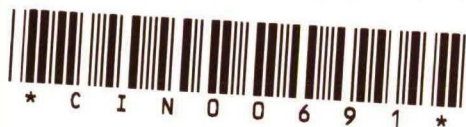


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**NEUTRAL STABILITY IN ASYMMETRIC  
EVOLUTIONARY GAMES**

by V. Bhaskar

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# NEUTRAL STABILITY IN ASYMMETRIC EVOLUTIONARY GAMES

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## Abstract

Selten (1980) showed that an evolutionary stable strategy must be a strict Nash equilibrium in a truly asymmetric game. Examples show that a neutrally stable strategy (NSS) may however be mixed. This paper shows that such examples are non-generic: in almost all truly asymmetric games, a mixed strategy cannot be a NSS, and a NSS is generically strict. Hence evolutionary stability and neutral stability are equivalent for almost all asymmetric games.

<sup>\*</sup> I started work on this paper while visiting the Wissenschaftszentrum Berlin, and completed this version at CentER, Tilburg. I am grateful to both institutions for their hospitality.

## 1. INTRODUCTION

The concept of an evolutionary stable strategy (ESS) was developed by Maynard Smith and Price (1973) in the context of two-player symmetric games. An ESS is a symmetric Nash equilibrium, which satisfies a stability condition, of being invulnerable to invasion by any other strategy which is also a best response to it. Many games are however asymmetric, and a player can be in a number of possible *roles*, or *information situations*. The ESS concept was extended to asymmetric games by Selten (1980, 1983), by considering the situation prior to a player being assigned a role, thereby symmetrizing the game. Selten showed that the ESS concept is very restrictive in truly asymmetric games, where two players are never in the same information situation (i.e. they always have different roles): an evolutionary stable strategy must be a strict Nash equilibrium of the agent normal form or normal form of the game. Since many games do not have strict Nash equilibria (eg. games with only mixed strategy Nash equilibria), this implies that ESS may not exist in a large class of truly asymmetric games.

The logic underlying Selten's result can be illustrated by considering the simplest case of a truly asymmetric game, where each player can be one of two roles, 1 and 2. A behavior strategy in the symmetrized game is simply a pair of strategies, one for each role. Let  $b = (b_1, b_2)$  be such a strategy, and let  $b'_1$  be an alternative best response to  $b_2$ . Consider the strategy  $b' = (b'_1, b_2)$ , which differs from  $b$  only with regard to the choice in role 1. It is clear that  $b'$  is a best response to  $b$ . Further, since the mutant strategy only differs in one information situation, it effectively never meets itself. Consequently, the payoffs of both strategies against  $b'$  are equal, so that  $b$  cannot be an ESS.

The above argument shows that  $b'$  and  $b$  have equal payoffs in the mixed population. Indeed, a mutant which differs only at one information situation cannot have a strictly greater payoff. It has therefore been suggested that weaker notions of evolutionary stability could be less restrictive in truly

asymmetric games. Selten proposed the notion of "limit ESS", but this was found to almost as restrictive - Samuelson (1991) showed that games with two possible roles, a limit ESS must be in pure strategies. Maynard Smith's (1982) neutrally stable strategy (NSS) is an even weaker concept. Van Damme (1987) provides an example of the "battle of the sexes" over the care of offspring, where a mixed strategy is neutrally stable, and is also dynamically stable. A similar example is presented in the game G1. This game has a unique Nash equilibrium in mixed strategies, where the row player plays T with probability  $b/(a+b)$ , and the column player plays L with probability  $b/(a+b)$ . It can be verified that this mixed strategy is neutrally stable. It is also possible to construct examples in 3x3 games. These examples suggest that weakening the ESS concept in the direction of neutral stability might alleviate the existence problem in asymmetric games.

Quite apart from its possible role in allowing existence of equilibrium in a larger class of games, neutral stability possesses an appeal in its own right. The literature on evolution and learning is in part a response to a dis-satisfaction with the requirements made on rationality and on knowledge by traditional game theory. Evolutionary theory discards these assumptions, but replaces them by appealing to asymptotic behaviour in the presence of the twin forces of natural selection/imitation and mutation/experimentation. How relevant are asymptotic results to a study of human societies? How seriously should one take results, such as the claim that evolutionary forces ensure efficiency in a large class of repeated games? In this context, neutral stability is a more appealing concept since it places less reliance upon random mutations, and more on the dynamics induced by payoff differences. Both Fudenberg and Maskin (1990) and Binmore and Samuelson (1992) employ the weaker concept of neutral stability in deriving their results on efficiency in undiscounted repeated games. In the context of cheap talk games, Warneryd (1991) and Bhaskar (1992) similarly use this weaker notion to derive their efficiency results. In these contexts, of games with a non-trivial extensive

form, neutral stability is qualitatively weaker than ESS, and in fact weaker in some respects than concepts such as the cyclically stable set (Gilboa and Matsui, 1991).

The main result of this paper however belies the hope that neutral stability will be significantly weaker than ESS in truly asymmetric games. We find that examples such as G1 are unusual, and the set of payoffs for which a given game has a mixed strategy NSS, is a closed set of Lebesgue measure zero. In other words, in almost all truly asymmetric games, a mixed strategy Nash equilibrium cannot be neutrally stable. Since pure strategy Nash equilibria are generically strict, this implies that a NSS must be a strict Nash equilibrium in almost all truly asymmetric games, and the concepts of NSS and ESS coincide. We also show that our results extend when we consider the special case of symmetric games where players can condition their choices upon the role they play, although there is no asymmetry in payoffs. These results contrast with the more positive results regarding repeated games and cheap talk games referred to earlier. They suggest that games where a (non-trivial) extensive form is induced by a move of nature are quite different from games where the extensive form is due to the moves of the players.

## 2. ASYMMETRIC GAMES: DEFINITIONS AND A LOCAL CHARACTERIZATION

In this section we introduce the asymmetric game set up, following Selten (1980) and Van Damme (1987) closely. Consider a random-matching situation where two players are chosen to play a bi-matrix game. Each of these players can be in one of several *information situations*. An information situation is a complete description of the state of the player, and may include some information regarding the other player's state. Let  $U$  be the (finite) set of information situations, and let  $C_u$  denote the finite set of choices available at  $u \in U$ . A *contest* is a pair,  $uv$ , of information situations. For each contest, there is a pair of matrices,  $A'_{uv}$  and  $A'_{vu}$ , the  $ij$ -th elements of which give

the payoff to the player at  $u$  and  $v$  respectively when they adopt the  $i$ -th and  $j$ -th pure strategy. A *local strategy* at  $u$  is an element of  $\Delta C_u$ , and will be denoted by  $b_u$ . A *behavior strategy* is a vector of local strategies, one for each information situation, and will be denoted by  $b = (b_1, b_2, \dots, b_u, \dots, b_{\#U})$ . We write  $B_u$  for the set of local strategies at  $u$ , and  $B$  for the set of behaviour strategies.

The two players are randomly allocated to information situations by a symmetric probability distribution which is consistent (see Van Damme, 1987). In other words,  $p$  is a symmetric probability measure over  $U \times U$ , with generic element  $p_{uv}$ , which denotes the probability of a  $uv$  contest with player 1 in situation  $u$ . Let  $p_u$  be the probability of player 1 being in information situation  $u$ , and assume that  $p_u > 0$  for all  $u$ .

Let  $b$  and  $b'$  be two behavior strategies. The expected payoff of  $b'$  against  $b$  is given by:

$$A(b', b) = \sum_u \sum_v p_{uv} b'_u A'_{uv} b_v \quad (2.1)$$

A strategy  $b \in B$  is said to be an *Evolutionary stable strategy* (ESS) if for all  $b' \neq b$ ,

$$bAb \geq b'Ab \quad (2.2)$$

and

$$bAb = b'Ab \Rightarrow bAb' > b'Ab' \quad (2.3)$$

(2.2) requires that  $b$  be a symmetric Nash equilibrium in the symmetrized game, and (2.3) is the stability condition - if  $b'$  is an alternative best response to  $b$ , then  $b$  does strictly better against  $b'$  than  $b'$  does against itself.

A game is said to be *truly asymmetric* if the two players are never in the same information situation, i.e. if  $p_{uu} = 0$  for all  $u$ . In this paper we shall concern ourselves only with truly asymmetric games. If  $\#U$  is the number of information situations, a truly asymmetric game can also be seen as a  $\#U$

person game with one player for each information situation. This is called the *agent normal form* of the game.

Selten (1980) defines the *local game at u induced by b* as the symmetric bi-matrix game with pure strategy sets  $C_u$  and fitness matrix  $A_u(b)$  defined by:

$$A_u(b) = p_{uu} A'_{uu} + \sum_{v \neq u} p_{uv} A'_{uv} b_v \quad (2.4)$$

Write  $A(b, b'; b, u)$  for the payoff of  $b$  against  $b'$  in the local game induced by  $b$  at  $u$ . Consider the special case where  $p_{uu}$  is zero, so that a player in role  $u$  never meets a  $u$ -player. In this case, (2.4) shows that  $A(b, b'; b, u)$  is independent of  $b'$ , and depends only upon  $b$ . Consequently, in a truly asymmetric game,  $A(b, b'; b, u)$  is independent of  $b'$  for all  $u$ , and for all  $b$ .

A strategy  $b$  is said to be a locally stable strategy (LSS) if  $b_u$  is an ESS of the local game at  $u$  for every  $u \in U$ . Obviously, a strategy must be a LSS if it is to be an ESS. Unfortunately, the reverse is not true - a LSS need not be an ESS, as the example in Van Damme (1987) shows. Intuitively, local stability checks for stability against mutants which vary their behavior at single information situations. This is insufficient, since a mutant may be able to do better by varying two or more information situations. Consequently, global analysis is required in order to check whether a strategy is ESS. Selten shows that one may obtain a local characterization of ESS in truly asymmetric games. This is possible since the ESS is a very restrictive concept in truly asymmetric games - any ESS must be a strict Nash equilibrium of the agent normal form of the game, and hence a strict Nash equilibrium of every local game. However, Selten's formulation does not allow a local characterization of NSS even in truly asymmetric games, since a NSS need not be a strict equilibrium.

The first result of this paper is to obtain a local characterization of NSS in truly asymmetric games. This requires that we check for stability only

against mutants which vary their behavior at two information situations.

Define the *local game at uv induced by b* as follows: the game consists of two information situations,  $u$  and  $v$ , with pure strategy sets  $C_u$  and  $C_v$  respectively, and payoff matrices  $A_u(b)$ ,  $A_v(b)$ .  $A_u(b)$  is defined by:

$$A_u(b, u, v) = p_{uu}A'_{uu} + p_{uv}A'_{uv} + \sum_{w \neq u, v} p_{uw}A'_{uw} b_w h \quad (2.5)$$

where  $h$  is the vector of ones,  $(1, 1, \dots, 1)$ .

(2.5) shows that the payoff matrix at  $u$  in the local game at  $uv$  defined by  $b$ ,  $A_u(b, u, v)$  is the probability weighted sum of payoff matrices of the bimatrix games at  $uu$  and  $uv$ , and a third matrix. This third matrix gives the expected payoff to the  $i$ -th pure strategy (row) in  $C_u$  in contests  $uw$  given the local strategies  $b_w$ . This has constant rows since the payoff in contests  $uw$  does not depend upon the choices made by the  $u$ -player or the  $v$ -player. The payoff of a strategy  $b'' = (b''_u, b''_v)$  against another strategy  $b' = (b'_u, b'_v)$  in the local game defined by  $b$  at  $uv$ , is given by :

$$\begin{aligned} A(b'', b'; b, u, v) = & p_{uu}b''_u A'_{uu} b'_u + p_{uv}b''_u A'_{uv} b'_v + p_{vv}b''_v A'_{vv} b'_v + p_{vu}b''_v A'_{vu} b'_u \\ & + \sum_{w \neq u, v} p_{uw}b''_u A'_{uw} b_w + \sum_{w \neq u, v} p_{vw}b''_v A'_{vw} b_w \end{aligned} \quad (2.6)$$

A strategy  $b$  will be called *pairwise neutrally stable strategy* (PNSS) if  $(b_u, b_v)$  is a NSS of the local game defined by  $b$  at  $(u, v)$  for every pair  $(u, v)$  in  $U \times U$ . Notice that the local game defined by  $b$  at  $(u, u)$  coincides with Selten's definition of the local game defined by  $b$  at  $u$ , so that pairwise neutral stability implies local neutral stability

**Theorem 1.** Let  $\Gamma$  be a truly asymmetric game.  $b$  is a NSS of  $\Gamma$  if and only if  $b$  is a PNSS of  $\Gamma$ .

**Proof:** Let  $b$  be a Nash equilibrium of the agent normal form. We can restrict attention to mutants which are best responses to  $b$ , so let  $b'$  be an alternative best response to  $b$ , so that  $A(b, b) = A(b', b)$ . Consequently,  $b'_u$  is

a best response to  $b_u$  in the local game at  $u$  for every  $u \in U$ . If  $b$  is a PNSS, we have  $A(b, b'; b, u) \geq A(b', b'; b, u)$  for every  $u$ . Consider the sum:

$$\begin{aligned}
 S &\equiv \sum_{u \neq v} \sum_v \{A(b, b'; b, u, v) - A(b', b'; b, u, v)\} \\
 &= 2 \sum_u \sum_v p_{uv} (b_u - b'_u) A'_{uv} b'_v \\
 &\quad + 2(\#U - 2) \sum_u \sum_v p_{uv} (b_u - b'_u) A'_{uv} b_v \\
 &\quad + 2(\#U - 2) \sum_u p_{uu} (b_u - b'_u) A_{uu} b'_u \quad (2.7)
 \end{aligned}$$

Since the first term on the right hand side of (2.7) is  $[A(b, b') - A(b', b')]$ , and since the second and third terms sum up to the difference in payoffs in all local games at  $u$ , we have:

$$A(b, b') - A(b', b') = S/2 - (\#U - 2) \sum_u \{A(b', b'; b, u) - A(b, b'; b, u)\} \quad (2.8)$$

If the game is truly asymmetric,  $A(b', b'; b, u) = A(v, b'; b, u)$  for every  $u$ , and hence:

$$A(b, b') - A(b', b') = S/2 \quad (2.9)$$

If  $b$  is a PNSS, each term in the summation (S) is non-negative and  $b$  is an NSS.  $\square$

The intuition behind theorem 1 suggests that it should be possible to generalize the result to games with more than two players. If  $m$  players are randomly allocated to  $\#U$  information situations, one needs to consider only possible  $m$ -tuples of deviations. However, since evolutionary game theory has focused on two-player games, we shall not pursue this generalization.

Theorem 1 will play an important role in our analysis: it allows to analyze neutral stability in the overall game at the level of pairwise neutral stability in local games.

## 3. GENERIC RESULTS FOR ASYMMETRIC GAMES

In this section of the paper we rely upon theorem 1 in order to analyze any  $\#U$  type asymmetric game at the level of 2 types. In other words, we shall analyze local games at pairs of information situations. Our aim is to prove the following theorem, which is the main result of the paper.

**Theorem 2.** Let  $\Gamma$  be a truly asymmetric game. For almost all payoff matrices,  $A_{uv}'$ ,  $\Gamma$  does not have a NSS in mixed strategies, and any NSS must be a strict Nash equilibrium.

We prove the theorem by a series of lemmata. The first lemma shows that if a NSS involves playing a mixed strategy in one information situation,  $u$ , then it must (generically) involve mixing in at least one other information situation,  $v$ . The lemma states this result somewhat more generally, in terms of the Nash equilibrium of the agent normal form of the game.

**Lemma 1.** Let  $b = (b_1, b_2, \dots, b_u, \dots, b_{\#U})$  be a Nash equilibrium of the agent normal form of the game. If  $b_u$  is a non-degenerate mixed strategy, then for almost all games, there exists a  $v$  different from  $u$  such that  $b_v$  is also a non-degenerate mixed strategy.

**Proof:** Let  $c_u, c_u' \in C(b_u)$ . If  $b_w$  is a pure strategy for all  $w$ , so that  $b_w = c_w$  other than  $u$ , then :

$$A_u'(c_1, c_2, \dots, c_u, \dots, c_n) = A_u'(c_1, c_2, \dots, c_u', \dots, c_n) \quad (3.1)$$

In other words, the payoff to player  $u$  from two pure strategies is identical, fixing the pure strategies all other players. The set of games where any two payoff entries of player at  $u$  are equal is a closed set one dimension less than the dimension of the space of payoffs. The set of games where payoffs to any player are equal is finite union of  $\#U$  closed sets of lower dimension, and is hence a closed set of Lebesgue measure zero, in the space of payoffs.  $\square$

Lemma 1 establishes that in almost all asymmetric games, if an equilibrium strategy involved mixing at one information situation, it must involve mixing in at least two information situations. A truly asymmetric games defines a  $n$  player agent-normal form game, and a neutrally stable

strategy is necessarily a Nash equilibrium of the agent normal form game. Consequently, a mixed strategy NSS in truly asymmetric game generically involves playing a mixed strategy in at least two information situations,  $u$  and  $v$ .

Consider the local game defined by  $b$  at the contest  $uv$ . A strategy in the local game,  $b_{uv}$ , is given by a pair  $(b_u, b_v)$  where  $b_u \in \Delta C_u$  and  $b_v \in \Delta C_v$ , where  $\Delta C_i$  is the set of probability measures on  $C_i$ . From (2.5), we write down the payoff matrices in the local game. Since the game is truly asymmetric,  $p_{uu} = 0$ .

$$A_u(b, u, v) = p_{uv} A'_{uv} + \sum_{w \neq u, v} p_{uw} A'_{uw} b_w h \quad (3.2)$$

$$A_v(b, u, v) = p_{uv} A'_{vu} + \sum_{w \neq u, v} p_{vw} A'_{vw} b_w h \quad (3.3)$$

We shall show that if  $b_{uv}$  is a (non-trivial) mixed strategy combination, then  $b_{uv}$  cannot be a NSS of the local game at  $uv$  for almost all payoff matrices. The strategy of our proof will be to assume that  $b_{uv}$  is a NSS, thereby deriving conditions on the payoff matrices which cannot be generically satisfied. However, some notational simplification is worthwhile at this stage. Note that if  $(b_u, b_v)$  is a NSS of the local game at  $uv$ , then  $(b_u, b_v)$  is a NSS of the local game where the player at  $u$  is restricted to mixed strategies which are used with positive probability by  $b_u$ , and the same holds for  $v$ . In other words, we consider the *restricted game* where players are restricted to probability measures over the set of pure strategies which are in the support of  $b_u$  and  $b_v$ . Write  $A$  for the restricted version of the payoff matrix  $A_u$  and  $B$  for the restricted version of the payoff matrix  $A_v$ . Write  $(p, q)$  for  $(b_u, b_v)$ : since we are only considering the restricted game, both  $p$  and  $q$  are in the interior of the simplex. A strategy,  $b_{uv} = (p, q)$  is a *neutrally stable strategy* (NSS) in the local game if for any other  $b'_{uv} = (p', q')$ :

$$pAq + pBq \geq p'Aq + pBq' \quad (3.4)$$

and

$$pAq + pBq = p'Aq + pBq' \Rightarrow pAq' + p'Bq \geq p'Aq' + p'Bq' \quad (3.5)$$

Let  $(p, q)$  be a completely mixed strategy NSS in the restricted local game.  $(p, q)$  is hence a mixed strategy Nash equilibrium, and let  $\varphi_1$  and  $\varphi_2$  be the equilibrium payoffs for the two roles. Since  $p$  and  $q$  are completely mixed strategy in the restricted game, they satisfy:

$$Aq = \varphi_1 h \quad (3.6)$$

$$pB = \varphi_2 h \quad (3.7)$$

where  $h$  is a vector of ones,  $(1, 1, \dots, 1)$ .

Lemma 2. For any mutant  $(p', q')$  in the restricted game, the expected payoff of the mutant and the expected payoff of the incumbent are equal in any mixed population, i.e.:

$$pAq + pBq = p'Aq + pBq' \quad (3.8)$$

$$pAq' + p'Bq' = p'Aq' + p'Bq' \quad (3.9)$$

Proof: Since  $(p, q)$  is a completely mixed strategy in the restricted game,  $p'$  is a best response to  $q$  and  $q'$  is a best response to  $p$ , so that (3.8) follows. Hence, if  $(p, q)$  is an NSS (3.10) must hold for all  $p', q'$  in the restricted game:

$$(p - p')Aq' + p'B(q - q') \geq 0 \quad (3.10)$$

(3.10) must hold with equality for all  $p', q'$ . Otherwise, if there exists  $p', q'$  such that the inequality is strict, there exists  $p'', q''$  such that the inequality is reversed. Since  $p, q$  are both in the interior of the simplex, there exist scalars  $\lambda_1 > 0$  and  $\lambda_2 < 0$  such that  $p'' = (1 - \lambda_1)p + \lambda_1 p'$ ,  $q'' = (1 - \lambda_2)q + \lambda_2 q'$  are both permissible.

$$(p - p'')Aq'' + p''B(q - q'') = \lambda_1 \lambda_2 [(p' - p)Aq' + p'B(q - q')] \quad (3.11)$$

Since the sign of (3.11) is the negative of the sign of (3.10), this implies that (3.9) must hold for all  $p', q'$ .  $\square$

Lemma 3. Let  $C = A + B$  be the sum of restricted payoff matrices of the local game at  $uv$  defined by  $b$ . If  $(p, q)$  is a NSS of the local game, then every  $2 \times 2$  sub-matrix of  $C$  is non-invertible.

Proof: Writing (3.9) for the case when  $p'$  is the  $i$ th pure strategy and  $q'$  is the  $j$ th pure strategy, this requires, that for all  $i, j$ :

$$pA_{jc} + B_{ir}q = a_{ij} + b_{ij} \quad (3.12)$$

where  $jc$  subscripts the  $j$ -th column and  $ir$  the  $i$ -th row of the matrix.

Re-write (3.10) for the  $h$ -th row and  $j$ -th column, and subtract to obtain:

$$(B_{ir} - B_{hr})q = c_{ij} - c_{hj} \quad (3.13)$$

$$\text{where } c_{ij} = a_{ij} + b_{ij}.$$

Notice that while the right hand side of (3.13) involves elements in the  $j$ -th row, the left hand side is independent of  $j$ . Re-writing (3.13) for column  $k$ , and equating, we get:

$$c_{ij} + c_{hk} = c_{ik} + c_{hj} \quad (3.14)$$

Since  $h, i, j$  and  $k$  were arbitrarily chosen, (3.14) holds for every row and every column., and every  $2 \times 2$  sub-matrix of the matrix  $C$  must have a vanishing determinant.  $\square$

Proof of theorem 2. Given a space of  $n \times n$  square matrices, the set of non-invertible matrices is a closed set of Lebesgue measure zero in this space (see Hoffman, 1975, for example). Lemma 3 establishes that every  $2 \times 2$  sub-matrix of the matrix  $C$  is non-invertible. From (3.2) and (3.3),  $C$  is the weighted sum of two-matrices each of which is the sum of a primitive payoff matrix of the agent normal form of the game, and a matrix with constant rows. It follows that the set of matrices  $A'_{uv}$  and  $A'_{vu}$  which satisfy pairwise local stability at the local game at  $uv$  is of measure zero. Since theorem 1 shows that pairwise neutral stability is equivalent to overall neutral stability, this establishes that almost all truly asymmetric games do not have a mixed strategy NSS. Since pure strategy NSS are generically strict, theorem 2 follows.  $\square$

Theorem 2 applies to games where the underlying game is asymmetric. A class of asymmetric games are those where the game itself is symmetric, but where players may condition their choice of strategy upon the role they fill (see, for example, the discussion in Van Damme, 1987 or Samuelson, 1991).

It may be thought that symmetric games of this class are a special case of asymmetric games more generally. However, for questions of genericity, the distinction could make a difference. In the case where the underlying game is symmetric, the dimension of the set of payoffs is  $\#S^2$ , where  $\#S$  is the number of pure strategies in each player's strategy set. This is one-half the dimensionality of the set of payoffs if we consider the underlying game to be asymmetric.

However, it is easy to show that a mixed strategy NSS is non-generic even in this class of games. If payoff functions are symmetric, it follows that in the local game defined by  $b$  at  $uv$ , the payoff matrix  $B$  equals the transpose of  $A$ . Hence, the matrix  $C$ , which is the sum of  $A$  and  $B$ , is symmetric. However,  $C$  must still satisfy the condition of lemma 3, i.e. every  $2 \times 2$  sub-matrix of  $C$  must be non-invertible. Hence, even within the class of symmetric matrices, the set of matrices satisfying lemma 3 is of measure zero. The implication of this result may be seen in  $2 \times 2$  games. If we consider the class of symmetric  $2 \times 2$  games, a game possesses a mixed strategy NSS only if it is zero-sum.

#### 4. CONCLUSIONS

This paper has shown that in generic truly asymmetric games, a NSS must be a strict Nash equilibrium. Consequently, the distinction between NSS and ESS is not important in such games. This result assumes significance in the context of the existing results analysing other classes of games with a non-trivial extensive form. In repeated games or games with pre-play communication, neutral stability is a significantly weaker notion than evolutionary stability, and is in fact weaker in many ways than other equilibrium concepts. This suggests that games where the extensive form is induced by a move of nature (such as the asymmetric games considered in this paper) differ significantly from games where the extensive form is induced by choices of players.

## REFERENCES

- Bhaskar, V., 1992, Noisy Communication and the Evolution of Cooperation, mimeo, Delhi School of Economics.
- Binmore, K., and L. Samuelson, 1992, Evolutionary Stability in Repeated Games Played by Finite Automata, *Journal of Economic Theory* 57, 278-305.
- Fudenberg, D., and E. Maskin, 1990, Evolution and Cooperation in Noisy Repeated Games, *American Economic Review Papers and Proceedings*, 80, 274-79.
- Gilboa, I., and A. Matsui, 1991, Social Stability and Equilibrium, *Econometrica*, 59, 859-868.
- Hoffman, K., 1975, *Analysis in Euclidean Space*, New Jersey: Prentice Hall.
- Maynard Smith, J., and G. Price, 1973, The Logic of Animal Conflict, *Nature*, 246, 15-18.
- Maynard Smith, J., 1982, *Evolution and the Theory of Games*, Cambridge: Cambridge University Press.
- Samuelson, L., 1991, Limit Evolutionary Stable Strategies in Two-Player, Normal Form Games, *Games and Economic Behavior* 3, 110-128.
- Samuelson, L., and J. Zhang, 1992, Evolutionary Stability in Asymmetric Games, *Journal of Economic Theory* 57, 363-391.
- Selten, R., 1980, A note on evolutionary stable strategies in asymmetric animal conflicts, *Journal of Theoretical Biology*, 84, 93-101.
- Selten, R., 1983, Evolutionary Stability in Extensive Two-Person Games, *Mathematical Social Science* 5, 269-363.
- Van Damme, E., 1987, *Stability and Perfection of Nash Equilibria*, Berlin: Springer-Verlag.
- Warneryd, K., 1991, Evolutionary Stability in Unanimity Games with Cheap Talk, *Economics Letters* 36, 375-8.

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