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THE DIVISION OF PROFIT IN SEQUENTIAL INNOVATION RECONSIDERED*

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Abstract

Sequential innovation with actual patent infringement and uncertainty in litigation is analyzed. Comparative statics shows that within a wide range of model parameters, a basic researcher holding a patent is able to extract all the profit facilitated by the basic innovation. The patent holder achieves this by offering a licencing contract which the subsequent innovator accepts. It is further demonstrated that under rather general circumstances, broader patent breadth may diminish the patent holder's chance to achieve the desired equilibrium outcome: that is to extract all the profit from the subsequent innovator marketing the product.

Keywords: Infringement, imperfect patent protection, patent breadth

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I. Introduction

The design of an optimal patent system has re-emerged recently as the subject of economic inquiry. An intriguing fact is that the patent law does not and, perhaps, cannot circumscribe its objects, individual patents, in a precise and unquestionable way. For instance, the most important statutory criteria for patentability are "novelty" and "nonobviousness". Novelty can be interpreted as the criterion to determine whether the new invention was not in "prior art", i.e., whether the inventor has really invented something. The nonobviousness criterion excludes patentability of inventions for which it is "obvious" that they could be invented with sufficient effort, even though no one has bothered to do so so far. Infringement of patent can be generally defined as the nonsanctioned manufacture, use, making or sale of an invention for which a valid patent has been issued. Typically, infringement constitutes a situation where a new invention significantly overlaps with the patented technology. Significant overlap in turn is determined again in terms of novelty and nonobviousness — which are subject to qualification and interpretation. Therefore, legal determination of infringement can be a difficult task. It is not too far-fetched to imagine a complicated infringement case where the legal institutions are incapable of sound judgment.[1]

One typical aspect of R&D is that a commercially profitable innovation results from basic research. Broadly speaking, basic research can be generated by individual researchers, independent research institutions such as universities, or industrial research laboratories. A rather unanticipated fact that has been noted by Jewkes et al. (1969) in their case study of seventy significant inventions is that more than one half of them could be attributed to individual inventors who had no capacity in commercializing their achievements. Thus the distribution of profit between basic researchers without directly marketable products and the vendors of marketable products derived from the basic technology should be of utmost importance and interest to economists. To illustrate, Robert W. Kearns, a former engineering professor who patented his intermittent wiper system in 1967, was awarded back royalties of $10.2 million in a settlement with Ford Motor Company in 1990 and $11.3 million by Federal Court in his patent infringing case against Chrysler Corporation in 1992. He has also sued the General Motors Corporation, the Toyota Motor Corporation, Fiat S.p.A and most large Japanese car manufacturers.[2]

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[1] Dreyfuss (1989): "In general, the court (Court of Appeals for the Federal Circuit, the specialized court established in 1982 to focus on patent jurisdiction) has been successful with issues like obviousness... issues that arise mainly in enforcement proceedings have not been nearly as well explicated.... (the court) has yet to announce clear tests for many of the issues involved in the infringement question." For instance, when asked to determine whether certain miniaturized calculators infringed Texas Instrument's pioneering calculator patent, the CAFC contradicted its statements by first recognizing significance of "pioneer status" of the patent but later rejecting the application of the doctrine of equivalents which favors the patentee in *Texas Instruments, Inc. v. United States International Trade Commission* (Federal Circuit, 1986). For a discussion of the case in detail, see also Merges and Nelson (1990).

These impressive episodes should not divert our attention from the fact that, as a rule, technological advancement nowadays demands more than just an ingeniously novel idea; that availability of sophisticated laboratory equipment and a substantial capital investment may also be essential. Nevertheless, companies and research institutions have realized by now that it can be profitable to sue for patent infringement for products they hold the patents to, but which they have never produced, never intended to produce, sometimes even considered non-producible or non-marketable. In a recent case involving commercial companies, Procter & Gamble sued Whitehall Laboratories and its parent, American Home Products Corporation over the cold remedy with ibuprofin although P.&G. has never had a product of this kind. Adopting similar strategies, Honeywell Inc. won a big patent case in 1992 against Minolta Camera Company of Japan over the auto-focus camera lens, a technology Honeywell never itself developed commercially.[3] Iowa State University, in yet another instance where independent research institutions try to claim the intellectual property right, was able to collect licensing fees of up to $18 million from Sharp Corp. of Japan, NEC Corp., and Canon Inc. in 1992 on its 1973 patent covering an encoding process in the fax machine.[4] Most research universities do have procedures and personnel to file patent applications and deal with licensing agreements as well as infringement suits.

Despite the economic significance of cumulative innovation, plausible theoretical models related to this issue are still rare. An exception is Green and Scotchmer (1995). They address the issue of optimal patent breadth and duration, and the role of different legal mechanisms when innovation takes place in two stages. The authors argue that the potential patent holder may lack incentives to invest in the first place, because not all the social value facilitated by basic research can be transferred from the second generation products. In their paper as in ours, quality improvement is the only indicator for patent protection and infringement.[5] Moreover, we are concerned with the division of profit due to imperfect patent protection. By imperfect patent protection we mean here that the outcome of infringement litigation is uncertain. Indeed, both parties may agree privately whether or not an infringement occurs. Yet the court may come to a different conclusion.

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[2] See *New York Times*, June 12, 1992.

[3] See Edmund L. Andrews, *New York Times*, Nov. 9, 1992 for these and other examples.

[4] See *Wall Street Journal*, Oct. 12, 1992.

[5] In general, however, this need not be the case. Development can occur in one or more of the many dimensions of product characteristics. With heterogeneous consumers, it may be impossible to single out an unambiguous direction of quality improvement. For instance, motivated by Klemperer's (1990) product variety model, Lerner (1994) develops a proxy for patent scope based on the International Patent Classification scheme which is more in parallel to the concept of product differentiation.

Our main conclusion is that very often the original patent holder may not lack incentives to invest in basic research in the first place! We arrive at this conclusion by identifying conditions on the probability of winning infringement litigation that guarantee the patent holder extraction of all the profit by offering a licencing contract to the subsequent innovator which the latter accepts. Our rather optimistic conclusion contrasts with the more pessimistic tenor of Green and Scotchmer (1995) who investigate the efficacy of various policies intended to insure that the patent holder receives a large enough profit share.

To put our contribution into broader context, we distinguish between two strands of literature related to patent protection: the "fencepost" system literature and the "signpost" system literature. Adopting the fencepost interpretation of patent scope, Hortsmann et al. (1985) look at patents as information transfer mechanism and assume "limited but exact patent coverage". Within the second strand of literature, Waterson (1990) looks at uncertainty in patent infringement litigation from a different angle and develops a model where like in ours the concept of "limited but inexact patent coverage" is employed. Whereas we explicitly require concavity of the patent holder's winning probability, he implicitly imposes an equivalent property on the "court cost function" defining litigation costs and damage fees awarded to the patent holder. While Waterson is primarily concerned with the impact of patent protection on product variety — and the implied consumer welfare — in a horizontal product differentiation model, our emphasis lies on appropriability and incentives to innovate in a vertical product differentiation model with sequential innovation.

The paper is organized as follows. In Section 2, we specify a model to be used to investigate the division of profit between an initial patent holder with no marketing power and a subsequent innovator of a derived product. For simplicity we assume that after the first innovation is made, the idea for each derivative improvement occurs to only one firm which is uniquely capable of developing it at a given cost. As advertised above, we show that under some circumstances the patent holder collects all the profit facilitated by its basic research. The necessary and sufficient conditions for the latter outcome suggest a simple intuitive explanation why patent holders tend to chase after those subsequent innovators whose products are sufficiently novel and highly profitable. Section 3 is devoted to comparative statics. One salient feature unearthed in the course of this investigation is the fact that the motion to postpone a patent infringement suit may have strategic reasons. The long lasting Intel-AMD suit over microprocessors and the recent Kodak-Sony patent dispute constitute typical examples.[6] Several concluding remarks are made in Section 4. The more technical or elaborate proofs are collected in an Appendix.

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[6] For details, see *Electronic News*, Nov. 22, 1993 and *Wall Street Journal*, March 23, 1994.

II. The Model

There are one research institution and one firm. The research institution is called the patent holder (PH) hereafter. It has acquired a patent on its invention with quality x. We set $x = 0$ without loss of generality. The patent breadth granted is y^*. Quality x is just a basic research outcome and has no market value *per se*. The firm is capable of developing a new product of quality y with $x \leq y \leq y^*$ so that it would surely infringe on the patent held by PH. The cost of developing quality y is c_y. Once developed, the new product can be produced at zero cost and has market value π_y.

The crucial elements of patent litigation can be described as follows: Each party incurs the same litigation cost $L > 0$. There is an objective probability f(y) of PH winning in litigation. The existence of such an f(y) can be defended on the grounds that there is no perfect patent protection due to the nature of current patent law and the process of infringement litigation. While both parties may agree privately whether or not an infringement occurs, the court may come to a different conclusion. Occasionally, we treat y as variable and f(y) as a decreasing function of $y \in [0,y^*]$ with $f(0) = 1$ and $f(y^*) = 0$. The further away from x a new invention is, the less likely is a verdict of infringement.

We model the strategic interaction as a <u>strategic</u> <u>game</u> between PH and the firm. The game lasts one period which is defined as the time interval beginning when PH makes the licensing offer and ending when the infringement issue is resolved. The two players take several steps during the period. There is no discounting within the period.

Both players enter the game with exogenously given and commonly known y. PH, as a first mover, makes a licensing agreement offer simply by specifying R with $R \in [0,\infty)$. We view R as a fixed-fee royalty: The number R represents the amount to be paid by the firm for the right to market its product. By offering R=0, PH tolerates the infringement without legal recourse. Facing the offer, the firm has three strategic alternatives: (i) quit the project; (ii) pay the royalty proposed by PH; (iii) challenge the patent infringement allegation. In the latter contingency, PH has to make one more move: take no action or litigate. In accordance with U.S. practice, we assume that even if it loses, the firm retains the profit from marketing this application while paying its litigation costs plus back royalties. Figure 1 summarizes the <u>extensive</u> <u>form</u> of the game, showing the order of decisions and the resulting (expected) payoffs.

[*Figure 1 about here*]

Set M={No-action, Litigation} and N={Take-it, Leave-it, Drop-out}. Then the <u>normal</u> <u>form</u> of the game has strategy spaces $S_{PH}=\mathbf{R}\times M^{\mathbf{R}}$ for PH and $S_F=N^{\mathbf{R}}$ for the firm. We consider strategy pairs that are Nash equilibria, i.e. each player chooses a strategy that maximizes its expected payoff given the other player's strategy. Moreover, we require subgame perfection: Equilibrium pairs of strategies induce equilibrium play in all subgames.

We distinguish four types of pure strategy equilibria. Which types occur, depends on the numerical specification of the model.

1. The Take-it equilibrium is characterized by an R_t with

   $\pi_y - c_y - R_t \geq 0,$

   $\pi_y - c_y - R_t \geq \pi_y - c_y - f(y)R_t - L$, and

   $f(y)R_t - L \geq 0.$

   The firm responds with Take-it to this offer. Should the firm play Leave-it in response to this offer, then PH would counter with Litigation.

2. The Leave-it equilibrium is characterized by an offer R_l with

   $\pi_y - c_y - f(y)R_l - L \geq 0,$

   $\pi_y - c_y - f(y)R_l - L \geq \pi_y - c_y - R_l$, and

   $f(y)R_l - L \geq 0.$

   The firm responds with Leave-it to this offer and PH counters with Litigation.

3. The No-Action equilibrium is characterized by an offer R_n with

   $f(y)R_n - L \leq 0$, and

   $\pi_y - c_y \geq 0.$

   The firm responds with Leave-it to this offer and PH counters with No-action.

4. The Drop-out equilibrium is characterized by an offer R_d with

   $\pi_y - c_y - R_d \leq 0,$

   $f(y)R_d - L \geq 0$, and

   $\pi_y - c_y - f(y)R_d - L \leq 0.$

   The firm responds with Drop-out to this offer. Should the firm respond with Leave-it to this offer, then PH would counter with Litigation.

PH, as a leader in this game, has the sole interest in manipulating the offer R so as to collect the highest possible profit share from the firm. Therefore we will not pursue any further the No-Action and

Drop-out equilibria where PH cannot generate any positive gain.[7]

We proceed with the following simplifying assumption:

(A1) $\pi_y = a \cdot y$, $c_y = c \cdot y$ where a and c are constants satisfying $a > c \geq 0$.

A constant marginal revenue occurs in a standard vertical (quality) differentiation problem. There consumers have utility functions of the form $U = \Theta y - P$ where Θ is a taste parameter and P is the price charged for the product of quality y. The distribution of tastes across consumers is given by the uniform distribution on the interval $[\theta', \theta]$ with $1 \geq \theta' \geq 0$ and $\theta = \theta' + 1$. Then, given y, the firm maximizes its gross profit by choosing the price level $P_y = \frac{y\theta}{2}$. The resulting gross profit is $\pi_y = \frac{y\theta^2}{4}$. Put $a = \theta^2/4$.

We first explore the possibility that <u>PH can extract all the profit from the firm</u>, i.e., where an offer $R_t = (a-c)y$ gets accepted in equilibrium. Necessary and sufficient conditions for such an equilibrium outcome are described in Proposition 1.

Proposition 1. PH can extract all the profit it facilitates from the firm if and only if

(a) $\quad y - yf(y) \leq \frac{L}{a - c}$ and

(b) $\quad yf(y) \geq \frac{L}{a - c}$.

<u>Proof</u>: For a licensing agreement to prevail, i.e., a Take-it equilibrium to exist, the following conditions must be satisfied:

$$\pi_y - c_y - R_t \geq 0,$$

$$\pi_y - c_y - R_t \geq \pi_y - c_y - f(y)R_t - L, \text{ and}$$

$$f(y)R_t - L \geq 0.$$

With the previous specification, they are equivalent to:

$$R_t \leq (a - c)y, \tag{1}$$

$$R_t \leq \frac{L}{1 - f(y)}, \text{ and} \tag{2}$$

$$R_t \geq \frac{L}{f(y)}. \tag{3}$$

[7] In a more complicated many-firm setting, however, these potential types of equilibria may significantly impact upon PH's decision-making.

Now assume (1) with equality. Then (2) and (3) are equivalent to

$$y[1 - f(y)] \leq \frac{L}{a - c} \text{ and} \tag{4}$$

$$yf(y) \geq \frac{L}{a - c}. \tag{5}$$

□□

Some simple comparative statics can help develop intuition for this result. First of all, suppose a, c, y, and L are given such that $y \geq \frac{L}{a - c}$ and f(y) is treated as a variable. Then (4) and (5) are simultaneously satisfied if and only if f(y) is sufficiently large. This can be seen from the game tree in Figure 1. To achieve a Take-it equilibrium with $R=(a - c)y$, PH's threat of litigation in case the firm rejects the offer has to be credible, that is, $f(y) \cdot R - L \geq 0$. If $y \geq \frac{L}{a - c}$ or, equivalently, $(a - c)y \geq L$ and if $R=(a-c)y$, then the inequality $f(y)R - L \geq 0$ holds trivially for $f(y)=1$. By continuity, a high and only a high winning probability f(y) helps PH achieve the licensing agreement where the firm obtains zero payoff. More specifically, we observe that $(a) \wedge (b)$ implies $f(y) \geq \frac{1}{2}$.

On the other hand, when a, c, y, and L are given with $y < \frac{L}{(a - c)}$, the situation changes drastically. Whereas (a) is always satisfied under this assumption, (b) breaks down for all $f(y) \in [0,1]$. Thus there does no longer exist the Take-it equilibrium PH is longing for. It is obvious from the game tree that PH is seriously concerned about the potential loss from litigation and therefore takes no action even if the firm dares to infringe.[8] Under this particular specification, a smaller y does not benefit PH in achieving its goal of exploiting the firm — even with a high winning probability. To sum up, the preceding comments suggest there might exist lower and upper bounds for those y which permit the type of Take-it equilibrium in question.

To extend the analysis one step further and arrive at such bounds, we next consider a situation with exogenously given L, a, c, y^*, and a function $f: [0, y^*] \mapsto [0, 1]$ satisfying:

(A2) $f(0) = 1$, $f(y^*) = 0$, and f is twice differentiable with $f' < 0$, $f'' \leq 0$.

[8] This could explain why some PHs never bother to file suit against the manufacturers of low-tech clones, targeting instead those subsequent prominent manufacturers whose products are sufficiently novel and making significant profit.

It is natural to assume f to be decreasing. It also appears plausible that PH's winning probability drops faster as y moves further away from x=0, or the more y approaches the delimiter of patent protection, y^*. From a different perspective, *ceteris paribus*, the firm is enjoying "increasing returns to litigation" as y varies.

We perform comparative statics with respect to $y \in [0,y^*]$. For this purpose, we introduce the functions

$$g_1(y) = y[1 - f(y)]$$

and

$$g_2(y) = yf(y),$$

which appear in (4) and (5) and, obviously, play a critical role in our analysis. Notice that $g_1' = 1 - f - y \cdot f' > 0$ and $g_1'' = -2f' - y \cdot f'' > 0$ in the interval $(0,y^*)$. Hence g_1 is strictly increasing and strictly convex in y with $g_1(0)=0$ and $g_1(y^*)=y^*$. Further notice that $g_2' = (y \cdot f)' = f + y \cdot f'$ and $g_2'' = (y \cdot f)' = 2f' + y \cdot f'' < 0$. Thus g_2 is strictly concave in y with $g_2(0) = g_2(y^*) = 0$. Consequently, g_2 has a unique maximizer $\hat{y}$ in $(0,y^*)$. This maximizer is given as the unique solution of the first order condition

$$g_2'(\hat{y}) = f(\hat{y}) + \hat{y} \cdot f'(\hat{y}) = 0.$$

Finally, let $\bar{y}$ denote the 'median' of f, i.e. $\bar{y}$ is implicitly given by the condition

$$f(\bar{y}) = \tfrac{1}{2}.$$

Lemma 1. $\hat{y} \leq \bar{y}$ and $0 < y^*/2 < \bar{y} < y^*$.

Proposition 2. Suppose

(c) $\qquad \hat{y}f(\hat{y}) \geq \frac{L}{a - c}.$

Then there exist y_l, y_r with the following properties:

(i) $0 < y_l \leq y_r < y^*$;

(ii) PH can extract all the profit it facilitates from the firm iff $y \in [y_l, y_r]$.

Having arrived at these conclusions, one still has to be very cautious in interpreting Propositions 1 and 2. First of all, the Take-it equilibrium described in the propositions is of a particular kind where an offer $R_t = (a - c)y$ gets accepted in equilibrium. When condition (c) holds as a strict inequality and y lies in the interval (y_l, y_r), there is always sufficient slack for a Take-it equilibrium with $R_t < (a - c)y$.[9] In other words, not all Take-it equilibria guarantee 100 per cent profit transfer. Secondly, one should wonder if a Take-it equilibrium guarantees the highest profit that PH can earn, since R_l, the offer made in a Leave-it equilibrium could well be above $(a - c)y$. To clarify that matter, we present the following proposition.

Proposition 3. Suppose (c) holds and PH is confronted with $y \in [y_l, y_r]$. Then the Take-it equilibrium with $R = (a - c)y$ generates the highest possible equilibrium payoff for PH.

Proof: By Proposition 2 we know that $[y_l, y_r]$ is a nonempty set when condition (c) is satisfied. Moreover, PH is capable of achieving the Take-it equilibrium in which the offer $R = (a - c)y$ gets accepted and, hence, $(a - c)y$ constitutes PH's equilibrium payoff. By (A1), the latter is guaranteed to be greater than the payoffs that can be obtained via a Drop-out or a No-Action equilibrium. On the other hand, a Leave-it equilibrium is subject to the following qualifications:

$$\pi_y - c_y - f(y)R_l - L \geq 0,$$
$$\pi_y - c_y - f(y)R_l - L \geq \pi_y - c_y - R_l, \text{ and}$$
$$f(y)R_l - L \geq 0.$$

With the previous specification, these are equivalent to:

$$(a - c)y - f(y)R_l - L \geq 0, \tag{6}$$

$$[1 - f(y)]R_l - L \geq 0, \text{ and} \tag{7}$$

$$f(y)R_l - L \geq 0. \tag{8}$$

Even if a Leave-it equilibrium exists for some $y \in [y_l, y_r]$, the corresponding expected payoff for PH is at most $f(y)R_l - L$. From (6) we can infer the following inequality:

$$f(y)R_l - L < f(y)R_l + L \leq (a - c)y. \qquad \square\square$$

It is also clear from the foregoing proof that a Leave-it equilibrium would always generate a profit less than $(a - c)y$ for PH. This observation combined with the assertion of Proposition 3 justifies our

[9]Namely, in the proof of Proposition 2, the bounds y_l and y_r are constructed such that $y \in (y_l, y_r)$ and $\hat{y}f(\hat{y}) > \frac{L}{a - c}$ imply $yf(y) > \frac{L}{a - c}$ or $(a - c)y > \frac{L}{f(y)}$ which in turn implies that $[\frac{L}{f(y)}, (a - c)y]$ is neither the empty set nor a singleton. Therefore there exists $R_t < (a - c)y$, say, $R_t = (a - c)y - \epsilon$ where ϵ is a small positive number, such that conditions (1), (2) and (3) are simultaneously satisfied.

almost exclusive focus on the properties of the complete-profit-transfer Take-it equilibrium.

It is obvious that our conclusions about the distribution of profit rely crucially on condition (c). The mathematical interpretation of (c) is straightforward:

$$\mathrm{Max}_{y\in[0\ ,\ y^{*}]}\, g_2(y) = \mathrm{Max}_{y\in[y_l,\ y_r]}\, g_2(y) \geq \frac{L}{a - c}.$$

The economic meaning of (c) is somewhat more subtle than that. Aiming at complete exploitation of the firm, PH has to balance two factors moving in opposite directions: what the firm is capable of, i.e., the magnitude of y, and how significant the chance is that he can win the infringement case, i.e., the range of f(y). More specifically, while making a higher offer to cope with a higher y, PH strives to maintain a credible threat to avoid the opportunism of the firm as f(y) declines. Under condition (c), PH masters this balancing act and extracts all profit.

So far we have shown in the previous propositions that PH is capable of achieving a Take-it equilibrium by offering a licensing agreement that transfers the entire profit (a - c)y from the firm when condition (c) is satisfied. This outcome would provide PH the maximal incentive to invent under the specification of our model. Meanwhile, PH's payoff from this type of agreement is always greater than that from a Leave-it equilibrium — even if the latter is feasible.

III. Comparative Statics

In this section we focus on the comparative statics with respect to several key variables — within their most interesting range in our set-up. In the sequel we denote $\mathcal{L} \equiv y_r - y_l$, the length of the interval of y where a Take-it equilibrium with all the profit being transferred can be obtained. We first investigate how $\mathcal{L}$ is responding to variations of the litigation cost L, the gross profit parameter a, and the product development cost parameter c. It suffices to see how $\mathcal{L}$ depends on the compound parameter $k \equiv \frac{L}{a - c}$.

Lemma 2. Suppose $\frac{\overline{y}}{2} \neq \hat{y}f(\hat{y})$, then $\frac{\partial \mathcal{L}}{\partial k} < 0$ for $k \in \left(\frac{\overline{y}}{2},\ \hat{y}f(\hat{y})\right)$. Moreover, the corresponding intervals $[y_l(k), y_r(k)]$ are strictly nested.

Intuitively, a higher litigation cost should have a stronger threatening effect on the firm who takes a chance when turning down the offer, since it will face a higher expected loss once it loses. However, Lemma 2 says that even when the litigation cost is in the "favorable" range where a complete-profit-transfer via a Take-it equilibrium can be assured, higher litigation cost will damage the manipulative power of PH.[10] What we observe is that when k rises above $\frac{\bar{y}}{2}$ while staying below the bound $\hat{y}f(\hat{y})$, $g_2(y)$ starts to effectively determine the boundaries of $\mathcal{L}$. Therefore the sensitivity of $\mathcal{L}(k)$ with respect to k depends only on the strict concavity of $g_2(y)$. Recall that $g_2(y) \geq k$ is essentially the same as condition (3), $R \geq \frac{L}{f(y)}$. It is then obvious that a higher L makes it harder to satisfy (3). A similar interpretation can be applied to the parameters a and c.

Lemma 3. $\mathcal{L}(k)$ is strictly concave in k and there exists a unique $\tilde{k} \in (0, \frac{\bar{y}}{2})$ such that $\mathcal{L}(\tilde{k}) \geq \mathcal{L}(k)$ for all $k \in \mathbf{R}_+$.

Lemmata 2 and 3 convey a complete picture of the comparative statics with respect to k. It is obvious that the convexity (concavity) of g_1 (g_2) plays a crucial role here. This property follows directly from the concavity of f(y), the probability of PH winning in litigation.

In a second type of comparative statics, we investigate how $\mathcal{L}$, the length of the interval where PH can extract all the surplus, is affected by a change of patent protection. Intuition may suggest that the best way to help PH transfer profit from the firm is to grant PH a broad patent protection. Intriguingly enough, this is a premature conclusion as the next proposition shows. One more simplifying assumption, (A3) is imposed to establish the result. Prior to that we have to extend the model appropriately by postulating that f takes the more general form $f(y; y^*)$, $0 \leq y \leq y^*$, where the patent breadth $y^* > 0$ is treated as variable in the sequel. The obvious notation $\mathcal{L}(k; y^*)$, $\tilde{k}(y^*)$, etc. will be used.

(A3) $f(y; y^*) = f(\frac{y}{y^*}; 1)$, i.e., $f(y; y^*)$ is homogeneous of degree 0.

[10] Although not specifically modeled here, a longer litigation process can also be viewed as increasing litigation costs. Therefore a motion to postpone a patent infringement suit may have strategic reasons.

(A3) stipulates that the winning probability for PH depends only on the ratio y/y^*, not on the absolute magnitude of y or y^*. An extremely high y^* might correspond to a very vague claim such as "All non-human transgenic mammals" or "All hand-use calculators."[11] The broader the patent protection, the easier is it for an allegedly infringing firm to challenge the patent claim. In a model with merely one-dimensional quality choice, imposing (A3) constitutes a simple attempt to capture that aspect of reality. (A3) has several immediate consequences:

Lemma 4. The functions $y_l(k;y^*)$, $y_r(k;y^*)$, and $\mathcal{L}(k;y^*)$ are homogeneous of degree 1 in $(k;y^*)$. The functions $\hat{y}(y^*)$ and $\bar{y}(y^*)$ are homogeneous of degree 1 in y^*.

Let us first state a result that conforms to intuition: As patent protection becomes broader, $\mathcal{L}(k;\ y^*)$ increases, i.e. the size of the interval where PH can extract all the surplus increases.

Proposition 4. The following three assertions hold:

(I) $\frac{\partial}{\partial k}\mathcal{L}(k;y^*)$ is strictly increasing in $y^* > 0$ as long as $0 < k < \bar{y}(y^*)/2$.

(II) $\mathcal{L}(k;y^*)$ is strictly increasing in $y^* > 0$ as long as $0 < k < \bar{y}(y^*)/2$.

(III) $\widetilde{k}(y^*)$ is strictly increasing in $y^* > 0$.

Let us now proceed to the promised, somewhat less intuitive result: As patent protection becomes broader, the relative size of the interval where PH can extract all the surplus may decrease.

Proposition 5. For any $0 < y^* < y^{**}$, there exists $\kappa(y^*, y^{**}) > 0$ such that

$$\frac{\mathcal{L}(k;\ y^{**})}{y^{**}} < \frac{\mathcal{L}(k;\ y^*)}{y^*} \quad \text{for all } 0 < k < \kappa(y^*, y^{**}).$$

Proposition 5 states that even though $\mathcal{L}(k,\ y^*)$ increases as patent protection becomes broader, $\mathcal{L}/y^*$, that is the relative size of the interval where PH can extract all the surplus, may be falling for certain k. The manipulative power of PH measured as the fraction of infringing y that provide maximal incentive to PH to innovate, can apparently diminish when the government institutes broader protection. Let us briefly explain how monotonicity, concavity and homogeneity of f can lead to such a conclusion. Homogeneity of degree zero of $f(y;\ y^*)$ yields homogeneity of degree one of $\mathcal{L}(k;\ y^*)$. The

[11] See U.S. Patent No. 4,736,866, issued Apr. 12, 1988. This patent is granted to Doctors Philip Leder and Timothy Stewart of the Harvard Medical School for their successful work on transgenic mice. For the hand-held calculators case, see *Texas Instruments Inc. v. United States International Trade Commission* (Federal Circuit 1986.)

impact of higher y^* on the ratio $\mathcal{L}/y^*$ is then immediate, since $\mathcal{L}(k;\ y^*)/y^*$ can be reduced to $\mathcal{L}(k/y^*;1)$. Moreover, $f(y;\ y^*)$ strictly decreasing and strictly concave in y implies that $\mathcal{L}(k;\ y^*)$ is strictly increasing and strictly concave in $k \leq \tilde{k}$. Thus for fixed k, the normalized $\mathcal{L}(\frac{k}{y^*};\ 1)$ is greater than the normalized $\mathcal{L}(\frac{k}{y^{**}};\ 1)$.

IV. Concluding Remarks

We have studied the division of profit between a patent holder and a derived product producer in an environment with uncertainty about the outcome of infringement litigation. Comparative statics with respect to several key parameters has been performed. Our analysis identifies the conditions on model parameters which permit a complete-profit-transfer equilibrium.

To reiterate, we have highlighted some of the most important differences between the "signpost" and "fencepost" interpretation of the patent system. In particular, the elements of uncertainty and concavity in the probability distribution function play prominent roles in arriving at conclusions that are quite different from those obtained by Green and Scotchmer (1995) for a similarly constructed quality improvement game. They claim that in general not all the profit can be transferred to the first innovator and therefore the patents should last longer when a sequence of innovations is undertaken by different firms. Their conclusion of profit erosion is derived primarily from the bargaining power of the second innovator whose threat of not bringing the product to market strengthens his position in the negotiation process. Extending their game one more stage further into the post-marketing period and addressing the issue of stochastic outcomes in litigation, we try to accomplish a better approximation of the current patent practice. By committing to the credible threat of court action, our PH enjoys the advantage of a first-mover with the occasional opportunity of extracting all the profit accruing to the second generation product. As a consequence, proposals to increase patent length in order to enhance incentives for basic research look less attractive for a "signpost" system of patents like ours.

As for the optimal patent breadth, Green and Scotchmer present a special case in which unlimited patent protection may not be optimal when the uncertainty on the exact development y is not resolved. In a quite different context, we arrive at another refutation of the argument that broader patent breadth unconditionally makes PH better off. However, unlike theirs our conclusion is not derived from uncertainty in development, but rests on a homogeneity assumption which renders the relative improvement measure y/y^* a main determinant of transferability of profit from the firm to the patent holder.

We believe that introducing uncertainty of the outcome of a patent infringement suit enriches and furthers the economic understanding of current patent systems. It opens a multi-facet, widely unexplored research area of law and economics. Several elements might be added to our model: for instance, endogenous choice of y and y^*; informational asymmetries discussed in the recent licensing literature (Gallini and Wright, 1990) and litigation literature (Bebchuk, 1984; Meurer, 1989; Reinganum and Wilde, 1986); different liability rules adopted by other countries; competition between PH and firm(s); etc.

There is certainly an element of uncertainty that we intentionally ignore in the present formal analysis, concerning the allocation of litigation costs. As Dreyfuss (1989) points out, the Court of Appeals for the Federal Circuit — the specialized court established in 1982 to focus on patent jurisdiction — has failed to clarify the law on pecuniary damages.[12] However, in our model we assume the smallest conceivable damages for PH: the licensing fee he is asking for. Given that PH can only do better under the prevalent practice, our qualitative results in favor of PH persist. Yet another qualification could be that outrageous licensing fee requests are corrected downward by the court or affect negatively f, the probability of winning litigation.

[12] The Patent Act permits the court to treble the damages, 35 U.S.C. § 284 (1982), and to award attorney's fees and cost in "exceptional cases" 35 U.S.C. § 285. These enhanced damages are typically awarded to penalize willful misbehavior. For example, Triarch Industries won treble damages against Trans Global Imports (*The Weekly Home Furnishings Newspaper*, Feb 21, 1994) and Exxon Corp. won award of $18 million in attorney's fees in its infringement case against Lubrizol Corp (*Wall Street Journal*, Feb. 19, 1993.)

APPENDIX

Proof of Lemma 1: Recall that $g_2(y)$ is strictly concave in y with $g_2(0) = 0$ and $g_2(y^*) = 0$. By Takayama(1985) Theorem 1.C.3: f is concave on $(0, y^*)$ if and only if for any $x, y \in (0, y^*)$:

$$f'(y)\cdot(x - y) \geq f(x) - f(y).$$

Evaluate this inequality at $y = \bar{y}$ and let $x \to 0$. Then

$$-f'(\bar{y})\cdot\bar{y} \geq 1 - \tfrac{1}{2} \text{ or } f'(\bar{y})\cdot\bar{y} \leq -\tfrac{1}{2}.$$

Adding $f(\bar{y}) = \frac{1}{2}$ to the latter inequality yields

$$g_2{}'(\bar{y}) = \bar{y}\cdot f'(\bar{y}) + f(\bar{y}) \leq 0.$$

Strict concavity of g_2 and $g_2{}'(\hat{y}) = 0$ imply the assertion $\hat{y} \leq \bar{y}$. Further, (A2) has the immediate implication $0 < y^*/2 < \bar{y} < y^*$. □□

Proof of Proposition 2: Recall that $g_1(y)$ is strictly increasing and strictly convex in y with $g_1(0) = 0$ and $g_1(y^*) = y^*$. Hypothesis (c) amounts to $g_2(\hat{y}) \geq \frac{L}{a - c}$.

Part (i):

$g_2(y)$ achieves its maximum at $\hat{y}$ where $g_2{}'(\hat{y}) = f(\hat{y}) + \hat{y}f'(\hat{y}) = 0$. By the hypothesis, the continuity and other properties of g_2, and the intermediate value theorem, there exist $z_l \in (0, \hat{y}]$ and $z_r \in [\hat{y}, y^*)$ such that $g_2(\hat{y}) \geq g_2(z_l) = g_2(z_r) = \frac{L}{a - c}$. If (c) holds with equality, then $z_l = z_r = \hat{y}$. If (c) holds with strict inequality, then $z_l < \hat{y} < z_r$.

Next note that (c) implies $y^* > g_2(\hat{y}) \geq \frac{L}{a - c} > 0$. Then, by the continuity and other properties of g_1 and the intermediate value theorem, there exists a unique $z \in (0, y^*)$ with $g_1(z) = \frac{L}{a - c}$. To compare the magnitudes of z_l and z, we consider two subcases.

Subcase (i-a): $\bar{y}/2 \geq \frac{L}{a - c}$. Now by definition, $f(\bar{y}) = \frac{1}{2}$ and therefore $\frac{L}{a - c} \leq \bar{y}/2 = g_1(\bar{y}) = g_2(\bar{y})$. Therefore, $\bar{y} \in [z_l, z_r]$, by the strict concavity of g_2. Also, $z \leq \bar{y}$, by the strict monotonicity of g_1. Hence $z \leq z_r$. Moreover, $0 = g_1(0) = g_2(0)$, $\frac{1}{2}\cdot\bar{y} = g_1(\bar{y}) = g_2(\bar{y})$, strict convexity of g_1 and strict concavity of g_2 imply $g_1(y) < \frac{1}{2}\cdot y < g_2(y)$ for $y \in (0, \bar{y})$. If $\bar{y} = z_l$, then $z = \bar{y} = z_l$. If $\bar{y} > z_l$, then $g_1(z_l) < g_2(z_l) = \frac{L}{a - c}$. Thus $z > z_l$. In any case, therefore, $z \in [z_l, z_r]$. •

Subcase (i-b): $\bar{y}/2 < \frac{L}{a - c}$. Then $\bar{y} \notin [z_l, z_r]$ and $z > \bar{y}$. By Lemma 1, $\bar{y} \geq \hat{y} \geq z_l$. Thus $z > z_l$. •

Now set $y_l = z_l$ and $y_r = \min[z_r, z]$. Then (i) is satisfied.

Part (ii):

We commence with the sufficiency proof. When condition (c) holds and $y \in [y_l, y_r]$, then $y \in [z_l, z_r]$ and the strict concavity of g_2 implies (b) $yf(y) \geq \frac{L}{a-c}$. Further $y \in [y_l, y_r]$ implies $y \leq z$. Since $g_1(y)$ is an increasing function in $y \in [0, y^*]$, condition (a) $y[1 - f(y)] \leq \frac{L}{a-c}$ holds as well.

Now we turn to the necessity proof. (b) implies that $y \in [z_l, z_r]$. (a) implies that $y \leq z$. Together (a) and (b) imply $y \in [z_l, \min\{z_r, z\}] = [y_l, y_r]$. Note that we know from Proposition 1 that by offering $R_t = (a - c)y$, PH can extract all the profit it facilitates from the firm if and only if (a) and (b) both hold. We have shown that under the hypothesis (c), the combination of (a) and (b) is equivalent to $y \in [y_l, y_r]$. This completes the proof. □

Proof of Lemma 2: Suppose a, c, y, and f(y) are given such that $k \in \left(\frac{\bar{y}}{2}, \hat{y}f(\hat{y})\right)$. This can be referred to subcase (i-b) in the proof of Proposition 2. Together with the supposition $\frac{\bar{y}}{2} \neq \hat{y}f(\hat{y})$ they imply that (c) holds as a strict inequality and thus $z_r > \hat{y} > z_l$. It is also known from (i-b) that $\bar{y} \notin [z_l, z_r]$ by the strict concavity of g_2 and $z > \bar{y}$ by the strict monotonicity of g_1. By Lemma 1, $\bar{y} \geq \hat{y}$ and $\bar{y} \notin [z_l, z_r]$ imply $\bar{y} > z_r$. Thus $z > \bar{y} > z_r$. So $\mathcal{L} = y_r - y_l = \min[z_r, z] - z_l = z_r - z_l$. The strict concavity and the other properties of g_2 imply that for all k_1, k_2 such that $\frac{\bar{y}}{2} < k_1 < k_2 < \hat{y}f(\hat{y})$ the corresponding $z_r(k_1)$, $z_l(k_1)$, $z_r(k_2)$ and $z_l(k_2)$ have the following order:

$$z_l(k_1) < z_l(k_2) < \hat{y} < z_r(k_2) < z_r(k_1) \text{ or}$$

$$\mathcal{L}(k_1) = [z_r(k_1) - z_l(k_1)] > [z_r(k_2) - z_l(k_2)] = \mathcal{L}(k_2).$$

This implies the assertion. □□

We need a technical auxiliary result to proceed:

Lemma A. Suppose that $k = g(y)$ is strictly increasing, concave (convex) and twice continuously differentiable in the interval (a, b) and suppose that $g'(y) \neq 0$ for $y \in (a, b)$. Then $y = g^{-1}(k)$ exists and is monotone, convex (concave), and twice continuously differentiable with respect to k.

Proof: The existence, monotonicity, and twice continuous differentiability of g^{-1} are assured by the inverse function theorem; see Flett (1966; Th. 10.9.5). Moreover, we have

$$g^{-1\prime}(k) = \frac{1}{g'(g^{-1}(k))}.$$

Now, the only task left is to prove the concavity (convexity) conversion. Differentiation of and application of the chain rule to the foregoing formula for $g^{-1\prime}(k)$ yields

$$g^{-1\prime\prime}(k) = -\frac{g''(g^{-1}(k))}{[g'(g^{-1}(k))]^3},$$

which has sign opposite to that of $g''(g^{-1}(k))$. This implies convexity (concavity) of $g^{-1}(k)$.

□□

Proof of Lemma 3: We consider three cases where $\bar{k}=g_2(\hat{y})=\frac{\bar{y}}{2}$.

Case(i): $k \in (\hat{y}f(\hat{y}), \infty)$. Then trivially $\mathcal{L}(k) = 0$, since condition (c) is violated, that is, there does not exist such interval $[y_l, y_r]$.

Case(ii): $k \in (\frac{\hat{y}}{2}, \hat{y}f(\hat{y})]$. Then, by Lemma 2, $\mathcal{L}(\bar{k}) \geq \mathcal{L}(k)$. (This is, however, a little more than what Lemma 2 states. When $k = \hat{y}f(\hat{y})$, $\mathcal{L}(k)$ is equal to zero since y_l and y_r coincide. So we include this boundary point in the statement.)

Case(iii): $k \in (0, \frac{\bar{y}}{2})$. Since both g_1 and g_2 are continuous, monotone, and twice differentiable, by the inverse function theorem, the following functions are well defined, unique, and twice differentiable:

$$h_1(k): [0, \tfrac{\hat{y}}{2}] \mapsto [0, \hat{y}] \text{ with } h_1\big(g_1(y)\big) = y \text{ for all } y \in [0, \hat{y}],$$

$$h_2(k): [0, \tfrac{\hat{y}}{2}] \mapsto [0, \hat{y}] \text{ with } h_2\big(g_2(y)\big) = y \text{ for all } y \in [0, \hat{y}].$$

Furthermore, by Lemma A, h_1 is monotone and strictly concave while h_2 is monotone and strictly convex. Therefore $\mathcal{L}(k) = h_1(k) - h_2(k)$ is strictly concave in k. Notice that h_1' is continuously decreasing from $h_1'(0) = \infty$ to $h_1'(\bar{k}) = \frac{1}{g_1'(\bar{y})}$ and h_2' is continuously increasing from $h_2'(0) = 1$ to $h_2'(\bar{k}) = \frac{1}{g_2'(\hat{y})}$. By Lemma 1 we already know $\bar{y} \geq \hat{y}$ which implies $g_2'(\bar{y}) = f(\bar{y}) + \bar{y}f'(\bar{y}) \leq 0$. Since $f(\bar{y}) = \frac{1}{2}$, $\bar{y}f'(\bar{y}) \leq -\frac{1}{2}$ or $-\bar{y}f'(\bar{y}) \geq \frac{1}{2}$. Then

$g_1'(\bar{y}) = 1 - f(\bar{y}) - \bar{y}f'(\bar{y}) \geq 1 - \frac{1}{2} + \frac{1}{2} = 1$. Thus $\frac{1}{g_1'(\bar{y})} = h_1'(\bar{k}) \leq 1 < \frac{1}{g_2'(\hat{y})} = h_2'(\bar{k})$. Set

$$H(k) = h_1'(k) - h_2'(k).$$

H is strictly decreasing and continuous with $H(0) > 0$ and $H(\bar{k}) < 0$. By the intermediate value theorem, there exists a unique $\tilde{k} \in (0, \frac{\bar{y}}{2})$ such that $H(\tilde{k}) = h_1'(\tilde{k}) - h_2'(\tilde{k}) = 0$, that is, $\mathcal{L}'(\tilde{k}) = 0$. By the strict concavity of $\mathcal{L}(k)$, such $\tilde{k}$ will be the unique global maximizer in $k \in [0, \frac{\bar{y}}{2}]$.

Cases (i), (ii), and (iii) together imply $\mathcal{L}(\tilde{k}) \geq \mathcal{L}(k)$ for all $k \in \mathbf{R}_+$. This completes the proof. □□

Proof of Lemma 4: Consider $\lambda > 0$, $y^* > 0$ and $k \geq 0$. Then:

$$\begin{aligned}\eta\in[\lambda y_l(y^*;k),\ \lambda y_r(y^*;k)] \quad &\Leftrightarrow\ \eta=\lambda y \text{ and } y\in[y_l(y^*;k),\ y_r(y^*;k)]\\ &\Leftrightarrow \eta=\lambda y \text{ and } y\cdot f(y;y^*)\geq k \text{ and } y\cdot(1-f(y;y^*))\leq k\\ &\Leftrightarrow \eta=\lambda y \text{ and } y\cdot f(\lambda y;\lambda y^*)\geq k \text{ and } y\cdot(1-f(\lambda y;\lambda y^*))\leq k\\ &\Leftrightarrow \eta=\lambda y \text{ and } \lambda y\cdot f(\lambda y;\lambda y^*)\geq \lambda k \text{ and } \lambda y\cdot(1-f(\lambda y;\lambda y^*))\leq \lambda k\\ &\Leftrightarrow \eta\cdot f(\eta;\lambda y^*)\geq\lambda k \text{ and } \eta\cdot(1-f(\eta;\lambda y^*))\leq\lambda k \ \Leftrightarrow\end{aligned}$$

$\eta\in[y_l(\lambda y^*;\lambda k),\ y_r(\lambda y^*;\lambda k)]$.

This shows that in the relevant range, $y_l(y^*;k)$ and $y_r(y^*;k)$ and, consequently, $\mathcal{L}(y^*;k)$ are homogeneous of degree 1 in $(y^*;k)$. Moreover, $f(\lambda\bar{y}(y^*);\lambda y^*)=f(\bar{y}(y^*);y^*)=1/2$ implies $\bar{y}(\lambda y^*)=\lambda\bar{y}(y^*)$. Finally, (A3) implies $f(y;\lambda y^*)=f(y/\lambda;y^*)$ and, hence, $\frac{\partial}{\partial y}f(y;\lambda y^*)=\frac{1}{\lambda}\frac{\partial}{\partial y}f(y/\lambda;y^*)$. Therefore, $f(\hat{y}(y^*);y^*)+\hat{y}(y^*)\cdot\frac{\partial}{\partial y}f(\hat{y}(y^*);y^*)=0$ if and only if $f(\lambda\hat{y}(y^*);\lambda y^*)+\lambda\hat{y}(y^*)\cdot\frac{\partial}{\partial y}f(\lambda\hat{y}(y^*);\lambda y^*)=0$. That means $\hat{y}(\lambda y^*)=\lambda\hat{y}(y^*)$. □□

Proof of Proposition 4: With (A3), $g_1(\lambda y;\ \lambda y^*) = \lambda y\cdot(1 - f(\lambda y;\ \lambda y^*)) = \lambda y(1 - f(y;\ y^*)) = \lambda g_1(y;\ y^*)$, i.e. g_1 is homogeneous of degree 1 in $(y;\ y^*)$. Similarly, g_2 is homogeneous of degree 1 in $(y;\ y^*)$. Furthermore h_1, the inverse function of g_1 inherits the homogeneity of degree 1 in $(k;\ y^*)$, since $g_1(\lambda y;\lambda y^*) = \lambda g_1(y;\ y^*) = \lambda k$ implies $h_1(\lambda k;\ \lambda y^*) = \lambda y = \lambda h_1(k;\ y^*)$. Similarly, it can be demonstrated that h_2, the inverse of g_2, is homogeneous of degree 1 in $(k;\ y^*)$. Therefore, by Euler's theorem,

$$0 = \frac{\partial^2 h_1}{\partial k\partial y^*}\cdot y^* + \frac{\partial^2 h_1}{\partial k^2}\cdot k,$$

With the strict concavity of h_1, we then have

$$\frac{\partial^2 h_1}{\partial k\partial y^*} = -\frac{\partial^2 h_1}{\partial k^2}\cdot\frac{k}{y^*} > 0. \tag{9}$$

Similarly, with the strict convexity of h_2,

$$\frac{\partial^2 h_2}{\partial k\partial y^*} = -\frac{\partial^2 h_2}{\partial k^2}\cdot\frac{k}{y^*} < 0. \tag{10}$$

Now $\mathcal{L}(k;y^*)=h_1(k;y^*)-h_2(k;y^*)$ with $\mathcal{L}(0;y^*)=0$. Clearly, h_1 and h_2 are C^2 so that (9) and (10) imply that $\frac{\partial^2\mathcal{L}(k;y^*)}{\partial y^*\partial k} = \frac{\partial^2\mathcal{L}(k;y^*)}{\partial k\partial y^*} > 0$; hence (I).

From $\mathcal{L}(0;y^*)\equiv 0$ follows $\frac{\partial}{\partial y^*}\mathcal{L}(0;y^*)\equiv 0$ which together with $\frac{\partial^2\mathcal{L}(k;y^*)}{\partial k\partial y^*} > 0$ yields $\frac{\partial}{\partial y^*}\mathcal{L}(k;y^*) > 0$ for all $k > 0$, $y^* > 0$. Therefore (II).

Finally, $\frac{\partial}{\partial k}\mathcal{L}(\widetilde{k}(y^*);\ y^*) = 0$ together with $\frac{\partial^2\mathcal{L}(k;y^*)}{\partial y^*\partial k} > 0$ and strict concavity of $\mathcal{L}$ in k implies that $\widetilde{k}(y^*) < \widetilde{k}(y^{**})$ for $0 < y^* < y^{**}$, i.e. (III). □□

Proof of Proposition 5. We divide the proof into three parts:

(i) From Lemma 3 and its proof, we know that for any $y^* > 0$, there exists a unique $\tilde{k}(y^*) \in (0, \bar{y}(y^*)/2)$ such that $\mathcal{L}(n;y^*) < \mathcal{L}(m;y^*)$ for $0 \leq n < m \leq \tilde{k}(y^*)$.

(ii) For $0 < y^* < y^{**}$, set $\kappa(y^*,y^{**}) \equiv \tilde{k}(1) \cdot y^*$.

Then $0 < k < \kappa(y^*,y^{**})$ implies $0 < k/y^{**} < k/y^* < \tilde{k}(1)$. Hence by (i),

$$\mathcal{L}\left(\frac{k}{y^{**}}; 1\right) < \mathcal{L}\left(\frac{k}{y^*}; 1\right).$$

(iii) Let $0 < y^* < y^{**}$ and $0 < k < \kappa(y^*,y^{**})$.

Then by Lemma 4 and (ii),

$$\mathcal{L}(k;y^{**})/y^{**} = \mathcal{L}\left(\frac{k}{y^{**}}; 1\right) < \mathcal{L}\left(\frac{k}{y^*}; 1\right) = \mathcal{L}(k;y^*)/y^*. \qquad \square\square$$

References

Bebchuk, L.: "Litigation and Settlement under Imperfect Information." RAND Journal of Economics 15 (1984), pp. 404-415.

Dreyfuss, R. C.: "The Federal Circuit: A Case Study in Specialized Courts." New York University Law Review 64 (1989), pp. 1-75.

Flett, T. M.: Mathematical Analysis. McGraw-Hill: New York. 1966.

Gallini, T.N., and B.D. Wright: "Technology Transfer under Asymmetric Information." RAND Journal of Economics 21 (1990), pp. 147-160.

Green, J. R., and S. Scotchmer: "On the Division of Profit in Sequential Innovation." RAND Journal of Economics 26 (1995), pp. 20-33.

Hortsmann, I., G.M. MacDonald, and A. Slivinski: "Patents as Information Transfer Mechanism: To Patent or (Maybe) Not to Patent." Journal of Political Economy 93 (1985), pp.837-858.

Jewkes, J., D. Sawers, and R. Stillerman: The Sources of Invention. 2nd ed. Norton: New York. 1969.

Klemperer, P.: "How Broad Should the Scope of Patent Protection Be?" RAND Journal of Economics 21 (1990), pp. 113-130.

Lerner, J.: "The Importance of Patent Scope: An Empirical Analysis." RAND Journal of Economics 25 (1994), pp. 319-333.

Merges P.M., and R.R. Nelson: "On the Complex Economics of Patent Scope." Columbia Law Review 90 (1990), pp. 839-916.

Meurer, M.: "The Settlement of Patent Litigation." RAND Journal of Economic 20 (1989), pp. 77-85.

Reinganum, J., and L. Wilde: "Settlement, Litigation and the Allocation of Legal Costs." RAND Journal of Economic 17 (1986), pp. 557-566.

Takayama, A.: Mathematical Economics. Cambridge University Press: Cambridge. 1985.

Waterson, M.: "The Economics of Product Patents." American Economic Review 80 (1990), pp. 860-867.

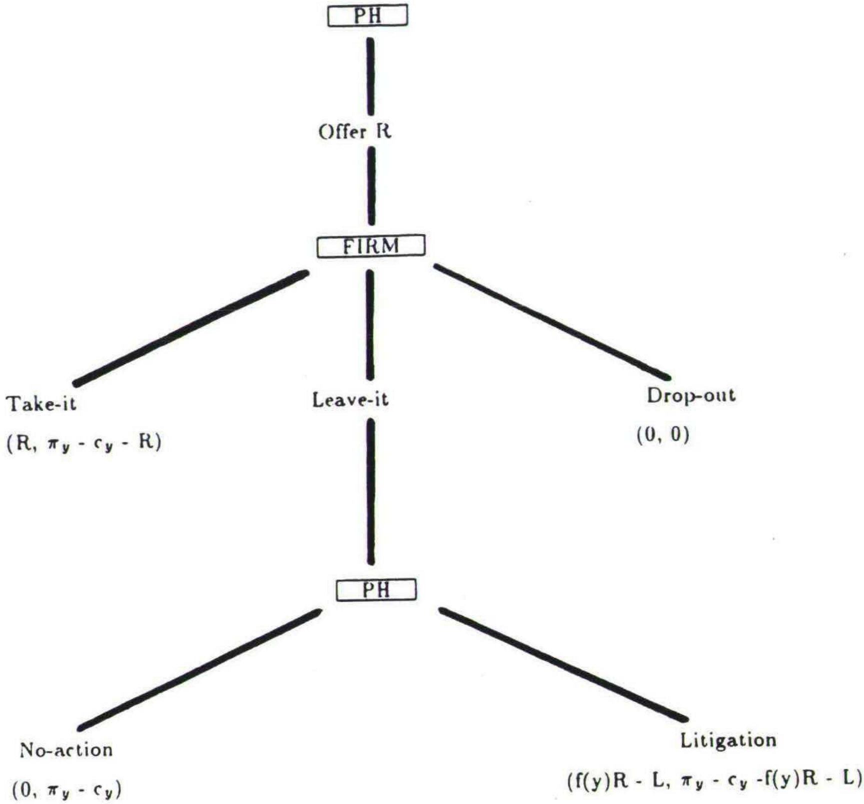
PH
Offer R
FIRM
Take-it
$(R, \pi_y - c_y - R)$
Leave-it
Drop-out
$(0, 0)$
PH
No-action
$(0, \pi_y - c_y)$
Litigation
$(f(y)R - L, \pi_y - c_y - f(y)R - L)$

Figure 1

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