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How useful is Structural VAR Analysis for Irish economics?

by

Daniel McCoy

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Economic Analysis, Research and Publications Department, Central Bank of Ireland, P.O. Box 559, Dublin 2.

Abstract

The purpose of this paper is to provide an introduction to the methodology known as Structural Vector Autoregression (SVAR) analysis and to examine its applicability in the context of Irish macroeconomics. The SVAR approach has been developed over the last decade to interpret business cycle fluctuations and to help identify the effects of different economic policies. It is an extension on the traditional atheoretic VAR approach in that it combines economic theory with time-series analysis to determine the dynamic response of economic variables to various disturbances. The main advantage with SVAR analysis is that the necessary restrictions on the estimated reduced form model, required for identification of the underlying structural model, can be provided by economic theory. These restrictions can be either contemporaneous or long-run in nature depending on whether the underlying disturbances are considered to be temporary or permanent in nature. Once the identification is achieved it is possible to recover the structural shocks. These shocks can then be used to generate impulse response and variance decomposition functions to assess the dynamic impacts on different economic variables. In addition these functions can be used to test whether such shocks affect the economic variables as economic theory would predict so providing a check on the theory. SVAR analysis has been used internationally to examine a variety of research topics, such as asymmetric shocks from monetary union and impacts of exchange rate movements. A number of research topics in the Irish context that could benefit from SVAR analysis are identified. These topics relate mainly to areas of inflation, exchange rate and monetary policy. The SVAR is an important and useful methodology that is worthy of more attention by the Irish economics community than it currently receives.

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1. Introduction

The aim of this paper is to examine the methodology known as Structural Vector Autoregression (SVAR) analysis with an emphasis on its usefulness in the context of Irish applied macroeconomics and to identify possible areas of applicability for future research. Structural VARs can be viewed as a bridge between economic theory and multiple time-series analysis in order to determine the dynamic response of variables to various disturbances, or shocks, that occur in the economy. As a consequence this methodology is sometimes referred to as the analysis of disturbances.

The two main uses of the SVAR approach are the interpretation of business cycle fluctuations and identifying the effects of different economic policies. It is illustrative to think of SVARs as being somewhere along a continuum between the atheoretical approaches of pure time series analysis, such as the traditional (unstructured) VARs, and the structured large scale macroeconomic models, such as the HERMES model used by the ESRI.

The paper is arranged as follows. Section 2 introduces the concept of SVARs as a natural extension of traditional VAR analysis. Section 3 describes the methodology of SVAR analysis focusing in particular on the type of decompositions used in much of the applied work of this technique. Section 4 outlines the type of topics that have been investigated internationally using the SVAR approach. Its applicability to the Irish context is discussed in Section 5, with emphasis on the type of data issues that are raised and some suggestions on how the SVAR approach can be used for further research. Section 6 concludes. A short appendix to the paper sets out the SVAR procedures contained in the RATS econometrics package.

2. What are Structural VARs?

Structural VARs are an extension of traditional VAR analysis. How they differ is that within a SVAR an attempt is made to identify a set of independent disturbances by means of restrictions provided by economic theory rather than by the (so-called) atheoretic restrictions used in traditional VARs. Therefore before we can proceed to discuss the SVAR it is necessary to outline the VAR methodology. The VAR approach, made popular by Sims (1980), has become an important tool in empirical macroeconomics. The popularity of this approach arose both out of the inability of economists throughout the 1970s to agree on the true underlying structure of the economy and from the Lucas critique, that changes in policy systematically alter the structure of econometric models, which led to a major shift away from the use of large-scale macroeconomic as tools for forecasting.

The criticism of existing large-scale models made by Sims is that that they imposed "incredible identification restrictions" in order to derive parsimony in structure. The modeler can choose which variables are to be included as determinants in each equation and make assumptions about whether a variable is to be considered exogenous or endogenous. These assumptions are informed by the modeler's "prior beliefs", or their understanding of economic theory, which can leave them open to the charge of "data mining". In an attempt to overcome these problems, particularly in determining which variables should be considered endogenous or exogenous, the VAR approach attempts to "let the data speak for themselves" by making all variables endogenous.

In the VAR framework each variable, whether measured in levels or first differences, is treated symmetrically in that all variables in the system contain the same set of regressors. There are no exogenous variables and no identifying restrictions. The only role for economic theory is in specifying the variables to be included. Apart from the need for theory for this specification, the technique is considered to be atheoretic.

Consider a system of simultaneous equations represented in vector form like in (1) below.

$$(1) \qquad \mathbf{A}\,\mathbf{y_t} = \mathbf{B(L)}\,\mathbf{y_{t-1}} + \mathbf{C}\,e_t$$

This is a general representation where $\mathbf{y_t}$ is a vector of endogenous variables, $\mathbf{y_{t-1}}$ is a vector of their lagged values, and e_t is a white noise vector of the disturbance terms for each variable. This disturbance term captures any exogenous factors in the model. The square n x n matrix **A,** where n is the number of variables, contains the structural parameters of the contemporaneous endogenous variables. The square n x n matrix **C** contains the contemporaneous response of the variables to the disturbances or innovations. **B(L)** is a pth degree matrix polynomial in the lag operator L, where p is the number of lagged periods used in the model. [1]

The problem with the representation in (1) is that because the coefficients in the matrices are unknown and the variables have contemporaneous effects on each other it is not possible to uniquely determine the values of the parameters in the model. The model in this form is not fully identified. However, it is possible to transform (1) into a reduced-form model to derive the standard VAR representation, as shown in (2), which facilitates estimation of the model parameters. Since there are no contemporaneous effects between variables in the standard VAR representation and each equation comprises a set of common regressors, this permits the use of OLS regression for estimation purposes, given

$$(2) \qquad \mathbf{y_t} = \mathbf{D(L)}\,\mathbf{y_{t-1}} + \mathbf{e_t}$$

The transformation of (1) into (2) implies that $\mathbf{D(L)}\ ^{o}\ A^{-1}B(\mathbf{L})$ and that $\mathbf{e_t}\ ^{o}\ A^{-1}\mathbf{C}e_t$. The error terms ($\mathbf{e_t}$) are linear combinations of the uncorrelated shocks (e_t) such that each individual error term is serially uncorrelated with a zero mean and a constant

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[1] The lag operator (L) works as follows: $Ly_t = y_{t-1}$, $L^2y_t = Ly_{t-1} = y_{t-2}$, ... ,$L^ny_t = y_{t-n}$. The matrix polynomial $B(L)y_{t-1} = B_0\,y_{t-1} + B_1Ly_{t-1} + B_3L^2\,y_{t-1} + ...+ B_PL^P\,y_{t-1}$ where all the matrices B_i are square.

variance. However, unlike the disturbance terms in e_t, the error terms in $\mathbf{e_t}$ are correlated with each other. This presents a problem in recovering the underlying structural disturbances from the estimated VAR.

The matrix S is the variance/covariance of the estimated residuals, $\mathbf{e_t}$, of the standard VAR. The σ^2 are the variance and σ_{ij} are the covariance terms where each $s_{ij} = \left(\frac{1}{T}\right)\sum_{t=1}^{T} e_{it}\, e_{jt}$ and where

$$S = \begin{bmatrix} s_1^2 & s_{12} & \dots & s_{1n} \\ s_{21} & s_2^2 & \dots & s_{2n} \\ . & . & . & . \\ . & . & . & . \\ s_{n1} & s_{n2} & \dots & s_n^2 \end{bmatrix}$$

The matrix Σ is symmetric, since $\sigma_{12} = \sigma_{21}$, and so it contains only $(n^2 + n)/_2$ distinct estimated parameters to use in recovering the structural parameters in (1). The matrix W is the variance/covariance matrix of the structural disturbances, e_t. The number of structural parameters to be estimated depends on the variance/covariance matrix W, which contains $(n^2 + n)/_2$ unique elements, and on the matrices A and C, each containing n^2 elements. The total number of parameters to be estimated is $2n^2 + (n^2 + n)/_2$ but there are only $(n^2 + n)/_2$ elements estimated from Σ. Therefore there are $2n^2$ restrictions required for identification.

Since the structural disturbances are assumed to be white noise with zero covariance terms, implying that each disturbance arises from independent sources, the Ω is a diagonal matrix. This provides $(n^2 - n)/_2$ restrictions. In addition, the matrices A and C are normally assumed to have main diagonal elements equal to unity. In A this implies a normalisation on a particular variable in each equation. In C this normalisation is a consequence of assuming a separate shock contained in each equation. These provide further 2n restrictions. Most studies, Bernanke (1986) is a notable exception, also impose exclusion restrictions on the C matrix to make it an identity matrix.[2] This adds

[2] The appendix to this paper contains a general framework provided by Giannini(1992) that embraces the different types of models used in the SVAR literature.

an additional $(n^2 - n)$ restrictions. Adding these together provides $[(n^2 - n)/_2 + 2n + n^2 - n]$ contemporaneous restrictions out of the $2n^2$ restrictions required. This leaves $(n^2 - n)/_2$ restrictions required, as otherwise the system is underidentified.

Identification necessitates the imposition of some structure on the system. It is on the imposition of this structure that SVARs differ from the traditional VAR analysis. Traditional VARs propose an identification restriction based upon on a recursive structure known as a Choleski decomposition. This statistical decomposition separates the residuals (e_t) into orthogonal (uncorrelated) shocks by restrictions imposed on the basis of an arbitrary ordering of the variables. The decomposition implies that the first variable responds only to its own exogenous shocks, the second variable responds to the first variable and to the second variable's exogenous shocks and so on. The structure that results is referred to as being lower triangular, where all elements above the principal diagonal are zero. such as in the system below where the $\mathbf{z_t}$ are the Choleski restrictions and the $w_\mathbf{t}$ is the vector of orthogonal shocks.

$$\begin{aligned}
\boldsymbol{e}_1 &= w_1 \\
\boldsymbol{e}_2 &= \mathbf{z}_1\boldsymbol{e}_1 + w_2 \\
\boldsymbol{e}_3 &= \mathbf{z}_2\boldsymbol{e}_1 + \mathbf{z}_3\boldsymbol{e}_2 + w_3 \\
\boldsymbol{e}_4 &= \mathbf{z}_4\boldsymbol{e}_1 + \mathbf{z}_5\boldsymbol{e}_2 + \mathbf{z}_6\boldsymbol{e}_3 + w_4
\end{aligned}$$

In the example of a four variable model given above, the Choleski decomposition provides $(4^2 - 4)/_2 = 6$ restrictions needed to exactly identify the system. However, this is just one possible ordering of the variables. There can be **n!** possible orderings, which in the four variable example would be 24 combinations. The choice of ordering is unlikely to be important if the correlation between the residuals is low but this is unlikely to be the case, given that variables included in a VAR will normally be chosen precisely because they have strong comovements. The results from VARs can be quite sensitive to the ordering imposed which makes their interpretation quite difficult.

The atheoretical approach of traditional VARs has come under quite strong attack from the economic profession. Cooley and LeRoy (1985) have criticised the VAR approach on the grounds that the ordering imposed by a Choleski decomposition is not in fact atheoretical. It implies a particular type of recursive contemporaneous structure for the economy which is not consistent with economic theory. Also the estimated shocks are not pure shocks but rather linear combinations of the structural disturbances.[3] Therefore it is difficult to assess the dynamic effects on variables because they will depend on all of the structural disturbances. The innovation (disturbance) accounting techniques of impulse response functions and variance decompositions associated with traditional VAR analysis have no obvious economic interpretation as a consequence of the atheoretical approach taken.

These criticisms led to the development of the SVAR approach. This work stemmed from the seminal contributions of Sims (1986), Bernanke (1986) and Blanchard and Watson (1986) who made use of economic theory to impose restrictions on the observed values of the estimated residuals (e_t) to recover the underlying structural disturbances (ε_t). In addition to recovering the structure of the disturbances it is necessary to preserve the assumed error structure to ensure independence between the shocks. Instead of the arbitrary method of restriction imposition used in traditional VARs, the SVAR approach, as advocated by these authors, estimates the structural parameters by imposing contemporaneous structural restrictions based on economic theory. These can be considered as short-run restrictions in that the shocks are considered to have temporary effects. A representation of the standard VAR in levels form as in (2) above would apply in this context.

An alternative SVAR approach, advanced by Shapiro and Watson (1988) and Blanchard and Quah (1989), is to consider the shocks as having permanent effects. This would imply that the variables are non-stationary since the shocks continue to accumulate through time given they are permanent. The presence of unit roots in the variables can give rise to spurious regression if the VAR is estimated in levels. Therefore it is necessary to use first differences to ensure stationarity in the case of shocks that have permanent effects. A standard VAR in first differences is outlined in (3) below.[4] The necessary restrictions in this approach are long-run which tends to more consistent with many economic theories.

[3] If the structural disturbances model is $e_t = A^{-1} C\varepsilon_t$ and the Choleski decomposition implies $e_t = Z\omega_t$ then since $\omega_t = Z^{-1} A^{-1} C\varepsilon_t$, the shocks of the Choleski are linear combinations of the structural disturbances (Keating, 1992).

[4] Δ is the first difference operator, where $\Delta = 1- L$

(3) $$\Delta \mathbf{y}_t = \mathbf{D(L)}\, D\mathbf{y}_{t-1} + \mathbf{e}_t$$

Alternatively, a cointegrated framework can be used to avoid the loss of information about the equilibrium relationships in the model that can result from first differencing. The stationary linear combinations of the non-stationary variables can be constructed prior to estimation (Keating, 1992). This cointegration constraint can then be imposed using a vector error correction model (VECM).

To sum up a Structural VAR is a standard VAR where the restrictions needed for identification of the underlying structural model are provided by economic theory. These can be either contemporaneous or long-run restrictions depending on whether economic theory suggests the shocks are either temporary or permanent in nature.

3. How do Structural VARs operate?

The procedure for operating a straightforward SVAR involves a number of discrete steps. Firstly, the user must determine whether the variables to be included are stationary I(0) or non-stationary I(1). This will determine whether a reduced form representation in levels (2) or in first differences (3) is required. Once the variables have been made stationary the next step involves estimating the reduced form VAR using OLS, ensuring that enough lags are incorporated to ensure no serial correlation from the residuals. Tests are needed to select the appropriate lag length given that VAR analysis, in trying to avoid exclusion restrictions, can quickly become overparameterised losing important degrees of freedom for estimation purposes.[5]

When the reduced form VAR is estimated it is then essential to impose sufficient restrictions to identify the structural parameters of the model. In some cases economic theory can suggest more than the necessary restrictions, such that the model is

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[5] If the model has *n* equations then with a lag length of *p* there would be $n^2 \times p$ coefficients plus n intercept terms. Typical tests for appropriate lag lengths include the Sims Likelihood Ratio test, the Akaike Information Criterion and the Schwarz Criterion

overidentified, but we will limit the discussion to the case of exact identification. In the case where the shocks are assumed to have temporary effects on the variables the restrictions are imposed on the contemporaneous elements contained in A, C and Σ. In contrast where the shocks are assumed to have permanent effects, the restrictions are imposed on the long-run multipliers in the impulse response functions, which in effect involves restrictions on D(L).

A relatively simple example may help clarify each type of restriction in turn. Take a model containing four variables - output (y), price level (p), interest rate (r) and money (m).[6]

3.1 Contemporaneous Restrictions

In a contemporaneous version of this four variable model there are six $((4^2 - 4)/_2 = 6)$ restrictions required for exact identification. These restrictions can be imposed on the structure of the error vector (e_t) on the basis of economic theory. Each equation contains an independent structural disturbance term. The first three restrictions can be deduced from the aggregate supply function. If the price level is predetermined, except that producers can respond instantly to aggregate supply shocks (ε^{as}), then the residual on the price variable is $e^p = \varepsilon^{as}$, which is independent of the shocks to the other variables. If the monetary authority is assumed to adjust short-term interest rates in response to disturbances in the money supply (ε^{ms}), independent of changes in output and prices, this would provide two more restrictions with $e^r = z_1 e^m + \varepsilon^{ms}$. The final restriction is obtained from the short-run money demand function. Nominal money holdings are assumed to depend upon the level of output and interest rates (LM relationship), consistent with the buffer stock theory of money, such that $e^m = z_2 (e^y + e^p) + z_3 e^r + \varepsilon^{lm}$.

The error structure incorporating these economic theory based contemporaneous restrictions is as follows

[6] A three variable version of this model using real variables is contained on page 57 of Frain (1995) dealing with SVAR. The four variable version presented here is based on Keating (1992).

$$e^{p} = \varepsilon^{as}$$
$$e^{r} = z_1 e^{m} + \varepsilon^{ms}$$
$$e^{m} = z_2 (e^{y} + e^{p}) + z_3 e^{r} + \varepsilon^{lm}$$
$$e^{y} = z_4 e^{p} + z_5 e^{r} + z_6 e^{m} + \varepsilon^{is}$$

The estimation procedure involves estimating the variance/covariance matrix (Σ) on the residuals conditional on these restrictions as in (4).

(4) $$\Sigma = \boldsymbol{E}[\boldsymbol{e}_t \boldsymbol{e}_t^{/}] = \boldsymbol{A}^{-1}\boldsymbol{C}\boldsymbol{E}[\varepsilon_t \varepsilon_t^{/}]\boldsymbol{C}^{/}\boldsymbol{A}^{-1/} = \boldsymbol{A}^{-1}\boldsymbol{C}\Omega\boldsymbol{C}^{/}\boldsymbol{A}^{-1/}$$

The estimation of (4) would require the use of non-linear algorithms to solve for the parameter values. The advantage of this technique, unlike the Choleski decomposition, is that the impulse response functions and the variance decompositions functions that result from these contemporaneous restrictions can be given a direct economic interpretation since they are derived using parameters from explicit economic models. The impulse response functions can be used to test whether the shocks affect each variable as economic theory would predict. So, for example, a positive aggregate supply shock would be expected to increase output. The dynamic response of the output variable can then be checked to assess its consistency with theory.

3.2 Long-run Restrictions

An alternative approach is placing the restrictions on the long-run multipliers for the structural shocks. This approach can be quite attractive for economists, given that there is more agreement on the long run properties of economic theory than with the short-run. This type of decomposition does not impose contemporaneous restrictions so it allows the data to determine the short-run dynamic properties of the model.

The long-run multipliers can be derived from a moving average representation of (2) given in (5) below.

(5) $$y_t = [I - D(L)L]^{-1} e_t = [I - D(L)L]^{-1} A^{-1} C \varepsilon_t = \mu(L)\varepsilon_t$$

The term $\mu(L) = \sum_{i=0}^{\infty} \mu_i L^i$, where each μ_i is the impact of changes in the shocks ε_t as reflected in the response of the variable y_{t+i}. These μ_i are the impact multipliers and the sum of these responses to infinity is the long-run multiplier for each variable. The set of these multipliers is the impulse response function.

When the shocks have permanent effects, the long-run effect of these impulses equals the sum of the coefficients in $\mu(L)$, when L is set equal to one. From (5) we have

$$\mu(1) = [I - D(1)]^{-1} A^{-1} C \quad \underline{\text{or}} \quad A^{-1} C = [I - D(1)]\mu(1)$$

and substituting for $A^{-1}C$ in (4) gives (6) which is used to parameterise the long-run restrictions.

(6) $$[I - D(1)]^{-1} \Sigma [I - D(1)]^{-1\prime} = \mu(1)\Omega\mu(1)'$$

As in the contemporaneous case there are $(n^2 - n)/_2$ restrictions required to identify this model, with the restrictions being imposed on [I - D(1)] matrix which contains the long-run multiplier. In the context of the four variable example above, economic theory can provide the six long-run restrictions necessary for exact identification. The long run output level is only affected by aggregate supply shocks, which have permanent effects, so $y = \varepsilon^{as}$ which provides three restrictions. From the IS relationship, the long run interest rate is affected by the level of output in the economy and by demand shocks, so $r = z_1 y + \varepsilon^{is}$, which provide a further two restrictions. The final restriction is derived from the money demand function that states that real money balances depend on output and the interest rate and money demand shocks, $m/p = z_2 y + z_3 r + \varepsilon^{lm}$. These could be represented in logs as follows

$$
\begin{aligned}
\mathbf{y} &= \mathbf{e}^{as} \\
\mathbf{r} &= z_1\mathbf{y} + \mathbf{e}^{is} \\
\mathbf{m} - \mathbf{p} &= z_2\mathbf{y} + z_3\mathbf{r} + \mathbf{e}^{lm} \\
\mathbf{m} &= z_4\mathbf{y} + z_5\mathbf{r} + z_6(\mathbf{m} - \mathbf{p}) + \mathbf{e}^{ms}
\end{aligned}
$$

These long-run restrictions allow for the recovery of the underlying structural disturbances which can be used to obtain the impulse response functions and the variance decompositions to analyse the dynamic (short-run) responses of the variables to the different shocks.

An alternative decomposition, which is now widely used in a number of branches of macroeconomics, has been put forward by Blanchard and Quah (1989). Given the extent to which this decomposition is used, it is worth looking at in a little more depth.

3.3 Blanchard and Quah Decomposition

Blanchard and Quah in their paper estimate a SVAR model where some variables are stationary and others contain unit roots. Instead of associating each disturbance (ε_t) directly with an individual variable, they consider the shocks as having either temporary or permanent effects. They then treat these shocks like exogenous variables. The objective is to decompose real GNP into its temporary and permanent components. Economic theory is used to associate aggregate demand shocks as being the temporary shocks and aggregate supply shocks as having permanent effects. There is no way to provide a unique decomposition into these two components with one variable so a second variable is necessary, in the Blanchard-Quah study unemployment is used.

Let y_t be real GNP, u_t be unemployment, ε_{1t} be the aggregate demand shock and ε_{2t} be the aggregate supply shock.[7] The bivariate moving average representation of these variables where both Δy and u are stationary[8] is

[7] See Enders (1995) for a comprehensive explanation of this model.
[8] The decomposition is only necessary because GNP contains a unit root.

$$\Delta y_t = \sum_{k=0}^{\infty} c_{11}(k)\varepsilon_{1t-k} + \sum_{k=0}^{\infty} c_{12}(k)\varepsilon_{2t-k}$$

$$u_t = \sum_{k=0}^{\infty} c_{21}(k)\varepsilon_{1t-k} + \sum_{k=0}^{\infty} c_{22}(k)\varepsilon_{2t-k}$$

or compactly

(7) $$\begin{bmatrix} \Delta y_t \\ u_t \end{bmatrix} = \begin{bmatrix} C_{11}(L) & C_{12}(L) \\ C_{21}(L) & C_{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

where the $C_{ij}(L)$ are polynomials in the lag operator. The standard VAR representation used for estimation of the bivariate model is

(8) $$\begin{bmatrix} \Delta y_t \\ u_t \end{bmatrix} = \begin{bmatrix} A_{11}(L) & A_{12}(L) \\ A_{21}(L) & A_{22}(L) \end{bmatrix} \begin{bmatrix} \Delta y_{t-1} \\ u_{t-1} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}$$

The residuals from this estimated VAR are composites of the structural disturbances. The one step ahead forecast error is e_{it}[9] and from the bivariate moving average representation the one step ahead forecast error is $c_{i1}(0)\varepsilon_{1t} + c_{i2}(0)\varepsilon_{2t}$, where i = 1,2. Since these are equivalent we get

(9) $$\begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = \begin{bmatrix} c_{11}(0) & c_{12}(0) \\ c_{21}(0) & c_{22}(0) \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

In order to recover the structural disturbances from the estimated residuals in (8) it is necessary to identify the four restrictions $c_{ij}(0)$ in (9). Three out of the four restrictions required can be obtained from the estimated VAR residuals.

Restriction 1: $\text{Var}(e_{1t}) = c_{11}(0)^2 + c_{12}(0)^2$

Restriction 2: $\text{Var}(e_{2t}) = c_{21}(0)^2 + c_{22}(0)^2$

[9] If $x_t = A_1x_{t-1} + e_t$, then the conditional expectation of x_t is $E_{t-1}x_t = A_1x_{t-1}$. The one step ahead forecast error is $x_t - E_{t-1}x_t = e_t$.

Restriction 3: $Cov(e_{1t},e_{2t}) = c_{11}(0)c_{21}(0) + c_{21}(0)c_{22}(0)$[10]

The fourth restriction is provided by the long-run theoretic restriction that aggregate demand shocks only have temporary effects on real GNP.

Restriction 4: $\left[1-\sum_{k=0}^{\infty} a_{22}(k)\right]c_{11}(0)+\sum_{k=0}^{\infty} a_{12}(k)c_{21}(0)=0$[11]

These provide the four equations necessary to solve for the four unknowns (c_{ij}) so using (9) the structural shocks can be recovered and the innovation accounting techniques of impulse response and variance decompositions can be applied to identify the dynamic responses of the variables.

4. What have Structural VARs been used for?

The Blanchard-Quah decomposition has been used in quite a wide range of macroeconomic topics in the last few years. The main uses of SVAR analysis have been in examining business cycle fluctuations and the impact of policy rules and regime changes. While most of the initial SVAR analysis was applied to U.S. macroeconomic data, the approach is becoming increasingly used in a European context to examine

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[10] $Var(e_{1t}) = c_{11}(0)^2 Var(\varepsilon_{1t}) + c_{12}(0)^2 Var(\varepsilon_{2t}) + 2\,Cov(\varepsilon_{1t},\varepsilon_{2t})$
By normalisation $E(\varepsilon_{1t}) = Var(\varepsilon_{1t}) = 1 = Var(\varepsilon_{2t}) = E(\varepsilon_{2t})$, where E is the expectations operator; Cov $(\varepsilon_{1t},\varepsilon_{2t}) = E(\varepsilon_{1t},\varepsilon_{2t}) = 0$. Therefore $Var(e_{1t}) = c_{11}(0)^2 + c_{12}(0)^2$. The $Var(e_{2t})$ is got similarly.

$Cov(e_{1t},e_{2t}) = E(e_{1t},e_{2t}) = c_{11}(0)c_{21}(0)E(\varepsilon_{it})^2 + c_{12}(0)c_{22}(0)E(\varepsilon_{2t})^2 + c_{11}(0)c_{22}(0)E(\varepsilon_{1t},\varepsilon_{2t}) + c_{12}(0)c_{21}(0)E(\varepsilon_{1t},\varepsilon_{2t})$. Putting in the values for the variance/covariane terms this reduces to $Cov(e_{1t},e_{2t}) = c_{11}(0)c_{21}(0) + c_{12}(0)c_{22}(0)$.

[11] Compacting (8) such that $x_t = A(L)x_{t-1} + e_t = A(L)Lx_t + e_t$. Gather the x_t terms to one side and premultiply by $[I - A(L)L]^{-1}$ to get $x_t = [I - A(L)L]^{-1} e_t$. Using the fact that the inverse of a matrix, M^{-1}, is equal to the inverse of its (non-zero) determinant, $|M|^{-1}$, multiplied by its adjunct, Adj M, we get $x_t = (1/|M|)\,[I - (A(L)L)']\,[e_t]$. Expanding this out for the Δy_t term only we get $\Delta y_t = (1/|M|)\left\langle [1 - \sum_{k=0}^{\infty} a_{22}(k)L^{k+1}]e_{1t} + \sum_{k=0}^{\infty} a_{12}(k)L^{k+1}e_{2t}\right\rangle$. Using (9) we have $e_{1t} = c_{11}(0)\varepsilon_{1t}$ and $e_{2t} = c_{21}(0)\varepsilon_{2t}$ which when we substitute in we get down to $\left[1-\sum_{k=0}^{\infty} a_{22}(k)\right]c_{11}(0)+\sum_{k=0}^{\infty} a_{12}(k)c_{21}(0)$. This is set equal to zero from our long-run restriction.

issues like the prospects for EMU through the degree of correlated shocks in potential member states (Bayoumi and Eichengreen, 1993) and through movements of exchange rates in response to variations in relative output (Canzoneri et al., 1996). In addition, SVAR analysis has been used to interpret whether exchange rate changes result in relative price changes (Astley and Garratt, 1996) and whether monetary authorities implicitly follow a particular type of policy rule (Clarida and Gertler, 1996). It has also been used to examine the impact of a regime change from money growth targets to price stability targets (Friedman and Kuttner, 1996).

In an Irish context SVAR analysis has not attracted the same attention, with the notable exception of Cosgrove and Roche (1994). They use SVAR to decompose GDP growth into components associated with major macroeconomic disturbances in order to identify the likely causes of Irish recessions over the period 1960-1992. They use a four variable VAR of output, interest rates, prices and money in an IS-LM-AS framework. Long run restrictions, consistent with the new classical view, are imposed to identify the aggregate demand and aggregate supply disturbances.[12] The aggregate supply shocks are assumed to have permanent impacts, while the aggregate demand shocks have transitory effects. The four variable model is used to examine how the variables respond to four underlying shocks. The shocks used are an aggregate supply (AS) shock and three aggregate demand shocks - autonomous demand (IS), money demand and money supply (LM).

The long-run restrictions imposed by Cosgrove and Roche provides them with the necessary six restrictions[13] for exact identification to use the estimated residuals to recover the underlying disturbances. They then use these disturbances to examine the impulse responses of a one standard deviation change to each of the four identified shocks in turn. They found that the responses were largely consistent with the predictions of economic theory using the IS-LM-AS model (see their paper for

[12] These restrictions and the four variable system used are based on Gali (1992).

[13] Three restrictions are provided by aggregate demand shocks (IS/LM) having no long-run impact on output. Two restrictions are obtained from distinction between real and financial shocks on real interest rates whereby monetary (LM) shocks have no long run impact. In addition long run money supply changes cause proportionate changes in the price level so that real money balances are unchanged with no impact on long run real interest rates. The final restriction is that real money balances are not affected by money supply shocks in the long run.

details). The variance decompositions carried out found that, in contrast to a Keynesian view, short run output variability is largely accounted for by both IS shocks and AS shocks. The historical decomposition of GDP growth in Ireland over the period 1960-1992 suggested that aggregate supply shocks predominated the recessions of the 1970s and 1981 while aggregate demand shocks were more important in the recessions of the 1980s and 1991. This would seem to accord with the consensus interpretation of these events.

Friedman and Kuttner (1996) make use of a SVAR, with the same four variables and structural disturbances as above, to test alternative hypotheses on why money has lost its predictive content in the U.S. over the last few decades. The evidence from this structural approach suggests that money demand fluctuations became so unstable as to be of little use to forecasting fluctuations in either output or price by the end of the 1980s. This was a key factor in the Federal Reserve's abandonment of monetary targets as a main plank of monetary policy. Clarida and Gertler (1996) use a SVAR approach to assess how the Bundesbank, despite having a public focus on monetary targeting, adjusts short-term interest rates to disturbances in the economy. They use an eight variable VAR divided into non-policy versus policy variables to organise the identifying assumptions.[14] A policy variable in this context means any variable which the Bundesbank can influence in the current period, for example, the interest rate is a policy variable while output is a non-policy variable.

The non-policy variables are treated separately with the residuals arranged in a recursive causal relationship. The ordering of the variables does not have a significant impact on the robustness of the results from the study. Disturbances in the non-policy variables are treated as exogenous in terms of identification of the policy variables. The restrictions, based on economic theory, on the key policy variables are contemporaneous restrictions. From the money supply function, the interest rate adjusts to contemporaneous disturbances in commodity prices, the money supply and the exchange rate, as these three variables are directly observable in the current period,

[14] The variables used include five non-policy variables: industrial production, retail sales, consumer prices, real world commodity prices and the US Federal Funds rate. The three policy variables include the short-term interest rate, the real money supply and the real DM/$ exchange rate.

unlike the other variables excluded that become available with a lag. The money demand function is restricted to depend on disturbances in real output and the nominal interest rate. The exchange rate is left unrestricted.

There are then three excluded non-policy variables which can be used to identify the exchange rate and the money supply terms, so the system is in fact overidentified. The dynamic responses of the variables to shocks contained in the impulse response functions suggest that the Bundesbank uses the short term interest rate quite aggressively to control inflationary pressures. However, it is clear that real economy factors also influence behaviour. Short term interest rates follow a feedback rule[15], which responds to anticipated, rather than past, inflation. Clarida and Gertler also find that the Bundesbank responds to exchange rate movements in a countercyclical manner, where a depreciation leads to an increase in interest rates, which would correspond with economic theory.

In the context of the UK, Astley and Garrett (1996) have used a SVAR to identify the sources of sterling exchange rate fluctuations and inflation. They estimate a three variable model in a two country world[16] using a Blanchard-Quah decomposition of shocks into aggregate supply shocks having long run permanent effects and IS-LM aggregate demand shocks with temporary effects. Their estimated impulse response functions were found to have been consistent with economic theory. The main conclusion drawn from their SVAR analysis is that IS shocks accounted for most of the variance in sterling exchange rates, while LM shocks account for most variance in prices. The most controversial, if somewhat misunderstood, implication presented to policy-makers from this work is that "sterling exchange rates movements per se do not appear to have been a major channel through which UK relative price pressure have arisen".

[15] This in fact is a modified Taylor rule which advocates a "lean against the wind" policy whereby the central bank raises interest rates as either inflation or output increase relative to their long run trend (Taylor, 1993).

[16] The underlying economic model is based on structural exchange rate model put forward by Obstfeld (1985).

The point that is important (and often missed) is that it is the type of shock that underlies both the exchange rate and price movements that matters. Exchange rate depreciation will increase inflationary pressures when the shock is from a monetary (LM) disturbance. In contrast IS and AS shocks leading to depreciating exchange rate will lead to price falls. These type of shocks have been the most prevalent types of shocks in the UK. This highlights the need to uncover the underlying sources of disturbances, a task that the SVAR methodology is particularly suitable for. This is demonstrated in recent work on the impact of asymmetric shocks in the context of EMU. Two studies in particular have utilised a SVAR to address this issue.

Canzoneri *et al*. (1996) pose the question of whether nominal exchange rates adjust to deal with asymmetric shocks. If they do, then joining a monetary union and giving up this adjustment instrument would be quite costly. Shocks are classified in this paper as "neutral" if they have no long run effect on relative output between countries and "non neutral" if they do. Using a simple two variable VAR of relative output and exchange rates, along with the Blanchard-Quah decomposition for identification, they find that the shock that explains 90-95 per cent of the variation in relative national outputs explains less than 20 per cent of the variation in nominal exchange rates. This is taken to imply that nominal exchange rates have not moved much in response to macroeconomic imbalances.

They use a three variable system to identify a "money and financial market" shock independent of a "goods market" shock. This identification draws on the Mundell-Fleming model, which is theoretical foundation of optimal currency areas, to provide the economic theory to impose long run restrictions. On the basis of this three variable system they find that the shocks that would be eliminated by joining a currency union do not seem to have been a major source international macroeconomic imbalances. Canzoneri *et al.* overall finding is that the costs of monetary union have been exaggerated in that exchange rates have not played an important role in absorbing shocks in the past. Bayoumi and Eichengreen (1993) use a similar SVAR setup with two types of shocks to identify disturbances for countries in Europe and for regions within the U.S. They find that only when a European core[17] is compared with the entire U.S. is there aggregate demand and supply disturbances of comparable magnitude. This core is more suitable in forming a monetary union than a wider EMU which would experience larger asymmetric shocks leading to greater macroeconomic imbalances.

5. Are Structural VARs of use for Irish economics?

Having examined a selection of the issues that SVAR analysis has been applied to internationally it is appropriate to consider its usefulness in an Irish context. It is clear that Structural VARs have a role to play in Irish economics as the Cosgrove and Roche (1994) paper demonstrates. The SVAR methodology can offer the researcher an alternative to a structural econometric model. The focus in a SVAR is not in the estimation of the equations but in decomposing the underlying disturbances into their different sources.

An obvious limitation is the availability and quality of the data sources given that any VAR model will be quickly overparameterised. SVAR analysis will generally require data with fairly high frequency which may pose a serious limitation in the Irish context given that only annual GNP figures are available. This necessitates the use of quite long time periods making stability of the estimates an issue given that the economy has undergone significant structural changes.

Another limitation of the SVAR is that it can only identify at most the same number of distinct shocks as there are variables. Typically the number of variables estimated is two or three, while in practice there may be many different types of shocks hitting the economy. The SVAR methodology therefore makes quite strong assumptions both on the econometric and the mathematical techniques required to provide the decomopositions. However, one of the main advantages of using this method is that

[17] The core includes Germany with its near neighbours. Ireland was not included as a desirable candidate, though more recent work by Eichengreen (1996) suggests that this situation may have since changed.

economic theory can provide over-identifying restrictions which can be used to test directly the structural model.

The following list of applications would seem to lend themselves to investigation by SVAR analysis in the Irish context, suitable data sets permitting:

Exchange Rates/EMU

- Identification of asymmetric shocks for Ireland to analyse the macroeconomic consequences of EMU entry.
- Assessment of the fundamentals that determine real and nominal exchange rates.
- Role of exchange rates in adjusting to macroeconomic disturbances.
- The relationship between the exchange rate and inflation in a small open economy.

Inflation

- The use of policy rules for price stability - price level targets or inflation targets.
- To identify the historical shocks that have determined the dynamics of inflation.

Money/Monetary Policy

- The analysis of different variables in the business cycle, for instance the role of credit.
- Examination of policy rules for monetary policy, such as the McCallum and Taylor rules in an Irish context in the presence of an exchange rate constraint.

This list is far from exhaustive but it hopefully illustrates the potential of the SVAR methodology to tackle research topics of direct importance for the Irish economy.

6. Conclusion

Structural VARs are an important and useful tool. The SVAR approach provides an intuitive method of identifying macroeconomic disturbances compared with the more atheoretical VAR methods and is a lot more simple to work with than large scale macroeconomic models. As a result it is worthy of more attention by the Irish economics community than it currently appears to receive. The next step must be to proceed to use a SVAR approach. With application in mind there are procedures to carry out SVAR analysis appearing in many econometric packages, such as *RATS*, which can be used to put the methodology into practice (see Appendix). This will be the task for future work

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Appendix

SVAR Procedures in RATS

The *RATS* Version 4 (Doan, 1992) econometric package contains a procedure for dealing with structural decompositions of VARs in Chapter 8.5 of the User's manual. This contains a procedure which draws on the contemporaneous restrictions approach of Sims (1986) and Bernanke (1986). It is titled the @BERNANKE but it embraces the specific type of model used by Sims also.

However, a more comprehensive RATS procedure for SVAR analysis by Antonia Lanzarotti and Mario Seghelini is available in Giannini (1992). This procedure checks for identification based on the constraints imposed by the different types of restrictions and then uses Full Information Maximum Likelihood (FIML) estimation to calculate the structural parameters. It is based on the general model of SVAR put forward by Giannini (1992). This methodological framework encompasses all the different models used in the applied SVAR literature. The three different ways in which the SVAR approach structures the VAR model are referred to as the K-model, the C-model and the AB-model.

If the vector of stochastic processes $\mathbf{y_t}$ is given in a finite order autoregressive representation:

$$\mathbf{A(L)\ y_t} = e_t$$

K Model

Premultiply the autoregressive representation by a n x n invertible matrix K such that

$$\mathbf{K\ A(L)\ y_t = K}\ e_t$$

$$\mathbf{K}\ e_t = \mathbf{e_t}$$

$$\mathbf{E(e_t, e_t^{/}) = I_n}$$

The vector $\mathbf{e_t}$ is a transformation of the ε_t disturbances into orthogonal shocks. The covariance matrix of the e_t are orthonormal $\mathbf{I_n}$ (that is it is a diagonal matrix with unity on the diagonal entries). Therefore if we assume to know the true variance/covariance matrix of the e_t disturbances(Ω) then

$$\mathbf{K}\ e_t\ e_t^{/}\ \mathbf{K}^{/} = \mathbf{e_t, e_t^{/}}$$

Taking expectations we get

$$\mathbf{K}\ W\ \mathbf{K}^{/} = \mathbf{I_n}$$

This imposes the necessary $n(n+1)/2$ restrictions on K leaving $n(n-1)/2$ free parameters to identify the model.

The contemporaneous restrictions SVAR models are represented by the K model.

C Model

Let C be a n x n invertible matrix where

$$\mathbf{A(L)\ y_t} = e_t$$

$$e_t = \mathbf{C\ e_t}$$

$$\mathbf{E(e_t,\ e_t^{/}) = I_n}$$

Therefore given the above

$$e_t\ e_t^{/} = \mathbf{C\ e_t,\ e_t^{/}\ C^{/}}$$

Taking expectations we get

$$W = \mathbf{C\ C^{/}}$$

which in turn leaves $n(n-1)/2$ free parameters in C to identify the model.

The long run restrictions SVARs, like the Blanchard and Quah decomposition, are represented by the C model.

AB Model

If A,B are n x n invertible matrices then

$$\mathbf{A\ A(L)\ y_t = A}\ e_t$$

$$\mathbf{A}\ e_t = \mathbf{B\ e_t}$$

$$\mathbf{E(e_t,\ e_t^{/}) = I_n}$$

Therefore given the above

$$\mathbf{A}\ (e_t\ e_t^{/})\ \mathbf{A^{/} = B\ (e_t,\ e_t^{/})\ B^{/}}$$

Taking expectations we get

$$\mathbf{A}\ W\ \mathbf{A^{/} = B\ B^{/}}$$

The AB model can be transformed into a contemporaneous restrictions version (K model) or a long-run restrictions (C model) version. It is on this AB model representation that the procedures in RATS are based in a file called SVAR.SRC.