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NEGOTIATION-PROOF CORRELATED EQUILIBRIUM

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Abstract: This article characterizes the set of correlated equilibria that result from open negotiations, which players make prior to playing a strategic game. A negotiation-proof correlated equilibrium is defined as a correlated strategy in which the negotiation process among all of the players prevents the formation of any improving coalitional deviation. Additionally, this notion of equilibrium is adapted to general games with incomplete information.

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1. Introduction

The study of pre-play communication in non-cooperative games has given rise to a number of interesting equilibrium notions, many of which can be categorized into two main areas of inquiry. Aumann (1959) initiated one area in which pre-play non-binding agreements are allowed, but there was not an explicit modeling of the communication process. In this setup, an agreement that is not subject to a beneficial deviation by any conceivable coalition of players is called a "strong Nash equilibrium". However, the fact that these deviations may not be stable against further coalitional deviations has lead to much criticism. To this end, Bernheim et al. (1987) proposed a coalition-proof Nash equilibrium. According to its definition, a coalition-proof agreement is one that is immune to self-enforcing deviations, or deviations for which no further beneficial deviations are available for any conceivable subset of the coalition. This notion has been studied extensively in the literature regarding strategic games, and some interesting applications of coalition-proof Nash equilibrium can be found in Bernheim and Whinston (1987).

Despite its applicability, coalition-proof Nash equilibrium, like strong-Nash equilibrium, was not immune to criticism. One major criticism is that the self-enforceability of the deviations is restricted only to the proper subsets of the deviating coalition and prohibits the non-members from participating in further attempts to block the plans. Another criticism is that coalition-proof Nash equilibrium ignores the players' foresight, or their ability to consider the consequences of their agreements. In other words, such an agreement completely overlooks the ability of the members of a coalition to see that a deviation, which at first may not lead to an improvement, can lead other players to further act in a way that leads to an increase in the payoffs for all the members of the initial coalition. To overcome these deficiencies, Xue (2000) introduced negotiation-proof Nash equilibrium (NPNE). To produce this equilibrium, all players voluntarily participate in an open negotiation process in which the members of a coalition openly announce their joint intention to play specific actions. Then, another coalition of players, which does not necessarily include only players from the initial coalition, can further deviate by openly announcing new strategies for its players. This process continues until there are no more positive pay-off deviations for any coalition. Additionally, the Nash equilibrium profile of the strategies is said to be negotiation-proof if and only if no coalition can implement a blocking sequence, which leads to another negotiation-proof Nash equilibrium that benefits all of its members. Hence, this equilibrium allows the non-members of a blocking coalition to participate as well as accounting for blocking deviations and allowing for the perfect foresight of rational players when they confer to determine whether a deviation will ultimately lead to increased payoffs. Different approaches to farsighted coalitional stability also appear in Greenberg (1989, 1990), Mariotti (1994) and Xue (1998).

The work of Aumann (1974, 1987) created the second area of inquiry. In this literature, instead of communicating in person, the players can use the recommendations of a correlation device (a mediator) that sends private extraneous signals to them, which allows the players to correlate their strategies in a way that yields beneficial agreements. Any agreement in which no player will decide to disobey the mediator and in which the other players will follow their recommendations is called a "correlated equilibrium". This notion has also been studied extensively, and one of its appealing features is that in all games other than the strategically zero-sum games of Moulin and Vial (1978), any Nash equilibrium payoff can be improved upon using correlated strategies.

In the mid-90s, a series of papers attempted to blend the two approaches by considering games that allow players to use a mediator to correlate their strategies but to also have the opportunity to form coalitions and deviate from the recommendations. The papers of Ray (1996), Moreno and Wooders (1996) and Milgrom and Roberts (1996) each consider a framework in which coalitions are allowed to plan deviations from a given profile of correlated strategies at the ex-ante stage, or the stage before the mediator announces his private recommendations to each player. In contrast, in the works of Einy and Peleg (1995), Ray (1998) and, more recently, Bloch and Dutta (2009), players are allowed to form coalitions at the interim stage, or the stage after receiving the private signals of the mediator and prior to play. Finally, Heller (2010) allows for different timing in the recommendations of the mediator; consequently, players that have different levels of information can form blocking coalitions. In all of these papers, we can find (different) notions of a strong correlated equilibrium (SCE) and a coalition-proof correlated equilibrium (CPCE).

In this paper, we study situations in which players are allowed to form coalitions and to block the recommendations of the mediator; however, any such deviations have to be stable in terms of open negotiations among the players. In other words, we introduce the concept of a negotiation-proof correlated equilibrium (NPCE), which occurs when players openly negotiate to determine which correlated equilibrium cannot be ultimately blocked by an improving stable blocking sequence. If a correlated equilibrium is indeed negotiation-proof, then the mediator should recommend strategies that implement this equilibrium. The idea of NPCE was actually suggested in the concluding remarks section in Xue (2000). However, our approach considers an alternative that does not employ von Neumann and Morgenstern's (1944) 'abstract stable set'. The main result of this paper is that if the strategic game has a correlated strategy that weakly Pareto dominates any other correlated strategy, this correlated strategy is the unique NPCE of the game. Finally, the open negotiation process is adapted to a setting in which players are of different privately known types; hence, we define the negotiation-proof correlated strategies for the games with incomplete information.

The main reason for analyzing this subject is because the various notions of CPCE also suffer from the 'nestedness' restriction on the formation of further deviations and the myopia of players in the notion of a coalition-proof Nash equilibrium. The approach proposed in this paper rectifies these deficiencies and also provides interesting equilibrium outcomes to well-known strategic games. In addition, we will use a number of examples to present some interesting characteristics of NPCE and the difference between it, SCE and CPCE. Hence, this attempt complements the papers mentioned in the previous paragraphs.

In the following section, we develop the context and several equilibrium notions. Next, we proceed to prove some results and present some examples that highlight the differences between the various equilibrium notions. In section 3, we study the negotiation-proof agreements in games with incomplete information. The concluding remarks and possible extensions are in the last section.

2. Model and definitions

In the first part of the paper, we combine the idea of the pre-play negotiation found in Xue (2000) with the theory of a correlated equilibrium. This approach will leads to the definition of a negotiation-proof correlated equilibrium. In our game, open negotiation among the players takes place before they receive the recommendations of the mediator. Moreover, the blocking coalitions are allowed to correlate their actions by employing new correlation devices. In this way, we follow closely the methodology of Moreno and Wooders (1996), which serves as a basis for our study.

Consider the game $G = (N, (A_i)_{i \in N}, (u_i)_{i \in N})$, where N is the set of players. The finite set of strategies of each player $i \in N$ is given by A_i, with a_i being the generic element and $A = \prod_{i \in N} A_i$ being the Cartesian product of the individual strategy sets. The utility of player i is given by $u_i : A \to \Re$.

Before the game is played, the mediator sends a private signal a_i to each player i, based on the probability distribution μ. Thus, a correlated strategy μ is a probability distribution over A. Additionally, let ΔA denote the set of probability distributions over A. The expected utility from obeying the correlated strategy μ is given by

$U_i(\mu) = \sum_{a \in A} \mu(a) u_i(a)$.

At the ex-ante stage (before the mediator sends the private signals a_i), players communicate and possibly plan deviations against their recommendations. According to the description of Moreno and Wooders (1996), the process takes place as if the deviating coalition employs a new mediator who received the undisclosed recommendations of the initial mediator and recommends a new correlated strategy to its members. Hence, a blocking plan against the correlated strategy μ for a coalition is an agreement to correlate their actions in a way different than prescribed by μ. For a coalition of players $S \subset N$ let $A_S = \prod_{i \in S} A_i$, and let$-S = \{i \in N | i \notin S\}$ be the complementary coalition and a blocking plan is defined as a mapping $\eta_S : A_S \to \Delta A_S$, which allows for the assignment of a correlated strategy over A_S to any possible recommendation a_S. The probability distribution over these actions is

$$\hat{\mu}(a) = \sum_{a_S \in A_S} \mu(a_S, a_{-S}) \eta(a_S | a_S)$$

and the expected utility of a blocking plan is

$$U_i(\eta_S) = \sum_{a \in A} \hat{\mu}(a) u_i(a).$$

Given a correlated equilibrium μ, the set of feasible blocking plans of coalition S is denoted by $F(S, \mu)$. The definition of a correlated equilibrium is as follows.

Definition 1. *A correlated strategy μ is a correlated equilibrium if no player $i \in N$ has a deviation $\hat{\mu} \in F(i, \mu)$ such that $U_i(\hat{\mu}) > U_i(\mu)$.*

In other words, a correlated equilibrium is a correlated strategy if no player has a feasible improving blocking plan, given that all other players follow their recommendations. A strong correlated equilibrium can now be defined, as a correlated strategy for which no coalition of players has a feasible improving blocking plan, given that the non-members of the coalition follow their recommendations.

Definition 2. *A correlated strategy μ is a strong correlated equilibrium (SCE) if a coalition S and a blocking plan η_S such that $U_i(\eta_S) > U_i(\mu)$ for every $i \in S$ does not exist.*

Like the notion of a strong Nash equilibrium, the notion of a strong correlated equilibrium also fails to take into account the possible sub-coalitions to be formed among the members of the coalition S to impose further blocking plans. To take this into account we define the blocking plan of a coalition S against μ as self-enforcing if no proper sub-coalition of S has a further self-enforcing improvement deviation. Obviously, any blocking plan by a one-player coalition is self-enforcing. In general, for coalitions of two or more players we have the following definition.

Definition 3. *Given the correlated strategy* μ, *a blocking plan* $\eta_S \in F(S,\mu)$ *for the coalition* S, *generating a distribution* $\hat{\mu}$, *is self-enforcing if there exists no coalition* $T \subset S$ *and a self-enforcing blocking plan* $\eta_T \in F(T,\hat{\mu})$ *generating a distribution* $\tilde{\mu}$ *such that* $U_i(\tilde{\mu})\ ) > U_i(\hat{\mu})$ *for all* $i \in T$.

Definition 4. *A correlated strategy* μ, *is a coalition-proof correlated equilibrium (CPCE) if there exist no coalition* S *and a self-enforcing blocking plan* η_S *such that* $U_i(\eta_S) > U_i(\mu)$ $\forall i \in S$.

The interpretation of this definition is that if a correlated strategy is coalition-proof, then this equilibrium is implemented by the strategies that the mediator privately recommends to the players. The new equilibrium notion introduced in this paper does not allow for private communication and all blocking agreements are common knowledge. Now, to define the NPCE, we need to examine blocking sequences. Blocking sequences are needed to the cope with the nestedness restriction and the myopia of players and because an initial blocking plan has to be immune to further deviations created by any conceivable coalition that may be beneficial at subsequent stages of the negotiation process.

A blocking sequence is denoted by $B_S = \{(S^k,\eta_k)\}_{k=1}^{K}$, where η_k is a feasible blocking plan for the coalition S^k at stage k. Moreover, a blocking sequence is termed stable if no other coalition, not necessarily consisting of only members from S, can counter-block this deviation with another stable blocking sequence.

Definition 5. *A blocking sequence* $B_S = \{(S^k,\eta_k)\}_{k=1}^{K}$ *of coalition* S *to the correlated strategy* μ *is stable, if there is no other coalition* T *and a stable blocking sequence* $B_T = \{(T^j,\eta_j)\}_{j=1}^{K+J}$ *to the correlated strategy* η_K, *such that* $U_i(\eta_J) > U_i(\eta_K)$ *for all* $i \in T$ *and* $U_i(\eta_J) < U_i(\mu)$ *for some* $i \in S$.

Finally, a negotiation-proof correlated equilibrium is a correlated strategy that cannot be blocked by a profitable blocking sequence.

Definition 6. *A correlated strategy* μ *is a negotiation-proof correlated equilibrium (NPCE) if there exists no coalition* S *and a stable blocking sequence* $B_S = \{(S^k,\eta_k)\}_{k=1}^{K}$, *such that* $U_i(\eta_K) > U_i(\mu)$ *for each* $i \in S$.

Some remarks are now in order. First, who makes the proposals within a coalition is not important because no rational farsighted player will make or accept an offer that would eventually lead to lower payoffs. Second, it is obvious that the mediator can only recommend a correlated strategy from the set of NPCE. Otherwise a coalition of players will benefit by not following the recommendations. However, if the set of NPCE contains more than one element, then the correlated strategy that will be recommended is not specified. Third, it is clear that a correlated equilibrium will not be negotiation-proof if it is ultimately blocked by another NPCE. Finally, the definition of a blocking sequence is intrinsically *circular* because it potentially allows for an infinite sequence of improving correlated strategies. In the sequel, we will define a game as being strictly acyclic if no infinite sequence of improving correlated strategies is possible.

Next we present a result that complements some of the results in the literature. Xue (2000) proved that if a game has a Nash equilibrium that weakly-dominates any other Nash equilibrium, then it is the unique negotiation-proof Nash equilibrium. Moreno and Wooders (1996) proved that if a game has a correlated equilibrium that weakly-dominates any other correlated equilibrium, then it is a CPCE. The following proposition establishes that if μ is a correlated strategy that weakly Pareto-dominates every other correlated strategy, i.e., $U_i(\mu) \geq U_i(\hat{\mu})$ for each $i \in N$, then μ is the unique NPCE.

PROPOSITION 1. *Let G be a strictly acyclic strategic game and $\mu \in \Delta A$ be a correlated equilibrium that weakly Pareto-dominates every other $\hat{\mu} \in \Delta A$. Then μ is the unique NPCE.*

Proof. Let $\mu \in \Delta A$ be a correlated equilibrium that weakly Pareto-dominates every other $\hat{\mu} \in \Delta A$, i.e., $U_i(\mu) \geq U_i(\hat{\mu})$ for each $i \in N$. Suppose that μ is not a NPCE. Then there is a coalition S and a stable blocking sequence $B = \{(S^k, \eta_k)\}_{k=1}^{K}$, such that $U_i(\eta_K) > U_i(\mu)$ for each $i \in S$, a fact that contradicts the Pareto-dominance of μ. Hence, μ forms a NPCE. The strategy is unique because there are no other correlated strategies $\tilde{\mu} \in \Delta A$ that are immune to the stable deviation of the grand coalition playing the weakly Pareto-dominant correlated strategy μ. □

Correlated play arises naturally when players are allowed to communicate. The correlation of an action leads to, on the one hand, an expansion of the set of strategies, but, on the other hand, it increases the blocking abilities of the coalitions. The following example illustrates a three-player game that admits a NPCE but has no NPNE, which is the equilibrium notion described in Xue (2000).

Example 1. *Consider the following three-player game, where player* 1 *chooses rows* (a_1, a_2), *player* 2 *chooses columns* (b_1, b_2) *and player* 3 *chooses matrices* (c_1, c_2).

c_1:

	b_1	b_2
a_1	$1,2,3$	$0,0,0$
a_2	$0,0,0$	$2,3,1$

c_2:

	b_1	b_2
a_1	$0,0,0$	$3,1,2$
a_2	$0,0,0$	$0,0,0$

This game admits no NPNE; however we will demonstrate that it allows for an NPCE. A correlated strategy for this game is a vector $\mu = (\mu_{ijk})_{i,j,k \in \{1,2\}}$, where $\mu_{ijk} \geq 0$ denotes the probability that players $1, 2$ and 3 are recommended actions a_i, b_j, c_k, respectively. Consider the following correlated strategy

c_1:

	b_1	b_2
a_1	$1/3$	0
a_2	0	$1/3$

c_2:

	b_1	b_2
a_1	0	$1/3$
a_2	0	0

(i.e., $\mu_{111} = 1/3$, $\mu_{221} = 1/3$ and $\mu_{122} = 1/3$), which forms a correlated equilibrium and results to expected utilities $u_i(\mu) = 2$. Moreover, μ cannot be blocked by any two-player coalition. Consider, for instance, the coalition $\{1, 2\}$ that deviates from this correlated strategy by choosing action a_2 and b_2, respectively. These actions yield the expected utilities $u_1(\hat{\mu}) = \frac{1}{3}2 + \frac{1}{3}2 < 2$ and $u_2(\hat{\mu}) = \frac{1}{3}3 + \frac{1}{3}3 = 2$. Thus, the coalition is not improving and is never formed. Similar arguments can be made for the deviations of coalitions $\{1, 3\}$ and $\{2, 3\}$.

2.1. **Relation between NPCE and CPCE.** Despite the close relationship between CPCE and NPCE there is no inclusion between the two notions. The following examples illustrate this point. Example 2 presents a three-player game where a CPCE is not a NPCE, whereas examples 3 and 4 present three-player games with no CPCE that admit an NPCE.

Example 2. *Consider the following three-player game, where player* 1 *chooses rows* (a_1, a_2), *player* 2 *chooses columns* (b_1, b_2) *and player* 3 *chooses matrices* (c_1, c_2).

c_1:

	b_1	b_2
a_1	$2,2,2$	$0,0,0$
a_2	$0,0,1$	$1,1,0$

c_2:

	b_1	b_2
a_1	$1,1,0$	$0,0,0$
a_2	$0,0,0$	$3,3,1$

As in the previous example, a correlated strategy for this game is a vector $\mu = (\mu_{ijk})$. Consider the correlated strategy $\mu_{111} = 1$. This correlated strategy forms a correlated equilibrium and because no coalition of two or three players can create profitable deviations, it also forms a SCE and a CPCE. However, $\mu_{111} = 1$ is not a NPCE. To see that, consider the coalition $\{1, 2\}$, which blocks the correlated strategy by deviating to (a_2, b_2, c_1). Initially, this blocking plan does not improve for players' utility; however it induces player 3 to further deviate to (a_2, b_2, c_2), which is stable against any deviation, and leads to an increase in the utilities for players 1 and 2.

Example 3. *Consider the following three-player game, where player* 1 *chooses rows* (a_1, a_2), *player* 2 *chooses columns* (b_1, b_2) *and player* 3 *chooses matrices* (c_1, c_2).

c_1:

	b_1	b_2
a_1	$2,2,2$	$0,0,0$
a_2	$0,0,0$	$3,3,0$

c_2:

	b_1	b_2
a_1	$0,0,0$	$0,0,0$
a_2	$0,0,0$	$1,1,1$

First, we show that this game has no SCE and no CPCE. Let μ be an arbitrary correlated equilibrium, yielding an expected utility of $u_i(\mu) = 2\mu_{111} + 3\mu_{221} + \mu_{222}$ for players $i = 1, 2$. The coalition of players $\{1, 2\}$ deviates by choosing actions a_2 and b_2, respectively, which results in an increase in the expected utilities that are equal to $u_i(\hat{\mu}) = 3\mu_{111} + 3\mu_{221} + \mu_{222}$ for $i = 1, 2$. Hence, μ is not a SCE; moreover, this deviation is self-enforcing because neither player 1 nor player 2 can further deviate. Therefore, μ is also not a CPCE.

However, this game admits a NPCE. Consider the correlated strategy $\hat{\mu}_{111} = 1$. This strategy is a NPCE because the only profitable deviation {1,2} who play a_2 and b_2, respectively, leads player 3 to further deviate to $(a_2,\ b_2, c_2)$, yielding utilities equal to

1 for each player. Hence, the blocking coalition {1,2} is never formed, and as a result, $\hat{\mu}_{111} = 1$ is a NPCE.

The next example has been extensively used in the literature and it appears in Einy and Peleg (1995), Moreno and Wooders (1996), Ray (1996) and Bloch and Dutta (2009).

Example 4. *Consider the following three-player game, where player* 1 *chooses rows* (a_1, a_2)*, player* 2 *chooses columns* (b_1, b_2) *and player* 3 *chooses matrices* (c_1, c_2)*.*

c_1	b_1	b_2
a_1	$3,2,0$	$0,0,0$
a_2	$2,0,3$	$2,0,3$

c_2	b_1	b_2
a_1	$3,2,0$	$0,3,2$
a_2	$0,0,0$	$0,3,2$

The game has no CPCE (as proved in Moreno and Wooders (1996)) but there is a NPCE. Consider the following correlated strategy μ

c_1	b_1	b_2
a_1	1/3	0
a_2	0	1/3

c_2	b_1	b_2
a_1	0	1/3
a_2	0	0

(i.e., $\mu_{111} = 1/3$, $\mu_{221} = 1/3$ and $\mu_{122} = 1/3$), which forms a correlated equilibrium and results in the expected utilities $u_i(\mu) = 5/3$ for $i = 1, 2, 3$. For this correlated strategy, three distinct improving coalitions can be formed. For example, if the coalition $\{1, 3\}$ deviates from μ by choosing actions a_2 and c_1, the expected utilities $u_1(\hat{\mu}) = 2$ and $u_3(\hat{\mu}) = 3$ are improved. However, $\hat{\mu}$ is blocked by coalition $\{1, 2\}$, which in turn is blocked by coalition $\{2, 3\}$, which in turn is again blocked by $\{1, 3\}$ using $\hat{\mu}$, which yields an unending series of coalitional deviations. Similar arguments can be made for the deviations of coalitions $\{1, 2\}$ and $\{2, 3\}$. Therefore, the correlated strategy μ forms a NPCE.

It should be noted that the last example highlights the intrinsic circularity in the definition of a blocking sequence because any deviation leads to an infinite sequence of improving deviations.

3. Games with incomplete information

Consider the game of incomplete information $G = (N, (T_i)_{i \in N}, (A_i)_{i \in N}, (p_i)_{i \in N}, (u_i)_{i \in N})$, where N is the set of players and T_i is the set of possible types for each player $i \in N$ and

$T = \prod_{i \in N} T_i$ is the Cartesian product of individual type sets. The finite set of strategies of each player is given by A_i, and $A = \prod_{i \in N} A_i$. Players i's probability distribution over the set of types of other players is denoted by $p_i : T_i \to \Delta T_{-i}$, where $T_{-i} = \prod_{j \in N \setminus \{i\}} T_j$. The utility of player i is given by $u_i : T \times A \to \Re$. For a coalition of players $S \subset N$, let $A_S = \prod_{i \in S} A_i$, $T_S = \prod_{i \in S} T_i$ and let $-S = \{i \in N | i \notin S\}$ denotes the complementary coalition.

Before the game is played, the mediator sends a private signal a_i to each player i, according to the probability distribution μ. Thus, a correlated strategy μ is a probability distribution over A. ΔA denotes the set of the probability distributions over A. The expected utility from obeying the correlated strategy μ is given by

$$U_i(\mu | t_i) = \sum_{t_{-i} \in T_{-i}} \sum_{a \in A} p_i(t_{-i} | t_i) \mu(a|t) u_i(a, t).$$

A coalition S deviates by employing a new mediator who, after receiving the reports on the types of the members of S, reports to the initial mediator a type profile for the coalition according to $f_S : T_S \to \Delta T_S$. Then, after receiving the recommendation of the initial mediator, the new mediator selects an action profile for each member of S according to $\eta_S : T_S \times A_S \to \Delta A_S$. The probability distribution over these actions is

$$\tilde{\mu}(a|t) = \sum_{\tau_S \in T_S} \sum_{a_S \in A_S} f_S(\tau_S | t_S) \tilde{\mu}(a_S, a_{-S} | \tau_S, t_{-S}) \eta(a_S | a_S, \tau_S, t_{-S})$$

Given a correlated equilibrium μ the set of feasible blocking plans of coalition S is denoted by $F(S, \mu)$. The definition of a Pareto dominant correlated strategy is as follows.

Definition 7. *A correlated strategy $\tilde{\mu} \in F(S, \mu)$ Pareto dominates the correlated strategy μ if*

(i) $U_i(\tilde{\mu} | t_i)\ ) \geq U_i(\mu | t_i)$ for each $i \in S$ and each $t_i \in T_i$ and

(ii) $U_i(\tilde{\mu} | \tilde{t}_i)\ ) > U_i(\mu | \tilde{t}_i)$ for each $i \in S$ and some $\tilde{t}_i \in T_i$.

Using the definition in Forges (1986) a communication equilibrium is a correlated strategy in which no player benefits by deviating, given that all other players follow their recommendations.

Definition 8. *A correlated strategy μ is a communication equilibrium if no player $i \in N$ has a deviation $\hat{\mu} \in F(i, \mu)$ that Pareto dominates μ.*

In view of that, the strong communication equilibrium is defined accordingly.

Definition 9. *A correlated strategy μ is a strong communication equilibrium (ScE) if there exists no coalition S and a blocking plan η_S that Pareto dominates μ.*

The deviating correlated strategy $\hat{\mu}$ of a coalition S from μ is self-enforcing if no proper sub-coalition of S has a further self-enforcing and improving deviation. Obviously, any blocking plan by a one-player coalition is self-enforcing. In general, for a coalition of two or more players we have the following definition.

Definition 10. *Given the correlated strategy μ, a blocking plan $\eta_S \in F(S,\mu)$ for the coalition S generating a distribution $\tilde{\mu}$ is self-enforcing if there exists no coalition $V \subset S$ and a self-enforcing blocking plan $\eta_T \in F(V,\tilde{\mu})$ that generates a distribution $\hat{\mu}$ such that $U_i(\hat{\mu})\ ) > U_i(\tilde{\mu})$ for all $i \in V$.*

Definition 11. *A correlated strategy μ is a coalition-proof communication equilibrium (CPcE) if there exists no coalition S and a self-enforcing blocking plan η_S that Pareto dominates μ.*

To define a negotiation-proof communication equilibrium (NPcE) we must examine blocking sequences because an initial blocking plan has to be immune to sequential deviations that may be beneficial to any conceivable coalition during the subsequent stages of the negotiation process. A blocking sequence is denoted by $B = \{(S^k,\eta_k)\}_{k=1}^K$, where η_k is a feasible blocking plan for the coalition S^k at stage k. Moreover, a blocking sequence is termed stable if no other coalition can counter-block this deviation with another stable blocking sequence.

Definition 12. *A blocking sequence $B_S = \{(S^k,\eta_k)\}_{k=1}^K$ of coalition S to the correlated strategy μ is stable, if there is no other coalition V and a stable blocking sequence $B_V = \{(V^j,\eta_j)\}_{j=1}^{K+J}$ that Pareto dominates η_K.*

Finally, the next definition identifies a correlated equilibrium as a correlated strategy that cannot be blocked by an improving stable blocking sequence.

Definition 13. *A correlated strategy μ is a negotiation-proof communication equilibrium (NPcE) if there exists no coalition S and a stable blocking sequence $B_S = \{(S^k,\eta_k)\}_{k=1}^K$ that Pareto dominates μ.*

The next example emphasizes the distinction between NPcE and CPcE. In particular, it presents a correlated strategy that is not a CPcE but it forms an NPcE and another correlated strategy which is a CPcE but not an NPcE.

Example 5. Consider the following three-player game, where player 1 has two possible types $\{H_1, T_1\}$ and no actions, player 2 has a single possible type and chooses rows (H_2, T_2) and player 3 has a single possible type and chooses columns (H_3, T_3). The priors over player 1's types are $p_2(H_1) = p_3(H_1) = 1/2$.

	H_3	T_3
H_2	$1, 1, -2$	$-1, -1, 2$
T_2	$-1, -1, 2$	$-1, -1, 2$

$t_1 = H_1$

	H_3	T_3
H_2	$-1, -1, 2$	$-1, -1, 2$
T_2	$-1, -1, 2$	$1, 1, -2$

$t_1 = T_1$

Consider the correlated strategy μ given by $\mu(H_2, T_3|H_1) = 1$ and $\mu(T_2, H_3|T_1) = 1$. This strategy is a correlated equilibrium and yields the expected utilities of $U_1(\mu|H_1) = U_1(\mu|T_1) = -1$, $U_2(\mu) = -1$ and $U_3(\mu) = 2$. As shown in Moreno and Wooders (1996) μ is not a CPcE because the coalition $\{1, 2\}$ can form a profitable deviation. The improving blocking plan is as follows: player 1 does not report his true type and player 2 plays T_2 when recommended H_2 and plays H_2 when recommended T_2. The correlated strategy $\tilde{\mu}$ that arises is given by $\tilde{\mu}(H_2, H_3|H_1) = 1$ and $\tilde{\mu}(T_2, T_3|T_1) = 1$. The expected utilities from $\tilde{\mu}$ are $U_1(\mu|H_1) = U_1(\mu|T_1) = 1$, $U_2(\mu) = 1$ and $U_3(\mu) = -2$. However, μ is an NPcE of this game because the correlated strategy $\tilde{\mu}$ is blocked by player 3 who can further deviate by playing T_3 when recommended H_3 and by playing H_3 when recommended T_3. The correlated strategy that arises is $\hat{\mu}$ with $\hat{\mu}(H_2, T_3|H_1) = 1$ and $\hat{\mu}(T_2, H_3|T_1) = 1$, and the expected utilities are $U_1(\mu|H_1) = U_1(\mu|T_1) = -1$, $U_2(\mu) = -1$ and $U_3(\mu) = 2$. Thus, the coalition $\{1, 2\}$ is never formed.

Finally, Moreno and Wooders (1996) illustrated that the game admits a CPcE, where player 2 plays H_2 when player 1 is of type H_1, player 2 plays T_2 when player 1 is of type T_1, and player 3 plays H_3 with the probability of $1/2$ for both types of player 1. This correlated strategy yields the expected utilities equal to zero for each player. However, it is not an NPcE because player 3 blocks this correlated strategy with the correlated strategy μ described above, which forms an NPcE.

4. Concluding remarks

This article is another study in the formation of coalitions within games that include communication, and it provides new insights to well-known strategic games. In the framework considered here, the proposed profiles of the correlated strategies can be objected by any conceivable coalition, and only those that cannot be discarded by the open negotiation among rational farsighted players can be admitted as equilibrium. Finally, it should be noted that the model of the ideas presented in this paper could be complemented by considering the possibility that the negotiation among the players and the formation of the coalitions occurs upon the reception of the recommendations of the correlating device.

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