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Alternative Models of the Irish Supply Side

by

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Abstract

This paper presents and estimates a suite of different models of the supply side of the Irish economy. In particular, various Cobb Douglas supply side systems are estimated for quarterly national income accounts data for the Irish economy between 1981 and 1998. Expressions for marginal cost, labour demand and the cost of capital are derived from the underlying technology and estimated. Using more sophisticated flexible functional form models as benchmarks for comparison, a particular Cobb Douglas specification is then preferred.

1   Introduction

The primary purpose of this paper is to outline a model of the supply side of the Irish economy. This exercise is conducted as part of the ongoing work on the Irish macro model, which is maintained at the Irish central bank. The approach builds on previous work by using the extended series of interpolated national income accounts data now available at the bank.[1] Additionally, alternative specifications are explored such as the use of 'flexible functional forms' as a means of better approximating the underlying technology of the Irish economy. Both 'primal' and 'dual' models are investigated and results from the different approaches are then compared.[2] The exercise is conditioned however, by two primary outputs which are required as part of the general modelling program. These are

(1) the necessity to generate standard macro indicators such as measurements of the output gap and technical progress,

(2) and the estimation of a consistent series of factor demands in keeping with the underlying technology assumption.

As will be seen, the necessity to satisfy both of these criteria can present difficulties for certain specifications. In broad terms, three different approaches are explored. The first approach is the generic Cobb Douglas model proposed by Allen and Mestre (1997) and already applied in an Irish case by Slevin (2001) amongst others. This approach, while highly tractable and universally popular in application, suffers from certain imposed restrictions.[3] Consequently, in approximating the underlying technology, this approach is complemented by the use of a translog flexible functional form. Both primal and dual functional forms are adopted. In both cases, key production parameters of the Irish economy are estimated.

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[1]Details of this data including its generation is available in McGuire, O'Donnell and Ryan (2002).

[2]'Primal' in this instance refers to the direct specification and estimation of a production function, while the 'dual' approach adopts a cost function, which is dual to the underlying production function.

[3]Unitary substitution between factor inputs being the most significant.

The relationship between the cost function and production function approach is relatively simple in the case of the Cobb Douglas form, owing to its self-dual nature.[4] This is of particular benefit in terms of satisfying the conditions listed above i.e. a series of factor demands and the simulation of key macro indicators can be achieved in a relatively straightforward manner. However, the situation is more complex in the case of a flexible functional form such as the translog. While certain indicators such as the rate of technical progress can be inferred from say a translog cost function, it is not possible to simulate the underlying production function, when tackling the issue from the dual perspective. Equally, it is not feasible to generate a consistent series of factor demands from the primal production function approach. In the context of the general modelling effort therefore, the results from the use of the more flexible functional form are used to condition and inform those from the Cobb Douglas approach.

The remainder of the paper is laid out as follows, the next section presents the Allen and Mestre (1997) supply side model using the Cobb Douglas form. This is followed by the specification of a translog cost and production function for the Irish economy. Results from all three approaches are subsequently examined and a number of permutations of the original Allen and Mestre (1997) model are considered in the light of the results. A final section offers some concluding comments and future areas of exploration.

2   Supply Side Representations

The supply-side of the Irish economy is treated as a representative firm operating under conditions of imperfect competition with two factor inputs - labour and capital. Factor prices for both labour and capital are treated as given and optimal levels for both inputs are determined for a given state of technology. A disembodied level of technical progress is also assumed. The following list of variables are used in the representation of the Irish supply side

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[4]The Cobb-Douglas cost function can be 'mapped' exactly onto its production function equivalent.

$$Q = \text{aggregate output,}$$

$$C = \text{total aggregate costs,}$$

$$X_1 = \text{aggregate labour,}$$

$$X_1^F = \text{aggregate full employment labour,}$$

$$X_2 = \text{aggregate capital,}$$

$$P_1 = \text{price of aggregate labour } X_1,$$

$$P_2 = \text{price of aggregate capital } X_2,$$

$$S_1 = \text{share of total costs attributable to labour,}$$

$$S_2 = \text{share of total costs attributable to capital,}$$

$$T = \text{technical progress,}$$

$$YED = \text{output price deflator.}$$

2.1 The Cobb Douglas Model

Under the Cobb Douglas approach outlined by Allen and Mestre (1997), the supply side of the Irish economy can be modelled as the following constant returns to scale production function

$$lnQ_t = ln(alpha) + (1 - beta)\, lnX_{1t} + beta\, lnX_{2t} + gamma\, (1 - beta)\, T \quad (1)$$

alpha, *beta* and *gamma* are parameters respectively denoting a scale factor, the exponent on the capital stock and the growth rate of Harrod-neutral technical progress. *ln* denotes the log of a variable and the full employment level of labour X_1^F is given by the following identity

$$X_1^F = (1 - 0.01\, URT)X_1 \quad (2)$$

where URT is the non-accelerating inflation rate of unemployment. Allen and Mestre (1997) have derived a system of equations for an output price deflator,

labour demand and the cost of capital consistent with (1).[5] The associated system allows for the estimation of all of the underlying parameters in (1) without having to estimate the production function itself. The system is given by the following

$$lnYED = ln(eta) - ln(1 - beta) + ln(P_1) + (beta/(1 - beta))\, ln(Q/X_2)$$
$$-ln(alpha)/(1 - beta) - gamma\, T \quad (3)$$
$$ln(X_1/Q) = -ln(alpha) - beta\, ln(X_2/X_1) - (1 - beta)\, gamma\, T \quad (4)$$
$$ln(P_2) = ln(beta/(1 - beta)) + ln(P_1) - ln(X_2/X_1) \quad (5)$$

The output price delator equation is derived by inverting the production function and obtaining the dual cost function. First order conditions yield an expression for marginal cost and output prices are then set equal to the marginal cost expression scaled by the parameter 'eta', which represents a mark-up over marginal cost.[6] Equations for labour demand and the cost of capital are obtained by applying Sheppard's lemma to the dual cost function. It should be pointed out that capital is treated as a quasi-fixed input in the present set-up as its value is assumed to respond only sluggishly through time.

Various macro indicators can be generated using the results from the system (3) - (5). For instance, a structural form expression for the output gap can be obtained by simulating the production function (1) with the parameter values achieved from (3) - (5), at the full employment level of labour - X_1^F. The gap may be defined as

$$GAP_t = ln(Q_t) - ln(Q_t^*) \quad (6)$$

where $ln(Q_t^*)$ is the simulated solution to

$$lnQ_t^* = ln(alpha) + (1 - beta)\, lnX_{1t}^F + beta\, lnX_{2t} + gamma\, (1 - beta)\, T \quad (7)$$

Similarly, an expression for Total Factor Productivity (TFP) can be calculated as a Solow Residual, again using the estimated parameter value *beta* from (3) - (5).

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[5]See the Appendix to Allen and Mestre (1997) for more details.

[6]Imperfect competition is therefore, explicitly assumed.

$$TFP_t = ln(Q_t) - beta\ ln(X_{2t}) - (1 - beta)\ ln(X_{1t}) \tag{8}$$

Results for the estimated system and the calculated value of the output gap are presented in section 3 of the paper.[7]

2.2 Flexible Functional Form Approximations of Technology

The concept of a flexible functional form has been formally defined by Diewert (1971) and has been elucidated by Chambers (1988), Hanrahan (2000) and Boyle (1982) amongst others. The idea is that if the true profit, cost or production function of a producer is represented by the unknown function $g(z)$, then a series of parameters ψ exists such that the known parameterized function $g_\psi(z)$ can approximate the unknown function at an arbitrary point z^0. This implies that for certain values of ψ, the following will hold

$$g_\psi\left(z^0\right) = g\left(z^0\right)$$
$$\nabla_z g_\psi\left(z^0\right) = \nabla_z g\left(z^0\right)$$
$$\nabla_{zz}^2 g_\psi\left(z^0\right) = \nabla_{zz}^2 g\left(z^0\right) \tag{9}$$

Therefore, the function value, the gradient and the Hessian of the known function $g_\psi(z)$ will be equivalent to those of the unknown function $g(z)$ at some arbitrary point z^0 for a certain series of parameters ψ. If a function satisfies the conditions outlined in (9), it may be referred to as a second-order differential approximation.[8] The fundamental advantage of a flexible functional form over a form such as the Cobb Douglas is that it is sufficiently flexible so as to not impose any restrictions on the relationships between the factor inputs. The elasticity of substitution is determined by the data and is not imposed *a priori* by the researcher. In the following sections, two approaches are followed. A dual translog cost function is specified

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[7]Diagnostic tests on all models estimated are also presented in Section3.

[8]Barnett (1983) has proven that this concept of flexibility is equivalent to the definition of second order approximation popular in mathematics.

with associated factor demands. This is then followed by the direct estimation of a translog production function.

2.3 Cost Function Approach

The adoption of duality circumvents the estimation of first order conditions by directly specifying suitable minimum cost functions or maximum profit functions rather than the actual production or transformation function itself. Duality theory provides a set of necessary properties for the cost and profit functions which are implied by a 'well behaved' production technology and by the corresponding behavioural assumptions. Behavioural response equations are obtained by differentiation of the dual functions with respect to input and/or output prices. The significant advantage of this, is, that it results in less algebraic manipulations and it also permits according to Lopez (1982) the specification of more complex functional forms which impose much less *a priori* restrictions on the estimated equations.

The cost function model presented in this section is similar to that presented in Hall and Nixon (1999). Aggregating across all firms within the domestic economy, the objective function for the imperfectly competitive firm may be summarised as

$$\textbf{min } \mathbf{C}_t = C_t\left(P_{1t}, P_{2t}, Q_t, T\right) \tag{10}$$

Applying Shephard's lemma, yields expressions for the optimal shares of total costs attributable to labour and capital. The functional form adopted here is the translog form. Consequently, the log (*ln*) of total costs (*C*) may be approximated as

$$lnC = \omega_0 + \omega_Q \, lnQ_t + \omega_T \, T + \sum_{i=1}^{2} \omega_i \, lnP_i + \frac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{2} \omega_{ij} \, lnP_i \, lnP_j$$

$$+ \sum_{i=1}^{2} \omega_{iQ} \, lnQ \, lnP_i + \sum_{i=1}^{2} \omega_{iT} \, T \, lnP_i + \omega_{QT} \, T \, lnQ$$

$$+ \frac{1}{2}\omega_{QQ} \, (lnQ)^2 + \frac{1}{2}\omega_{TT} \, (T)^2 \tag{11}$$

where the ω's are parameters to be estimated. The corresponding input share equations are derived

$$S_i = \frac{\partial lnC}{\partial lnP_i} = \omega_i + \sum_{j=1}^{2} \omega_{ij} \, lnP_j + \omega_{iQ} \, Q + \omega_{iT} \, T \tag{12}$$

It should be noted that in this specification, capital is now treated as a variable cost unlike in the previous Cobb Douglas application where it was quasi-fixed. This results from the specification of a *long run* rather than a *short run* cost function. The following restrictions associated with the regularity conditions of the cost function[9] are imposed during estimation

(1) Linear homogeneity in input prices (i.e. the shares add to 1): $\sum_{i=1}^{2} \omega_i = 1$, $\sum_{j=1}^{2} \omega_{ij} = 0$, & $\sum_{i=1}^{2} \omega_{iQ} = 0$.

(2) Symmetry: $\omega_{ij} = \omega_{ji}$.

(3) Linear homogeneity in output: $\omega_Q = 1$, $\omega_{QQ} = 0$ & $\omega_{QT} = 0$.

(4) Homotheticity: $\omega_{iQ} = 0 \; \forall \; i$.

While these restrictions are imposed during the estimation stage, the validity of the imposition may of course be tested through standard likelihood ratio (LR) tests. Two commonly reported sets of results associated with systems estimation such as (11) and (12) are the Allen-Uzawa partial elasticity of substitution (σ_{ij}) and the price elasticity of demand (η_{ij}). For the translog, the relevant expressions for both sets of elasticities are

$$\sigma_{ii} = \frac{\omega_{ii} + S_i^2 - S_i}{S_i^2}, \quad \sigma_{ij} = \frac{\omega_{ii} + S_i S_j}{S_i S_j} \tag{13}$$

and

$$\eta_{ii} = S_i \sigma_{ii} \quad \eta_{ij} = S_j \sigma_{ij} \tag{14}$$

[9]The regularity conditions of the cost function are set out by Mandy (2000) amongst others.

One point of note in relation to the use of a dual flexible functional form such as the translog is the inability to map exactly back to the underlying production function. Consequently, it is not possible to generate the structural form representation of the output gap analguous to that in (6). In terms of technical progress, estimates of both the nature and scale of the rate of technical progress can be inferred from the dual cost function under certain conditions. The relationship between the rate of cost diminuition $\phi(P_1, P_2, Q, T)^{10}$ and the rate of technical progress $TP(X_1, X_2, T)$ is given by the following

$$TP(X_1, X_2, T) = -\epsilon(P_1, P_2, Q, T) \phi(P_1, P_2, Q, T) \qquad (15)$$

where $\epsilon(P_1, P_2, Q, T)$ is the elasticity of size.[11] Under the imposition of linear homogeneity in output, $\epsilon = 1$ and hence $TP()$ and $\phi()$ are equivalent. Again, all empirical results are reported in section 3. The next section directly specifies and estimates a translog production function.

2.4   Translog Production Function

In this section a translog production function of the supply side of the Irish economy is specified and estimated. As such, the results from the direct estimation of the production function can be compared with the simulated estimates from the equivalent Cobb Douglas production function in section 2.1.[12] However, the use of a flexible functional form for the production function renders it exceedingly difficult to generate consistent factor demands for this specification. Therefore, only the production function is presented and estimated here. The translog production function $f()$ is given by

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[10]Given by $\partial lnC / \partial T = \omega_T + \sum_{i=1}^{2} \omega_{iT} lnP_i + \omega_{QT} lnQ + \omega_{TT} T$

[11]Given by $\partial lnC / \partial lnQ$

[12]As already noted, it is not possible to simulate the underlying production function associated with the dual translog cost function in section 2.2.

8

$$lnQ = \theta_0 + \sum_{i=1}^{2} \theta_i lnX_i + \theta_T T + \frac{1}{2}\theta_{TT} \ (T)^2$$

$$+\frac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{2} \theta_{ij} \ lnX_i \ lnX_j + \theta_{jT} \ T \ lnX_j \tag{16}$$

where the θ's are now the parameters to be estimated. The previous section out-lined the regularity conditions for the cost function. The equivalent 'well behaved' conditions for the production function are that

(1) the function $f\,()$ is monotonic - $\partial f\,()\,/\partial \mathbf{x} \geq 0$ and

(2) the function $f\,()$ is concave in $\mathbf{x}$ - $\partial^2 f/\partial \mathbf{x}^2 \leq 0$.

Neither condition is imposed in the estimation, although both can be checked *ex post*.[13]

For the dual cost function an expression for technical change was derived under certain conditions. In the primal case, the equivalent expression for the rate of technical change $TP\,(X_1, X_2, T)$ is given by

$$\frac{\partial lnQ}{\partial T} = \theta_T + \theta_{TT}T + \sum_{j=1}^{2} \theta_{jT}lnX_j \tag{17}$$

As noted by Tzouvelekas, Giannakas, Midmore and Mattas (1999), depending on the relative usage of labour and capital, the rate of technical change can be decomposed into pure and biased technical change. These correspond to the first two terms and the last term of (17) respectively. The pure effect of technical change demonstrates the effect of technical change *per se*, while the biased effect shows the effect through the use of labour and capital.

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[13]The symmetric generalised McFadden functional form as presented by Diewert and Wales (1987) allows for the imposition of both monotonicity and concavity.

3 Data and Empirical Results

All data used in the analysis is taken from the interpolated series of national income accounts prepared at the Central Bank of Ireland. Estimation was conducted over the period 1981:1 to 1999:4. Output is in constant 1995 prices and labour is measured as the actual numbers of people employed. Wages are derived by dividing total compensation of employees in the economy by labour. The derivation of capital and the cost of capital is presented in Appendix A to this paper. Full details of the data along with details of its interpolation may be obtained from McGuire, O'Donnell and Ryan (2002). All estimations were conducted using the nonlinear systems estimator (NLSYSTEM) in WinRats-32 5.0.[14]

The parameter estimates from the Cobb Douglas system in (3) through (5) are presented in Table 1. Also presented in the table are the initial results for the same system conducted in 1999 - denoted as 'Previous' in the table. It is evident that the parameter results have not differed significantly between both estimations.[15] The gamma parameter, which indicates productivity growth, is still quite high at just less than one per cent per quarter. The parameter estimate for beta is not that much different from the share of capital in national income. As mentioned previously, the eta parameter represents a mark-up over marginal cost and as such, the greater its difference from unity, the stronger the evidence of imperfect competition in the Irish market. Its value has increased marginally between the present and previous estimation - (1.457 versus 1.378). Initially, the results are encouraging owing to the feasibility of the parameter estimates and their similarity to the estimates achieved in the previous estimation. However, closer inspection of the macro indicators associated with the system estimates reveal one or two potential problems. Figure 1 plots the output gaps associated with all of the approaches in the paper. The gap associated with (6) and (7) (CD1), clearly increases substantially towards the end of the sample, reaching values of almost 20 per cent by 1999. Actual output therefore, was considerably above the potential level of output denoted by (7) for this period. In the mid 1980's the gap is persistently negative reaching values of over 10 per

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[14] All programs are available from the author upon request.

[15] The data is not exactly the same, capital in the more recent estimation refers to non-housing capital stock whereas in the previous estimation, housing was included. See Appendix A for more on this point.

cent. The presence of such large output gaps suggests that the underlying residuals associated with the simulated production function may not be stable or white-noise. After all, the residuals for the production function are obtained by substituting X_1 for X_1^F in (7) and then calculating the difference between (7) and actual output. Miss-specification tests for the underlying Cobb Douglas production function are available in row 1 of Table 4 (CD1). The tests clearly demonstrate that for the Cobb Douglas form, the residuals from the underlying production function display clear evidence of both heteroscedasticity and autocorrelation. Unsurprisingly, there is evidence of non-stationarity in the residuals aswell. The results are to be expected given the nature of the estimated output gap. However, they do suggest that alternative models of the supply side should at the very least be explored with the results used to check/condition those from the Cobb Douglas specification.

Table 2 presents the parameter results for both the cost function and production function given by (12) & (17).[16] For the cost function system, almost 73% of the parameters are significant at the 1 per cent level, while 90% of the production function parameters are significant at the same level. The miss-specification tests in Table 4, suggest that the residuals of the primal approach (PF), while displaying evidence of serial correlation, can at least reject the presence of heteroscedasticity and non-stationarity amongst the residuals. Therefore, the residuals would appear to be 'better behaved' than those of the Cobb Douglas approach. The Jarque-Bera statistic calculated for the residuals is also considerably less for the direct estimation of the production function. The output gap associated with the production function approach (PF) can be compared with that of the Cobb Douglas in Figure 1. The translog production function gap is much more moderate than that of the Cobb Douglas reflecting the more 'normal' nature of the residuals associated with that approach. Persistent positive gaps exist for the 1988-1991 time period, however, these gaps reach a maximum value of less than six per cent for the period. More significantly, there would appear to be a sharp contrast in the nature of the gap predicted for the end of the sample period (1994 - 1999) between the Cobb Douglas model and the translog approach. In particular, the Cobb Douglas

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[16]Note, as the cost function is estimated, only one of the factor demand equations needs to be estimated in order to fully identify all of the underlying parameters. In this case, the labour demand function is the one estimated.

approach suggests actual output in the economy is considerably greater than that of potential output, while the translog gap indicates that potential and actual output are converging towards one another for the period. As noted in section 2.3, because the translog cost function cannot be 'mapped' exactly back to the underlying production function, it is not possible to generate a corresponding output gap with the dual translog form. In the context of the overall modelling exercise, this is a considerable drawback of this approach.

The elasticities associated with the cost function approach ((13) & (14)) are summarised in Table 3. It is evident that labour and capital are substitutes, as one would have expected. Concavity in input prices is observed for capital but not for labour. The own Allen-Uzawa elasticity for labour - σ_{LL} is also positive. These results can be contrasted with the case of the Cobb Douglas, where unitary elasticity of substitution is assumed between the factor inputs. In terms of the magnitudes of the results, it should be noted that the elasticities achieved are quite large relative to other results. For instance, the long-run Allen-Uzawa own elasticity for capital is approximately three times as large as that achieved by Hall and Nixon (1999). Similarly, the price elasticities in Table 3 are over four times as large as those in Hall and Nixon (1999).[17]

3.1 The Cobb Douglas Revisited

The preceeding has outlined the results from three different models of the Irish supply-side. In general, the Cobb Douglas approach is favoured owing to the consistent series of factor demands associated with it and the ease with which certain key macro indicators can be generated owing to the function's 'self dual' nature. However, the residual estimates and the closely related output gap generated with the Cobb Douglas model do present certain difficulties. This is particularly the case, when the results are compared with output gaps from the translog production function. Consequently, in this section, the Cobb Douglas system is revisited with certain modifications made in the light of previous results.

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[17]It should be noted however, that the elasticities in Hall and Nixon (1999) are not exactly comparable to those in this paper. They generate elasticities for a long-run static model equivalent to (11) and (12), however, they are generated within a dynamic, short-run specification.

Three different variations on the original Cobb Douglas model are considered and estimated. These are as follows

(1) the inclusion of a structural change indicator (CD2),

(2) the inclusion of a seperate time trend (CD3),

(3) the adoption of a non-linear productivity growth rate (CD4).[18]

(1) and (2) are essentially seeking to capture the same phenomenan - the considerable increase in growth rates in the domestic Irish economy for the latter part of the 1990's - the 'Celtic Tiger' effect. The first approach seeks to provide a more structural explanantion for this, while the second adopts an additional trend which commences in 1995:1 and continues until the end of the sample. The structural change indicator is the proportion of output from the hi-tech[19] sector to total manufacturing output. This proportion has increased considerably over the sample period ranging from 33 per cent in 1981:1 to almost 74 per cent by 1999:4.

The third change concerns the use of a linear trend as a proxi for technical progress. This assumption may well be inappropriate overtime, particularly, given the significant changes in the Irish economy over the period 1981 - 1999. Therefore, the estimate of TFP from (8) is subsequently filtered (TFP^*) with the Hodrick-Prescott (HP) filter.[20] In the subsequently, revised Cobb Douglas system, $ln(alpha) + (1 - beta)gamma\,T$ is replaced by TFP^*

[18]This latter proposal was kindly suggested by Geraldine Slevin, Central Bank of Ireland, and the approach follows that in Slevin (2001)

[19]Using the definitions of these sectors from the Central Statistics Office (CSO).

[20]The HP filter fits a trend through all the observations of the series in question. This is achieved by obtaining a trend series estimate that simultaneously minimises a weighted average of the gap between the series and the trend series, at any point in time and the rate of change in the trend series at that point in time. As the data is quarterly, the λ coefficient in the filter is set to 1600.

$$lnYED = ln(eta) - ln(1 - beta) - 1/(1 - beta)\ TFP^*$$

$$+ln(P_1) + (beta/(1 - beta))\ ln(Q/X_2) \tag{18}$$

$$ln(X_1/Q) = -beta\ ln(X_2/X_1) - TFP^* \tag{19}$$

$$ln(P_2) = ln(beta/(1 - beta)) + ln(P_1) - ln(X_2/X_1) \tag{20}$$

The associated output gap is obtained by re-specifying (7) as

$$lnQ_t^* = (1 - beta)\ lnX_{1t}^{F} + beta\ lnX_{2t} + TFP^* \tag{21}$$

and taking the simulated value from actual output. The three output gaps ((CD2), (CD3) and (CD4)) are plotted in Figure 1, miss-specification results are in Table 4 and parameter estimates are summarised in Table 5. From Figure 1, it is apparent that the presence of a structural break variable in CD2 does not greatly affect the size of the output gap relative to the initial one estimated. Indeed, Table 6, which lists the correlation coefficients between the different gaps illustrates this quite clearly. Significantly, the parameter sign on the structural break variable is not as one would have hypothesised (-) and the variable is insignificant in explanatory power terms. Therefore, this permutation of the original Allen and Mestre (1997) model would not appear to characterise much of an improvement on the original model.

The diagnostics for model CD3 are a marginal improvement on those in CD1 and CD2, however, not by much. Table 4 reveals that the Jarque-Bera statistic is considerably lower for CD3 than it is for either CD1 or CD2. Autocorrelation and heteroscedasticity tests still indicate evidence of both in the residuals of CD3. The output gap for CD3 is reduced in magnitude relative to the initial Cobb Douglas model, with smaller negative output gaps in the early 1980's and smaller positive gaps in the late 1990's. From Table 6, the output gap for the additional trend model is still highly correlated with the initial two Cobb Douglas models. The diagnostics for the final Cobb Douglas model estimated (with the HP filtered TFP term) would appear to be the most satisfactory of all of the Cobb Douglas models. Both the presence of ARCH errors and non-stationarity in the residuals can be rejected - the latter at the 5 per cent. Autocorrelation in the same residuals persists however. Significantly, the output gap for CD4 is closet to that of the translog production

14

function (with a correlation coefficient of 84 per cent) and the Jarque Bera statistic is the lowest of all of the production models. The *relatively* robust nature of the translog's residuals, the high correlation between CD4's output gap and that of the translog production function, all lend considerable credence to its selection as the optimal specification of the Irish supply-side.

4   Conclusions

This paper has presented a suite of different supply side models of the Irish economy. These consist of the generic Cobb Douglas model with associated factor demands, and two flexible functional form specifications - a dual cost function and a production function. The results from the more flexible specifications are then used to re-examine the original Cobb Douglas supply side model. Unfortunately, neither the flexible form cost, or, production function were in themselves, able to service the requirement of the overall modelling execise.

The original Allen and Mestre (1997) model is expanded to allow for the possibility of a structural break in the Irish economy over the sample period as well as the specification of a more flexible productivity measure. This re-examination is, in part, warranted by the results achieved with the original Cobb Douglas model. Unsatisfactory diagnostic results for the underlying production function coupled with very large output gaps pose questions for the basic specification. Using the translog production function as a benchmark model, the results of three different permutations of the initial model were then compared. The results suggest that the model, allowing for a more flexible specification for technical progress, generates residuals and macro indicators closet to that of the translog model. The results would also appear to suggest that the issue of technical progress in supply-side models of the Irish economy warrants further examination.

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A Capital and the Cost of Capital

Almost all data used in this paper is taken from the interpolated series of national income accounts maintained at the Central Bank. However, certain adjustments have been made to the capital and cost of capital series.

The Irish housing market experienced a sharp increase in prices throughout the latter part of the 1990's and the early part of the new century. Many reasons have been advanced for this including institutional factors which resulted in a relatively slow adjustment in the Irish housing stock to these new market conditions. Consequently, non-housing capital is used as the total capital stock. A value of housing stock was obtained for the initial time-period of 1980:1 and an associated non-housing capital stock generated. This was rolled forward using the perpetual inventory method with a non-housing investment series.

The cost of capital series (CC0) is based on the standard Jorgenson, Gollop and Fraumeni (1987) expression

$$CC0_t = I_t[r_t + \sigma_t - (I^e_{t+1} - I_t)/I_T]$$ (22)

where I is an investment deflator, e denotes expected value, r_t is the cost of borrowing funds and σ_t is a depreciation factor. Two adjustments were made to this series. Firstly, a split depreciation schedule was used with the level of depreciation increasing from 6.25 per cent prior to 1996 to 9 per cent thereafter. This, in part, reflected the changing nature of the Irish capital stock with anecdotal and investment evidence of movements towards a faster depreciating stock. It also reflected the exclusion of housing from the capital stock. The depreciation rate in previous applications had been 4 per cent. This rate appeared quite low, particularly, when compared with rates used by the Bureau of Economic Analysis (BEA) in the United States.

The second adjustment made to (22) was to increase the cost of borrowing funds. Previously, r_t had been equivalent to the AAA or prime rate - the rate charged to large commercial customers for short-term borrowings. However, a simple average of the AAA rate and the AA rate is now used. The latter is the rate to charged to more medium sized enterprises. Given, that these enterprises usually face larger

rates, the new r_t is persistently above the level of the older rate.

Table 1: Initial Cobb Douglas Parameter Estimates

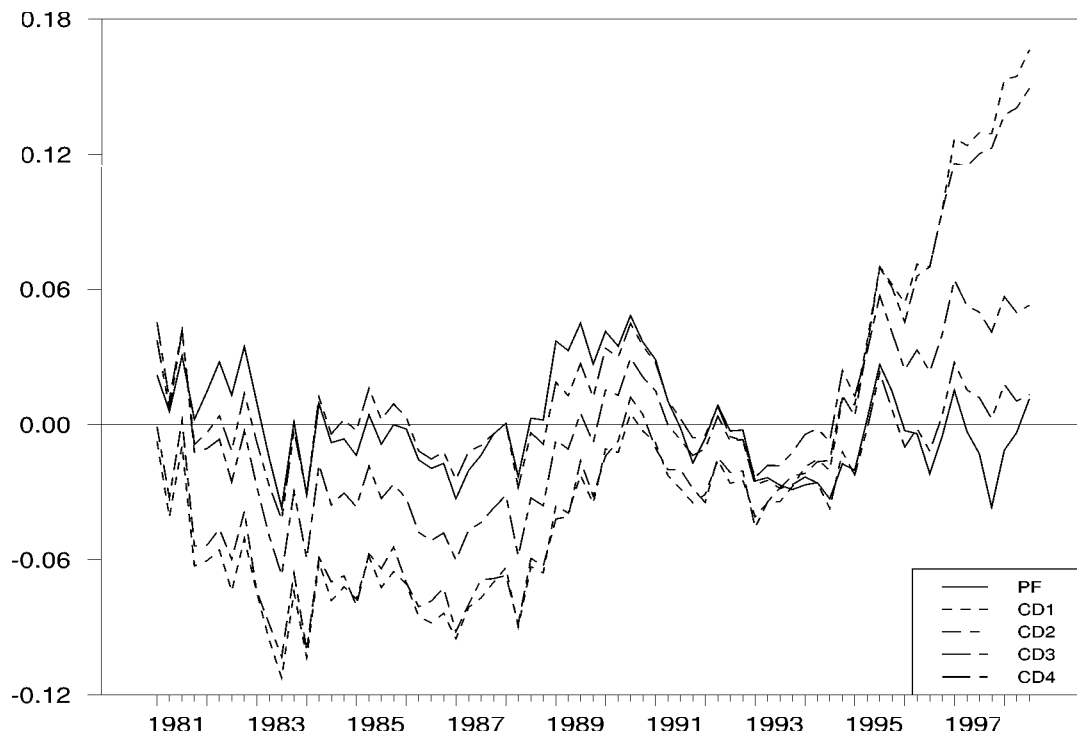| Parameter	Present Value	Present P-Value	Previous Value	Previous P-Value
alpha	1.431	0.000	1.252	0.046
beta	0.335	0.000	0.321	0.009
gamma	0.009	0.000	0.009	0.000
eta	1.457	0.000	1.378	0.020

Sample size $= 76$

Figure 1: Alternative Output Gaps

Table 2: Flexible Functional Form Parameter Estimates

| | Cost Function | | | Production Function | |
Parameter	Estimate	P-Value	Parameter	Estimate	P-Value
ω_0	9.722	0.000	θ_0	9339.463	0.000
ω_Q	1.000	0.000	θ_1	384.516	0.000
ω_T	-0.001	0.000	θ_2	-1934.248	0.000
ω_1	-0.476	0.000	θ_T	2.498	0.008
ω_2	1.476	0.000	θ_{TT}	0.0001	0.429
ω_{11}	0.248	0.000	θ_{11}	11.012	0.005
ω_{12}	-0.239	0.000	θ_{22}	201.454	0.000
ω_{22}	0.230	0.000	θ_{12}	-41.603	0.000
ω_{1Q}	0.000	0.302	θ_{1T}	0.043	0.000
ω_{2Q}	0.000	0.999	θ_{2T}	-0.252	0.003
ω_{1T}	0.133e-03	0.169			
ω_{2T}	-0.886e-03	0.000			
ω_{QT}	0.000	0.004			
ω_{QQ}	0.000	0.999			
ω_{TT}	0.198e-03	0.000			

Sample size = 76

Table 3: Cost Function Elasticity Results

Elasticity	Estimate
σ_{LL}	0.746
σ_{LC}	2.094
σ_{CC}	-2.307
η_{LL}	0.479
η_{LC}	0.739
η_{CC}	-0.814
η_{CL}	1.295

Note: All elasticities are evaluated at the sample mean

Table 4: Production Function Miss-Specification Tests and Statistics

| | P-Values | | | | Statistic |
	AR(1)	AR(4)	ARCH	ADF	Jarque-Bera
CD(1)	0.000	0.000	0.000	0.324	15.623
PF	0.004	0.000	0.959	0.000	0.764
CD(2)	0.000	0.000	0.000	0.403	14.064
CD(3)	0.000	0.000	0.007	0.070	1.267
CD(4)	0.000	0.000	0.961	0.000	0.094

Note: CD(1) refers to the initial Cobb Douglas model, PF is the translog production, CD(2) refers to the second Cobb Douglas model with a stuctural change component, CD(3) is the Cobb Douglas model with the additional trend term and CD(4) is the Cobb Douglas model with a filtered TFP used as a proxi for technical change. AR tests are from Godfrey (1978) and Breusch (1978), ARCH tests are from Engle (1982) and ADF are standard augmented dickey fuller tests.

Table 5: Cobb Douglas Parameter Estimates

| | CD2 | | CD3 | | CD4 | |
Parameter	Value	P-Value	Value	P-Value	Value	P-Value
alpha	1.329	0.000	1.346	0.000		
beta	0.335	0.000	0.336	0.000	0.335	0.000
gamma	0.007	0.000	0.010	0.000		
gamma1	0.331	0.116	0.012	0.000		
eta	1.457	0.000	1.453	0.000	1.457	0.000

Sample size = 76

Note: gamma1 is the coefficient on the structural change variable in CD(2) and the coefficient on the additional trend in CD(3).

Table 6: Correlation Coefficients of the Different Output Gaps

	PF	CD1	CD2	CD3	CD4
PF	1	0.08	0.09	0.36	0.84
CD1		1	1	0.87	0.27
CD2			1	0.89	0.28
CD3				1	0.56
CD4					1