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# *EMPLOYMENT AND INFLATION RESPONSES TO AN EXCHANGE RATE SHOCK IN A CALIBRATED MODEL*

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**Abstract**

Ireland has no ability to affect the exchange rate through interest rates since the adoption of the euro. This paper provides a theoretically transparent method for analysing the impact of an exchange rate shock on employment and inflation in this context. The split between the tradable and non-tradable sectors of the economy is highlighted. A small, calibrated model adapted from Barry (1997) is used in the paper. The equations in this paper are derived under less restrictive assumptions making the results more widely applicable. The parameters of the model can be changed easily to reflect the structure of the economy and to conduct scenario analyses. A practical application is provided using a specific calibration and set of assumptions and the sensitivity of the results to the calibrated parameters and assumptions is discussed.

# 1 Introduction

The Irish economy fits the description of a "small open economy" more than most other economies. Irish GNP is a small fraction of euro zone GNP and the value of total trade is greater than GNP. For an economy of this type, the exchange rate is of great importance. Exchange rate fluctuations have a significant influence on key economic variables such as the level of employment and the inflation rate.

Ireland has participated in a number of different exchange rate regimes over the past decades. There have been periods of fixed exchange rates and flexible exchange rates. For a brief period, with the European snake in the early seventies and more recently in the Exchange Rate Mechanism (ERM) of the European Monetary System (EMS), Ireland participated in currency regimes that had some of the properties of a target zone. The ERM was a precursor to the single currency, which has been adopted by certain European countries that satisfied a number of economic convergence criteria. Amongst these countries, there is obviously no exchange rate. However, the value of the euro still floats against other currencies. Therefore, the exchange rate still has the potential to influence the domestic economy, albeit to a lesser extent now than prior to monetary union.

The aim of this paper is to provide a theoretical model of the economy that is capable of explaining the effect of a change in the exchange rate on employment and the aggregate price level. The model emphasises the distinction between the tradable and non-tradable sectors of the economy, which is important from both a theoretical and practical viewpoint. A second aim is to provide a practical demonstration of the use of the model. This is based on a specific set of parameters and assumptions. As such, the results are meant to be illustrative rather than definitive.

Various modelling strategies are available. There are many types of economic models e.g. structural versus astructural, partial versus general equilibrium, estimated versus calibrated. A small, calibrated model is used here. The model is adapted from Barry (1997). The tradable sectors in Barry's model are designated as trading with either the UK or Germany. In this paper, the tradable sectors are designated as trading with either euro area countries or the rest of the world, giving a more complete and updated picture of the tradable side of the economy. The model is re-calibrated to reflect the current structure of the economy and current trading patterns. Further enhancements are also made to Barry's

model. A distinction is drawn between long-run and short-run effects and an attempt is made to put an explicit time-frame on these effects where possible. Furthermore, the model is extended to identify explicitly the effects of the exchange rate shock on the aggregate price level although this, in itself, is an extension of work carried out on price effects in a similar two-sector model in Barry (2001). Most importantly, the equations in the model are derived under less restrictive assumptions.

The layout of the remainder of the paper is as follows. Section 2 contains a review of some of the relevant literature. Section 3 provides an overview of the model. Section 4 explains how the employment and price effects are derived from the equations in the model Section 5 provides practical examples of the usefulness of the model using calibrated data. Finally, section 6 provides the summary and conclusions.

## 2 Literature Review

In order to measure the effect of the exchange rate on the price level, it is necessary to construct an open economy model. It is also necessary to identify how the exchange rate will influence the price level in the model. Several open economy models exist that relate the behaviour of the exchange rate and the inflation rate. These models differ considerably in terms of the importance that they place on the role of the exchange rate in determining inflation.

At one end of the spectrum, Purchasing Power Parity (PPP) posits that the domestic inflation rate is determined entirely by changes in the exchange rate and foreign inflation rates. As such, it attaches great importance to the exchange rate in explaining inflation. There is complete pass-through from the exchange rate to the domestic inflation rate. Empirical support for this strict form of PPP in relation to consumer prices is quite weak in the short-term. However, there is widespread evidence in favour of PPP as a long-run proposition. (See Froot and Rogoff (1995) for a survey on PPP.) This is particularly true in relation to tradable prices.

At the other end of the spectrum, some recent literature on "new open economy macroeconomics" uses an assumption called "pricing-to-market", which attaches no importance to

the exchange rate in the determination of consumer price inflation. (See Lane (2001) for a survey.) In this family of model, import price setters set the import price equal to the domestic price. This price fixing practice is maintained regardless of shifts in the exchange rate. There is no pass-through at all. Devereux and Engel (2002) have termed this as "exchange rate disconnect". Neither of these polar-case models is appropriate to the Irish case.

It is necessary to consider an intermediate model that places partial but not absolute importance on the exchange rate in the analysis of domestic consumer prices. One such model is the "Scandinavian" model of inflation. This type of model advocates separate treatment for the tradable and non-tradable sides of the economy in recognition of the different price determination mechanisms in operation. Inflation in the tradable sectors of the economy is determined by PPP. In the non-tradable sectors, the inflation rate is a mark-up over costs, wages or some other appropriate variable. Although the approach adopted in this paper is similar to the Scandinavian model of inflation, it differs in its treatment of non-tradable prices, to the extent that non-tradable prices are not modelled strictly as a mark-up on another variable. Furthermore, full pass-through is not assumed for tradable prices but can be specified using the pass-through parameter if desired. In effect, a certain degree of pricing-to-market is assumed.

Models of this variety, which make the distinction between tradable and non-tradable prices, are useful in explaining the price determination process in an open economy. Kenny and McGettigan (1996) use a multivariate cointegration analysis to demonstrate the empirical relevance of the distinction between tradable and non-tradable prices in Ireland. They show the strongest form of PPP to be consistent with the data on tradable prices in the long-run. Slevin (2003) obtains the same result using the autoregressive distributed lag approach to cointegration. Slevin also finds that prices in the non-tradable sector are determined by wages and productivity growth. The empirical evidence from these studies demonstrates clearly that the distinction between tradable and non-tradable prices is warranted in the Irish case.

The model adopted in this paper is from Barry (1997). Barry uses his model in order to determine the likely costs of EMU membership in the event that Britain does not join. In Barry's model, there is a non-tradable sector and two tradable sectors. One tradable sector competes with the UK and the other with Germany. He calculates the number of

jobs that would be lost because of a 20 per cent decrease in the value of sterling (Britain being Ireland's largest trade partner), under the assumption that Ireland is in EMU and Britain is not. Barry (1997) focuses on the employment effects of a shock to the exchange rate under different assumptions regarding nominal wage flexibility and demand conditions.

This paper first sets out the model formulated by Barry (1997) but with the tradable sectors defined differently. In this paper, prices can vary in all sectors and the desired level of pass-through can be specified. Using the same methodology but with fewer assumptions, an expression for non-tradable prices is derived. Furthermore, a distinction is drawn between long-run and short-run effects and an explicit time horizon is put on these effects. The paper goes beyond issues considered in Barry (1997) by considering the implications of an exchange rate change on the aggregate price level. It draws on a theoretical framework from Barry (2001), which considers the trade-off between price-level and employment responses to a nominal shock in an analogous two-sector model. The second part of the paper provides a demonstration of the use of the model in analysing both employment and price level effects.

## 3 Model Description

The model used is adopted from Barry (1997). The economy produces three types of goods: euro area tradable goods, foreign tradable goods and non-tradable goods. Variables are subscripted with E, F and N. The subscript E refers to euro area tradable goods, which are tradable goods consumed domestically that compete on euro area international markets. The subscript F refers to foreign tradable goods, which are tradable goods consumed domestically that compete on international markets outside the euro area. Tradable goods can be produced domestically or abroad. The subscript N refers to domestically produced and consumed non-tradable goods. The superscript * denotes foreign variables.

Tradable goods prices are determined on international markets. The price of goods in the euro area sector is simply equal to euro area prices on international markets. In setting euro area prices, euro area producers engage in a certain degree of pricing-to-market behaviour following an exchange rate change. The weight given to this behaviour is measured by $\sigma$. Prices in the foreign sector of the economy are determined by Purchasing Power Parity

(PPP) so that the euro price of these goods is determined by the price on international markets, the exchange rate and the assumed level of pass-through, $\beta$:

$$
\begin{aligned}
p_e &= p_e^* && (1)\\
p_e^* &= e^{\sigma} && (2)\\
p_f &= e^{\beta} \left(p_f^*\right)^{\gamma} && (3)
\end{aligned}
$$

In addition, $p_f^*$ is constant so that the only source of price changes in the foreign sector is from the exchange rate. The nominal value of the marginal product of labour is equal to the nominal wage rate:

$$w = p_i \, MPL_i \tag{4}$$

Total employment in the model refers to total private non-agricultural employment. Sectoral employment is a function of the real wage in each sector:

$$L = L_e(w_e) + L_f(w_f) + L_n(w_n) \tag{5}$$

where $w_i$ refers to the real product wage in sector i; $(w_i = w/p_i)$. The derivative of labour employment in each sector with respect to the real wage in that sector is negative:

$$\frac{d}{dw_i} L_i < 0$$

The three-good utility function is specified as follows:

$$U = Y_f^{\psi} \, Y_e^{\chi} \, Y_n^{\theta} \tag{6}$$

with

$$\psi + \chi + \theta = 1$$

The nominal demand for non-tradables is a fixed proportion of nominal income[1]:

[1] See Appendix 2, section 1 for more details.

$$p_n Y_n = \theta \; M \tag{7}$$

Real output, $Y$, is expressed in terms of European prices:

$$Y = \frac{p_n Y_n}{p_e} + \frac{p_f Y_f}{p_e} + Y_e \tag{8}$$

Under the assumption that the utility function is log-linear, the fraction of income spent on non-tradable goods, $\theta$, is also the elasticity of the consumer price index with respect to non-tradable goods prices. Similarly, the elasticity of the consumer price index with respect to prices of goods in the other sectors of the economy is equal to the exponent of that sector's good in the utility function. The equilibrium in the tradable sector of the economy is given by[2]:

$$p_n Y_n = \frac{\theta}{1-\theta} \left( p_e Y_e + p_f Y_f \right) \tag{9}$$

As mentioned, non-tradable prices are not modelled as a mark-up on another variable.

# 4 Analysing the Exchange Rate Shock

The equations in the model allow the effect of an exchange rate shock on employment and the aggregate price level to be examined. In this section, these effects are explained and the relevant equations are presented. However, the full derivations are contained in the appendix.

## 4.1 Effect of a Shock to the Exchange Rate on Employment

The level of employment in each sector is dependent on the real wage. In turn, the real wage is determined by the nominal wage and price. The assumed level of nominal wage adjustment in response to a change in the exchange rate is taken from an ESRI analysis of a very similar problem.[3] This means that knowledge of how the exchange rate will affect

[2] See Appendix 2, section 2 for more details.

[3] The wage response is taken from Baker et al (1996). This will be discussed in more detail in the next section.

prices in each sector is necessary in order to determine the effects on employment. Prices in the euro sector vary with the exchange rate in accordance with the pass-through parameter and prices in the foreign sector obey a more traditionally framed PPP relationship with foreign prices. However, the effect of the change in the exchange rate on non-tradable prices is not immediately clear.

From the equilibrium condition above, non-tradable prices depend on prices in the other sectors and output in all sectors. The price effects in the other sectors have just been discussed but in order to determine the output effects it is necessary to make assumptions regarding how output is affected by the change in the exchange rate. Production is dependent on the level of capital and labour employed in each sector. Thus, in order to determine the change in output, it is first necessary to find how the assumed level of nominal wage adjustment in response to an exchange rate change affects labour and capital demand.

The change in the amount of labour employed as a result of a change in the wage is determined by the wage elasticity of labour demand in each sector. For the baseline scenario, these elasticities are calibrated based on previous estimates in the literature.[4] However, the values of these parameters are subsequently varied. The response of capital employed to a change in the exchange rate is more difficult to determine. Two different assumptions regarding the level of capital adjustment are employed. Specifically, different values for these labour and capital elasticities are used to draw a distinction between long-run effects and short-run effects.

For simplicity, it is assumed that production technology is approximated by Cobb-Douglas production functions. It is initially assumed that capital stocks adjust fully so that the elasticity of capital demand with respect to wages equals 1 in each sector. This is viewed as a long-run assumption. In reality, it is unlikely that capital demand will be this sensitive to the wage rate but this assumption will be varied. It is also assumed that the elasticity of labour demand in each sector is not equal to -1. Otherwise, it would mean that changes in the real wage have no effect on sectoral output levels.[5] With these assumptions, the response of output in each sector to the exchange rate change can be calculated. Barry (1997) derives his equations under the assumption that the economy starts from an equilibrium condition where all prices are equal. This assumption is not

[4]See Appendix 1 on parameter calibration for more details.

[5]See Appendix 2, section 3 for more details.

used to derive the equations in this version of the model. Therefore, the results in this case are more general. With this first set of assumptions, the total derivative of equation (9) relates changes in non-tradable prices to changes in the nominal wage, euro area prices and foreign prices:[6]

$$\frac{dp_n}{p_n} = \left[1 - \frac{\epsilon\left(L_f, w_f\right)}{\epsilon\left(L_n, w_n\right)}\right]\frac{dw}{w} + \left[\frac{\epsilon\left(L_f, w_f\right)}{\left(1-\theta\right)\epsilon\left(L_n, w_n\right)}\right]\left\{\chi\,\frac{dp_e}{p_e} + \psi\,\frac{dp_f}{p_f}\right\} \tag{10}$$

where $\epsilon(L_i, w_i)$ is the elasticity of labour demand in sector i with respect to the real wage in that sector.

A restriction on the choice of labour demand elasticities is that the elasticity in the non-tradable sector cannot be equal that in the foreign sector. It can be seen from equation (10) above that changes in the wage will have no effect on non-tradable prices if $\epsilon(L_f, w_f) = \epsilon(L_n, w_n)$. It is further assumed that the elasticity in the non-tradable sector is greater than in the tradable sectors. This is based on previous work by Bradley, Fitzgerald and Kearney (1991) which finds that labour elasticites are higher in services sectors than in manufacturing.

Equation (10) is a long-run relationship because it is derived under the assumption that capital stocks adjust fully. If it is instead assumed that capital stocks are fixed, then the short-run relationship can be derived. In this case, the derivative of equation (9) leads to the following non-tradable price equation:[7]

$$\frac{dp_n}{p_n} = \left[\frac{s_f\epsilon\left(L_f, w_f\right) - s_n\epsilon\left(L_n, w_n\right)}{1 - s_n\epsilon\left(L_n, w_n\right)}\right]\frac{dw}{w} + \left[\frac{1 - s_f\epsilon\left(Y_f, w_f\right)}{\left(1-\theta\right)\left[1 - s_n\epsilon\left(L_n, w_n\right)\right]}\right]\left\{\chi\,\frac{dp_e}{p_e} + \psi\,\frac{dp_f}{p_f}\right\} \tag{11}$$

where $s_i$ represents the labour share in sector $i$. The calibrated parameters are substituted into this equation to determine the change in non-tradable prices.

Once the effect on non-tradable prices has been evaluated, the price effects in each sector will have been determined. In conjunction with the assumed level of nominal wage adjustment in the economy as a whole, this allows the level of real wage adjustment in each

[6] See Appendix 2, section 4 for more details.

[7] See Appendix 2, section 5 for more details.

sector to be calculated. It can then be established what effects a change in the exchange rate has on the level of employment in the economy.

### 4.2 Effect of a Shock to the Exchange Rate on the Overall Price Level

Next, the impact that a change in the exchange rate has on the aggregate price level is examined. It was mentioned that preferences are described using a Cobb-Douglas utility function. Thus, the exponent on the consumption of an individual sector's good in the utility function gives the elasticity of the HICP with respect to prices in that sector.

$$\epsilon(P, e) = \psi\epsilon(p_f, e) + \chi\epsilon(p_e, e) + \theta\epsilon(p_n, e)$$

Using the definitions of elasticities:

$$\frac{dP}{de}\left(\frac{e}{P}\right) = \psi\left(\frac{dp_f}{de}\right)\frac{e}{p_f} + \chi\left(\frac{dp_e}{de}\right)\frac{e}{p_e} + \theta\left(\frac{dp_n}{de}\right)\frac{e}{p_n}$$

Multiply both sides by $de/e$:

$$\frac{dP}{P} = \psi\,\frac{dp_f}{p_f} + \chi\,\frac{dp_e}{p_e} + \theta\,\frac{dp_n}{p_n}$$

From the price equations:

$$\frac{dp_f}{dp_f} = \beta\,\frac{de}{e} \quad \text{and} \quad \frac{dp_e}{p_e} = \sigma\,\frac{de}{e}$$

where $\beta$ is the level of pass-through and $\sigma$ is the pricing-to-market parameter. The change in the price level due to an exchange rate shock can be expressed:

$$\frac{dP}{P} = (\psi\,\beta + \chi\,\sigma)\,\frac{de}{e} + \theta\,\frac{dp_n}{p_n} \tag{12}$$

This equation relates changes in the domestic price index to changes in the exchange rate and non-tradable prices. Changes in non-tradable prices are calculated using equation (10). The tradable price equations in this paper are based on PPP, which is only supported by the data in the long-run. In their analysis of inflation in a small open economy, Kenny and McGettigan (1996) find that PPP for tradable prices is supported by Irish data with full pass-through in three to four years. Accordingly, the price effects in the paper are assumed to take place within four years.

# 5 A Practical Illustration

Having outlined the theoretical model and derived the necessary equations, an example of how the model can be implemented is now provided. The effects are examined using parameters that are calibrated based on data from 2002 and from parameter estimates in previous studies. Full details of how the parameters are calibrated can be found in Appendix 1. In addition, the parameters that are used are in Table 1 at the end of the paper. The results of the model are sensitive to the chosen assumptions and calibrated parameters. However, this means that the model is quite flexible in the sense that it can be easily calibrated to reflect different views of the true structure of the economy. The scenarios that follow provide an example of how this is accomplished.

## 5.1 Employment Response

In the employment scenario, the effects of a change in the exchange rate on the level of employment are discussed. Both long-run and short-run effects are discussed. The level of nominal wage adjustment is taken from Baker et al (1996), an ESRI analysis of the likely implications for Ireland of participating in monetary union. As part of the analysis, the response of Irish wages to a 20% devaluation of sterling is calculated.[8] The ESRI analysis suggests that nominal wages in Ireland should fall by slightly over 5% in four years in response to a 20% devaluation of sterling. Although this paper considers a devaluation of 15% for foreign currencies rather than 20%, it is a larger shock given that it is against all currencies. Nonetheless, the level of nominal wage adjustment is still set at 5%.

The long-run for employment effects in this paper is considered to be 3-4 years because the majority of the adjustment in nominal wages takes place inside this time frame, again based on the ESRI analysis. The short-run is more difficult to specify. The short-run is usually defined as a period of time in which at least one of the factors of production is fixed. Given that employment responses are under consideration, and labour is consequently variable, the only other input specified in the production function is capital. Therefore, the short-run is defined as the period in which capital remains fixed. It is possible to make this definition operational based on the equations in the last section. Non-tradable price

[8] This is achieved in two steps. The NiGEM model of the UK economy is first used to calculate wage and price effects in the UK. A quarterly econometric model of prices and wages is then employed to determine the likely effects in Ireland.

equations were derived in the case of both fixed and fully adjustable capital stocks - these equations correspond to short-run and long-run equations respectively. We now have a theoretical and operationally consistent definition of the short-run but not an explicit time horizon. We will associate this with a period of less than one year.

A final point to note is that the employments effects in the tradable sector are based on the export shares of our trading partners. Consequently, from an employment perspective, the exchange rate shock is export weighted rather than a shock to the nominal effective exchange rate, which is based on overall trade figures.

### 5.1.1 Employment Scenario: Long-run Employment Effects with Flexible Wages

The exchange rate shock that is considered is a 15% increase in the value of the euro against all currencies simultaneously. Based on the ESRI analysis, it is assumed that nominal wages fall by 5% in all sectors in response to the exchange rate shock. Although it is unrealistic to assume equal wage adjustment in each sector, it simplifies the derivation of the non-tradable price equations. The change in the level of employment in each sector is examined separately. The effects are summed to give the total change in employment in the economy as a whole. This scenario indicates how to use the model to examine long-run employment effects.

*Foreign sector:* In the non-euro area tradable sector, prices fall by 7.5%, assuming 50% pass-through from the exchange rate to prices in the foreign sector ($\beta = 0.5$). In addition, wages fall by 5%. Thus, the change in the real product wage $w/p_f$ is an increase of 2.5%. The impact of a 2.5% rise in the real wage in the foreign sector on the total percentage employed in the economy is given by:[9]

$$\epsilon\left(L_f, w_f\right)\left(\frac{L_f}{L_t}\right)\left(\frac{L_t}{L}\right)\left(\frac{dw_f}{w_f}\right) = -0.28\%$$

The first term of this product is the elasticity of labour demand with respect to the real wage in the foreign sector of the economy. The second and third terms together give the fraction of the total labour market employed in the foreign sector of the economy. The final term is the percentage change in the real wage rate. Thus, the whole product expresses the

[9]The percentage change is generally displayed to 2 decimal places but calculations are based on six significant figures.

reduction in the numbers employed in the foreign sector of the economy as a percentage of the total number employed. Consequently, the reduction in employment due to the increase in the real product wage in the foreign sector of the economy is:

$$\Delta L_f = (-0.0028)L = -4638$$

The increase in the real wage in the foreign sector leads to a reduction of approximately 4600 in those employed.[10]

*Euro sector:* Prices in the euro sector depend on the level of pricing-to-market. Anderton (2003) finds that extra-euro area producers assign a 30 to 50 per cent weight to shadowing euro area prices following an exchange rate change i.e. if the effective exchange rate of the euro decreases by 10%, producers will reduce prices by 3% to 5% to maintain competitiveness. Assuming that euro area producers behave in the same way as extra-euro area producers, it is assumed that pricing-to-market has a 40% weight so that a 15% rise in the exchange rate leads to a 6% fall in prices. In conjuction with the 5% fall in nominal wages, this means that the real wage in the euro sector increaes by 1% :

$$\epsilon\left(L_e, w_e\right)\left(\frac{L_e}{L_t}\right)\left(\frac{L_t}{L}\right)\left(\frac{dw_e}{w_e}\right) = -0.067\%$$

The change in the real wage in the euro sector leads to an reduction in employment of 1100.

*Non-tradable sector:* In the long-run, non-tradable prices are related to foreign prices according to equation (10). Based on the calibrated parameters:

$$\frac{dp_n}{p_n} = 0.25\,\frac{dw}{w} + 0.5966\,\frac{dp_f}{p_f} + 0.1534\,\frac{dp_e}{p_e}$$

The reduction in foreign prices is 7.5%, the reduction in euro area prices is 6% and there is a 5% fall in the nominal wage. We now have all the necessary figures to calculate the change in the real wage:

[10] In the text, employment changes are reported to the nearest hundred.

$$\begin{aligned}\frac{dw_n}{w_n} &= \frac{dw}{w} - \frac{dp_n}{p_n} \\ &= \frac{dw}{w} - \left[0.25\,\frac{dw}{w} + 0.5966\,\frac{dp_f}{p_f} + 0.1534\,\frac{dp_e}{p_e}\right] = 1.64\%\end{aligned}$$

This leads to the following change in the level of employment:

$$\begin{aligned}\epsilon\left(L_n, w_n\right)\left(\frac{L_n}{L}\right)\left(\frac{dw_n}{w_n}\right) &= -0.92\% \\ \Rightarrow \Delta L_n &= -15143\end{aligned}$$

The change in overall employment is given by the sum of the individual changes in each sector:

$$\Delta L = \Delta L_f + \Delta L_e + \Delta L_n = -20884$$

In this scenario, with the baseline calibration and under the assumption of fully adjustable capital stocks, a 15% rise in the value of the euro coupled with a 5% reduction in nominal wages in the economy leads to a reduction in employment of approximately 20900. In an analysis of the employment effects of a 15% fall in the value of sterling, Barry (1997) finds that job losses would amount to 30000. One might expect more job losses here given that Barry's results relate to a shock to the euro/sterling exchange rate only. However, the labour demand elasticities are lower in this paper and there is not full pass-through from the exchange to prices in the foreign sector. These factors are highly influential and indicate the sensitivity of the employment results to the parameters and assumptions.

A number of qualifications need to be made regarding the results in this section. One problem is that the analysis is a partial rather than general analysis. There are numerous other factors that are likely to influence the level of employment, particularly over the time frame considered. The Irish labour force is quite mobile internationally and, in adverse conditions, the level of unemployment could be less than expected due to migration. Baker et al (1996) point out that adjustment to the exchange rate shock will take place abroad also so it can be misleading to focus solely on domestic factors. Government policies might also be implemented to boost job creation.

### 5.1.2 Short-run employment effects

It is also possible to use the model to calculate short-run employment effects. The methodology is the same as with the long-run calculations with the exception that non-tradable prices are described using equation (11) instead of equation (10). Short-run labour demand elasticities are also necessary but using the econometric estimates of the short-run elasticities in the literature also leads to problems. In particular, there is little agreement on the appropriate value for these elasticities. (See Barry (1998) and Fitz Gerald (1998) for a more detailed discussion.) Another significant problem in terms of the calibration of these elasticities in the current context is that sectoral distinctions in this paper are different from those in previous papers so that econometric estimates of the elasticities can only be used as a guide. It is also necessary to estimate the level of wage and price adjustment in the short-run and this will differ from the amount of adjustment in the long-run. A numerical example of short-run calculations is not provided as the method is identical to that in the previous scenario.

## 5.2 Aggregate Pricel Level Response

The following scenario indicates how the model can be used to examine the effect of an exchange rate shock on the aggregate price level. As with the employment scearios, the results are sensitive to the assumptions and parameters and, again, it is the procedure rather than the results that is of interest. The shock considered is a 15% rise in the value of the euro against all currencies - identical to the employment scenarios. It is worth noting that the effect of the exchange rate change on the aggregate price level is calibrated by weighting the tradable goods in the HICP according to import shares. Thus, from the perspective of aggregate prices, the exchange rate shock is import weighted.

### 5.2.1 Price Level Scenario: 15% Rise in the Value of the Euro

The responsiveness of prices in the individual sectors of the economy with respect to the exchange rate differs considerably. The level of responsiveness in the foreign sector is set according to the pass-through parameter $\beta$ while responsiveness in the euro sector is determined by the pricing-to-market parameter. In the non-tradable sector, the level of responsiveness is determined by the parameters in equation (10). Calibrating equation (10) with the appropriate parameters:

$$\frac{dp_n}{p_n} = 0.25\ \frac{dw}{w} + 0.5966\ \frac{dp_f}{p_f} + .1533\ \frac{dp_r}{p_e} = -0.066$$

In this scenario, non-tradable prices fall by 6.6% due to the strength of the euro. If this is substituted back into the expression for the HICP, the change in the aggregate price level is:

$$dP/P = 0.30(-0.15) + 0.38(-0.066) = -0.07$$

Thus, in this scenario, a 15% rise in the value of the euro leads to a 7.0% reduction in the aggregate price level, suggesting a 47% pass-through from the exchange rate to the aggregate price level. As with the employment results, the figures should not be taken as "estimates" because some parameters were chosen arbitrarily and may not be entirely reflective of reality. In a sense, the results are qualitative rather than quantitative. It is shown that the reduction in prices in the tradable sectors and the reduction in the nominal wage put downward pressure on prices in the non-tradable sector of the economy.

# 6 Summary and Conclusions

The aim of this paper has been to examine how the exchange rate impacts on the level of employment and the aggregate price level in the economy. The paper uses a simple framework in the form of a small, calibrated model adapted from Barry (1997). The equations in this paper are derived under less restrictive assumptions making them more widely applicable. Within the model framework, the distinction between the tradable and non-tradable sectors of the economy is of vital importance. A distinction between long-run and short-run effects is also drawn. The simplicity of the framework means that the parameters can be adjusted easily as the structure of the economy changes. This provides significant flexibility. The paper also demonstrates how the model can be implemented practically based on a specific calibration. The results of the model vary according the chosen assumptions and parameters.

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# A Appendix 1 Parameter Values used for Calibration:

The parameters in the model are calibrated based on data from the end of the year 2002. This is the most recent period for which data is available for the parameters of interest. In some instances, the end of year figure was taken and in others, an annual average is taken.

*Share of each class of good in the CPI:*

The distinction between tradable and non-tradable goods is, to a certain extent, arbitrary. Barry (1997) makes the distinction based on economic sectors. Services, housing and a certain proportion of wholesale and retail items in the CPI are classed as non-tradable. The remainder are classed as tradable. The drawback of such broad categories is that there are likely to be tradable goods being classified as non-tradable and visa versa. To address this shortcoming, the distinction between the tradable and non-tradable component of the HICP is made according to the methodology of Kenny and McGettigan (1997). Each individual item in the HICP is categorised as either tradable or non-tradable. The examination of the composition of the HICP in such detail allows the proportion of tradable and non-tradable goods in the HICP to be calculated as accurately as is possible.

Based on the expenditure weights of goods in the calculation of the CPI, tradable goods are seen to represent 59.4 per cent of the CPI with the remaining 40.6 per cent being accounted for by non-tradable goods, based on the method cited above. However, the HICP excludes certain items in both the tradable and non-tradable categories that are not excluded in the CPI. Having accounted for these excluded items, tradable goods are seen to represent 62.1 per cent of the HICP and non-tradables represent 37.9 per cent. Thus, the elasticity of the HICP with respect to non-tradable goods prices, $\theta$, is equal to 0.379. The tradable goods component is itself broken down further between the euro-area sector and the remaining foreign sector.

In order to assess the proportion of the tradable goods component of the HICP determined by euro-area and other foreign prices, the tradable component of the HICP is weighted according to our share of imports from these areas. According the "Budgetary and Economic Statistics 2003", published by the Department of Finance, imports from the euro area in 2002[11] accounted for 20.5 per cent of overall imports. This

[11] Figures apply to the period from January to November 2002.

means that goods produced and consumed domestically that compete with euro area goods together with imported euro area goods account for 12.7 per cent of the HICP. Thus, the elasticity of the HICP with respect to euro area goods prices, $\chi$, is equal to 0.127. This implies that goods produced and consumed directly that compete in other foreign markets together with imports from these markets comprise 49.4 percent of the consumer price index. Thus, the elasticity of the consumer price index with respect to foreign goods prices, $\psi$, is equal to 0.494.

The use of import shares to weight the tradable component of the HICP suffers from the drawback that the tradable components that this paper aims to identify consist of not only imports but also domestically produced goods that compete on foreign markets. However, it is difficult to ascertain which items in the tradable component of the HICP were produced by domestic companies that compete on international markets. Consequently, the import shares were used as weights.

*Share of labour employed in each sector*

The share of labour employed in the tradable and non-tradable sectors of the economy was obtained from the Quarterly National Household Survey (QNHS). Following the methodology in Barry (1997), agricultural employment was excluded in the analysis on the basis that it is largely constrained by the Common Agricultural Policy. Tradable employment is defined as employment in industry, where industry is defined as NACE economic sectors C-F, as outlined in tables 2a and 2b of the QNHS. Non-tradable employment is defined as employment in services, where the services industry is defined as NACE economic sectors G-O, also outlined in tables 2a and 2b of the QNHS. According to these definitions, employment in the tradable sector accounts for 30 per cent of total non-agricultural employment with the remaining 70 per cent accounted for by employment in the non-tradable.

The percentage of tradable employment in the euro area sector is taken to be equal to the volume of euro area exports as a percentage of total exports. The data on exports is taken from the "Budgetary and Economic Statistics 2003", published by the Department of Finance. According to export volumes, tradable employment in the euro area sector accounts for 37.32 per cent of tradable employment. The remaining 62.68 per cent of tradable employment is accounted for by the foreign sector.

*Sectoral Labour Demand Elasticities:*

Bradley, Fitzgerald and Kearney (1991) estimate various sectoral elasiticites. They find that labour elasticities in the services sector are generally higher than in the manufacturing sectors. Based on this evidence, Barry (1997) uses a value of -1 for the labour elasticity in the non-tradable sector and one of the tradable sectors. A value of -0.6 is assigned to the remaining tradable sector. Labour elasticities in the tradable sectors differ in his paper due to differences in the level of production technology employed. These differences are not necessarily preserved in the definition of the tradable sectors in this paper. For that reason, the labour elasticities in the tradable sectors are assumed to be equal in this paper and both are assigned a value of -0.6. In addition, the elasticity in the non-tradable sector is set equal to -0.8 rather than -1 on the basis that a value of -1 means that wage changes have no effect on output in the fully adjustable capital stock case.

In terms of the labour shares, it is assumed that the labour share in the tradable sectors is equal to 0.5, implying an equal split between labour and capital in the production process in the tradable sector. This seems to be a reasonable assumption for the production of commodities in an aggregate sense although it will obviously vary from sector to sector. In the non-traded sector, which has been calibrated based on the services side of the economy, the labour share is assumed to be 0.7, reflecting the fact that the provision of services is more labour intensive than the production of commodities.

# B Appendix 2 Mathematical Results used in the Model

The model uses a 3-good utility function. The mathematics of 2-good utility functions are widely known from undergraduate economics. The 3-good utility function is a simple extension of the 2-good case. However, for the sake of completeness, a few of the elementary results for 3-good utility functions that are used in the paper are first presented here. In addition, results that are specific to this model and have been used in the paper are also presented.

1. *If utility functions are Cobb-Douglas, the nominal demand for any good is a fixed proportion of nominal income (equation 7):*

   In a two-good world, it is not difficult to show that the nominal demand for any good is a fixed proportion of nominal income. This is also true for a three-good world and this well-known result is presented here.

   The three-good utility function is specified as follows:

   $$U = Y_f^{\psi}\, Y_e^{\chi}\, Y_n^{\theta}$$

   with

   $$\psi + \chi + \theta = 1$$

   The Lagrangian is set up to maximize utility subject to the constraint that all income is spent:

   $$L = {Y_f}^{\psi} {Y_e}^{\chi} {Y_n}^{\theta} + \lambda\,(M - p_f Y_f - p_e Y_e - p_n Y_n)$$

   Take the derivative with respect to the quantity demanded in each sector of the economy:

   $$\begin{aligned} \frac{\partial}{\partial Y_f}\left({Y_f}^{\psi} {Y_e}^{\chi} {Y_n}^{\theta} + \lambda\,(M - p_f Y_f - p_e Y_e - p_n Y_n)\right) &= \frac{{Y_f}^{\psi}\psi\, {Y_e}^{\chi} {Y_n}^{\theta}}{Y_f} - \lambda\, p_f \\ \Rightarrow \lambda p_f &= \left(\frac{\psi}{Y_f}\right) U \end{aligned}$$

   Similarly,

   $$\begin{aligned} \lambda p_e &= \left(\frac{\chi}{Y_e}\right) U \\ \lambda p_n &= \left(\frac{\theta}{Y_f}\right) U \end{aligned}$$

Using these expressions for prices, take the ratio of prices in the euro sector of the economy to prices in the non-tradable sector of the economy to find an expression for nominal income in the euro sector in terms of nominal income in the non-tradable sector:

$$\begin{aligned} \frac{p_e}{p_n} &= \Big(\frac{\chi}{\theta}\Big)\Big(\frac{Y_n}{Y_e}\Big) \\ \Rightarrow p_e Y_e &= \Big(\frac{\chi}{\theta}\Big) p_n Y_n \end{aligned} \tag{13}$$

Repeat the process to with foreign prices to get an expression of nominal income in the foreign sector in terms of nominal income in the non-tradable sector:

$$\begin{aligned} \frac{p_f}{p_n} &= \Big(\frac{\psi}{\theta}\Big)\Big(\frac{Y_n}{Y_f}\Big) \\ \Rightarrow p_f Y_f &= \Big(\frac{\psi}{\theta}\Big) p_n Y_n \end{aligned} \tag{14}$$

From the lagrangian above, the expression for income is given by:

$$M = p_e Y_e + p_f Y_f + p_n Y_n$$

Substitute the expressions for nominal income in foreign and non-tradable sectors into the expression for income:

$$\begin{aligned} M &= \Big(\frac{\psi}{\theta}\Big) p_n Y_n + \Big(\frac{\chi}{\theta}\Big) p_n Y_n + p_n Y_n \\ &= p_n Y_n \Big(\frac{\psi}{\theta} + \frac{\chi}{\theta} + 1\Big) \\ &= p_n Y_n \Big(\frac{\psi + \chi + \theta}{\theta}\Big) \\ &= p_n Y_n \left(\frac{1}{\theta}\right) \\ \Rightarrow p_n Y_n &= \theta\, M \end{aligned}$$

This final equation is equivalent to equation (7) in the paper. This expression shows that nominal demand for non-tradable goods is a fixed proportion of nominal income. Furthermore, the exponent on the non-tradable variable in the utility function gives that fixed proportion. It is readily apparent that this result is equally applicable to nominal demand in the other sectors of the economy.

2. *Expression for equilibrium condition in non-tradable sector (equation 8):*

   The equilibrium condition for prices in the non-tradable sector of the economy follows very simply from the previous result. Recall equations (13) and (14) from the previous section:

$$\begin{aligned} p_e Y_e &= \left(\frac{\chi}{\theta}\right) p_n Y_n \\ p_f Y_f &= \left(\frac{\psi}{\theta}\right) p_n Y_n \end{aligned}$$

   Add these two expressions together:

$$\begin{aligned} p_e Y_e + p_f Y_f &= \left(\frac{\psi}{\theta}\right) p_n Y_n + \left(\frac{\chi}{\theta}\right) p_n Y_n \\ &= p_n Y_n \left(\frac{\psi + \chi}{\theta}\right) \\ &= p_n Y_n \left(\frac{1-\theta}{\theta}\right) \\ \Rightarrow p_n Y_n &= \left(\frac{\theta}{1-\theta}\right) p_e Y_e + p_f Y_f \end{aligned}$$

   This is equation (9) from the body of the paper.

3. *The elasticity of output with respect to the sectoral real wage is equal to one plus the elasticity of labour demand with respect to the sectoral real wage i.e.* $\epsilon(Y_i, w_i) = 1 + \epsilon(L_i, w_i)$

   Production technology is approximated with Cobb-Douglas functions:

$$\begin{aligned} Y &= K^{\alpha} L^{1-\alpha} \\ \Rightarrow \hat{Y} &= \alpha \hat{K} + (1-\alpha)\hat{L} \\ \Rightarrow \hat{Y} &= \alpha(\hat{K} - \hat{L}) + \hat{L} \end{aligned}$$

   It has been assumed that capital stocks adjust fully so that $\hat{K} \neq 0$. Now, using the first-order condition that the marginal product of labour equals the real wage:

$$\begin{aligned} \frac{w}{p} &= (1-\alpha)\left(\frac{K}{L}\right)^{\alpha} \\ \Rightarrow \left(\frac{\hat{w}}{p}\right) &= \alpha\left(\hat{K} - \hat{L}\right) \end{aligned}$$

Substitute this into the expression for $\hat{Y}$ :

$$\begin{aligned} \Rightarrow \hat{Y} &= \left(\frac{\hat{w}}{p}\right) + \hat{L} \\ \Rightarrow \frac{d\hat{Y}}{d\left(\frac{\hat{w}}{p}\right)} &= 1 + \frac{d\hat{L}}{d\left(\frac{\hat{w}}{p}\right)} \\ \Leftrightarrow \epsilon(Y_i, w_i) &= 1 + \epsilon(L_i, w_i) \end{aligned}$$

4. *Derivation of equation (10)*

Equation (9) from the body of the paper states that:

$$p_n Y_n = \frac{\theta}{1-\theta}\left(p_e Y_e + p_f Y_f\right)$$

where

$$Y_i = f\left(\frac{w}{p_i}\right) = f(w_i)$$

Taking the total derivative of equation (9):

$$p_n dY_n + Y_n dp_n = \frac{\theta}{1-\theta}\left(p_e dY_e + Y_e dp_e + p_f dY_f + Y_f dp_f\right)$$

Given that each Y is a function of the sectoral real wage, this is re-written:

$$p_n\left(\frac{dY_n}{dw_n}\right)dw_n + Y_n dp_n = \frac{\theta}{1-\theta}\left\{p_e\left(\frac{dY_e}{dw_e}\right)dw_e + Y_e dp_e + p_f\left(\frac{dY_f}{dw_f}\right)dw_f + Y_f dp_f\right\}$$

Consider the sectoral real wage differentials; $w_i = w/p_i$. Thus,

$$dw_i = \left(\frac{d}{dw}w_i\right)dw + \left(\frac{d}{dp_i}w_i\right)dp_i = \frac{dw}{p_i} - \left(\frac{w}{{p_i}^2}\right)dp_i$$

Substituting these results back into the original equation:

$$\begin{aligned} p_n\left[\frac{dY_n}{dw_n}\right]\left(\frac{dw}{p_n} - \left[\frac{w}{{p_n}^2}\right]dp_n\right) + Y_n dp_n = \frac{\theta}{1-\theta}\left\{p_e\left[\frac{dY_e}{dw_e}\right]\left(\frac{dw}{p_e} - \left[\frac{w}{{p_e}^2}\right]dp_e\right) + Y_e dp_e\right\} \\ + \frac{\theta}{1-\theta}\left\{p_f\left[\frac{dY_f}{dw_f}\right]\left(\frac{dw}{p_f} - \left[\frac{w}{{p_f}^2}\right]dp_f\right) + Y_f dp_f\right\} \end{aligned}$$

Algebraic manipulation yields:

$$dp_n\left(Y_n - \frac{dy_n}{dw_n}\left[\frac{w}{p_n}\right]\right) = dw\left(\frac{\theta}{1-\theta}\left\{\frac{dY_f}{dw_f} + \frac{dY_e}{dw_e}\right\} - \frac{dY_n}{dw_n}\right) + \frac{\theta}{1-\theta}\left\{dp_f\left[Y_f - \frac{dY_f}{dw_f}\left(\frac{w}{p_f}\right)\right] + dp_e\left[Y_e - \frac{dY_e}{dw_e}\left(\frac{w}{p_e}\right)\right]\right\}$$

Barry (1997) proceeds similarly up to this point. However, he uses the assumption that $p_e = p_f = p_n$ to derive a non-tradable price equation in terms of wages and British prices in his model. Consequently, the accuracy of the model depends on a PPP relationship holding between sectoral prices. This is not necessary. The model already has a sufficient number of assumptions to derive a non-tradable equation. Here, a non-tradable price equation is derived without a sectoral PPP assumption, thereby making the model more widely applicable.

In the previous section, it was found that $\epsilon(Y_i, w_i) = 1 + \epsilon(L_i, w_i)$. If sectoral labour demand elasticities equal -1, $\epsilon(Y_i, w_i) = 0$, and thus sectoral real wage changes have no effect on sectoral output. This is an unrealistic scenario and therefore labour demand elasticities are restricted so that they cannot equal -1. In this case, $dY_i/dw_i \neq 0$ . Using this result and the fact that $dY_i/dw_i = \epsilon(Y_i, w_i)(Y_i/w_i)$, the equation above can be written:

$$dp_n\left(Y_n - \epsilon\left(Y_n, w_n\right)\frac{Y_n}{w_n}\left[\frac{w}{p_n}\right]\right) = dw\left(\frac{\theta}{1-\theta}\left\{\epsilon\left(Y_f, w_f\right)\frac{Y_f}{w_f} + \epsilon\left(Y_e, w_e\right)\frac{Y_e}{w_e}\right\}\right) - dw\,\epsilon\left(Y_n, w_n\right)\frac{Y_n}{w_n} + \frac{\theta}{1-\theta}\left\{dp_f\left[Y_f - \epsilon\left(Y_f, w_f\right)\frac{Y_f}{w_f}\left(\frac{w}{p_f}\right)\right]\right\} + \frac{\theta}{1-\theta}\left\{dp_e\left[Y_e - \epsilon\left(Y_e, w_e\right)\frac{Y_e}{w_e}\left(\frac{w}{p_e}\right)\right]\right\}$$

This is equivalent to:

$$\frac{dp_n}{p_n}\left(p_nY_n - \epsilon\left(Y_n, w_n\right)p_nY_n\right) = \frac{dw}{w}\left(\frac{\theta}{1-\theta}\left\{\epsilon\left(Y_f, w_f\right)p_fY_f + \epsilon\left(Y_e, w_e\right)p_eY_e\right\}\right) - \frac{dw}{w}\left\{\epsilon\left(Y_n, w_n\right)p_nY_n\right\} + \left(\frac{\theta}{1-\theta}\right)\left[\frac{dp_f}{p_f}\left(p_fY_f - \epsilon\left(Y_f, w_f\right)p_fY_f\right)\right] + \left(\frac{\theta}{1-\theta}\right)\left[\frac{dp_e}{p_e}\left(p_eY_e - \epsilon\left(Y_e, w_e\right)p_eY_e\right)\right]$$

It was shown in the first proof that the nominal demand in the non-tradable sector is proportional to aggregate income $p_n Y_n = \theta M$. The constant of proportionality is the exponent on non-tradable goods in the utility function. Similarly, for the tradable sectors $p_f Y_f = \psi M$ and $p_e Y_e = \chi M$. Every term in the previous equation contains a nominal demand expression:

$$\frac{dp_n}{p_n}\left(\theta M - \epsilon\left(Y_n, w_n\right)\theta M\right) = \frac{dw}{w}\left(\frac{\theta}{1-\theta}\left\{\epsilon\left(Y_f, w_f\right)\psi M + \epsilon\left(Y_e, w_e\right)\chi M\right\} - \epsilon\left(Y_n, w_n\right)\theta M\right) + \left(\frac{\theta}{1-\theta}\right)\left[\frac{dp_f}{p_f}\left(\psi M - \epsilon\left(Y_f, w_f\right)\psi M\right) + \frac{dp_e}{p_e}\left(\chi M - \epsilon\left(Y_e, w_e\right)\chi M\right)\right]$$

Dividing accross by $\theta M$:

$$\frac{dp_n}{p_n}\left[1 - \epsilon\left(Y_n, w_n\right)\right] = \frac{dw}{w}\left(\frac{1}{1-\theta}\left\{\epsilon\left(Y_f, w_f\right)\psi + \epsilon\left(Y_e, w_e\right)\chi\right\} - \epsilon\left(Y_n, w_n\right)\right) + \frac{dp_f}{p_f}\left(\frac{\psi}{1-\theta}\right)\left[1 - \epsilon\left(Y_f, w_f\right)\right] + \frac{dp_e}{p_e}\left(\frac{\chi}{1-\theta}\right)\left[1 - \epsilon\left(Y_e, w_e\right)\right]$$

The tradable sectors of the economy are considered to be broadly similar. Specifically, they have the same labour shares and the same labour demand elasticities. This means that $\epsilon\left(Y_e, w_e\right) = \epsilon\left(Y_f, w_f\right)$. This allows the equation to be simplified even further:

$$\frac{dp_n}{p_n}\left[1 - \epsilon\left(Y_n, w_n\right)\right] = \frac{dw}{w}\left[\epsilon\left(Y_f, w_f\right) - \epsilon\left(Y_n, w_n\right)\right] + \left[\frac{1 - \epsilon\left(Y_f, w_f\right)}{1-\theta}\right]\left\{\chi\,\frac{dp_e}{p_e} + \psi\,\frac{dp_f}{p_f}\right\} \quad (15)$$

In the last derivation, it was seen that the assumption that capital stocks fully adjust means that $\epsilon(Y_i, w_i) = 1 + \epsilon(L_i, w_i)$. Making this substitution into the equation above means that changes in non-tradable prices can be expressed:

$$\frac{dp_n}{p_n} = \left[1 - \frac{\epsilon\left(L_f, w_f\right)}{\epsilon\left(L_n, w_n\right)}\right]\frac{dw}{w} + \left[\frac{\epsilon\left(L_f, w_f\right)}{(1-\theta)\,\epsilon\left(L_n, w_n\right)}\right]\left\{\chi\,\frac{dp_e}{p_e} + \psi\,\frac{dp_f}{p_f}\right\}$$

This is equation (10) from the body of the paper.

5. *Derivation of equation (11)*

Equation (11) again finds the derivative of equation (9) but under the assumption that capital stocks are fixed. This means that $\hat{K} = 0$. Examining the Cobb-Douglas production function with capital fixed:

$$\begin{aligned} Y &= K^{\alpha} L^{1-\alpha} \\ \Rightarrow \hat{Y} &= (1-\alpha)\,\hat{L} \\ \Rightarrow \frac{dY_i}{Y_i} &= s_i \frac{dL_i}{L_i} \end{aligned}$$

where $s_i$, the sectoral labour share, equals $1 - \alpha$. Therefore,

$$\epsilon(Y_i, w_i) = s_i \epsilon(L_i, w_i)$$

Substituting this expression for the output elasticites into equation (15):

$$\frac{dp_n}{p_n}\left[1 - s_n \epsilon\left(L_n, w_n\right)\right] = \frac{dw}{w}\left[s_f \epsilon\left(L_f, w_f\right) - s_n \epsilon\left(L_n, w_n\right)\right] + \left[\frac{1 - s_f \epsilon\left(Y_f, w_f\right)}{1-\theta}\right]\left\{\chi\,\frac{dp_e}{p_e} + \psi\,\frac{dp_f}{p_f}\right\}$$

This means that non-tradable prices in the fixed capital stock case can be expressed:

$$\frac{dp_n}{p_n} = \left[\frac{s_f \epsilon\left(L_f, w_f\right) - s_n \epsilon\left(L_n, w_n\right)}{1 - s_n \epsilon\left(L_n, w_n\right)}\right]\frac{dw}{w} + \left[\frac{1 - s_f \epsilon\left(Y_f, w_f\right)}{(1-\theta)\left[1 - s_n \epsilon\left(L_n, w_n\right)\right]}\right]\left\{\chi\,\frac{dp_e}{p_e} + \psi\,\frac{dp_f}{p_f}\right\}$$

This is equation (11) from the body of the paper.

Table 1: Parameters of Calibrated Model

| Parameter | Value | Symbol |
|---|---|---|
| Share of E-goods in the HICP | 0.127 | $\chi = \epsilon(P, p_e)$ |
| Share of F-goods in the HICP | 0.494 | $\psi = \epsilon(P, p_f)$ |
| Share of N-goods in the HICP | 0.379 | $\theta = \epsilon(P, p_n)$ |
| E share in tradable employment | 0.373 | $L_e/L_t$ |
| F share in tradable employment | 0.627 | $L_f/L_t$ |
| Tradable share in total employment | 0.300 | $L_t/L$ |
| Non-tradable share in total employment | 0.700 | $L_n/L$ |
| Total employment | 1 644 000 | L |
| Labour elasticity in E sector | -0.6 | $\epsilon(L_e, w_e)$ |
| Labour elasticity in F sector | -0.6 | $\epsilon(L_f, w_f)$ |
| Labour elasticity in N sector | -0.8 | $\epsilon(L_n, w_n)$ |