Expected Profitability of Capital under Uncertainty – a Microeconomic Perspective

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ABSTRACT

Hartman (1972) and Abel (1983) showed that when firms are competitive and there is flexibility of labour relative to capital, marginal profitability of capital is a *convex* function of the stochastic variable (e.g., price); by Jensen's inequality, this means that uncertainty increases the expected profitability of capital, which increases the incentive to invest. We argue that, besides factor substitutability, the relevant assumption for the convexity property to hold is the implicit assumption about the choice variable in the representative firm's maximisation problem: the assumption of perfect competition implies that the choice variable is output and that price is exogenous. However, in the case of a firm facing a downward-sloping demand curve, both output and output price emerge as the possible choice variable. We show that, when price is the choice variable, marginal profitability of capital is a *concave* function of the stochastic variable; hence, by Jensen's inequality, an increase in uncertainty decreases the expected profitability of capital. We also show that keeping the assumption of factor substitutability but changing the share of labour in the production function has an important impact on the degree of concavity/convexity of the capital profit function.

Keywords: Expected Profitability; Uncertainty; Jensen's Inequality.

JEL Classification: D21; D24

[*] The author is grateful to Pedro Cosme Vieira and Manuel Luís Costa for their valuable suggestions.

1. INTRODUCTION

Hartman (1972) and Abel (1983) were among the first to establish a theoretical result with respect to the impact of increased uncertainty on the expected profitability of capital of a risk-neutral firm. These authors demonstrated that uncertainty over future prices or input costs can increase the expected present value of the marginal unit of capital, which increases the incentive to invest. By Jensen's inequality,[1] this only requires that the marginal profitability of capital be a *convex* function of the stochastic variable (prices or costs); convexity is ensured by the flexibility of labour relative to capital, where the latter is chosen knowing only the probability distribution of the stochastic variable and the former is chosen after the realisation of its actual value.

In contrast, other microeconomic theories, such as the theory of irreversible investment, predict that the profit generated by the marginal unit of capital is a *concave* function of the stochastic variable, in which case an increase in uncertainty will decrease the expected marginal profitability of capital and thus discourage investment (see, e.g., Pindyck, 1988).

Leahy and Whited (1995) performed empirical testing both on the sign of the investment - uncertainty relationship and on its alternative channels and found no evidence for a positive correlation between investment and uncertainty, namely through the convexity of the marginal profitability of capital, as predicted by the Hartman-Abel theory. Instead, the results in Leahy and Whited (1995) indicated that an increase in uncertainty decreases investment, which led the authors to point to "irreversible investment as the most likely explanation for the observed correlation between investment and uncertainty" (Leahy and Whited, 1995, p. 15).

However, one can show that the Hartman-Abel approach can also produce a *concave* function of the stochastic variable and, hence, by Jensen's inequality, an investment - uncertainty relationship with a negative sign. Note, in the first place, that the models by Hartman (1972) and Abel (1983) were built on the assumption of production technology with flexibility of labour relative to capital, as well as of perfect competition, so that the representative firm faced an infinitely elastic demand curve. It is well known the importance of the assumption of factor substitutability for the convexity of the marginal profitability of capital in output price

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[1] According to Jensen's inequality, from the general theory of choice under uncertainty, greater uncertainty will increase the expected value of an action if the payoff is convex in the random variable and decrease it if the payoff is concave.

and input costs, but how does this interact with the assumption of perfect competition and the degree of price elasticity of demand?

This paper shows that, besides factor substitutability, the relevant assumption for the convexity property to hold is the one concerning the choice variable (price or quantities) in the firm's profit maximisation problem. The assumption of perfect competition in Hartman (1972) and Abel (1983) has it implicit that the choice variable of the representative firm is output and that price is exogenous. However, in the case of a firm facing a downward-sloping demand curve, both output and output price emerge as the possible choice variable. We show that, when price is the choice variable, marginal profitability of capital is a *concave* function of the stochastic variable; hence, by Jensen's inequality, an increase in uncertainty decreases the expected marginal profitability of capital. We also show that keeping the assumption of flexibility of labour relative to capital but changing the share of labour in the production function has an important impact on the degree of concavity/convexity of the profit function.

This paper is organised as follows. Section 2 introduces the representative firm's maximisation problem. Section 3 explores a closed-form solution to the firm's problem that is compatible with both convex and concave profit functions in the stochastic variable. Section 4 elaborates on the economic reasons behind the results obtained in the previous section. Section 5 shows analytically how the assumption concerning the choice variable in the firm's maximisation problem is determinant for the behaviour of the expected marginal profitability of capital when the degree of uncertainty varies. Section 6 analyses the impact of changes in the price elasticity of demand and in the share of labour in the production function on the degree of the concavity/convexity of the profit function. Section 7 concludes.

2. THE FIRM'S OPTIMISATION PROBLEM

Let us consider a risk-neutral firm and its investment decisions in a context of variable capacity of production and where uncertainty affects demand conditions faced by the firm. Our exposition is embedded in a dynamic programming approach.

We assume that the firm produces output at time t using perfectly variable factors of production and its capital stock, K_t and that the firm sells all of its output, Q_t. The firm retains some pricing power, in the sense that the output price, P_t, is determined by a downward-sloping demand curve. The position of the demand curve depends on the value of the stochastic variable X_t. Since the model is developed in a continuous-time and infinite-

time horizon framework, henceforth we omit the time subscripts.[2] Thus, the (inverse) demand function can be defined by:

$$P = D(X,Q) \tag{1}$$

The demand shock X evolves exogenously accordingly to the following geometric Brownian motion:

$$dX = \alpha\, X\, dt + \sigma\, X\, dz \ , \ \ \sigma > 0 \tag{2}$$

where αX is the expected instantaneous drift rate and $(\sigma X)^2$ is the instantaneous variance rate of the stochastic process; dz is the increment of a Wiener process.[3] An important property of a Brownian motion is that it generates continuous but non-differentiable (with respect to time) paths; see Dixit and Pindyck (1994, p. 70) for a formal proof. The current value of the demand shock is known (the firm observes X changing), but its future values are always uncertain – the firm only knows its distribution of probability. We assume the firm has rational expectations about the underlying stochastic process, so that the firm's decisions are optimal given (2). The operating profit of the firm, i.e., revenues minus the cost of the perfectly variable factors of production, is:

$$\pi = H(K,X) \tag{3}$$

where π is assumed to account for whatever optimisation the firm can do at every instant on dimensions other than its choice of K, given the level of X. Thus, we can regard π as the outcome of an instantaneous optimisation problem.

Given the initial capital stock K and the initial level of the stochastic demand shock X, the firm wants to choose the path of its stock of capital in order to maximise the expected present value of its cash flows, that is, its operating profit less the cost of investing (the cost of purchasing capital), over an infinite horizon. The firm is risk-neutral and discounts future cash flows at the constant positive rate r, with $r > \alpha$ – otherwise, since X grows exponentially at a deterministic rate α, waiting longer would always be a better policy and the optimum would not exist. Therefore, the maximised value of the firm's objective function is:

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[2] If there is no fixed time horizon for the decision problem, dynamic programming obtains a recursive structure and the calendar date t no longer matters (see, for details, Dixit and Pindyck, 1994, p. 101).

[3] The increment of a Wiener process in continuous time can be represented as $dz = \varepsilon_t \sqrt{dt}$, where ε_t is a normally distributed random variable with a mean of zero and a standard deviation of one. It follows that the Wiener process has $E(dz) = 0$ and $Var(dz) = dt$.

$$V(K,X) \equiv \max_{dK} E\left\{\int_0^\infty e^{-rt}\left[H(K(t),X(t))dt - \kappa\,dK(t)\right]\right\}$$

where κ is the price of a unit of capital. Since $K(t)$ is not differentiable with respect to time, the last term in the right-hand side is to be interpreted as a Riemman-Stieltjes integral.[4]

3. A CLOSED-FORM SOLUTION TO THE FIRM'S PROBLEM

3.1. Convexity *versus* Concavity of the Profit Function in *X*

We will now assume specific functional forms for demand and production functions, common in the literature. Consider a firm that faces an isoelastic demand curve:

$$Q = X P^{-\varepsilon}, \quad \varepsilon > 1 \tag{4}$$

where Q is the quantity of output demanded and ε is the price elasticity of demand. The firm produces non-storable output Q according to the Cobb-Douglas production function:

$$Q = L^w K^v, \quad w > 0, v > 0, \quad w + v \leq 1 \tag{5}$$

where L is labour, K is the capital stock, w is the labour share and v is the capital share. At every instant the firm chooses L to maximise its operating profits $PQ - WL$, given the levels of K and X; the wage rate W is exogenous and assumed to be constant over time. Notice that K has to be chosen knowing only the probability distribution of X, while L can be chosen after the realisation of its actual value. The instantaneously maximised value of operating profit and marginal operating profit are given, respectively, by:

$$H(K,X) = \frac{C}{\theta} X^\gamma K^\theta \tag{6}$$

$$H_K(K,X) = C X^\gamma K^{\theta-1} \tag{7}$$

where $C = \theta\left(\dfrac{1}{\gamma\varepsilon}\right)^{\gamma\varepsilon} (\gamma\varepsilon - 1)^{\gamma\varepsilon-1} W^{1-\gamma\varepsilon}$ is a positive constant (this expression is based on the calculations by Abel and Eberly, 1995, p. 7) and where, it is easy to show, the elasticity parameters θ and γ depend on ε, v and w as follows:

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[4] Since *X* follows a continuous non-differentiable time path (because it is governed by a Brownian motion) and there are only linear costs to capital adjustment (κ), then *K* will also follow a continuous non-differentiable time path.

$$\theta \equiv \frac{v(\varepsilon - 1)}{\varepsilon(1 - w) + w} \tag{8}$$

$$\gamma \equiv \frac{1}{\varepsilon - w(\varepsilon - 1)} < 1 \tag{9}$$

(γ will take values below unit provided $\varepsilon > 1$). Since the power of X is $\gamma < 1$, we see from (6) and (7) that both $H(K,X)$ and the marginal profit $H_K(K,Y)$ are *concave* in the stochastic variable X. This result is similar to that derived by Abel and Eberly (1995), although they assume a constant-return-to-scale production function.

If instead we choose to represent the demand-side of the model by:[5]

$$P = Q^{-1/\varepsilon} X \tag{10}$$

the optimal instantaneous operating profit $H(K,X)$ and the marginal profit $H_K(K,X)$ continue to be linear functions of X^γ, as in (6) and (7), but with the elasticity parameter given by:

$$\gamma \equiv \frac{\varepsilon}{\varepsilon - w(\varepsilon - 1)} > 1 \tag{11}$$

Therefore, $H(K,X)$ and $H_K(K,X)$ are now *convex* in X. If we let $\varepsilon \to \infty$ (i.e., the perfect competition case), then (11) becomes $\gamma \equiv 1/(1-w)$. This corresponds to the specification in the seminal articles by Hartman (1972) and Abel (1983). One can see that the unambiguous convexity of the marginal profit $H_K(K,X)$ with respect to X derives from the fact that, given Q, the relation between P and X in (10) depends neither on technology (supply) nor on demand parameters.

3.2. The Expected Present Value of the Marginal Operating Profit

We will now calculate the expected present value of $H_K(K,X)$ holding K fixed, which we will represent by Π_K hereafter. Suppose first that $F(X) = X^\gamma$ and recall that $dX = \alpha X \, dt + \sigma X \, dz$.

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[5] An alternative equation is discussed in Appendix A.

Then, applying Ito's Lemma[6], we get:

$$dF = \frac{\partial F}{\partial X} dX + \frac{1}{2} \frac{\partial^2 F}{\partial X^2} (dX)^2 =$$

$$= \frac{\partial F}{\partial X} (\alpha\, X\, dt + \sigma\, X\, dz) + \frac{1}{2} \sigma^2 X^2 \frac{\partial^2 F}{\partial X^2} dt =$$

$$= \left(\gamma\alpha + \frac{1}{2}\gamma(\gamma-1)\sigma^2 \right) F\, dt + \sigma\,\gamma\, F\, dz\,.$$

Notice that since higher-order terms go to zero faster than dt as it becomes infinitesimally small, we ignore them and write $(dX)^2 = \sigma^2 X^2 dt$. Thus, we see that F follows a geometric Brownian motion with variance rate $(\sigma\gamma)^2$ and expected drift rate $\frac{1}{dt} E\left[\frac{dF}{F} \right] \equiv \gamma\alpha + \frac{1}{2}\gamma(\gamma-1)\sigma^2$. The latter is an ordinary differential equation that, as it is easily shown, has the solution:

$$E[F(X,t)] = F(X_0) e^{\left[\gamma\alpha + \frac{1}{2}\gamma(\gamma-1)\,\sigma^2 \right]t} = X^\gamma\, e^{\left[\gamma\alpha + \frac{1}{2}\gamma(\gamma-1)\,\sigma^2 \right]t}.$$

Notice that, given the recursive structure of the infinite horizon problem, we can assume that $X_0 = X$. The expected present value of $F(X)$ is thus:

$$E\left[\int_0^\infty F(X,t)\, e^{-rt} dt \right] = \frac{X^\gamma}{r - \gamma\alpha - \frac{1}{2}\gamma(\gamma-1)\sigma^2}$$

provided the denominator is positive. Then, substituting in (7), we see that $\Pi_K(K,X)$, the expected present value of the flow of marginal profit $H_K(K,X)$, is:

$$\Pi_K(K,X) = \frac{C}{r - \gamma\alpha - \frac{1}{2}\gamma(\gamma-1)\sigma^2} X^\gamma K^{\theta-1} \qquad (12)$$

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[6] Consider a function *F[x, t]* and suppose that *x* follows the process of equation (2). Then, Ito's Lemma, also known as the Fundamental Theorem of stochastic calculus, gives the differential *dF* as: $dF = (\partial F / \partial t)dt + (\partial F / \partial x)dx + (1/2)(\partial^2 F / \partial x^2)(dx)^2$. In an infinite horizon problem this expression loses its first term on the right-hand side (see fn. 2)

4. ECONOMIC REASON BEHIND THE SHAPE OF THE PROFIT FUNCTION

In this section, we try to elucidate the economic reason for the convexity/concavity of the profit function in the stochastic variable.

The rationale for the convexity property is already well established in the literature of microeconomic analysis. If the demand shock is represented as in (10), meaning that it takes the form of changes in the price associated with any given level of output demanded, the firm sees price as an exogenous stochastic variable and thus focuses on its variance; therefore, it will produce more output when the price is high and less when the price is low. As a result, profit will exhibit increasing marginal returns in prices, which is to say the profit function is convex in the stochastic variable. See, e.g., Varian (1993, pp. 42-43) for a formal proof.

However, if the demand shock is represented as in (4), meaning that it comes about as changes in the quantity demanded at any given price, the firm sees quantity demanded as an exogenous stochastic variable and thus focuses on its variance. *As the firm optimally increases L (the instantaneously variable factor), for a given K, to take advantage of the demand shock (higher Q for a given P), the firm runs into the decreasing marginal productivity of factor L. As a result, profit will exhibit decreasing marginal returns in quantity demanded, which is to say the profit function is concave in the stochastic variable.*

A simple verification consists of analysing the role of the elasticity of L in the production function (5). If we set $w = 1$ in each specification of the demand function, meaning that L is characterised by constant marginal productivity, we see that the elasticity of X represented by (9) becomes $\gamma = 1$, i.e., the profit function becomes linear in the stochastic variable. By substituting $\gamma = 1$ in equation (12), we see that, in this case, increased uncertainty has no effect on the expected present value of capital profitability. In contrast, the elasticity of X represented by (11) becomes $\gamma = \varepsilon$, which means that γ continues to be greater than one, i.e., the profit function is convex in the stochastic variable as before (Section 6.1, below, further elaborates on this point).

To conclude, in both cases we assist to the endogenous response of perfectly variable production factors to exogenous demand shocks.[7] Whether that generates a profit function which is convex (with increasing marginal returns) or concave (decreasing marginal returns)

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[7] Recall that in the model presented here, the firm continuously chooses L (the perfectly variable production factor) to maximise its operating profit taking into account, at each point of time, the level of the demand shock variable.

in the stochastic variable depends on the type of demand shock we are assuming: shock on prices or on quantities demanded.

5. THE EFFECT OF INCREASED UNCERTAINTY

The choice between (4) and (10) is not just a normalisation, as Abel and Eberly (1995, p. 15) recognised; it has both qualitative and quantitative importance, namely by determining the sign of the 'Jensen's inequality' effect of uncertainty on the expected marginal profitability of capital.[8] Therefore, we are interested in examining the effects of an increase in uncertainty for each specification of the demand function. In order to study these effects we would like to focus on mean-preserving increases in the variance of the stochastic process.[9]

Let us first consider the demand function in (10), according to which P depends linearly on X. In this case, since (exogenous) demand shocks take the form of changes in the price associated with any given level of output demanded, X is the relevant shock variable. Implicitly, we are assuming that the choice variable of the firm facing uncertain demand is output (Q) and that price is exogenous. We must thus study the effects of an increase in uncertainty (variance) that leaves the expected value of X unchanged, so as to obtain a mean-preserving increase in the variance. Given the stochastic process that governs the demand shock, represented by (2), this increase in uncertainty simply corresponds to an increase in the instantaneous variance rate parameter σ^2 holding the instantaneous drift parameter α fixed. Using equations (10)-(12) above, we derive the effect of σ^2 on the expected present value of the marginal profit, Π_K:

Proposition 1: If the choice variable of the firm is output (Q), meaning that the demand shock is represented as in (10) and $\gamma > 1$, then $\dfrac{d\Pi_K}{d\sigma^2} > 0$.

Proof: Inspect equations (10) and (11) and observe from the definition of Π_K in equation (12) that, when $\gamma > 1$, the denominator on the right-hand side of (12) decreases with a mean-preserving increase in σ^2, which in turn increases Π_K.

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[8] According to equation (12), Π_K depends on both K and X. However, Abel and Eberly (1993, p. 23) show under rather general assumptions that the level of the existing capital stock does not qualitatively affect the impact of uncertainty on investment through the expected marginal profitability of capital.
[9] This is similar but not equal to the well-known Stiglitz and Rothschild's concept of a mean-preserving spread over a probability distribution function. The increased variance criterion for measuring uncertainty is weaker, since it does not require second-order stochastic dominance of probability distribution functions.

This is the 'Jensen's inequality' effect of uncertainty on investment with a positive sign, which results from the convexity of $H_K(K,X)$ in X (i.e., $\gamma > 1$). This effect means that greater uncertainty increases the incentive to investment, since the marginal unit of capital generates a higher expected flow of future (operating) profits, in present value.

Proposition 1 also applies to the case of a competitive firm (see, below, Section 6), as in Hartman (1972) and Abel (1983). Indeed, in this case, the price facing the firm is the natural demand shock variable to focus on, since firms are price-takers and output is the only possible choice variable of the representative firm. Of course, in this context, equation (10) must be seen as the industry-wide demand curve, while the individual competitive firm focuses on mean-preserving increases in the variance of price (remember that P is proportional to X in the inverse demand function in (10), which means the stochastic processes that governs P when Q is fixed is $dP = \alpha P\, dt + \sigma P\, dz$).

Nevertheless, in the case of a firm facing a downward-sloping demand curve, both output (Q) and output price (P) emerge as the possible choice variable of the firm facing uncertain demand. If the choice variable *is P*, then expression (4) should be appropriate to represent the firm's demand curve, because, in this case, (exogenous) demand shocks come about as changes in the quantity demanded at any given price. Since, in (4), Q depends linearly on X, the relevant shock variable is X; thus, the mean-preserving increase in the variance of the stochastic variable is again an increase in σ^2 holding α fixed. Using equations (4), (9) and (12) above, we establish:

Proposition 2: If the choice variable of the firm is output price (P), meaning that the demand shock is represented as in (4) and $\gamma < 1$, then $\dfrac{d\,\Pi_K}{d\sigma^2} < 0$.

Proof: Inspect equations (4) and (9) and observe from the definition of Π_K in equation (12) that, when $\gamma < 1$, the denominator on the right-hand side of (12) increases with a mean-preserving increase in σ^2, which in turn decreases Π_K.

This is the 'Jensen's inequality' effect of uncertainty on investment with a negative sign, which results from the concavity of $H_K(K,X)$ in X (i.e., $\gamma < 1$).[10] This effect means that

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[10] Abel and Eberly (1995) also obtain a marginal profit function concave in the stochastic variable. However, they do not study the 'Jensen's inequality' effect of an increase in uncertainty.

greater uncertainty implies less willingness to invest, since the marginal unit of capital generates a lower expected flow of future (operating) profits, in present value.

6. SENSITIVITY ANALYSIS OF THE ELASTICITY PARAMETER γ

6.1. Sensitivity of γ to ε

Assuming that (4) and (10) are alternative expressions for the firm's demand curve, we analyse now the impact of changes in the price elasticity of demand ε (i.e., the pricing power of the firm) on the concavity/convexity of the profit function, which, as we have seen, is measured by the coefficient γ.

Proposition 3: If $\gamma < 1$, then the profit function becomes more concave in the demand shock X as ε increases.[11]

Proof: From (9) we see that $\dfrac{d\gamma}{d\varepsilon} < 0$.

Proposition 4: If $\gamma > 1$, then the profit function becomes more convex in the demand shock X as ε increases (eventually approaching the perfect competition case, i.e., $\varepsilon \to \infty$).

Proof: From (11) we see that $\dfrac{d\gamma}{d\varepsilon} > 0$

Figure 1, below, illustrates these results. We conclude that the convexity/concavity of the marginal profit function in the stochastic variable does not depend on any given magnitude of the price elasticity of demand (in our model provided that $\varepsilon > 1$), and thus – as far as the response of the discounted value of marginal profits to increased uncertainty is concerned – neither does the sign of the investment-uncertainty relationship. The crucial factor that distinguishes the non-competitive firm case (which we have analysed in the previous sections) from the competitive firm limiting case, (studied by Hartman, 1972, and Abel, 1983) is the way exogenous demand shocks are modelled and the implicit assumption about the choice variable of the firm.

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[11] However, since we are applying (9) only to the non-competitive case, we must have $\varepsilon < \infty$.

Figure 1 - Sensitivity Analysis of γ to ε, with w = 0,6
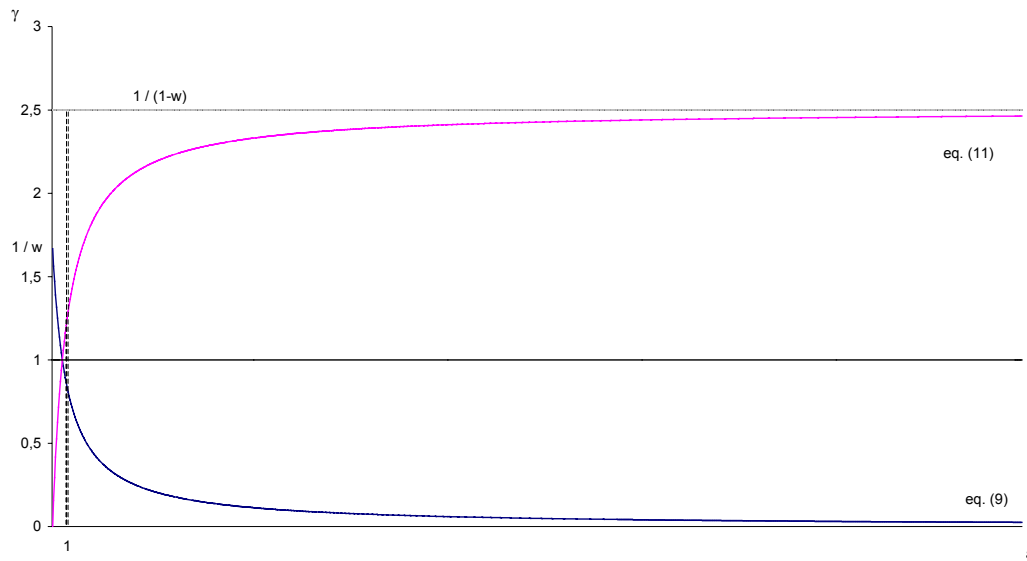
6.2. Sensitivity of γ to w

Notice that the particular values chosen for w and v, the elasticities of labour and capital in the Cobb-Douglas production function, do not alter the results described in Section 6.1, since we are assuming $0 > w > 1$; $0 > v > 1$. Nevertheless, the specific value taken by w, which may be seen as the contribution of labour to production, will be relevant for the degree of concavity/convexity of the profit function in the stochastic variable and thus for its sensitivity to changes in uncertainty.

As shown in figure 2, below, when the firm faces a production technology with w assuming a value near zero, γ (eq. 9) rapidly approaches zero as the elasticity-price of demand ε grows, whereas γ (eq. 11) rapidly stabilises just above unity. When the former is true, we fall back into the deterministic case – the position of the profit function does not depend on the stochastic variable; the latter means that, being the profit function almost linear in the stochastic variable, the expected present value of marginal profitability of capital is rather insensitive to changes in the degree of uncertainty. In both cases, what happens is that *the effect of the endogenous response of labour (the perfectly variable production factor) to exogenous demand shocks is dampened by its very low contribution to production (i.e., the very small w)* (see Section 5, above). Figures 4 and 5 in Appendix B further illustrate these results, by comparing the sensitivity of the expected marginal profitability of capital to changes in the degree of uncertainty for selected values of γ.

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Figure 2 - Sensitivity Analysis of γ to ε, with w = 0,1
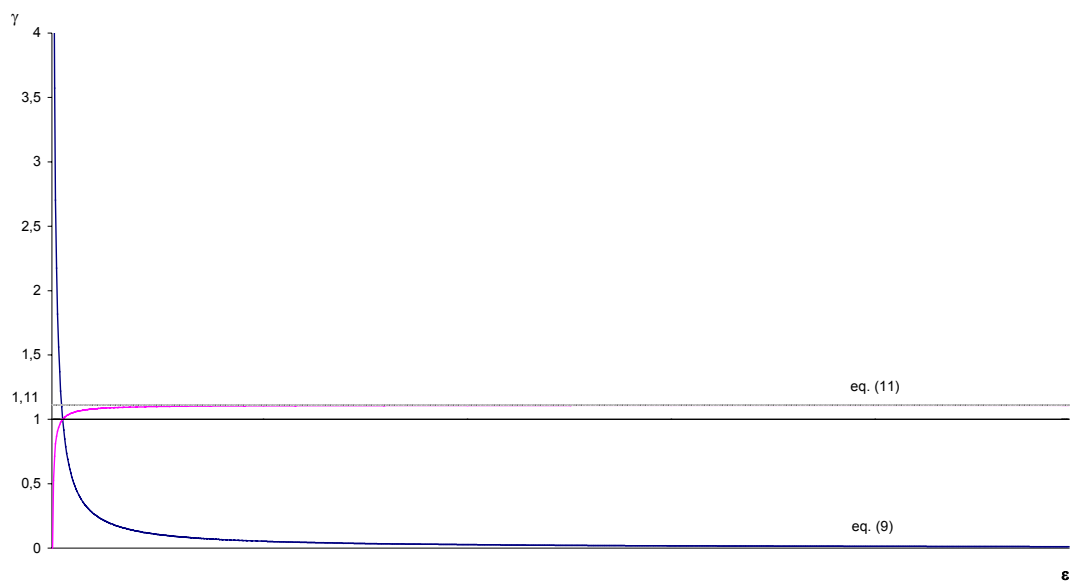
However, as shown in figure 3, below, if $w = 1$, then γ (eq. 9) equals one for every value of ε, whereas γ (eq. 11) grows linearly with ε – this is the case we have referred to above, in Section 5. Note also that, for γ (eq. 11), the higher the share of labour in the production function, the grater is the convexity of the profit function, for a given ε; this result is implicit in the model by Abel (1983).

Figure 3 - Sensitivity Analysis of γ to ε, with w=1

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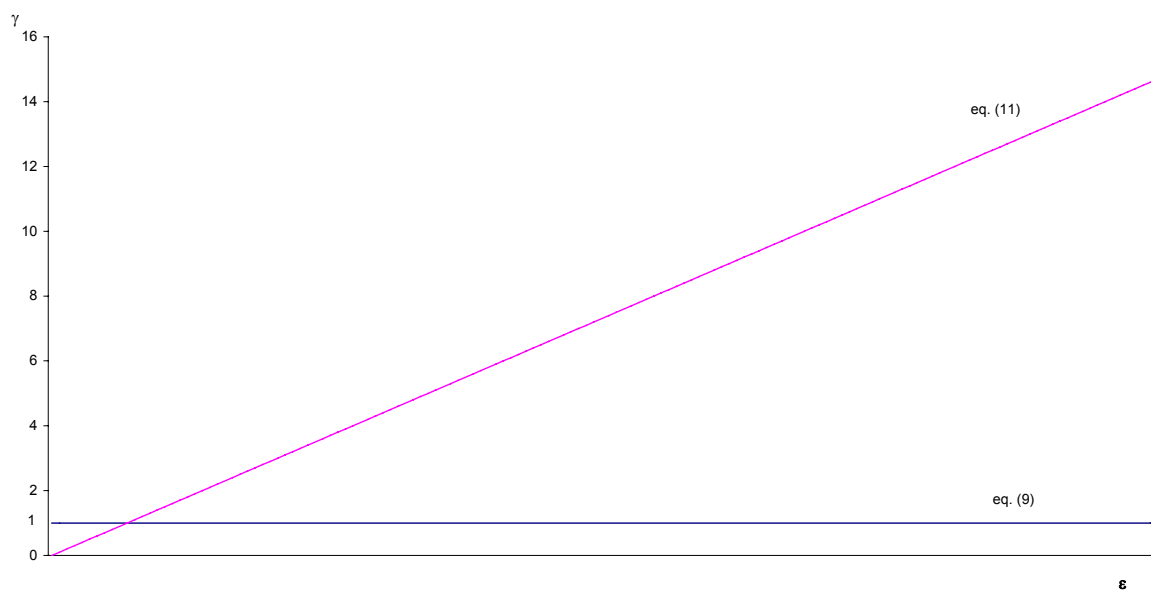
The table below makes the synthesis of the results for the extreme values of *w*:

	γ (eq. 9) < 1 (demand shock variable: Q)	γ (eq. 11) > 1 (demand shock variable: P)
$w \to 0$	$\gamma \to 0$ (deterministic case)	$\gamma \to 1$ (neutral JIE)
$w \to 1$	$\gamma \to 1$ (neutral JIE)	$\gamma > 1$, grows with ε (positive JIE)

Note: JIE = Jensen's Inequality Effect

7. CONCLUSION

Hartman (1972) and Abel (1983), the two seminal papers that studied the impact of price and cost uncertainty on firms' investment, showed that when firms are competitive and there is flexibility of labour relative to capital, marginal profitability of capital is a convex function of the stochastic variable. By Jensen's inequality, this means that uncertainty increases the expected present value of the marginal unit of capital, which increases the incentive to invest.

In this paper, we have shown that the Hartman-Abel approach can also produce a marginal profit function *concave* in the stochastic variable, in which case an increase in uncertainty decreases the expected marginal profitability of capital. In particular, we have shown that:

- the relevant assumption for the convexity/concavity of the profit function, besides factor substitutability, is the variable of choice in the firm's maximisation problem: if the choice variable is price (quantities), then the profit function is concave (convex).

- the price elasticity of demand and the share of labour in the production function may be determinant for the degree, but not the sign, of the relationship between marginal profitability of capital and uncertainty.

In this light, a possible goal for future research would be to try to find empirical evidence on the shape of the marginal profitability of capital, testing specifically for the variable of choice in the firms' optimisation problem and analysing its interaction with both the price elasticity of demand facing the firms and the share of labour in the production functions. For instance, our theoretical results imply that within an industry characterised by a production technology

with a high share of labour, where output is the choice variable in the firm's optimisation problem (implying a convex profit function), there should be a positive correlation between firms' marginal operating profits and price uncertainty; furthermore, if the elasticity price of demand is relatively high, the former should also be very sensitive to the latter (think, e.g., of a clothing industry facing exchange rate uncertainty). Instead, if price is the choice variable of firms in that industry (implying a concave profit function), then firms' marginal operating profits should be rather insensitive to changes in the level of uncertainty, whatever the degree of the elasticity price of demand.

APPENDIX A

In this appendix we study the possibility of modelling demand shocks as changes in the price associated with a given level of output by using the inverse demand function obtained from (4). Thus, we assume:

$$P = Q^{-1/\varepsilon} X^{1/\varepsilon}.$$

But, in this case, the relevant shock variable is $X^{1/\varepsilon}$. It is easy to show that in order to leave the expected value of $X^{1/\varepsilon}$ unchanged, the increase in σ must be accompanied by an increase in α: repeating the calculations for X^{γ} presented above, we see that the expected drift rate of $X^{1/\varepsilon}$ is:

$$\frac{1}{\varepsilon}\alpha + \frac{1}{2\varepsilon}\left(\frac{1}{\varepsilon} - 1\right)\sigma^2 \equiv M(\varepsilon).$$

Setting $M(\varepsilon) = M_0$, where M_0 is a constant, and applying the implicit function theorem, we get:

$$\frac{\partial M}{\partial \alpha}\frac{d\alpha}{d\sigma^2} + \frac{\partial M}{\partial \sigma^2} = 0 \quad \Leftrightarrow \quad \frac{d\alpha}{d\sigma^2} = \frac{1}{2}\left(1 - \frac{1}{\varepsilon}\right) > 0.$$

The impact of an increase in σ on the discounted present value of the marginal profit can be found by analysing the way the denominator in (12) changes when σ (and consequently α) is increased. Total differentiation yields:

$$-\gamma\frac{1}{2}\left(1 - \frac{1}{\varepsilon}\right)d\sigma^2 - \frac{1}{2}\gamma(\gamma - 1)d\sigma^2 =$$

$$= \frac{\gamma}{2}d\sigma^2\left(\frac{1}{\varepsilon} - \gamma\right),$$

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which is negative, since $\gamma > 1$, and $\varepsilon > 1$ by assumption. Thus, a mean-preserving increase in the variance of the stochastic variable $X^{1/\varepsilon}$ contributes to increasing the discounted value of the marginal profit, just like when $\gamma > 1$ and the relevant stochastic variable is simply X. Nevertheless, that partially reflects an *ad hoc* effect, that is, the increase in the drift necessary to preserve the expected value of the shock variable.[12] For this, we find more appropriate to focus on the two cases in the text, where the relevant shock variable is X.

APPENDIX B

In this appendix, we illustrate the impact of changes in the degree of uncertainty on the expected present value of marginal profitability of capital, Π_K, performing a simulation exercise with $r = 3$ and $\alpha = 0,5$.

Figure 4, below, depicts Π_K as a function of σ when $\gamma = 1,11$ and $\gamma = 2,50$; these are the limiting values of γ (eq. 11) in figures 1 and 2 in the text, corresponding respectively to a low value (0,1) and a high value (0,6) of w. As we can see, when γ is above but near to one, the positive reaction of Π_K to changes in uncertainty only becomes evident for rather high values of σ.

Figure 4 - Sensitivity Analysis of Π_k to σ

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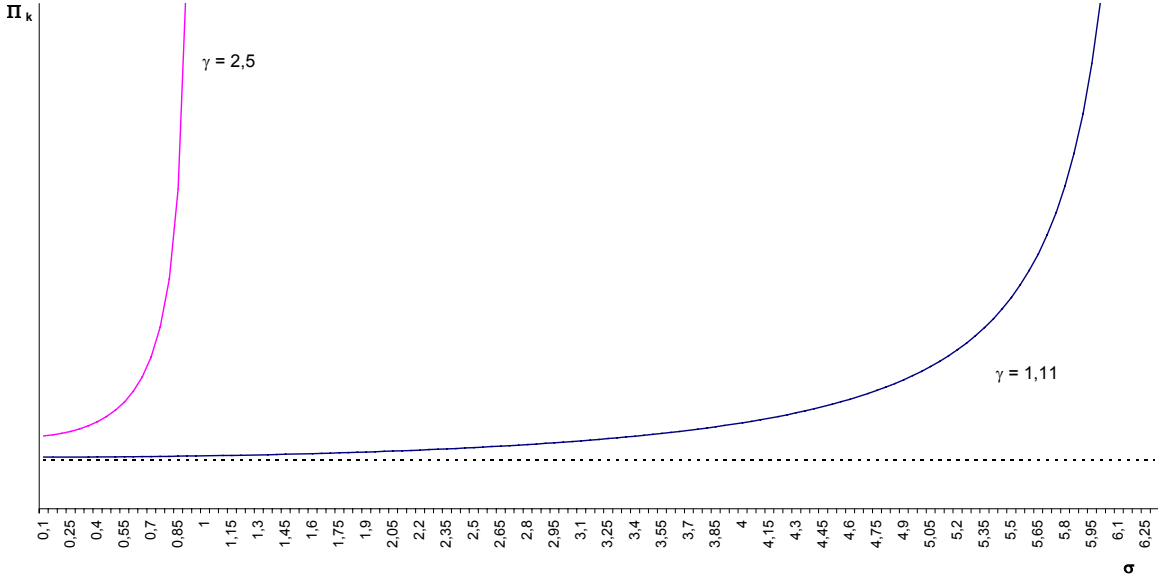
[12] This approach was followed, e.g., by Abel and Eberly (1995, p. 19-20).

Figure 5, below, depicts Π_K as a function of σ when $\gamma = 0,1$ and $\gamma = 0,9$ (the expected marginal profitability of capital is normalised to unity); these may be seen as the values of γ (eq. 9) corresponding, respectively, to extreme low values (near zero) and high values (near one) of w, for a given ε. We also include $\gamma = 0,5$ with the purpose of comparison. Notice that when the profit function is concave in the stochastic variable, the expected marginal profitability of capital displays maximum sensitivity to the degree of uncertainty when $\gamma = 0,5$. For values of γ above that level, the sensitivity of the expected marginal profitability of capital to changes in uncertainty levels decreases because the profit function becomes ever more linear in the stochastic variable. For values of γ below 0,5, the position of the profit function tends to be independent of the stochastic variable and thus becomes closer to the deterministic case.

Figure 5 - Sensitvity Analysis of Π_k to σ
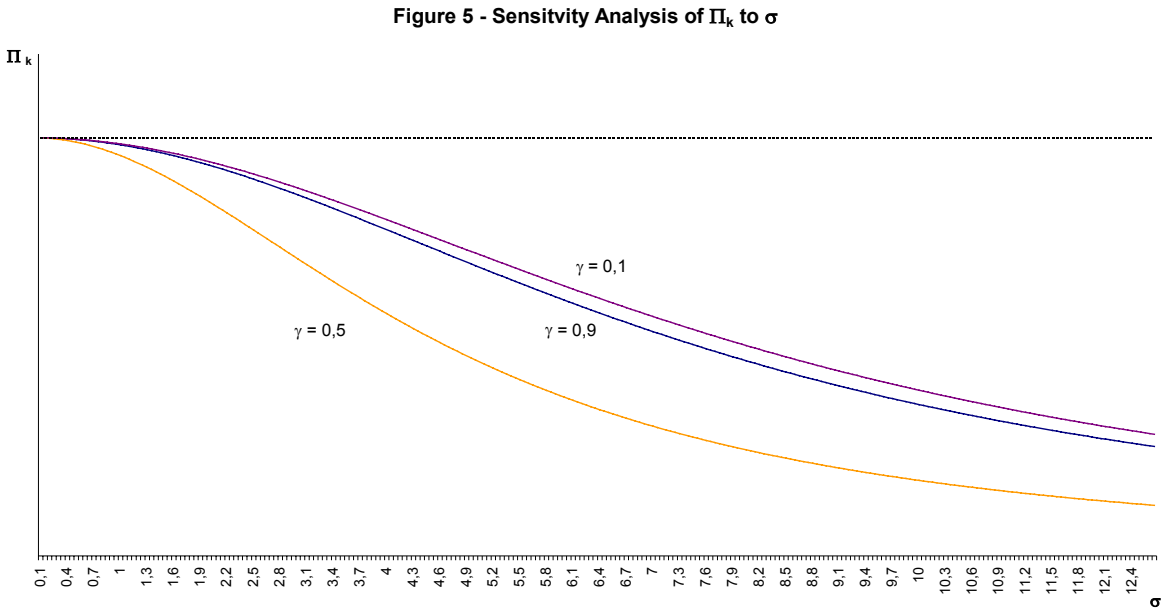
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