

A Model of Firm Behaviour with Equity Constraints and Bankruptcy Costs

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ABSTRACT

Based on Greenwald and Stiglitz (1988,1990), this work explores a simple model of microeconomic behaviour which incorporates the impact of capital markets imperfections generated by asymmetric information on firms' optimal investment decision rules. In particular, this paper analyses how a specific form of asymmetric information problem (adverse selection) *may* imply lower investment than otherwise through the reduction of the firms' ability to raise external financing – either in the form of credit rationing or the 'voluntary' reduction of firms' borrowing activity.

The natural follow-up to this work would be to *formally* show how a loan market where both contractual interest rates *and* loan sizes are (*a priori*) variable may be characterised by a credit rationing equilibrium.

Keywords: Asymmetric Information, Firm Behaviour, Investment Financing

RESUMO

Partindo de Greenwald e Stiglitz (1988,1990), este trabalho explora um modelo simples de comportamento microeconómico que tem em conta o impacto das imperfeições dos mercados de capitais originadas por assimetrias de informação nas regras óptimas de decisão de investimento das firmas. Em particular, é analisado o modo como uma forma específica de problemas de informação assimétrica (selecção adversa) *poderá* levar a menores níveis de investimento devido à redução da capacidade de financiamento externo das firmas – tanto sob a forma de racionamento de crédito como de redução 'voluntária' da actividade mutuária das firmas.

O seguimento natural do presente trabalho será mostrar *formalmente* como um mercado de empréstimos no qual tanto as taxas de juro contratuais como as dimensões dos empréstimos são (*a priori*) variáveis pode apresentar um equilíbrio com racionamento de crédito.

Palavras-chave: Informação Assimétrica, Comportamento da Firma, Financiamento do Investimento.

1. Introduction

Traditionally, the models of imperfect financial markets have focused on imperfections related to asymmetric information between lenders and borrowers and between outside investors and managers. These models assume that there exist either *ex post* or *ex ante* information asymmetries of different types, associated with both equity and debt financing, so that firms are much better informed about their investment projects than outside investors and creditors are. Globally, the models predict the existence of frictions associated with external financing: external funds will carry a premium cost against the cost of internal funds (e.g., Williamson, 1987; Bernanke and Gertler, 1990) and there may even arise quantitative constraints on both equity and debt financing (e.g., Jaffee and Russell, 1979; Stiglitz and Weiss, 1981).

By means of a simple model, based on Greenwald and Stiglitz (1988, 1990a), we study analytically how a firm's optimal decision rule of investment must be redefined to take into account the premium on external finance. This model focuses on debt finance by assuming that firms have limited access to the equity market (informational problems make them equity-constrained), just like the majority of the models dedicated to credit market imperfections does.¹ However, unlike the latter, Greenwald and Stiglitz assume that lenders in the debt market (banks) are perfectly informed.

These authors note that debt imposes a particularly heavy risk burden on the firm because the contractual payments to the lender are fixed and thus the firm must absorb all fluctuations in profits. As a firm's ability to raise equity funds is restricted (and the growth potential of cash-flow from existing operations wears out), higher levels of investment or production are only feasible with increased debt; the ensuing fixed obligations imply that as debt is increased, the bankruptcy probability gets larger (Stiglitz, 1992, p. 280). This is true irrespective of whatever kind of information imperfection (if any) exists in the capital market. On the other hand, one may attach a cost to the bad state of nature (bankruptcy). Greenwald and Stiglitz (1988, p. 2, fn 8) suggest an informational justification for this: when a firm becomes 'financially distressed', it is usually impossible to tell whether this is due to bad luck with projects

¹ The models that focus on the imperfections in the credit market usually either assume that the contract between borrowers and lenders is a debt contract, *a priori* excluding the possibility of any other

which were *ex ante* properly undertaken or to bad management. As a result, failure will stigmatise managers whether it is deserved or not. Greenwald and Stiglitz (1990a, p. 17) also refer to a “punishment” imposed to managers in case of failure. In turn, this cost may induce some kind of ‘bankruptcy’ avoidance behaviour.

In the model presented below (Sections 2 and 3), firms have limited access to equity market (they are equity-constrained) but not to the debt market (information problems only exist in the former – eventually we will relax this assumption). The output and investment decisions of firms are assumed to be made by managers who are averse to the occurrence of ‘bankruptcy’. In concrete, we will think of firms as maximising expected end-of-period equity² minus perceived expected bankruptcy costs,³ so that firms act explicitly to avoid bankruptcy. In Section 4, we proceed to explicitly consider *ex ante* asymmetric information in the model of equity-constrained firms, so that banks are not able to distinguish among potential borrowers. This implies that the contractual rate of interest will be set at the same level for all firms. We re-analyse the firm’s optimal decision rule of investment in the light of this. Section 5 makes a synthesis of the results of our model and Section 6 concludes.

2. The Basic Model

The model is based on Greenwald and Stiglitz (1990a; Section 1.1) and Greenwald and Stiglitz (1988) and built on the following assumptions.

Firms make decisions at discrete time intervals t . Inputs must be paid before output, q_t , is available for sale (there are no stocks) and before output price, \tilde{p}_t , is known.⁴ The price of output is a random exogenous variable with probability distribution function $F(\tilde{p}_t)$, where $E(\tilde{p}_t) = 1$. Firms produce output using only working capital, K , as an input, with $K_t = \phi(q_t)$; ϕ is a ‘capital requirements’ function with $\phi' > 0$, $\phi'' > 0$

form of contract (namely equity), or derive debt contracts as the optimal contract form given some type of asymmetric information but again assuming that firms are equity-constrained.

² Henceforth, this will be alternatively referred to as ‘terminal wealth’ or ‘terminal value of the firm’.

³ Alternatively, we could think of firms as maximising an expected utility (or valuation function) of end-of-period equity, with the utility function characterised by decreasing absolute risk aversion and declining marginal utility of end-of-period equity (see Greenwald and Stiglitz, 1990b);

⁴ It is assumed that future markets are not a “significant factor”. The justification for this kind of assumption is again informational; see Greenwald and Stiglitz (1988, p. 3, fn 9).

(note that ϕ^{-1} is the usual production function). The price of capital, p_k , is constant and exogenous to any of the firm's decisions.

Both borrowers (firms) and lenders (banks) are perfectly informed and risk-neutral. The contract between borrowers and lenders takes the form of a debt contract. The contractual level of interest rate the firm promised to pay debtholders at the beginning of period t , r_t , is endogenously determined. The debt incurred by the firm at the beginning of period t is $b_t = p_k \phi(q_t) - a_{t-1}$, where a_{t-1} is the level of equity inherited from period $t-1$, i.e., $a_{t-1} = p_{t-1} q_{t-1} - (1 + r_{t-1})b_{t-1}$. Bankruptcy occurs if the end-of-period value of the firm is below zero, which is to say if $\tilde{p}_t q_t < (1 + r_t)b_t$; in this case the entire proceeds from the sale of output are distributed to debtholders (there exist no reorganisation or liquidation costs to debtholders). The level of price at which the firm is just solvent is:

$$(1) \quad \bar{u}_t \equiv (1 + r_t) \frac{p_k \phi(q_t) - a_{t-1}}{q_t}.$$

Then, the rate of return to lenders is a random variable $(1 + \tilde{r}_t)$, that equals:

$$\begin{cases} (1 + r_t) & \text{if } \tilde{p}_t \geq \bar{u}_t \\ \frac{\tilde{p}_t q_t}{b_t} & \text{if } \tilde{p}_t < \bar{u}_t. \end{cases}$$

Thus, the lenders' expected rate of return from a loan is:

$$(2) \quad E(1 + \tilde{r}_t) = (1 + r_t) \cdot (1 - F(\bar{u}_t)) + \frac{q_t}{b_t} \int_0^{\bar{u}_t} \tilde{p}_t dF(\tilde{p}_t).$$

Now, assume that lenders have access to elastic supply of funds at a cost of \bar{r}_t . If lenders are competitive, \bar{r}_t is, in equilibrium, the required expected rate of return on loans in period t , and the appropriate level for the contractual interest rate r_t is found by equating:

$$(3) \quad E(1 + \tilde{r}_t) = (1 + \bar{r}_t).$$

Equations (2) and (3) together constitute the lender's expected break-even condition. Hence, making use of equations (1), (2) and (3), we can solve for the equilibrium levels

of r_t , \bar{u}_t and the probability of bankruptcy, P_B (which equals $F(\bar{u}_t)$), as functions of q_t , a_{t-1} and \bar{r}_t .

Each firm's decision maker chooses q_t in order to maximise, in each period t :

$$E(\tilde{a}(q_t)) - c(q_t)P_B,$$

where $\tilde{a}(q_t) = \tilde{p}_t q_t - (1 + \tilde{r}_t)b_t$ is the end-of-period t value of the firm (a random variable) and $c(q_t) = c q_t$ is the cost of bankruptcy, which increases with the level of a firm's output.⁵

Substituting we have:

$$(4) \quad \max q_t - E(1 + \tilde{r}_t) \cdot (p_k \phi(q_t) - a_{t-1}) - c q_t F(\bar{u}_t),$$

subject to:

$$(5) \quad E(1 + \tilde{r}_t) \cdot \frac{p_k \phi(q_t) - a_{t-1}}{q_t} = \bar{u}_t \cdot (1 - F(\bar{u}_t)) + \int_0^{\bar{u}_t} \tilde{p}_t dF(\tilde{p}_t) \quad \text{and}$$

$$(3) \quad E(1 + \tilde{r}_t) = (1 + \bar{r}_t).$$

Note that (5) is just equivalent to (2). Making use of equation (3), we see that the left-hand side represents the expected return required by lenders per unit of output. The right-hand side represents the actual expected return to lenders per unit of output as a function of \bar{u}_t .

Following Aizenman and Powell (1997, p. 8), we combine equations (1), (3) and (5) to obtain:

$$r_t = \bar{r} + \frac{\int_0^{\bar{u}_t} (\bar{u}_t - \tilde{p}_t) q_t dF(\bar{u}_t)}{b_t}.$$

The contractual interest rate can thus be seen as the sum of two components: the bank's required rate of return and a 'mark-up' that compensates for the expected revenue lost due to (partial) default in bad states of nature. Notice that this result is common to many models of imperfect financial markets, although they may differ in the way the 'mark-up' term is defined. When lenders are imperfectly informed about the borrowers' type, the mark-up is calculated to compensate for the *average* rate of default of the pool of borrowers, as perceived by the lenders (see, e.g., Jaffee and Russell, 1976, and Mankiw,

⁵ Greenwald and Stiglitz (1988, p.10) give some economic justifications for this assumption.

1986), whilst when lenders can distinguish between borrowers (as in the model presented in this section), the mark-up should compensate for each borrower's rate of default (see, e.g., Greenwald and Stiglitz, 1988, and Bernanke and Gertler, 1990).

Before solving for q_t it would be useful to extend the basic model in order to allow for some kind of informational imperfections. This is what we will do in the next sections, where we develop the model sketched in the Appendix by Greenwald and Stiglitz (1990a, pp. 34-5).

3. Heterogeneous Firms

We now include in the basic model an additive productivity factor, θ_t , which is unobservable to outside investors, but known with certainty by a firm's managers and – in a first stage – by banks. One can also interpret θ_t as a net cash flow, describing the 'quality' or 'value' of a particular firm associated with existing operations (Greenwald, Stiglitz and Weiss, 1984, p. 196). At the beginning of each period, each firm receives an independent θ_t drawn from a distribution that is the same to all firms, with a range $[\theta_x, \theta_y]$ and $E(\theta_t) = 0$. Thus, at the beginning of t , a firm knows $p_k, q_t, a_{t-1}, \theta_t$ with certainty, as well as $F(\tilde{p}_t)$ (but not the realised value of \tilde{p}_t).

With this new feature, the end-of-period value of the firm is $\tilde{a}(q_t) = \tilde{p}_t q_t - (1 + \tilde{r}_t) b_t + \theta_t$. The debt incurred by the firm continues to be $b_t = p_k \phi(q_t) - a_{t-1}$, but now bankruptcy occurs if $\tilde{p}_t q_t + \theta_t < (1 + r_t) b_t$.⁶ The level of price at which the firm is just solvent is:

$$(1') \quad \bar{u}_t \equiv \frac{(1 + r_t)(p_k \phi(q_t) - a_{t-1}) - \theta_t}{q_t}.$$

The problem of the firm becomes:

$$\max q_t - E(1 + \tilde{r}_t) \cdot (p_k \phi(q_t) - a_{t-1}) - c q_t F(\bar{u}_t) + \theta_t,$$

subject to:

$$E(1 + \tilde{r}_t) = (1 + \bar{r}_t)$$

and

$$(2') \quad E(1 + \tilde{r}_t) = (1 + r_t) \cdot (1 - F(\bar{u}_t)) + \int_0^{\bar{u}} \frac{q_t \tilde{p}_t + \theta_t}{b_t} dF(\tilde{p}_t),$$

which is equivalent to

$$(5') \quad h \equiv E(1 + \tilde{r}_t) \cdot \frac{\left(p_k \phi(q_t) - a_{t-1} - \frac{\theta_t}{E(1 + \tilde{r}_t)} \right)}{q_t} = \bar{u}_t (1 - F(\bar{u}_t)) + \int_0^{\bar{u}} \tilde{p}_t dF(\tilde{p}_t) \equiv z(\bar{u}_t)$$

The first order condition (for an interior maximum) is:⁷

$$(6) \quad 1 - (1 + \bar{r}) p_k \phi' = \Phi(\bar{u}(q)),$$

where:

$$(7) \quad \Phi(\bar{u}(q)) \equiv cF(\bar{u}(q)) + c q f(\bar{u}(q)) \frac{d\bar{u}(q)}{dq}.$$

The last equation represents a firm's expected marginal bankruptcy cost in period t , which equals the expected average bankruptcy cost, holding output q fixed, *plus* the total cost of the change in the probability of bankruptcy due to a change in output. This sum is easily shown to be positive.

3.1 The Optimal Decision Rule for an Equity-Constrained Firm

Using (6) and (7), we can see the optimal investment rule as:

$$(1 - p_k \phi') = \bar{r} p_k \phi' + \Phi,$$

which is to say that output (and capital) must be increased to the point where the expected marginal return product of K equals the expected user cost of capital. This, in turn, equals the standard user cost of capital (in the case of no depreciation and no capital gains, since we have only a one-period horizon⁸) augmented by a 'premium' that takes into account the marginal bankruptcy risk induced by external (debt) financing. Thus, the equity-constrained firm will demand an 'excess' return which will induce a lower level of output (and investment) than in the standard case, with no bankruptcy costs ($\Phi = 0$). This result is valid whatever the level of θ , provided that (3) and (5') are satisfied.

⁶ θ may be interpreted as collateral, since it does not enter the computation of b but helps to determine the default threshold (\bar{u}) and adds to the lender's revenue in case of default (see equations (1') and (2')).

⁷ Henceforth, we suppress the time subscripts for the sake of expositional convenience.

⁸ This is the 'user cost of capital' concept as defined in Jorgenson (1963).

Alternatively we can rewrite the optimal decision rule in order to equate the expected marginal revenue to the expected marginal costs (in present value terms), so that:

$$1 = (1 + \bar{r})p_k \phi' + \Phi .$$

As we will see, the wedge between marginal revenue and standard marginal costs (which translates the impact of bankruptcy risks) is increased by an increase in uncertainty, reducing further output and capital.

3.2 The Equilibrium Level of Output and Comparative Statics

To solve for the equilibrium level of output we will make use of the fact that constraint (5') defines \bar{u} as an implicit function of q . We will look at the constant-returns-to-scale case, in which, with a suitable choice of units, $K \equiv \phi(q) = q$. By deriving equation $h(q) - z(\bar{u}(q)) = 0$ in order to q , we find that \bar{u} is a positive function of q :

$$(8) \quad \frac{d\bar{u}}{dq} = \frac{1}{1-F} \frac{(1+\bar{r})a + \theta}{q^2} > 0$$

The first order condition⁹ can now be written as:

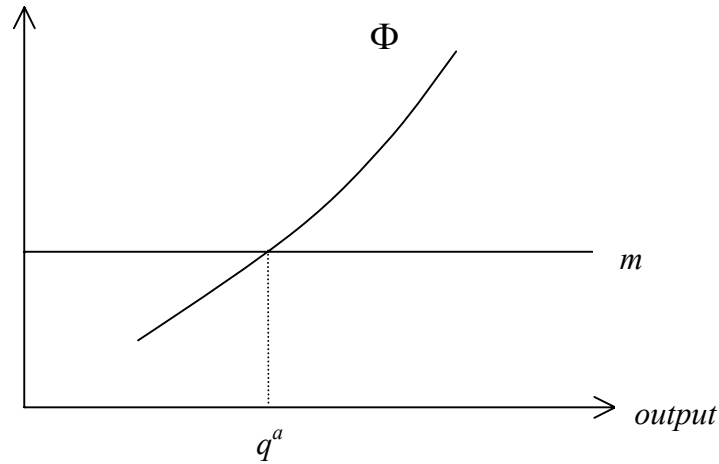
$$(4) \quad m \equiv 1 - (1 + \bar{r})p_k = c \left[F + \frac{f}{1-F} \frac{(1+\bar{r})a + \theta}{q} \right] \equiv \Phi(\bar{u}(q)),$$

where the distribution and density functions are evaluated at \bar{u} (note that m can be seen as the marginal return product to production, ignoring bankruptcy costs; this interpretation parallels those given in Section 3.1 above). Rewriting the above equation gives us the output (investment) function of a typical firm:

$$q = g(p_k, \bar{r}, v, a, \theta),$$

where v represents a measure of riskiness of the distribution F (note that this does not constitute a reduced-form solution for q , since the right-hand side of this equation is a function of \bar{u} through F and f). Importantly, it is easy to see that, under the current assumptions, q is linear in $\left(a + \frac{\theta}{(1+\bar{r})} \right) \equiv a'$. Making use of equations (1'), (2') and (3), we can solve for the equilibrium level of r , \bar{u} and of course the probability of bankruptcy, $F(\bar{u})$.

Fig. 1 – Determination of firm output level with perfectly informed lenders.



Plotting m and Φ in order to q (the first is a constant, the latter is increasing with q , as long as the second-order condition is satisfied) allows us to construct the graphical solution for q (see figure 1). The simplicity of the structure of the first-order conditions makes the comparative statics analysis fairly simple. Following Greenwald and Stiglitz (1988, pp. 36-37), we focus on three cases.

First, we consider an increase in θ . What impact does it have in q , r and \bar{u} ? As q is linear in a' and the bankruptcy price, \bar{u} , depends on q only through (a'/q) (see (5')) using $\phi(q) = q$, this implies that \bar{u} is independent of a' . The same way, $\Phi(\bar{u})$ depends on q only through (a'/q) and consequently is independent of a' . Now see that $da' = \left(\frac{1}{1+\bar{r}}\right)d\theta$, which is an exogenous quantity, known to the firm. Then, the optimal response will be an equiproportionate increase in q (i.e., $d \ln q / d \ln a' = 1$), leaving \bar{u} and Φ unchanged, as well as r . This means that firms characterised by different values of θ – which are perfectly known to the lenders – face identical levels of contractual interest rates, but the size of the loans varies (a lower q means a lower b , *ceteris paribus*), in order to ensure that the actual expected return equals the expected return required by lenders (\bar{r}).¹⁰

⁹ Some restrictions have to be imposed to ensure that the second-order conditions are satisfied; besides, it must be assumed that the bankruptcy cost c is “sufficiently large” so that there is a finite optimal level of output (see Greenwald and Stiglitz, 1988, pp. 34-6, for derivation).

¹⁰ The general case, for $\phi'' > 0$, can be derived as in Greenwald and Stiglitz (1988, pp. 37-40). The comparative statics properties parallel those described in the text, although now we get $m(q)$ as a

Secondly, we consider an increase in the required rate of return, \bar{r} . This will directly move the m curve down (see equation (9)), while the curve Φ goes up – this happens because, from equation (5'), at any q , \bar{u} increases, which implies that the marginal bankruptcy cost is increased (as long as the second-order condition is satisfied). Thus, q is reduced, while the contractual rate r may be increased (this follows from (1'), if we solve it in order to r ¹¹).

Finally, we consider an increase in uncertainty, defined as a mean-preserving spread in the probability distribution function F .¹² This shifts the values of f and F corresponding to any set of values of the other variables and parameters. If an increase in uncertainty increases the likelihood of bad events through both higher F and higher f , then Φ will unambiguously increase and output (investment) will be reduced. Now, even if F increases but f decreases, Φ will increase (reducing output) as long as the 'hazard rate', $f/(1-F)$, is increased (see equation (9)). Thus, the wedge between marginal revenue and marginal costs (defined in the traditional sense) will indeed be increased by an increase in uncertainty.

4. The Model with Imperfectly Informed Lenders

So far we have assumed that lenders are perfectly informed and risk-neutral. Greenwald and Stiglitz (1990a, p. 37) argue that if we relax the informational assumption and assume instead that banks are not able to distinguish among potential borrowers (i.e., they cannot observe q or θ , neither can they infer θ from the level of firm borrowing), then the contractual rate of interest, r , must be set at the same level for all firms (r is a 'one-size-fits-all' contractual rate). In this section we explore this possibility, modifying accordingly the basic model. Formally, r becomes exogenous to the firm's optimisation problem and $E(\tilde{r})$ becomes endogenously determined. We will hereafter refer to this

decreasing function of q , which implies that an increase (decrease) in a' leads to a less than proportional increase (decrease) in output.

¹¹ In fact, the sign and magnitude of the change in r depends, in general, on the actual values of the parameters and on the precise shape of f and F , and, in particular, on the magnitude of $d\bar{u}/dq$ (the smaller this is, the more likely is that a decrease in q leads to an increase in r).

¹² The concept of mean-preserving spread as a notion of increasing risk is due to Rothschild and Stiglitz (1970). We limit our analysis to the case of a mean-preserving spread in the price distribution about \bar{u} , the bankruptcy price, so that the cumulative probability of bad states is increased by increased uncertainty, $F^*(\bar{u}) = F(\bar{u}) + \nu S(\bar{u})$, with $0 < \nu \leq 1$ (that is, $F^*(\bar{u}) > F(\bar{u})$).

setting as ‘regime II’ (*versus* ‘regime I’, where $E(\tilde{r})$ is exogenously set and r is an endogenous variable).

For a firm ‘endowed’ with a net cash flow θ , the optimisation problem continues to be to:¹³

$$\max q - E(1 + \tilde{r}) \cdot (p_k q - a) - c q F(\bar{u}) + \theta.$$

However, the first order condition is:

$$(10) \quad 1 - E(1 + \tilde{r}) p_k - (p_k q - a) \frac{dE(1 + \tilde{r})}{dq} = c F(\bar{u}(q)) + c q f(\bar{u}(q)) \frac{d\bar{u}(q)}{dq}.$$

Note, also, that \bar{u} is no longer defined as an implicit function of q in equation $h(q) - z(\bar{u}(q)) = 0$ (see (5')). We must resort to the definition of \bar{u} , represented by (1'), but now taking r as exogenous:

$$\bar{u} \equiv (1 + r) p_k - \frac{(1 + r)a + \theta}{q}.$$

4.1 The Optimal Decision Rule under ‘Regime II’ (r exogenous)

By deriving (1') in order to q , we find that \bar{u} continues to be a positive function of q :

$$(11) \quad \frac{d\bar{u}}{dq} = \frac{(1 + r)a + \theta}{q^2} > 0.$$

If we compare this result with (8), we verify that $\frac{d\bar{u}}{dq}$ is no longer ‘geared’ by $1/(1 - F)$.

On the other hand, the expected rate of return to lenders is found from equation (5'). So, by solving it in order to $E(1 + \tilde{r})$, we have now:

$$(12) \quad E(1 + \tilde{r}) = \left(\frac{\theta}{q} + z(\bar{u}) \right) \left(\frac{q}{p_k q - a} \right).$$

In a ‘regime’ of r exogenous, there is no *a priori* reason for this to equal the required return \bar{r} . By deriving in order to q , substituting for (11) and simplifying, we get:

$$(13) \quad \frac{dE(1 + \tilde{r})}{dq} = \frac{1}{b} \left[(1 - F) \frac{(1 + r)a + \theta}{q} - \frac{p_k \theta}{b} + z \cdot \left(1 - \frac{p_k q}{b} \right) \right],$$

¹³ We continue to focus on the constant-return-to-scale case.

with $b = p_k q - a$, as before. This expression will be negative if the following sufficient condition is satisfied: in the right-hand side of (13), inside the square brackets, the third term (which is negative) must be larger than the first term (which is positive) minus $(1-F)(\theta/q)$ (we subtract this last term because, it is easy to see, it is smaller than the second term, which is negative). Proceeding like this we get:

$$a > p_k q - \frac{z q}{(1-F)(1+r)} \Leftrightarrow \frac{p_k q - a}{q} (1+r)(1-F) < z,$$

which, as it happens, is guaranteed *a priori* by the definition of expected return to lenders (see (5')). The first order condition can thus be written as:

$$(14) \quad m^2(q) \equiv 1 - E(1+\tilde{r})p_k = cF + cf \frac{[(1+r)a + \theta]}{q} + \frac{dE(1+\tilde{r})}{dq} (p_k q - a) \equiv \Phi^2(q),$$

where, as before, the distribution and density functions are evaluated at \bar{u} . We must note that, in this case, the ‘wedge’ in the optimal investment rule is smaller than before due to the negative effect of q on $E(1+\tilde{r})$, which, by decreasing the expected (standard) user cost of capital, partially counterbalances the effect of the marginal bankruptcy risk ‘premium’.¹⁴

We can also derive (12) separately in order to the exogenous variables θ and r , holding q fixed (without forgetting the impact through $d\bar{u}$). As it should be expected:

$$(15) \quad \frac{dE(1+\tilde{r})}{d\theta} = \frac{F}{p_k q - a} > 0.$$

$$(16) \quad \frac{dE(1+\tilde{r})}{dr} = (1-F) > 0.$$

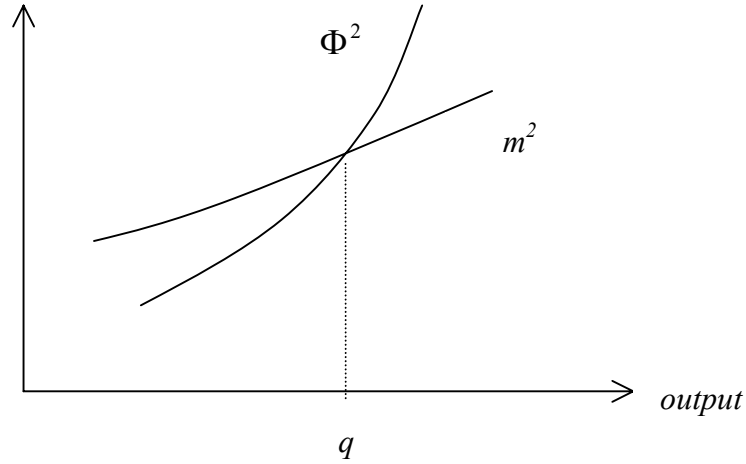
4.2 Graphical Solution and Comparative Statics

It follows from (13) that $m^2(q)$ is an increasing function of q . However, the second-order condition ensures that, at any maximum, the $\Phi^2(q)$ curve is positively sloped and cuts $m^2(q)$ from below.¹⁵ As in Section 3.2 above, plotting m^2 and Φ^2 in order to q allows us to construct the graphical solution for q (see figure 2).

¹⁴ Equation (14) also differs from (9) through the second term in the right-hand side. Yet, it is not possible to tell *a priori* if this accounts for a positive or a negative effect, since $1/(1-F) > 1$ but $\tilde{r} < r$.

¹⁵ The restrictions that have to be imposed to ensure that the second-order conditions are satisfied are clearly more demanding than before, since now both functions are positively sloped. Note also that a

Fig. 2 – Determination of firm output level under ‘regime II’.



Now, imagine that a particular bank is somehow aware of the average θ (denoted hereafter by θ^a) that characterises the pool of potential borrowers, but still cannot observe a particular firm's θ . Or, alternatively, the bank may have a historical knowledge about a pool of ‘old’ customers (firms), indeed characterised by θ^a , but still is not able to distinguish among potential new borrowers. The bank will set the contractual level of interest rate r^a (hypothetically corresponding to an output q^a) by equating $E(\tilde{r}^a) = \bar{r}$. However, this turns out to be true only for those firms with $\theta = \theta^a$ (operating in ‘regime I’). In particular, a firm operating under ‘regime II’, with $\theta < \theta^a$ but $q = q^a$, will face $E(\tilde{r}) < \bar{r}$ (see (15) above) – this amounts to the ‘subsidisation’ of low quality borrowers (and the correspondent ‘taxation’ of high quality borrowers) implied by contract homogeneity which we alluded to in Section 2, above.

Suppose that, in the meantime, due to some exogenous factor, the bank observes an increase in the required rate of return, \bar{r} ,¹⁶ which in turn leads to an increase in the contractual interest rate, r^a (this result was shown in the comparative statics exercise in Section 3.2 above). Then, those firms in ‘regime II’ will view this last movement as an ‘exogenous’ increase in r^a (recall that \bar{r} is not a factor in their optimisation problem). We will illustrate the consequences of these changes by making use of comparative

¹⁶‘sufficiently large’ c is still necessary so that there is a finite optimal level of output. This also guarantees that $\Phi^2 > 0$ (see the Appendix for details).

statics analysis, as in ‘regime I’, seen above. An increment in r^a increases \bar{u} at any q , which reduces the marginal return from production m^2 through $E(1+\tilde{r})$ (see (16) above), while (as long as the second-order condition is satisfied¹⁷) pushing the marginal bankruptcy cost Φ^2 up. It should be noticed that because m^2 is positively sloped (see figure 2), an upward movement in Φ^2 induces a larger reduction in output (investment) than otherwise. However, the downward movement in m^2 may in fact be smaller than that of m (the former corresponds to a fraction $p_k(1-F)$ of dr while the latter corresponds to $p_k d\bar{r}$), while the upward movement in Φ^2 is partially dampened by the countervailing effect on $dE(1+\tilde{r})/dq$ (which did not happen with Φ as defined by (7)).¹⁸ Intuitively, we can say that as \bar{r} and r^a increase, q^a (for firms with $\theta = \theta^a$, under ‘regime I’) tends to decrease more than q (for firms with $\theta < \theta^a$, under ‘regime II’). Under these circumstances, the pool of borrowers changes, with a bias towards firms under ‘regime II’, which, as we have seen, do not necessarily guarantee the lender the required expected return (i.e., $E(1+\tilde{r}_i) = (1+\bar{r}_i)$ may not be satisfied).

Now, consider an increase in uncertainty, again defined as a mean-preserving spread in the probability distribution function F . As before, we assume that the cumulative probability of bad states is increased by increased uncertainty, thus we have $dF > 0$ for every value of the other parameters (see Section 3.2, fn. 12, above). Nevertheless, the density function, f , may decrease or increase. From equation (12) it follows that:¹⁹

$$(17) \quad \frac{dE(1+\tilde{r})}{dF} = \frac{dE(1+\tilde{r})}{dz} \frac{dz}{dF} = -\frac{q}{b} \bar{u} < 0.$$

Thus, m^2 is increased at any q . However, the impact over the marginal bankruptcy cost, Φ^2 , is ambiguous. Recall from (14) that:

$$\Phi^2(q) \equiv cF + cf \frac{[(1+r)a + \theta]}{q} + \frac{dE(1+\tilde{r})}{dq} (p_k q - a).$$

¹⁶ This may be due, for instance, to an increase in the opportunity cost of loanable funds.

¹⁷ The second-order condition is sufficient for $d\Phi^2/dr > 0$ to be true.

¹⁸ It is easily shown that $d^2E(1+\tilde{r})/dq dr = -f[(1+r)a + \theta]/q^2 < 0$.

¹⁹ We made use of the result $z(\bar{u}) = \bar{u} - \int_0^{\bar{u}} F(\tilde{p}) d\tilde{p}$, obtained by integrating $z(\bar{u})$, in (5’), by parts.

The first term increases, by assumption, but we do not know what happens with f (note that an ‘increasing hazard rate’, $f/(1-F)$, will no longer be a sufficient condition – nor necessary, for that matter – to ensure an upward movement in Φ^2). Furthermore, the third term will decrease with increased uncertainty, again holding q fixed, since:

$$(18) \quad \frac{d}{dF} \left(\frac{dE(1+\tilde{r})}{dq} \right) = -\frac{\theta}{bq} \left(1 + \frac{a}{b} \right) < 0.$$

This particular result shows that an environment of increased uncertainty reinforces the negative effect of $\frac{dE(1+\tilde{r})}{dq}$ on the ‘wedge’ between marginal revenue and marginal costs in the traditional sense. However, the overall effect on marginal bankruptcy costs depends on the behaviour of F and f . In any case, even if we assume that an increment in uncertainty increases the likelihood of bad events, increasing both F and f in a such a way that the Φ^2 curve moves upward,²⁰ we can conclude that ‘regime II’ is very likely characterised by a smaller reduction – if not an increase – in output than ‘regime I’ (remember, from Section 3.2, that in that case output unambiguously decreases following an increase in uncertainty). The main difference rests on the induced tendency to observe, in ‘regime II’, an increase in output due to the ensuing negative effect on the expected interest rate; as it happens, this effect is stronger when uncertainty is higher.²¹

Note that an environment of increased uncertainty may also make q , the output of a firm in ‘regime II’, less responsive to changes in the contractual interest rate r^a . See, for instance, that the downward movement in m^2 is less pronounced when F increases (from equation (16) we see that $d^2E(1+\tilde{r})/dr dF < 0$); besides, if an increased uncertainty corresponds to an increased f , then the impact of an increase in r^a on $\frac{dE(1+\tilde{r})}{dq}$ is exacerbated,²² which in turn reinforces its negative effect on the marginal bankruptcy costs.

Finally, consider a decrease in θ . We see from (15) that $E(1+\tilde{r})$ will decrease and thus the marginal return from production m^2 will move up. At the same time, as long as

²⁰ Here, too, a sufficiently high cost of bankruptcy, c , may provide a sufficient condition.

²¹ For simplicity, we are ignoring the (hypothetical) effect of increased uncertainty on the contractual interest rate via ‘regime I’. This effect would reinforce the impact from increased uncertainty on firms’ decisions in ‘regime II’.

the second-order condition is satisfied,²³ we observe an increment in \bar{u} at any q , which pushes the marginal bankruptcy cost Φ^2 up. However, at least for some range of values of θ , the upward movement in Φ^2 due to a decrease in θ is partially dampened by the countervailing effect on $dE(1+\tilde{r})/dq$ (which did not happen with Φ as defined by (7)).²⁴ It follows from here that ‘regime II’ will be very likely characterised by a smaller reduction in output than ‘regime I’ after a reduction in the ‘quality’ parameter θ (in ‘regime I’, output unambiguously decreases in response to a decrease in θ). Indeed, it may even happen that a ‘poor’ firm (with a low θ) exhibits a higher output – and so a lower expected return to lenders (this follows from (13) and (15), above) – than a ‘good’ firm (with a large θ), given the homogeneous contractual interest rate r^a .

Now, it would be valuable to know how do changes in θ affect the impact of variations in the contractual rate and uncertainty on output (investment). Some straightforward calculations show that, holding q fixed:²⁵

$$(19) \quad \frac{d^2 E(1+\tilde{r})}{dr d\theta} = f \frac{1}{q} > 0,$$

$$(20) \quad \frac{d^2 E(1+\tilde{r})}{dF d\theta} = \frac{1}{p_k q - a} > 0.$$

Thus, think of an increase in r : a smaller θ implies a smaller increase in $E(1+\tilde{r})$ and so a smaller downward movement in m^2 . Now in the case of an increase in uncertainty: a smaller θ implies a larger decrease in $E(1+\tilde{r})$ and so a larger upward movement in m^2 .

As far as movements in Φ^2 are concerned, the results are not so clear-cut. We find that:

$$(21) \quad \frac{d^2 \Phi^2}{dr d\theta} = (cf' - f) \frac{b}{q^2} a - cf' \frac{p_k}{q} + (f' - cf'') \frac{b}{q} \frac{d\bar{u}}{dq}.$$

If this is positive, changes in the contractual rate will imply smaller movements in Φ^2 for smaller θ . Remember that, from the second-order condition, $cf' - f > 0$ and

²² It can be shown that the derivative of $d^2 E(1+\tilde{r})/dq dr$ in order to f is negative.

²³ The second-order condition is sufficient for $d\Phi^2/d\theta < 0$ to be true.

²⁴ It can be shown that $d^2 E(1+\tilde{r})/dq d\theta > 0$ for sufficiently large values of θ .

²⁵ We suppress the superscript from the r variable for expositional convenience.

$f' > 0$; for (21) to be positive we must also assume that $f'' < 0$, and that the values of θ (and thus $d\bar{u}/dq$; see (11) above) are ‘sufficiently large’ (note that this was also a sufficient condition for $d^2E(1+\tilde{r})/dq d\theta > 0$ to be true; see above the comparative statics analysis for θ). Finally:

$$(22) \quad \frac{d^2\Phi^2}{dv d\theta} = c \frac{df}{dv} \frac{1}{q} - \frac{p_k}{b},$$

where dv represents the change in the probability distribution. A ‘large’ value of c will be a sufficient condition for (22) to be positive, but of course only if we assume that f increases with increased uncertainty (i.e., if uncertainty increases the likelihood of bad events). In this case, changes in the degree of uncertainty will imply smaller movements in Φ^2 for smaller θ .²⁶

5. Synthesis of Results

The first model analysed above, where firms have limited access to the equity market but not to the debt market, is characterised by rather straightforward results. Both lower levels of current cash flow from existing operations and higher uncertainty over output prices lead to lower levels of production and investment. These variables do not affect investment when financial markets are perfect, but play an important role in this context of imperfect financial markets due to their impact on the expected marginal bankruptcy cost. In turn, a decrease in the banks’ required return leads to an increase in production and investment; in this case, two channels operate: the ‘conventional’ impact on the standard user cost of capital and the impact on the marginal bankruptcy cost, which reinforces the ‘conventional’ effect.²⁷

The model of an equity-constrained firm and imperfectly informed lenders is mainly characterised by two sets of results. First, an increase in the contractual level of interest rate r^a (after an increase in the lenders’ required rate of return \bar{r}) leads to a decrease in output q (i.e., for firms in ‘regime II’, where asymmetric information prevails), but less pronounced than in q^a (for firms in ‘regime I’); in parallel, there is an increase in the

²⁶ Note that changes in θ also affect the slope of both m^2 and Φ^2 curves. However, the precise way they are affected depends on the actual values of the several parameters.

²⁷ It is easy to show that a similar result applies to the price of capital goods.

lenders' expected rate of return, $E(\tilde{r})$. In this sense, the 'voluntary' reduction of firms' borrowing activity (i.e., a change in the levels of q and b chosen by the firm) as a response to increased contractual interest rates is weakened. Yet more important, in 'regime II', firms characterised by a lower net cash-flow from existing operations, θ , and thus very likely with a lower $E(\tilde{r})$ ('poor' firms) experience a smaller decrease in output (investment) than firms with a larger θ and a higher $E(\tilde{r})$ ('good' firms). This effect may indeed get stronger as r^a increases,²⁸ at the same time, it tends to be less likely when the levels of θ are globally small.

Thus, we can say that, *under asymmetric information concerning firms' prospects* (i.e., θ and q), *lenders face a negative adverse selection effect when they decide to increase the level of contractual interest rates*: there is an induced tendency for 'poor' firms to borrow more than 'good' firms (for a given level of inherited equity a and of cost c), because they face lower expected interest costs for any given contractual interest rate r^a – in other words, their high expected default rates are not matched by comparably high contractual interest rates (recall that r^a is assumed the same for the two types).

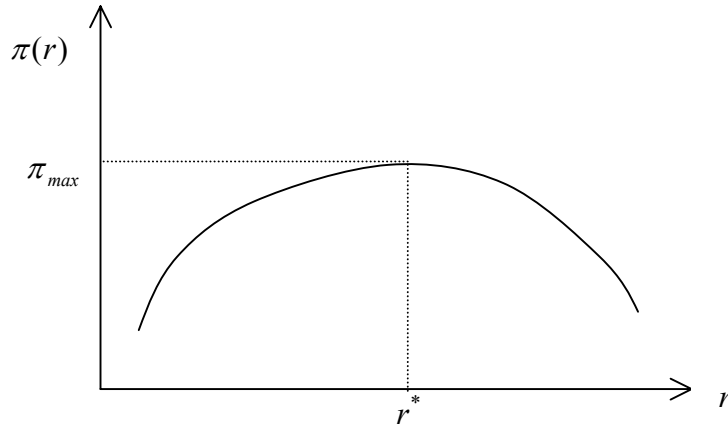
As in Stiglitz and Weiss (1981), the banks – under some circumstances²⁹ – may face, after a point, decreasing expected returns on loans, because the cost of the deterioration in the borrowers pool outweighs the direct gains from higher contractual rates. Nevertheless, in the present model, this change in the quality mix comes about through changes in the relative size of loans, while in the model in Stiglitz and Weiss (1981), as well as in Greenwald and Stiglitz (1990a, Section 1.2), we observe a change in the mix of applicants – the amount borrowed for each project/firm is assumed identical and projects indivisible. Likewise, banks may apply credit rationing as a way to eschew adverse selection inherent to changes in contractual interest rates and its negative effects on expected returns on loans. To see this more clearly, suppose that $\pi(r)$ is the mean rate of return to the bank from its set of borrowers at the contractual interest rate r , so that:

²⁸ The derivative of (19) in order to r is positive, provided that $f' > 0$.

²⁹ As we have seen, adequately large values for c and θ are required (as sufficient conditions) in our model.

$$\pi(r) \equiv \int_{\theta_x}^{\theta_y} \tilde{r}^e(\theta, r, q(\theta, r)) dG(\theta),$$

where $G(\theta)$ is the probability distribution of firms by ‘quality’ θ , with a range $[\theta_x, \theta_y]$, and $\tilde{r}^e \equiv E(\tilde{r})$ is defined by equations (12) and (1’) above (for simplicity, we assume that q is always strictly positive, whatever the value of θ ³⁰). Drawing from Stiglitz and Weiss’ work, we conclude (though do not prove) that, with adverse selection, the bank may face $d\pi(r)/dr < 0$ for some value of r . If this would be the case, then there will be an interest rate that maximises the bank’s expected return on its set of loans (see the figure below).



We assume, as in the model with perfectly informed lenders (see Section 2, above), that banks are competitive and have access to elastic supply of funds, at a cost of \bar{r} . Thus, in equilibrium, the banks’ required rate of return on loans will be \bar{r} . However, since banks are imperfectly informed about the borrowers’ type, the expected break-even condition is given by $\bar{r} = \pi(r)$, so that the cost of funds equals the bank’s expected return from its *set* of loans (instead of the expected return from a loan $E(\tilde{r})$, as when lenders are perfectly informed). Solving for r gives the level of the contractual interest rate that compensates for the perceived *average* rate of default of the pool of borrowers (at the end of Section 2, above, we have contrasted this result with the one with perfectly informed lenders). In this setting, the level of the contractual rate will be r^*

³⁰ Otherwise we would have to calculate the expected value $\pi(r)$ conditional on $1 - G(\theta^*)$, where θ^* is the critical value for which a firm’s expected profit is zero (and is indifferent between a strictly positive production q and no production), and thus below which the firm does not apply for loans.

only if $\bar{r} \equiv \pi_{max}$. Alternatively, Stiglitz and Weiss (1981) assume banks choose the level of the contractual rate in order to maximise the expected return on their set of loans (given the perceived *average* rate of default of the pool of borrowers); i.e., they set $r \equiv r^*$ to obtain $\pi(r^*) = \pi_{max}$. If banks compete freely for depositors, then π_{max} is the interest rate received by depositors and banks' net returns are zero. Notice that either in the model presented by Stiglitz and Weiss (1981) and in the one we study in Section 4, the contractual rate r is endogenous to the bank's optimisation problem but it is exogenous to each borrower's optimisation problem.

If, at interest rate r^* , there is an excess demand for loanable funds, there are no competitive forces leading supply to equate demand; borrowers would not receive a larger loan (if any at all) even if they offered to pay a higher interest rate, thus credit is rationed in equilibrium. Note that, in this case, credit rationing could take the form of restrictions on loan size – just like in Jaffee and Russell (1979), although the mechanism at work is quite different.³¹ Stiglitz and Weiss (1981) formally show how in equilibrium a loan market may be characterised by credit rationing, being $d\pi(r)/dr < 0$ for some value of r a sufficient condition for this result; however, in their model credit rationing is solely defined as an exclusion of potential borrowers.³²

The second set of results concerns changes in the level of uncertainty and their effect on firms' production and investment decisions. We have seen that an increase in the degree of uncertainty results in an increase in q ('regime II') or, alternatively, in a smaller decrease in q than the one observed by q^a ('regime I'). Indeed, in 'regime II', increased uncertainty may not leverage the 'wedge' (the marginal bankruptcy risk 'premium') in the firm's optimal investment rule, as it clearly happened in 'regime I' of perfectly informed lenders, where it depressed the firm's production and investment.

³¹ In their model, the adverse selection problem that arises from the fact that lenders cannot distinguish between the two types of borrowers ('honest' and 'dishonest') on an *a priori* basis implies a negative effect on the 'honest' borrowers' utility; this effect induces 'honest' borrowers to choose contracts with smaller loan sizes. The lenders 'adjust' the size of the loans they offer, given the 'honest' borrowers' preferences.

³² Stiglitz and Weiss (1981, p. 399) show that the expression for $d\pi(r)/dr$ comprises two terms of opposing signs. The first term is negative and represents the change in the mix of applicants, whereas the second term is positive and represents the increase in bank's returns, holding the applicant pool fixed, from raising the interest charges. The first term is large if, for example, a small change in the contractual interest rate induces a large change in the applicant pool, i.e., if $g(\theta^*)/(1-G(\theta^*))(d\theta^*/dr)$ is large (where θ^* denotes the critical value for which a firm's expected profit is zero; see fn. 30).

And even in case it does, the effect will tend to be smaller than in ‘regime I’. In this sense, the ‘voluntary’ reduction of firms’ borrowing activity (i.e., a change in the levels of q and b chosen by the firm) as a response to increased uncertainty is weakened. Furthermore, in a context of increased uncertainty, the ‘voluntary’ change in firms’ borrowing activity as a response to changes in other exogenous variables, say contractual interest rates, is weakened further. *In other words, the combined effect of imperfectly informed lenders over borrowers’ prospects and increased uncertainty counterbalances the negative impact of bankruptcy risks on investment decisions that arises in a context of imperfect equity markets.*

On the other hand, the effect of increased uncertainty described above for ‘regime II’ is stronger for the ‘poor’ firms. We also observe that, comparatively with ‘good’ firms, their $E(\tilde{r})$ is more likely to decrease in response to an increase in uncertainty. Hence, an environment of increased uncertainty may exacerbate the ‘adverse selection effect’ through its impact on the relative size of loans from ‘poor’ and ‘good’ firms; the interest rate which maximises the bank’s expected return on his set of loans (if that rate exists) may then be lower than otherwise, leading to a stronger credit rationing result. Intuitively, we can conclude that *the more the mechanism of ‘voluntary’ reduction of ‘poor’ firms’ borrowing activity is weakened (vis-à-vis ‘good’ firms’), the more likely is the adverse selection effect, and thus the credit rationing equilibrium* (this is the same reasoning that applies to changes in the contractual interest rate).

Greenwald and Stiglitz (1990a, pp. 23-24) refer to the negative impact of increased uncertainty on the quality of the borrowers pool when lenders are imperfectly informed, although under different assumptions. In their model (just as in Stiglitz and Weiss, 1981), the amount borrowed by each firm is assumed identical and projects are indivisible, i.e. q and b are fixed. In this case, increased uncertainty unambiguously implies a decrease in the expected rate of return from a loan, $E(\tilde{r})$, whatever the borrower type (this result can be seen in (17) above, which is calculated assuming that q is fixed). Hence, the mean rate of return to the bank from his set of borrowers is decreased at any level of the contractual interest rate. In our model, the result is not so clear-cut, because increased uncertainty also implies a change in q . Depending on the values of parameters (namely θ), this change will be upwards or downwards: if q increases, comparative statics tells us that $E(\tilde{r})$ will certainly decrease; however, if q

decreases, $E(\tilde{r})$ may increase or decrease.³³ Instead, as seen above, in our model the main impact of increased uncertainty on the bank's mean return from loans comes about through changes in the quality mix of borrowers (i.e., changes in the relative size of loans from 'poor' and 'good' firms), holding the contractual interest rates fixed.

6. Conclusion

By means of a simple model based on Greenwald and Stiglitz (1988, 1990a), we have analysed how both an explicit premium on external finance and credit rationing may arise when financial markets are characterised by asymmetric information problems, such as adverse selection.

In a first stage (Section 3), the central role of informational imperfections has been simply to restrict a firm's ability to raise equity funds in capital markets, while lenders in the debt market (i.e., banks) have been assumed to be perfectly informed. In this setting, the premium on external financing emerges in the form of an expected bankruptcy cost, as perceived by the firm's managers, who are assumed to find the 'bankruptcy state' costly to themselves. The firm adds the expected bankruptcy cost to the standard user cost of capital in order to compute its optimal decision rule of investment; the higher total cost of the marginal unit of investment induces the firm to choose a lower level of output and investment than in the standard case, with no bankruptcy costs.

In a second stage (Section 4), we have explicitly considered asymmetric information in the model of equity-constrained firms, so that banks are not able to distinguish among potential borrowers ('good' *versus* 'poor' firms); this implies that the contractual rate of interest will be set at the same level for all firms. Importantly, unlike other authors, we have done this without constraining firms' projects to a common fixed size – i.e., the size of the project of investment continues to be the choice variable in each firm's optimisation problem. In this context, we have shown that the existence of a 'one-size-fits-all' interest rate weakens the impact of the bankruptcy cost on investment decisions. This happens because firms now compute their optimal decision rules taking into account that higher expected default rates may not be matched by comparably high

³³ The positive impact of a lower q on the expected rate of return from a loan counterbalances the

contractual interest rates. This counterbalancing effect weakens the ‘voluntary’ reduction of firms’ borrowing activity as a response, for instance, to increased uncertainty or increased contractual interest rates and, importantly, it becomes more intense as firms’ quality gets poorer (in our model, as firms have lower current cash flows). But, then, this means that banks face a negative adverse selection effect when they decide to increase the level of contractual interest rates. We have seen that, under some circumstances, this may induce banks to apply credit rationing as a way to eschew adverse selection inherent to changes in contractual interest rates and the negative effects on banks’ expected return from loans associated with it. An environment of increased uncertainty will display a credit rationing equilibrium with higher likelihood.

To sum up, the model illustrates how the existence of bankruptcy costs implies ‘deadweight losses’ on investment through the reduction of the firms’ ability to raise external financing. Whether this arises predominantly in the form of credit rationing or the ‘voluntary’ reduction of firms’ borrowing activity (i.e., the firm chooses a lower level of output and investment in response to the premium on external finance) is a matter of which types of asymmetric information we assume to exist.

The natural follow-up to this work would be to *formally* show how a loan market where both contractual interest rates *and* loan sizes are (*a priori*) variable may be characterised by a credit rationing equilibrium. In parallel, it might be of interest to perform some empirical research in order to assess the practical relevance of symmetric information (and its different forms) in the equity and debt market. This may also constitute a relevant issue for monetary policy purposes.

On the other hand, since our comparative statics results often depend on the precise values of the parameters considered, it would be desirable to perform some calibration exercises in order to compute numerical solutions. These would help to illustrate the results more clearly and show how they depend on the values of the various parameters.

Finally, we note that the analytical tractability of our model depends a great deal on the assumptions of constant returns to scale and a single-period time horizon. However, they may not be innocuous to some of our results. Thus, an objective for future research would be to move towards a model of imperfectly informed lenders where firms optimise over a multiperiod horizon and face decreasing returns to scale.

negative direct impact of increased uncertainty.

Appendix

Existence of a Finite Solution to the Firm's Maximisation Problem

We follow the same reasoning of Greenwald and Stiglitz (1988, pp. 34-5). First note that as q increases towards infinite, \bar{u} tends toward $(1+r)p_k \equiv \bar{u}_0$ (see the definition of \bar{u} in (1'), using $\phi(q) = q$). Thus, as q goes toward infinity, the probability of bankruptcy $F(\bar{u})$ approaches a finite limit $F(\bar{u}_0) \equiv F_0$, while $f(\bar{u})$ approaches $f(\bar{u}_0) \equiv f_0$. Likewise, it is easy to see that, in (4.14), as q goes toward infinity:

$$E(1+\tilde{r})p_k \rightarrow z(\bar{u}_0) \quad \text{and} \quad \Phi^2 \rightarrow cF_0.$$

These results mean that the first derivative of the firm's objective function in order to q approaches $1 - z(\bar{u}_0) - cF_0$ as q increases toward infinity. Note that unless this is negative, the firm's objective function can be increased without bound. Thus, we will assume that c is sufficiently large so that $1 - z(\bar{u}_0) - cF_0 < 0$ is satisfied and consequently that there is a finite optimal level of output. In 'regime I', the condition that must be satisfied is $1 - (1+\bar{r})p_k - cF_0 < 0$ (Greenwald and Stiglitz, 1988, p. 35); versus 'regime II', this condition requires a not so large bankruptcy cost c when $(1+\bar{r})$ exceeds the *limit* of $E(1+\tilde{r})$, which is more likely for lower levels of contractual rate r .

Second-Order Condition

With a constant-returns-to-scale technology, the second-order condition for firms under 'regime II' is:

$$\left(\frac{(1+r)a + \theta}{q^2} \right)^2 q(f - cf') < 0,$$

where f' is the first derivative of the density function f evaluated at \bar{u} , the optimal bankruptcy point (we assume that F is sufficiently smooth so that it is twice differentiable at the optimal level of output).

The restrictions that have to be imposed to ensure that the second order conditions are satisfied are clearly more demanding than before. Indeed, at the optimal level of output the second-order condition implies in 'regime I' that $f' > -f^2/(1-F)$ (Greenwald and Stiglitz, 1988, p. 35). In 'regime II', we must have $f' > f/c$. Note

that the larger the bankruptcy cost c is, the less restricting the second-order condition becomes on the slope of f ; but it will have to be non-negative in any case (which did not happen in ‘regime I’). Notice that if firms operate with bankruptcy levels in the lower tail of the price distribution, and if that distribution is single peaked, then f' will be positive at the relevant levels of output (i.e., at the levels of q compatible with \bar{u} in the lower tail of F ; see Appendix 2, above).

References

- Aizenman, J. and Powell, A. (1997) “Volatility and financial intermediation”, NBER Working Paper 6320, Cambridge, Mass.
- Bernanke, B. and Gertler, M. (1990) “Financial fragility and economic performance”, *Quarterly Journal of Economics* (Feb.), 87-114.
- Greenwald, B. and Stiglitz, J. (1988) “Financial market imperfections and business cycles”, NBER Working Paper 2494, Cambridge, Mass.
- Greenwald, B. and Stiglitz, J. (1990a) “Macroeconomic models with equity and credit rationing”, *Asymmetric Information, Corporate Finance and Investment*, ed. Glenn Hubbard, The University of Chicago Press, NBER Series, 15-42.
- Greenwald, B. and Stiglitz, J. (1990b) “Asymmetric information and the new theory of the firm: financial constraints and risk behaviour”, NBER Working Paper 3359, Cambridge, Mass.
- Greenwald, B. and Stiglitz, J. and Weiss, A. (1984) “Informational imperfections in the capital market and macroeconomic fluctuations”, *American Economic Review Papers and Proceedings*, 74, 194-99.
- Hirshleifer, J. and Riley, J. (1992) *The analytics of uncertainty and information*, Cambridge University Press.
- Jaffee, D. and Russell, T. (1976) “Imperfect information, uncertainty and credit rationing”, *Quarterly Journal of Economics*, 90, 651-666.
- Jorgenson, D. (1963) “Capital theory and investment behavior”, *American Economic Review Papers and Proceedings*, 53 (May), 247-259.

- Mankiw, N. G. (1986) "The allocation of credit and financial collapse", *Quarterly Journal of Economics*, 101, 455-470.
- Rothschild, M. and Stiglitz, J. (1970) "Increasing risk: I. A definition.", *Journal of Economic Theory*, 2, 225-243.
- Stiglitz, J. (1992) "Capital markets and economic fluctuations in capitalistic economies", *European Economic Review*, 36, 269-306.
- Stiglitz, J. and Weiss, A. (1981) "Credit rationing in markets with imperfect information", *American Economic Review*, 71 (3), 393-410.
- Williamson, S. (1987) "Costly monitoring, loan contracts and equilibrium credit rationing", *Quarterly Journal of Economics*, 102, 135-146.

