# Ignorance Is Bliss: Matching in Auctions with an Uninformed Seller

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**Abstract**

In many auctions, a good match between the bidder and seller raises the value of the contract for both parties. However, information about the quality of the match may be incomplete. We consider the case in which each bidder observes the quality of his match with the seller but the seller does not observe the quality of his matches with the bidders. Our objective is to determine whether it is in the seller's interest to observe the matches before selecting the winner. It is shown that the seller's value for the information may be negative: the seller's knowledge of the matches generates an asymmetry across bidders which depresses bids. The more matching matters, the greater the penalty associated with observing the matches.

**Keywords** Asymmetries, Auctions, Auction Theory, Bidding, Information Revelation, Matching, Signaling

**JEL Classification** C72, C78, D44, D82

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# 1 Introduction

In a wide range of commercial arrangements, compatibility between the contracting parties is a primary determinant of contract value. A good match between an author and his editor may generate a better book. An athletic team is more likely to win games if the players have compatible skills. An assistant professor is prone to flourish in a department with an active research group in his field of interest, and a husband and wife are more likely to enjoy a successful marriage if their temperaments are congruent.

Given the importance of a good match, it is not surprising that we observe sellers using matching as a factor in their choice of buyer. During the 2002 auction for the rights to his second novel, Charles Frazier, author of *Cold Mountain*, asserted that "money was not the only consideration and that he was keen to choose the right editor to help him shape his book from the beginning" (Gumbel, 2002). Venezuela's state-owned oil company, Petróleos de Venezuela (PDVSA), accounted for technological compatibility when it selected private partners for the development of marginal fields in the early 1990s (Chalot, 1996).

In this paper, we consider a setting in which a seller auctions off a contract, taking both price and match quality into account. Each bidder is assumed to be better informed about the quality of his match with the seller than the seller is. We then introduce an opportunity for the seller to observe the quality of the matches before selecting the winner and ask, "What is the seller's value for this information?" We find that, in many reasonable cases, the value is negative. Moreover, in these cases, the more the seller cares about matching, the stronger his incentive *not* to observe the matches.

We develop a stylized model in which a single seller seeks to contract with one of several bidders. Each pairing of seller and bidder is characterized by a match. It is assumed that match quality is the sole determinant of contract value. The higher the quality of the match, the more each party values the contract. A fitting example is that of the author and editor: a good match between the two not only makes the working process more pleasant but also results in a superior manuscript which raises the revenues to be shared. Note that the relationship between match quality and contract value induces a positive correlation between the valuations of the bidder and seller.[1]

This paper focuses on the case in which the bidder is better informed about his compatibility with the seller than the seller is.[2] In particular, it is assumed that each bidder observes his match with the seller but that the seller does not observe his matches with the bidders.

To capture the idea that the seller takes both price and match quality into account, we assume the seller administers a *first-score* auction. In a first-score auction, each bidder submits a price offer. After observing these offers, the seller updates his beliefs about the quality of his match with each of the bidders and identifies the bidder whose combination of price and expected match is most attractive. If contracting with that bidder exceeds the reserve score announced by the seller at the outset, the contract is awarded to that bidder.

[1] In this sense, our model is reminiscent of the literatures on affiliated values (e.g., Milgrom and Weber, 1982a) and interdependent valuations (e.g., Jehiel and Moldovanu, 2001). But while these literatures are primarily concerned with linking the bidder's valuations, our paper focuses on linking the valuations of the bidder and seller.

[2] The case in which the seller is better informed than the bidder is addressed in Lamping (2005) and is the subject of ongoing research.

We find this mechanism to be representative of the way contracts are generally allocated: review the bids or proposals submitted and simply select the one you like best.[3]

Since match quality factors into the allocation decision and the seller is uninformed about the quality of the matches, well matched bidders have an incentive to transmit their information. And since contract value increases with match quality, a well matched bidder can credibly signal his status by raising his bid beyond the point at which it is profitable for poorly matched bidders to mimic him.[4] With higher bids signaling better matches, the contract goes to the bidder submitting the highest bid.

Equilibrium bidding behavior changes dramatically once we introduce an opportunity for the seller to observe the matches before selecting the winner. When the seller observes the matches, the incentive to signal no longer exists. Moreover, the seller's knowledge introduces a bias in favor of well matched bidders, which causes them to compete less vigorously on price. In particular, the bias permits a well matched bidder to bid less than a poorly matched counterpart and still win the auction. Since the best-matched bidder still wins the contract

[3]Che (1993), Branco (1997), Zheng (2000), and Asker and Cantillon (2006) analyze a similar auction format in which the winning bidder is selected on the basis of price and quality. But while the bidders in these papers bid directly on both factors, the bidders in our paper bid only on price, leaving the seller to estimate the quality of the matches on his own. The mechanism in our paper is more closely linked to the biased procurement problem studied by Rezende (2006), in which each bidder submits a price offer and the seller selects the winner on the basis of both price and some pre-existing bias. But while the bias in Rezende's paper is determined by the seller's private information, the bias in our paper is determined by the bidders' private information.

[4]Bikhchandani and Huang (1989), Katzman and Rhodes-Kropf (2002), Das Varma (2003), Goeree (2003), Haile (2003), and Molnár and Virág (2006) examine signaling in auctions, but these paper are concerned with bidders signaling their private information to other bidders so as to affect future strategic interactions. In contrast, the signaling behavior in our paper is motivated by the structure of the auction game itself: bidders are interested in signaling their private information to the seller in order to influence the seller's choice of winner. In this sense, our paper is more similar to Avery (1998), which addresses the use of jump bids to signal a high valuation and encourage competing bidders to withdraw.

but at a lower price, the seller is better off *not* observing the matches. That is, the seller's value for information about the matches is negative.

In addition, we note that the greater the effect of matching on the seller's utility, the greater the seller's bias in favor of well matched bidders and the greater the margin by which a well matched bidder can reduce his bid and still win.[5] Therefore, the more the seller cares about matching, the stronger his incentive not to observe the matches.

The remainder of this paper is organized as follows. Section 2 lays out the model. In Section 3, we solve for the equilibria of the first-score auction when the seller cannot observe the matches. In Section 4, we introduce an opportunity for the seller to observe the matches before selecting the winner and solve for the (unique) equilibrium of this modified first-score auction game. Section 5 compares the two sets of equilibria to assess the seller's value for information about the matches. Concluding remarks are offered in Section 6. All proofs are relegated to the Appendices.

# 2 The Model

A seller offers a contract to $n$ risk-neutral bidders ($n \geq 2$). Every potential pairing of seller and bidder has an associated match. We denote the match between the seller and bidder $i$ by

[5]There is a vast literature on the negative effect of asymmetries on price competition, the majority of which examines asymmetries in the distributions from which valuations are drawn. See Milgrom (1981), Myerson (1981), Milgrom and Weber (1982b), Graham and Marshall (1987), McAfee and McMillan (1987), Bulow and Roberts (1989), McAfee and McMillan (1989), Mailath and Zemsky (1991), McAfee and McMillan (1992), Marshall, Meurer, Richard, and Stromquist (1994), Tschantz, Crooke, and Froeb (1997), Waehrer (1999), Dalkir, Logan, and Mason (2000), Kaplan and Zamir (2002), Waehrer and Perry (2003), Cantillon (forthcoming), and Bulow and Levin (2006). The nature of the asymmetry in our paper is more closely related to that in Che (1993) and Rezende (2006), where the source of asymmetry is the seller's use of factors other than price to determine the auction winner.

$\theta_i \in [\underline{\theta}, \overline{\theta}\,] \subset \mathbb{R}$, where $\theta_i > \theta_j$ indicates that bidder $i$ has a better match than bidder $j$ does. We assume the $\theta_i$'s are independently and identically distributed according to a commonly known cumulative distribution function (cdf) $F$ with $F(\underline{\theta}) = 0$ and $F\left(\overline{\theta}\right) = 1$.

**Assumption 1** *$F$ has positive density $f$ at every $\theta \in [\underline{\theta}, \overline{\theta}\,]$.*

**Assumption 2 (regularity condition)** *$F$ satisfies*

$$\frac{\mathrm{d}}{\mathrm{d}\theta_i}\left(\frac{f(\theta_i)}{1 - F(\theta_i)}\right) \geq 0$$

*for all $\theta_i \in [\underline{\theta}, \overline{\theta}\,]$.*

Bidder $i$'s utility from contracting with the seller is

$$\theta_i - b_i,$$

where $\theta_i$ is bidder $i$'s value for the contract and $b_i \in \mathbb{R}$ is the bid submitted by bidder $i$. Bidder $i$'s utility is zero if he does not win the contract.

The seller derives utility from both the bid payment and his match with the winning bidder. We assume the seller's utility from contracting with bidder $i$ is

$$V(\theta_i) + b_i,$$

where $V(\theta_i)$ represents the seller's value for his match with bidder $i$. The following assumption captures the notion that a good match raises the value of the contract for both the seller and the bidder:

**Assumption 3** *$V : [\underline{\theta}, \overline{\theta}\,] \to \mathbb{R}$ is twice continuously differentiable with $V'(\theta_i) > 0$ and $V''(\theta_i) \leq 0$ for all $\theta_i \in [\underline{\theta}, \overline{\theta}\,]$.*

The seller's utility is zero if he does not contract with any bidder.[6]

The following assumption is imposed so as not to rule out the possibility of a mutually beneficial trade:

**Assumption 4 (participation condition)** $V\left(\overline{\theta}\right)+\overline{\theta}$ *is positive.*

We assume bidder $i$ is better informed about his match than the seller is. Bidder $i$ observes his match $\theta_i$ but not the matches of his opponents.[7] The seller does not observe the matches directly, and therefore, his beliefs about the matches are determined by the prior $F$ and the observed bids.

The seller and bidders play the following auction game, the structure of which is assumed to be common knowledge:

1. Each bidder submits a price offer independently and simultaneously.

2. The seller contracts with the bidder whose combination of price and expected match maximizes the seller's expected utility provided that the combined value is not less than the reserve score $s_* \in \mathbb{R}$. That is, bidder $i$ wins the contract if

$$E\left[V(\theta_i) \mid b_i\right] + b_i \geq s_*$$

and

$$E\left[V(\theta_i) \mid b_i\right] + b_i > E\left[V(\theta_j) \mid b_j\right] + b_j \quad \forall j \neq i,$$

[6]When $V(\theta_i) = v\theta_i$, where $v$ is a positive constant, our model can be mapped to the interdependent valuations framework outlined in Section 5 of Jehiel and Moldovanu (2001): simply let the agents be indexed by $i \in \{0, 1, 2, ..., n\}$, where agent $i$ is the seller if $i = 0$ and bidder $i$ otherwise; let $p_k^i$ be the probability the contract is awarded to agent $i$ in alternative $k$; and let $s^0 = 0$, $s^i = \theta_i$, $a_{k0}^i = vp_k^i$, $a_{ki}^i = p_k^i$, and $a_{ki}^j = 0$ for all $j \neq i$.

[7]Bidder $i$'s beliefs about $\theta_j$, $j \neq i$, are determined by the prior, $F$.

where $b_i$ denotes the price offer (bid) submitted by bidder $i$. Ties are resolved by a random draw with equal probability.

3. If the contract is allocated to bidder $i$, $\theta_i$ is revealed. The seller's utility is $V(\theta_i) + b_i$, bidder $i$'s utility is $\theta_i - b_i$, and all other bidders get zero utility. If the contract is not allocated, every agent gets zero utility.

We call this game a *first-score* auction, where the term "score" refers to the combination of price and expected match. For instance, bidder $i$'s score is given by $E[V(\theta_i) \mid b_i] + b_i$. As indicated in the timeline above, the contract is allocated to the bidder with the highest score provided that the score is not less than $s_*$. The winning bidder pays the price he offered $b_i$, thereby delivering his true score $V(\theta_i) + b_i$.

# 3 Equilibria with an Uninformed Seller

Our objective in this section is to characterize the equilibria of the first-score auction game. In order to define the equilibrium concept, we introduce some additional notation.

Let $\mathrm{B}_i : \mathbb{R} \times [\underline{\theta}, \overline{\theta}] \to [0,1]$ represent bidder $i$'s equilibrium bidding strategy (possibly a mixed strategy), where $\mathrm{B}_i(b \mid \theta_i)$ is the cdf from which bidder $i$ draws a bid of $b$ when his type is $\theta_i$. Let $\mathrm{B}$ represent the profile of bidding strategies $(\mathrm{B}_1, \mathrm{B}_2, \ldots, \mathrm{B}_n)$, and let $\mathrm{B}_{-i}$ represent the vector of competing strategies $(\mathrm{B}_1, \ldots, \mathrm{B}_{i-1}, \mathrm{B}_{i+1}, \ldots, \mathrm{B}_n)$. Finally, let $\beta_i(\cdot \mid \theta_i)$ be the density function associated with $\mathrm{B}_i(\cdot \mid \theta_i)$.

Recall that the seller does not observe the matches directly and as such, his beliefs about the matches are determined by the prior $F$ and the observed bids. We define the seller's posterior beliefs by $\mathrm{M}_i : [\underline{\theta}, \overline{\theta}\,] \times \mathbb{R} \rightarrow [0,1]$, where $\mathrm{M}_i(\theta \mid b_i)$ is the probability the seller assigns to bidder $i$'s type being in $[\underline{\theta}, \theta]$ when bidder $i$ offers a price of $b_i$. Let M represent the profile of posterior beliefs $(\mathrm{M}_1, \mathrm{M}_2, \ldots, \mathrm{M}_n)$.

Given the seller's posterior beliefs M, bidder $i$'s score is given by

$$s_i(b_i) \equiv \int_{\underline{\theta}}^{\overline{\theta}} V(x)\,\mathrm{dM}_i(x \mid b_i) + b_i$$

when he submits a bid of $b_i$. Bidder $i$ wins the contract if his score is the highest among the $n$ bidders ($s_i(b_i) = \max\{s_1(b_1), \ldots, s_n(b_n)\}$) and is at least as great as the reserve score ($s_i(b_i) \geq s_*$). Ties are resolved by a random draw with equal probability.

Let $P_i(b_i \mid \mathrm{B}_{-i}, \mathrm{M})$ represent bidder $i$'s probability of winning the contract when bidder $i$ bids $b_i$, every other bidder follows the bidding strategy prescribed by $\mathrm{B}_{-i}$, and the seller's posterior beliefs are given by M. Bidder $i$'s expected utility can then be written as

$$U_i(b_i \mid \theta_i, \mathrm{B}_{-i}, \mathrm{M}) \equiv (\theta_i - b_i)\, P_i(b_i \mid \mathrm{B}_{-i}, \mathrm{M})\,.$$

We are now ready to define the equilibrium concept.

**Definition 1** *A perfect Bayesian equilibrium of the first-score auction game with an uninformed seller is a pair of the bidders' bidding strategies and the seller's posterior beliefs* $(\mathrm{B}^*, \mathrm{M}^*)$ *such that the following two conditions hold:*

(1) *For any $b_i^*$ in the support of $\beta_i^*$,*

$$U_i(b_i^* \mid \theta_i, \mathrm{B}_{-i}^*, \mathrm{M}^*) \geq U_i(b_i \mid \theta_i, \mathrm{B}_{-i}^*, \mathrm{M}^*)$$

*for all $b_i \in \mathbb{R}$, $\theta_i \in [\underline{\theta}, \overline{\theta}\,]$, and $i \in \{1, \ldots, n\}$.*

(2) *For any* $i \in \{1, \ldots, n\}$ *and* $b_i \in \mathbb{R}$,

$$\mathrm{M}_i^*(\theta \mid b_i) = \frac{\int_{\underline{\theta}}^{\theta} \beta_i^*(x \mid b_i) f(x) \, \mathrm{d}x}{\int_{\underline{\theta}}^{\overline{\theta}} \beta_i^*(x \mid b_i) f(x) \, \mathrm{d}x}$$

*if* $\beta_i^*(x \mid b_i) > 0$ *for some* $x \in [\,\underline{\theta}, \overline{\theta}\,]$ *and* $\mathrm{M}_i^*(\theta \mid b_i)$ *is any cdf on* $[\,\underline{\theta}, \overline{\theta}\,]$ *if* $\beta_i^*(x \mid b_i) = 0$ *for all* $x \in [\,\underline{\theta}, \overline{\theta}\,]$.

Condition (1) stipulates that bidder $i$ can do no better than to follow his equilibrium strategy $\mathrm{B}_i^*$ whenever his competitors follow their equilibrium strategies $\mathrm{B}_{-i}^*$ and the seller updates his beliefs according to $\mathrm{M}^*$. Condition (2) requires that $\mathrm{M}^*$ satisfy Bayes' rule on the equilibrium path.

Given the lack of restrictions on off-equilibrium-path beliefs, it is not surprising that there exist multiple equilibria. For the remainder of the section, we restrict our attention to equilibria that are symmetric and separating. We do so for two reasons: these equilibria hold up under standard equilibrium refinements, and we find them to be the most intuitive.

**Lemma 1** *In any symmetric separating equilibrium of the first-score auction game, there exists a* $\theta_* \in [\,\underline{\theta}, \overline{\theta}\,]$ *such that*

(1) *any bidder with type* $\theta \in (\theta_*, \overline{\theta}\,]$ *bids according to the function*

$$b(\theta) \;=\; \theta - \frac{\int_{\theta_*}^{\theta} F^{n-1}(x) \, \mathrm{d}x}{F^{n-1}(\theta)}$$

*and wins with positive probability.*

(2) *any bidder with type* $\theta \in [\,\underline{\theta}, \theta_*)$ *wins with zero probability.*

*Proof* See Appendix A.

Lemma 1 establishes that in any symmetric separating equilibria, there exists a threshold $\theta_*$ such that any bidder whose type exceeds $\theta_*$ has a positive probability of winning and any bidder whose type is below $\theta_*$ has zero probability of winning. The lemma also establishes that any bidder whose type exceeds $\theta_*$ plays the pure strategy $b(\theta)$.

Note that $b(\theta)$ is increasing on $(\theta_*, \overline{\theta}\,]$. Since the seller does not directly observe the matches, each bidder would like to project that the quality of his match is as high as possible, thereby raising his probability of winning the contract. However, since contract value increases with the quality of the match, a truly well matched bidder can afford to bid more than a poorly matched bidder. As such, a higher bid signals a better match, and the contract goes to the bidder submitting the highest bid.

The following proposition completes our characterization of the symmetric separating equilibria of the first-score auction game.

**Proposition 1** *A symmetric separating equilibrium exists for every* $\theta_* \in [\,\theta_*^L, \theta_*^H]$, *where*

$$\theta_*^L = \begin{cases} \underline{\theta} & \text{if } s_* < V(\underline{\theta}) + \underline{\theta} \\ \left\{x \in [\,\underline{\theta}, \overline{\theta}\,] : V(x) + x = s_*\right\} & \text{if } s_* \in \left[V(\underline{\theta}) + \underline{\theta},\, V(\overline{\theta}) + \overline{\theta}\,\right] \\ \overline{\theta} & \text{if } s_* > V(\overline{\theta}) + \overline{\theta} \end{cases}$$

*and*

$$\theta_*^H = \min\left\{\theta_*^L + \left[V(\theta_*^L) - V(\underline{\theta})\right], \overline{\theta}\right\}.$$

*Moreover, there are no symmetric separating equilibria such that* $\theta_* \notin [\,\theta_*^L, \theta_*^H]$.

*Proof* See Appendix B.

In a standard first-price auction, the choice of reserve price uniquely identifies the threshold type $\theta_*$ (Riley and Samuelson, 1981; Maskin and Riley, 1986), but in a first-score auction, multiple $\theta_*$'s can be sustained due to the lack of restrictions on off-equilibrium-path beliefs. $\theta_* < \theta_*^L$ cannot be supported because bidders with types in $(\theta_*, \theta_*^L)$ are not willing to bid high enough to meet the reserve score $s_*$. $\theta_* > \theta_*^H$ cannot be supported because bidders with types in $(\theta_*^H, \theta_*)$ are willing to bid enough to meet $s_*$ – even if the seller believes them to be the lowest possible type $\underline{\theta}$.

In the next section, we consider the value of information about the matches and ask whether the seller can do better if he observes the matches in advance.

# 4 Equilibria with an Informed Seller

Suppose the seller could observe the vector of matches $(\theta_1, \ldots, \theta_n)$ after selecting the mechanism (a first-price auction with reserve score $s_*$) but before selecting the winner and that it would be commonly known if he chose to do so. For instance, the novelist Frazier could meet with the editors at the various publishing houses before receiving bids or the Ph.D. job candidate could ask his prospective colleagues about their research programs before receiving offers. Our objective in this section is to solve for the (unique) equilibrium of the first-score auction given that the seller elects to observe the matches.

Let $\mathrm{B}_i$, $\mathrm{B}$, $\mathrm{B}_{-i}$, and $\beta_i$ be defined as in Section 3. Since the seller observes $(\theta_1, \ldots, \theta_n)$ before selecting the winner, there is no need to specify his posterior beliefs. Bidder $i$'s score is simply given by

$$s_i(b_i \mid \theta_i) \equiv V(\theta_i) + b_i \, .$$

As before, bidder $i$ wins the contract if his score is the highest among the $n$ bidders and is at least as great as the reserve score $s_*$ with ties resolved by a random draw with equal probability. Let $P_i(b_i \mid \theta_i, \mathrm{B}_{-i})$ represent bidder $i$'s probability of winning when he has type $\theta_i$, bids $b_i$, and faces competing bidders who follow the bidding strategies prescribed by $\mathrm{B}_{-i}$. Bidder $i$'s expected utility can then be written as

$$U_i(b_i \mid \theta_i, \mathrm{B}_{-i}) \equiv (\theta_i - b_i)\, P_i(b_i \mid \theta_i, \mathrm{B}_{-i})\,.$$

**Definition 2** *A Nash equilibrium of the first-score auction game with an informed seller is a strategy profile* $\mathrm{B}^*$ *such that for any* $b_i^*$ *in the support of* $\beta_i^*$*,*

$$U_i(b_i^* \mid \theta_i, \mathrm{B}_{-i}^*) \geq U_i(b_i \mid \theta_i, \mathrm{B}_{-i}^*)$$

*for all* $b_i \in \mathbb{R}$*,* $\theta_i \in [\underline{\theta}, \overline{\theta}\,]$*, and* $i \in \{1, \ldots, n\}$*.*

We proceed by reformulating the problem in terms of bidder $i$'s score rather than bidder $i$'s bid. This reformulation permits us to directly apply the standard independent private values results.

Let $s_i \equiv V(\theta_i) + b_i$ denote the score offered by bidder $i$. Since both bidder and seller observe $\theta_i$, the choice of bid unambiguously determines the score. Let $\Sigma_i : \mathbb{R} \times [\underline{\theta}, \overline{\theta}\,] \to [0, 1]$ represent bidder $i$'s equilibrium score strategy (possibly a mixed strategy), where $\Sigma_i(s \mid \theta_i)$ is the cdf from which bidder $i$ draws a score of $s$ when his type is $\theta_i$. Let $\Sigma$ be the profile of score strategies $(\Sigma_1, \ldots, \Sigma_n)$, $\Sigma_{-i}$ be the vector of competing strategies $(\Sigma_1, \ldots, \Sigma_{i-1}, \Sigma_{i+1}, \ldots, \Sigma_n)$, and $\sigma_i(\cdot \mid \theta_i)$ be the density function associated with $\Sigma_i(\cdot \mid \theta_i)$. Finally, let $Q_i(s_i \mid \Sigma_{-i})$ represent bidder $i$'s probability of winning when he offers a score of $s_i$

and his competitors follow the strategies prescribed by $\Sigma_{-i}$. Note that $Q_i(s \mid \Sigma_{-i})$ is simply the probability that $s$ is the highest score offered if $s \geq s_*$ and zero otherwise.

Using this notation, bidder $i$'s expected utility can be written as follows:

$$\hat{U}_i(s_i \mid \theta_i, \Sigma_{-i}) \equiv [V(\theta_i) + \theta_i - s_i]\, Q_i(s_i \mid \Sigma_{-i})\,.$$

A Nash equilibrium is then a strategy profile $\Sigma^*$ such that for any $s_i^*$ in the support of $\sigma_i^*$, $\hat{U}_i(s_i^* \mid \theta_i, \Sigma_{-i}^*) \geq \hat{U}_i(s_i \mid \theta_i, \Sigma_{-i}^*)$ for all $s_i \in \mathbb{R}$, $\theta_i \in [\underline{\theta}, \overline{\theta}\,]$, and $i \in \{1, \ldots, n\}$. By interpreting $V(\theta_i) + \theta_i$ as the bidder's type, $s_i$ as the bidder's bid, and $s_*$ as the seller's reserve price, we can map this formulation into the standard independent private values framework.[8] We can then invoke Maskin and Riley (1986) and Riley and Samuelson (1981) to obtain the following result:

**Lemma 2** *Let $\theta_*^L$ be defined as in Proposition 1. There exists a unique equilibrium in which*

(1) *any bidder with type $\theta \in (\theta_*^L, \overline{\theta}\,]$ offers a score of*

$$s(\theta) = V(\theta) + \theta - \frac{\int_{\theta_*^L}^{\theta} F^{n-1}(x)\,\mathrm{d}x}{F^{n-1}(\theta)} - \frac{\int_{\theta_*^L}^{\theta} V'(x) F^{n-1}(x)\,\mathrm{d}x}{F^{n-1}(\theta)}\,.$$

(2) *any bidder with type $\theta = \theta_*^L$ offers a score of $V(\theta) + \theta$ if $s_* \leq V(\overline{\theta}) + \overline{\theta}$ and a score less than $s_*$ otherwise.*

(3) *any bidder with type $\theta \in [\underline{\theta}, \theta_*^L)$ offers a score less than $s_*$.*

Lemma 2 indicates that equilibrium scores increase with the quality of the match over the range $[\theta_*^L, \overline{\theta}\,]$. Since a better match raises the value of the contract for both the bidder

[8] This technique is similar to that used in Asker and Cantillon (2006). In their terminology, $V(\theta_i) + \theta_i$ is bidder $i$'s "pseudotype."

and the seller, well matched bidders can raise their price offers by enough to outscore their poorly matched counterparts. Although higher types offer higher scores, it need not be the case that higher types offer higher *prices*. Using Lemma 2 and the fact that in equilibrium $s(\theta) = V(\theta)+b(\theta)$, we back out the equilibrium bidding function in the following proposition.

**Proposition 2** *Let $\theta_*^L$ be defined as in Proposition 1. There exists a unique equilibrium in which*

(1) *any bidder with type $\theta \in (\theta_*^L, \overline{\theta}\,]$ bids according to the function*

$$b(\theta) = \theta - \frac{\int_{\theta_*^L}^{\theta} F^{n-1}(x)\,\mathrm{d}x}{F^{n-1}(\theta)} - \frac{\int_{\theta_*^L}^{\theta} V'(x)F^{n-1}(x)\,\mathrm{d}x}{F^{n-1}(\theta)}\,.$$

(2) *any bidder with type $\theta = \theta_*^L$ bids $\theta$ if $s_* \leq V(\overline{\theta}) + \overline{\theta}$ and bids less than $s_* - V(\theta)$ otherwise.*

(3) *any bidder with type $\theta \in [\,\underline{\theta}, \theta_*^L)$ bids less than $s_* - V(\theta)$.*[9]

The first two terms of the bidding function are identical to the corresponding function in Lemma 1 (with the exception of the lower limit of integration). The third term, however, is a novel addition. The seller's value for matching enters the bidding function and depresses bids. The greater the importance of matching $V'$, the lower the bids. Consequently, the bidding function need not be well behaved. In fact, for $\theta$ sufficiently close to $\theta_*^L$, bids decrease in type.

Two conflicting effects are at play:

[9]Despite the multiplicity of bids less than $s_* - V(\theta)$, we assert the equilibrium is unique in the sense that these bids can be classified as "non-participating."

**The value effect:** A well matched bidder has a higher value for the contract, and therefore, the opportunity cost of not increasing his bid is higher;

**The asymmetry effect:** A well matched bidder is preferred by the seller, and therefore, he need not bid as aggressively to win.[10]

The value effect causes bids to increase with the quality of the match. It is the reason we observe bids increasing monotonically in the first-score auction with an uninformed seller. This effect is represented by the first two terms of the bidding function outlined in Proposition 2. The asymmetry effect is represented by the third term. Since the seller's utility increases with the quality of the match and the matches are known to the seller, a bidder with a good match can bid less than a bidder with a bad match and still win the contract. In other words, the asymmetry across bidders dampens price competition. The greater the importance of matching $V'$, the greater the asymmetry and the lower the bids.

# 5 The Value of Information

In this section, we investigate whether it is in the seller's interest to observe the matches in advance. We show that, in many reasonable cases, the seller is better off *not* observing the matches.

[10] The asymmetry effect is similar to "the competition effect" in Rezende (2006).

We begin by deriving the seller's expected utility for each of the two information structures. Following Riley and Samuelson (1981), we obtain

$$U_0^U \equiv n\int_{\theta_*}^{\overline{\theta}} \left[V(\theta)+\theta-\frac{1-F(\theta)}{f(\theta)}\right] F^{n-1}(\theta)\, f(\theta)\,\mathrm{d}\theta$$

$$U_0^I \equiv n\int_{\theta_*^L}^{\overline{\theta}} \left[V(\theta)+\theta-\big(V'(\theta)+1\big)\frac{1-F(\theta)}{f(\theta)}\right] F^{n-1}(\theta)\, f(\theta)\,\mathrm{d}\theta$$

where the superscript $U$ refers to the case in which the seller is *uninformed* and the superscript $I$ refers to the case in which the seller is *informed*. Let $t_*^U$ be the $\theta_*$ that maximizes $U_0^U$ and $t_*^I$ be the $\theta_*^L$ that maximizes $U_0^I$. A straightforward application of Assumptions 1 through 4 delivers the following result:

$$t_*^U \equiv \begin{cases} \underline{\theta} & \text{if } V(\underline{\theta})+\underline{\theta} \geq \dfrac{1}{f(\underline{\theta})} \\ \left\{x\in(\underline{\theta},\overline{\theta}\,) : V(x)+x=\dfrac{1-F(x)}{f(x)}\right\} & \text{otherwise} \end{cases}$$

$$t_*^I \equiv \begin{cases} \underline{\theta} & \text{if } V(\underline{\theta})+\underline{\theta} \geq \dfrac{V'(\underline{\theta})+1}{f(\underline{\theta})} \\ \left\{x\in(\underline{\theta},\overline{\theta}\,) : V(x)+x=\big(V'(x)+1\big)\dfrac{1-F(x)}{f(x)}\right\} & \text{otherwise} \end{cases}$$

Note that $t_*^I \geq t_*^U$; that is, the seller's incentive to restrict the set of participating bidders is stronger when he expects to observe the matches.

We will now identify the cases in which the seller benefits from not observing the matches. First, suppose the reserve score $s_*$ is fixed across information structures. If $\theta_*=\theta_*^L$, the set of participating bidders is the same, but these bidders bid less when the matches are observed. Hence, the seller prefers to remain uninformed. But what if $\theta_*>\theta_*^L$? In this case, the seller also prefers to remain uninformed as long as $\theta_*\leq t_*^U$. Since $U_0^U$ is increasing in $\theta_*$ over the range $[\underline{\theta},t_*^U]$, choosing $\theta_*\in(\theta_*^L,t_*^U]$ is strictly better than choosing $\theta_*=\theta_*^L$. The following proposition summarizes these observations.

**Proposition 3** $U_0^U > U_0^I$ *if either* $\theta_* = \theta_*^L < \overline{\theta}$ *or* $\theta_* \in (\theta_*^L, t_*^U]$ .

We elect to disregard the case in which $\theta_* > t_*^U$ because there is no agent (bidder or seller) who prefers $\theta_* > t_*^U$ to $\theta_* = t_*^U$.

Now suppose $s_*$ varies across information structures. In particular, suppose the seller knows whether or not he will subsequently observe the matches and therefore chooses $s_*$ such that $\theta_* = t_*^I$ if the matches will be observed and $\theta_*^L = t_*^U$ if they will not. Proposition 4 establishes that even when the set of participating bidders is chosen optimally for each information structure, the seller still prefers to remain uninformed.

**Proposition 4** *If* $\theta_* = t_*^U$ *and* $\theta_*^L = t_*^L$, $U_0^U > U_0^I$ .

The proof is trivial and therefore omitted.

We have established that, in many reasonable cases, the seller's value for information about the quality of the matches is negative. This result is driven primarily by the fact that for any given set of participating bidders, bids are lower when the seller observes the matches than when he does not. In addition, Proposition 2 indicates that the greater the magnitude of $V'(\cdot)$, the greater the reduction in bids. Hence, the counterintuitive result that the more the seller cares about matching, the stronger his incentive *not* to observe the matches.

# 6 Concluding Remarks

In many commercial arrangements, a good match between the buyer and seller raises the value of the contract for both parties. However, at the time the terms of the contract are set, the parties may not be fully informed about the degree to which they match. This paper has addressed the case in which the quality of the match is the private information of the bidder and provided answers to the natural questions regarding the effects of information asymmetry, namely (1) What is the seller's value for the bidders' information? and (2) How would the equilibrium bids change if the seller were to observe the matches in advance?

If the seller is uninformed about the quality of the matches, bidders signal favorable information via higher bids. If, instead, the seller observes the matches, bidders exploit their favored status by reducing their bids. The more the seller cares about matching, the greater the advantage enjoyed by well matched bidders, and the larger the margin by which they can reduce their bids and still win. Provided that certain reasonable restrictions on the reserve score are met, we have shown that the seller's value for the information is not only negative but decreasing in the importance of matching.

A desirable extension would be to allow both the bidder and seller to observe a signal about the quality of their match. This would capture a situation in which neither party knows the quality of the match but each party has an impression about how well he matches with the other party. This problem is substantially more difficult than the one presented here because it features private information on both sides of the market.

However, this paper provides a good starting point in that it clarifies some of the issues at play. It suggests that well matched bidders experience a tension when selecting their bids. On the one hand, well matched bidders have an incentive to raise their bids in order to transmit their private information, cause the seller to update his beliefs in their favor, and raise their probability of winning. On the other hand, these bidders have an incentive to reduce their bids in order to capitalize on what they expect to be a bias in their favor: since the signal observed by these bidders is favorable, they expect the signal observed by the seller to be favorable as well. Note that this paper has nothing to say about how the seller's choice of winner transmits his private information and affects equilibrium bidding behavior. Developing these ideas further would be an interesting area for future research.

# Appendix A. Proof of Lemma 1

The proof proceeds via a series of five claims.

**Claim 1** *Suppose there exist* $\theta_i \in [\underline{\theta}, \overline{\theta}\,)$ *and some* $b$ *in the support of* $\beta_i^*(\cdot \mid \theta_i)$ *such that* $P_i(b \mid \mathrm{B}_{-i}^*, \mathrm{M}^*) > 0$*. Then for all* $\hat{\theta}_i > \theta_i$ *and all* $\hat{b}$ *in the support of* $\beta_i^*(\cdot \mid \hat{\theta}_i)$*, it is the case that* $P_i(\hat{b} \mid \mathrm{B}_{-i}^*, \mathrm{M}^*) > 0$*.*

*Proof* Suppose $\hat{b}$ is in the support of $\beta_i^*(\cdot \mid \hat{\theta}_i)$ and $b$ is in the support of $\beta_i^*(\cdot \mid \theta_i)$. If $\hat{\theta}_i > \theta_i$ and $P_i(b \mid \mathrm{B}_{-i}^*, \mathrm{M}^*) > 0$, then by Definition 1,

$$\begin{aligned}(\hat{\theta}_i - \hat{b})\, P_i(\hat{b} \mid \mathrm{B}_{-i}^*, \mathrm{M}^*) &\geq (\hat{\theta}_i - b)\, P_i(b \mid \mathrm{B}_{-i}^*, \mathrm{M}^*) \\ &> (\theta_i - b)\, P_i(b \mid \mathrm{B}_{-i}^*, \mathrm{M}^*) \\ &\geq 0\end{aligned}$$

Since $(\hat{\theta}_i - \hat{b})\, P_i(\hat{b} \mid \mathrm{B}_{-i}^*, \mathrm{M}^*) > 0$, it must be the case that $P_i(\hat{b} \mid \mathrm{B}_{-i}^*, \mathrm{M}^*) > 0$. ■

**Claim 2** *Let* $\mathrm{B}_i^*$ *be a separating equilibrium bidding strategy. Suppose there exist* $\theta_i \in [\underline{\theta}, \overline{\theta}\,)$ *and some* $b$ *in the support of* $\beta_i^*(\cdot \mid \theta_i)$ *such that* $P_i(b \mid \mathrm{B}_{-i}^*, \mathrm{M}^*) > 0$*. If* $\hat{\theta}_i > \theta_i$ *and* $\hat{b}$ *is in the support of* $\beta_i^*(\cdot \mid \hat{\theta}_i)$*, then* $\hat{b} > b$*.*

*Proof* Since we are constraining ourselves to separating equilibria, $\theta_i \neq \hat{\theta}_i$ implies $b \neq \hat{b}$. Hence, it is sufficient to show that $\hat{\theta}_i > \theta_i$ implies $\hat{b} \geq b$. By Definition 1, we obtain

$$(\theta_i - b)\, P_i(b \mid \mathrm{B}^*_{-i}, \mathrm{M}^*) \;\geq\; (\theta_i - \hat{b})\, P_i(\hat{b} \mid \mathrm{B}^*_{-i}, \mathrm{M}^*) \tag{.1}$$

$$(\hat{\theta}_i - \hat{b})\, P_i(\hat{b} \mid \mathrm{B}^*_{-i}, \mathrm{M}^*) \;\geq\; (\hat{\theta}_i - b)\, P_i(b \mid \mathrm{B}^*_{-i}, \mathrm{M}^*)\,. \tag{.2}$$

Combining the two inequalities yields

$$\begin{aligned} \theta_i \left[P_i(b \mid \mathrm{B}^*_{-i}, \mathrm{M}^*) - P_i(\hat{b} \mid \mathrm{B}^*_{-i}, \mathrm{M}^*)\right] \;&\geq\; b\, P_i(b \mid \mathrm{B}^*_{-i}, \mathrm{M}^*) - \hat{b}\, P_i(\hat{b} \mid \mathrm{B}^*_{-i}, \mathrm{M}^*) \\ &\geq\; \hat{\theta}_i \left[P_i(b \mid \mathrm{B}^*_{-i}, \mathrm{M}^*) - P_i(\hat{b} \mid \mathrm{B}^*_{-i}, \mathrm{M}^*)\right]. \end{aligned}$$

Since $\hat{\theta}_i > \theta_i$, $P_i(\hat{b} \mid \mathrm{B}^*_{-i}, \mathrm{M}^*) \geq P_i(b \mid \mathrm{B}^*_{-i}, \mathrm{M}^*) > 0$. Inequality (.1) can then be written as

$$\hat{b} - b \;\geq\; (\theta_i - b)\, \frac{P_i(\hat{b} \mid \mathrm{B}^*_{-i}, \mathrm{M}^*) - P_i(b \mid \mathrm{B}^*_{-i}, \mathrm{M}^*)}{P_i(b \mid \mathrm{B}^*_{-i}, \mathrm{M}^*)}\,.$$

The right-hand side is nonnegative since by Definition 1, $(\theta_i - b)\, P_i(b \mid \mathrm{B}^*_{-i}, \mathrm{M}^*) \geq 0$. ■

**Claim 3** *Let* $\mathrm{B}^*_i$ *be a symmetric separating equilibrium bidding strategy. If* $\theta_i \in [\underline{\theta}, \overline{\theta}\,]$ *and there exists some* $b$ *in the support of* $\beta^*_i(\cdot \mid \theta_i)$ *such that* $P_i(b \mid \mathrm{B}^*_{-i}, \mathrm{M}^*) > 0$*, then* $b$ *is the only bid in the support of* $\beta^*_i(\cdot \mid \theta_i)$*.*

*Proof* By Claim 1, there exists $\theta_* \in [\underline{\theta}, \overline{\theta}\,]$ such that bidders with types in $(\theta_*, \overline{\theta}\,]$ win with positive probability, while bidders with types in $[\underline{\theta}, \theta_*)$ win with zero probability. We begin by addressing the case in which bidders with type $\theta_*$ win with positive probability. Suppose $\theta_i \in [\theta_*, \overline{\theta}\,]$. Given our definition of $\theta_*$, bidder $i$ beats any bidder whose type is less than $\theta_*$. Since the equilibrium is separating, the seller can infer types from bids. By Assumption 3 and Claim 2, scores increase with type over the range $[\theta_*, \overline{\theta}\,]$. It follows that bidder $i$ wins the auction with probability $F^{n-1}(\theta_i)$. Now suppose the support of $\beta_i^*(\cdot \mid \theta_i)$ includes the bids $b$ and $b'$, where $b \neq b'$. Since bidder $i$ is indifferent among bids in the support of $\beta_i^*(\cdot \mid \theta_i)$,

$$(\theta_i - b)F^{n-1}(\theta_i) = (\theta_i - b')F^{n-1}(\theta_i).$$

Since $\theta_i \geq \theta_*$, $F^{n-1}(\theta_i)$ is positive, which implies that $b$ and $b'$ are equal – a contradiction. An analogous argument can be used to prove the claim for the case in which bidders with type $\theta_*$ win with zero probability. ■

Claim 3 indicates that bidder $i$ follows a pure strategy when $\theta_i > \theta_*$. Let $b : (\theta_*, \overline{\theta}\,] \to \mathbb{R}$ represent that pure strategy, where $b(\theta_i) = \{b \in \mathbb{R} : \beta_i^*(b \mid \theta_i) > 0\}$.

**Claim 4** *In any symmetric separating equilibrium, the strategy* $b : (\theta_*, \overline{\theta}\,] \to \mathbb{R}$ *is continuous.*

*Proof* Suppose $b$ is not continuous at some $\theta \in (\theta_*, \overline{\theta}\,]$. By Claim 2, $b$ is increasing on $(\theta_*, \overline{\theta}\,]$. Hence, there exists $\epsilon > 0$ such that at least one of the following conditions holds:

$$b(\theta) - b(\hat{\theta}) \;\; \geq \;\; \epsilon, \quad \forall\, \hat{\theta} \in (\theta_*, \theta) \tag{.3}$$

$$b(\hat{\theta}) - b(\theta) \;\; \geq \;\; \epsilon, \quad \forall\, \hat{\theta} \in \left(\theta, \overline{\theta}\right]. \tag{.4}$$

Definition 1 requires that $[\theta - b(\theta)]\, F^{n-1}(\theta) \;\geq\; [\theta - b(\hat{\theta})]\, F^{n-1}(\hat{\theta})$ for all $\hat{\theta} \in (\theta_*, \overline{\theta}\,]$. If condition (.3) holds,

$$[\theta - b(\hat{\theta})]\; [F^{n-1}(\theta) - F^{n-1}(\hat{\theta})] \;\; \geq \;\; \epsilon\, F^{n-1}(\theta) \tag{.5}$$

for all $\hat{\theta} \in (\theta_*, \theta)$. Since $\theta \in (\hat{\theta}, \overline{\theta}\,]$ and $\epsilon > 0$ are fixed, $\epsilon\, F^{n-1}(\theta)$ is both positive and fixed. However, since $F$ is continuous, $F^{n-1}(\theta) - F^{n-1}(\hat{\theta})$ can be brought arbitrarily close to zero by selecting a $\hat{\theta}$ sufficiently close to $\theta$, and since $b$ is increasing on $(\theta_*, \overline{\theta}\,]$, $\theta - b(\hat{\theta})$ is decreasing as $\hat{\theta}$ approaches $\theta$. Hence, for $\hat{\theta}$ sufficiently close to $\theta$, inequality (.5) is violated, which implies that condition (.3) cannot hold. An analogous argument can be used to show that condition (.4) cannot hold either. ∎

**Claim 5** *In any symmetric separating equilibrium, $b : (\theta_*, \overline{\theta}\,] \to \mathbb{R}$ is given by*

$$b(\theta) \;=\; \theta - \frac{\int_{\theta_*}^{\theta} F^{n-1}(x)\,\mathrm{d}x}{F^{n-1}(\theta)}\,.$$

*Proof* We will first establish that $\lim_{\theta \to \theta_*^+} b(\theta) = \theta_*$. By Claim 4, the limit exists. Suppose $\lim_{\theta \to \theta_*^+} b(\theta) \neq \theta_*$. Then there exists $\epsilon > 0$ such that one of the following conditions holds:

$$\lim_{\theta \to \theta_*^+} b(\theta) \;=\; \theta_* + \epsilon \tag{.6}$$

$$\lim_{\theta \to \theta_*^+} b(\theta) \;=\; \theta_* - \epsilon\,. \tag{.7}$$

Suppose condition (.6) holds and consider a bidder with type $\theta \in (\theta_*, \theta_* + \epsilon)$. Since $b$ is increasing on $(\theta_*, \overline{\theta}\,]$, $b(\theta) > \theta_* + \epsilon$, and since $\theta > \theta_*$, $F^{n-1}(\theta) > 0$. It follows that the bidder's expected utility, $[\theta - b(\theta)]\, F^{n-1}(\theta)$, is negative, which contradicts Definition 1. Now suppose condition (.7) holds and consider a bidder with type $\theta \in (\theta_* - \epsilon, \theta_*)$. Since $\theta < \theta_*$, the bidder earns zero utility in equilibrium. Since $b$ is continuous and increasing, there exists $x > \theta_*$ such that $b(x) \in (\theta_* - \epsilon, \theta)$. If the bidder deviates to $b(x)$, his expected utility is $[\theta - b(x)]\, F^{n-1}(x) > 0$, which contradicts Definition 1.

We will now derive the bid function $b(\cdot)$. Since scores increase with type over $(\theta_*, \overline{\theta}\,]$, $U_i\,[\,b(x)\mid\theta, \mathrm{B}^*_{-i}, \mathrm{M}^*] = [\theta - b(x)]\,F^{n-1}(x)$ for all $x \in (\theta_*, \overline{\theta}\,]$ and $\theta \in [\underline{\theta}, \overline{\theta}\,]$. Since $b$ and $F$ are continuous, it must be the case that for all $\theta \in (\theta_*, \overline{\theta}\,)$

$$\frac{\partial U_i\,[\,b(x)\mid\theta, \mathrm{B}^*_{-i}, \mathrm{M}^*]}{\partial x} \;=\; 0$$

when $x = \theta$. Taking the derivative of $U_i\,[\,b(x)\mid\theta, \mathrm{B}^*_{-i}, \mathrm{M}^*]$ with respect to $x$, substituting $\theta$ for $x$, and setting the resulting expression equal to zero yields

$$\frac{\mathrm{d}b(\theta)}{\mathrm{d}\theta}\,F^{n-1}(\theta) \;+\; b(\theta)\,\frac{\mathrm{d}F^{n-1}(\theta)}{\mathrm{d}\theta} \;=\; \theta\,\frac{\mathrm{d}F^{n-1}(\theta)}{\mathrm{d}\theta}\,.$$

After integrating both sides, evaluating the integrals from $\theta_*$ to $\theta$, and applying the boundary condition, we obtain the bidding function

$$b(\theta) \;=\; \theta - \frac{\int_{\theta_*}^{\theta} F^{n-1}(x)\,\mathrm{d}x}{F^{n-1}(\theta)} \tag{.8}$$

for $\theta \in (\theta_*, \overline{\theta}\,)$. Since $b$ is continuous over $(\theta_*, \overline{\theta}\,]$, equation (.8) gives the equilibrium bid for type $\overline{\theta}$ as well. ■

Lemma 1 follows trivially from Claims 1 and 5. ■

# Appendix B. Proof of Proposition 1

The proof proceeds via a pair of claims.

**Claim 6** *A symmetric separating equilibrium exists for every* $\theta_* \in [\,\theta_*^L, \theta_*^H\,]$.

*Proof* Fix an arbitrary $\theta_* \in [\,\theta_*^L, \theta_*^H\,]$. Suppose that any bidder with type $\theta$ bids according to the function

$$b(\theta) = \begin{cases} \min\{0, s_* - [V(\theta_*) + \theta_*]\} + \theta & \text{if } \theta \in [\,\underline{\theta}, \theta_*) \\ \theta - \dfrac{\int_{\theta_*}^{\theta} F^{n-1}(x)\,\mathrm{d}x}{F^{n-1}(\theta)} & \text{if } \theta \in [\,\theta_*, \overline{\theta}\,] \end{cases}$$

Suppose further that the seller's posterior beliefs are given by

$$\mathrm{M}_i(\theta \mid b_i) = \begin{cases} 0 & \text{if } \theta < b^{-1}(b_i) \\ 1 & \text{if } \theta \geq b^{-1}(b_i) \end{cases}$$

if $b_i = b(\theta)$ for some $\theta \in [\,\underline{\theta}, \overline{\theta}\,]$ and by $\mathrm{M}_i(\theta \mid b_i) = 1$ for all $\theta \in [\,\underline{\theta}, \overline{\theta}\,]$ otherwise. We will verify that this is a symmetric separating equilibrium.[11]

The equilibrium is clearly symmetric and separating. Since the equilibrium is separating, Bayes' rule requires the seller infer the bidder's type is $\theta$ when the bid submitted is $b(\theta)$. As this is precisely what is prescribed by M, condition (2) of Definition 1 is satisfied. It remains to show that $b(\theta)$ satisfies condition (1) of Definition 1.

Since the seller infers $\theta$ from $b(\theta)$, the bidder's score can be written as $V(\theta) + b(\theta)$. If $\theta < \theta_*$, the bidder's score is less than $s_*$, and his utility is zero. If $\theta = \theta_*$, the bidder's bid equals his valuation, and his utility is zero. If $\theta > \theta_*$, the bidder's score is at least $s_*$ and increasing in $\theta$; his expected utility is thus $[\theta - b(\theta)]F^{n-1}(\theta) > 0$.

Deviating to a bid of $b(x)$, where $x \in [\underline{\theta}, \theta_*)$, yields a score less than $s_*$ and utility of zero. Deviating to a bid of $b(x)$, where $x \in [\theta_*, \overline{\theta}\,]$, yields expected utility of $[\theta - b(x)]F^{n-1}(x)$. Substituting for $b(x)$ yields

$$(\theta - x)\,F^{n-1}(x) + \int_{\theta_*}^{x} F^{n-1}(y)\,\mathrm{d}y$$

[11]It can also be shown that this equilibrium satisfies the Intuitive Criterion of Cho and Kreps (1987).

which is increasing in $x$ for $x < \theta$ and decreasing in $x$ for $x > \theta$. Hence, for any $\theta > \theta_*$, bidding $b(\theta)$ is more profitable than bidding $b(x)$, and for any $\theta \leq \theta_*$, bidding $b(\theta)$ is at least as profitable as bidding $b(x)$.

Now consider the deviating bid $b > b(\overline{\theta})$. If $\theta < b$, the expected utility associated with $b$ is nonpositive, and $b$ is not a profitable deviation. If $\theta \geq b$, the expected utility associated with $b$ is at best $\theta - b$, which is less than the expected utility associated with $b(\overline{\theta})$. Since deviating to $b(\overline{\theta})$ is not profitable, deviating to $b$ is not profitable. Finally, consider the deviating bid $b < \theta_*$, where $b$ is assumed to be off the equilibrium path. Bidding $b$ yields a score of $V(\underline{\theta}) + b$. If $s_* < V(\underline{\theta}) + \underline{\theta}$, then $\theta_* = \underline{\theta}$ and the lowest score that occurs in equilibrium is $V(\underline{\theta}) + \underline{\theta}$. Since $V(\underline{\theta}) + b < V(\underline{\theta}) + \underline{\theta}$, deviating to $b$ is not profitable. If $s_* \geq V(\underline{\theta}) + \underline{\theta}$, then $V(\underline{\theta}) + b < V(\underline{\theta}) + \theta_*^H \leq s_*$ and deviating to $b$ is not profitable. ■

**Claim 7** *There are no symmetric separating equilibria such that $\theta_* \notin [\,\theta_*^L, \theta_*^H\,]$.*

*Proof* Suppose there exists a symmetric separating equilibrium such that $\theta_* < \theta_*^L$. If $s_* \leq V(\underline{\theta}) + \underline{\theta}$, then $\theta_*^L = \underline{\theta}$ and there is no $\theta_* < \theta_*^L$. If $s_* > V(\underline{\theta}) + \underline{\theta}$, then $\theta_*^L > \underline{\theta}$. By Lemma 1, any bidder with type $\theta \in (\theta_*, \theta_*^L)$ bids according to the function

$$b(\theta) \;=\; \theta - \frac{\int_{\theta_*}^{\theta} F^{n-1}(x)\,\mathrm{d}x}{F^{n-1}(\theta)}$$

and wins with positive probability. Since the seller infers $\theta$ from $b(\theta)$, the bidder's score is $V(\theta) + b(\theta)$. Since $V$ and $b$ are both increasing, $V(\theta) + b(\theta) \;<\; V(\theta_*^L) + \theta_*^L \;\leq\; s_*$. Since his score falls short of the reserve, the bidder's probability of winning is zero – a contradiction.

Now suppose there exists a symmetric separating equilibrium such that $\theta_* > \theta_*^H$. If $s_* \geq V(\overline{\theta}) + \overline{\theta}$, then $\theta_*^H = \overline{\theta}$ and there is no $\theta_* > \theta_*^H$. If $s_* < V(\overline{\theta}) + \overline{\theta}$, then $\theta_*^H =$

$\min\left\{\theta_*^L + [V(\theta_*^L) - V(\underline{\theta})],\, \overline{\theta}\right\}$. It follows that either $\theta_*^H \geq s_* - V(\underline{\theta})$ or $\theta_*^H = \overline{\theta}$ or both. As there is no $\theta_* > \overline{\theta}$, we focus on the case in which $\theta_*^H \in [s_* - V(\underline{\theta}), \overline{\theta}\,)$. Consider a bidder with type $\theta \in (\theta_*^H, \theta_*)$ who bids $b \in (\theta_*^H, \theta)$. By Lemma 1, the bidder wins with zero probability, earning zero utility in equilibrium. However, the score associated with $b$ is at least $V(\underline{\theta}) + b$, and since $b > s_* - V(\underline{\theta})$, the score exceeds $s_*$. Moreover, Lemma 1 stipulates that any bidder with type less than $\theta_*$ wins with zero probability. It follows that our bidder's probability of winning is at least $F^{n-1}(\theta_*)$ and his expected utility is at least $(\theta - b)\, F^{n-1}(\theta_*)$. Since $b < \theta$ and $\theta_* > \theta_*^H \geq \underline{\theta}$, his expected utility is positive – a contradiction. ■

Proposition 1 follows trivially from Claims 6 and 7. ■

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