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Katrin Springer

Institut für Weltwirtschaft an der Universität Kiel
The Kiel Institute of World Economics

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Katrin Springer*
✉ The Kiel Institute of World Economics
D-24100 Kiel, Germany
e-mail: kspringer@ifw.uni-kiel.de

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Abstract. This paper provides a technical description of the **D**ynamic **A**pplied **R**egional **T**rade (DART) General Equilibrium Model. The DART model is a recursive dynamic, multi-region, multi-sector computable general equilibrium model. All regions are fully specified and linked by bilateral trade flows. The DART model can be used to project economic activities, energy use and trade flows for each of the specified regions to simulate various trade policy as well as environmental policy scenarios, and to analyze the allocational and distributional impacts of these policies.

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1. Introduction

This paper gives an algebraic documentation of the **D**ynamic **A**pplied **R**egional **T**rade General Equilibrium Model, hereafter referred to as DART model. The DART model is a global, computable general equilibrium (CGE) model with regional as well as sectoral detail. Within every region household and industry behavior is fully specified based on microeconomic foundations. All regions are linked by bilateral trade flows. This multi-regional, multi-sectoral trade model is recursive-dynamic. That means, the evolution of the economies over time is described by a sequence of single-period static equilibria connected through capital accumulation. This version of the DART model runs through the year 2050.

The DART model can be used to project economic activities, energy use and trade flows for each of the specified regions according to exogenous assumptions about the dynamics of the model. The model can also be used to simulate policy scenarios in various economic fields, like for example, trade policy or environmental policy, and to analyze the allocational and distributional impacts of these policies.

In the international environmental discussion one main topic deals with the increasing concentration of greenhouse gases in the atmosphere and the resulting climate change. As the Kyoto environment conference in December 1997 has shown, there exists a broad consensus about the necessity of reducing greenhouse gas emissions considerably in order to protect world climate, but the concrete aims, policy measures and instruments for reducing emissions are still under debate. (UN 1997) Alternative climate policy measures can be evaluated within a simulation model that integrates economic and natural science considerations. The integrated assessment approach allows to consider the impact of climate change on the economic development of economies, and to analyze the cost and benefits of climate protection policies (Fankhauser, 1995). This can be done by linking an economic computable general equilibrium (CGE) model with an climate model, e.g. an ocean atmosphere model.

In a joint research project with the Max-Planck-Institute of Meteorology (Hamburg) the DART model serves as a component of an integrated assessment project for evaluating global climate change impacts on economies. The DART model's projections of energy use (and in the forthcoming version of the DART model of anthropological carbon dioxide emissions) are inputs to a climate model of the Max-Planck-Institute and thereby form the first link in the integrated analysis of global climate change.

For evaluating climate change policies the economic model should include dynamics in order to cover dynamic economic effects and the time dependent effects of greenhouse gas emissions and accumulation in the atmosphere, and the resulting climate change impacts. In the last years the number of dynamic CGE models has increased. However, most of them are single country models. Most of the existent multi-regional CGE models consider only one sector, i.e. they have a macroeconomic production function, or ignore international trade relations. Very few deal with all that. One prominent example is the General Equilibrium Environmental (GREEN) model by the OECD (Burniaux et al. 1992), which is a global, multi-regional multi-sectoral trade model.

The DART model stands in the tradition of the GREEN model, but will include in addition impacts from the climate system on the economy. The model is programmed in the GAMS / MPSGE language (Brooke et al. 1992 / Rutherford 1994). This version of the model is calibrated on the Global Trade Analysis Project (GTAP) data base version 3 for the year 1992 and covers up to 30 regions and 37 industry sectors (Table 1).

The present model leaves much room for improvements. The calibration of the dynamics in the recursive-dynamic, multi-regional trade model will be the next step. Some of the main fields of further developments concern the detailed modeling of the energy sector, the calibration of the dynamics including technological and productivity improvements, the implementation of international capital mobility, and the specification of sectoral climate impact functions.

This paper is organized as follows. Section 2 presents a brief non-technical overview of the DART model. Section 3 gives a more complete description of the static part of the CGE model. A short mathematical introduction into the used equilibrium conditions precedes the algebraic description of the economic behavior of the agents. Section 4 presents the dynamics incorporated in the DART model. Section 5 briefly discusses the solution concept applied in this CGE model. Finally, in section 6 future developments of the DART model are summarized.

2. Overview of the Model Structure

The **D**ynamic **A**pplied **R**egional **T**rade (DART) General Equilibrium Model is a multi-region, multi-sector, recursive-dynamic computable general equilibrium (CGE) model. The economic structure is fully specified for each region and covers production, consumption, investment and governmental activity. All markets are perfectly competitive. The world is divided into regions, which are linked by bilateral trade flows. All goods are traded among regions, except the investment good.

For each region, the model incorporates three types of agents: the producers, distinguished by production sectors, the representative private household and the government. The agents of the model have myopic expectations. The DART model is dynamic, meaning that it solves for a sequence of equilibria for future time periods connected through capital accumulation. The dynamics of the model are based on assumptions concerning exogenous growth rates for population and technological change, as well as savings behavior. The DART model is recursive in the sense that it is solved stepwise in time without any ability to anticipate possible future changes in relative prices or in constraints. Therefore, the description of the CGE model can be divided into a static part, i.e. the ART model, and a dynamic part, i.e. the DART model.

Each region has a production structure described by industry production functions, which include both primary factors and intermediate products, provided by other production sectors, as inputs. Each production sector is modeled by a nested, i.e. hierarchical production function. That means, that special functional forms as, for instance, constant elasticity of substitution (CES), Cobb-Douglas or Leontief functions can be contained within CES functions, and many layers of hierarchy can be employed (Shoven, Whalley 1992, p. 97). This allows a flexible representation of the degree of substitution between inputs to the production process.

The output of one production sector is produced by the combination of energy, intermediate goods, and the primary factors labor, capital and land. Labor and capital are mobile across industries, but internationally immobile. Land is only used in the agricultural sectors.

The differentiation between energy and non-energy intermediate products is useful in the context of climate change policy. Energy use in production and consumption produces varying amounts of the greenhouse gas (GHG) carbon dioxide (CO_2) depending on the fossil source and the policies assumed to be in place. Carbon dioxide, with large emission levels and a long lifetime in the atmosphere is the largest single contributor to the greenhouse effect. Thus, other GHGs as methane, nitrous oxide, ozone, and halocarbons, as well as emissions of CO_2 from deforestation are not considered in this model.[1]

Producer goods are directly demanded by regional households, governments, the investment sector, other industries, and the export sector.

The representative household receives all income generated by value added by providing primary factors to the production process. Disposable income is used for maximizing utility by purchasing goods after taxes and savings are deducted. The household wants to realize a certain consumption level. The household decides between different primary energy inputs and non-energy inputs depending on their relative prices in order to receive this consumption with the lowest expenditures. In other words, there is a household production function in the model. The private household saves a fixed share of income in each time period. These savings are invested in the production sectors.

The third agent, the government, provides a public good by demanding goods produced in the production sectors. These purchases are financed by tax revenues.

The static part of the CGE model, the ART model, is calibrated on the Global Trade Analysis Project (GTAP) database version 3 for 1992 (McDougall 1997). This version of the GTAP database contains 30 regions and 37 sectors, all shown in Table 1, and can be aggregated to any preferred aggregation level as a microeconomic consistent database. The dynamic part of the DART model is calibrated on exogenous assumptions concerning growth rates of population, human capital accumulation and technological change, savings rates, and capital to GDP ratios.

[1] For a comprehensive overview of the science and politics of climate change see Mabey et al., 1997.

3. Single Period Equilibrium

This section describes the economic structure of the static part of the computable general equilibrium model, i.e. the ART model. It is an "equilibrium" model because it finds a set of product and factor prices that balances supplies and demands in each time period. The ART model is general in the sense that it clears all markets taking the interactions of factor and product markets into account. The economic structure of the ART model is consistent with the Arrow-Debreu equilibrium framework. Therefore the equilibrium problem of an economy can be described by classes of conditions associated with the Arrow-Debreu general equilibrium.

First, a short mathematical introduction to the equilibrium conditions used here is given because there are several ways to characterize an economic equilibrium. Then, the economic behavior of the agents is described algebraically.

3.1 General Equilibrium - a mathematical introduction

This section defines equilibrium conditions for a simple CGE model within the Arrow-Debreu framework as a reference for the algebraic exposition below. Following Shoven and Whalley (1992) the number of consumers is specified in that framework. Each household has an initial factor endowment and a set of preferences, resulting in demand functions for each commodity. The sum of each agent's utility maximizing demand yields the market demand. On the production side, the technology is described by constant-returns-to-scale activities. Producers maximize profits. Markets are perfectly competitive.

The standard Arrow-Debreu equilibrium is then characterized by a set of prices and activity levels in each industry such that market demand equals supply for each commodity (market clearance condition). Profit maximization in the constant-returns-to-scale case implies that no activity earns a positive profit (zero profit condition). On the consumer's side, in equilibrium income restricts expenditure, i.e. there is no excess demand of the "household", including government (income balance).

Mathiesen (1985) demonstrated that an Arrow-Debreu general equilibrium model can be formulated and efficiently solved as a complementarity problem. The equilibrium in complementarity format (CP) is as follows (Mathiesen 1985):

(CP) Find $z \in R^{N \times M \times H}$ that solves $F(z) \geq 0,\ z \geq 0$ and $z^{\mathbf{T}} F(z) = 0.$

where z represents a vector of decision variables and $F(\cdot)$ the corresponding equilibrium condition.

The vector z consists of three vectors of decision variables associated with the solution of the economic equilibrium problem:

p is a non-negative vector of N commodity prices, indexed by i, including all final goods, intermediate goods and primary factors of production:

p_i with $i=1,...,L,K,B,...,N$
where $i=L$ is the index for the primary factor labor,
$i=K$ is the index for the factor capital,
$i=B$ indexes the factor price for agricultural land;

a represents a non-negative vector of M activity levels, indexed by k, for constant-return-to-scale production sectors in the economy:

a_k with $k=1,...,M$;

a_k is divided into four subsets: the industrial production, Y, with J industry sectors, indexed by j, the investment sector, Inv, the consumption sector, C, and the government sector, G:[2]

[2] The consumption and the government activity are interpreted as production activities as well. For further explanation see the subsequent section.

$$a = \begin{bmatrix} Y_j \\ Inv \\ C \\ G \end{bmatrix} \qquad \text{with } j=1,...,J \qquad \text{and} \qquad J=M\text{-}3;$$

Inc denotes a vector of *H* income levels, indexed by *r*, one for each household including the government. The model assumes a representative consumer for each region, which comprises the total regional private consumption. Hence, the index *r* denotes a region in the model.

Inc_r with $r=1,...,H$

Therefore, $z = [p, a, Inc]^T$. Under perfect competition, these decision variables constitute an equilibrium if they satisfy a system of the above mentioned three classes of nonlinear inequalities: the zero profit condition, the market clearance condition, and the income balance condition. The complementarity problem (CP) states then that in equilibrium to each decision variable, there corresponds an equilibrium condition, $F(\cdot)$, and each of the variables, z, must be nonnegative. And, in optimum each variable is characterized by complementary slackness, $z^T F(z) = 0$. This means that, for each decision variable, we must find in the optimal solution that either the equilibrium condition holds as an equality or the decision variable in question must take a value of zero, or both.

Hence, in the complementarity format, the equilibrium problem is defined as a system of weak inequalities and complementary slackness conditions. This system is equivalent to the first order necessary conditions of a general equilibrium problem.

For further details of definition and proof of competitive equilibria see for example Shoven and Whalley (1992), Rutherford (1987), Mathiesen (1985). Woodland (1982, pp. 134) derives the general equilibrium conditions in terms of unit value added cost functions for the case of joint output and intermediate input.

Compared to the standard optimization approach the formulation of an equilibrium problem in the complementarity format has the advantage that it represents the general equilibrium conditions and features in its most general form.[3] The complementarity format allows for weak inequalities and complementary slackness. This can be used to implement price and quantity constraints into the model.

In this paper, the dual form is used for presenting the equations, i.e. the independent variable is the price and not the quantity as in the primal case.[4] Furthermore, the duality between cost and production functions on the production side, and between expenditure and utility function on the consumption side is exploited.[5] Hereafter, the equilibrium conditions, $F(\cdot)$, are specified in more detail.

Zero Profit Condition: [6]

The zero profit condition holds for all activities: industrial production, private consumption, public consumption and investment. The producers minimize production costs in order to get a certain value of output. Consumption is determined by the household's decisions in order to maximize the utility subject to a budget constraint, i.e. the disposal income, or to minimize expenditures in order to achieve a certain utility level.

Let $y_{\hat{i}}$ and x_i denote output and input quantities respectively with $p_{\hat{i}}^y$ and p_i^x the corresponding prices, where the index $i=1,...,L,K,B,...,N$ for the factors and intermediate inputs, and the index $\hat{i}$ goes from 1 to q with $q=N\text{-}L,K,B$. The economic agents are minimizing cost or expenditures, $\sum x_i p_i^x$, respectively

[3] For a general discussion of alternative methods for the formulation of an economic equilibrium see Mathiesen (1987) or Böhringer (1996).
[4] For further details about the concept of duality, proofs etc. see Diewert (1982), Varian (1992), Cornes (1992).
[5] In the primal formulation preferences and technology are represented by utility and production function, whereas in the dual form they are represented by expenditure and cost functions. Cornes (1992) gives a good overview about these terms.
[6] The amplifications of the three equilibrium conditions are based on Woodland (1982, pp. 136) who presents the equilibrium conditions by using the concepts of a value added function, and a unit value added cost function.

under competitive factor markets. The production function in industry sector j $y_j = f\left(x_{j,1},\ldots,x_{j,i},\ldots,x_{j,n}\right) = f_j(x)$ is assumed to be a strictly increasing, twice continuously differentiable, quasi-concave function, possessing a unique dual cost function $C\left(p^x, y^*\right)$. The cost function is twice continuously differentiable, concave and linear-homogenous in p^x, and increasing with y from zero to infinity.

Of all four activity sectors only the industry production sectors Y_j are permitted to produce joint output. By joint production is meant the situation where the technology facing the industry sector cannot be described by separate production functions for each of the produced goods. Therefore, the technology for the production of a good is influenced by the amount of another good being produced. Hence, an industry sector can produce more than one output. The output of sector j, y_j, is a vector of q produced goods in industry sector j:

$$y_j = \begin{bmatrix} y_{j,1} & \cdots & y_{j,\hat{i}} & \cdots & y_{j,q} \end{bmatrix}^{\mathrm{T}}.$$

Besides the j industry sectors, an investment activity sector, denoted by *Inv*, a government activity sector, denoted by G, and a consumption activity sector, denoted by C, are distinguished.

Furthermore, it is assumed that, if y_j is a vector of q outputs and x a vector of N inputs, the production frontier is separable between outputs and inputs. This is the case, if y_j is weakly separable in F, i.e. there exists a function $g(y_j)$ such that

$$F\left(y_j, x\right) = F\left[g\left(y_j\right), x\right] \equiv 0$$

with $\partial F / \partial g > 0$. Since $g(y_j)=f(x)$ may be solved from the latter equation applying the implicit function theorem (cf. Chiang 1984, pp. 206), this implies strong separability as well. The aggregate $\tilde{y}_j=g(y_j)$ may be substituted for a single output, $\tilde{y}_j$, due to the separability assumption between outputs and inputs and the assumed functional form of constant elasticity of substitution or transformation respectively. This implies that all transformations are applicable in analogy to the one-output case. (cf. Fuss and McFadden 1978, p. 302)

The behavior of the agents (producers, consumers, government) can be described in compact form by the profit function. We can think of the agents as choosing a quantity of output, i.e. the quantity level of an activity k in region r[7], $a_{k,r}$, to maximize profits. At any point $\left(y^*, x^*\right)$ the profit function gives the maximum profits Π_k at given prices $\left(p^y, p^x\right)$, while costs are minimized, subject to fixed y^*,[8] and revenue maximized for fixed x^* (cf. Fuss and McFadden 1978, p. 292). Hence the profit function $\Pi_{k,r}$ is defined as:

$$\Pi_{k,r}\left(p^y, p^x\right) = R_{k,r}\left(p^y, x^*\right) - C_{k,r}\left(p^x, y^*\right).$$

The profit function $\Pi_{k,r}\left(p^y, p^x\right)$ for activity k is the difference between the revenue, $R_{k,r}\left(p^y, x^*\right)$, and the cost function, $C_{k,r}\left(p^x, y^*\right)$. Linear homogeneity of profits in prices is assumed, that means doubling all prices doubles money profits.

Because of the constant returns to scale assumption in production the profit function may be written as follows (see Varian 1992, p.67):

$$\Pi_{k,r}\left(p^y, p^x\right) = a_{k,r} \cdot r_{k,r}\left(p^y, x^*\right) - a_{k,r} \cdot c_{k,r}\left(p^x, 1\right) = a_{k,r} \cdot \pi_{k,r}\left(p^y, p^x\right)$$

where $r_{k,r}\left(p^y, x^*\right)$ denotes the unit revenue function, $c_{k,r}\left(p^x, 1\right)$ the unit cost function and hence $\pi_{k,r}\left(p^y, p^x\right)$ the unit profit function. $a_{k,r}$ is the associated dual activity level of the profit maximization problem, i.e. the profit maximizing

[7] In the subsequent section all equations are region-specific. For simplification the reference to the region r is omitted.

[8] The cost function is defined for a fixed vector y_j, hence a fixed scalar $\tilde{y}_j^* = g(y_j)$. The same applies to the revenue function. (Fuss and McFadden 1978, p. 302)

net (joint) output of activity k (cf. foot note 8). The unit revenue and the unit cost function of activity k are defined as:

$$r_{k,r}\left(p^{y},x^{*}\right)\equiv \max_{y}\left\{\sum_{\hat{i}=1}^{q} p_{\hat{i},r}^{y}\cdot y_{\hat{i},r}\middle|\tilde{a}_{k,r}=g_{k,r}\left(y_{j}\right)=1\right\}$$

$$c_{k,r}\left(p^{x},1\right)\equiv \min_{x}\left\{\sum_{i=1}^{n} p_{i,r}^{x}\cdot x_{i,r}\middle|f_{k,r}(x)=1\right\}$$

where g and f denote the associated production functions characterizing the feasible output y_j and input x per unit of activity k. The function g applies only in the case of joint output (i.e. q>1). The revenue function gives the maximum revenue per unit of activity k at fixed input quantity vector x^* when output prices p^y are given. The revenue function is linear homogeneous and convex in p^y. The cost function gives the minimum cost of producing one unit of joint output $\tilde{y}_{j,r}=g_r\left(y_j\right)$ in industry sector j when input prices p^x are given (cf. Fuss and McFadden 1978, p. 292).

The dual decision problem for the household is to minimize expenditure in order to achieve a given utility.[9] In this case $c_{k,r}\left(p^{x},1\right)$ can be interpreted as the unit expenditure function and $r_{k,r}\left(p^{y},x^{*}\right)$ as the utility valued with the marginal cost of utility. The expenditure function gives the minimum cost of achieving a fixed level of utility (Varian 1992, p. 104). Utility can be thought of as a kind of output resulting from consuming goods, i.e. the output of consumption.

Maximizing profits with respect to $a_{k,r}\geq 0$ yields an unbounded solution when $c_{k,r}\left(p^{x},1\right)<r_{k,r}\left(p^{y},x^{*}\right)$, i.e. when it costs less to produce one unit of aggregate output of activity k than one can earn for that unit. This unbounded solution is inconsistent with perfect competition; so it is ruled out. Thus,

[9] For deriviation of the profit function for the consumption side see Cornes (1992, p. 164).

$c_{k,r}(p^x,1) \geq r_{k,r}(p^y,x^*)$ and the solution for $a_{k,r}$ must satisfy the Kuhn-Tucker conditions:

$$\text{(I)} \qquad c_{k,r}(p^x,1) - r_{k,r}(p^y,x^*) \geq 0 \qquad \text{and} \qquad 0 \leq a_{k,r} \; ; \qquad k = 1,\ldots,M$$

If $c_{k,r}(p^x,1) > r_{k,r}(p^y,x^*)$, then condition (I) indicates that the activity level of sector k is zero , $a_{k,r} = 0$, whereas if $c_{k,r}(p^x,1) = r_{k,r}(p^y,x^*)$, then any $a_{k,r} \geq 0$ maximizes profit. Thus, in competitive equilibrium no activity *k* earns an excess profit, i.e. the value of inputs per unit of activity must be equal or greater than the value of outputs (zero profit condition).

Market clearance condition

The second class of equilibrium conditions deals with market clearance. Two types of markets are distinguished: the factor market and the good market.

Applying Hotelling's lemma to the unit profit function (i.e. differentiating the unit profit function with respect to factor prices; Varian 1992, p. 43), the factor market clearance condition can be derived:

$$\text{(IIa)} \qquad \sum_k -\frac{\partial \pi_{k,r}(p^y,p^x)}{\partial p^x_{i,r}} a_{k,r} \leq \omega_{i,r} \qquad i=L,K,B \qquad \text{and} \qquad p^x_{i,r} \geq 0$$

where $-\partial \pi_{k,r}(p^y,p^x)/\partial p^x_{i,r}$ is the demand for factor *i=L,K,B* per unit of aggregate output in sector *k*. $a_{k,r}$ denotes the activity level of joint output in sector *k* which is derived from the profit maximizing condition (I). Hence the factor market clearance conditions (IIa) says that at equilibrium prices and equilibrium activity levels, the total demand of factor *i* must balance or fall short of the factor endowment $\omega_{i,r}$ supplied in region *r*.

The final good output of industry sector *j=1* is demanded by the other *J-1* industry sectors as intermediate input, by the household, the government , or the investment sector. The market clearance condition says then that at equilibrium prices and equilibrium activity levels, the supply of any good must balance or exceed the demand by agents. By applying Hotelling's lemma (i.e. differentiating the unit profit function with respect to price of good $\hat{\imath}$; Varian 1992, p. 43), the market clearance condition for good $\hat{\imath}$ (excluding the primary factors *L,K,B*) can be expressed as:

(IIb) $$\sum_{j=1}^{J} \frac{\partial \pi_{j,r}\left(p^y, p^x\right)}{\partial p_{\hat{\imath},r}} \tilde{y}_{j,r} \geq d_{\hat{\imath},r}\left(p^y, Inc_r\right) \qquad \text{and} \qquad p_{\hat{\imath},r} \geq 0$$

with $\hat{\imath}=1,\dots,q$ and $q=N\text{-}L,K,B$

where $\partial \pi_{j,r}\left(p^y, p^x\right)/\partial p_{\hat{\imath},r}$ is the optimal net supply of good $\hat{\imath}$ per unit of activity of the constant-returns to scale industry sector *j*. The left-hand-side of the constraint then gives the net output of good $\hat{\imath}$ for the whole economy where each term in the sum is the net output or net input of good $\hat{\imath}$ in sector *j* respectively.[10] The-right-hand-side denotes the uncompensated market demand by the household (including the government) in region *r* for good $\hat{\imath}$, $d_{\hat{\imath},r}\left(p^y, Inc_r\right)$, given market prices p^y and income levels *Inc*, i.e. the Marshallian demand function.[11]

The uncompensated demand is derived from the budget-constrained utility maximization:[12]

$$d_{\hat{\imath},r}\left(p^y, Inc_r\right) = arg\,max\left\{ U_r\left(y_{\hat{\imath}}\right) \middle| \sum_{\hat{\imath}} p^y_{\hat{\imath},r} \cdot y_{\hat{\imath},r} \leq Inc_r \right\}$$

[10] Note that the intermediate inputs are subtracted due to the negative sign by differentiating the unit profit function with respect to input prices (Hotelling's lemma). Hence, we get the *net* supply.
[11] The market clearance condition for the investment good, *cgd*, is not included here. But without loss of generality it can be assumed that the index *j* includes the investment sector *Inv*. Then, the investment good *cgd* is demanded as intermediate input by all other industry sectors *j* and is included in the sum on the left-hand side of the inequation. For further details see the concrete model description below.
[12] The market demand function can be derived by the Lagrangian for the utility maximizing problem or by using Roy's identity for the indirect utility function that gives the maximum utility achievable at given prices and income (Varian 1992, pp. 99).

in which U_r is the utility function of the household in region r. Market demand depends on all output prices, and is continuous, nonnegative, and homogenous of degree zero (i.e. there is no money illusion).

The market clearance conditions for factors and goods (IIa and IIb) can be integrated into one inequality constraint and written as:

$$\text{(II)} \qquad \sum_{k=1}^{M} \frac{\partial \pi_{k,r}\left(p^y, p^x\right)}{\partial p_{i,r}} a_{k,r} \geq \xi_{i,r}(p) \qquad \forall i$$

$$\text{with} \qquad \xi_{i,r}(p) = d_{i,r}\left(p^y, Inc_r\right) - \varpi_{i,r} \qquad \text{and} \qquad p_{i,r} \geq 0$$

where $\partial \pi_{k,r}\left(p^y, p^x\right)/\partial p_{i,r}$ gives the optimal input demand and output supply of commodity i per unit of activity k by exploiting Hotelling's lemma (remember that $\left[\partial \pi_{k,r}\left(p^y, p^x\right)/\partial p^y_{i,r}\right] a_{k,r} = y_{i,r}$ gives the output supply, $y_{i,r}$, and $\left[\partial \pi_{k,r}\left(p^y, p^x\right)/\partial p^x_{i,r}\right] a_{k,r} = -x_{i,r}$ the input demand, $x_{i,r}$, of activity k). The sum on the left-hand-side of the constraint is then the economy wide net output or net input of commodity i (including factors, intermediate and final goods) respectively in region r. The-right-hand-side represents the excess demand of the household in region r for commodity i, $\xi_{i,r}(p)$, which is defined as the difference between the uncompensated market demand for good i by household in region r, $d_{i,r}\left(p^y, Inc_r\right)$, and the endowment of factor i=L,K,B associated with the household in region r, $\varpi_{i,r}$.

Income balance

The third condition says that in equilibrium the value of each household's income must equal the value of factor endowments (income definition). This is written as:

$$\text{(III)} \qquad Inc_r = \sum_{i=L,K,B} \omega_{i,r} p^x_{i,r}$$

As usual, non-satiation for the utility function is assumed, which implies that the excess demand function will always satisfy Walras' law, i.e. for any set of prices, the value of excess demand is equal to zero. In other words, the value of a consumer's expenditures equals that consumer's income (cf. Varian 1992, p. 317)):

$$\sum_i \xi_{i,r}(p) \cdot p_{i,r} = 0 \qquad\qquad \forall p_{i,r}.$$

By exploiting the definition of excess demand, $\xi_{i,r}$, and equation (III), Walras' law can also be written as:

$$\sum_{\hat{i}} d_{\hat{i},r}\left(p^y, Inc_r\right) \cdot p^y_{\hat{i},r} = \sum_{i=L,K,B} \omega_{i,r} p^x_{i,r} = Inc_r$$

which means that the value of consumer's expenditures exhausts the consumer's budget.

Referring to the complementary format (CP) above, *F(z)* can then be written as:

$$F\begin{bmatrix} p \\ a \\ Inc \end{bmatrix} = \begin{bmatrix} a_{k,r}\dfrac{\partial \pi_{k,r}(p)}{\partial p} - \xi(p) \\ \pi_r(p) \\ \omega p - Inc \end{bmatrix}.$$

When excess demand satisfies Walras' law (III) and the zero profit condition (I) and the market clearance condition (II) are satisfied using equilibrium prices and activity levels, it then follows that:

$$\text{(IV)} \qquad p_{i,r}\left(\sum_{k=1}^{M} \frac{\partial \pi_{k,r}\left(p^y, p^x\right)}{\partial p_{i,r}} a_{k,r} - \xi_{i,r}(p)\right) = 0 \qquad\qquad \forall i$$

(V) $$a_{k,r} \cdot \pi_{k,r}\left(p^{y}; p^{x}\right) = 0 \qquad \forall k$$

(VI) $$Inc_r \cdot \left(\sum_{\hat{i}} d_{\hat{i},r}\left(p^{y}, Inc_r\right) \cdot p^{y}_{\hat{i},r} - \sum_{i=L,K,B} \omega_{i,r} p^{x}_{i,r} \right) = 0 \qquad \forall r$$

or in (CP) Format: $z^{\mathbf{T}} F(z) = 0.$

Therefore, Walras' law implies complementarity between decision variables and the three conditions.[13] First, equation (IV) shows that commodity prices (including final and intermediate goods, and primary factors), $p_{i,r}$, exhibit complementary slackness with the market clearance condition (II). This means that in equilibrium any factor or good which commands a positive price has a balance between market supply and demand, and that there are zero prices on commodities which are in excess supply. Second, equation (V) says that an activity level, $a_{k,r}$, is complementary to the zero profit constraint (I), i.e. in equilibrium an operating production activity makes zero profit, and there are zero activity levels on activities with negative profits. The third equation (VI) shows, that a consumer's income variable is complementary to the income definition equation (III), i.e. positive incomes are associated with a balanced budget. (cf. Hoster et al. 1997, pp. 35)

Due to the homogeneity properties of cost and demand functions, the solution is not uniquely determined; only relative prices matter. A convenient normalization is to fix the income value of one regional household (Inc=1) and omit the income constraint (VI). (Lau et al. 1997)

In the subsequent sections the ART model will be described by these three classes of conditions, which characterize an economic equilibrium. An equilibrium allocation determines market production, prices, and incomes.

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[13] Note that complementary slackness is a feature of the equilibrium allocation and not an a priori equilibrium condition.

3.2 Production Behavior

Producer behavior is characterized by cost minimization for a given output in sector j, $\tilde{y}_j = g(y_j)$. Markets are perfectly competitive and output and factor prices are fully flexible. All industry sectors are assumed to operate at constant-returns-to-scale and share a common production structure. That means, the functional form of the production function, and therefore of the cost function, is the same for each industry sector, but the value shares differ between industry sectors depending on the benchmark dataset.

For each industry, a multi-level nested separable constant elasticity of substitution (CES) function describes the technological substitution possibilities in domestic production.[14] Figure 1 shows the nested production structure. The top level of the production function is a linear function, i.e. Leontief function, of non-energy intermediate goods and a value added composite. The intermediate input of good i in sector j corresponds to a so-called Armington aggregate of non-energy inputs from domestic production and imported varieties (see section 3.4). The value added composite is a CES function of the energy aggregate and the aggregate of the primary factors. The substitution possibilities of the primary factors are described by a Cobb-Douglas function on the lowest level of the production function. On the output side, the output in sector j, $\tilde{y}_j$, represents composite production of domestically used goods, d_i, and exported goods, x_i[15]. Therefore, products destined for domestic and international markets are treated as imperfect substitutes produced subject to a constant elasticity of transformation (see section 3.4).

Profit maximization under constant returns to scale implies that revenue, $Py \cdot Y$, equals cost. The unit cost function is a nested CES function and dual to the

[14] The nesting structure and nest elasticities of the production cost functions are based on the ETA-MACRO model (See Manne and Richels 1992, pp. 130).
[15] Because we are in the explicit description of the CGE model now, x_i denotes no longer inputs but exported goods.

production function. In equilibrium, competition eliminates excess profits and the resulting intra-period zero profit condition for production of one unit of composite joint output of good *i* in sector *j* of region *r* holds:

$$\pi_{j,r}^{Y} = Py_{i,r} - \sum_{i \in NE} b_{i,j,r}^{a} Pa_{i,j,r} - b_{j,r}^{EKLB}\left[\theta_{j,r}^{E} Pe_{j,r}^{1-\sigma_{EKLB}} + \left(1-\theta_{j,r}^{E}\right)\left(Pl_r^{\beta_L} \cdot Pk_r^{\beta_K} \cdot Pb_{Ag,r}^{(1-\beta_L-\beta_K)}\right)^{1-\sigma_{EKLB}}\right]^{\frac{1}{1-\sigma_{EKLB}}} = 0 \qquad (3.1)$$

with: $$Py_{i,r} = \left(\theta_{j,r}^{y} Px_{i,r}^{1-\tau} + \left(1-\theta_{j,r}^{y}\right) Pd_{i,r}^{1-\tau}\right)^{\frac{1}{1-\tau}}$$

where:

$\pi_{j,r}^{Y}$ [16]	is the unit profit function of industry sector *j* in region *r*,
$Py_{i,r}$	is the output price of the composite good *i* in region *r*,
$b_{i,j,r}^{a}$	is the benchmark value share of non-energy (*NE*) input *i* within the Leontief aggregate of Armington goods, denoted by *a*, in sector *j* of region *r*,
$Pa_{i,j,r}$	represents the Armington price aggregate of the non-energy (*NE*) intermediate input *i* in industry sector *j* of region *r*,
$b_{j,r}^{EKLB}$	is the aggregate value share of energy, *E*, capital, *K*, labor, *L*, and agricultural land, *B*, inputs (*EKLB* aggregate) in sector *j* of region *r* in the Leontief nest,
$\theta_{j,r}^{E}$	stands for the energy composite input value share in the CES-*EKLB* aggregate of sector *j* in region *r*,
$Pe_{j,r}$	is the composite price of energy composite inputs into industry sector *j* in region *r*,

[16] The notation $\pi_{j,r}^{Y}$ is used to denote the unit profit function of industry sector *j* in region *r*, where *j* (index of production activity *Y*) stands as an example for an activity *k* and the subscript *Y* denotes the considered subset of the activity, here the production.

σ_{EKLB} is the elasticity of substitution between the energy aggregate and the aggregate of capital, labor and agricultural land,

Pl_r represents the real wage rate in region r,

Pk_r is the rate of return on capital in region r,

$Pb_{Ag,r}$ is the real factor price for land in agricultural sectors, denoted by Ag, in region r,

β_z denotes the value shares of labor ($z=L$) and capital ($z=K$) in the value-added Cobb-Douglas aggregate of region r,

$\theta^y_{j,r}$ represents the export value share of the composite output in sector j of region r,

$Pd_{i,r}$ is the domestic output price of good i in region r,

$Px_{i,r}$ is the export price of good i in region r,

τ denotes the elasticity of transformation between production for the domestic and production for the export market,

$\tilde{y}_{j,r}$ is the associated complementary aggregate which indicates the activity level of joint production in industry sector j in region r.

Figure 2 shows the production structure of the investment activity. In each region, composite investment is a Leontief aggregation of Armington inputs (see section 3.4) by each industry sector. There is no sector-specific investment activity in this version of the model. The ART model does not contain cross border investment activities, i.e. the investment good is treated as a non-tradable. Therefore, the price index for regional investment activity, $Py_{cgd,r}$, is equal to the price index of the domestically produced investment good, $Pd_{cgd,r}$, in this region. Investment does not require direct primary factor inputs. The resulting unit zero profit condition for the investment activity, Inv, in region r is given by:

(3.2) $$\pi_r^{Inv} = Pd_{cgd,r} - \sum_i b^a_{i,Inv,r} Pa_{i,Inv,r} = 0 \qquad i \neq L,K,B$$

where:

π_r^{Inv} is the unit profit function for the investment activity, *Inv*, in region *r*

$Pd_{cgd,r}$ is the price index, i.e. the domestic price index, for the investment good *cgd* in region *r*,

$b_{i,Inv,r}^{a}$ is the fixed value share of Armington good input *i* in the investment sector *Inv*,

$Pa_{i,Inv,r}$ represents the Armington price aggregate of input good *i* in the investment sector *Inv* of region *r*,

Inv_r is the associated complementary variable which indicates the activity level of the investment sector in region *r*.

3.3 Consumption and Government Expenditure

Beside the industry sectors, there are two further agents in the ART model: private households and governments.

Each region has a representative household. The household wants to realize a certain consumption level with the lowest possible expenditures. The household chooses among various commodity bundles, depending on relative prices, to achieve this consumption level.

Figure 3 shows the structure of household and government behavior. The expenditure function of the representative household is assumed to be a CES composite which combines consumption of an energy aggregate and a non-energy bundle. Within the non-energy consumption composite, substitution possibilities are described by a Cobb-Douglas function of Armington goods (see section 3.4). The resulting unit zero profit condition for „producing" consumption of the private household, *C*, in region *r* is given by:

$$(3.3) \quad \pi_r^C = Pc_r - \left\{ \theta_{C,r}^E Pe_{C,r}^{1-\sigma_{ec}} + \left(1-\theta_{C,r}^E\right)\left[\prod_{i \in NE} Pa_{i,C,r}^{\beta_{i,r}}\right]^{1-\sigma_{ec}} \right\}^{\frac{1}{1-\sigma_{ec}}} = 0$$

with $\sum_{i \in NE} \beta_{i,r} = 1$

where

π_r^C	is the unit profit function for the private consumption activity, *C*, in region *r*,
Pc_r	is the composite price for consumption demand in region *r*,
$\theta_{C,r}^E$	represents the energy composite *(E)* input value share in the CES aggregate of the expenditure function in region *r*,
$Pe_{C,r}$	is the price of the energy composite in the consumption sector *C* in region *r*,
σ_{ec}	is the elasticity of substitution between the energy aggregate and the non-energy Armington composite in the consumption sector,
$Pa_{i,C,r}$	represents the Armington price aggregate of the non-energy (*NE*) good *i* in the consumption sector *C* in region *r*,
$\beta_{i,r}$	denotes the value share of Armington good *i* in the Cobb-Douglas consumption aggregate of region *r*,
C_r	is the associated complementary variable which indicates the activity level of consumption in region *r*.

The government provides a public good which is produced with commodities purchased at market prices. The public good is produced with the same two level nesting structure as the household „production" function (see Figure 3); that means, on the top level there is a CES composite of energy and non-energy bundles and on the second level is a Cobb-Douglas aggregation of non-energy Armington goods. The corresponding zero profit condition for the provision of one unit of public good *G* in region *r* is given by:

$$(3.4) \quad \pi_r^G = Pg_r - \left\{ \theta_{G,r}^E Pe_{G,r}^{1-\sigma_{eg}} + \left(1-\theta_{G,r}^E\right) \left[\prod_{i \in NE} Pa_{i,G,r}^{\beta_{i,r}} \right]^{1-\sigma_{eg}} \right\}^{\frac{1}{1-\sigma_{eg}}} = 0$$

with $\sum_{i \in NE} \beta_{i,r} = 1$

where:

π_r^G	is the unit profit function for the provision of public good G in region r,
Pg_r	is the composite price for government demand in region r,
$\theta_{G,r}^E$	represents the energy composite (E) value share in region r for public good provision,
$Pe_{G,r}$	is the price of the energy composite in the government sector G in region r,
σ_{eg}	is the elasticity of substitution between the energy aggregate and the non-energy composite in the government sector,
$Pa_{i,G,r}$	represents the composite Armington price of the non-energy good i in the government sector G in region r,
$\beta_{i,r}$	denotes the Cobb-Douglas value share of the non-energy Armington good i in region r,
G_r	is the associated complementary variable which indicates the level of government activity G in region r.

3.4 Foreign trade

After having described the economic behavior of the agents in one region, we now open the regional economy and come to the trade relations among regions. The world is divided into economic regions, which are linked by bilateral trade flows. Following the proposition of Armington (1969), domestic and foreign goods are imperfect substitutes, and distinguished by country of origin. Despite the imperfections and shortcomings of the Armington assumption[17], it

[17] For example, Haaland et al (1988, pp. 36) criticize the Armington approach for the following reasons: First, the Armington assumption introduces an arbitrary rigidity into production and trade patterns , thus assuming a particular division of labor between countries rather than explaining it. Second, the empirically estimated elasticities in Armington-type models are typically low which tends to produce large price effects, thereby giving even smaller countries a substantial influence on their terms of trade. Therefore, in CGE modeling Armington elasticities are usually adjusted to reasonable trade reactions due to changes in relative prices. Sensitivity analysis on Armington elasticities are necessary. Third, the Armington approach is inconsistent with microeconomic

accommodates intra-industry trade and is convenient in picturing world trade flows by employing only one exogenous parameter, which is the Armington elasticity of substitution.

In the ART model, foreign trade is modeled by expanding the activity vector and adding an Armington activity, denoted by *A*, an import activity, denoted by *M*, and an international transport activity, represented by *IT*. The Armington activity describes the production of an Armington good, which is a composite of domestically produced and imported goods. The import of one region *r* is also an aggregate which comprises the imports from all other regions. That means, the imports distinguished by regions are imperfect substitutes, too. This fact is modeled by the import activity. The international transport activity determines the transport costs entailed by foreign trade.

Three Armington good markets per region *r* are distinguished in the ART model: one for intermediate demand by the industry production sector and the investment sector, *I*, one for final demand of the private consumption sector, *C*, and one for final demand of the government sector, *G*. On these markets domestic and imported varieties of the same commodity *i* are aggregated to equate final demand. Therefore, there are different import value shares for industrial intermediate demand (including investment), final demand of private households and final demand of governments.

Hence, for each Armington commodity *i*, *A* represents a non-negative vector of Armington activity levels, indexed by *s*; *s* includes three elements: the demand for intermediate goods, *I*, which comprises the intermediate demand by the industrial production activity and the investment activity, the private consumption demand, *C*, and the public consumption (i.e. government) demand,

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theory in imperfectly competitive industries. The third argument does not apply in the ART model because perfect competition is assumed.

G. Hence, Armington activities in region r are denoted by $A_{i,s,r}$ with $s=I,C,G$.[18]

As described above, import demand stems from cost minimizing producer behavior and expenditure minimizing household or government behavior. Import demand is derived from a three stage, nested, separable CES cost or expenditure function respectively. The structure of foreign trade is shown in Figure 4. On the top level, the import aggregate and the domestic output of good i can be substituted with a constant elasticity. The resulting zero profit condition for the production of a unit of the Armington good i in region r used by the agent (activity sector) s, $a_{i,s,r}$, is:

$$(3.5) \qquad \pi^A_{i,s,r} = Pa_{i,s,r} - \left(\theta^M_{s,r} Pm_{i,r}^{1-\sigma_{DM}} + \left(1-\theta^M_{s,r}\right) Pd_{i,r}^{1-\sigma_{DM}}\right)^{\frac{1}{1-\sigma_{DM}}} = 0$$

$s=I,C,G$

where:

$\pi^A_{i,s,r}$	is the unit profit function for the production of the Armington good i in region r used by the activity sector s, $a_{i,s,r}$,
$Pa_{i,s,r}$	denotes the price of the composite Armington good i in region r, which differ among demand types s, where s comprises intermediate demand by industry and investment sectors, I, (cf. foot note 18) private (household) consumption, C, and public (government) consumption, G,
$\theta^M_{s,r}$	represents the value share of the import aggregate, M, in the overall value of the Armington aggregate s in region r,
$Pm_{i,r}$	is the import price index of good i imported to region r,
σ_{DM}	is the elasticity of substitution between domestic and imported goods,
$Pd_{i,r}$	is the domestic price of good i in region r,

[18] Because there is only one Armington good market for intermediates, there is also only one Armington price index for each good i no matter by which industry sector j or the investment sector Inv this good i is demanded. Therefore, $Pa_{i,j,r}=Pa_{i,Inv,r}=Pa_{i,I,r}$.

$A_{i,s,r}$ is the associated complementary variable which indicates the quantity level of producing an Armington good i in region r for each demand type s.

On the second level of the CES cost function of foreign trade the import activity, M, is described which produces a composite import good, m. This import composite of a good i into region r, $m_{i,r}$, is an aggregate of imports from all other regions rr with $RR=H-r$ where rr is an alias for r. These imports of good i from all other regions rr can be substituted for each other with a constant elasticity. The imports of one region r are equivalent to the exports, X, of all other regions rr into that region r including transport. Transport costs, distinguished by commodity and bilateral flow, apply to international trade but not to domestic sales. The exports are connected to transport costs by a Leontief function. The corresponding zero profit condition for the production of one unit import composite m of good i to region r, $m_{i,r}$, is given by:

$$\text{(3.6)} \qquad \pi_{i,r}^{M} = Pm_{i,r} - \left[\sum_{rr \neq r} \theta_{i,rr}^{MM} \left(b_{i,rr}^{ex} Px_{i,rr} + b_{i,rr}^{t} PT \right)^{1-\sigma_{mm}} \right]^{\frac{1}{1-\sigma_{mm}}} = 0$$

where:

$\pi_{i,r}^{M}$ is the unit profit function of the import composite m of good i of region r, $m_{i,r}$,

$Pm_{i,r}$ is the import price index of good i imported to region r,

$\theta_{i,rr}^{MM}$ represents the benchmark value share of the export from region rr into region r in the whole import aggregate of good i,

$b_{i,rr}^{ex}$ is the Leontief value share of the exported good i in the export aggregate of region rr into region r relative to transport costs,

$Px_{i,rr}$ is the export price of good i from region rr where rr is different from r, the importing region,

$b_{i,rr}^{t}$ is the fixed value share of international transport costs of good i in the export aggregate from region rr,

PT is the price index for international transport activity,

σ_{mm} is the elasticity of substitution between imports from different countries,

$M_{i,r}$ is the associated complementary variable which indicates the activity level of demand in region r for the aggregate import variety of commodity i.

Transport costs which are connected to international trade are considered by specifying an international transport activity, *IT*. International transports are treated as a worldwide activity which is financed by domestic production proportionately to the trade flows of each commodity. There is no special sector for transports related to international trade in the model. Therefore, the world market price index for international transport services is a Cobb-Douglas aggregate of domestically produced goods over all commodities and regions. The zero profit condition for one unit of international transport activity, *IT*, is given by:

$$(3.7) \qquad \pi^{IT} = PT - \prod_{r=1}^{H}\prod_{i=1}^{N} Pd_{i,r}{}^{\beta^t_{i,r}} = 0 \qquad \text{with } \sum_i \beta^t_{i,r} = 1$$

where:

π^{IT} is the unit profit function of the interantional transport activity,

PT is the price index for international transportation activity,

$Pd_{i,r}$ is the domestic price of good i in region r,

$\beta^t_{i,r}$ denotes the Cobb-Douglas value shares of good i in region r,

IT is the associated complementary variable which indicates the level of international transport activity.

On the export side, the Armington assumption applies to final output of the industry sectors destined for domestic and international markets (see equation 3.1). Here, produced commodities for the domestic and for the international market are no perfect substitutes. Exports are not differentiated by country of destination.

3.5 Market Clearance, Income Balance and Closures

After having defined the zero profit conditions for all activities in the ART model, now the market clearance conditions and the income balance will be derived by using Hotelling's lemma.

Factor markets are perfectly competitive and full employment of all factors is assumed. Hence, factor prices adjust so that supply equals demand.

Labor is assumed to be a homogenous good, mobile across industries within regions but internationally immobile. The equilibrium condition of the solution requires that the sum of all sectoral demands for labor is equal to the exogenous labor supply in each region. Hence, the intra-period supply-demand balance for the labor market is written:

$$(3.8) \qquad \bar{L}_r = -\sum_j \tilde{y}_{j,r} \frac{\partial \pi^Y_{j,r}}{\partial Pl_r}$$

where $\bar{L}_r$ denotes the exogenously given labor supply in region r for each time period.[19]

In this version of the ART model, capital is inter-sectorally but not internationally mobile. There is no sector-specific capital. The capital stock is given at the beginning of each time period and results from the capital accumulation equation (see section 4.2). In every time period the regional capital stock earns a correspondent amount of income measured as physical units in terms of capital services. The supply-demand balance for capital in one time period is written:

$$(3.9) \qquad K_r = -\sum_j \tilde{y}_{j,r} \frac{\partial \pi^Y_{j,r}}{\partial Pk_r}$$

[19] The bar over the variable name indicates the exogeneity.

where K_r is the aggregate supply of capital services of region r for domestic production in one time period. (For derivation of capital supply cf. section 4.2 and 6.)

The primary factor land is only used in agricultural sectors, Ag, and exogenously given. The intra-period supply-demand balance for land is written:

$$(3.10) \qquad \overline{B}_r = -\sum_{Ag} \tilde{y}_{Ag,r} \frac{\partial \pi^Y_{Ag,r}}{\partial Pb_{Ag,r}} \qquad \text{with} \qquad Ag \subset j$$

where $\overline{B}_r$ denotes the exogenous supply of land in agricultural sectors, i.e. Ag is a subset of the sector set j.

Besides the factor markets, there are three Armington good markets, s, in the model; one for each demand type: the intermediate demand in industry production and investment, I, and the final demand by the private consumption sector, C, and the public consumption (government) sector, G.

The intra-period market clearance condition for the Armington composite used as an intermediate good by the industry sectors j and the investment sector Inv is written as:

$$(3.11a) \qquad A_{i,I,r} = -\left[\sum_j \tilde{y}_{j,r} \frac{\partial \pi^Y_{j,r}}{\partial Pa_{i,I,r}} + Inv_r \frac{\partial \pi^{Inv}_r}{\partial Pa_{i,I,r}}\right]$$

The supply of the Armington intermediate good i in region r, $A_{i,I,r}$, equals the total intermediate demand for this Armington good i in all industry sectors j and in the investment sector Inv. Remember that $Pa_{i,I,r} = Pa_{i,j,r} = Pa_{i,Inv,r}$.

The market clearance condition for the Armington commodity demanded by the private household is:

(3.11b) $$A_{i,C,r} = -C_r \frac{\partial \pi_r^C}{\partial Pa_{i,C,r}}$$

where $A_{i,C,r}$ denotes the supply of Armington consumption good i in region r. The right-hand-side of equation (3.11b) gives the private demand of the household for Armington good i in region r.

The market clearance condition for the Armington composite used by the government is written:

(3.11c) $$A_{i,G,r} = -G_r \frac{\partial \pi_r^G}{\partial Pa_{i,G,r}}$$

where $A_{i,G,r}$ denotes the supply of Armington composite i in region r, demanded by the government sector.

Commodities produced for the domestic market in the industrial and the investment sector go into the Armington good production and are used as intermediate goods in the production of other industry goods or the investment good, and as final goods by the private household and the government. The market clearance condition for each commodity i (excluding the investment good *cgd*) produced for the domestic market is written as:

(3.12a) $$D_{i,r} = \underbrace{\tilde{y}_{j,r} \frac{\partial \pi_{j,r}^Y}{\partial Pd_{i,r}}}_{Supply} = \underbrace{-\left(A_{i,I,r} \frac{\partial \pi_{i,I,r}^A}{\partial Pd_{i,r}} + A_{i,C,r} \frac{\partial \pi_{i,Cr}^A}{\partial Pd_{i,r}} + A_{i,G,r} \frac{\partial \pi_{i,G,r}^A}{\partial Pd_{i,r}} \right)}_{Demand}$$

$\forall i \neq cgd$

and for the investment good, *cgd*, is:

(3.12b) $$\underbrace{D_{cgd,r} = Inv_r \frac{\partial \pi_r^{Inv}}{\partial Pd_{cgd,r}}}_{Supply} = \underbrace{-\left(A_{cgd,I,r} \frac{\partial \pi_{cgd,I,r}^{A}}{\partial Pd_{cgd,r}} \right)}_{Demand}$$

where $D_{i,r}$ denotes the domestic supply of produced good i in region r which is equal to the total demand for domestically used commodity i in region r.

The market for imported goods is analogous to the market of domestic outputs. Imported goods are demanded as intermediates by industrial sectors and the investment sector, and as final goods by the private household, and the government. The supply-demand balance for each imported good is written:

(3.13) $$\underbrace{M_{i,r}}_{Supply} = \underbrace{-\left(A_{i,I,r} \frac{\partial \pi_{i,I,r}^{A}}{\partial Pm_{i,r}} + A_{i,C,r} \frac{\partial \pi_{i,Cr}^{A}}{\partial Pm_{i,r}} + A_{i,G,r} \frac{\partial \pi_{i,G,r}^{A}}{\partial Pm_{i,r}} \right)}_{Demand} \quad \forall i$$

where $M_{i,r}$ is the supply of the composite imported good i in region r.

Export supply of commodity i in region r ,$X_{i,r}$, equals the import demand of good i in all other regions rr. Hence, the market clearance condition for exports of commodity i of region r is given as:

(3.14) $$\underbrace{X_{i,r} = \tilde{y}_{j,r} \frac{\partial \pi_{j,r}^{Y}}{\partial Px_{i,r}}}_{Supply} = \underbrace{-\left(\sum_{rr \neq r} M_{i,rr} \frac{\partial \pi_{i,rr}^{M}}{\partial Px_{i,r}} \right)}_{Demand}.$$

The market for the international transport activity is connected to the export or import activity respectively. The supply of international transport activity, IT, is equal to a fixed proportion of all import flows among regions. The market clearance condition for the international transport market is given as:

$$(3.15) \qquad IT = -\left(\sum_r \sum_i M_{i,r} \frac{\partial \pi_{i,r}^M}{\partial PT} \right).$$

Because of the dynamic nature of the economy also a static model has to reflect the possibility for agents to save or dissave. In order to solve a general equilibrium model which is not based on an explicit intertemporal optimization framework, one of the constraints of the model must be relaxed because the system of equations is overdetermined (cf. Dewatripont and Michel 1987). This requires to choose closure rules which determine the exogenous and endogenous variables in the macroeconomic equilibrium conditions of the model. The choice of macroeconomic closure rules follows from the *impossibility to warrant the ex-post identity between savings and investment* although all markets are in equilibrium. (Conrad 1997, p. 21) The choice of the exogenous parameters in the model determines the adaptation of the economic system to exogenous shocks.

Hence, for the complete determination of the CGE model closure rules for the foreign trade activity, for the government activity, and for the ex-post savings and investment identity have to be added.

In this version of the model the closure concerning the government activity states that the levels of government activities, $\overline{G}_r$, are exogenously fixed at a constant share of total GDP for each region. The production of the public good is completely financed by tax revenues from household taxes and from taxes on production and trade. The public good is redistributed lump sum to the private household. In the benchmark the government budget is balanced in every region, that is tax payments are equal to government expenditures.

The total income of the representative household in each region , Inc_r, is the sum of factor incomes minus tax payments, which are equal to government expenditures, plus current account deficit, $\overline{CA}_r$:

$$(3.16) \qquad Inc_r = Pl_r \cdot \overline{L}_r + Pk_r \cdot K_r + Pb_r \cdot \overline{B}_r - Pg_r \cdot \overline{G}_r + \overline{CA}_r.$$

The current account of region r, $\overline{CA}_r$, is defined as:

$$(3.17) \qquad \overline{CA}_r = \sum_i Pm_{i,r} M_{i,r} - \sum_i Px_{i,r} Ex_{i,r}$$

where $\overline{CA}_r$ denotes the exogenously given current account balance of region *r*, i.e. a trade deficit if $\overline{CA}_r$ is greater than zero (and a surplus if it is negative).

Total regional income is employed in constant fractions on household demand and savings, i.e. savings are determined by the exogenously given constant marginal propensity to save. Therefore, given the price indices for consumption, Pc_r, and investment, $Pd_{cgd,r}$, consumption of region *r* is derived by:

$$(3.18) \qquad C_r = \frac{(1 - s_r) Inc_r}{Pc_r}$$

where s_r denotes the constant marginal propensity to save in region *r*. Investment for region *r* is then given by:

$$(3.19) \qquad Inv_r = \frac{s_r \cdot Inc_r}{Pd_{cgd,r}}.$$

Due to Walras' law (i.e. the sum of the excess demands over all markets is identically zero) the equilibrium conditions are not independent of each other in a static trade model. Hence, consumer's budget balance is implied by the trade balance and vice-versa. According to the dependence of the equilibrium conditions one condition has to be dropped - here it is the regional closure concerning the ex-post identity between savings (inclusive capital imports, that is current account deficit) and investment.

In the ART model, as in most CGE models, the identity of private gross domestic production from both the flow of cost approach with the flow of product approach is used, and a residual variable is chosen to close the model (cf. Conrad 1997, p. 21). Here, the current account is computed residually to ensure the ex-post identity of gross investment to net savings (the sum of household savings, government budget deficit and current account balance).

The ART model is closed by using the ex-post savings-investment identity and determining the current account deficit $\overline{CA_r}$ as a residual for each region r:

(3.20) $$\overline{CA_r} = Inv_r - S_r - Depr_r.$$

where Inv_r is the total gross investment, S_r denotes the total savings, defined as the sum of private household savings, government budget (in the benchmark zero) and net foreign capital inflows (i.e. trade deficit $\overline{CA_r}$), and $Depr_r$ denotes the depreciation of capital stock in region r. The regional current account $\overline{CA_r}$ remains fixed at the level of the benchmark share of GDP over the time horizon of the model.

Furthermore, the world budget constraint must hold, whereby the total value of world imports is equal to the total value of world exports.[20] Hence, the sum of all regional trade imbalances must be zero:

(3.21) $$\sum_r \overline{CA_r} = \sum_r \left(Inv_r - S_r - Depr_r\right) = 0$$

Up to here the economic behavior of the agents was specified for a single time period. Now the model will be expanded to include time dependent aspects.

4. Dynamics

This section deals with the implementation of dynamics into the **D**ynamic **A**pplied **R**egional **T**rade (DART) General Equilibrium Model. Therefore, another subscript denoting the time period t has to be added to the variables in

[20] In the DART model the current account of a region is measured in terms of price of labor of the region with the highest national income. Thus, trade imbalances of regions have a common price and are comparable.

the model.

Dynamic modeling means, we have a time dependent process in which capital stocks available for use in year $t+1$ are determined by investment which takes place before year $t+1$. There are two main approaches how dynamic aspects have been incorporated into CGE models: the dynamic sequencing of static equilibria (also called the recursive dynamic approach) and the completely dynamic approach.[21] Recursive CGE models do not consider intertemporal aspects of decision making. Therefore, only economic agents with myopic or adaptive expectations can be modeled in these kinds of models.

The DART model is a recursive dynamic model. The dynamics of the DART model are defined by equations which describe how the endowments of the primary factors evolve over time. The major driving exogenous factors in the model are population change, the rate of labor productivity growth, change in human capital, savings rates, initial capital stocks, the gross rate of return on capital, and thus the endogenous rate of capital accumulation.

The agents have myopic expectations, which is consistent with the in principle static nature of the DART model. Hence, no consideration is given to potential changes in future prices or intertemporal possibilities of substitution in determining each period's equilibrium. The savings behavior of regional households is characterized by an exogenously given savings rate over time. The savings rate is allowed to adjust to income changes in regions in some calibration runs. This rather ad-hoc assumption seems consistent with empirical observable, regional different, but nearly constant savings rates of economies, which adjust according to income developments over very long time periods (for savings rates cf. Schmidt-Hebbel and Servén 1997).

[21] Recent trade models for quantifying the welfare effects of trade liberalization like Harrison et al. (1995) consider „dynamic" effects by implementing a steady state condition as a constraint on the static CGE model. Nevertheless, the model remain static. Therefore, the term „dynamic" as used in this paper refers to a multiperiod setting.

Government expenditures, $\overline{G}_r$, and current account, $\overline{CA}_r$, as above described, are set as a fixed benchmark share of GDP over the time horizon. The development of the primary factors labor, land, and capital is described below.

4.1 Supply of Labor and Agricultural Land

Like Hall and Jones (1998) we assume that labor, measured in physical units $\tilde{L}_{r,t}$ (here, number of workers instead of hours per worker is used as a unit), is homogenous within a region r and that each unit of labor has been trained with F_r years of schooling (education). The amount of human capital-augmented labor, HK_r, used in production in region r is given by:

$$HK_r = e^{\phi(F_r)}\tilde{L}_r$$

where the function $\phi(F_r)$ reflects the efficiency of a unit of labor with F years of schooling relative to one with no schooling ($\phi(0) = 0$). Note that if $\phi(F) = 0$ for all F we have the standard case with undifferentiated labor in a region. The derivative $\phi'(F_r)$ yields the return to schooling estimated in a Mincerian wage regression (Mincer 1974), i.e. an additional year of schooling raises a worker's efficiency proportionately by $\phi'(F_r)$.

According to Hall and Jones (1998) who have based their assumptions on Psacharopoulos' (1994) summery of Mincerian wage regressions, the function $\phi(F_r)$ is piecewise linear. Decreasing returns on schooling are assumed. That means, the first 4 years of schooling achieve a rate of return of 13.4 percent, corresponding to the average Psacharopoulos reports for sub-Saharan Africa. For the next 4 years a value of 10.1 percent, the average for the world as a whole, and beyond the 8th year 6.8 percent, the value Psacharopoulos reports for the OECD, are used.

Furthermore, we assume labor-augmenting technical progress with constant rates in order for the model to possess a steady state.[22] Hence, labor supply in region r , $L_{r,t}$, is derived by:

$$L_{r,t} = HK_{r,t} \cdot A_r$$

where $HK_{r,t}$ denotes the amount of human capital-augmented labor and A_r is a labor-augmenting measure of productivity specific for region *r*. That means labor is measured in efficiency units or in effective amount of labor.

In the DART model, the labor supply in efficiency units, $L_{r,t}$, evolves exogenously over time. Therefore, exogenous labor supply $\bar{L}$ for each region *r* at the beginning of time period $t+1$ is given by:

(4.1) $$\bar{L}_{r,t+1} = \bar{L}_{r,t} \cdot \left(1 + gp_{r,t} + ga_r + gh_r\right).$$

An increase of effective labor implies either growth of the human capital accumulated per physical unit of labor, gh_r, population growth, gp_r, or total factor productivity improvement, ga_r, or the sum of all.

In the basic version of the DART model we assume constant, but regionally different labor productivity improvement rates, ga_r, constant but regional different growth rates of human capital, gh_r, and declining population growth rates over time, $gp_{r,t}$, according to the World Bank population growth projections (World Bank, 1998). Because of the lack of data for the evolution of the labor participation rate in the future we use the growth rate of population instead of the labor force which implies that the labor participation rate is constant over time.

The regional rates of change in human capital per worker, gh_r, are derived exogenously by using the assumption that in the year 2050 all regions will achieve a maximal obtainable level of education, i.e. 12 years of schooling.

[22] For the proof see Barro and Sala-i-Martin (1995), pp. 54.

Therefore, the at maximum achievable human capital in region r, $HK_{r,max}$, is given by:

$$HK_{r,max} = \tilde{L}_{r,90}(4 \cdot 0{,}134 + 4 \cdot 0{,}101 + 4 \cdot 0{,}068)$$

where $\tilde{L}_{r,90}$ is the labor force of region r in the year 1990 taken from the Penn World Tables Mark 5.6a revision of Summers et al. (1995). Every 4 years of schooling are multiplied by their correspondent rate of return of schooling (cf. reference above).

The average educational attainment, F_r, is measured in 1990 for the population aged between 25 and 65, as reported by Hall and Jones (1998). Then, the actual amount of human capital-augmented labor in 1990 is derived for each region, $Hk_{r,90}$ by using the same rates of return of schooling as above. The annual growth rate for human capital per worker, gh_r, is equivalent to $\phi(F_r)$ and derived by:

$$gh_r = \phi(F_r) = \left(\frac{HK_{r,max}}{\tilde{L}_{r,90}} - \frac{HK_{r,90}}{\tilde{L}_{r,90}} \right) / 60$$ [23].

The supply of the sector-specific primary factor land, $\overline{B}_{r,t}$, is held fixed to its benchmark level over time in the basic version of the DART model. Therefore, the regional supply of agricultural land over the whole simulation horizon is exogenously given by:

(4.2) $$\overline{B}_{r,t+1} = \overline{B}_{r,t} = \overline{B}_{r,0} = const.$$

This assumption of a fixed supply of agricultural land over the time horizon should be improved in the next version of the DART model. Otherwise land becomes the scarce factor in the model leading to substantial increases in the factor price of agricultural land.

[23] The 60 in the denominator is the time span between 1990 and 2050, the year in which all regions are supposed to achieve the highest obtainable education level.

4.2 Capital Formation

Current period's investment augments the capital stock in the next period. The aggregated regional capital stock, *Kst*, in each time period *t* is updated by an accumulation function equating the next-period capital stock, Kst_{t+1}, to the sum of the depreciated capital stock of the current period and the current period real gross investment, $Inv_{r,t}$. In each time period gross investment is determined by equation (3.19). The equation of motion for capital stock $Kst_{r,t+1}$ in region *r* is given by:

(4.3) $$Kst_{r,t+1} = (1 - \delta_t) Kst_{r,t} + Inv_{r,t}$$

where δ_t denotes the exogenously given constant depreciation rate in period *t*. Note that the depreciation rate is the same over all regions and is 0.04 according to the version 3 GTAP database. The allocation of capital among sectors follows from the intra-period optimization of the firms as described in section 3.

5. The Solution Concept

This section gives a short overview of the algorithm used for implementing and solving the CGE model and doing counterfactual analysis. The DART model, as described in sections 3 and 4, is programmed in GAMS (Generalized Algebraic Modeling System - Brooke et al., 1992). Under this platform, the static part of the General Equilibrium model, the ART model, is written in MPSGE language (Mathematical Programming System for General Equilibrium analysis - Rutherford, 1994). Through programming in MPSGE / GAMS the economic model and the solution algorithm can be separated. Because of this separation, changes in the model structure and assumptions can be easily implemented.

For specifying and using CGE models the following steps are commonly used: The economic regions under consideration are assumed to be in equilibrium, i.e. the benchmark equilibrium. The behavior of the economic agents has been analytically formulated. Then, the parameters of the model are chosen through a calibration procedure. Calibration means that the specified model is solved from equilibrium data, i.e. the benchmark data base, for its parameter values. These parameter values can then be used to solve the model for alternative equilibria associated with changes in policy variables. The so-called „counterfactual" scenarios are imposed on the model to explore and evaluate the impacts of different policy measures and shocks by comparing the results of the benchmark equilibrium with the counterfactual.[24]

The ART model, i.e. the static part of the CGE model, is calibrated on the Global Trade Analysis Project (GTAP) database version 3 for 1992. The GTAP data base contains 30 regions and 37 sectors (Table 1). Regions and sectors can be aggregated suitable for the research task. The current version of the DART model runs in a 5 regions 6 sectors aggregation (see Table 2).

In the dynamic part of the model, factor endowments are updated after every time step. The equilibria in any sequence are connected to each other through capital accumulation. Each single period equilibrium calculation begins with an initial capital services endowment resulting from the end of the period t-1. A new equilibrium of supply, demand and relative prices is calculated for the next time period t based on the exogenous and endogenous changes in endowments. Savings of the current period t will augment the capital-services endowment at the end of period t available in the next period t+1. (cf. Shoven and Whalley 1992) The way in which the DART model is calibrated is specified in more detail in section 6. Therefore, the benchmark solution of the DART model consists of the benchmark equilibrium for 1992 and the calculated equilibria for the following one year time steps through 2050, i.e. the baseline path.

When an alternative policy measure, for instance an energy tax on production, is imposed, the DART model derives a new path of sequential equilibria that is

[24] For further details see Shoven and Whalley (1992).

consistent with the alternative policy constraint. This counterfactual path can then be assessed relative to the baseline path.

So far, there are only taxes imposed as policy instruments. In the DART model they occur as price wedges, such as factor taxes in production, value-added taxes, import and export taxes. The implementation of further policy instruments like CO_2 emission permits is in progress.

6. Calibration

Normally, the CGE literature amplifies on the calibration of the single period equilibrium, but is very short in describing the calibration procedure of dynamic models and the underlying assumptions. Calibration in a dynamic context is generally interpreted as requiring two properties: first, replication of the benchmark data base; second, the model is parameterized in such a way that the balanced growth path is simulated when the base policy is maintained (cf. Pereira and Shoven 1988).

A recursive dynamic CGE model can also be interpreted as a sequence of counter-factuals to the base year run by altering the factor endowments holding everything else constant. The first step in calibration of a recursive dynamic model is the same as in the static case (for the calibration procedure of a single period model see Shoven and Whalley 1992, pp. 115). The data in the benchmark dataset are in value terms, i.e. they are the products of prices and quantities. Unit conventions are adopted to deal separately with prices and quantities. These unit conventions tell us, for example, what constitutes a unit of primary factor. Usually, physical units of factors of production are taken as the amount that earns a reward of one currency unit, e.g. $1, in equilibrium, abstracting from taxes. Units of commodities are then defined as those amounts that sell for $1 net of all taxes and subsidies in equilibrium. The assumption that in equilibrium marginal revenue products of factors are equalized in all uses permits the direct incorporation of factor payments data by industry as observations of physical quantities of factors in the calibration.

After having calibrated the first, i.e. benchmark, period of the model, the next step is to update the factor endowments. Often the data for regional capital stocks are not available or not reliable. Therefore, equation (4.3) can be rearranged by exploiting the unit price convention and using capital earnings, $K_{r,t}$ as physical capital services instead of the capital stock. Using the stock-to-flow-conversion, the capital earnings for region r are given by:

$$(6.1) \qquad K_{r,t} = rk_{r,t} \cdot Kst_{r,t}.$$

where $rk_{r,t}$ denotes the gross return on capital.

If data for regional capital stocks are not available the initial (benchmark) gross rate of return, $rk_{r,0}$, can be derived by dividing the regional capital value share, $KVSH_{r,0}$, by the exogenously assumed capital stock to GDP ratio, $\overline{KGDP}_{r,0}$:

$$(6.2) \qquad rk_{r,0} = \frac{KVSH_{r,0}}{\overline{KGDP}_{r,0}}.$$

where the zero in the subscript stands for the benchmark period.

Now, equation (4.3), the capital accumulation equation, can be written in terms of physical units of capital services by using the stock-to-flow-conversion (equation (6.1)). Therefore, the value of real gross investment has to be scaled by the initial gross rate of return, $rk_{r,0}$. This gives:

$$(6.3) \qquad Kq_{r,t+1} = (1-\delta_t)Kq_{r,t} + Inv_{r,t} \cdot rk_{r,0}.$$

where $Kq_{r,t}$ denotes the physical unit of the factor capital in period t which earns \$1 in the initial time period. $Inv_{r,t}$ is the value of real gross investment in period t, i.e. the product of the actual quantity and actual price in period t. Once the variables have been scaled, the physical (or quantity) units of capital services can be updated according to equation (6.3); whereas the actual value of gross investment has to be scaled with the benchmark gross rate of return in every time step. A detailed discussion of the dynamic benchmarking is given in Klepper and Springer (1998).

7. Further Development of the DART Model

This paper describes technically the basic version of the DART model. The DART model can be applied for policy analysis in various fields, like trade or environmental policy. The DART model serves as a component of an integrated assessment project for evaluating global climate change impacts on economies. The model can be used to project economic activities, energy use and trade flows for each of the specified regions according to exogenous assumptions about the dynamics of the model. Anthropogenic carbon dioxide emissions can be directly derived from the energy use projections by the DART model. These projections of energy use are inputs to a climate model and thereby form the first link in the integrated analysis of global climate change.

For the purpose of climate policy evaluation the economic CGE model should be multi-regional, multi-sectoral and dynamic. Regional detail enables the model user to analyze distributional impacts of different climate protection policies by considering of terms of trade effects. Furthermore, regional differences in climate vulnerability and adaptation levels can be taken into account. The sectoral disaggregation allows the analysis of structural change as a consequence of climate change and climate policies. The implementation of dynamic features into the model allows to cover accumulation mechanisms in the economy and the time dependent effects of greenhouse gas emissions and accumulation in the atmosphere, and the resulting climate change impacts.

The DART model is such a global CGE model with regional as well as sectoral detail. Within every region the household and industry behavior is fully specified based on microeconomic foundations. All regions are linked by bilateral trade flows. It is recursive dynamic, i.e. evolution over time of the economies is described by a sequence of single-period static equilibria connected through capital accumulation. The current version of the DART model is calibrated on the GTAP data base version 3 for the year 1992 and covers up to 30 regions and 37 industry sectors (Table 1) and runs through the year 2050.

The model can be expanded in several ways to fulfill the demands of the integrated assessment project and other applications. One important aspect is the detailed specification of the energy sector allowing substitutions between primary energy inputs in order to model the reactions of the economic agents to CO_2 emission policy measures. For this, the data for production and use of primary energy factors in the GTAP data base have to be revised. On the supply side an resource extraction model of fossil fuels could be implemented in order to get some realistic price and behavior reactions of resource owners to climate policies.

Furthermore, the modeling of technological development and productivity amelioration could be improved. This would also affect the modeled growth and convergence process[25] since labor productivity improvement is an important determinant of the growth process in each region. Furthermore, the technological development within the energy sector could be modeled. The assumption about autonomous improvements in the energy efficiency or the availability of backstop technologies within the energy sector determines the energy intensity of economic activity which affects energy use / emissions and therefore the cost of CO_2 emission reduction.

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[25] For the controversy on the convergence and divergence of growth rates see Durlauf (1996) and the following papers in *The Economic Journal* 106:1016-1069. Ventura (1997) provides another explanation for different growth rates. There, trade is the source of long persistent growth.

Further potential improvements of the DART model could include a more sophisticated update of the regional endowment with agricultural land, and the incorporation of other policy instruments than taxes, as for instance quantity restrictions or a tradable emission certificate system for CO_2 emissions which is hotly debated among the UNFCCC parties because of potential efficiency gains and distributional consequences.[26]

The model would be suited also to represent climate impacts on the economic system or international capital mobility. The modeling of climate change impacts would allow to incorporate regionally differentiated feedbacks of the climate system to the economic system, and to analyze the cost and benefits of climate protection policies. In the course of globalization international cross-ownership of capital has become more extensive in the past two decades. Changes in such cross-holdings can only be examined in models with international capital movements (cf. Bovenberg and Goulder 1991) which have consequences for the distributional results of policy simulations.

[26] The UNFCCC fourth conference of the parties in Buenos Aires on 2-13 November 1998 will deal with flexibility mechanisms such as emissions trading or activities implemented jointly.

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Appendix - Model Dimensions, Data, and Production structure

Table 1 Regions and Sectors in the GTAP Data Base Version 3 (1992)

	Region		Sector
1	Australia	1	Paddy rice
2	New Zealand	2	Wheat
3	Japan	3	Grains
4	Republic of Korea	4	Non grain crops
5	Indonesia	5	Wool
6	Malaysia	6	Other livestock
7	Philippines	7	Forestry
8	Singapore	8	Fisheries
9	Thailand	9	Coal
10	China	10	Oil
11	Hong Kong	11	Gas
12	Taiwan	12	Other minerals
13	India	13	Processed rice
14	Rest of South Asia	14	Meat products
15	Canada	15	Milk products
16	United States of America	16	Other food products
17	Mexico	17	Beverages and tobacco
18	Central America and Caribbean	18	Textiles
19	Argentina	19	Wearing apparels
20	Brazil	20	Leather etc.
21	Chile	21	Lumber
22	Rest of South America	22	Pulp paper etc
23	European Union 12	23	Petroleum and coal
24	Austria, Finland and Sweden	24	Chemicals rubbers and plastic
25	European Free Trade Area	25	Nonmetallic minerals
26	Central European Associates	26	Primary ferrous metals
27	Former Soviet Union	27	Nonferrous metals
28	Middle East and North Africa	28	Fabricated metal products
29	Sub Saharan Africa	29	Transport industries
30	Rest of World	30	Machinery and equipment
		31	Other manufacturing
		32	Electricity, water and gas
		33	Construction
		34	Trade and transport
		35	Other services (private)
		36	Other services (government)
		37	Ownership of dwellings

Table 2: Regions and Commodities

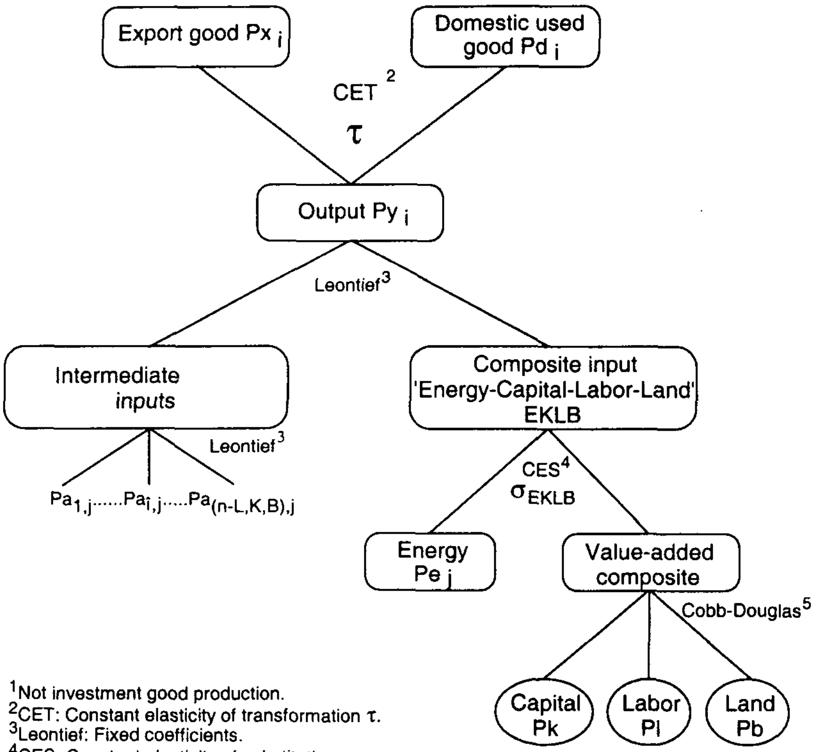
Regions in the 5 by 6 GTAP Aggregation

OECD:	**OECD** (Australia, Austria, Canada, EFTA, European Union 12, Finland, Japan, New Zealand, Sweden and United States)
EEX:	**Energy Exporting Countries** (Indonesia, Malaysia, Mexico, Middle East & North Africa and Rest of South America)
DYN:	**Dynamic Asien Countries** (Korea, Philippines, Singapore, Taiwan and Thailand)
CHN:	**China** (China and Hong Kong)
ROW:	**Rest of World** (Argentina, Brazil, Central America & the Caribbean, Central European Associates, Chile, Former Soviet Union, India, Rest of South Asia, Rest of the World and Sub Saharan Africa)

Commodities in the 5 by 6 GTAP Aggregation

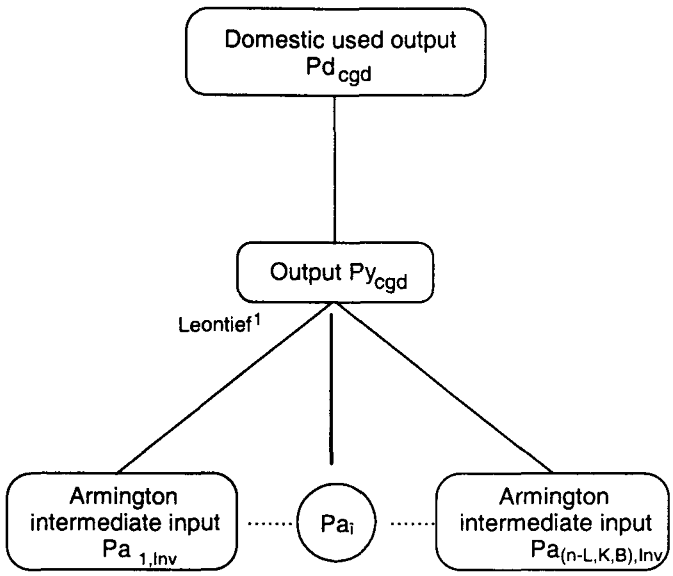
AGR:	**Agriculture** (beverages & tobacco, fisheries, forestry, grains, meat products, milk products, non grain crops, other food products, other livestock, paddy rice, processed rice, wheat and wool)
ENINT:	**Energy Intensive Goods** (chemicals, rubbers & plastic, nonferrous metals, primary ferrousmetals and pulp & paper)
FUEL:	**Fossil Fuels** (coal, gas, oil and petroleum & coal)
ELEC:	**Electricity** (electricity, water & gas)
SER:	**Services** (construction, government services, ownership of dwellings, private services and trade & transport)
OTHER:	**Other Industries** (fabricated metal products, leather, lumber, machinery & equipment, other manufacturing, other minerals, nonmetallic minerals, textiles, transport industries and wearing apparels)

Figure 1: Production structure of industry sector j in region r[1]



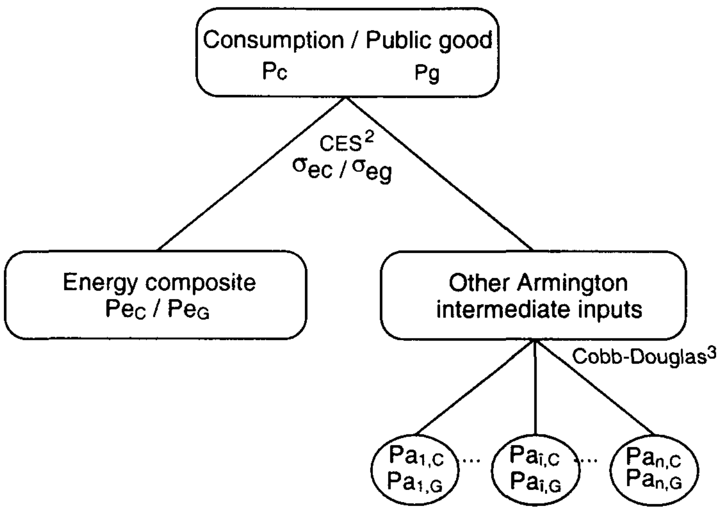
[1]Not investment good production.
[2]CET: Constant elasticity of transformation τ.
[3]Leontief: Fixed coefficients.
[4]CES: Constant elasticity of substitution σ.
[5]Cobb-Douglas: $\sigma = 1$.

Figure 2: Production structure of the investment good sector Inv in region r



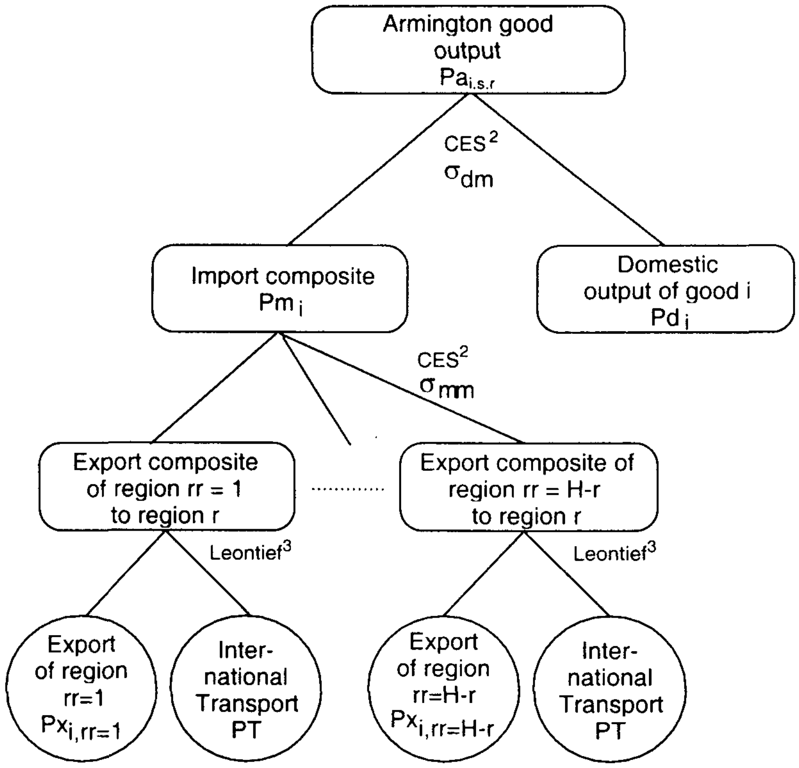
[1] Leontief: Fixed coefficients.

Figure 3: Household / Government production structure[1]



[1] *c* stands for household and *g* for government.
[2] CES: Constant elasticity of substitution σ_{ec}/σ_{eg}.
[3] Cobb-Douglas: $\sigma = 1$.

Figure 4: Structure of foreign trade
(Armington good production of good i in region r)



[1]Armington output is distinguished by agent with s = {I, C, G}
[2]CES: Constant elasticity of substitution.
[3]Leontief: Fixed coefficients.