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Fitting Broadband Diffusion by Cable Modem in Portugal

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Abstract The purpose of this article is to described the evolution of the number of residential subscribers of broadband fixed access by cable modem, in Portugal, on the period from 2000–2009. The pattern of evolution is estimated by fitting several models to the series, namely the following: exponential, Gompertz, Logistic, Bass and Michaelis-Menten. We fit the models to the data by nonlinear least squares, except in the exponential model where the linear version is fitted by ordinary least squares, using the internet freely available program R. This comparative study is in line with many others on the diffusion of technological innovations in the telecommunications sector, where the point is finding out if there is an early or a late take-off phenomenon. The Michaelis-Menten model is introduced for the first time in this approach. It allows to predict the later evolution in the series and reveals a qualitatively different behavior.

1 Introduction

Broadband Internet access is an infrastructure indispensable to a society of information and knowledge. Its development is stimulated by successive processes of technological innovations in the telecommunications sector. The benefits of expanding the access structure to high-speed Internet are recognized either by firms and countries, as is recognized in OECD studies. We can find in *[6]* a descriptive analysis of the penetration of broadband in Portugal, where most technological platforms of fixed broadband access are ADSL and cable model.
The purpose of this article is to describe the evolution of the number of subscribers of broadband fixed access by cable modem, in Portugal, on the period from 2000–

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2009. The graphic representation suggest at first sight a S-shaped curve similar to those observed in many studies on the diffusion of technological innovations in the telecommunications sector. This behavior is appropriate for services based on a network where it is expected that the take-off occurs even later than for durable goods. The pattern of evolution is estimated by fitting several models to the series, namely the models based on exponential, Gompertz and Logistic functions and Bass model as other authors have done, for instance *[4]*, *[7]*, *[2]*. We also fit a Michaelis-Menten growth model which, to our knowledge, hasn't been used in diffusion modeling until now. Although this model seems to be less well fitted than most of the others (as will be seen), it is more effective in predicting the latest evolution in the sector. This model also adjusts better to describe the behavior in sectors where subscription bases previously constituted in other sectors may be used, as happens in services related to Internet such as cable model. In this case early take-off arises.

2 Data

We analyze a series referring to the number of non residential subscribers to the broadband access to the Internet by cable modem from 2000 to 2009 (obtained from Anacom).

In the data series figure (section 3), the number of subscribers is plotted against time. If there is a take-off it seems to happen from the fourth to the fifth observations, but that's not absolutely clear. The rhythm seems to slowdown at the final period, but that too is not beyond all doubt, as may be found from the last observation available (out of the sample), which is much larger than the previous ones.

3 The Models

Some diffusion models compatible with S-shaped form are used to fit the data. The pattern of evolution for each of the models is described through a differential equation in which the growth rate of the dependent variable is a function of the saturation level and the variable itself.

In the following table we present in the first column, the functional form and in the second one the corresponding differential equation for the growth models such as the exponential, logistic and Gompertz, Bass and Michaelis-Menten.

Originally, Bass and Michaelis-Menten models were introduced respectively in the study of the diffusion of durable goods (*[1]*) and in the study of the kinetics of enzymes in Chemistry. The last one is also used in growth models in Biology (see for instance *[3]*). As far as the authors are aware this model hasn't be used before to study the diffusion of goods or services.

In the exponential model, the growth rate is constant; there will be a saturation level only if that rate is negative. In the logistic model the growth rate depends on the

Table 1

Models	Functional Form	Differential Equation
Exponential	$y = e^{a+bt}$	$\frac{y'}{y} = b$
Logistic	$y = \frac{A}{1+e^{(v-t)/\sigma}}$	$\frac{y'}{y} = \sigma^{-1}\left(1-\frac{y}{A}\right)$
Gompertz	$y = Ae^{-ab^t}$	$\frac{y'}{y} = \ln b \ln\left(\frac{y}{A}\right)$
Bass	$y = A\frac{1-e^{-(p+q)t}}{1+(q/p)e^{-(p+q)t}}$	$\frac{y'}{y} = q\left(1-\frac{y}{A}\right)+p\left(\frac{A}{y}-1\right)$
Michaelis-Menten	$y = \frac{At}{k+t}$	$\frac{y'}{y} = k^{-1}\left(\frac{A}{y}-1\right)\left(1-\frac{y}{A}\right)$

proportion of potential subscribers which did not yet join the service. The Gompertz model can be written as a limit case $(\alpha \to +\infty)$ of the generalized logistic given by the differential equation:

$$\frac{y'}{y} = \alpha lnb\left(1-(\frac{y}{A})^{1/\alpha}\right) ,\alpha > 0$$

In the Bass model it is assumed that the population of potential subscribers is constituted by innovators and imitators. Therefore, a distinction holds between new adoptions of the product as a result of the influence of advertising and information (the innovation part to which is associated the coefficients p) and new adoptions resulting from the influence of others subscribers, word to mouth effect, (the imitation part to which is associated the coefficient q). Note that the logistic model can be seen as a particulary case of Bass model: if we make $p = 0$ and $q = 1/\sigma$ we obtain the differential equation of the logistic model, so this is a model without innovation part. In the Michaelis-Menten model the differential equation can be obtained from the Bass equation by setting $p = 1/kA$ and $q = 0$. In this model it's not easy to visually identity the take-off moment and, as y increases, the growth rate decreases faster than the logistic model. As we will see later these features are adequate for the characterization of a network base service where there are subscribers bases proceeding other services already founded. The estimated saturation level is much grater in this model than in the others.

4 Estimation

We fit the models to the data by nonlinear least squares except in the exponential model where the linear version is fitted by ordinary least squares. Estimations are made using the Internet freely available program R. In the estimation of the Bass model, we use Srinivasan-Mason method (*[8]*).

For each model we present the estimated coefficients and the corresponding value of the t-statistics, and the residual standard deviation.

Table 2

Models	Coefficients	T-statistic	Res. Standard Deviation
Exponential	$a = 8.3209$	46.195	7769.072
	$b = 0.06996$	8.017	
Logistic	$A = 25935.86$	64.66	1262
	$\nu = 10.27$	33.25	
	$\sigma = 4.19$	14.71	
Gompertz	$A = 26677.66$	71.47	941.9
	$a = 3.5948$	13.8	
	$b = 0.8502$	144.42	
Bass	$A = 24256.5$	4.801	1480.033
	$p = 0.039$	2.166	
	$q = 0.182$	1.896	
Michaelis-Menten	$A = 50985.67$	12.304	1640
	$k = 27.37$	6.766	

For all estimated coefficients, we have a negligible p-value except for the Bass model. At a p-value of 0.05, only the q is not significant, which confirms that the restriction $q = 0$ may be correct in this case.

The following points must be emphasized:

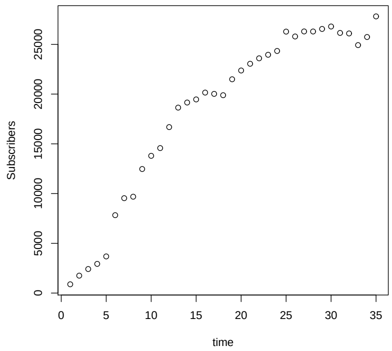
- In the exponential model there is a poor adjustment, as becomes evident by looking at the plot of this model: the increasing rhythm of growth clearly doesn't describe the data;
- There is a better adjustment in the other models, in particulary, Gompertz model;
- In the Bass model the estimated saturation level seems to be undervalued; this is consistent with the fact that the corresponding estimator is biased, which is pointed out in the literature;
- In the Michaelis-Menten model, thou the residual standard deviation is greater than in the others models, it seems to capture the long run behavior and provide a better prediction for the latest observations (first and second semesters of 2010) not included in the sample used in the estimation. See the next table:

Table 3

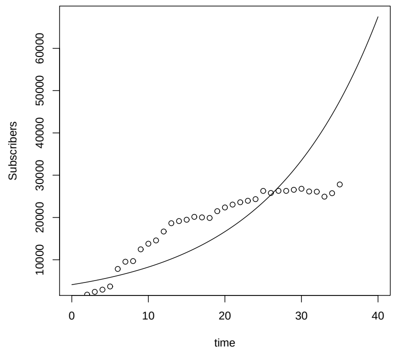
Models	First Semester of 2010	Second Semester of 2010
Exponential	58652.18	62902.43
Logistic	25901.52	25908.81
Gompertz	26477.22	26507.15
Bass	24225.04	24231.27
Michaelis-Menten	29638.3	29959.94
Observations	27846.00	32737.00

We present the joint plot of the estimated curves and the observations.

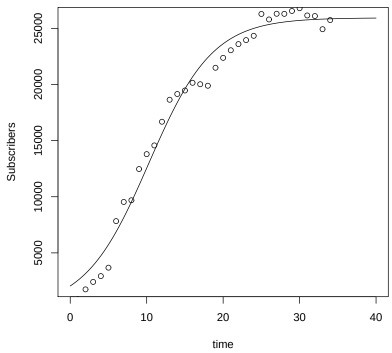
Data Series



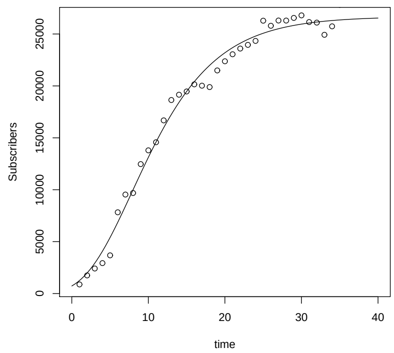
Exponential



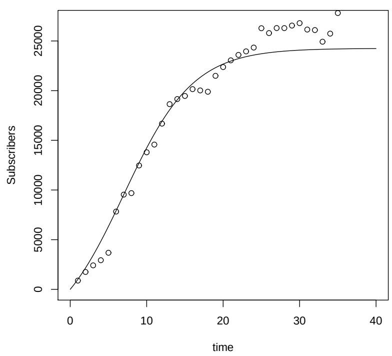
Logistic



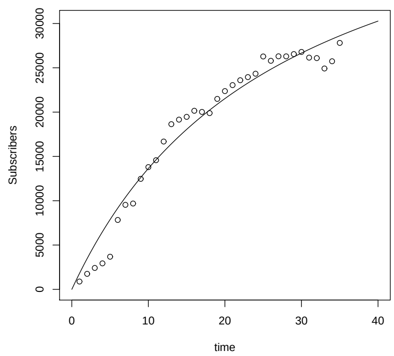
Gompertz



Bass



Michaelis-Menten



5 Final Remarks

The ability of the Michaelis-Menten curve to capture the recent evolution in the series can be explained by taking into account the following analysis of telecommunications diffusion patterns (see for instance (*[5]*).
ADSL and cable modem are high speed Internet access services which have a generally widespread use both by households and firms. As all these services are based on a Network, network externalities (or effects) are relevant. We must distinguish between direct and indirect effects. The direct effects appears on the demand side: a network is more desirable for a potential adopter the more adopters are already in the Network. The indirect effect is a supply side effect, it's the result of returns to scale potentiated by the size of the network.
A central concept in this context is the critical mass. Critical mass is the number of adopters which allows the adoption of innovation to be self-sustainable. This is even more important when the network is interactive, that is, the adopter can play an active role in the communication with other users so that he wishes to have a big number of members on the Network with whom he can communicate. In this kind of Network analysis arising for instance in telephone, fax, mobile phone, the number of adopters has a S-shaped curve, like in durable goods, but the take-off occurs later due to the very slow of diffusion before the critical mass is attained; then there is a faster evolution after the critical mass is attained.
However, Internet is a packet switching network, meaning that many different packets share the bandwidth, so there is no need to dedicate a communication path between CPE's when they communicate. In services related to Internet such as ADSL and cable modem, what happens is that there is a faster diffusion from the start since the adopters bases previously constituted in services as telephone, fax, are available to be used by the new services. In this case, the critical mass may already exist at the first moment. Take-off occurs earlier than for durable goods("early take-off") and the diffusion is very fast since the beginning. Both externalities are present in ADSL and cable modem. The competition in this sector and the fact that there is no need for another terminal to perform the new services for the adopter make its contribution for the fast diffusion of these services.
When Network effects apply, it is expected that, on the Bass model, the imitation coefficient be very significant. This may justify the estimation obtained for the model. On the other hand, in a packet switching network, this imitation coefficient may not be relevant, and the Michaelis-Menten model can be obtained as a transformation of the Bass model where q is set equal to zero. This new formulation has a huge implication on the value estimated for the saturation level. Thou this model appears to have a poorer adjustment in comparison to some of the other models, it seems to forecast more accurately the recent evolution of the series. An explanation for the gap between the series and the estimated curve, can be made by doing a case study.

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