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Characterizations of a Game Theoretical Cost Allocation Method

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Abstract

In the 1930's the Tennessee Valley Authority developed several methods to allocate the costs of multipurpose water projects. One of these methods is the alternate cost avoided method. This paper provides two characterizations of the alternate cost avoided method, one on a class of cost games with a fixed player set, the other on a class of cost games with a variable player set using a reduced game property.

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1 Introduction

Cost allocation problems occur in many practical situations, where individuals work together in a joint project. In these cases the problem arises of allocating the joint costs to the participants in the project in a "fair" way. A mathematical tool to analyse this type of problems is provided by cooperative game theory.

Examples of cost allocation problems studied in a game theoretical context are the setting of airport landing fees (e.g. Littlechild and Owen (1973), Littlechild and Thompson (1977)), the allocation of joint overhead costs of a firm among its different divisions (e.g. Shubik (1962), Jensen (1977), Hamlen et al. (1977)), and the apportioning of costs of multipurpose water development projects (e.g. Ransmeier (1942), Suzuki and Nakayama (1976), Loughlin (1977), Straffin and Heaney (1981), Young et al. (1982)).

Especially the last type of cost allocation problems has a rich history dating back to the 1930's in which the Tenessee Valley Authority (TVA) was established (see Ransmeier 1942, Parker (1943)). The problem TVA engineers were confronted with was the apportioning of costs of projects in the Tennessee River among the different 'purposes' to be served (mainly navigation, flood control, and hydro-electric power). TVA engineers made several proposals to allocate the costs of projects to these purposes. Almost all these methods begin by allocating the so-called separable cost, to each 'participant' (purpose), and then dividing the remaining nonseparable cost.

Two of the methods developed by the TVA are the egalitarian nonseparable cost (ENSC) method, which allocates the nonseparable cost equally among the participants, and the alternate cost avoided (ACA) method, which allocates the nonseparable cost among the participants in proportion to the 'cost savings' made by including a participant in the joint project instead of developing a separate project only to serve the purposes of that participant.

A modification of the ACA-method is the separable cost remaining benefit (SCRB) method. This has become the principal method used by civil engineers to allocate the costs of multipurpose water projects (see e.g. Inter-Agency Committee on Water Resources (1958)).

A game theoretical base for the ACA-method was established by Gately (1974). Gately proposed a new solution concept for cooperative games based on a player's "propensity to disrupt" the solution. This solution concept has been further generalized by Fischer and Gately (1975), Littlechild and Vaidya (1976) and Charnes et al. (1978). It was shown by Straffin and Heaney (1981) that the allocation method proposed by Gately corresponds precisely to the ACA-method.

The purpose of this paper is to provide an axiomatic characterization of the ACA-method on a certain class of cost games with a fixed player set as well as on a class of cost games with a variable player set, using a reduced game property. This is the subject of section 3. First, in section 2 we discuss the cost allocation problem in a formal game theoretical context, and recall some of the cost allocation methods proposed by the TVA.

2 Game theory and cost allocation problems

To formulate a cost allocation problem in terms of cooperative game theory, it is modelled as a *cost game* (N, c). Here, N represents a finite set of participants among which the costs of a joint project are allocated. For example, N can be a set of potential customers of a public facility, the divisions of a firm, or municipalities which share a joint water system, etc. The elements of N are called *players* and subsets of N are called *coalitions*. For any coalition $S \subset N$, the minimal costs of designing a project for the purposes of S only are denoted by $c(S)$. In particular, $c(\emptyset) := 0$, where $\emptyset$ denotes the empty set. The function $c : 2^N \rightarrow \mathbf{R}$ is called the *(joint) cost function*. Let CG^N denote the set of all cost games with player set N.

Example 1: As an example of a joint cost game based on a cost allocation problem, we consider the cost allocation problem for the TVA ten dam system. Here the purposes navigation, flood control and hydro-electric power are denoted as players 1,2, and 3 respectively. Table 1 is adapted from Ransmeier (1942, p. 329).

coalitions S	$\emptyset$	{1}	{2}	{3}	{1,2}	{1,3}	{2,3}	{1,2,3}
cost c(S)	0	163,520	140,826	250,096	301,607	378,821	367,370	412,584

Table 1. The cost game for the TVA ten dam project (costs in \$ 1000).

Given a cost game (N,c), the cost allocation problem now becomes to choose a cost allocation in a "fair" manner. A *cost allocation for* (N,c) is a vector $x \in \mathbf{R}^N$ such that $\sum_{i\in N} x_i = c(N)$. Here, x_i is the cost allocated to player $i \in N$. The TVA engineers proposed several cost allocation methods. If A^N is a subset of CG^N, then a *(cost) allocation method* on A^N is a map $f : A^N \to \mathbf{R}^N$, which assigns to every cost game $(N,c) \in A^N$ a cost allocation $f(c) \in \mathbf{R}^N$.

Almost all cost allocation methods proposed by the TVA begin by charging every player a minimal cost, called separable cost, which are the additional cost of including the player in the project already designed for the other players. Thus, for a cost game (N,c), *the separable cost* $SC_i(c)$ *of player* $i \in N$ are formally defined by

$$SC_i(c) := c(N) - c(N\backslash\{i\}).$$

To use methods based on the idea above it is reasonable to make the following two assumptions on the underlying cost game.

$$SC_i(c) \leq c(\{i\}) \quad \text{for all } \ i \in N, \tag{1}$$

$$\sum_{i\in N} SC_i(c) \leq c(N) \leq \sum_{i\in N} c(\{i\}). \tag{2}$$

Conditions (1) and (2) are well-known balancedness conditions for cost games. If $SC_i(c) > c(\{i\})$ for some $i \in N$, then it is not favourable to include player i in the joint project. Condition (2) implies that after each player is charged his minimal costs there is still a positive amount of cost remaining which should be allocated. These remaining cost are called the *nonseparable cost* and are given by

$$NSC(c) := c(N) - \sum_{i\in N} SC_i(c).$$

The easiest way to allocate the nonseparable cost is to divide these cost equally among the players. This method is called the *egalitarian nonseparable cost* ($ENSC$) *method,* and it is one of the first allocation methods proposed by the TVA. Thus, for a cost game (N,c) the cost allocated to player $i \in N$ by the ENSC-method are

$$ENSC_i(c) = SC_i(c) + \frac{1}{|N|}NSC(c).$$

An alternative allocation method is the *alternate cost avoided ACA) method*, which was first proposed by Martin Gleaser, a TVA consultant in 1938 (see Ransmeier (1942)). By this method the nonseparable cost are divided in proportion to $c(\{i\}) - SC_i(c)$. Hence,

$$ACA_i(c) := SC_i(c) + \frac{c(\{i\}) - SC_i(c)}{\sum_{j\in N} c(\{j\}) - SC_j(c)}NSC(c) \quad \text{for all } i \in N$$

The number $c(\{i\}) - SC_i(c)$ represents the *alternate cost avoided* by including player i in the joint project.

A modification of the ACA-method is the *separable cost remaining benefit (SCRB) method.* If $b(i)$ is the benefit of the project to player i, then i would not be willing to pay more than $\min\{b(i), c(\{i\})\}$. The remaining benefit to player i is defined by $\min\{b(i), c(\{i\})\} - SC_i(c)$. The SCRB-method allocates the nonseparable cost proportional to the remaining benefits. Since in many situations the benefits exceed the alternate costs, the SCRB-method often coincides with the ACA-method.

The mayor drawback of the cost allocation methods mentioned above is that they only take into account the values of the coalitions with 1, $|N| - 1$ and $|N|$ players. In particular, there is no guarantee that the corresponding allocations of these methods are core elements of the cost game, which means that there might be subcoalitions that have an incentive to split of from the grand coalition.

From a practical viewpoint however, the advantage of these methods is that in general they are much easier to compute than game theoretical solution concepts as the Shapley value (Shapley (1953)), the nucleolus (Schmeidler (1969)) and the cost gap method (Driessen en Tijs (1985), Tijs en Driessen (1986)), which take into account the values of *all* coalitions.

Moreover, as is shown in e.g. Suzuki and Nakayama (1976), Legros (1982) and Driessen and Tijs (1985) there are (large) classes of cost games for which some of the solution concepts mentioned above coincide with one (or more) of the game theoretical solution concepts.

Example 2: For the TVA cost game of example 1 the cost allocations of the ENSC- and ACA-method are given in table 2 together with the cost allocations corresponding to the game theoretical solutions mentioned above. Note that in this case the cost allocations by the ACA-method and the cost gap method coincide.

	1	2	3
ENSC-method	119,424	107,973	185,187
ACA-method	117,476	99,157	195,951
Shapley value	117,829	100,756	193,999
nucleolus	116,234	93,540	202,810
cost gap method	117,476	99,157	195,951

Table 2. Cost allocation for the TVA cost game by five methods (cost in \$ 1000).

3 Characterizations of the ACA-method

This section further investigates the ACA-method. Attention is restricted to the class of cost games (N, c) for which (1) and (2) hold. This class is denoted by F^N and F_m denotes the class of cost games with m or more players satisfying (1) and (2).

Geometrically, for a cost game $(N, c) \in F^N$ the cost allocation $ACA(c)$ is the unique element in the hyperplane $\{x \in \mathbf{R}^N | \sum_{i \in N} x_i = c(N)\}$ which lies on the line segment with end points $(SC_i(c))_{i \in N}$ and $(c(\{i\}))_{i \in N}$ (see figure 1).

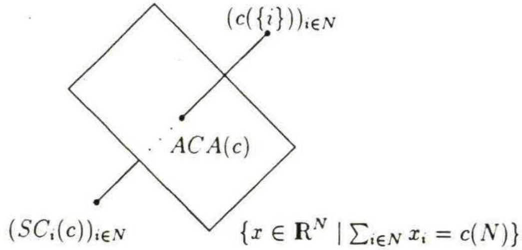

figure 1.

Let $A \subset F_1$. Clearly, the ACA-method satisfies *individually rationality on* A, i.e., $ACA_i(c) \leq c(\{i\})$ for all $i \in N$ and all $(N, c) \in A$.

Furthermore, the ACA-method satisfies the *symmetry property on* A, i.e., for all $(N, c) \in A$ and all players i and j that are symmetric in (N, c), i.e., $c(S \cup \{i\}) = c(S \cup \{j\})$ for all $S \subset N \backslash \{i, j\}$, it holds that $ACA_i(c) = ACA_j(c)$.

The ACA-method also satisfies *invariance w.r.t. strategic equivalence on* A, i.e., for all $(N, c) \in A$, all $k > 0$ and all $a \in \mathbf{R}^N$, such that $(N, kc + a) \in A$, we have that $ACA(kc + a) = kACA(c) + a$. Here the game $(N, kc + a)$ is defined by $(kc + a)(S) := kc(S) + \sum_{i \in S} a_i$ for all $S \subset N$.

Another property of the ACA-method on A is *weak proportionality* which says that if $(N, c) \in A$ is such that $SC_i(c) = 0$ for all $i \in N$, then $ACA(c)$ is proportional to the vector $(c(\{i\}))_{i \in N}$ of individual costs.

This weak proportionality property shows great resemblance to the restricted proportionally property of the τ-value (Tijs (1981), (1987)). Cost games for which each player's separable cost are zero arise when the increase in the total costs of adding an extra player can be neglected compared to the total cost of the project.

Similar to the characterization of the τ-value by Tijs (1987) one can prove

Theorem 1: The ACA-method is the unique cost allocation method on F^N which satisfies invariance w.r.t. strategic equivalence and weak proportionality.

Proof: Suppose that $f : F^N \rightarrow \mathbf{R}^N$ satisfies the two mentioned properties. Let $(N, c) \in F^N$. It suffices to show that $f(c) = ACA(c)$.

Define the game $(N, \hat{c}) \in F^N$ by

$$\hat{c}(S) := c(S) - \sum_{i \in S} SC_i(c) \quad \text{for all } S \subset N.$$

Then $SC_i(\hat{c}) = 0$ for all $i \in N$. From the weak proportionality property it follows that there exists an $\alpha \in \mathbf{R}$ such that for all $i \in N$

$$f_i(\hat{c}) = \alpha \hat{c}(\{i\}) = \alpha(c(\{i\}) - SC_i(c)).$$

From the strategic equivalence property it follows that for all $i \in N$

$$f_i(c) = SC_i(c) + f_i(\hat{c}) = SC_i(c) + \alpha(c(\{i\}) - SC_i(c)).$$

Using the fact that $\sum_{i \in N} f_i(c) = c(N)$, it easily follows that $f(c) = ACA(c)$. □

The last part of this section provides a characterization of the ACA-method on the class F_1 using a reduced game property. In the literature several types of reduced games have been considered to provide a foundation of game theoretic solution concepts based on the consistency principle. We mention, Hart and Mas-Colell (1989) for the Shapley value, Sobolev (1975), Snijders (1991) for the (pre)nucleolus, Peleg (1986) for the core, and recently, Driessen (1992) for the τ-value. Also the ENSC-method has been characterized by means of a reduced game property (Moulin (1985), Driessen and Funaki (1993)). For a detailed survey on consistency see e.g. Driessen (1991).

The idea behind consistency is the following. Given a cost game, and a cost allocation for this game, determined by a cost allocation method, imagine that a coalition decides to renegotiate the allocation within their subgroup. The new situation is described by a reduced game. A cost allocation method is consistent w.r.t this reduced game if the new cost allocation within this subgroup is the same as in the original game.

Let (N, c) be a cost game, $k \in N$ and $x \in \mathbf{R}^N$ a cost allocation. The *reduced game* $(N\backslash\{k\},\ c^{k,x})$ *corresponding to* (N, c) is defined as follows. For $S \subset N\backslash\{k\}$

$$c^{k,x}(S) := \begin{cases} c(S) & \text{if } |S| \leq 1 \\ c(S \cup \{k\}) - x_k & \text{if } 2 \leq |S| \leq |N| - 1. \end{cases}$$

It should be noted that the reduced game introduced here coincides with the reduced game of Moulin (1985) except for the 1-person coalitions.

The interpretation of this reduced game is as follows. In the reduced game the cost of a 1-person coalition is the same as in the original game. However, if in the reduced situation the players want to cooperate in a coaltion S, then player k should be involved and, therefore, the cost of coalition S in the reduced game is the cost of coalition $S \cup \{k\}$ in the original game minus the original cost x_k allocated to player k.

Let $A \subset F_1$ and let $m(A) := \min\{|N| \mid (N, c) \in A\}$. A cost allocation method f on A satisfies the *reduced game property on* A if for all $(N, c) \in A$ with $n > m(A)$ and all

$k \in N$ it holds that

(i) $(N\backslash\{k\},\ c^{k,f(c)}) \in A$, and

(ii) $f_i(c^{k,f(c)}) = f_i(c) \quad$ for all $\ i \in N\backslash\{k\}$.

The ACA-method satisfies the reduced game property on the class F_3. This is shown in

Lemma 2: The ACA-method satisfies the reduced game property on F_3.
Proof: Let $(N,c) \in F_3$ with $|N| \geq 4$, and let $k \in N$. We first show that the reduced game $(N\backslash\{k\}, c^{k,ACA(c)})$ is an element of F_3. Herefore note that for all $i \in N \setminus \{k\}$

$$c^{k,ACA(c)}(\{i\}) = c(\{i\}) \tag{3}$$

and since $|N| \geq 4$ also

$$SC_i(c^{k,ACA(c)}) = c(N) - ACA_k(c) - (c(N\backslash\{i\}) - ACA_k(c)) = SC_i(c). \tag{4}$$

Since $(N,c) \in F_3$, it follows that $SC_i(c^{k,ACA(c)} \leq c^{k,ACA(c)}(\{i\})$ for all $i \in N\backslash\{k\}$.
It remains to show that

$$\sum_{i\in N\backslash\{k\}} SC_i(c^{k,ACA(c)}) \leq c^{k,ACA(c)}(N \setminus \{k\}) \leq \sum_{i\in N\backslash\{k\}} c^{k,ACA(c)}(\{i\}). \tag{5}$$

Note that for $i \in N \setminus \{k\}$

$$SC_i(c) \leq ACA(c) \leq c(\{i\}).$$

Then, using (3), (4), and the fact that $c^{k,ACA(c)}(N\backslash\{k\}) = \sum_{i\in N\backslash\{k\}} ACA_i(c)$ the required inequality (5) is easily obtained.
Now we show that $ACA_i(c^{k,ACA(c)}) = ACA_i(c)$ for all $i \in N \setminus \{k\}$.
Since $ACA_i(c) = SC_i(c) + \alpha(c(\{i\}) - SC_i(c))$ for all $i \in N$, where α is such that

$$c(N) = \sum_{i\in N} SC_i(c) + \alpha \sum_{i\in N} (c(\{i\} - SC_i(c)). \tag{6}$$

Similarly, using (3) and (4), we obtain that $ACA_i(c^{k,ACA(c)}) = SC_i(c)+\beta(c(\{i\})-SC_i(c))$ for all $i \in N \setminus \{k\}$, where β is such that

$$c(N) - ACA_k(c) = \sum_{i\in N\setminus\{k\}} SC_i(c) + \beta \sum_{i\in N\setminus\{k\}} (c(\{i\} - SC_i(c)). \tag{7}$$

Subtracting (7) from (6) we obtain

$$ACA_k(c) = SC_k(c) + \alpha(c(\{k\}) - SC_k(c)) + (\alpha - \beta) \sum_{i\in N\setminus\{k\}} (c(\{i\} - SC_i(c)).$$

Hence,

$$(\alpha - \beta) \sum_{i\in N\setminus\{k\}} (c(\{i\} - SC_i(c)) = 0. \tag{8}$$

We now distinguish two cases.
If $\sum_{i\in N\setminus\{k\}}(c(\{i\}) - SC_i(c)) = 0$, then $ACA_i(c) = SC_i(c)$ for all $i \in N \setminus \{k\}$. Since, in this case,

$$c^{k,ACA(c)}(N \setminus \{k\}) = \sum_{i\in N\setminus\{k\}} ACA_i(c) = \sum_{i\in N\setminus\{k\}} SC_i(c) = \sum_{i\in N\setminus\{k\}} SC_i(c^{k,ACA(c)})$$

it easily follows that $ACA_i(c^{k,ACA(c)}) = ACA_i(c)$ for all $i \in N \setminus \{k\}$.
If $\sum_{i\in N\setminus\{k\}}(c(\{i\}) - SC_i(c)) \neq 0$, then by (8) $\alpha - \beta = 0$. Hence, $ACA_i(c^{k,ACA(c)}) = ACA_i(c)$ for all $i \in N \setminus \{k\}$. □

Example 3 illustrates that the ACA-method does not satisfy the reduced game property on the set F_2. This is due to the fact that by reducing a 3-person game to a 2-person game the separable costs of the players may change.

Example 3: Let $N := \{1,2,3\}$ and define (N,c) as follows. For $S \subset N$

$$c(S) = \begin{cases} 2 & \text{if } \{2,3\} \not\subset S \\ 4 & \text{if } \{2,3\} \subset S. \end{cases}$$

Clearly, $(N, c) \in F_2$ and $ACA(c) = (0, 2, 2)$. The reduced game $(\{1,2\}, c^{3,ACA(c)}) \in F_2$ is given by $c^{3,ACA(c)}(\{1\}) = c^{3,ACA(c)}(\{2\}) = 2$ and $c^{3,ACA(c)}(\{1,2\}) = 2$.
Hence, $ACA(c^{3,ACA(c)}) = (1,1) \neq (0,2) = (ACA_1(c), ACA_2(c))$.

Lemma 3: Let f be a cost allocation method on F_3 which satisfies weak proportionality on $F_3 \setminus F_4$ and the reduced game property on F_3. Then f satisfies weak proportionality on F_3.

Proof: Let $(N, c) \in F_3$ with $|N| \geq 4$ be such that $SC_i(c) = 0$ for all $i \in N$ and suppose that f satisfies the weak proportionality property on $F_3 \setminus F_{|N|}$.
Let $k \in N$ and let $(N \setminus \{k\}, c^{k,f(c)})$ be the $(|N|-1)$-person reduced game of (N, c). Then $(N \setminus \{k\}, c^{k,f(c)}) \in F_3 \setminus F_{|N|}$. Since $SC_i(c^{k,f(c)}) = 0$ for all $i \in N \setminus \{k\}$ (cf. (4)), there exists an $\alpha \in \mathbf{R}$ such that

$$f_i(c^{k,f(c)}) = \alpha c^{k,f(c)}(\{i\}) = \alpha c(\{i\}) \quad \text{for all } i \in N \setminus \{k\}.$$

Since f satisfies the reduced game property on F_3 it follows that

$$f_i(c) = f_i(c^{k,f(c)}) = \alpha c(\{i\}) \quad \text{for all } i \in N \setminus \{k\}.$$

Varying $k \in N$ leads to

$$f(c) = \alpha(c(\{1\}), \ldots, c(\{n\})).$$

□

Now we can formulate our main theorem which characterizes the ACA-method on F_1.

Theorem 4: The ACA-method is the unique cost allocation method on F_1 which satisfies

(i) symmetry on F_1,

(ii) invariance w.r.t. strategic equivalence on F_1,

(iii) weak proportionality on $F_3 \setminus F_4$,

(iv) the reduced game property on F_3.

Proof: Clearly, the ACA-method satisfies (i)-(iv).
Let f be a cost allocation method, defined on F_1, satisfying (i)-(iv). Let $(N, c) \in F_1$. We show that $f(c) = ACA(c)$. Herefore we distinguish three cases.
If $|N| = 1$, then $f(c) = c(\{1\}) = ACA(c)$.
If $|N| = 2$, then (i) and (ii) imply that $f_i(c) = c(\{i\}) + \frac{1}{2}(c(N) - c(\{i\}) - c(N \setminus \{i\})) = ACA_i(c)$ for $i = 1, 2$.
If $|N| \geq 3$, then theorem 1 and lemma 3 imply that $f(c) = ACA(c)$. □

It may be noted that also the the ENSC-method satisfies symmetry, invariance w.r.t strategic equivalence and the reduced game property on the set F_3. However, this cost allocation method does not satisfy weak proportionality.

For a cost game $(N, c) \in F_1$, the *center of imputation set (CIS) value* is defined by

$$CIS_i(c) := c(\{i\}) + \frac{1}{|N|}(c(N) - \sum_{j \in N} c(\{j\})) \quad \text{for all } \; i \in N.$$

If in theorem 4 condition (iii) is omitted and condition (iv) is replaced by the reduced game property on F_2 then a characterization of the CIS-value on F_1 is obtained. It is left to the reader to show that the CIS-value is indeed the unique cost allocation method on F_1 which satisfies symmetry, invariance w.r.t. strategic equivalence, and the reduced game property on F_2.

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