and check later that the last one will always be satisfied afterwards. We should also notice that if the regulator induces a truthful report, none of the unregulated firms is willing to pay any bribe to  $F^A$ , meaning that

 $b_i(\overline{\theta}) = b_i(\underline{\theta}) = 0, \forall i=1,...,n$  and that the incentive compatibility constraint for the inefficient  $F^A$  can be rewritten as  $\pi^a(\overline{\theta},\overline{\theta}) \ge \pi^a(\underline{\theta},\underline{\theta}) - \Delta\theta \underline{q}^a$ , which is exactly the same constraint as the incentive compatibility constraint for an inefficient  $F^A$  in the regulator's problem. Hence, if this constraint is not binding in the regulator's problem for the best collusion-proof contract, the multiplier  $\delta^a(\overline{\theta})$  equals to zero, and the collusion-proof constraint for the efficient  $F^A$  is the same as with perfect information within the coalition:  $\pi^a(\underline{\theta},\underline{\theta}) \ge \pi^a(\overline{\theta},\overline{\theta}) + \Delta\theta \overline{q^a} + \sum_{i=1}^n \Delta \pi_i^b$ .

### The best separating collusion-proof contract

Lets start by assuming that  $\delta^a(\overline{\theta})$  is not equal to zero, which means that the incentive compatibility constraint for the inefficient  $F^A$  is not binding in the regulator's problem. Hence the collusion-proofness constraint is more stringent that the incentive compatibility constraint and the only binding conditions for the regulator's problem are the participation constraint for the inefficient  $F^A$  and the collusion-proofness constraint for the efficient  $F^A$ . We have now to check that the equilibrium quantities will satisfy the incentive compatibility constraint for the inefficient  $F^A$ , i.e.,  $\underline{q}^a \ge \overline{q}^a$ . Knowing that  $\pi_i^b = (q_i^b)^2$ , we get that:

$$\sum_{i=1}^{n} \Delta \pi_i^b = \frac{ns}{(n+1)^2} \left(\underline{q}^a - \overline{q}^a\right) \left[ 2\left(d^b + \sum_{i=1}^{n} \theta_i^b\right) - s\left(\underline{q}^a + \overline{q}^a\right) \right] - \frac{2s}{n+1} \left(\underline{q}^a - \overline{q}^a\right) \sum_{i=1}^{n} \theta_i^b$$

Plugging this expression into the binding constraints we have that:

$$\pi^{a}\left(\overline{\theta}\right) = 0$$

$$\pi^{a}\left(\underline{\theta}\right) = \Delta\theta\overline{q^{a}} + \frac{ns}{(n+1)^{2}}\left(\underline{q}^{a} - \overline{q^{a}}\right) \left[2\left(d^{b} + \sum_{i=1}^{n}\theta_{i}^{b}\right) - s\left(\underline{q}^{a} + \overline{q^{a}}\right)\right] - \frac{2s}{n+1}\left(\underline{q}^{a} - \overline{q^{a}}\right)\sum_{i=1}^{n}\theta_{i}^{b}$$

$$-\frac{\delta^{a}(\overline{\theta})}{\underline{p} + v^{b}(\underline{\theta})}\left(\underline{q}^{a} - \overline{q^{a}}\right)\Delta\theta$$

Substituting these conditions into the regulator's problem and rearranging the first-order conditions, we obtain the best collusion-proof regulated quantities:

$$\frac{q_{cp}^{a}}{q_{cp}^{a}} = \frac{1}{(n+1)^{2} - ns^{2}(2+n)} \left[ (n+1)^{2} \left( d^{a} - \underline{\theta} + \frac{\delta^{a}(\overline{\theta})}{\underline{p} + v^{b}(\underline{\theta})} \Delta \theta \right) - (2+n)s \left( nd^{b} - \sum_{i=1}^{n} \theta_{i}^{b} \right) \right]$$

$$\overline{q_{cp}^{a}} = \frac{1}{(n+1)^{2} - ns^{2} \left( n-2\frac{\underline{p}}{\underline{p}} \right)} \left[ (n+1)^{2} \left( d^{a} - \overline{\theta} - \frac{\underline{p}}{\underline{p}} \left( 1 + \frac{\delta^{a}(\overline{\theta})}{\underline{p} + v^{b}(\underline{\theta})} \right) \Delta \theta \right) - s \left( n-2\frac{\underline{p}}{\underline{p}} \right) \left( nd^{b} - \sum_{i=1}^{n} \theta_{i}^{b} \right) \right]$$

To check if the incentive compatibility constraint for the inefficient  $F^A$  is satisfied we need to prove that  $q_{cp}^a \ge \overline{q_{cp}^a}$ , hence:

$$\underline{q_{cp}^{a}} - \overline{q_{cp}^{a}} = \frac{(n+1)^{2}}{\overline{p}\left[(n+1)^{2} - ns^{2}(2+n)\right]\left[(n+1)^{2} - ns^{2}\left(n-2\frac{p}{\overline{p}}\right)\right]}} \begin{cases} 2ns^{2}\left(d^{a} - \underline{\theta}\right) + \left((n+1)^{2} - ns^{2}(2+n)\right)\Delta\theta + \left((n+1)^{2} - ns^{2}\right)\frac{\delta^{a}(\overline{\theta})}{\underline{p} + v^{b}(\underline{\theta})}\Delta\theta + s\underline{p}\left(\sum_{i=1}^{n}\theta_{i}^{b} - nd^{b}\right) \end{cases}$$

Which is always greater than zero for a small enough *s*. Therefore, if this condition is satisfied then the incentive compatibility constraint for the best collusion-proof contract for an inefficient  $F^A$  is not binding and the relevant collusion proofness constraint is indeed the same as if the coalition was under complete information. This proves the claim that  $\delta^a(\overline{\theta})$  is equal to zero and the equilibrium quantities will simplify as in the text. However if this is not satisfied, then the best collusion-proof contract will entail pooling.

### The best pooling collusion-proof contract

If the previous conditions does not hold, the best collusion-proof contract will entail a pooling equilibrium where the regulator imposes always the same quantity  $\overline{q^a}$ and therefore no collusion will take place. Hence, the best pooling quantity is the full information one for an inefficient firm  $F^A$ .

### A.4. Application of the Revelation Principle to Collusion-Allowing Contracts

We will now consider the contracts that do not impose a passive response from the coalition: the Collusion-Allowing contracts. First we will prove that the Revelation Principle still applies in the present framework and we can concentrate in direct and truth-telling mechanisms. Finally we will determine the best Collusion-Allowing Contract.

### Direct Mechanisms

Let us start by considering the mediator's problem. The Revelation Principle applies and we can consider only direct truthful mechanisms  $m: \{\underline{\theta}, \overline{\theta}\} \rightarrow \mu \times$ , mapping a truthful report  $\theta^a$  by  $F^A$  into an allocation  $\{m(\theta^a), b(\theta^a)\}$ . For simplicity, let us denote  $m(\overline{\theta}) \equiv \overline{m}$  and  $m(\underline{\theta}) \equiv \underline{m}$ . The mediator's problem can then be written as:

$$\begin{split} & \max_{\left\{m(\theta^{a}),b_{i}(\theta^{a})\right\}_{i=1}^{n}} E_{\theta^{a}} \left[\pi^{a}\left(m(\theta^{a}),b(\theta^{a})\right) + \sum_{i=1}^{n} \pi_{i}^{b}\left(m(\theta^{a})\right)\right] \\ & \text{s.t.} \\ & \pi^{a}\left(\underline{m},\underline{\theta}\right) + B(\underline{\theta}) \geq \pi^{a}\left(m^{nc}(\underline{\theta}),\underline{\theta}\right) \quad \text{PC for efficient A} \\ & \pi^{a}\left(\overline{m},\overline{\theta}\right) + B(\overline{\theta}) \geq \pi^{a}\left(m^{nc}(\overline{\theta}),\overline{\theta}\right) \quad \text{PC for inefficient A} \\ & \pi^{a}\left(\underline{m},\underline{\theta}\right) + B(\underline{\theta}) \geq \pi^{a}\left(\underline{m},\underline{\theta}\right) + B(\overline{\theta}) \quad \text{ICC for efficient A} \\ & \pi^{a}\left(\overline{m},\overline{\theta}\right) + B(\overline{\theta}) \geq \pi^{a}\left(\underline{m},\overline{\theta}\right) + B(\underline{\theta}) \quad \text{ICC for inefficient A} \\ & \pi^{a}\left(\overline{m},\overline{\theta}\right) - b_{i}(\underline{\theta}) \geq \pi_{i}^{b}\left(m^{nc}(\underline{\theta})\right) \quad \text{PC for each firm in B} \\ & \pi_{i}^{b}\left(\overline{m}\right) - b_{i}\left(\overline{m}\right) \geq \pi_{i}^{b}\left(m^{nc}(\overline{\theta})\right) \quad \text{PC for each firm in B} \end{split}$$

The next definition will allow us to distinguish clearly the incentive constraints coming from the mediator's problem from the ones coming from the regulator's problem.

Definition 1: A message response m(.) that associates some messages  $\underline{m}$  and  $\overline{m}$  in  $\mu$  to  $\underline{\theta}$  and  $\overline{\theta}$  respectively is said to be *C*-incentive feasible if and only if there exists a couple  $\{b_i(\underline{\theta}), b_i(\overline{\theta})\} \in {}^{2n}$  such that all the constraints of the mediator's problem are simultaneously satisfied for  $\{\underline{m}, \overline{m}, b_i(\underline{\theta}), b_i(\overline{\theta})\}_{i=1}^{n}$ .

Let us denote by  $\mathcal{P}$  a subset of  $\mu \times \mu$  as the set of C-incentive feasible message responses and by  $\{\underline{m}^T, \overline{m}^T\}$  the solution of the maximization of the aggregated firms' profits. Then we have that:

$$\left\{\underline{m}^{T}, \overline{m}^{T}\right\} \in \underset{\left\{\underline{m}, \overline{m}\right\} \in \mathcal{G}}{\operatorname{arg\,max}} \underline{p}\left[\pi^{a}(\underline{m}, \underline{\theta}) + \sum_{i=1}^{n} \pi_{i}^{b}(\underline{m})\right] + \overline{p}\left[\pi^{a}(\overline{m}, \overline{\theta}) + \sum_{i=1}^{n} \pi_{i}^{b}(\overline{m})\right]$$

*Lemma 1:* If  $\{\underline{m}, \overline{m}\}$  is C-incentive feasible given an initial message space  $\mu$ , then it remains so when the message space is reduced to  $\{\underline{m}^T, \overline{m}^T\}$ .

*Proof:* All the 2n+4 constraints cane be satisfied for the restricted message space. Considering the participation constraint for the efficient  $F^A$ , if  $\{\underline{m}, \overline{m}\}$  is feasible for  $\mu$  then that constraint will be satisfied and there exists a vector given by  $\{b_i(\underline{\theta})\}_{i=1}^n$  such that  $\pi^a(\underline{m},\underline{\theta}) + B(\underline{\theta}) \ge \pi^a(\underline{m}^{nc}(\underline{\theta}),\underline{\theta})$ . By definition of the non-collusive best response we have that  $\pi^a(\underline{m}^{nc}(\underline{\theta}),\underline{\theta}) \ge \max\{\pi^a(\underline{m},\underline{\theta}),\pi^a(\overline{m},\underline{\theta})\}$ . Hence, the participation constraint for the efficient  $F^A$  is also satisfied when the message space is restricted. There exists a  $\{b_i(\underline{\theta})\}_{i=1}^n$  such that  $\pi^a(\underline{m},\underline{\theta}) + B(\underline{\theta}) \ge \max\{\pi^a(\underline{m},\underline{\theta}),\pi^a(\overline{m},\underline{\theta})\}$ . The same reasoning applies for an inefficient  $F^A$  and for all the firms in market B. The collusive incentive compatibility constraints are unaffected and all the constraints are therefore satisfied.

Hence, from the regulator's perspective there is no restriction in offering a message space  $\{\underline{m}^{T}, \overline{m}^{T}\}$  instead of  $\mu$  since the pair is incentive feasible and all other messages are never played.

### Truthful Mechanisms

Now we need to prove that there is no restriction on focusing on truthful mechanisms. From before we have seen that there are only two possible messages  $\{\underline{\theta}, \overline{\theta}\}$ . Therefore we have three possible responses from the coalition when it comes to choosing the report to the regulator: either it always announces the same type, independently of  $F^A$ 's true costs (which can also be implementable through a truthful mechanism), it may decide to make a different announcement given  $F^A$ 's type (which directly corresponds to a truthful mechanism) or the coalition randomizes on the two messages in at least one state of nature. Hence, it is sufficient to prove that no randomization will take place to prove that there is no loss of generality to focus only on truthful mechanisms.

Randomization only occurs if the coalition's total profits are identical for both messages in each state of nature, i.e.,

$$\pi^{a}(\underline{\theta},\theta^{a}) - \pi^{a}(\overline{\theta},\theta^{a}) = \sum_{i=1}^{n} \left[ \left( \overline{q_{i}^{b}} \right)^{2} - \left( \underline{q_{i}^{b}} \right)^{2} \right], \quad \forall \theta^{a}$$

Since the regulator's Welfare only depends on  $q^a$  (since  $q_i^b$  is a function of this quantity) and on the transfers  $t^a$ , we have that only one of the possible two pairs  $\{q^a(\underline{\theta}), t^a(\underline{\theta})\}\$  and  $\{q^a(\overline{\theta}), t^a(\overline{\theta})\}\$  is therefore preferred by the regulator. Hence, if we assume that the coalition is indifferent between the two pairs, the regulator can always offer some additional transfer of  $\varepsilon$  (which s approximately zero) for the message that will entail a higher Welfare. Hence, there is no restriction in considering truthful mechanisms.

*Lemma 2:* There is no loss of generality in considering that only direct and truthful (C-incentive feasible) mechanisms are offered to the coalition.

### A.5. Computational Simulation

Although comparing equilibrium quantities is not as algebraically demanding as comparing Social Welfare and Consumer Surplus for the Collusion-Proof and Collusion-Allowing equilibria, we have decided to use graphical representations to illustrate them which result from computational simulation using Matlab.

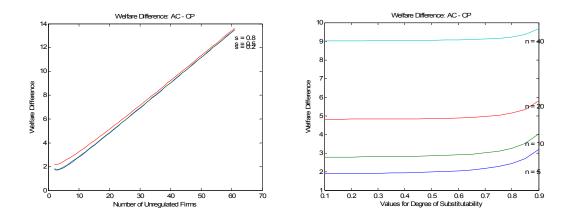
For parameters we have defined for firm  $F^A$  that  $d^a - \underline{\theta} = 2$  and that  $d^a - \overline{\theta} = 1$ and furthermore that  $\underline{p} = 0.4$ , as in Aubert and Pouyet (2006). For the unregulated sector a vector of costs was assumed such that for the most efficient firm we had that  $d^b - \theta_1^b = 2$  and for the most inefficient unregulated firm we had that  $d^b - \theta_n^b = 1$ . This means that the regulated firm  $F^A$  would always at the same level of the most efficient or the most inefficient firm of the unregulated sector. Later on the same simulation was done for a wider range of costs for the unregulated sector where  $d^b - \theta_1^b = 2.3$  and  $d^b - \theta_n^b = 1.7$ .

The simulation was made for a range of values for the substitutability degree such that  $0.1 \le s \le 0.9$  and for a number of unregulated firms between 2 and 61. After running

the program we ended up with equilibrium quantities matrices of size  $(s \times n)$ , which fulfilled non-negativity constraints. We have then used these values to plug into the Social Welfare and Consumer Surplus functions to compare them over this two dimensions.

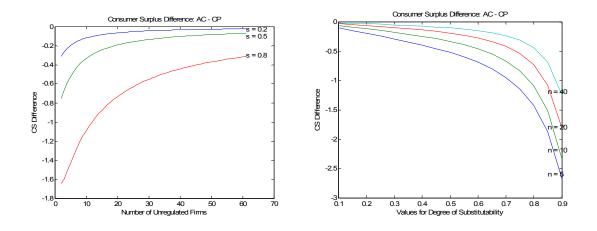
The results for the first vector of marginal costs for the unregulated sector are present in the text. When we consider the wider range of costs for theses firms, the graphical results are as follows:

### Social Welfare Comparison



As we can see from the graphs presented above, having a wider range of costs fro the unregulated sector would translate into higher levels of Social Welfare differences. This means that when firm  $F^A$  is no longer the most efficient nor the least efficient in the market, the Collusion-Allowing Contracts will yield an even better Social Welfare value, when compared to the Colusion-Proof Contracts.

### Consumer Surplus Comparison



The above diagrams show that also in this case we have negative values for the Consumer Surplus difference, which are increasing in the number of unregulated firms in the market and decreasing with the substitutability degree, similarly to the results obtained previously for a smaller range of the unregulated firms' marginal costs. Furthermore, the Consumer Surplus differences are approximately the same as before since the equilibrium are also the same.

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