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# Alternative Tests of the Expectations Hypothesis of the Term Structure of Interest Rates

By

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## **Abstract**

This paper reports the results for a number of alternative tests of the term structure of interest rates using Monte Carlo analysis. The study provides an extension to previous work done on the term structure using Irish money market rates. The aim of the paper is to provide evidence that rejections of the expectations hypothesis may be due to inaccurate statistical testing rather than the data being inconsistent with the theory. The study first focuses on some traditional methods of testing the EH using single equations. Then some alternative tests, which are frequently used in the literature, are discussed. However, we show using Monte Carlo experiments that the rejection of the EH using these alternative tests may be due to poor statistical properties in finite samples rather than a rejection of the theory.

## 1. Introduction

The expectations hypothesis (EH) of the term structure implies that the yield spread between the long rate and short rate is an optimal predictor of future changes in short rates over the life of the 'long bond'.

There is a great deal of evidence on the EH for the US, based on the Campbell and Shiller (1991) VAR methodology using monthly data on spot rates (e.g. Hardouvelis, 1994). In general, for a wide variety of maturities from 1 to 12 months and for 2, 3, 4, ... 10-year maturities, Campbell and Shiller (1991) reject the EH. The (long-short) interest rate spread does not predict the direction of changes in the long-term interest rate that is consistent with the EH, and future changes in short-rates are not often closely correlated with the long-short spread (Campbell and Shiller, 1991).

Kugler (1988) using US, German and Swiss monthly data on one and three month Euromarket deposit rates found support for the EH only on German data (for the period of March 1974 to August 1986). Similarly, Engsted (1994) using Danish money market rates and Engsted and Tanggaard (1994) for longer maturity bonds find considerable support for the EH when the variation in interest rates is relatively large, such as in the post-1992 ERM 'crisis period'. This is to be expected following the analysis of Mankiw and Miron (1986), for if interest rate stabilisation results in random walk behaviour for short rates, then the expected change in short rates is zero and the spread has no predictive power for future short rates (See also Rudebusch, 1995).

Using the Campbell-Shiller VAR methodology on data at the short end of the maturity spectrum (i.e. up to one year) Cuthbertson (1996) finds reasonable support for the EH on UK data. This is in contrast to Taylor (1992) who uses maturities of 5, 10 and 15 years and finds strongly against the EH (see also MacDonald and Speight, 1991). Two recent studies have tested the EH using Irish data. Cuthbertson and Bredin (2000 and 2001) using a VAR methodology find evidence in support of the EH using a range of interest rate maturities.

The main aim of the paper is to present results based on alternative methods and to test the validity of these procedures. Although the theory suggests that the long-short spread should have some predictive power for the future change in the short rate, ordinary least squares (OLS) estimation of the actual change in the short rate on the lagged yield spread plus a constant, results in coefficients which are often both the wrong size and sign (Mankiw and Miron, 1986). However, empirical evidence has found that, when the same relationship is estimated using instrumental variables (IV) to allow for errors in variables or a random error in the term structure relationship, the rejection of the model is less conclusive. I will also estimate this model using generalised methods of moments (GMM) to account for any heteroscedasticity in the error term or serial correlation due to the use of overlapping errors. The model is also estimated the other way around, i.e. the yield spread is regressed on the expected future changes in the short rate, where the actual future change in the short rate proxy the expected change.

In the final section of the paper, I present the results from a number of Monte Carlo (MC) experiments. The experiments focus on the fact that the alternative single equation tests based on the regression of the change in the short-term rate on the lagged spread are prone to severe over-rejection of the EH, even when it is true. However tests of the spread on the first difference of the short-rate reject at the correct rate.

## **2 Theoretical Review**

### **2.1 Traditional Single Equation Tests<sup>1</sup>**

The Expectations hypothesis (EH) of the term structure posits that the return on an  $n$ -period bond  $R_t^{(n)}$  is determined solely by expectations of (current and) future rates on a set of  $m$ -period assets  $r_t^{(m)}$  where  $n > m$ . Using the continuously compounded spot rates the ‘fundamental term structure’ relationship is:

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<sup>1</sup> This section of the paper draws on Cuthbertson and Bredin (2000).

$$R_t^{(n)} = (1/k) \sum_{i=0}^{k-1} E_t r_{t+im}^{(m)} \quad (1)$$

where  $k = n/m$  is an integer. Reparameterising (1) enables the spread to be interpreted as the optimal predictor of future changes in short rates,  $r_t^{(m)}$ :

$$S_t^{(n,m)} = E_t \sum_{i=1}^{k-1} (1-i/k) \Delta^m r_{t+im}^{(m)} = E_t [PFS_t^{(n,m)}] \quad (2)$$

where  $S_t^{(n,m)} = (R_t^{(n)} - r_t^{(m)})$  is the yield spread. For example, if  $m = 1$  the above equation states that the spread between an  $n$ -period bond and a one-period bond equals the expectation of the change in one-period rates over the  $n$ -period horizon. The *Perfect foresight spread* is the spread that would be predicted by agents if they had perfect foresight about future movements in interest rates. A testable implication of equation 2 is that the spread Granger causes future changes in short rates.<sup>2</sup> Assuming that  $(R_t^{(n)}, r_t^{(m)})$  are  $I(1)$ , then  $\Delta r_t^{(m)}$  is  $I(0)$ , which from equation 2 implies that the spread  $S_t^{(n,m)}$  should also be  $I(0)$ . and therefore  $(R_t^{(n)}, r_t^{(m)})$  should be co-integrated with a co-integrating parameter of unity.<sup>3</sup>

If we now add the assumption of rational expectations (RE):

$$r_{t+im}^{(m)} = E_t r_{t+im}^{(m)} + \varepsilon_{t+im} \quad (3)$$

we obtain the following single equation test of the null of the ‘expectations hypothesis plus rational expectations’, EH + RE:

$$PFS_t^{(n,m)} = \alpha + \beta S_t^{(n,m)} + \gamma \Omega_t + \varepsilon_t^* \quad (4)$$

$$H_0 : \alpha = \gamma = 0, \beta = 1$$

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<sup>2</sup> Strictly, failure of Granger causality does not constitute a rejection of the EH, but a failure to confirm it.

where  $\boldsymbol{\varepsilon}_t^*$  is a moving average error of order (n-m-1) consisting of a weighted sum of future values of  $\boldsymbol{\varepsilon}_{t+im}$ . Under RE,  $\boldsymbol{\varepsilon}_t^*$  is independent of information at time t,  $\boldsymbol{\Omega}_t$ , and in particular is independent of the yield spread. If there is a constant term premia or if there are differential yet constant transactions costs (between investing ‘long’ and in a series of rolled-over short-term investments) then  $\alpha \neq 0$ . Under RE the right hand side variables in equation 4 are independent of  $\boldsymbol{\varepsilon}_t^*$  and hence we do not require IV estimation. However a GMM estimator is employed to correct the covariance matrix for the moving-average error of order (n-m-1) and possible heteroscedasticity (Hansen, 1982; Newey and West, 1987).

## 2.2 Alternative Single Equation Tests

In this section a number of alternative single equation tests will be described. I focus on the 6 month – 3 month combination. Under the EH the 6 month rate is equal to an average of current plus expected future 3 month rates plus a term premium.

$$R_t^{(6)} = (1/2)[r_t^{(3)} + E_t r_{t+3}^{(3)}] + \theta \quad (5)$$

While the 6 month - 3 month spread is equal to a weighted average of the expected change in future 3 month rates.

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<sup>3</sup> Strictly, for this to hold, forecast errors and any term premia must also be I(0). Shea (1992) examines the possibility of multiple co-integrating vectors, an issue not explored here since we concentrate on tests of bilateral relationships.

$$S_t^{(6,3)} = (1/2)E_t \Delta^3 r_{t+3}^{(3)} + \theta \quad (6)$$

One can see that an appropriate test would be, the regression of the change in the short rate on the lagged yield spread and a constant (Mankiw and Miron, 1986). Taking the inverse of equation 6 and from theory the short rate should differ from its predicted value only by a forecast error, which would be orthogonal to all information at t or earlier, yields;

$$\Delta^3 r_{t+3}^{(3)} = -2\theta + 2S_t^{(6,3)} + \eta_{t+3} \quad (7)$$

where  $\eta_{t+3}$  is the forecast error.

The EH implies that the coefficient on  $\Delta R_{t+3}$  should be insignificantly different from 1/2, in equation 6, while the coefficient on  $S_t$ , in equation 7, should be insignificantly different from 2. Empirical evidence would suggest that the EH does not hold; OLS estimation yields coefficients that are of the wrong magnitude and sometimes the wrong sign (Shiller, 1990).

There are two possible explanations for the empirical rejection of the theory. The first assumes that market expectations are rational but that the information contained in the in the term structure is affected by non-stationary risk premia. The second explanation assumes that risk premia are stationary, but that market expectations are not strictly rational and so long rates tend to overreact to future short rates. I will focus on the weaker form of the EH where we have a term premium that contains elements which vary randomly over time, independent of the short-term rates. Equation 5 is replaced by;

$$R_t^{(6)} = (1/2)[r_t^{(3)} + E_t r_{t+3}^{(3)}] + \theta + \varepsilon_t \quad (8)$$

and equation 6 by

$$S_t^{(6,3)} = (1/2)E_t \Delta^3 r_{t+3}^{(3)} + \theta + \varepsilon_t \quad (9)$$

The  $\varepsilon_t$  is a zero-mean random term and is uncorrelated with  $r_{t+3}$ . The  $\varepsilon_t$  may be thought of as representing a time varying term premium or some other error.

Equation 7 may be re-written as;

$$\Delta^3 r_{t+3}^{(3)} = -2\theta + 2S_t^{(6,3)} + \eta_{t+3} + 2\varepsilon_t \quad (10)$$

### 3. EMPIRICAL EVIDENCE

#### 3.1 Evidence based on the Traditional Single Equation Tests

Campbell and Shiller (1991) use a wide variety of maturities from 1 to 12 months and for 2,3,4 ... 10-years to test the EH on monthly data from January 1952 to February 1987. For the perfect foresight spread equations Campbell and Shiller find slope coefficients ranging between 0 and 0.5 for maturities up to 2 years and for maturities greater than 2 years the slope coefficients increase to around 1. Overall they find little support for the EH at the short end, but do find some support at the long end of the maturity spectrum.

However for my purposes the studies carried out using UK data will prove important for comparison purposes<sup>4</sup>. Hurn, Moody and Muscatelli (1995) also test the

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<sup>4</sup> Although the Sterling/Irish pound link has been broken, with Ireland joining the ERM in March 1979, the two countries financial systems are still very closely linked, Walsh (1996).



term structure of interest rates at the short end of the maturity spectrum using UK LIBOR for 1, 3, 6, and 12 month maturities. In contrast to Cuthbertson (1996), the authors use monthly data over a longer sample period covering January 1975 to December 1991. The findings from the perfect foresight spread regression are supportive of the EH, with slope coefficients ranging between 0.816 for the S(12,3) months spread and 1.168 for the S(3,1) months spread combination.

Two recent studies give evidence in favour of the EH using UK data, Cuthbertson (1996) and Cuthbertson, Hayes and Nitzsche (1996). Cuthbertson (1996) uses London Interbank (offer) rates (LIBOR) with maturities of 7 days, 1, 3, 6, and 12 month to test the EH at the short end of the maturity spectrum. The data set is sample weekly beginning on the 2<sup>nd</sup> Thursday in January 1981 and ending on the 2<sup>nd</sup> Thursday of February 1992. The perfect foresight spread equations yield evidence in favour of the economic theory. The null  $H_0 : \beta=1$  (given  $\gamma = 0$ ) is not rejected in all cases except that for the 4-week/1-week spread.

Cuthbertson, Hayes and Nitzsche (1996) using UK Certificates of Deposit (CD) rates for maturities of 1, 3, 6, 9 and 12 months also find evidence in favour of the EH + RE. The data set used in the study is sampled weekly and covers the period from the 1<sup>st</sup> of October 1975 to 14<sup>th</sup> of October 1992<sup>5</sup>. In all cases the authors do not reject the null of  $H_0: \beta=1$ , or that information, available at  $t$  or earlier does not incrementally add to the predictions of future interest rates. Therefore the results are consistent with the EH + RE and previous empirical evidence on UK rates, Cuthbertson (1996)<sup>6</sup>.

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<sup>5</sup> The only exception here is the 9 month CD rate which ends on the 27<sup>th</sup> of January 1988.

<sup>6</sup> Cuthbertson and Bredin (2000) also report the perfect foresight spread results using Irish data, but for comparison we also report them in this paper.

### 3.2 Evidence based on the Alternative Single Equation Tests

Mankiw and Miron (1986) run an OLS regression of the change in the short-run rate  $\Delta R_t$  on the lagged yield spread  $S_{t-1}$  and a constant over a number of different sample sizes. The total sample period runs from 1890 – 1958, however the authors take into account the different monetary regimes over the total sample period. In all the predictive power of the spread is tested over 4 different regimes for the (6m, 3m) maturity combination using quarterly data<sup>7</sup>.

$$(r_{t+1} - r_t) = \alpha + \beta(R_t - r_t) + v_{t+1} \quad (11)$$

Initially Mankiw and Miron (1986) test the predictive power of the spread over a relatively recent sample period, 1959 – 1979. The authors find a coefficient on the spread that is insignificantly different from zero and significantly different from the theoretical value of 2. The authors also find an adjusted R-squared of 0.01, which implies that the spread has negligible predictive power. Overall the results are not supportive of the expectations hypothesis and suggest that the slope of the yield curve does not contain information regarding the future path of the short rate.

The results for the various sub-samples offer some interesting comparisons. For the sub-samples over the period 1915 – 1958 the results are remarkably similar to the recent sample and the spread is not significantly different from zero. However for the sample period 1890 – 1914, the results are in marked contrast and, although the coefficient on the spread is still statistically different from its theoretical value, it is three times larger than that for the recent sample. The predictive power is also much

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<sup>7</sup> The 4 sub-samples for each of the different regimes are the following. 1890Q4 – 1914Q4 which ends with the founding of the Federal Reserve System. 1915Q1 – 1993Q4 ends with both the introduction of the New Deal banking reforms and the approximate end of the gold standard and approximate beginning of interest rate pegging. 1934Q1 – 1951Q1 which ends with the Fed no longer pegging interest rates and

greater, the adjusted  $R^2$  is 0.40 compared to 0.01 for the recent sample. Therefore the sample period 1890-1914 confirm that expectations are an important factor of fluctuations in the yield curve.

Two recent studies include Sola and Driffill (1994) and Driffill, Psaradakis and Sola (1997). Sola and Driffill (1994) test the expectations hypothesis for the 3 and 6 month combination using quarterly data for US treasury bills for the sample 1962Q1 – 1987Q3. Given the change in the Federal Reserves operating procedure towards monetary base control in the late 70's the authors initially test the theory from 1962Q2 – 1979Q3 and so avoid any possible rejection based on the shift in regime<sup>8</sup>. The authors initially estimate equation 7 using OLS and find the coefficient on the lagged spread is significantly different from the theoretical value, 2. However as has been discussed above, the failure of equation 7 may be due to time varying term premia, fads, measurement errors, or other random deviations from the pure expectations hypothesis.

The authors re-estimate the model using instrumental variables (IV), using  $S_{t-i}$ ,  $i=2, \dots, 4$ , and  $\Delta r_{t-i}$ ,  $i=1, \dots, 4$  as instruments. The results do not find evidence against the weak version of the EH. Sola and Driffill (1994) also estimate equation 10 using  $S_{t-i}$ ,  $i=1, \dots, 4$ , and  $\Delta r_{t-i}$ ,  $i=0, \dots, 4$  as instruments. The coefficient on the change on the short rate is not statistically different from the theoretically correct value of 0.5. The authors conclude that equation 10 is the correct model and there is a measurement or other error in equation 7. In a Monte Carlo study Driffill, Psaradakis and Sola (1998) confirm that the results found in real data are likely to emerge when equation 10 is the correct model. The authors also estimate the models over the full sample period 1962Q2 – 1987Q3 and find unsurprisingly that the theory is rejected when there is no allowance made for the regime switch.

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finally 1951Q2 – 1958Q4 ending at the time when the active market in 3 and 6 month Treasury bills begins.

<sup>8</sup> There are 2 possible reasons why the theory may be rejected given the switch in regime. There may be a break in the time series properties of the data given the regime switching. Secondly there may be a perception of future shifts which may affect the behaviour of market participants, i.e. 'peso problem'.

Driffill, Zacharias and Sola (1997) test the expectations hypothesis for UK and US 1 and 3 month combinations using monthly data. The sample period for UK covers the period 1975M2 - 1994M12 and 1982M12 - 1991M2 for the US. The authors also analyse the theory for the UK using a shorter sample period, starting 1982M11, given the possible structural breaks in the preceding period<sup>9</sup>. The authors recognise the possible measurement or other error in the relationship between the spread and the expected future change in the short rate and so focus on;

$$S_t = (2/3)E_t \Delta r_{t+2} + (1/3)E_t \Delta r_{t+4} + \theta + \varepsilon_t$$

Again the authors run the single equation tests using IV, and find that the results are well within the statistically significance level. Also of note is the fact that non-rejection of the theory was found in both samples for the UK.

## 4.0 EMPIRICAL RESULTS

### 4.1 THE SPREAD AND THE PREDICTABILITY OF CHANGES IN SHORT RATES<sup>10</sup>

The regression of the perfect foresight spread,  $PFS_t^{(n,m)}$  on the actual spread  $S_t^{(n,m)}$  and the limited information set  $H_t$  (consisting of lags of  $S_t^{(n,m)}$  and  $\Delta r_t^{(m)}$ ) are shown in table 1. Under the null hypothesis of PEH + RE, we expect  $H_0 : \alpha = \gamma = 0, \beta=1$ . The method estimation is GMM with a correction for heteroscedasticity and moving average errors using the Newey-West (1987) declining weights.

<sup>9</sup> There were a number of causes for these structural changes; the change in monetary policy regime that accompanied the Medium Term Financial Strategy, and the elimination of all foreign exchange controls.

<sup>10</sup> Unit root tests on the individual series  $R_t$  and  $S_t^{(n,m)}$  indicate that we cannot reject the null hypothesis that changes in short rates  $\Delta r_t^{(m)}$  and the yield spread  $S_t^{(n,m)}$  are  $I(0)$ . A detailed discussion of the unit root properties of the data is discussed in Cuthbertson and Bredin (2000), where a similar data set is used.

I first run the model including the information set and test,  $H_2: \gamma = 0$ . As can be seen from table 1, I cannot reject the null and so is consistent with the theory. In all cases I do not reject the null of  $H_0 : \beta=1$  or that information, available at time  $t$  or earlier does not incrementally add to the predictions of future interest rates. This is the case for the each of the chosen interest rate maturity combinations, at the short end of the spectrum. The results therefore do not reject the EH + RE.

## **4.2 ALTERNATIVE SINGLE EQUATION TEST RESULTS**

I now focus on alternative single equation tests, described earlier in the paper. I focus on the 6 month and the 3 month interest rate combination, as a comparison with Sola and Driffill (1994). The method of estimation includes IV and GMM. Based on previous empirical evidence, Shiller (1990), OLS estimation yields coefficients that are often both the wrong sign and magnitude. Therefore I focus on the errors in variables, IV estimation and GMM. I also run a Wald test of the restriction implied by the EH.

I first consider equation 6, where the 6 month – 3 month spread ( $S_t$ ) is regressed on the change (3 month) in the future 3 month short rate ( $\Delta r_{t+3}$ ) and a constant. The numbers in parenthesis are the standard errors.

Regressing  $S_t$  on a constant and  $\Delta r_{t+3}$ , using  $S_{t-i}$ ,  $i = 1, \dots, 4$  and  $\Delta r_{t-i}$ ,  $i = 0, \dots, 4$ , as instruments yields;

***Instrumental Variables (IV)***

$$S_t = 0.0005 + 0.37\Delta R_{t+3} \quad (12)$$

$$[0.0005] + [0.05]$$

**Instruments:**  $S_{t-1}, S_{t-2}, S_{t-3}, S_{t-4}, \Delta r_t, \Delta r_{t-3}, \Delta r_{t-6}, \Delta r_{t-9}, \Delta r_{t-12}$ .

Sample Size = 147, Standard Error = 0.005,  $R^2 = 0.31$

AR[ $\chi^2(12)$ ] = 45.77, HET[ $\chi^2(1)$ ] = 0.31

Wald Test of the Expectations Hypothesis Restriction:  $\chi^2(1) = 5.41$

I also give a summary of the diagnostic tests for each regression. The AR test is a test for serial correlation up to order 12, while the HET test is a test of unconditional heteroscedasticity. As can be seen from the above equation the estimated coefficient on the change in the short rate appears to be different from the theoretically correct value. The Wald test result rejects the restriction implied by the EH. As has already been discussed these results have been taken as evidence against the EH.

Given that  $\varepsilon_t^*$  may have a moving average error and be possibly heteroscedastic, I also estimate the model using a GMM estimator to correct the covariance matrix for the moving-average error of order (n-m-1) and possible heteroscedasticity (Hansen, 1982; Newey and West, 1987).

***Generalised Method of Moments (GMM)***

$$S_t = 0.001 + 0.39\Delta r_{t+3} \quad (13)$$
$$[0.0004] + [0.06]$$

**Instruments:**  $S_{t-1}, S_{t-2}, S_{t-3}, S_{t-4}, \Delta r_t, \Delta r_{t-3}, \Delta r_{t-6}, \Delta r_{t-9}, \Delta r_{t-12}$ .

Sample Size = 147, Standard Error = 0.006,  $R^2 = 0.31$

AR[ $\chi^2(12)$ ] = 92.64, HET[ $\chi^2(1)$ ] = 41.11

Wald Test of the Expectations Hypothesis Restriction:  $\chi^2(1) = 4.08$

Overall the results are quite similar to those using the IV estimation. As can be seen from both the regression results and the Wald test, the estimated coefficient on the change in the short rate appears to be different from the theoretically correct value.

I also estimate equation 7, by regressing the change (3 month) in the 3 month short rate ( $\Delta R_t$ ) on the 3 month lagged spread ( $S_{t-3}$ ). Again, I estimate using both IV and GMM estimation.

***Instrumental Variables (IV)***

$$\Delta r_t = - 0.001 + 3.02S_{t-3} \quad (14)$$
$$[0.001] + [0.71]$$

**Instruments:**  $\Delta r_{t-1}, \Delta r_{t-2}, \Delta r_{t-3}, S_{t-6}, S_{t-9}, S_{t-12}$ .

Sample Size = 150, Standard Error = 0.01,  $R^2 = 0.31$

AR[ $\chi^2(12)$ ] = 87.23, HET[ $\chi^2(1)$ ] = 2.91

Wald Test of the Expectations Hypothesis Restriction:  $\chi^2(1) = 293.18$

As can be seen from the above regression results, the Wald test overwhelmingly rejects the restriction implied by the EH. I also report the results with a correction for possible heteroscedasticity and moving average error.

***Generalised Method of Moments (GMM)***

$$\Delta r_t = -0.002 + 3.05S_{t-3} \quad (15)$$

$$[0.001] + [0.78]$$

**Instruments:**  $\Delta r_{t-1}, \Delta r_{t-2}, \Delta r_{t-3}, S_{t-6}, S_{t-9}, S_{t-12}$ .

Sample Size = 150, Standard Error = 0.01,  $R^2 = 0.31$

AR[ $\chi^2(12)$ ] = 92.64, HET[ $\chi^2(1)$ ] = 41.11

Wald Test of the Expectations Hypothesis Restriction:  $\chi^2(1) = 298.30$

Overall the results are quite similar to those using the IV estimation. As can be seen from both the regression results and the Wald test, the estimated coefficient on lagged spread is significantly different from the theoretically correct value.

## **5.0 MONTE CARLO EXPERIMENTS**

In the previous section of this paper, I used two alternative methods to test the weak version of the EH, considering time varying term premium, using single equation estimation. I now focus on the findings that the tests based on the regression of the change in the short-term rate on the lagged spread is prone to severe over-rejection of the EH. However tests of the spread on the first difference of the short-rate reject at the correct rate. I will use Monte Carlo (MC) experiments to show this point. This fact has been briefly discussed in the section dealing with the empirical results on these alternative single equation tests and can be seen at first hand from the reported results



in the previous section. The procedure used will be to estimate both methods of the single equation tests for 3 alternative samples;  $T = 100, 200$  and  $500$ <sup>11</sup>. A 1000 series of these regressions will be produced. I set up the experiments based on the Irish data set already analysed here. In the experiments I take the long rate as the 2 period rate (6 month), and the short rate as the 1 period rate (3 month). The generating process is determined by the following equations;

$$\mathbf{R}_t = (1/2)[\mathbf{r}_t + \mathbf{E}_t \mathbf{r}_{t+1}] + \boldsymbol{\theta} + \boldsymbol{\sigma}_\varepsilon \boldsymbol{\varepsilon}_t \quad (16)$$

$$\mathbf{r}_{t+1} = \boldsymbol{\mu} + \sum_{i=0}^2 \mathbf{b}_{i+1} \mathbf{r}_{t-i} + \boldsymbol{\sigma}_u \mathbf{u}_{t+1}$$

As has been mentioned the simulations are carried out based on the previously used Irish data set. Based on the data set,  $\boldsymbol{\theta} = 0.0077$ ,  $\boldsymbol{\mu} = 0.0088$ ,  $b_1 = 1.11$ ,  $b_2 = -0.47$ ,  $b_3 = 0.24$ ,  $\boldsymbol{\sigma}_\varepsilon = 0.011$ ,  $\boldsymbol{\sigma}_u = 0.012$ . As has been mentioned 1000 series of regressions will be generated, and the pseudo-random deviates  $u_t$  and  $\varepsilon_t$  will be obtained using the RNDN function in GAUSS. As a direct comparison with the previous section, estimation will be by IV and GMM.

**Model 1: The DS Test**

$$\Delta r_t = \alpha_1 + \beta_1 S_{t-1} + e_{1t}$$

**Instruments:**  $S_{t-2}, S_{t-3}, S_{t-4}$ .

$$\beta_1=2$$

**Model 2: The SD Test**

$$S_t = \alpha_2 + \beta_2 \Delta r_{t+1} + e_{2t}$$

**Instruments:**  $\Delta r_t, \Delta r_{t-1}, \Delta r_{t-2}$ .

$$\beta_2=0.5$$

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<sup>11</sup> The sample size will be  $T + 50$ , in each replication. Then the first 50 data points will be dropped in order to take account of start-up effects.

In table 2, I present the results for the mean bias for models. As can be clearly seen from the table of results, model 1 is prone to a large amount of over-rejection of the EH, even when it is true. This is the case for both the IV and the GMM estimator. As can be seen the bias for model 1 continues to be sizeable, even with a larger sample size. On the other hand, model 2's bias is much smaller and is not significant in any of the cases. In table 3, I report the findings for the test rejection frequencies for the 2 models. Again the results are consistent with that in table 2. The results are based on both the t test and the Wald test at the 5% significance level, that the beta value is equal to its theoretical value. As can be seen the fraction that reject for model 1, is much greater than that for model 2. Therefore, model 1 rejects the EH even when it is true. The results reported in this section are consistent with those reported in a similar study by Driffill, Psaradakis and Sola (1998), which uses MC experiments<sup>12</sup>.

## 6.0 CONCLUSIONS

This study looks at a number of traditional and alternative tests of the EH and uses MC experiments to test the validity. We have focused on tests of whether the spread predicts future changes in short rates at a number of interest rate maturities. In all cases we do not reject the null of  $H_0 : \beta=1$  or that information, available at time t or earlier does not incrementally add to the predictions of future interest rates. This is the case for the each of the chosen interest rate maturity combinations, at the short end of the spectrum. The results therefore do not reject the EH + RE.

This paper has also looked at some alternative single equation tests of the EH. Focusing in particular on the 6 month and 3 month maturities, I initially test the model using OLS. However given the previous evidence, that OLS estimation of the actual change in the short rate on the lagged yield spread plus a constant, results in coefficients which are often both the wrong size and sign (Mankiw and Miron, 1986), I also use IV and GMM estimation. Although both regressions reject the EH, the

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<sup>12</sup> In the Driffill, Psaradakis and Sola (1998) paper the authors set up the MC experiments using US

regression of the change in the short-term rate on the lagged spread appears to reject to a much larger degree.

Finally, I also report the results for the MC experiments which show that the single equation tests based on the regression of the change in the short-term rate on the lagged spread are prone to severe over-rejection of the EH. However the tests of the spread on the first difference of the short-rate reject at the correct rate. These findings are consistent with those from Driffill, Psaradkis and Sola (1998) using US data.

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**Table 1 :**  
**Does the Spread Predict Future Changes in Short-Rates ?**

$$\text{Regression : } PFS_t^{(n,m)} = \alpha + \beta S_t^{(n,m)} + \gamma \Omega_t$$

Spread (n,m)	Coefficients		Wald	Test	
	$\alpha$ s.e.( $\alpha$ )	$\beta$ s.e.( $\beta$ )	$H_0:\beta=1$ [p-value]	$H_1:\alpha=0, \beta=1$ [p-value]	$H_2:\gamma=0$ [p-value]
<b>(6,3) Month</b>	-0.0006 (0.0008)	1.02 (0.23)	0.007 [0.93]	0.007 [0.93]	2.88 [0.09]
<b>(3,1) Month</b>	-0.001 (0.001)	0.87 (0.21)	0.38 [0.54]	0.39 [0.53]	3.75 [0.06]
<b>(6,1) Month</b>	-0.001 (0.001)	1.04 (0.15)	0.09 [0.77]	0.08 [0.78]	0.32 [0.58]

Notes:

The regression coefficients reported in columns 2 and 3 are from the regression with  $\gamma = 0$  imposed. The method of estimation is GMM with a correction for heteroscedasticity and moving average errors using the Newey-West (1987) declining weights. The last 3 columns report Wald statistics and marginal significance levels for the null hypothesis stated. For  $H_0: \gamma = 0$  the reported results are for an information set which includes 4 lags of the change in the interest rates and the interest rate spread. The null  $H_0:\beta=1$ , is conditional on  $\gamma=0$  while the null  $H_1:\alpha=0, \beta=1$  is also conditional on  $\gamma=0$ .

**Table 2 :**  
**Monte Carlo Bias**

<b>Mean Bias of the IV Estimators</b>		
<b>T =</b>	<b>Model 1</b>	<b>Model 2</b>
<b>100</b>	<b>-1.896</b> <b>(0.0396)</b>	<b>-0.0370</b> <b>(0.0709)</b>
<b>200</b>	<b>-1.844</b> <b>(0.0398)</b>	<b>-0.0172</b> <b>(0.0702)</b>
<b>500</b>	<b>-1.830</b> <b>(0.0336)</b>	<b>-0.0095</b> <b>(0.0667)</b>
<b>Mean Bias of the GMM Estimators</b>		
<b>T =</b>	<b>Model 1</b>	<b>Model 2</b>
<b>100</b>	<b>-1.888</b> <b>(0.0363)</b>	<b>-0.0356</b> <b>(0.069)</b>
<b>200</b>	<b>-1.868</b> <b>(0.0338)</b>	<b>-0.0197</b> <b>(0.0675)</b>
<b>500</b>	<b>-1.785</b> <b>(0.0365)</b>	<b>-0.0119</b> <b>(0.0669)</b>

**Notes:**

The reported results give the mean bias of the slope estimators and its Monte Carlo standard error in parenthesis.

**Table 3 :**  
**Test Rejection Frequencies:**

<b>Test Rejection Frequencies – IV Estimators (T-Test)</b>		
<b>T =</b>	<b>Model 1</b>	<b>Model 2</b>
<b>100</b>	<b>0.540</b>	<b>0.063</b>
<b>200</b>	<b>0.548</b>	<b>0.054</b>
<b>500</b>	<b>0.524</b>	<b>0.051</b>
<b>Test Rejection Frequencies – GMM Estimators (Wald-Test)</b>		
<b>T =</b>	<b>Model 1</b>	<b>Model 2</b>
<b>100</b>	<b>0.580</b>	<b>0.069</b>
<b>200</b>	<b>0.575</b>	<b>0.065</b>
<b>500</b>	<b>0.540</b>	<b>0.058</b>

**Notes:**

The reported results give the fraction of MC replications that fail the t-test and the Wald test at the 5% significance level. Both the t-test and the Wald test, test whether the value for the beta for both model 1 and model 2 are equal to the theoretical value.