TECHNICAL PAPER

3/RT/99 **APRIL 1999**

ASYMMETRIC ADJUSTMENT COSTS AND THE DYNAMICS OF HOUSING SUPPLY

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* Paper presented at the 13th Annual Conference of the Irish Economic Association, Wesport, Co. Mayo, 23-25 April, 1999. The views expressed in this paper are not necessarily held by the Central Bank of Ireland and are the personal responsibility of the author. Comments and criticisms are welcome. The author would like to thank John Frain, Danny McCoy, Aidan Meyler and participants at a seminar in the Central Bank of Ireland for helpful comments. All errors and omissions are the authors.

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ABSTRACT

This paper examines aggregate industry dynamics on the supply side of the housing market. The representative firm's profit maximisation problem is considered in a dynamic framework which assumes asymmetric adjustment costs. This provides microfoundations for the divergence between long and short run supply elasticities and also predicts asymmetric adjustment whereby expansions are associated with slower adjustment as compared with contractions. The hypothesis of asymmetric adjustment costs is also examined empirically using data on the Irish housing market. A number of interesting insights into the dynamics of housing supply have been uncovered. These including support for the proposition that the adjustment costs associated with an expansion in housing output are greater than the adjustment costs associated with a contraction, evidence that there are threshold points beyond which output adjustment starts to speed up and also the existence of a continuum of equilibria between these thresholds where no adjustment occurs at all.

1. Introduction

Relative to the price of other goods and services, the price of both owner-occupied and rented housing has risen sharply in Ireland over recent years. These developments have attracted a good deal of commentary from which there has emerged a broad consensus concerning the fundamental factors which lie behind this event. The demand-side of the market would in particular appear to be well understood: declining mortgage interest rates, strong growth in personal incomes and demographic developments have resulted in a dramatic increase in the desired stock of dwellings. The supply of dwellings is, however, inelastic over the short-term. As a result, the relative price of housing has risen in order to "clear" the market, i.e. to choke off the excess demand and thereby equate the desired stock with the relatively fixed supply.

The medium to long-term response of the home-building sector to these developments has, however, received less attention. This is unfortunate since future outcomes in the housing market, particularly price developments, will ultimately depend on the dynamic response of firms in the home-building sector over the medium to long-run. In general, it is commonly believed that while supply is highly inelastic over the short-run a much greater supply response is forthcoming as firms in the construction sector gradually react to changes in the profitability of home-building activity. However is the dynamic response of housing supply symmetric over the business cycle? In particular, in light of the growing empirical literature which suggest that economic behaviour is not symmetric over the business cycles, this paper will investigate whether the supply response differs depending on whether or not housing output is above or below its equilibrium level. For example, are firms slower to expand output following a positive shock to demand (such as has recently been experienced in Ireland) than they are to reduce the level of output when demand contracts ? Early work on asymmetric business cycles was undertaken by Neftci (1984). Hamilton (1989) at an aggregate level finds that "the dynamics of recessions are qualitatively distinct from those of normal times in a clear statistical sense" (p. 359). In an analysis of factor demands, Pfann (1996) cites evidence that asymmetric adjustment mechanisms "give rise to unbalanced demand for capital and labour between peaks and troughs of the business cycle" (p.328). Finally, in the housing economics literature, Holly and Jones (1997) find empirical evidence that the dynamic adjustment of house prices is asymmetric depending on whether house prices are above or below their equilibrium path.

A priori, given the procyclical nature of housing market activity, expansions are constrained by the availability of skilled labour and serviced land, and also by the fixed capital available to firms which supply materials to the construction sector. Hence, firms may be forced to incur significant adjustment costs in the form of a diversion of resources away from production toward various planning, installation and search activities. In the case of the average building firm these adjustment costs might include searching for skilled labour and land which is fit for housing production (i.e. zoned and serviced), the investment of human resources and possibly capital in an attempt to secure planning permission on such land, drawing up of site development plans, site installation costs etc. When the average construction firm is scaling down its level of activity, however, such adjustment costs either simply do not arise or at least they are less likely to be significant. The likelihood that such an asymmetry would exist was suggested by Topel and Rosen (1988) in an earlier paper which applied the theory of adjustment costs to the analysis of housing supply in a dynamic profit maximising setting.[1] This paper extends that analysis by allowing for asymmetric adjustment costs using the flexible adjustment cost function introduced by Pfann and Verspagen (1989). The model provides explicit microfoundations for the distinction between the short- and the long-run supply of housing by superimposing internal adjustment costs on the representative home-building firm. However, since the adjustment costs associated with an expansion in housing output need not coincide with those of a contraction, the speed with which output adjusts differs depending on whether or not housing output is above or below its long-run equilibrium level. The implications of the model are then tested using Irish data.

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[1]See Topel and Rosen (1988), footnote 2, p. 723.

The layout of the rest of the paper is as follows. To motivate the empirical analysis, section 2 examines the housing supply decision in a dynamic profit maximising setting which assumes asymmetric adjustment costs. Section 3 proposes the asymmetric or non-linear error correction model, originally applied by Granger and Lee (1989) and recently extended by Escribano and Pfann (1998), as an approximate closed form solution to the firm's profit maximisation problem under asymmetric adjustment costs. Section 4 tests for asymmetries in the dynamics of Irish housing supply using quarterly data from the period 1975-1998. Finally section 5 summarises and concludes.

2. Asymmetric Adjustment Costs and Housing Supply

This section undertakes a partial equilibrium analysis of the housing supply decision in a dynamic intertemporal setting. The model is intended as a description of the supply side of the market for new homes. The price of housing and the costs facing the average construction firm are taken as exogenous.[2] Gross housing investment, $I(t)$, refers to the output of the representative firm in the home-building sector and adjustment costs are imposed by allowing the firm's total

2 Topel and Rosen (1988) append a demand side and also consider how the market for new homes interacts with the market for the existing stock of dwellings.

costs depend on both its level and its rate of change, $I'(t)$. The representative firm is assumed to maximise discounted profits over an infinite horizon,

$$\underset{\{I(t)\}}{Max} \int_{t=0}^{\infty} \left[P(t)I(t) - TC(I(t), I'(t), y(t)) \right] e^{-rt} dt$$

(2.1)

where r is a positive constant representing the interest rate and *y(t)* denotes a set of cost shift variables. To clearly distinguish the firms costs of production from the adjustment costs associated with a change in output, total costs (*TC*) are decomposed into the firms costs of production (*C*), which depend only on the level of output and exogenous cost shifters, and adjustment costs (*AC*) which depend on the rate of change in output.[3]

$$TC(I(t), I'(t), y(t)) = C(I(t), y(t)) + AC(I'(t))$$

(2.2)

The marginal costs of production are assumed positive and increasing in output. Hence, using subscripts to denote partial derivatives, the

[3] This decomposition implies that marginal adjustment costs depend only on the rate of change in output and not on either the level of output or the cost shift variables.

cost function must have the properties $C_I > 0$ and $C_{II} > 0$. Adjustment costs are modelled using a flexible functional form, defined in Pfann and Verspagen (1989).

$$AC(I'(t)) = \frac{1}{2} gI'(t)^2 - dI'(t) + \exp(dI'(t)) - 1 \tag{2.3}$$

For $g > 0$, equation (2.3) can be shown to satisfy the following properties.

$$AC_{I'} > 0 \quad iff \quad I'(t) > 0\,, \qquad AC_{I'} < 0 \quad iff \quad I'(t) < 0\,, \qquad AC_{I'I'} > 0$$
$$and \quad AC(I'(t)) = 0 \quad iff \quad I'(t) = 0$$

Hence, adjustment costs are strictly convex and are minimised for $I'(t)$ equal to zero. For δ = 0, equation (2.3) reduces to the common quadratic form of adjustment costs effectively assumed in Topel and Rosen (1988). However, the adjustment costs represented by equation (2.3) are not symmetric in the case of δ ≠ 0. For δ > 0, the marginal adjustment costs (MAC) associated with an increase in *I(t)* exceed the costs of reducing *I(t)*. Conversely, for δ < 0, the marginal adjustment costs associated with a fall in *I(t)* exceed the costs of increasing *I(t)*. Again using subscripts to denote partial derivative, the necessary first order condition for the firm's optimisation problem is given by (2.4a) below.

$$P(t) - C_I = r\, AC_{I'} - AC_{I't}$$

(2.4a)

or

$$P(t) - C_I = r\left[gI'(t) + d\exp(dI'(t)) - d\right] - I''(t)\left[g + d^2 \exp(dI'(t))\right]$$

(2.4b)

The interpretation of equation (2.4a) and (2.4b) is similar to the first order condition under quadratic adjustment costs except that the absolute value of the term on the right hand side will now depend on the sign of $I'(t)$. In the absence of adjustment costs, all derivatives on the right hand side equal zero and the first order condition implies that the representative firm chooses the level of housing output such that price equates with marginal cost. Away from the static optimum, however, the costs associated with changing output drive a wedge between price and marginal cost. This wedge gives rise to a less elastic supply response in the short-run than in the long-run. Ordinarily, when the term on the right hand side of (2.4a) is positive the firm would have an incentive to expand the level of output because marginal revenue exceeds marginal cost. Similarly, a negative wedge suggests that the current level of output is too high and the firm should scale down its level of activity. However, in this dynamic intertemporal setting it may not be optimal in a present value sense to adjust output in a single time period. Instead, the representative firm may find it optimal to spread the adjustment process out over several time periods, and thus reduce current adjustment costs at the expense of having a less efficient level of production.[4] Moreover since adjustment costs are asymmetric, from the properties of the asymmetric adjustment cost function the absolute size of this wedge and, hence, the nature of the adjustment process, will not be invariant with respect to positive and negative changes in output.[5]

From expression (2.4b) it is clear that the above first order condition takes the form of a second order *non-linear* differential equation. It is in general not possible to solve this equation analytically for the path of investment which maximises discounted profits. As a result, Pfann (1996) proposes direct econometric estimation of the first order condition using the Generalised Method of Moments. Alternatively, the implications of (2.4a) and (2.4b) for the dynamics of housing supply can be worked out analytically using the piecewise quadratic

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[4] In an intertemporal setting adjustment costs will therefore be amortised, i.e. the optimising firm will compare the costs of adjusting today with the costs of adjusting tomorrow. In present value terms, abstracting from any direct effect on marginal costs, higher interest rates make current adjustment less attractive when compared with future adjustment. Hence, if the interest rate is very high relative to the growth rate in marginal adjustment costs, which is completely deterministic and assumed to be known in this model, the firm will have an incentive to prolong the adjustment process.

[5] More precisely, the optimising firm will not be indifferent with respect to the adjustment costs arising from positive and negative changes in output, $I(t)_1 > 0$ and $I(t)_2, < 0$ even when $| I(t)_1 | = | I(t)_2 |$.

approximation to (2.3) which has been suggested by Pfann (1996) and Escribano and Pfann (1998). This is given in equation (2.5) below.

$$AC(I'(t)) = \begin{cases} \frac{1}{2}\, g_1\, I'(t)^2 \quad iff \quad I'(t) < 0 \\ \\ \frac{1}{2}\, g_2\, I'(t)^2 \quad iff \quad I'(t) > 0 \\ \\ 0 \qquad otherwise \end{cases} \tag{2.5}$$

There is a clear correspondence (see Escribano and Pfann, 1998 , p. 205) between the adjustment cost parameters (γ_1, γ_2) in this piecewise approximation and the structural asymmetry parameter (δ).

$$\begin{aligned} d < 0 &\Leftrightarrow g_1 > g_2 \\ d > 0 &\Leftrightarrow g_1 < g_2 \\ d = 0 &\Leftrightarrow g_1 = g_2 \end{aligned}$$

The two necessary first order conditions associated with this piecewise approximation are given by

$$P(t) - C_I = r\, g_1 I'(t) - g_1 I''(t) \qquad for\ I'(t) < 0 \tag{2.6a}$$

and

$$P(t) - C_I = r\, g_2 I'(t) - g_2 I''(t) \qquad for\ I'(t) > 0$$

(2.6b)

both of which take the form of second order *linear* differential equations. From Topel and Rosen (1988), the closed form solution to each piecewise linear-quadratic approximation is therefore known. Using (2.6a) and (2.6b) it is possible to solve for the path of housing output depending on whether output is decreasing or increasing. The derivation is given in the Appendix. According to the solution, the current level of forcing variables *P(t)* and *y(t)* are not sufficient to determine the level of housing output at any point in time. Instead, housing output at time *t* is shown to be a function of past, present and future forcing variables with exponentially declining weights. Moreover, conceptual experimentation with the solution yields an asymmetric flexible accelerator model. Consider for example a fall in house prices from P_1 to P*. Holding the other cost shift variables fixed, this gives rise to a decline in the target level of output from I_1 to I*. The path over which $I(t)$ travels from I_1 to I^* is given by

$$I(t) = I^* - (I^* - I_1)\exp(a_1 t) \qquad for \quad I^* < I_1$$

(2.7a)

where $\alpha_1 < 0$. Similarly, in the case of a rise in house prices from P_2 to P^* and a consequential increase in the target level of output from I_2 to I^*, the path over which output travels is given by.

$$I(t) = I^* - (I^* - I_2)\exp(a_2 t) \qquad for \quad I^* > I_2$$

(2.7b)

where $\alpha_2 < 0$. From (2.7a) and (2.7b) it is clear that the model converges to a unique equilibrium in the sense that output will tend toward the same level I* determined by the forcing variable P* and independent of any initial starting level. This can be seen by taking the limit of (2.7a) and (2.7b) as $t \rightarrow \infty$.[6] Furthermore, differentiating (2.7a) and (2.7b) with respect to time gives an asymmetric flexible accelerator model where the speed of adjustment is asymmetric depending upon whether or not output is above or below its target level, i.e.

$$I'(t) = a_1\left[I(t) - I^*\right] \qquad for \quad I(t) > I^*$$

(2.8a)

and

$$I'(t) = a_2\left[I(t) - I^*\right] \qquad for \quad I(t) < I^*$$

[6] Since both α_1 and α_2 are negative these limits both tend to *I** as time extends to infinity.

(2.8b)

According to equations (2.8a) and (2.8b), the change in output bears a proportionate relationship with the deviation in output from its target level I*. Since $|\alpha_1| < \infty$ and $|\alpha_2| < \infty$, the presence of adjustment costs causes the firm to close the discrepancy between the target and actual level of output but only with a lag. Moreover, in the case where the adjustment costs associated with an expansion in output exceed those associated with a contraction, it is straightforward (see the Appendix) to show that $|\alpha_1| > |\alpha_2|$. Hence, the representative firm will adjust output more slowly when it is below its target level and expanding than when it is above and contracting.

The foregoing analysis provides microfoundations for sluggish adjustment on the supply side of the housing market and it also predicts asymmetric adjustment whereby expansions are associated with slower adjustment compared with contractions. In testing the empirical implications of this theory it is important to bear in mind a number of ways in which the representative agent's problem under convex adjustment costs may not carry over into observed industry behaviour. One serious qualification comes to mind. In particular, the continuous and smooth adjustment which derives from the convexity of the assumed asymmetric adjustment cost function cannot be taken as a description of actual firm behaviour. In practice, as outlined in Bertola and Caballero (1990), it is likely that adjustment costs at the firm level are non-convex, non-differentiable and also discontinuous. There may, for example, be fixed or lumpy costs of adjustment which arise regardless of the size of the actual adjustment being undertaken and these may not be symmetric with respect to expansions and contractions. Under such circumstances, it may no longer be optimal for the firm to continuously adjust every infinitesimal deviation of output from its target level. Instead, because the adjustment cost function is such that continuous small reactions are penalised, inaction can become an optimal policy.[7] Moreover, in the stochastic setting considered by Bertola and Caballero (1990), when individual firms face this form of adjustment costs optimal inaction can carry over into aggregate industry behaviour if aggregate shocks are large relative to firm specific shocks. In the next section, a class of econometric models which can test for asymmetric adjustment is reviewed. However, the models are sufficiently flexible to allow for the qualitative dynamic effects of adjustment cost non-convexities such as inertial supply behaviour and threshold points beyond which aggregate adjustment starts to speed up.

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[7] Bertola and Caballero (1990) consider a stochastic dynamic optimisation problem with discontinuous, non-differentiable and non-convex adjustment costs. They find that it is suboptimal to correct small deviations of the choice variable from its static optimum. Instead, the optimal policy involves allowing the choice variable wander some finite distance from the target before adjusting. In a stochastic setting, there may also be an option value to waiting or "optimal inertia" and, as described in Dixit (1992), this may be significant at an aggregate industry level also. Grenadier (1996) provides a real options approach to analysing industry dynamics in the US property market.

3. Asymmetric Equilibrium Correction[8]

The asymmetric flexible accelerator model derived above predicts that the speed with which output changes following a change in the target level of housing depends on whether or not output is above or below its equilibrium or target level. Escribano and Pfann (1998) have suggested the non-linear or asymmetric error correction model as a reasonable closed form approximation to the firm's dynamic optimisation problem under asymmetric adjustment costs.[9] Below, this model is described in brief and various types of asymmetric adjustment mechanism are illustrated. More extensive treatments are provided in Escribano (1997) and Escribano and Pfann (1998). Escribano and Mira (1997) provide a partial generalisation of the Granger Representation Theorem to the case of a non-linear error correction model with linear cointegrated variables. They also provide the sufficient conditions for the parameters of such models to be estimated consistently. Granger and Lee (1989), Hendry and Ericson (1991) and Escribano and Granger (1998) provide empirical application of both non-symmetric and non-linear error correction.

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[8] In what follows, the terms error correction model and equilibrium correction model will be used interchangeably.

[9] In earlier work, Nickel (1985) relates the quadratic or symmetric adjustment cost literature to the standard linear error correction model.

According to the predictions of the model described in section 2, the target or equilibrium level of output is posited to be a linear function of real house prices and exogenous cost shift variables. The problem faced by the firm was greatly simplified, however, insofar as a completely deterministic setting was assumed. In an empirical application, it is necessary to take account of the random variation in the optimal level of output (I*) that will take place in response to shocks to real house prices and changes in cost shift variables. Hence, it would seem reasonable to assume that the target level of gross housing investment (I_t*) is linearly related to the firms forcing variables (θ_t), and a stationary stochastic shock $z_{t.}$

$$I_t^* = \beta\ \theta_t + z_t \tag{3.1}$$

where β' is a vector of constant parameters. If $I_t^* = I_t$, and both I_t and θ_t are nonstationary variables, the above equation can be interpreted as a cointegrating relationship with (1, -β) being the cointegrating vector. If such a cointegrating vector can be shown to exist, it follows that gross investment will systematically react in order to correct past deviations from the target level implied by (3.1). [10] According to the

[10] If the forcing variables are endogenous, equation (3.1) would imply other error correction equations where the elements of θ_t react in order to restore the cointegrating relationship. In the next section the assumption that the elements of

asymmetric error correction model, however, the extent of this correction differs depending on the sign of z_{t-1}. This can be written as equation (3.2) below,

$$\Delta I_t = m + lagged\,(\Delta I_t\,, \Delta q_t) + a_1 (z_{t-1})^+ + a_2 (z_{t-1})^- + e_t \qquad (3.2)$$

where μ is a constant term and ε_t is a white noise error term. $(z_{t-1})^+$ and $(z_{t-1})^-$ represent positive and negative deviations from the target, i.e.

$$z^+_{t-1} = \begin{cases} (I_{t-1} - bq_{t-1}) & iff \quad I_{t-1} - bq_{t-1} > 0 \\ 0 \; otherwise \end{cases}$$

and

$$z^-_{t-1} = \begin{cases} (I_{t-1} - bq_{t-1}) & iff \quad I_{t-1} - bq_{t-1} < 0 \\ 0 \; otherwise \end{cases}$$

The two adjustment parameters, α_1 and α_2, capture the size of the response of output when it is, respectively, above or below its target level. If the adjustment costs of expanding output are greater than the costs associated with a contraction then, according to the asymmetric

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θ_t can be considered as exogenous "forcing" variables is subjected to empirical flexible accelerator discussed in section 2, one would expect $|\alpha_1| > |\alpha_2|$. The specification in (3.2) was originally suggested in Granger and Lee (1989). In order to generalise it further, Escribano and Pfann (1998) have proposed an alternative given in (3.3) below.

$$\Delta I_t = m + lagged\,(\Delta I_t, \Delta q_t) + a_1 D_1 z_{t-1} + a_2 D_2 z_{t-1} + a_3 D_3 z_{t-1} + e_t \tag{3.3}$$

where the D_i are zero/one dummies defined by

$$D_1 = \begin{cases} 1 & iff \quad (I_{t-1} - bq_{t-1}) > C^+ \\ 0 & otherwise \end{cases}$$

$$D_2 = \begin{cases} 1 & iff \quad I_{t-1} - bq_{t-1} < C^- \\ 0 & otherwise \end{cases}$$

$$D_3 = \begin{cases} 1 & iff \quad C^- \leq (I_{t-1} - bq_{t-1}) \leq C^+ \\ 0 & otherwise \end{cases}$$

This specification allows for both asymmetries and threshold points beyond which output becomes more sensitive to deviations from its target level. For example, the model facilitates empirical testing of the hypothesis that between the two thresholds (C^+ and C^-) output adjusts

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scrutiny.

relatively slowly (i.e. $|\alpha_3| < |\alpha_1|$ and $|\alpha_3| < |\alpha_2|$). However, for deviations below C^- or above C^+ output may adjusts more quickly and/or asymmetrically (i.e. $|\alpha_1| \neq |\alpha_2|$) By setting $C^+ = C^- = 0$, the restricted Granger and Lee (1989) specification is obtained. Finally, while the model in section 2 predicted continuous but partial adjustment following a change in exogenous forcing variables, in an empirical application it is important to consider the possibility as in Bertola and Caballero (1990) that no adjustment takes place when there are only small deviations from equilibrium. This situation of optimal inaction for small deviations from equilibrium occurs when $|\alpha_3| = 0$. The resulting equilibrium is neither unique nor centred at zero: there is a range of implied equilibria over the interval $[C^-,C^+]$.

One problem associated with the piecewise linear asymmetric error correction in equations (3.3) and (3.2), however, is the requirement that the unknown threshold points C^- and C^+ must be specified prior to estimation. It would be preferable to let the data determine these thresholds endogenously.[11] In addition the "kinked" nature of the change in the speed of adjustment is not entirely appealing from an economic point of view. For example, if the housing market is populated by a number of heterogeneous firms with distinct adjustment cost specifications and, hence, different threshold points one might expect a smooth rather than a kinked change in the speed of

[11] One could possibly consider the use of a grid search procedure which selected the thresholds based on an in-sample goodness of fit criteria.

adjustment.[12] In order to incorporate smoother dynamics, Escribano and Pfann (1998) suggest a more general cubic polynomial adjustment function which captures non-linearities and/or asymmetries in the adjustment of the endogenous variable back to equilibrium. This is given in equation (3.4) below,

$$\Delta I_t = \mu + lagged\,(\Delta I_t\,, \Delta q_t) + f\,(z_{t-1}) + \varepsilon_t \tag{3.4}$$

where

$$f\,(z_{t-1}) = \alpha_1 z_{t-1} + \alpha_2 (z_{t-1})^2 + \alpha_3 (z_{t-1})^3 \tag{3.5}$$

The non-linear error correction in (3.4) allows the data determine endogenously the threshold points beyond which the error starts to speed up based on the coefficient estimates α_1, α_2 and α_3. However, since the concept of cointegration is based on a linear time series framework the introduction of non-linearity is not trivial. Theorem 2.1 in Escribano (1997) describes the regularity and stability conditions under which the variables in z_t of equation (3.5) are cointegrated. In addition, for the regression in (3.4) to be balanced, it is necessary that

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[12] The intuition here is that the further output deviates from its target level the greater is the proportion of firms that are pushed over their respective threshold points and, hence, the quicker is the speed of adjustment. Anderson (1997) has taken this idea and applied it using an error correction model of the US T-Bill market.

if z_t is stationary then so too is $f(z_t)$. The implied dynamics of I_t will of course depend on the estimated coefficients and adjustment may be both non-linear and/or asymmetric.

Unfortunately, the above cubic specification does not guarantee either stability or uniqueness of the implied equilibrium. However, using the concept of mixing errors, Escribano and Mira (1997) and Escribano (1997) have outlined the conditions required for stability in non linear error correction models. In general, with linear cointegrated variables, stability requires $-2 < df(z_{t-1})/dz_{t-1} < 0$.[13] While an empirical model may satisfy this condition in-sample, the general cubic polynomial does not satisfy it when $z_{t-1} \rightarrow \infty$. This can however be overcome by making α_3 time dependent for very large values of z_{t-1}. Alternatively, Escribano and Pfann (1998) have advocated the class of rational polynomial functions. These non-linear models, which satisfy the above stability condition, replace the cubic polynomial given in (3.5) with the rational polynomial functions given in (3.6) and (3.7) below.

$$f(z_{t-1}) = \left\{(z_{t-1} + g_1)^3 + g_2\right\} / \left\{(z_{t-1} + g_3)^2 + g_4\right\} \tag{3.6}$$

$$f(z_{t-1}) = \left\{(z_{t-1} + g_1)^3 + g_2\right\} / \left\{(1/(z_{t-1} + g_3)^2) + g_4\right\} \tag{3.7}$$

From an examination of (3.6) it is clear that if $\gamma_2 = -(\gamma_1)^3$ and $(\gamma_3)^2 + \gamma_4 \neq 0$ then the equilibrium is unique and given by $f(z_{t-1} = 0) = 0$. In the case of the rational polynomial adjustment given by equation (3.7), if $\gamma_2 = -(\gamma_1)^3$ then $f(z_{t-1} = \gamma_3) = 0$, $f(z_{t-1}=0) = 0$ and $f(z_{t-1}) \approx 0$ for all $z_{t-1} \in [0, \gamma_3]$. In other words, the model implies a continuum of equilibria where no adjustment takes place over the interval $z_{t-1} \in [0, \gamma_3]$. In the next section, all of the above models are fitted to Irish data and the *a priori* hypothesis of asymmetric adjustment in the housing market is subjected to empirical scrutiny.

4. An Empirical Application to Irish Housing Supply

In this section the hypothesis of asymmetric adjustment is tested on Irish data over the period 1975Q4-1998Q3. To begin with, the Engle-Granger two step estimation procedure is employed in order to identify a linear error correction model for gross housing investment. This linear model is then subsequently tested against a number of non-linear and/or asymmetric specification including (i) the piecewise asymmetric specification of Granger and Lee (1989) given in equation (3.2), (ii) the piecewise asymmetric error correction model with threshold points given in equation (3.3), (iii) the cubic polynomial adjustment with endogenous threshold points in (3.4) and (iv) the rational polynomials

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[13] See the proof of Theorem 2.1, Appendix A in Escribano (1997).

recently advocated in Escribano and Pfann (1998). The quarterly number of private new houses completed is taken as the empirical measure of gross housing investment (I_t). The vector of exogenous forcing variables (θ_t) is comprised of the "real" price of new housing (p_t), an index of real costs in the average construction firm (y_t) and the real interest rate (r_t). As in Topel and Rosen (1988) the inclusion of the latter is intended to capture the cost of working capital which is normally considered to be a significant factor in the construction sector. All variables, except the real interest rate, are logged and seasonally adjusted. Data sources are described in detail at the end of the paper. Augmented Dickey Fuller tests also reported at the end of the paper suggest that I_t, p_t, y_t, r_t all contain a unit root but are stationary in first difference. Hence, it is meaningful to proceed and test for cointegration as in equation (3.1) using the two step Engle-Granger methodology.[14]

Table 1 below presents the first step static OLS estimates of the long-run housing supply curve. The levels regression also includes a deterministic time trend (T) to proxy for the impact of any unobserved exogenous growth factors (e.g. technological change, increases in the

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[14] The use of the Engle-Granger methodology amounts to the assumption that there exists only a single cointegrating relationship among I_t, p_t, y_t, r_t. In the absence of any economic rationale for additional cointegrating relationships, the adoption of modelling techniques such as the Johansen methodology which explicitly allows for multiple cointegrating vectors would seem inappropriate.

land input etc.).[15] As originally pointed out by Engle and Granger (1987) if the residuals of this static regression are I(0), the long-run parameters are super-consistent in the sense that they converge rapidly to their true values. The augmented Dickey-Fuller statistic testing the hypothesis of a unit root in the residuals was -5.17. The 95% critical value from the Dickey-Fuller distribution is -4.60 and hence the unit root hypothesis is rejected. Based on these results the OLS regression has the interpretation of a long-run housing supply curve. According to the estimated parameters, the long-run supply of housing is unit price elastic, decreasing in cost shift variables and the real interest rate. The finding of a unit elastic long-run supply schedule is contrary to other empirical results which suggest a more elastic long-run supply response.[16] In addition, while correctly signed, the sensitivity of housing supply to the costs of production is somewhat lower than expected. The real interest rate is shown to have a significant and negative effect on new housing supply. However, as is common in static regressions of this form, due to the omission of any short-term

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[15] While a deterministic trend is not an ideal proxy for these effects, it was considered to be the only option. Any model of the supply side which did not attempt to take account of such exogenous growth factors would clearly be misspecified. When the model was estimated without a deterministic trend, an ADF test on the residuals again indicated that the variables were cointegrated. However, the real cost variable was incorrectly signed (positive) but statistically insignificant.

[16] See Table 2E in Appendix E in Bacon *et al* (1998). An earlier study by Keneally and McCarthy (1982) suggested a long-run supply elasticity of about 1.6 which is more consistent with the results reported here. The evidence in Kenny (1999) is also consistent with significant long-run constraints on the supply side of the Irish housing market.

dynamics the residuals fail standard tests for serial correlation and normality.[17] As a result statistical inference using the absolute t-statistics may not be valid. To check the validity of the t-statistics, Table 1 also reports the long-run solution from an autoregressive distributed lag (ARDL) model. The selected model contains 3 lags of each of the variable plus a time trend and passes various tests for serial correlation, heteroskedasticity and functional form.[18] Moreover, the long-run coefficients are very close to those in the OLS regression with all variables except the intercept being significant and correctly signed. Finally, the third column in Table 1 contains generalised instrumental variables (IV) estimates of the static long-run supply equation to shed light on the possible existence of simultaneous equation bias due to, for example, the endogeneity of p_t in the full market equilibrium.[19] The IV estimates closely resemble the OLS estimates discussed above suggesting that simultaneous equation bias is not a significant problem in the static regression.

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[17] These tests are described in detail in Pesaran and Pesaran (1997).

[18] Normality of the residuals remains a problem in the ARDL model. However, if the true data generating process contains systematic asymmetries then this lack of normality is only to be expected.

[19] Costs, y_t, may also be endogenously determined by shifts in the derived demand for labour, materials etc.

Table 1: Long-run Analysis, Dependent Variable: I_t

Regressor	OLS	ARDL	IV
Intercept	-2.06	0.25	2.16
	(1.24)	(0.08)	(0.91)
T	0.006	0.015	0.014
	(2.87)	(3.11)	(3.95)
$\mathbf{p_t}$	1.02	0.83	0.72
	(7.62)	(3.37)	(4.00)
$\mathbf{y_t}$	-0.16	-0.36	-0.48
	(1.42)	(1.64)	(2.64)
$\mathbf{r_t}$	-1.16	-2.19	-1.83
	(2.41)	(2.67)	(2.57)
Diagnostics			
R^2	0.72	0.84	0.78*
SE Regression	0.13	0.11	0.14
DW	1.37	2.06	1.20
Heteroskedasticity	1.07 [.301]	1.01 [.314]	5.14 [.203]
Serial Correlation	12.4 [.015]	6.73 [.150]	15.8 [.003]
Functional Form	2.98 [.084]	0.80 [.368]	0.08 [.777]
Normality	7.90 [.019]	15.8 [.000]	0.97 [.616]

Absolute T-values in () and P-values given in [ ]. * For the IV regression the Generalised R-squared suggested in Carthy and Smith (1994) is reported.

The results in Table 1 are supportive of the proposition that there exists a long-run housing supply curve which defines the equilibrium level of housing completions consistent with the prevailing level of real house prices, costs and real interest rates. It is now possible to proceed with the second stage and estimate a dynamic linear error correction model in order to examine the extent to which any deviations from the above implied target level of housing supply are corrected. Table 2 , column (1), reports this linear specification where the dependant variable is the first difference of the log of new private completions and two lags of the first difference of each variable together with previous periods disequilibrium are included as regressors. The estimated adjustment coefficient, which is correctly signed (negative) and statistically significant, illustrates that the home building sector adjusts in order to close any deviation from its target level of output. Furthermore, Table 2 also reports Lagrange multiplier tests which supports the hypothesis that real house prices, costs and the real interest rate are all weakly exogenous with respect to the adjustment parameter and the parameters of the long-run cointegrating relationship.[20] Hence statistical inference concerning the estimated adjustment coefficient using *only* the equation for housing completions would appear to be valid. In addition, it should be noted that the equation for housing completions passes a number of misspecification tests apart from a test for normality of the residuals. However, as noted previously, if the true adjustment mechanism on the supply side of the housing market is characterised by systematic asymmetries then such a lack of normality is only to be expected in a model which imposes symmetric adjustment.

[20] This involves testing the hypothesis that the error correction term does not enter as a significant regressor in identical regressions (not reported) where the first difference of real house prices, costs and the real interest rate were used as the dependant variables. In the case of p_t, y_t and r_t the null hypothesis of exclusion of the long-run relationship cannot be rejected. In other words, the distributions for p_t, y_t and r_t conditional on the lagged short term dynamics would appear to contain no information about the adjustment parameter and the long-run parameters of the cointegrating relationships.

Table 2: Piecewise Linear Asymmetric ECMs, Dependent Variable DI_t [$Z_t = I_t + 2.06 - 0.006T - 1.02p_t + 0.17*y_t + 1.16\ r_t$]

Regressors	(1)	(2)	(3)	(4)
Intercept	0.02	-	-	-
	(0.76)			
DI_{t-1}	-0.24	-	-	-
	(2.15)			
DI_{t-2}	-0.14	-	-	-
	(1.43)			
Dp_{t-1}	-0.28	-	-	-
	(0.71)			
Dp_{t-2}	-0.20	-	-	-
	(0.49)			
Dy_{t-1}	-0.33	-	-	-
	(0.37)			
Dy_{t-2}	-0.11	-	-	-
	(0.13)			
Dr_{t-1}	1.05	-	-	-
	(1.70)			
Dr_{t-2}	-0.70	-	-	-
	(1.15)			
Z_{t-1}	**-0.44**	**-**	**-**	**-**
	(3.38)			
$(Z_{t-1})^+$	**-**	**-0.52**	**-**	**-**
		(2.54)		
$(Z_{t-1})^-$	**-**	**-0.36**	**-**	**-**
		(1.95)		
$D_1 Z_{t-1}$	**-**	**-**	**-0.59**	**-0.57**
			(2.82)	**(2.74)**
$D_2 Z_{t-1}$	**-**	**-**	**-0.38**	**-0.39**
			(2.17)	**(2.19)**
$D_3 Z_{t-1}$	**-**	**-**	**-0.32**	
			(1.09)	
R^2	0.554	0.556	0.560	0.553
S.E. Regression	0.111	0.111	0.112	0.112
Serial Correlation	2.22 [0.694]	1.44 [.837]	1.12 [.890]	2.34 [.673]
Functional Form	0.06 [0.796]	.007 [.929]	.008 [.966]	.019 [.889]
Normality	8.12 [0.017]	9.17 [.010].	8.38 [.015]	4.22 [.121]
Heteroskedasticity	0.47 [0.494]	0.45 [.501]	.468 [.493]	.535 [.464]
Symmetry	**-**	**0.281 [.596]**	**0.881[.644]**	**.526 [.468]**
Weak Exogeneity c^2(1)	**I**	**p**	**y**	**r**
	12.18 [.000]	0.029 [.864]	0.759 [.383]	0.989 [.320]

Absolute T-values in () and P-values given in [ ]. To take account of an outlier, the regression for ΔI_t includes a dummy variable which takes a value of unity in 1977Q3 and zero otherwise. The coefficients on the lagged short term dynamics in columns (2), (3) and (4) have been omitted but are virtually identically to those reported in column (1).

Column (2), Table 2, reports the adjustment coefficients on the piecewise asymmetric specification due to Granger and Lee (1989). The estimated adjustment coefficients given by $\alpha_1 = -0.52$ and $\alpha_2 = -0.36$ are consistent with the *a priori* hypothesis that the adjustment costs associated with an expansion in housing output exceed the costs of contracting. As described by the asymmetric flexible accelerator model in section 2, firms in the home building sector appear to expand output more slowly when it is below its target level and, conversely, contract output more quickly when it is above its target level. The implied asymmetric adjustment is depicted graphically in Figure 1. Column (3), Table 2, reports the estimated adjustment coefficients using the piecewise asymmetric specification due to Escribano and Pfann (1998). This specification allows for threshold points beyond which output becomes more sensitive to deviations from its target level. The threshold points are set exogenously at ±10% above or below the target level of output, i.e. $C^+ = 0.10$ and $C^- = -0.10$. The estimated equation again supports the conclusion that positive (negative) deviations from the target level give rise to faster (slower) adjustment of gross housing investment. However, between the two threshold points close to the target, the adjustment is slower and not significantly different from zero. Finally column (4) reports the estimated adjustment coefficients using the piecewise asymmetric specification but imposing the restriction that housing supply does not adjust between the threshold points. Consistent with the results in column (3), this restriction - which implies non-uniqueness of the equilibrium level of output - is easily accepted by the data.

The results reported in Table 2 are of interest insofar as they support the prior theoretical predication that there are asymmetric adjustment costs in the housing market. However, it is difficult to argue on statistical grounds that the asymmetric models are preferable. This problem derives from a general shortcoming associated with the application of non-linear methods in empirical economics. Namely, the class of non-linear and/or asymmetric models is much larger than the class of linear models and it is difficult to distinguish competing specifications from each other and from competing linear models. The piecewise asymmetric models in Table 2 are generally associated with only a very modest improvement of in-sample fit and, furthermore, in the case of all three models examined the nested hypothesis of symmetric adjustment (also reported in Table 2) cannot be rejected at standard levels of significance.

One potentially significant deficiency of the models of Table 2 is the implied discrete kink in the adjustment process which is unlikely to be compatible with aggregate industry behaviour. The cubic and rational polynomial adjustment mechanisms discussed in Section 3 do not impose such discrete kinks and they may therefore be preferable. Table 3 reports the estimated adjustment coefficients from these non-

Table 3: Non-linear Asymmetric ECMs, Dependent Variable: DI_t

Regressor	(1)	(2)	(3)
Z_{t-1}	**-0.298** **(2.11)**	-	-
Z^2_{t-1}	**-0.620** **(1.55)**	-	-
Z^3_{t-1}	**-3.10** **(2.44)**	-	-
$\{Z_{t-1}\}^3 / \{(Z_{t-1})^2+1\}$	-	**-4.28** **(3.29)**	-
$\{(Z_{t-1}+1)^3-1\} / \{1/(Z_{t-1}+0.05)^2 + 1\}$	-		**-1.71** **(3.59)**
R^2	0.589	0.552	0.562
S.E. Regression	0.108	0.111	0.110
Serial Correlation	2.78 [.595]	8.67 [.070]	7.74 [.101]
Functional Form	1.20 [.272]	1.65 [.198]	1.23 [.266]
Normality	9.23 [.010]	5.08 [.079]	6.14 [.050]
Heteroskedasticity	0.85 [.365]	.400 [.527]	0.29 [.289]
Symmetry	**6.72 [.035]**	-	-

Absolute T-values in () and P-values given in [ ]. To take account of an outlier, the regression for ΔI_t includes a dummy variable which takes a value of unity in 1977Q3 and zero otherwise. The coefficients on the lagged short term dynamics have been omitted but are virtually identical to those reported in column (1) of Table 2.

linear asymmetric equilibrium correction models. Column (1), Table 3, reports the estimated coefficients (α_1, α_2, α_3) from the cubic polynomial adjustment mechanism. In contrast to the models described in Table 2, on this occasion the nested hypothesis of symmetric equilibrium correction is rejected in favour of the alternative hypothesis of both non-linear and asymmetric adjustment. An insight into the implied adjustment can be obtained from the plot, given in Figure 2, of $f(z_{t-1})$ as a function of the estimated disequilibrium. From the graph it

Figure 1: Piecewise Asymmetric Adjustment

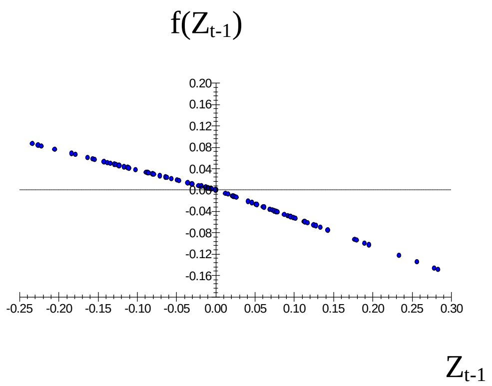


Figure 2: Cubic Polynomial Adjustment

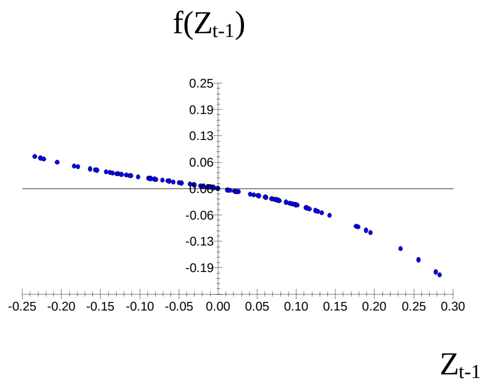


Figure 3: Rational Polynomial Adjustment

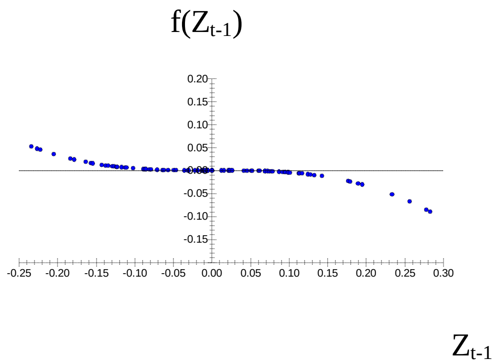
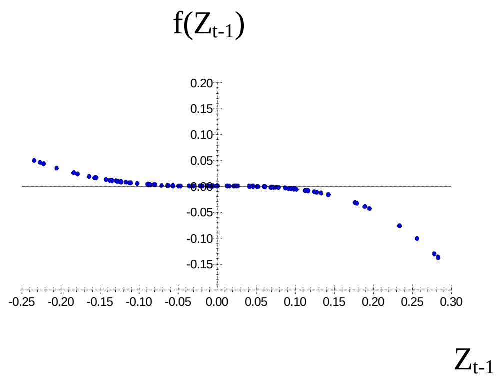


Figure 4: Restricted Rational Polynomial Adjustment



is clear that the polynomial adjustment is both non-linear and asymmetric. Consistent with the previous results the model implies that positive deviations from equilibrium give rise to faster adjustment when compared with corresponding negative deviations. It is also clear from Figure 2 that the implied equilibrium is unique although this is not imposed a priori and - at least in sample - it can be shown that the polynomial function satisfies the stability condition $-2 < df(z_{t-1}/dz_{t-1}) < 0$. Moreover, on this occasion, the hypothesis of symmetric error correction is rejected by the data. Columns (2) and (3) of Table 3 report the estimated adjustment coefficients using the rational polynomials advocated by Escribano and Pfann (1998). The first of these in column (2) imposes uniqueness of the equilibrium at $z_{t-1} = 0$, the second in column (3) imposes a continuum of equilibria where no adjustment takes place over the interval $z_{t-1} \in [0, 0.05]$. The implied adjustment which is depicted graphically in Figures 3 and 4 is again supportive of the *a priori* prediction of asymmetric adjustment costs. However, while both models appear to overcome the problem of non-normal residuals associated with the restricted linear specification, a variety of non-nested tests were unable to discriminate between the linear and non-linear specifications. Against this, a number of goodness of fit criteria such as the AIC and the SBC clearly favour the rational polynomial model in column (3) over the restricted linear model.

5. Concluding Remarks

This paper has considered the potential effects of asymmetric adjustment costs on the dynamics of housing supply. The theoretical model of section 2 extended the dynamic framework employed in Topel and Rosen (1988) to allow for asymmetric adjustment cost using the flexible adjustment costs function advocated in Pfann (1996). The model provides explicit microfoundations for the divergence between long and short run supply elasticities and also predicts asymmetric adjustment whereby positive deviations from equilibrium are associated with faster adjustment as compared with corresponding negative deviations. The paper also tests for asymmetric adjustment costs by estimating a number of asymmetric and/or non-linear equilibrium correction models using data on the Irish housing market. A number of interesting insights into the dynamics of housing supply have been uncovered.

Firstly, and most importantly, the empirical section estimated a unit elastic equilibrium housing supply curve which suggest Irish housing supply is significantly less elastic than housing supply in other economies such as the US. The finding of only a unit elastic long-run housing supply curve means that there would appear to be significant constraints on the supply side of the market *even in the long-run*. Secondly, of the six models considered, all are supportive of the proposition that the adjustment costs associated with an expansion in housing output are greater than the adjustment costs associated with a contraction. This gives rise to relatively slow upward adjustment of housing output in response to a surge in demand. Conversely, in a downturn, adjustment is faster and this mitigates the likelihood that a building boom would continue in the context of declining demand. Thirdly, a number of the estimated models support the belief that there are threshold points on the supply side of the housing market: large deviations from equilibrium appear to be associated with faster adjustment when compared with small deviations from equilibrium. Indeed, over a small interval about the estimated equilibrium, the adjustment of housing supply is not significantly different from zero. As in the model of Bertola and Caballero (1990), such inertial supply behaviour is consistent with optimising behaviour under adjustment costs non-convexities.

In conclusion, it appears that the above models with both asymmetries and non-linearities can capture important empirical features of the supply side of the housing market. One not insignificant shortcoming associated with these models, however, is that it is very difficult to distinguish them in-sample from corresponding linear symmetric specifications. Only in the case of the cubic polynomial adjustment mechanism was it possible to statistically distinguish the asymmetric non-linear adjustment from a nested model with symmetric linear adjustment. Future research should therefore examine the extent to which it is possible to distinguish between competing models in terms of out-of-sample forecasting.

Description of Data

Gross housing investment (I_t)

Quarterly new private housing completions taken from the Housing Statistics Bulletin, Department of the Environment. Seasonally adjusted using the Tramo/Seats macro

Real house prices (p_t)

Quarterly countrywide new house prices taken from the Housing Statistics Bulletin, Department of the Environment.

Real house building costs (y_t)

The building cost index published in the Housing Statistics Bulletin and compiled by the Department of the Environment. The building cost index includes only material and labour costs and is estimated to account for no more than 65 per cent. of the cost base of the average construction firm. It does not include the interest expenses associated with development financing or the cost of development land. Hence the inclusion of the real interest rate and the deterministic trend as separate regressor in the static long-run regression.

Real interest rates(r_t)

An estimate of the real interest rate for period *t* was computed as the difference between the prime lending rate on AA loans in period *t* less the year-on-year rate of consumer price inflation up to period *t*. The prime lending rate on AA loans was obtained from the International Financial Statistics database.

The Time Series Properties of the Data

The Table below reports Augmented Dickey-Fuller tests for unit roots in the levels and first difference of each of the above variables. ADF(n) indicates that there were *n* lags of the first difference of the dependent variable included in the ADF regression. All variables except the interest rate were logged. A deterministic trend is included in each of the levels regressions (except for the real interest rate) but not in the tests on the first difference. Based on the evidence in the Table each of the variables can be considered nonstationary I(1) in levels but stationary I(0) when differenced once.

Augmented Dickey-Fuller Test statistics

	I_t	p_t	y_t	r_t
Level	ADF(2) = -0.81	ADF(3) = -0.92	ADF(2) = -2.11	ADF(2) = -2.87
First Difference	ADF(2)=-7.74*	ADF(1) = -6.16*	ADF(1) = -4.91*	ADF(2)=-5.09*

*** Indicates that the null hypothesis of a unit root is rejected at the 95% level**

Appendix

Below, the solution to the piecewise approximation to the firm's maximisation problem under asymmetric adjustment costs is presented. Linearising the marginal cost term on the left hand side using a first order Taylor expansion and rearranging gives

$$\left[D^2 - r\,D - \frac{1}{b_1}\right] I(t) = -\frac{q_1(t)}{b_1} \qquad for \quad I'(t) < 0$$

(A.1)

and

$$\left[D^2 - r\,D - \frac{1}{b_2}\right] I(t) = -\frac{q_2(t)}{b_2} \qquad for \quad I'(t) > 0$$

(A.2)

where *DI(t)* denotes the time derivative of *I*(*t*), $\beta_1 = \gamma_1 / C_{I\,I}$, $\beta_2 = \gamma_2 / C_{I\,I}$ and both $q_1(t)$ and $q_2(t)$ are linear functions of real house prices and cost shift variables, i.e.

$$q_1(t) = \frac{b_1}{g_1} P(t) + \frac{b_1}{g_1}\left[C_I + C_{I\,y}\, y(t)\right]$$

$$q_2(t) = \frac{b_2}{g_2} P(t) + \frac{b_2}{g_2}\left[C_I + C_{I\,y}\, y(t)\right]$$

Using (A.1) and (A.2) it is possible to solve for the path of housing output depending on whether output is decreasing or increasing. Defining (α_1,λ_1) as the negative and positive roots of the characteristic equation X^2 - r X - (1/β_1)= 0 and (α_2,λ_2) as the corresponding roots of X^2 - r X - (1/β_2) = 0, each piecewise solution takes the negative stable root backward and the positive root forward. For I'(t) < 0, the implied path for investment is therefore

$$I(t) = e^{\alpha_1 t}\left\{ I_1 - \frac{1}{\lambda_1 - \alpha_1}\int_0^{\infty}\frac{\theta(\tau)}{\beta_1}\exp(-\lambda_1 \tau)d\tau \right\}$$

$$+\frac{1}{\lambda_1 - \alpha_1}\left\{ \int_0^{t}\frac{\theta(\tau)}{\beta_1}\exp(\alpha_1(t-\tau))d\tau + \int_t^{\infty}\frac{\theta(\tau)}{\beta_1}\exp(\lambda_1(t-\tau))d\tau \right\}$$

(A.3)

and for I'(t) > 0

$$I(t) = e^{\alpha_2 t}\left\{ I_2 - \frac{1}{\lambda_2 - \alpha_2}\int_0^{\infty}\frac{\theta(\tau)}{\beta_2}\exp(-\lambda_2 \tau)d\tau \right\}$$

$$+\frac{1}{\lambda_2 - \alpha_2}\left\{ \int_0^{t}\frac{\theta(\tau)}{\beta_2}\exp(\alpha_2(t-\tau))d\tau + \int_t^{\infty}\frac{\theta(\tau)}{\beta_2}\exp(\lambda_2(t-\tau))d\tau \right\}$$

(A.4)

where I_1 and I_2 represent initial output levels. From (A.3) and (A.4), it is clear that the current level of housing output depends on past, present and future values of the forcing variables in $\theta(t)$. In addition, the weights on past and future values of $\theta(t)$ are exponentially declining. Conceptual experimentation with (A.3) and (A.4) yields a flexible accelerator model. Consider for example a fall in house prices from P_1 to P*. Holding the other cost shift variables fixed, this gives rise to a decline in θ from θ_1 to θ^*. Substituting $q(t) = q^* = I^*$ in (A.3) and performing the integration gives the path over which $I(t)$ travels from I_1 to I^*.

$$I(t) = I^* - (I^* - I_1)\exp(a_1 t) \qquad for \quad I^* < I_1 \tag{A.5}$$

where I_1 is the initial level of housing investment associated with price level P_1. Similarly, in the case of (A.4) consider a rise in house prices from P_2 to P* and a consequential increase in θ_2 to θ^* . Substituting $q(t) = q^* = I^*$ and evaluating the integrals gives

$$I(t) = I^* - (I^* - I_2)\exp(a_2 t) \qquad for \quad I^* > I_2 \tag{A.6}$$

From (A.5) and (A.6) it is clear that the model converges to a unique equilibrium in the sense that output will tend toward the same level I* determined by the exogenous forcing variables θ^* and independent of any initial starting level. This can be seen by taking the limit of (A.5) and (A.6) as $t \rightarrow \infty$. Since both α_1 and α_2 are negative these limits both tend to I^* as time extends to infinity. Furthermore, differentiating (A.5) and (A.6) with respect to time gives a flexible accelerator model where the speed of adjustment is asymmetric depending upon whether or not output is above or below its target level, i.e.

$$I'(t) = a_1 \left[I(t) - I^* \right] \qquad for \quad I(t) > I^* \tag{A.7}$$

and

$$I'(t) = a_2 \left[I(t) - I^* \right] \qquad for \quad I(t) < I^* \tag{A.8}$$

According to equations (2.8a) and (2.8b), the change in output bears a proportionate relationship with the deviation in output from its target level I*. Since $|\alpha_1| < \infty$ and $|\alpha_2| < \infty$, the presence of adjustment costs causes the firm to close the discrepancy between the target and actual level of output but only with a lag. Given the dependence of β on C_{II}, the model in no way implies that α_1 and α_2 will be time-invariant. In addition, as noted by Maccini (1987), if interest rates change over time the adjustment coefficients will also change. Moreover, in the case where the adjustment costs associated with an expansion in output exceed those associated with a contraction, it is straightforward to show that $|\alpha_1| > |\alpha_2|$. From the definition of β_1 and β_2, $\gamma_1 < \gamma_2$ if and only if $\beta_1 < \beta_2$. Noting that α_1 and α_2 are the negative roots of $X^2 - r X - (1/\beta_1)$ and $X^2 - r X - (1/\beta_2)$, it follows automatically that, for a given interest rate, $\alpha_2 > \alpha_1$ or $|\alpha_1| > |\alpha_2|$.

References

Anderson, H. (1997) "Transaction Costs and Non-Linear Adjustment Toward Equilibrium in the US Treasury Bill Market", Oxford Bulletin of Economics and Statistics, 59, 4, pp. 465- 484.

Bacon, P., F. McCabe and A. Murphy (1998) "An Economic Assessment of Recent House Price Developments" *Report submitted to the Minister for Housing and Urban Renewal,* April.

Bertola, G. and R. J. Caballero (1990) "Kinked Adjustment Costs and Aggregate Dynamics", NBER Macroeconomics Annual 1990, pp. 237-288.

Dixit, A. (1992) "Investment and Hysterisis", Journal of Economic Perspectives, 6, pp. 107-132.

Engle, R. F and C. W. L. Granger (1987) "Cointegration and Error Correction: Representation, Estimation and Testing", Econometrica, 55, pp. 251-276.

Escribano, A. and C. W. L. Granger (1998) "Investigating the Relationship between Gold and Silver Prices", Journal of Forecasting, 17, pp. 81-107.

Escribano A. and G. A. Pfann (1998) "Non-linear error correction, asymmetric adjustment costs and cointegration", Economic Modelling, 15, pp. 197-216.

Escriabano, A. and S. Mira (1997) "Non-linear Cointegration with Mixing Errors", WP 9726-13, Universtad Carlos III de Madrid.

Escribano, A. (1997) "Non linear error correction: The case of money demand in the UK (1878-1970)", WP 96-55-24 Universtad Carlos III de Madrid.

Grenadier, S. R. (1996) “The Strategic Exercise of Options: Development Cascades and Overbuilding in Real Estate Markets”, Journal of Finance, Vol. LI, No. 5, pp. 1653-1679.

Granger, C. W. and T. H. Lee (1989) “Investigation of Production, Sales and Inventory Relationships Using Multicointegration and Non-symmetric Error Correction models”, Journal of Applied Econometrics, 4, pp. s145-s159.

Hamilton, J. D. (1989) “A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle”, Econometrica, 57, pp. 357-384.

Hendry D. and N. Ericson (1991) “An Econometric Analysis of the UK Money Demand in *Monetary Trends in the United States and the United Kingdom* by Milton Freedman and Anna Schwartz”, The American Economic Review, 81, pp. 8-38.

Holly S. and N. Jones (1997) “House prices since the 1940s: cointegration, demography and asymmetries”, Economic Modelling, 14, 549-565.

Keneally and McCarthy (1982) “A Preliminary econometric investigation of the Irish housing market 1969-1976”, The Economic and Social Review, 14, pp. 41-69.

Kenny, G. (1999) “Modelling the Demand and Supply Sides of the Housing Market: Evidence from Ireland”, *forthcoming*, Economic Modelling.

Maccini, L. J. (1987) “Adjustment Costs”, in J. Eatwell, M. Milgate and P. Newman (eds.) The New Palgrave: A Dictionary of Economics, (London, McMillan Press).

Neftci, S. N. (1984) “Are Economic Time Series Asymmetric Over the Business Cycle”, Journal of Political Economy, 92, pp. 307-328.

Nickel, S. (1985) "Error correction, partial adjustment and all that: An expository note", Oxford Bulletin of Economics and Statistics, 47, pp. 119-129.

Pesaran, M. H. and B. Pesaran (1997) Working with Microfit 4.0, Interactive Econometric Analysis, Oxford University Press (Oxford).

Pesaran, M. H. and R.J. Smith (1994) "A Generalised R^2 Criterion for Regression Models Estimated by Instrumental Variables Method", Econometrica, 62, pp. 705-710.

Pfann, G. A. and B. Verspagen (1989) "The Structure of Adjustment Costs for labour in the Dutch Manufacturing Sector", Economic Letters, 29, pp. 365-371.

Pfann, G. A. (1996) "Factor Demand Models with non-linear short-run fluctuations, Journal of Economic Dynamics and Control, 20, pp. 315-333.

Topel, R. and S. Rosen (1988) "Housing Investment in the United States", Journal of Political Economy, 96, pp. 718-740.