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DISCRIMINATION BETWEEN NESTED TWO- AND THREE-PARAMETER DISTRIBUTIONS: AN APPLICATION TO MODELS OF AIR POLLUTION *

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Abstract

The purpose of this paper is to discriminate between 2- and 3-parameter nested alternatives for the gamma, Weibull and lognormal distributions. Monte Carlo experiments are conducted to evaluate the likelihood ratio test, Akaike's information criterion, Schwarz's information criterion, the Chi-square test and the Kolmogorov-Smirnov test. The performance of the tests and criteria depends on the types of nested distributions under consideration, the parametric values of the parent distributions, the confidence levels used (if applicable), and the sample sizes. The practical usefulness of the techniques is illustrated by observing the errors of the models in fitting the upper percentiles of the parent distribution. Two sets of air pollution data, namely hourly pollutant observations of β-scattering and nitrogen dioxide, from an urban airshed are used to examine the similarities and differences in fitting 2- and 3-parameter distributions where historical practice suggests there is a preference for the more parsimonious model.

Keywords: discrimination criteria; likelihood ratio test; gamma, Weibull and lognormal distributions; estimating upper percentiles; β-scattering and nitrogen dioxide.

1 Introduction

Several statistical criteria have been developed to discriminate among alternative parametric probability distributions. This paper deals with discrimination between 2- and 3-parameter nested alternatives for three common shape-scale-location parametric distributions, namely the gamma, Weibull and lognormal distributions. These 2- and 3-parameter distributions have frequently been used to model air pollution and environmental quality data; for example, see Jakeman and Taylor (1989) and the references cited therein. In the Monte Carlo experiments, we evaluate the well-known likelihood ratio (LR) test, Akaike's (1974) Information Criterion (AIC), Schwarz's (1978) Information Criterion (SIC), the Chi-square test, and the Kolmogorov-Smirnov test. Using extensive Monte Carlo simulations from 2- and 3-parameter parent distributions, we investigate the performance of these tests and information criteria. The performance of the tests and criteria depends to some extent on the types of nested distributions being considered, the parametric values of the parent distributions, the confidence levels used (if applicable), and the sample sizes. The parameter space investigated covers an extensive range of values which might arise in practice. For an illustrative example, the sensitivity of the results to the values of the location and shape parameters is evaluated.

Selection of an appropriate criterion should depend upon the intended use of the model. The practical usefulness of the techniques is illustrated by observing the errors of the models in fitting the upper percentiles of the parent distribution. Two sets of air pollution data from an urban airshed are used to examine the similarities and differences in fitting 2- and 3-parameter distributions where there is a preference for the more parsimonious model.

The paper also considers the relationship between the LR test and the two information criteria. The former is an hypothesis test which implicitly assumes that one of the distributions being tested is true, while the latter makes no such assumption and attempts to discriminate among alternatives in terms of the maximized log-likelihood value, with an allowance made for the number of parameters and observations used in estimation. Since the LR test performs quite well, it is useful to interpret the equivalence of the test and the information criteria at a given confidence level in terms of a generalised information criterion which relates directly to the critical region of the LR test.

The plan of the paper is as follows. In Section 2 the distribution functions and log-likelihood equations are presented. The discrimination criteria and loss functions are given in Sections 3 and 4, respectively. Sections 5 and 6 contain discussions of the simulation procedure and Monte Carlo results, respectively. An empirical application on hourly pollutant observations of β-scattering and nitrogen dioxide is outlined in Section 7. Some concluding remarks are given in Section 8.

2 The Distributions

Standardized probability density functions for the 3-parameter gamma, Weibull and lognormal distributions for a random sample are given by:

Gamma:

$$f(x) = \frac{1}{\beta\Gamma(\alpha)}(\frac{x-\gamma}{\beta})^{\alpha-1}exp[-(\frac{x-\gamma}{\beta})] \tag{1}$$

Weibull:

$$f(x) = \frac{\alpha}{\beta}(\frac{x-\gamma}{\beta})^{\alpha-1}exp[-(\frac{x-\gamma}{\beta})^{\alpha}] \tag{2}$$

Lognormal:

$$f(x) = \frac{1}{\alpha\sqrt{2\pi}}(x-\gamma)^{-1}exp\{-\frac{[log(x-\gamma)-\beta]^2}{2\alpha^2}\}. \tag{3}$$

In equations (1), (2) and (3), α represents the shape parameter, β the scale parameter, γ the location parameter, and Γ is the gamma function. The 2-parameter versions of the density functions of the gamma, Weibull and lognormal distributions are the same as in (1), (2) and (3), with $\gamma = 0$ in each case. In the above equations, $\beta > 0$, $\alpha > 0$ and γ is less than the minimum observed sample value.

The properties of these three distributions and the asymptotic behaviour of estimators depend very heavily on the values of the parameters, particularly that of the shape parameter. Figures 1 and 2 show that the resulting density functions of the gamma and Weibull distributions are similar to the exponential distribution at $\alpha = 1$, reverse 'J' shaped for $\alpha < 1$, and 'bell' shaped for $\alpha > 1$. Figure 3 shows that the curves for the lognormal distribution change from nearly symmetric to heavily skewed as α is increased from 0.3 to 1.2. These values span a large range of shapes which arise in the analysis of real data, such as air pollutant concentrations. In order to assess the different criteria for discriminating among competing descriptions of the data, the shape parameter is examined over an extensive range of possible cases where the density functions vary from being skewed to symmetric.

The maximized value of the likelihood function is an essential statistic employed in many criteria used to discriminate among alternative models. For a sample $x_1, x_2, \ldots, x_n$ of n independently and identically distributed random observations, the log-likelihood functions for the 3-parameter gamma, Weibull and lognormal distributions are given as follows:

Gamma:

$$logL = -n\alpha log\beta - nlog\Gamma(\alpha) + (\alpha - 1)\sum_{i=1}^{n} log(x_i - \gamma) - \sum_{i=1}^{n}(\frac{x_i - \gamma}{\beta}) \quad (4)$$

Weibull:

$$logL = nlog\alpha - n\alpha log\beta + (\alpha - 1)\sum_{i=1}^{n} log(x_i - \gamma) - \sum_{i=1}^{n}(\frac{x_i - \gamma}{\beta})^{\alpha} \quad (5)$$

Lognormal:

$$logL = -\frac{n}{2}log(2\pi\alpha^2) - \sum_{i=1}^{n} log(x_i - \gamma) - \frac{1}{2\alpha^2}\sum_{i=1}^{n}[log(x_i - \gamma) - \beta]^2. \quad (6)$$

The parameters of the three log-likelihood functions are estimated by maximum likelihood methods. Since the general maximum likelihood procedure for the 3-parameter gamma and Weibull distributions will frequently fail to converge when the (unknown) shape parameter is less than or equal to unity, a computationally efficient approach that circumvents this problem is used (for further details, see Bai et al. (1989)).

3 Discrimination Criteria

Let $x_1, x_2, \ldots, x_n$ represent a random sample of n observations. Interest here lies in discriminating among nested 2- and 3-parameter distributions in which the null hypothesis of interest is $H_0 : \gamma = 0$ against the alternative $H_1 : \gamma \neq 0$. The standard LR test can be employed for this problem. Denoting the maximized values of the 2- and 3-parameter variants of a particular log-likelihood function as $logL_0$ and $logL_1$, respectively, the LR test can be expressed as:

$$LR = -2(logL_0 - logL_1) \overset{a}{\sim} \chi^2(1) \tag{7}$$

under the null hypothesis that the location parameter is zero. The AIC and SIC may be expressed, respectively, as:

Choose the $\left\{ \begin{matrix} 2 \\ 3 \end{matrix} \right\}$ parameter distribution if

$$AIC : logL_0 - 2 \left\{ \begin{matrix} > \\ < \end{matrix} \right\} logL_1 - 3 \tag{8}$$

$$SIC : logL_0 - logn \left\{ \begin{matrix} > \\ < \end{matrix} \right\} logL_1 - 3logn/2. \tag{9}$$

When H_0 holds for testing a 2-parameter distribution against a 3-parameter alternative, rearranging (8) and (9), and defining $\Delta L = logL_0 - logL_1$, corresponds to choosing the $\left\{ \begin{matrix} 2 \\ 3 \end{matrix} \right\}$ parameter distribution if

$$AIC : -\Delta L \left\{ \begin{matrix} < \\ > \end{matrix} \right\} 1 \tag{10}$$

$$SIC : -\Delta L \left\{ \begin{matrix} < \\ > \end{matrix} \right\} \frac{logn}{2}. \tag{11}$$

Since the information criteria and the LR test are based on the maximized value of the likelihood function, it is possible to compare the information criteria and hypothesis test in terms of the probability of accepting the underlying null distribution. By comparison with (10) and (11), the LR test will accept H_0 if

$$LR : -\Delta L < \frac{c}{2} \tag{12}$$

where c is the critical value of the $\chi^2(1)$ statistic. It is easy to see that an equivalence among the LR test, AIC and SIC can be found when the nested model is regarded as the true distribution. Use of the AIC criterion is equivalent to the LR statistic at the 84.2 per cent confidence level (i.e. when $c = 2$) and SIC, for a sample size of 365, is equivalent to the LR statistic at the 98.5 per cent confidence level (i.e. when $c = logn$). When the sample size is decreased to $n = 100$, SIC is equivalent to the LR statistic at the 96.81 per cent confidence level, but will be increased to the 99.14 per cent confidence level for $n = 1000$.

The equivalence demonstrated above could be used to construct generalised information criteria (GIC) which, when the nested distributions is true, is equivalent to the LR test at different confidence levels. In this paper, we use two such criteria, GIC1 and GIC2 which can be regarded as LR analogues at the 40 per cent and 99 per cent confidence levels, respectively, and indicate two extreme cases: the lowest and highest confidence levels that might reasonably be considered in applications. These two cases can also help to illustrate the tradeoff between the confidence level and power of a test. An appropriate confidence level for air pollutant concentrations will be recommended in a later section when examining real data.

The performance of two well-known procedures for testing goodness of fit are also considered, namely the chi-square (CHI) test and Kolmogorov-Smirnov (KS) test. Classifying the n observations into k categories, the chi-square statistic is of the form (see Pearson (1900)):

$$CHI = \sum_{i=1}^{k} \frac{(f_i - np_i)^2}{np_i} \tag{13}$$

which has an asymptotic χ^2 distribution with $(k - l - 1)$ degrees of freedom when H_0 holds. The p_i are hypothetical probabilities, the f_i are empirical frequencies and l is the number of parameters estimated for each distribution (for further details, see Kendall and Stuart (1979)). For the experiments conducted in Section 6 below, $k = 10$ and $l = 2$ or $l = 3$. The KS test, which is defined in terms of the maximum absolute difference between the sample distribution function $S_n(x)$ and the hypothetical distribution function $F_0(x)$ (see e.g. Bury (1975, p. 204)), is given by

$$D_n = \sup_x |S_n(x) - F_0(x)|. \tag{14}$$

Large observed values of the D_n statistic lead to rejection of the hypothesis $F_0(x)$.

4 Loss Functions

An assessment of the performance of different tests and criteria requires some form of loss function or performance criterion which should rely on the nature of the problem and the major purpose of the application. Standard performance criteria for assessing nested hypothesis tests are size and power. In this Monte Carlo study, loss functions recommended for assessing air quality models have also been chosen (see Fox (1981)) to establish the effect of discrimination criteria on the intended use of the model. These functions are the relative bias (bias) and the relative root mean square error (rrmse) which are evaluated at the upper percentiles of the distributions. For an estimate $\hat{q}_i$ of a quantity of interest q, these loss function are defined in terms of deviations from the true or parent value q in each replication of the Monte Carlo experiments. The definitions used for the loss functions are:

$$bias(q) = \frac{1}{N} \sum_{i=1}^{N} (\frac{\hat{q}_i - q}{q}) \tag{15}$$

$$rrmse(q) = [\frac{1}{N}\sum_{i=1}^{N}(\frac{\hat{q}_i - q}{q})^2]^{0.5} \qquad (16)$$

where N is the number of replications of the experiment. For present purposes, the quantity q denotes the upper percentiles of the underlying distributions.

5 Simulation Procedure

In order to assess the various criteria for discriminating between models over different independently and identically distributed random samples, simulation over an extensive range of possible cases is considered. For all parameter sets in the tables and figures reported here, one thousand simulation experiments are processed. The main sample size used is $n = 365$, since it represents a common case: a full year of 24-hour average observations. For two extreme cases associated with possible applications, $n = 100$ and $n = 1000$ are considered here as illustrative examples. The shape parameters take the values 0.5, 1, 2, 4, 6 for the gamma distribution; 0.5, 1, 2, 3, 4 for the Weibull distribution; and 0.3, 0.5, 0.7, 0.9, 1.2 for the lognormal distribution. It should be noted that the lognormal distribution has the opposite behaviour to the other two as the shape parameter is increased. In all of the cases investigated in this paper, the arbitrary scale parameter is set at unity. In most cases the location parameter is also set to unity, but the sensitivity of our results to other values is also examined.

The random sample generators used for the Monte Carlo experiments are DRNGAM for the gamma, DRNLNL for the lognormal and DRNWIB for the Weibull distribution. These are available as subroutines in the International Mathematical and Statistical Library (IMSL) in version 1.0 of April 1987. The same seed number (1234) is used to obtain the first random sample of the first of the 1000 simulations. Varying the initial seed produces similar results to those given in the paper. For maximum likelihood estimation, a golden section search algorithm is used with final estimates being accepted when the relative error between two successive approximations is less than 10^{-6}. Two subroutines, namely DCHIGF and DKSONE, are chosen from IMSL to perform the

CHI and KS tests. All results are obtained on a VAX8700 mainframe computer at ANU.

6 Monte Carlo Results

Consider initially an investigation of the performance of the discrimination criteria for random samples of size $n = 365$ from the gamma distribution. In this paper, the scale parameter is always set at $\beta = 1$, and the values of the location parameter are $\gamma = 0$ or $\gamma = 1$. It should be noted that, for a fixed value of the location parameter, it becomes increasingly difficult to reject the false null hypothesis that $\gamma = 0$ as the value of the shape parameter increases (i.e. power decreases). Table 1 shows the results in two situations: first, the null hypothesis H_0 is true, so that the samples for each Monte Carlo experiment are taken from a 2- parameter distribution; second, the alternative hypothesis H_1 is true so that the samples are taken from a 3-parameter distribution.

When $\alpha \geq 2$, $\gamma = 0$ and $n = 365$, the empirical performance of the LR test is not significantly different from the nominal level of 0.05 given by asymptotic theory. The empirical probabilities vary only slightly with the shape parameter and with the initial seed used for the random number generator. Acceptance rates for AIC, SIC, GIC1 and GIC2 are also similar to those expected from the derived equivalent (LR) confidence intervals reported in Section 3, namely the 84.2, 98.5, 40.0 and 99.0 per cent levels, respectively. The CHI test has rejection frequencies which are very similar to those predicted by theory, while the KS test rarely rejects the true null hypothesis.

The power of any of the first five tests is inversely related, in general, to the acceptance rate when $\alpha \geq 2$. The lower the confidence level imposed for acceptance of the null hypothesis, the higher is the power. Quantifying this inverse relationship for different parameter values is a major concern in terms of how often we can expect underfitting of 2-parameter distributions to occur in samples taken from 3-parameter parent distributions. For a fixed value of the location parameter, power decreases as the shape parameter is increased.

Notice that, for the gamma distribution when $\alpha = 0.5$ and $\alpha = 1.0$, the LR test and the four discrimination criteria tend to overfit, a 3-parameter distribution being generally preferred when H_0 is true (i.e. $\gamma = 0$), especially for $\alpha = 0.5$, and always preferred when H_0 is false (i.e. $\gamma = 1$). This behaviour is due to the fact that the distributions approach the exponential when $\alpha \leq 1$, and likelihood values increase when the location parameter is set near the first order statistic. However, the CHI test has empirical sizes that are unaffected by whether the value of the shape parameter is less than or greater than unity, and the KS test still rarely rejects a true null hypothesis.

Figure 4 portrays the cumulative frequency over 1000 experiments of the differences between the maximized log-likelihood values of the 2- and 3-parameter gamma distributions when the samples are taken from a 3-parameter parent distribution. The figure demonstrates why power decreases as the value of the shape parameter increases. Figure 5 shows the differences when the samples are taken from a 2-parameter parent distribution. Whatever the value of the shape parameter, the differences are large in only a small proportion of the 1000 cases.

Consider the power of the LR test at the 95 per cent confidence level for different values of the location parameter. Figure 6 shows the results for $\alpha = 2, 4$ and 6. For $\alpha = 2$, the power of the LR test is high for quite low location values; for example, power is 0.98 when $\gamma = 0.34$. Power is also 0.98 for the combinations ($\alpha = 4, \gamma = 2.4$) and ($\alpha = 6, \gamma = 6.5$).

Table 1 also provides the results for rejection probabilities of the null hypothesis and powers of the tests and discrimination criteria over a range of parameter values for the Weibull and lognormal distributions. The conformity with theory of the LR test and AIC, SIC, GIC1 and GIC2 is good for the Weibull distribution when $\alpha \geq 2$ and for the lognormal distribution for all values of α when the sample size is $n = 365$. The empirical sizes of the CHI and KS tests of the Weibull and lognormal distributions are very similar to those of the gamma distribution for all values of the shape parameter. Sizes for the CHI test are close to the nominal size of 0.05, while the sizes for the KS test are almost zero in all cases. Not surprisingly, the powers of the CHI and KS tests are much lower than for the LR test.

The power of the LR test, when applied at the 95 per cent confidence level for the Weibull and lognormal distributions, is shown in Figures 7 and 8, respectively, as a function of the shape and location parameters. Compared with the case of the gamma distribution in Figure 6, a similar pattern of power as a function of the shape and location parameters is observed for the Weibull distribution. High power will be obtained when the shape parameter is 2 for quite low values of the location parameter, as shown in Figure 7. For example, power is 0.98 when $\alpha = 2$ and $\gamma = 0.17$. When the value of the shape parameter is increased, large values of the location parameter will be required to maintain high power. For instance, power is also 0.98 for the combinations ($\alpha = 3$, $\gamma = 0.49$) and ($\alpha = 4$, $\gamma = 1.48$). Figure 8 provides similar results for the lognormal distribution, except that the lognormal has the opposite behavour to the other two as the shape parameter is increased. For power to be 0.98, the combinations of shape and location parameters are ($\alpha = 0.9, \gamma = 0.41$), ($\alpha = 0.7, \gamma = 1.25$) and ($\alpha = 0.5, \gamma = 3.91$).

We now turn to an evaluation of the performance of the discrimination criteria for other sample sizes using the gamma distribution as a guide. Table 2 provides the analogous results to those in Table 1 where the sample sizes are $n = 100$ and $n = 1000$. As expected, at $n = 1000$ the criteria perform according to asymptotic theory in terms of correctly accepting 2-parameter models, since a similar result is achieved at the lower sample size of $n = 365$. The empirical sizes for the KS test are still very low, and the powers of the CHI and KS tests are considerably lower than for the LR test. Again, power declines with the shape parameter for a fixed value of the location parameter, but at the larger sample size the power is much higher for any specific shape parameter and criterion. At $n = 100$, the acceptance rates of 2-parameter distributions for the LR and CHI tests and the four criteria are lower than those predicted by theory, while power is consistently lower than at $n = 365$ for any specific shape parameter and criterion. The acceptance rates for the KS test vary with the value of the shape parameter, being too high when $\alpha = 2$ and too low when $\alpha = 4$ and $\alpha = 6$. The powers of the CHI and KS tests are considerably lower than those of the LR test for all values of the shape parameter.

7 Application to Models of Air Pollution

Hourly pollutant observations of β-scattering and nitrogen dioxide for Melbourne, Australia, are available at state site numbers 11 (Museum), 27 (Alphington), 34 (Dandenong) and 81 (Camberwell) for the years indicated in Tables 3 and 4. These data are converted into samples of 24-hour averages for those years and sites where the number of the resultant daily samples available is greater than 100. These data sets are used to illustrate an application of the discrimination criteria for the situation where the intended use of the model is predicting extreme concentrations and historical practice suggests there is a preference for 2-parameter models.

Table 3 gives results for β-scattering when the 2- and 3-parameter lognormal distribution is estimated. For these pollutant data, β-particle beams emitted from a radioactive source are detected by integrating nephelometer equipment which measures light scattering through all angles (see Finlayson-Pitts and Pitts (1986)). Notice that, in general, the probabilities of rejecting the 2-parameter lognormal model are very high and the maximized log-likelihood values are much lower for the 2-parameter lognormal distribution than for its 3-parameter counterpart. Indeed, the lognormal distribution yields much larger maximized log-likelihood values for the 3-parameter models than the gamma and Weibull distributions in 18 of the 20 cases considered. Omitting the single case in 1977 for site 11 where the 2- and 3-parameter log-likelihoods are equal, the minimum value of the acceptance threshold for the null hypothesis is 0.9860 for site 11 in 1976. If the parent distribution is the 3-parameter lognormal, then fitting the 2-parameter lognormal to samples from this parent yields substantial errors which can be quantified by simulation. For example, for the 3-parameter lognormal distribution, the rrmse obtained by simulation over 1000 experiments is 0.092 for the maximum percentile (MAX1), 0.078 for the second-highest percentile (MAX2), and 0.058 for the 98'th percentile (98%), while for the the 2-parameter lognormal these values are 0.136, 0.112 and 0.076, respectively. Admittedly, if we do not wish to risk obtaining errors of these magnitudes in such air quality applications, we should set our acceptance threshold for the null hypothesis below 98.6 per cent.

In order to fine-tune the estimate of where this threshold should be, given a preference for 2-parameter models, consider the results for daily nitrogen dioxide samples in Table 4. The 3-parameter gamma and lognormal distributions have the highest maximized log-likelihood values. However, the 3-parameter lognormal distribution generally has a negative location parameter, which is regarded as physically unrealistic. Let us, therefore, assume that the gamma distribution is appropriate. The simulation results reported in Table 1 indicate that, even when the parent distribution is a 2-parameter model, the probability of rejecting the 2-parameter gamma distribution with the LR test is unity when the shape parameter is less than unity. In such cases, the true underlying model may not be determined even when the estimate of the location parameter for the 3-parameter gamma distribution is very small.

Let us now re-examine the data sets and evaluate the errors in percentiles when we obtain an acceptance threshold below the value of .9860 found to be too high in the β-scattering case. The 1978 data set at site 11 yields an acceptance threshold of 0.9782. Again we can calculate the errors assuming that the 3-parameter gamma is the underlying parent distribution. The rrmse values for MAX1, MAX2 and 98% for fitting the 3-parameter gamma distribution are 0.063, 0.060 and 0.055, respectively, compared with 0.081, 0.075 and 0.065, respectively, when fitting the 2-parameter gamma distribution. These results provide further information as to where to set the confidence levels, given the errors that can be tolerated. If there is seen to be a strong need to use a 2-parameter model, such as might be set by historical precedent, then from the results presented here, it can be observed how often and by how much the use of such a model is likely to exceed tolerable errors.

Any criterion used to discriminate between nested models will involve a trade-off between acceptance of the true null hypothesis and rejection of the false null hypothesis. What level of bias should be chosen for overfitting? This should depend on answers to the following two basic questions: (i) Under what conditions would it be inaccurate to assume that the true model is the 2-parameter null? (ii) When would it be inaccurate to assume that the true model is the 3-parameter alternative? Of course, the precise answers depend on the acceptable levels of inaccuracy. Basically, for the first question, inaccuracy is greatest for those parameter sets where the powers of the discrimination criteria are around unity. The answer to the second question is when the information content of the sample is too low to give reasonably efficient estimates of the three parameters.

The answers given above can be refined in specific cases. Consider, for example, predicting the upper percentiles of the underlying parent distribution. This is a motivation in analysing data sets for environmental quality. Environmental guidelines for air and water pollutants can be written in terms of allowable excesses of some extreme concentration. In this paper we confine attention to the annual maximum concentration MAX1, the second highest value MAX2, and 98% values. More detailed results for the comparative errors in fitting 2- and 3-parameter alternatives to the distributions with parameters within the range of Table 1 are given in Bai et al. (1990). However, some indication of the errors is warranted here. We use the gamma distribution as a guide and begin with the situation where the underlying parent is a 3-parameter distribution. For $\alpha = 2$, the comparative errors begin to diverge for $\gamma > \frac{1}{2}$. For instance, when $\alpha = 2$, $\gamma = 1$ and $n = 365$, the rrmse of MAX1, MAX2 and 98% is more than double that of the 3-parameter estimates when the location parameter is not estimated but is set to zero. When the parent distribution is 2-parameter and the sample size is 365, there is little additional error in fitting a 3-parameter over a 2-parameter model.

8 Concluding Remarks

The purpose of this paper has been to discriminate between 2- and 3-parameter nested alternatives for the gamma, Weibull and lognormal distributions. Monte Carlo experiments were conducted to evaluate the likelihood ratio test, Akaike's information criterion, Schwarz's information criterion, the Chi-square test and the Kolmogorov-Smirnov test. The performance of the tests and criteria was shown to depend on the types of nested distributions under consideration, the parametric values of the parent distributions, the confidence levels used (if applicable), and the sample sizes. The practical usefulness of the techniques was illustrated by observing the errors of the models in fitting the upper percentiles of the parent distribution. Two sets of air pollution data, namely hourly pollutant observations of β-scattering and nitrogen dioxide, from an urban airshed were used to examine the similarities and differences in fitting 2- and 3-parameter distributions where historical practice suggests there is a preference for the more parsimonious model.

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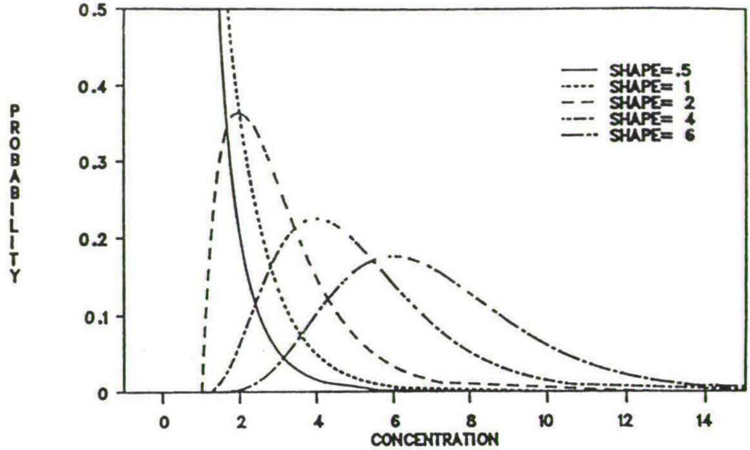

Figure 1: Profile of the gamma distribution for a range of shape parameters and unit scale and location parameters

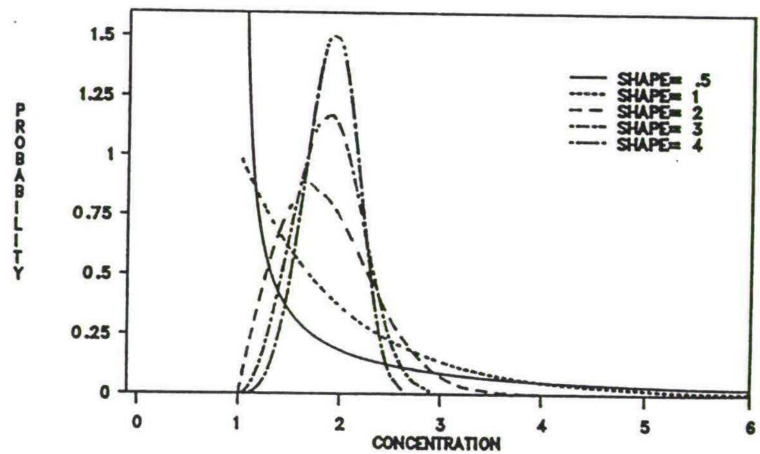

Figure 2: Profile of the Weibull distribution for a range of shape parameters and unit scale and location parameters

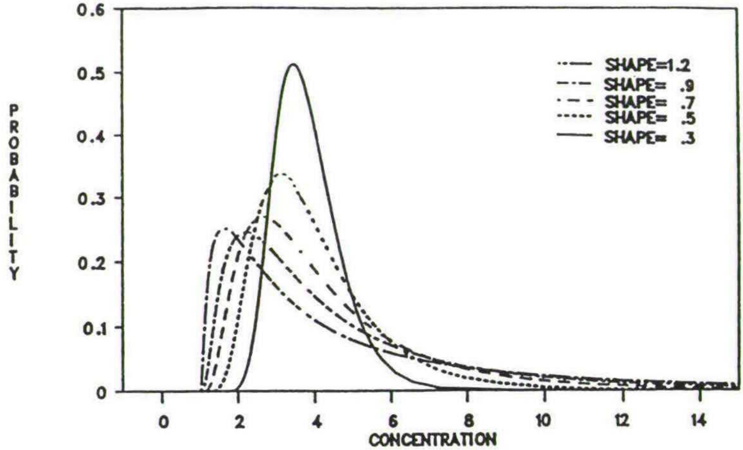

Figure 3: Profile of the lognormal distribution for a range of shape parameters and unit scale and location parameters

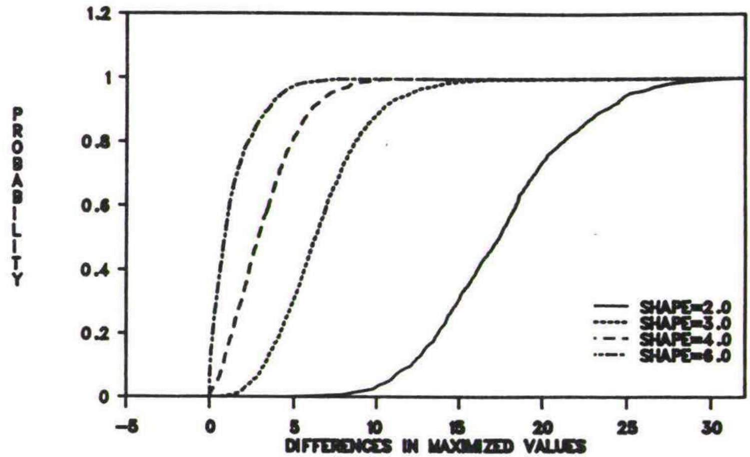

Figure 4: Cumulative frequency over 1000 experiments of the differences between the maximized log-likelihood values of the 2- and 3-parameter gamma distributions when the samples are taken from a 3-parameter parent distribution

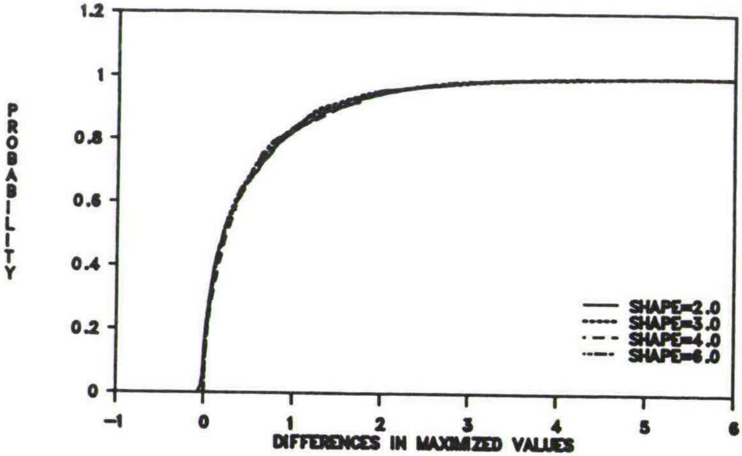

Figure 5: Cumulative frequency over 1000 experiments of the differences between the maximized log-likelihood values of the 2- and 3-parameter gamma distributions when the samples are taken from a 2-parameter parent distribution

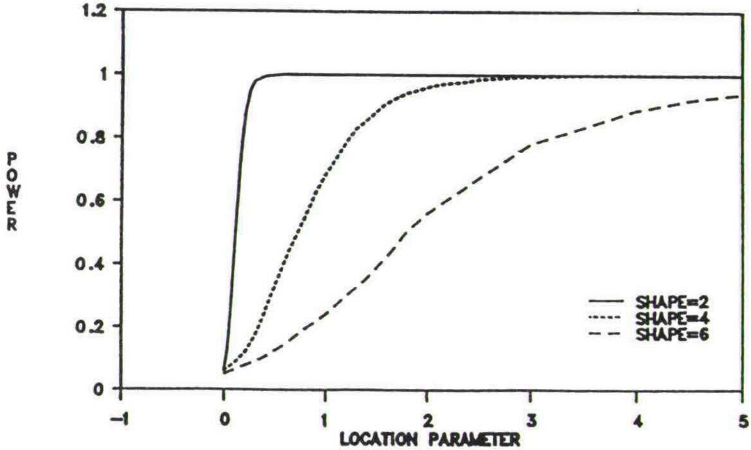

Figure 6: The power of the LR test at the 95 per cent confidence level for different values of the location parameter γ for testing between 2- and 3-parameter gamma distributions

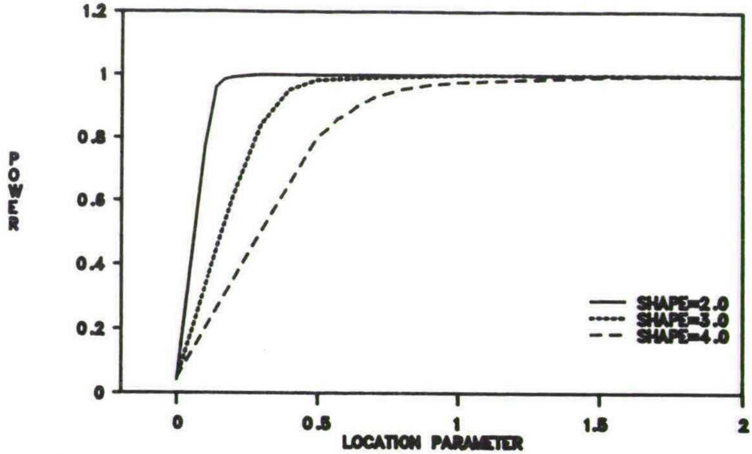

Figure 7: The power of the LR test at the 95 per cent confidence level for different values of the location parameter γ for testing between 2- and 3-parameter Weibull distributions

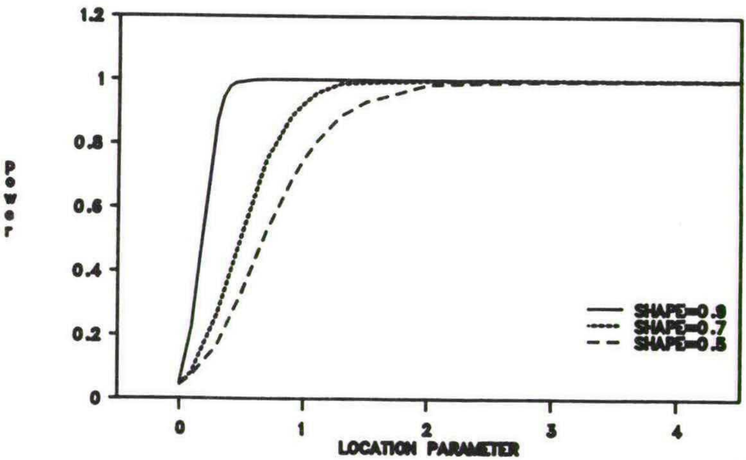

Figure 8: The power of the LR test at the 95 per cent confidence level for different values of the location parameter γ for testing between 2- and 3-parameter lognormal distributions

Table 1

Probabilities of rejecting the null hypothesis that $\gamma = 0$ using seven tests and discrimination criteria over 1000 replications of random samples of size $n = 365$ ($\beta = 1$)

		Gamma					Weibull					Lognormal				
		Values of Shape Parameter α					Values of Shape Parameter α					Values of Shape Parameter α				
True Distribution	Criteria	0.5	1.0	2.0	4.0	6.0	0.5	1.0	2.0	3.0	4.0	1.2	0.9	0.7	0.5	0.4
Two Parameter ($\gamma = 0$)	LR	1.000	0.332	0.061	0.061	0.050	1.000	0.217	0.041	0.045	0.049	0.053	0.049	0.048	0.043	0.043
	AIC	0.883	0.627	0.175	0.164	0.167	1.000	0.506	0.129	0.136	0.192	0.163	0.153	0.146	0.145	0.140
	SIC	0.904	0.148	0.017	0.020	0.015	1.000	0.087	0.010	0.010	0.043	0.017	0.012	0.016	0.015	0.016
	GIC1	1.000	0.884	0.573	0.632	0.589	1.000	0.843	0.574	0.581	0.601	0.588	0.598	0.596	0.597	0.597
	GIC2	1.000	0.107	0.010	0.013	0.012	0.999	0.059	0.007	0.003	0.008	0.011	0.009	0.009	0.010	0.010
	CHI	0.055	0.058	0.056	0.056	0.053	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.062
	KS	0.001	0.001	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.000
Three Parameter ($\gamma = 1$)	LR	1.000	1.000	1.000	0.690	0.243	1.000	1.000	1.000	1.000	0.977	1.000	1.000	0.997	0.925	0.752
	AIC	1.000	1.000	1.000	0.865	0.462	1.000	1.000	1.000	1.000	0.975	1.000	1.000	1.000	0.986	0.896
	SIC	1.000	1.000	1.000	0.479	0.128	1.000	1.000	1.000	0.999	0.925	1.000	1.000	0.993	0.815	0.575
	GIC1	1.000	1.000	1.000	0.987	0.840	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.996	0.992
	GIC2	1.000	1.000	1.000	0.420	0.103	1.000	1.000	1.000	0.999	0.912	1.000	1.000	0.992	0.781	0.412
	CHI	1.000	1.000	0.586	0.105	0.071	1.000	1.000	0.986	0.602	0.312	0.963	0.743	0.447	0.222	0.134
	KS	1.000	1.000	0.211	0.003	0.001	1.000	1.000	0.510	0.071	0.027	0.705	0.281	0.105	0.025	0.006

Note: The LR, CHI and KS tests have a nominal level of significance of 0.05.

Table 2

Probabilities of rejecting the null hypothesis that $\gamma = 0$ for the gamma distribution using seven tests and discrimination criteria over 1000 replications of random samples of sizes $n = 100$ and $n = 1000$ ($\beta = 1$)

		$n = 100$			$n = 1000$		
		Shape Parameter α			Shape Parameter α		
True Distribution	Criteria	2.0	4.0	6.0	2.0	4.0	6.0
Two Parameter ($\gamma = 0$)	LR	0.088	0.066	0.051	0.042	0.043	0.048
	AIC	0.199	0.178	0.168	0.143	0.152	0.143
	SIC	0.056	0.039	0.041	0.005	0.013	0.014
	GIC1	0.648	0.628	0.609	0.546	0.576	0.575
	GIC2	0.026	0.018	0.014	0.005	0.016	0.014
	CHI	0.086	0.083	0.091	0.055	0.049	0.065
	KS	0.002	0.112	0.087	0.019	0.000	0.000
Three Parameter ($\gamma = 1$)	LR	0.930	0.256	0.114	1.000	0.987	0.569
	AIC	0.979	0.473	0.276	1.000	0.996	0.789
	SIC	0.898	0.189	0.079	1.000	0.931	0.279
	GIC1	0.999	0.844	0.695	1.000	1.000	0.963
	GIC2	0.767	0.090	0.040	1.000	0.942	0.299
	CHI	0.286	0.112	0.087	0.965	0.227	0.193
	KS	0.019	0.000	0.000	0.896	0.040	0.103

Note: **The LR, CHI and KS tests have a nominal level of significance of 0.05.**

Table 3

Maximized log-likelihood values, estimated parameter values and probabilities of rejecting the null hypothesis that $\gamma = 0$ for the 3- and 2-parameter lognormal distributions fitted to n daily β-scattering samples over different years and sample sizes ($\beta = 1$)

Site	Year	n	Probabilities of rejecting $\gamma = 0$	Max(log L) Lognormal (3) Lognormal (2)	Shape α	Scale β	Location γ
11	1975	156	0.9993	-458.89	0.73	1.84	1.84
				-464.68	0.56	2.14	0.00
	1976	311	0.9860	-855.78	0.57	1.89	1.25
				-858.80	0.47	2.09	0.00
	1977	251	0.0000	-644.62	0.43	1.99	0.01
				-644.62	0.43	2.00	0.00
	1978	215	1.0000	-644.42	0.87	1.71	2.10
				-659.12	0.63	2.11	0.00
	1979	257	0.9999	-655.32	0.57	1.70	1.99
				-663.19	0.42	2.04	0.00
	1980	199	1.0000	-478.81	0.68	1.38	1.79
				-487.36	0.46	1.80	0.00
	1981	277	1.0000	-587.44	0.81	0.92	1.57
				-602.63	0.49	1.48	0.00
	1982	272	1.0000	-563.11	0.91	0.75	1.77
				-586.93	0.49	1.45	0.00
	1983	324	0.9999	-735.19	0.61	1.35	1.25
				-742.98	0.45	1.66	0.00
27	1979	291	1.0000	-812.74	0.61	1.86	1.69
				-821.60	0.49	2.13	0.00
	1980	304	1.0000	-686.81	0.69	1.21	3.30
				-722.44	0.37	1.95	0.00
	1981	302	1.0000	-755.21	0.72	1.41	2.74
				-790.78	0.46	1.98	0.00
	1982	279	1.0000	-686.98	0.72	1.37	1.94
				-708.51	0.49	1.82	0.00
	1983	326	0.9948	-824.74	0.55	1.70	1.18
				-828.64	0.45	1.92	0.00
34	1981	272	1.0000	-589.83	0.68	1.14	1.44
				-600.24	0.46	1.56	0.00
	1982	298	1.0000	-694.29	0.68	1.29	1.43
				-703.41	0.48	1.67	0.00
	1983	280	0.9991	-628.90	0.59	1.35	1.23
				-634.44	0.44	1.66	0.00
81	1981	160	1.0000	-319.99	0.76	0.85	2.89
				-343.30	0.37	1.72	0.00
	1982	312	1.0000	-985.45	0.78	1.99	3.05
				-1002.13	0.54	2.40	0.00
	1983	301	1.0000	-827.81	0.73	1.64	2.54
				-842.02	0.49	2.10	0.00

Note: Sites 11, 27, 34 and 81 are Museum, Alphington, Dandenong and Camberwell, respectively. Lognormal (3) and Lognormal (2) are the 3- and 2-parameter lognormal distributions, respectively.

Table 4
Maximized log-likelihood values, estimated parameter values and probabilities of rejecting the null hypothesis that $\gamma = 0$ for the 3- and 2-parameter gamma distributions fitted to n daily nitrogen dioxide samples over different years and sample sizes ($\beta = 1$)

Site	Year	n	Probabilities of rejecting $\gamma = 0$	Max(log L) Gamma (3) Gamma (2)	Shape α	Scale β	Location γ
11	1975	116	0.5457	-253.42	2.68	1.50	0.25
				-253.70	3.14	1.37	0.00
	1976	280	0.1936	-687.36	2.06	2.36	0.03
				-687.39	2.10	2.33	0.00
	1977	297	0.5835	-678.94	3.44	1.43	0.13
				-679.27	3.20	1.49	0.00
	1978	227	0.9782	-613.74	1.32	4.26	0.25
				-616.37	1.57	3.74	0.00
	1979	271	0.9996	-586.23	1.76	1.98	0.53
				-592.42	2.60	1.54	0.00
	1980	175	0.7666	-365.56	2.30	1.52	0.18
				-366.27	2.65	1.39	0.00
	1981	276	0.9600	-518.99	2.05	1.33	0.14
				-521.10	2.35	1.22	0.00
	1982	292	0.9698	-773.78	1.59	3.47	0.22
				-776.13	1.80	3.18	0.00
	1983	313	0.5024	-741.46	2.67	1.81	0.12
				-741.69	2.85	1.74	0.00
27	1979	317	1.0000	-525.33	0.64	3.29	0.04
				-556.82	0.95	2.24	0.00
	1980	302	0.9787	-477.91	1.19	1.52	0.03
				-480.56	1.29	1.43	0.00
	1981	245	0.9632	-355.64	1.22	1.31	0.03
				-357.82	1.34	1.22	0.00
	1982	188	1.0000	-380.20	0.61	5.04	0.17
				-408.71	0.92	3.53	0.00
	1983	241	1.0000	-470.45	0.84	3.13	0.09
				-480.57	1.10	2.46	0.00
34	1981	193	1.0000	-244.10	0.67	2.07	0.04
				-260.46	1.15	1.24	0.00
	1982	256	0.9636	-426.33	0.96	2.04	0.04
				-428.52	1.23	1.62	0.00
	1981	139	0.1585	-219.45	1.52	1.23	0.01
				-219.47	1.55	1.21	0.00
	1982	251	0.9995	-536.92	1.35	2.37	0.25
				-543.01	1.74	1.99	0.00
	1983	230	0.8945	-477.96	1.17	2.53	0.03
				-479.27	1.24	2.42	0.00

Note: Sites 11, 27 and 34 are Museum, Alphington and Dandenong, respectively. Gamma (3) and Gamma (2) are the 3- and 2-parameter gamma distributions, respectively.

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