CentER for Economic Research

Discussion paper

[redacted]

TILBURG UNIVERSITY

No. 9014

THE CONTINUITY OF THE EQUILIBRIUM PRICE DENSITY: THE CASE OF SYMMETRIC JOINT COSTS, AND A SOLUTION TO THE SHIFTING-PATTERN PROBLEM

by Anthony Horsley
and Andrzej Wrobel

February 1990

ISSN 0924-7815

THE CONTINUITY OF THE EQUILIBRIUM PRICE DENSITY: THE CASE OF SYMMETRIC JOINT COSTS, AND A SOLUTION TO THE SHIFTING-PATTERN PROBLEM *

By Anthony Horsley[†] and Andrzej J. Wrobel

Center for Economic Research, University of Tilburg, Hogeschoollaan 225, P. O. Box 90153, 5000 LE Tilburg, The Netherlands.

Abstract: We give a continuity result for the price density in a competitive equilibrium model with L^∞ as the commodity space. The result applies to commodities -- differentiated over characteristics such as time and events of delivery -- with symmetric joint costs (i.e., with a joint cost of production that is a symmetric, viz., rearrangement-invariant, function of the output bundle). Cost symmetry is characteristic of peak-load pricing problems, which motivate the analysis. Our price continuity result applies to the case of a separably additive utility function on L^∞, in which Jones' method (1984, p. 524) -- based on finding a suitable extension of the utility function from L^∞ to the space of measures, $\mathcal{M}$ -- fails. No price continuity result for an economy with L^∞ as the commodity space can be obtained from Richard's (1989) general analysis, either. An extension of our result to the case of non-symmetric costs is possible if costs are additively separable, and it is outlined. Our approach can also be used for other L^p-spaces (but not for $\mathcal{M}$, since both the concept of cost symmetry and that of additive separability are based on the presence of an underlying measure). We apply our analysis to a multi-station electricity pricing problem (with constant returns to scale) to prove that, though the variable costs of the stations are different, the equilibrium price is a continuous function of time. This result implies that the equilibrium output profile contains offpeak plateaux, in addition to a peak plateau. This equilibrium configuration is the basis of an example, given by Horsley and Wrobel (1989b, Example 1 in Section 5), of a short-run equilibrium with the capital inputs optimal (for the short-run equilibrium output) that is not a long-run equilibrium -- for the given equipment prices -- even though the long-run cost is exactly covered.

Our purpose in this paper is to prove the continuity of the density for equilibrium prices for a commodity differentiated over time, random events, etc., for which the joint cost of production is symmetric in the output bundle, i.e., apart from its dependence on input prices, the cost depends only on the distribution of output over the set of commodity characteristics. We use this result to solve the "shifting-pattern problem" in peak-load pricing with more than one production technique in the technology, e.g., in a two-station model of electricity generation.

Our price continuity result is obtained for a variant of Bewley's (1972) competitive equilibrium model. This model, with L^∞ as the commodity space, is developed by Horsley and Wrobel (1989a, Section 3; 1990a, Section 2) for the study of an industry producing a time-differentiated commodity, which is usually also differentiated over events of delivery. The commodity may also be differentiated over locations, but a "continuous" description of locations may require the use of a commodity space different from that of Bewley's model, such as the space of measures, $\mathcal{M}$, for which a price continuity result is given by Jones (1984). Our approach is specialized to the commodity space L^∞ (and other L^p-spaces). It applies, however, to the important case of a separably additive utility function on L^∞, in which Jones's (1984, pp. 524-525) approach fails because such a utility function on L^∞ has no weak*-continuous extension to the commodity space $\mathcal{M}$. (There is no such extension because, in general, a separably additive utility function is not even weak*-continuous on L^∞, i.e., it is not continuous even in the weak topology on L^∞ generated by L^1, let alone in the weak topology generated by space of (bounded) continuous functions, C, which is contained in L^1.) Relationship to other work is discussed at the end of this Introduction.

The characteristics of the industry's products are taken to form a measure space, Ξ. For example, in continuous-time, cyclic, deterministic pricing problems, Ξ can be taken as the unit time interval, representing one cycle. (This interval, $[0, 1]$, is taken with the Lebesgue measure. In the case of a stochastic demand, Ξ is taken to be the product of the interval, $[0, 1]$, and of the set, Ω, of the states of the world, with the product of the Lebesgue measure and of the probability measure on Ω.) A bundle of the differentiated commodity is modelled as an (essentially) bounded function, $x \in L^{\infty}(\Xi)$, on Ξ, and a price for the differentiated commodity is represented as an integrable function, $p \in L^{1}(\Xi)$, on Ξ. For simplicity, all other commodities in the economy are taken to be homogeneous, and their number to be finite, but this is not essential. (In Section 3, households' initial endowments consist solely of these other commodities and contain none of the differentiated commodity produced by the industry, so the pure exchange case is of no interest. When there are nonzero initial endowments of the differentiated commodity, the price continuity result also holds, provided that the total initial endowment is continuous on Ξ: see Remark 4.2. This case includes pure exchange economies.) The main result of this paper (Theorem 3.3) is that, in every equilibrium, the price density for the differentiated commodity, p^{*}, is a continuous function on the space of commodity characteristics, Ξ. On the supply side of the market, the key property for this result is that output bundles with the same distribution (with respect to the underlying measure on Ξ, e.g., over time and/or events) require the same quantities of inputs. In more formal terms, the section of the industry's production set by any given input bundle is a symmetric (or rearrangement-invariant) subset of $L^{\infty}(\Xi)$: see Assumption (a.1). Equivalently, the joint cost of production, $C(y)$, is a symmetric function of y (i.e., it depends only on the distribution of y over Ξ, and <u>not</u> on the way in which the values $y(\xi)$, for $\xi \in \Xi$, are arranged on Ξ), for every input price vector. Cost symmetry is a distinctive feature of peak-load pricing problems, e.g., as with utilities like electricity and water: see Horsley and Wrobel (1986a, 1987b). The crucial implication of cost symmetry, derived in Lemma 2.3, is that a product price profile, p, and a corresponding revenue-maximizing output bundle, y, are always similarly arranged on Ξ, i.e., the output is not higher when the price is lower. Equivalently, in terms of production costs, a marginal cost profile, p, is is always arranged similarly to the output bundle, y, at which it is calculated. (N.B.: The comparison of quantities and prices is between <u>different</u> product characteristics, i.e., the similarity of arrangement of p and y means that for each ξ and ξ' from Ξ, if $p(\xi) > p(\xi')$, then $y(\xi) \geq y(\xi')$.)

On the demand side of the market for the differentiated commodity, we assume that the that households' marginal utilities (as well as the marginal productivities of firms using the differentiated commodity as an input, when these are included in the model) are continuous in the product characteristics. This is formalized as Assumption (a.6), for utility functions of the additively separable form specified in Formula (3.2). Given this assumption and the symmetry of costs, a heuristic argument for the continuity of the equilibrium price density, p^*, goes as follows. Suppose that $p^*(\xi)$ is discontinuous in ξ. Since marginal utilities (or, in the case of input demand, marginal productivities) are continuous in ξ, an upward jump in $p^*(\xi)$ -- as ξ varies -- would bring about a downward jump in the trajectory of each household's demand, $x_h^*(\xi)$, and also in input demand (when it is included in the model). On the supply side, however, the price, p^*, and the output, y^*, are similarly arranged (as a result of cost symmetry, as pointed out above), and this means that $y^*(\xi)$ is higher (or at least not lower) for those ξ for which $p^*(\xi)$ is higher. In particular, the output is not lower after the upward price jump than before it. Since $\sum_h x_h^* = y^*$, the downward jump in the trajectory of demand is contradictory to the the lack of decrease in supply. Therefore, the equilibrium price density, $p^*(\xi)$, is continuous in ξ.

Cost symmetry is a characteristic of peak-load pricing problems, and in Example 3.5 of this paper we continue the development of a rigorous theory of the kind called for by Dreze (1964, pp. 16-17) which we started in earlier papers (Horsley and Wrobel (1986a, 1987b, 1988a, 1988b, 1988c, 1989a, 1989b, 1990a)). In the context of peak-load pricing with more than one production technique, our price continuity result removes the doubts about the existence of an equilibrium that originate from an apparent discontinuity of marginal cost. The difficulty, which we call "the shifting-pattern problem" (in view of some similarities to the shifting-peak problem solved definitively by Horsley and Wrobel (1989a, 1990a)), can be described as follows. For simplicity, we look at a deterministic, two-station model of electricity generation with given unit capital costs per period, r_1 and r_2, and given unit fuel costs, w_1 and w_2, for the two types of station. Assume that $r_1 > r_2$, $w_1 < w_2$, and that the (positive) number $\rho = (r_1 - r_2)/(w_2 - w_1)$ is smaller than 1. Thus, the first station is the base-load station, and the second station is the peak station (since, to minimize long-run costs, a station designed to be operated for a total time of more than ρ per period must be of the first type, and a station designed to be operated for a time shorter than ρ must be of the second type). Assume also that $r_2 = 0$, to simplify the notation. Consider some time-profile of demand, y. Denote by k_1 and k_2, respectively, the capacities of the two types of station in the long-run cost-minimizing plant mix for the production of y (explicit formulae for k_1 and k_2 are given in Example 3.5 in Section 3 but these are not needed here). With $r_2 = 0$, the marginal cost, i.e., the cost, $p(t)$, of supplying an additional unit at time t is equal to the base-load unit fuel cost, w_1, when $y(t) < k_1$, and it is equal to the peak unit fuel cost, w_2, when $y(t) > k_1$. These conditions describe $p(t)$ completely if $y(t)$ does not remain at the level k_1 for a positive time. Since $p(t)$ takes only two values, it is discontinuous at at least one instant, t_0. Suppose, for the sake of argument, that there is an upward jump in the marginal cost for y, $p(t)$, as t increases passing through t_0, i.e., that the right-hand limit, $p(t_0+)$, is larger than the left-hand limit, $p(t_0-)$. This is the case if y is (strictly) increasing around t_0 (with $y(t_0) = k_1$), which implies $y(t_0-) \le y'(t_0+)$. (For a globally increasing output, the marginal cost is illustrated in Figure 3.) Faced with these marginal costs as prices, electricity users may reduce their consumption immediately after the price jump to a level lower than that before the price jump. That is, for the new time-profile of demand, y', i.e., the demand at prices p for which $p(t_0-) < p(t_0+)$, one has $y'(t_0-) > y'(t_0+)$, in which case p is not a marginal cost price system for y' (since marginal cost is not higher when output is lower). Thus, demand changes its pattern around a point of price discontinuity, and, as a result, around t_0, the price charged, p, is equal to the wrong fuel cost. If a long-run marginal cost price system, p', for the new demand profile, y', is then tried, the pattern may well shift again in a similar way. The resulting iterative sequences of demands and marginal costs need not converge. However, an equilibrium, in which prices are equal to marginal costs, exists. Under the assumption that electricity consumption is harmlessly interruptible, this follows from the result of Horsley and Wrobel (1989a, Section 3; 1990a, Section 2) which, unlike Bewley's analysis (1972, Theorem 3), is applicable also in the presence of input demand (in addition to household demand) for electricity. The most interesting case is that in which both types of station are used in equilibrium, i.e., their equilibrium capacities, k_1^* and k_2^*, are both positive. By the preceding argument, the equilibrium price is continuous, but this raises the question of how the continuity of marginal fuel cost is possible with just two (or, more generally, any finite number) of unit fuel costs, w_1 and w_2. Our solution (presented in detail in Example 3.5, in Section 3) is that in equilibrium the time-profile of output, y^*, has a plateau (perhaps consisting of a number of intervals, as illustrated in Figure 2) at a level equal to the equilibrium capacity of the base-load type of station, k_1^*. During this plateau, the output stays at a kink of the intantaneous fuel cost curve ($c(\ell)$ in Figure 6), and there are multiple short-run marginal costs, which take, at any time, any value between the unit fuel cost of the first station, w_1, and that of the second station, w_2. (Long-run marginal costs are also multiple on the plateau, although they are more specified than the short-run marginal costs, since, in addition to being between w_1 and w_2, their integrals over the plateau all have the same value.) As a result of this multiplicity of marginal costs (i.e., of cost nondifferentiability), a gradual, continuous transition of marginal costs from w_1 to w_2 is possible, and this takes place in the equilibrium price (as illustrated in Figure 5). The gradual price change prevents the "shifting-pattern" problem that would arise if the price were always equal either to w_1 or to w_2. Note that this is an <u>off-peak</u> plateau (since $k_2^* > 0$). (In the case $r_2 > 0$ the equilibrium output, y^*, also has a peak plateau, over which the peak capacity cost is charged. This is Boiteux's (1964, pp. 81-82) conjecture on the form of a solution to the shifting-peak problem, which is formalized and proved by Horsley and Wrobel (1989a, 1990a).) Similarly, in an M-station model, generally there are $M-1$ off-peak plateaux in the equilibrium output, in addition to a peak plateau. (More precisely, the number of offpeak plateaux is one less than the number of stations actually used in the equilibrium generating system.) A difference between the roles of the peak and the off-peak plateaux should be noted: whereas the peak plateau is necessary for the existence of an equilibrium price, in a multi-station model the off-peak plateaux are necessary for the equilibrium price to be continuous.

We next point to possible extensions of the analysis presented here. First, competitive firms with an input demand for the differentiated commodity can be included, along with households, in the model (under assumptions on the production functions similar to those on the utility functions). Second, in the case of a stochastic demand, a variant of the result proving the continuity of price in, say, time alone is of interest (since there may even be no suitable topology on the probability space), and it can be given. Third, the assumption of a separably additive utility is made for convenience when formulating the continuity assumption about marginal utilities, and this form of preferences is not essential. Fourth, an extension to the case of non-symmetric costs is possible if costs are additively separable, and this is outlined in Section 4 and given in detail by Horsley and Wrobel (1990d). Fifth, some extension to the case of increasing returns to scale appears to be possible.

In the literature, continuity properties of an equilibrium price (as a linear functional on the commodity space), viz., the result that the equilibrium price belongs to some specified dual, L', of the commodity space, L, is usually obtained under assumptions which include the lower semicontinuity of preferences in some topology (on the commodity space), τ, that is compatible with the given duality, i.e., for which the continuous dual, $(L, \tau)^*$, of L is equal to L'. The reasons for this can be explained as follows. With monotone preferences on the consumption set taken to be the nonnegative cone, L_+, the lower semicontinuity of preferences alone guarantees that any equilibrium price is in $(L, \tau)^*$ if L_+ has a nonempty τ-interior. (This is because a linear functional is τ-continuous if it is bounded, either below or above, on a set with a nonempty τ-interior.) In many examples, however, the τ-interior of L_+ is empty. The above, "quick" argument for price continuity can be extended to this case if: (i) preferences are convex and extensible to convex, τ-continuous preferences defined on a τ-neighbourhood of L_+, and (ii) the equilibrium consumption of the household in question is in the algebraic interior of L_+ (relative to the whole space, L). For exchange economies, the second of these conditions holds if the total initial endowment is in the algebraic interior of L_+, as is assumed, for $L = L^\infty$, by Bewley (1972, Theorem 2). Bewley's argument is more precise than the above "quick" one, since he does not assume extensible preferences: for example, an additively separable, concave utility cannot be extended beyond L^∞_+ if the marginal utility at zero consumption level is infinite, as noted by Back (1988, pp. 97-98). The first of the two conditions used for the "quick" proof of τ-continuity, i.e., the condition that preferences be extensible to a neighbourhood of L_+, is somewhat stronger than the extensibility condition of Richard and Zame (1986, Theorems 2 and 4). As they show, the latter property is essentially equivalent to the "τ-uniform properness" of preferences, introduced by Mas-Colell (1986a).

The above Conditions (i) and (ii) for the "quick" argument are too restrictive for at least two reasons. First, in some commodity spaces, e.g., in the space of measures, $\mathcal{M}$, the algebraic interior of the nonnegative cone is empty. Second, even if the algebraic interior of L_+ is nonempty, these conditions are too restrictive for economies with production: for a detailed discussion of the problem in the case $L = L^\infty$, see Horsley and Wrobel (1989a, Introduction; 1990, Subsection 3.2). The position is then more complicated, and, as a result, it is generally not true that all equilibrium prices are in the τ-dual, even if assumptions, additional to the τ-closedness, are imposed on the production sets. What one aims to prove in this case is the existence of a τ-continuous price. For $L = L^\infty$, such a result is given by Bewley (1972, Theorem 3). Bewley's analysis is improved upon by Horsley and Wrobel (1989a, 1990a) whose equilibrium pricing model for time-differentiated commodities is based on this extension.

For an economy with the commodity space L^∞, no continuity result for the equilibrium price density can be deduced from the price-continuity results given by Jones (1984) for the commodity space $\mathcal{M}$ and by Richard (1989) for a class of commodity spaces including $\mathcal{M}$. This is because such an extension of their analyses would have to be obtained by embedding L^∞ in $\mathcal{M}$ and by suitably extending the preferences (and the production sets). In the case of Jones' model, a separably additive utility function on L^∞ is not, in general, continuous in the weak*-topology, $\sigma(L^\infty, L^1)$, on L^∞, and, therefore, does not have a $\sigma(\mathcal{M}, C)$-continuous extension to the commodity space $\mathcal{M}$, as noted by Jones' (1984, pp. 524). Richard's (1989) result on the existence of a τ-continuous equilibrium price is given for a production economy with a commodity space, L, that is both an ordered topological vector space and a vector lattice, with the topology denoted by τ and the nonnegative cone denoted by L_+. In addition to the τ-continuity of preferences, Richard (1989, Theorem 1) assumes the τ-uniform properness of preferences and production sets (to deal with the problem of an empty τ-interior of L_+). Unlike Mas-Colell (1986b), whose work Richard extends, he does not assume that L is a topological vector lattice[1], i.e., the lattice operations may be τ-discontinuous, as is the case with, e.g., L equal to $\mathcal{M}$ with the weak* topology, $\tau = \sigma(\mathcal{M}, C)$. In Richard's model, the best result for the commodity space $\mathcal{M}$ is obtained, however, by taking the Mackey topology, $\tau = \tau(\mathcal{M}, C)$, for the duality with the given price space, C. Although we do not know[2] whether an additively separable, concave utility function on L^∞_+ with continuous marginal utilities, as in our Assumption (a.6), is continuous in the restriction to L^∞_+ of the topology $\tau(\mathcal{M}, C)$, the embedding approach for L^∞ cannot succeed in Richard's model, either. This is shown by the one-consumer, no-production example of Jones (1984, Example 4). To put it briefly, the character of the assumptions made for an economy with the commodity space $\mathcal{M}$ is unsuitable for deriving a continuity result for the equilibrium price density by embedding L^∞ into $\mathcal{M}$: if this approach were applicable to any L^∞-economy at all, then it would also apply to the economy of Jones' example, which is impossible (since that is a counterexample to the continuity of the price density). Though we are not sure which of Richard's conditions fails in the process of trying to extend a separably additive utility function from L_+^∞ to $\mathcal{M}_+$, we know that at least one of them fails. This can be shown in detail as follows. In Jones' example, at time, t, the instantaneous utility is a continuously differentiable, strictly concave function, $u(x(t))$, of the consumption level, $x(t)$, for $t \in [0, 1]$. (That is, the instantaneous marginal utility is not only continuous when the consumption level, $x(t)$, varies continuously, but also stays constant when the consumption level stays constant; a more general instantaneous utility has the form $u(x(t), t)$.) The initial endowment in Jones' example, $\bar{x}(t)$, is discontinuous, over t, and this causes the (unique) equilibrium price, $p(t) = Du(\bar{x}(t))$, to be discontinuous. By Richard's result, it follows that the separably additive utility function, $U(x) = \int u(x(t))\,dt$, which is defined on L_+^∞, has no extension to $\mathcal{M}_+$ that is concave, $\tau(\mathcal{M}, C)$-continuous and uniformly proper in the direction $\bar{x}$. Since $\bar{x}$ is is bounded away from zero, it follows that U has no concave and continuous extension that is uniformly proper in any direction in L^∞. (This is because, for monotone preferences, uniform properness in a direction, x, implies uniform properness in any direction x' with $x' \geq x$ and, also, in the direction λx for every positive scalar λ.) Thus, even for the no-production, one-consumer economy with the utility function U, specified above, and with a <u>continuous</u> initial endowment, $\bar{x} \in C$, the continuity of the equilibrium price density, which <u>holds</u> in this case, cannot be deduced from Richard's result. This is because either the $\tau(\mathcal{M}, C)$-continuity, or the uniform properness, or both of these conditions fail for any concave extension of the utility function to $\mathcal{M}_+$. (In the case of infinite marginal utility at zero, i.e., if $Du(0+) = +\infty$, one knows that the uniform properness condition fails: as noted above, such preferences are not uniformly proper on L_+^∞ even in the topology $\tau(L^\infty, L^1)$, which is stronger than $\tau(\mathcal{M}, C)$.)

In each section (or appendix), the numbering of formulae, etc., is independent of other sections. For example, (3.1) is the first formula of Section 3, and (B.1) is the first formula of Appendix B. Assumptions are numbered separately, as (a.1), etc. The other formal paragraphs (definitions, theorems, etc.) are numbered consecutively within each section (or appendix). Appendices A and B contain mathematical definitions and results needed for Sections 2 and 3. Appendix C contains the proof of the main result, Theorem 3.3 given in Section 3.

2. Revenue Maximization with Symmetric Production Sets

In the next Section we give a continuity result for the equilibrium price of a differentiated commodity with a symmetric joint cost of production (i.e., with a joint cost that depends only on the distribution of the output level over the set of commodity characteristics). To prepare the ground for this result, in this Section, after introducing the commodity space, we study the implications of cost symmetry. The analysis is set up in the framework of Horsley and Wrobel's (1990a, Section 2) variant of Bewley's (1972) competitive equilibrium model. This framework is designed for the study of marginal cost pricing for an industry producing a time-differentiated commodity (which is usually also differentiated over events of delivery and over locations), e.g., as with utilities like electricity and water. The characteristics of the industry's products are taken to form a set, Ξ, with a finite, nonnegative measure, μ, on a sigma-algebra, $\mathfrak{A}$, of subsets of Ξ. (For example, in continuous-time, deterministic pricing problems, Ξ can be taken as the unit time interval, $[0, 1]$, with the Lebesgue measure on the sigma-algebra of Borel subsets of the interval.) The commodity space for the industry's products is $L^{\infty}(\Xi, \mathfrak{A}, \mu)$, abbreviated to $L^{\infty}(\Xi)$ or to L^{∞}. Every price system for the industry's products that we consider in this paper can be represented by a density, p, which is a μ-integrable function on Ξ, i.e., $p \in L^1(\Xi)$. Every such function, p, has a natural interpretation as a list of prices, with the value of any differentiated commodity bundle, $x \in L^{\infty}(\Xi)$, calculated as the integral, $\int_{\Xi} x(\xi)p(\xi)\,\mu(d\xi)$, of the quantity of the commodity for each characteristic, $\xi \in \Xi$, multiplied by its price; for brevity, this integral is denoted by $\langle x, p \rangle$. In setting up the model, the norm dual of $L^{\infty}(\Xi)$, which is larger than $L^1(\Xi)$, is used as the price space for the industry's products: see Horsley and Wrobel (1989a, Section 3; 1990a, Section 2). However, with this commodity space only those price systems with a density have a useful economic interpretation, and the singular term is excluded from the equilibrium price system by the imposition of appropriate assumptions. For economies with production, a result of this kind was first given by Bewley (1972, Theorem 3), and an extension that applies to the important case of a differentiated commodity that is used as an <u>input</u> is given by Horsley and Wrobel (1989a, Theorem 2; 1990a, Theorem 3.4).

All commodities in the economy other than the given industry's products are taken to be homogeneous, i.e., nondifferentiated. It is assumed that their number is finite, and they are numbered by $n = 1, 2, \ldots, N$. Therefore, the full commodity space is $L^\infty(\Xi) \times R^N$, and a commodity bundle is written as a pair (x, m), where $x \in L^\infty(\Xi)$ and $m \in R^N$. A price system is written as (p, q), where $p \in L^1(\Xi)$ and $q \in R^N$.

The industry's production possibilities are specified in terms of a set, Y, consisting of commodity bundles,(y, a), each of which represents a nonnegative output of the differentiated commodity, $y \in L^\infty(\Xi)$, that the industry can produce from an input bundle, $a \in R^N$, of the homogeneous commodities. (The possibility that the industry can also produce some of the homogeneous commodities, in which case some components of a are positive, is not excluded.) The production set Y is taken to include free disposal, i.e., $Y - L^\infty_+(\Xi) \times R^N_+ = Y$. If, as is usual in practical examples, the industry's production possibilities (or, equivalently, its production costs), are originally specified only for nonnegative product bundles, then Y is taken to be the free-disposal hull of the original production set. A closedness result for the free-disposal hull, which is needed in this case, is given by Horsley and Wrobel (1990a, Appendix A; 1990b). Note, also, that the symmetry property stated in (a.1) is preserved under the operation of taking the free-disposal hull.

We assume that the production set Y has the following symmetry property with respect to the output of the differentiated commodity:

(a.1) For every $y \in L^\infty(\Xi)$ and every $a \in R^N$, if $(y, a) \in Y$, $\tilde{y} \in L^\infty(\Xi, \mathfrak{X}, \mu)$ and $\tilde{y}$ has the same distribution as y, with respect to the measure μ, then $(\tilde{y}, a) \in Y$.

In other words, for each $a \in R^N$, the section of Y by a, viz., the set $Y^a = \{y \in L^\infty(\Xi) \mid (y, a) \in Y\}$, is symmetric, or rearrangement-invariant, in the sense defined formally in Appendix A.

A key result about revenue maximization with a symmetric production set is that, given an input bundle and an output price system, in every revenue-maximizing output bundle the output of a product with a lower price is not higher than the output level of a product with a higher price (note that the comparison of quantities and prices is between _different_ products, i.e., different characteristics of the differentiated commodity). Stated more precisely, this means that for each pair of product characteristics, ξ and ξ', if, at an output price system, p, with $p(\xi) > p(\xi')$, an output bundle, y, yields the maximum revenue among those output bundles producible from a given input bundle, $-a$, then $y(\xi) \geq y(\xi')$. (Equivalently, in terms of the production cost of y, given an input price system, q, the marginal cost is not higher for those products with lower output levels, i.e., for each ξ and ξ' with $y(\xi) > y(\xi')$, if p is a marginal cost profile at an output bundle, y, then $p(\xi) \geq p(\xi')$). This is stated formally in Lemma 2.3 below in terms of the property of _similar arrangment_, defined as follows by Day (1972, p. 932).

Definition 2.1: Two $\mathfrak{X}$-measurable, real-valued functions, p and y, on Ξ, are said to be _similarly arranged_ if for every $A \in \mathfrak{X}$ and every $B \in \mathfrak{X}$, the condition $\operatorname{ess\,sup}_{\xi \in A} p(\xi) < \operatorname{ess\,inf}_{\xi \in B} p(\xi)$ implies that $\operatorname{ess\,sup}_{\xi \in A} y(\xi) \leq \operatorname{ess\,inf}_{\xi \in B} y(\xi)$.

Remark 2.2: Although in Definition 2.1 the roles of the two functions, p and y, are not formally symmetric, similarity of arrangment is a symmetric relation, i.e., p and y are similarly arranged if and only if y and p are similarly arranged. This follows from, e.g., an equivalent condition for similarity of arrangement, given by Day (1972, Proposition 5.6), which is discussed in Remark A2.

Lemma 2.3: Assume that the measure μ is nonatomic on $\mathfrak{A}$, that Y is a subset of $L^\infty(\Xi, \mathfrak{A}, \mu)$ satisfying Assumption (a.1). Take any output price, $p \in L^1(\Xi, \mathfrak{A}, \mu)$, and any input, $-a \in R^N$. For any $y \in L^\infty(\Xi)$ with $(y, a) \in Y$, if $\langle y, p\rangle \geq \langle y', p\rangle$ for every y' with $(y', a) \in Y$ (i.e., if y maximizes the revenue among outputs producible from the given input), then y and p are similarly arranged.

Remark 2.4: Lemma 2.3 can be generalized by dropping the assumption that μ is nonatomic and replacing Assumption (a.1) by the following assumption on Y:

(a.1') For every $y \in L^\infty(\Xi)$ and every $a \in R^N$, if $(y, a) \in Y$, $\tilde{y} \in L^\infty(\Xi, \mathfrak{A}, \mu)$ and $\tilde{y}$ is majorized by y, then $(\tilde{y}, a) \in Y$;

where the term "majorized" is used in the sense of Hardy, Littlewood and Polya, discussed, e.g., by Ryff (1965) and Day (1972, 1973). Condition (a.1') is equivalent to Assumption (a.1) if: (i) the section, Y^a, of Y by any $a \in R^N$ is convex, and (ii) μ is either a nonatomic measure on $\mathfrak{A}$, or isomorphic to the counting measure (in which case the space $L^\infty(\Xi)$ is finite-dimensional). In general, (a.1') is stronger than (a.1).

Remark 2.5: Production-supporting prices for the industry's outputs can be calculated as marginal costs. This is convenient when, as in Example 3.5 below, the relationship between the properties of the output prices and the properties of the output bundle is to be studied. For any input prices, $q \in R^N_+$, and for any output, $y \in L^\infty(\Xi)$, the production cost is defined as

$$C(y, q) = \inf \{-\langle a, q\rangle \mid (y, a) \in Y\}.$$

In many examples (e.g., that of electricity generation, discussed in Section 3) the cost, $C(y)$, is a nondifferentiable function of the output bundle, y, and, to give a precise meaning to the notion of "marginal cost", we use the subdifferential, i.e., the collection of all subgradients, $\partial C(y)$, as the concept of a generalized derivative, with respect to output. Properties of subdifferentials are discussed by, e.g., Ioffe and Tihomirov (1979). Under Assumption (a.1), the cost is a symmetric function of the output bundle, i.e., if $\tilde{y}$ and y have the same distribution, then $C(\tilde{y}) = C(y)$. If C is symmetric and $p \in \partial C(y)$, then p and y are similarly arranged (Horsley and Wrobel (1988b, Theorem 1)), and (for the convex case) this is an equivalent statement of Lemma 2.3.

3. The Continuity of Equilibrium Price

To obtain the price continuity result we assume in this Section that the set of product characteristics, Ξ, is a topological space -- in addition to being a measure space described in Section 2 -- with the sigma-algebra of the Borel subsets, $\mathcal{B}$, of Ξ that is contained in the sigma-algebra $\mathfrak{A}$.

There is a finite number of households, which are the industry's customers. (For simplicity we assume that there are no other producers in the economy.) Households are numbered by $h = 1, 2, \ldots, H$. For each household, h, the set of feasible consumption plans is taken to be the nonnegative orthant,

$$(3.1) \qquad X_h = L_+^{\infty}(\Xi, \mathfrak{A}, \mu) \times R_+^N.$$

The initial endowment of household h is denoted by $(0, \bar{m}_h)$, i.e., it consists of an amount, $\bar{m}_{hn}$, of each homogeneous commodity, n. Preferences of household h are represented by a utility function of the form

$$(3.2) \qquad U_h(x, m) = \int_{\Xi} u_h(x(\xi), \xi)\, d\xi + v_h(m),$$

where $u_h\colon \Xi \times R_+ \to R$ and $v_h\colon R_+^N \to R$. The utility function, defined by (3.2) on X_h, is extended to the whole commodity space by setting $U_h(x, m) = -\infty$ for $(x, m) \notin X_h$, as is standard in convex analysis. The share of household h in the industry is denoted by s_h, with $s_h \geq 0$ and $\sum_h s_h = 1$ (in the case of constant returns to scale, the distribution of shares is irrelevant for the competitive equilibrium solution).

A <u>feasible allocation</u> is a list of: consumption plans, $(x_h, m_h) \in X_h$ for each household, h, and a production plan for the industry, $(y, a) \in Y$, such that:

(i) $\Sigma_h x_h = y$, and (ii) $\Sigma_h m_h + a = \Sigma_h \bar{m}_h$. In writing sums, etc., we follow the convention that the range of an index is understood to be the largest possible, with any restrictions specified; e.g., in Condition (iii) of Definition 3.1 below, j ranges from 1 to J.

Definition 3.1: A pair consisting of a feasible allocation, $((x^*_h, m^*_h)_{h=1}^{H}, (y^*, a^*))$, and a price system, $(p^*, q^*) \in L^1(\Xi) \times R^N$, is termed a <u>competitive equilibrium</u> if: the industry maximizes profits, and the consumption plan of each household maximizes its utility subject to the budget constraint, i.e., in formal terms, if:

(i) $\langle y^*, p^*\rangle + \langle a^*, q^*\rangle = \sup \{\langle y, p^*\rangle + \langle a, q^*\rangle \mid (y, a) \in Y\}$;

(ii) for each h, $\langle x^*_h, p^*\rangle + \langle m^*_h, q^*\rangle = \langle s_h y^*, p^*\rangle + \langle \bar{m}_h + s_h n^*, q^*\rangle$;

and

(iii) for each h and every $(x, m) \in X_h$, if $\langle x, p^*\rangle + \langle m, p^*\rangle \le \langle x^*_h, p^*\rangle + \langle m^*_h, q^*\rangle$, then $U_h(x, m) \le U_h(x^*_h, m^*_h)$.

Remark 3.2: If Y is a cone, with polar Y°, then Condition (i) in Definition 3.1 can equivalently be replaced by the conditions that $\langle y^*, p^*\rangle = \langle -a, q^*\rangle$ and that $(p^*, q^*) \in Y^\circ$, i.e., that $\langle y, p^*\rangle \le \langle -a, q^*\rangle$ for every $(y, a) \in Y$.

The following assumptions on households and the industry are made, in addition to (a.1) of Section 2. Assumption (a.2) is a rudimentary form of the adequacy (or survival) assumption. It ensures that in equilibrium each household has a positive income, and, therefore, that prices are equal to marginal utilities: see Step 1 in the proof of Theorem 3.3. The rest of the assumptions are assumptions on u_h (and v_h), in particular, the continuity, over Ξ, of marginal utility over the set of characteristics, Ξ, of the differentiated commodity is assumed in (a.6). The infinite marginal utility at zero is assumed, in (a.8), only to ensure that demand is positive and bounded away from zero in equilibrium, and it can be dropped if, e.g., the positivity of equilibrium demand is assumed directly.

(a.2) Each household is endowed with a positive amount of each homogeneous commodity, i.e., $\bar{m}_{hn} > 0$ for each h and n.

(a.3) For each $\ell > 0$, the function $\xi \to u_h(\ell, \xi)$ is μ-integrable on Ξ, i.e., $u_h(\ell, \cdot) \in L^1(\Xi)$;

(a.4) For each $\xi \in \Xi$, the function $\ell \to u_h(\ell, \xi)$ is: continuous, nondecreasing and concave on R_+, differentiable on R_{++}, and $u_h(0, \xi) = 0$;

(a.5) For every ℓ, the function $\xi \to D_1u_h(\ell, \xi)$, i.e., the partial derivative of u_h with respect to its first variable, considered as a function of the second variable of u_h, is bounded on Ξ;

(a.6) The function $(\ell, \xi) \to D_1u_h(\ell, \xi)$, is continuous in its second variable uniformly over any compact range for the first variable;

(a.7) $D_1u_h(\ell, \xi) > 0$ for every $\xi \in \Xi$ and for every $\ell > 0$;

(a.8) $D_1u_h(\ell, \xi) \to +\infty$ uniformly for $\xi \in \Xi$ as $\ell \to 0+$.

(a.9) The function v_h is continuous, nondecreasing and concave on R^N_+.

Assumptions (a.5) and (a.6) hold if the function $(\ell, \xi) \to D_1 u_h(\ell, \xi)$ is jointly continuous on $\Xi \times R_{++}$ and Ξ is compact.

Theorem 3.3: Under (a.1) to (a.9), if (p^*, q^*) is an equilibrium price density with $q^* \neq 0$, then p^* is continuous on Ξ.

Remark 3.4: (i) Since p^*, as an element of $L^1(\Xi)$, is an equivalence class of functions (equal to each other almost everywhere) rather than a single function, the assertion of Theorem 3.3, stated more formally, is that there exists an equivalent modification of p^* that is continuous. (This property should not be confused with "continuity almost everywhere", which holds, e.g., for every step function on an interval of the real line.)

(ii) From the continuity of price it follows that the equilibrium output is also continuous, under the assumption that the inverse marginal utility is continuous. This identifies a class of models with, in effect, the commodity space of continuous functions, $\mathcal{C}(\Xi)$, and with consumption sets equal to the nonegative orthant (and, therefore, having a nonempty norm-interior), in which the existence of an equilibrium can be proved by embedding $\mathcal{C}(\Xi)$ in $L^\infty(\Xi)$. Note, however, a basic difference between the continuity-in-equilibrium result obtained in this framework and the framework of Horsley and Wrobel (1988a, 1988c), in which commodity bundles are modelled as continuous functions and preferences are norm-continuous. For the case $\Xi = [0, 1]$, in the (L^∞, L^1)-framework, the Mackey-continuity assumptions on the demand side mean that consumption can be harmlessly interrupted, and this is needed for the existence of an equilibrium price density, although the equilibrium time-paths of consumption are continuous. The framework with the space $\mathcal{C}(\Xi)$ is designed to accommodate a richer price structure, including concentrated charges, to deal with the case of preferences that are norm continuous but not Mackey-continuous. In peak-load pricing, when interruptions in consumption are not harmless, pointed peaks (rather than a peak plateau) occur in equilibrium, and peak charges, represented mathematically as Dirac measures, are levied at particular instants (rather than being spread over a peak plateau). The two set-ups are compared in detail by Horsley and Wrobel (1990c).

Example 3.5: Theorem 3.3 can be applied as follows to solve the shifting-pattern problem in peak-load pricing for electricity, described in the Introduction. We show that the equilibrium marginal cost price is continuous over time, and that this is only possible in the presence of some offpeak plateaux in the equilibrium output (see Figures 1 and 5). For simplicity, consider the deterministic, two-station model of electricity generation. (It is straightforward to extend the analysis to the case of a technology with more than two stations and, also, to the analysis of marginal <u>expected</u> costs in the case of uncertainty.) We assume that each kind of generating station can be run only on one kind of fuel: thus, there are only two generating techniques, which correspond to the two kinds of station, denoted by $\theta = 1, 2$. Capital equipment is taken to be perfectly divisible, and constant returns to scale are assumed. As a result, every generating station has a specified maximum level of output, called its capacity, and the rate of fuel consumption is proportional to the station's instantaneous level of output, which cannot exceed the capacity. Hence, the production of an output bundle $y(t)$, $t \in [0, 1]$, by the use of any single technique requires an amount of the relevant capital equipment equal to the maximum of $y(t)$ over t. The amount of the appropriate fuel that is consumed is proportional to the total amount of energy in the bundle, viz., $\int_0^1 y(t)\,dt$. (This is usually referred to as the case of "rigid capacity", cf. Boiteux (1964, pp. 63-67) and Drèze (1964, pp. 9 ff).) Besides electricity, generating equipment and fuel, there may be other (homogeneous) goods in the model (their number is $N-4$), and a vector of their quantities is denoted by $g = (g_1, g_2, \ldots)$). In this example, the set of commodity characteristics, Ξ, is the unit interval of the real line, [0, 1], taken with the Lebesgue measure, mes, on the sigma-algebra of all Borel subsets, $\mathcal{B}$, of [0, 1]. It represents the relevant time period (usually a year), and an annual output bundle, y, is represented as an element of $L^\infty[0, 1]$. The installed capacity of station type θ is denoted by k_θ, and it is measured in the same units as the output level, $y(t)$, say, in MW. The amount of fuel input of type θ is denoted by v_θ, and it is measured in MWyears (one MWyear of fuel is the amount needed to run a unit station continuously for a year). With this notation, the production set for electricity, Y, which is a subset of $L^\infty[0, 1] \times R^N$, can be written as the sum

$$(3.3) \qquad Y = Y^{(1)} + Y^{(2)},$$

where

$$Y^{(1)} = \{(y, -k_1, -k_2, -v_1, -v_2, -g) \mid \operatorname*{ess\,sup}_{t \in [0,1]} y^+(t) \le k_1, \int_0^1 y^+(t)\,dt \le v_1, (k_2, v_2, g) \in R_+^{N-2}\},$$

and, similarly,

$$Y^{(2)} = \{(y, -k_1, -k_2, -v_1, -v_2, -g) \mid \operatorname*{ess\,sup}_{t \in [0,1]} y^+(t) \le k_2, \int_0^1 y^+(t)\,dt \le v_2, (k_1, v_1, g) \in R_+^{N-2}\}.$$

(The symbol "ess sup" stands for the essential supremum with respect to the Lebesgue measure.) The production set Y is Mackey closed, or, equivalently in view of its convexity, weak*-closed. (The proof of this, which is not given in detail here, follows from the Mackey lower semicontinuity of ess sup y, the Mackey continuity of the mapping $y \to y^+$, and from the capacity constraints in the definitions of Y^θ together with the relative weak*-compactness of bounded subsets of L^∞.) The set Y also satisfies the other assumptions of Horsley and Wrobel (1990a, Theorem 2.1), and, therefore (with the demand side that satisfies the assumptions of that result) there exists an equilibrium price system, $(p^*; r_1^*, r_2^*, w_1^*, w_2^*, \ldots)$, with a density, i.e., with $p^* \in L^1[0, 1]$. The other prices, r_1^*, etc., are scalars, and, since the analysis is concentrated on p^*, they are written as r_1, etc., to avoid clutter. We take the equipment prices to be expressed as annual rental prices, i.e., r_θ is measured, say, in £/MW p.a., and it is equal to the (equilibrium) purchase price of a unit station of type θ multiplied by the sum of the interest rate and the capital depreciation rate for equipment of type θ (the depreciation rate is assumed to be independent of the degree of equipment utilization). With amounts of fuel of each type are measured in MWyears (one MWyear of fuel is defined as the amount needed to run a unit station continuously for a year), w_θ is also expressed in £/MW p.a. Thus, r_θ is the capital cost per period, and w_θ is the fuel cost per period of operation, for a unit station of type θ.

Directly from the definition, (3.3), of Y it follows that this set satisfies the symmetry assumption, (a.1). Therefore, with firms and households that satisfy the other assumptions of Theorem 3.3, the equilibrium price density for electricity, p^*, is continuous on $[0, 1]$. The corresponding equilibrium output of electricity is y^*, with $\operatorname{ess\,inf} y^* > 0$. The most interesting case is when both types of station are needed to produce the equilibrium output, y^*, at a minimum (long-run) cost, i.e., when (possibly, after re-numbering the techniques): (i) $w_1 < w_2$, $r_1 > r_2$, and $0 < \rho < 1$, where

$$(3.4) \qquad \rho = \frac{r_1 - r_2}{w_2 - w_1},$$

and (ii) $y^*_\uparrow((1-\rho)+) < \operatorname{ess\,sup} y^*$, where $y^*_\uparrow$ denotes the nondecreasing rearrangement of y^*, defined as the nondecreasing function on $[0, 1]$ with the same distribution (with respect to the Lebesgue measure, mes) as that of y^*. The second of these conditions ensures that the second, peak type of station is needed; the need for the first, base-load type of station follows from $y_{\uparrow}^{*}((1-\rho)-) \geq \operatorname{ess\,inf} y^{*} > 0$. (We show below that it then follows from Theorem 3.3 that $y_{\uparrow}^{*}$ is continuous at $1-\rho$, and even constant on an interval, from some $\underline{t}^{*}$ to some $\bar{t}^{*}$, that contains $1-\rho$: see Figure 1.

The amounts of base-load and of peak types of station in the optimal plant mix are: $k_1^{*} = y_{\uparrow}^{*}(1-\rho)$ and $k_2^{*} = \operatorname{ess\,sup} y^{*} - k_1^{*}$, respectively (if $r_2 > 0$).) This is the case we consider below. As a function of output, y, the long-run cost derived from the production set Y is given by

$$(3.5) \qquad C(y) = C_0(y) + r_2 \operatorname*{ess\,sup}_{t \in [0,1]} y(t),$$

and

$$(3.6) \qquad C_0(y) = w_1 \int_0^{\rho} y_{\uparrow}(t)\,dt + w_2 \int_0^{1-\rho} y_{\uparrow}(t)\,dt,$$

where $y_{\uparrow}$ denotes the nondecreasing rearrangement of y, which is defined as the nondecreasing function on $[0, 1]$ with the same distribution (with respect to the Lebesgue measure, mes) as that of y. (The generalization of Formula (3.6) to the case of an arbitrary number of stations is derived by Horsley (1982) and is also given by Horsley and Wrobel (1986a).) The equilibrium price for electricity, p^{*}, is a time-profile of marginal cost, i.e., by Horsley and Wrobel (1990a, Remark 2.2), it belongs to the subdifferential of C, which is equal to the sum of the subdifferentials of C_0 and of $r_2 \operatorname{ess\,sup}$. Since C_0 is a Mackey continuous function of y by Horsley and Wrobel (1986b, Theorem; 1988b, p. 468), each of its subgradients has a density. In formal terms, it follows that

$$(3.7) \qquad p^{*} = p^{0*} + r_2 \nu^{*},$$

for some

(3.8) $$p^{0*} \in \partial C_0(y^*) \subset L^1[0,1]$$

and some $\nu^* \in L^1[0,1]$ with

(3.9) $$\int_0^1 \nu^*(t)\,dt = 1,$$

and

(3.10) $\nu^*(t) = 0$ for (almost) every $t \in [0,1]$ with $y^*(t) <$ ess sup y^*.

(The description of those subgradients of ess sup with a density is given by, e.g., Ioffe and Tihomirov (1979, Section 4.5.1, on p. 219).) The existence of such a ν^* implies that y^* has a peak plateau, illustrated in Figure 2. We next show that the continuity of $p^*(t)$ in t implies that y^* has an off-peak plateau. Under rearrangement, this plateau in y^* corresponds to a plateau in $y^*_\uparrow$ that extends from some $\underline{t}^*$ to some $\bar{t}^*$ and contains $1-\rho$, i.e., with $\underline{t}^* < 1-\rho < \bar{t}^*$. In the absence of such a plateau, p^{0*} would have to take the value w_1 on a set of instants, t, of measure $1-\rho$ (which is positive and less than 1), whereas at all other times p^{0*} would have to take the value w_2, which is greater than w_1. This follows from the description of $\partial C_0(y)$ given by Horsley and Wrobel (1988b, Theorem 4 and Remark 5; 1989b, Proposition 4), which is illustrated in Figure 3 (for the case of a nondecreasing output, y). As a result, the function $t \to p^{0*}(t)$ would be discontinuous (more precisely, it would have no continuous equivalent modification). Then $t \to p^*(t)$ would also be discontinuous (since $\nu^*(t) > 0$ only at peak, when $p^{0*}(t) = w_2$), but by Theorem 3.3, this is not the case.

To see in detail how a gradual transition of the equilibrium price density, $p^{0*}(t)$, from the fuel cost of one station to that of the other is made possible by the presence of the off-peak plateau, note that, by Horsley and Wrobel (1988b, Theorem 4 and Remark 5; 1989b, Proposition 4), $p^0 \in \partial C_0(y^*)$ if and only if:

(i) $w_1 \le p^0(t) \le w_2$ for (almost) every t on the off-peak plateau containing $1-\rho$, i.e., for every t with $y^*(t) = y^*_\uparrow(1-\rho)$, and the integral of $p^0(t)$ over the plateau is equal to $w_1(1-\rho-\underline{t}^*) + w_2(\bar{t}^* - (1-\rho))$,

(ii) $p^0(t) = w_1$ for those $t \in [0, 1]$ with $y^*(t) < y^*_\uparrow(1-\rho)$, and

(iii) $p^0(t) = w_2$ for those $t \in [0, 1]$ with $y^*(t) > y^*_\uparrow(1-\rho)$.

A time-continuous variant of marginal fuel cost, $p^0 \in \partial C_0(y)$ is illustrated in Figure 4 (for the case of a nondecreasing output, y). This should be compared with Figure 3 which illustrates the no-plateau case.

The equilibrium price density, p^*, is illustrated in Figure 5. As shown above, its existence implies that the equilibrium output, y^*, has a peak plateau, and its continuity implies that y^* has an off-peak plateau. This is illustrated in Figure 5.

Remark 3.6: The above analysis of electricity generation is set up in the long run, but symmetry of costs also holds in the short run, i.e., in the case of an a priori specified plant mix, (k_1, k_2). Hence, in a short-run equilibrium the price, p^*_{SR}, is also continuous. As before, assume $w_1 < w_2$. At output levels lower than k_1 the marginal fuel cost is w_1, and at output levels higher than k_1 the marginal fuel cost is w_2. When output level is equal to k_1, a marginal short-run fuel cost can take any value between w_1 and w_2. This is illustrated in Figure 6. (Unlike the long-run case, in the short run there is no additional restriction on the integral of a marginal fuel cost over the output plateau at the level k_1 -- if there is one -- since equipment prices are irrelevant in the short run.) It follows from the continuity of p^*_{SR}, that also the short-run equilibrium output, y^*_{SR}, has an off-peak plateau at the level k_1 (on the assumptions that $k_1 > 0$ and at that some of the available capacity of the second type of station is utilized, i.e., that ess sup $y^*_{SR} > k_1$).

4. Extensions to Non-symmetric, Additively Separable Costs

A similar argument to that given in the Introduction (immediately before the discussion of the the shifting-pattern problem for the two-station model of electricity generation) shows that the price continuity result also holds if the production cost is a convex integral functional, i.e., if

$$(4.1) \qquad C(y, q) = \int_{\Xi} c(y(\xi), \xi, q)\, d\xi ,$$

where $c: R_+ \times \Xi \times R^N_+ \to R \cup \{+\infty\}$. This cost function, C, may be non-symmetric in y, since c may depend on its second variable. The assumptions on c are similar to the assumptions on u_h made in Section 3, namely:

(i) For each ℓ and q, the function $\xi \to c(\ell, \xi, q)$ is μ-integrable on Ξ;

(ii) For each ξ and q, the function $\ell \to c(\ell, \xi, q)$ is: lower semicontinuous, convex and nondecreasing on R, and $c(\ell, \xi, q) = 0$ for $\ell \le 0$. Note that we do not assume that c is differentiable with respect to ℓ (since the differentiability of u_h suffices for our purpose), and we use the subdifferential as the concept of a generalized derivative (so that, e.g., the case of the instantaneous short-run cost, $c(\ell, t, w)$, of Example 3.5, shown in Figure 6, is included);

(iii) For every ℓ and every q, the correspondence $\xi \to \partial_1 c(\ell, \xi, q)$, i.e., the partial subdifferential of c with respect to its first variable, considered as dependent on the second variable of c, is bounded on Ξ;

(iv) For every q, the correspondence $(\ell, \xi) \to \partial_1 c(\ell, \xi, q)$, is continuous in its second variable uniformly over any compact range for the first variable.

The equilibrium price density, p^*, is continuous because, as before, an upward (discontinuous) jump in $p^*(\xi)$ -- as ξ varies -- would bring about a downward jump in the trajectory of each household's demand, $x_h^*(\xi)$, and also in input demand (when it is included in the model). On the supply side, however, in view of the continuity of the marginal cost, $\partial_1 c$, in ξ, an upward jump in price cannot bring about a downward (discontinuous) jump in the trajectory of output, $y^*(\xi)$, so p^* cannot be discontinuous.

In Example 3.5, the short-run cost of electricity generation (i.e., the fuel cost over the period, for given capacities of both types of station) has the additively separable form, (4.1). It is also a symmetric function of y, since the integrand, $c(y(t), t, w)$ does not dependent directly on time, t. It follows that price continuity in this example can be proved either by the method given in Section 3 for symmetric cost (long-run or short-run), or by the method outlined above which is developed in detail by Horsley and Wrobel (1990d). The latter method is for the short-run cost, but the long-run case follows, since every long-run marginal cost is a short-run marginal cost (i.e., $p \in \partial C_{LR}(y)$ implies $p \in \partial C_{SR}(y)$, or, equivalently, $p \in \partial c(y(t))$ for almost all $t \in [0, 1]$).

Remark 4.1: Every (Mackey lower semicontinuous) convex, symmetric function can be represented as a supremum of symmetric functions of the form: $y \to \int_0^1 y_\tau(t) e(t)\, dt \; + \;$ const., where $e(\cdot)$ is a nondecreasing function on $[0, 1]$: see Luxemburg (1967, Formula 13.4). This form is that of the long-run electricity cost function (with an infinite number of types of station, unless $e(\cdot)$ is a step function), for which the equilibrium price continuity can be proved by arguing in terms of the short-run cost, which is additively separable. By combining these facts it might be possible to reduce the general case of symmetric costs to the case of additively separable costs. However, this way of reasoning would be a little akward, since the formula for the subdifferential, at y^*, of the supremum of a family of convex functions involves, in general, points from an (arbitrarily small) neighbourhood of y^* (and, also, it requires the supremum in question to be continuous rather than only lower semicontinuous): see, e.g., Valadier (1969, Theorem 1). The method developed in Sections 2 and 3, based on the inequality of Hardy, Littlewood and Polya, is simpler and more direct.

Remark 4.2: The price continuity result also holds if households have (nonzero) initial endowments of the differentiated commodity, provided that: (i) the total initial endowment is a continuous function of ξ, and (ii) the marginal utility, $D_1 u_h(\ell, \xi)$ is jointly continuous in (ℓ, ξ), for each h. In Jones's (1984, p. 524) one-consumer, no-production example, the equilibrium price discontinuity is caused by the discontinuity of the initial endowment.

Appendix A: Symmetric Production Sets and Cost Functions

A subset, $\hat{Y}$, of $L^{\infty}(\Xi, \mathfrak{A}, \mu)$ is said to be symmetric if it contains every rearrangement of each of its elements, i.e., if it contains every function with the same distribution (with respect to μ) as a function that belongs to $\hat{Y}$. In formal terms, two (real-valued) functions, y and $\tilde{y}$, defined on some measure space, $(\Xi, \mathfrak{A}, \mu)$, are said to have the <u>same distribution</u> if $\mu(y^{-1}(B)) = \mu(\tilde{y}^{-1}(B))$ for every Borel subset, B, of the real line, in which case we write $y \sim \tilde{y}$. A set, $\hat{Y}$, is called <u>symmetric</u> if, for every $y \in \hat{Y}$ and for every $\tilde{y} \in L^{\infty}(\Xi)$ the condition $y \sim \tilde{y}$ implies that $\tilde{y} \in \hat{Y}$. In these terms, the symmetry assumption, (a.1), means that for each $a \in R^N$, the section of Y by a, viz., the set $Y^a = \{y \in L^{\infty}(\Xi) \mid (y, a) \in Y\}$, is symmetric. The key result about revenue maximization on symmetric production sets can be stated in terms of the following property of similar arrangment for functions.

Definition A1: For any y and p from $L^{\infty}(\Xi, \mathfrak{A}, \mu)$, the functions y and p are <u>similarly arranged</u> if for every pair, A and B, of sets from $\mathfrak{A}$, the condition $\operatorname{ess\,sup}_{\xi \in A} y(\xi) < \operatorname{ess\,inf}_{\xi \in B} y(\xi)$ implies that $\operatorname{ess\,sup}_{\xi \in A} p(\xi) \leq \operatorname{ess\,inf}_{\xi \in B} p(\xi)$.

Remark A2: The above definition of similar arrangement is given by Day (1972, p. 932). For the case of a nonatomic measure μ, an equivalent condition for similarity of arrangement is also given by Day (1972, Proposition 5.6). For any measurable function, y, on Ξ, define the nondecreasing rearrangement, $y_{\uparrow}$, of y as the nondecreasing function on $[0, 1]$ with the distribution, with respect to the Lebesgue measure on $[0, 1]$, equal to the distribution of y with respect to the measure μ. Since μ is nonatomic, there exists a measure-preserving mapping from Ξ into $[0, 1]$ such that $y = y_{\uparrow} \circ S$: see Day (1973, Proposition (3.3)) or, for the case $\Xi = [0, 1]$, Ryff (1965, Lemma 1). Any such mapping is called a <u>pattern</u> (or an <u>arrangement</u>) of y; the set of all the arrangements of y is denoted by $\mathcal{S}_y$. In these terms, y and p are similarly arranged if and only if $\mathcal{S}_y \cap \mathcal{S}_p \neq \emptyset$, i.e., if the two functions have a pattern in common. This condition is used by Horsley and Wrobel (1988b, p. 469 ff.) to calculate subdifferentials of symmetric functions and their extreme points (using results of Ryff (1967) and of Horsley and Wrobel (1987a)).

Lemma A3: Assume that the measure μ is nonatomic on $(\Xi, \mathfrak{A})$ and that $\hat{Y}$ is a symmetric subset of $L^{\infty}(\Xi, \mathfrak{A}, \mu)$. For every $p \in L^1(\Xi, \mathfrak{A}, \mu)$ and for every $y \in \hat{Y}$, if $\langle y, p\rangle = \sup\{\langle y', p\rangle \mid y' \in \hat{Y}\}$, then y and p are similarly arranged.

Proof: We first find a rearrangement of y that maximizes $\langle \cdot, p\rangle$ over the set of all rearrangements of y. To this purpose take any $S \in \mathcal{S}_p$, i.e., S is a measure-preserving mapping from $(\Xi, \mathfrak{A}, \mu)$ into $[0, 1]$ with $p = p_\dagger \cdot S$. (Such a mapping exists by a result of Ryff (1965, Lemma 1) and Day (1973, Proposition (3.3)), since μ is nonatomic.) Then $y_\dagger \cdot S \in \hat{Y}$ by the symmetry of $\hat{Y}$, and, since y maximizes $\langle \cdot, p\rangle$ on $\hat{Y}$,

$$\langle y, p\rangle \geq \langle y_\dagger \cdot S, p\rangle = \langle y_\dagger \cdot S, p_\dagger \cdot S\rangle = \langle y_\dagger, p_\dagger\rangle.$$

This, together with the inequality of Hardy, Littlewood and Polya (stated, e.g., by Day (1972, Theorem 5.1)), implies that $\langle y, p\rangle = \langle y_\dagger, p_\dagger\rangle$. Hence, by Day's characterization of the case of equality in the inequality of Hardy, Littlewood and Polya, it follows that y and p are similarly arranged. Q. E. D.

Remark A4: In the case of a μ with atoms, the assertion of Lemma A3 is true either if: Ξ consists of atoms of equal measure (i.e., μ is equal to the counting measure on Ξ, and $L^{\infty}(\Xi)$ is a finite-dimensional space), or if: the conditions y' is majorized (in the sense of Hardy, Littlewood and Polya) by y and $y \in \hat{Y}$ together imply that

$y' \in \hat{Y}$. (The latter condition on $\hat{Y}$ is equivalent to symmetry of $\hat{Y}$ if $\hat{Y}$ is convex and μ is either a nonatomic or a counting measure, but in general it is stronger than symmetry.)

Appendix B: Upper and Lower Essential Limits

Assume that Ξ is a topological space, and that μ is a measure on a sigma-algebra, $\mathfrak{A}$, of subsets of Ξ that contains the Borel sigma-algebra.

Definition B1: The upper essential limit of p at ξ_0, denoted by $\bar{p}(\xi_0)$, is defined as the infimum of all the essential suprema of p taken over neighbourhoods of ξ_0. Formally,

$$\text{(B.1)} \qquad \bar{p}(\xi_0) = \inf\{\operatorname*{ess\,sup}_{\xi \in W} p(\xi) \mid W \in \mathcal{N}(\xi_0)\},$$

where $\mathcal{N}(\xi_0)$ denotes the family of all neighbourhoods of ξ_0. Similarly, define the lower essential limit of p at ξ_0 by

$$\text{(B.2)} \qquad \underline{p}(\xi_0) = \sup\{\operatorname*{ess\,inf}_{\xi \in W} p(\xi) \mid W \in \mathcal{N}(\xi_0)\}.$$

Lemma B2: (i) The extended real-valued function $\bar{p}$ is upper semicontinuous;
(ii) The extended real-valued function $\underline{p}$ is lower semicontinuous;

If the topology of Ξ has a countable base of open sets, then

(iii) $\bar{p} \geq p$ μ-almost everywhere; and
(iv) $\underline{p} \leq p$ μ-almost everywhere.

Proof: To prove Part (i), take any $\xi_0 \in \Xi$ and any number $\delta > 0$. By the definition, (B.1), of $\bar{p}$, there exists a neighbourhood, W, of ξ_0 with

$$\text{(B.3)} \qquad \operatorname*{ess\,sup}_{\xi \in W} p(\xi) \leq \bar{p}(\xi_0) + \delta.$$

Also by (B.1), $\bar{p}(\xi) \le \text{ess sup}_{\xi \in W} p(\xi)$ for every $\xi \in W$, and from this and from (B.3) it follows that $\bar{p}(\xi) \le \bar{p}(\xi_0) + \delta$. This shows that $\bar{p}$ is upper semicontinuous. To prove Part (iii), take any number $\delta > 0$, and define $G(\delta) = \{\xi \in \Xi \mid p(\xi) \ge \bar{p}(\xi) + \delta\}$. For every $\xi \in G(\delta)$, there exists a neighbourhood, W, of ξ such that $\mu(G(\delta) \cap W) = 0$ (if there were no such W, then $\bar{p}(\xi) \ge \bar{p}(\xi) + \delta$, which is false). Furthermore, such a neighbourhood can be chosen from the countable base of the topology. It follows that there exists a countable covering of Ξ, $(W_\alpha)_{\alpha=1}^{\infty}$, such that $\mu(G(\delta) \cap W_\alpha) = 0$, for each $\alpha = 1, 2, \ldots$. Hence $\mu(G(\delta)) = 0$ for every $\delta > 0$, and it follows that $\mu(\{\xi \in \Xi \mid p(\xi) > \bar{p}(\xi)\}) = 0$. The proofs of Parts (ii) and (iv) are similar to those of Parts (i) and (iii), respectively. Q. E. D.

Remark B3: (i) For the case of Ξ equal to the real line, variants of Lemma B2 for one-sided (e.g., right) upper and lower essential limits are given by Dellacherie and Meyer (1978, p. 106, Theorem IV.37).

(ii) The upper essential limit as defined by (B.1), is a special case of the upper limit concept in topology theory, since it is equal to the upper limit (at ξ_0 but excluding the point ξ_0 in passing to the limit) in the so-called essential topology on Ξ (on the assumption that no singleton has a positive measure). For the case of Ξ equal to the real line (with the Lebesgue measure), this is remarked by Dellacherie and Meyer (1978, p. 105). A similar observation applies to the lower essential limit.

(iii) In view of Lemma B2, the functions $\bar{p}$ and $\underline{p}$ are also called "the upper and lower essential envelopes" of p, respectively.

Appendix C: Proofs

Proof of Theorem 3.3: Let $(x_h^*, m_h^*)_{h=1}^H$ and (y^*, a^*) be an equilibrium allocation associated to (p^*, q^*). Then $y^* = \sum_{h=1}^H x_h^*$. The proof is done in a number of steps. In Step 1 we characterize the optimality of households' choices as the equality of prices to marginal utilities. In Step 2 we show that the equilibrium price for the differentiated commodity, p^*, is bounded. The idea in Step 2 is that, on the demand side, very high values of $p^*(\xi)$ would depress each household's demand, $x_h^*(\xi)$, so much so as to contradict the similarity of arrangement between y^* (which is equal to $\sum_{h=1}^H x_h^*$) and p^*, which holds by Lemma A3 and by the symmetry assumption, (a.1). In Step 3 we use the boundedness of p^* to show that x_h^* is bounded away from zero, for each h. In Step 4, which is the main part of the proof, we formalize the argument that a (discontinuous) jump in price results in a jump in demand, leading to dissimilarity of arrangement between market demand and price. This cannot be the case in equilibrium, since, the output, y^*, is arranged similarly to the price, p^*. (The result of Step 3 is needed in Step 4 because a downward jump in demand cannot occur when demand is zero.)

Step 1 (Proportionality of prices to marginal utilities): Slater's condition holds for each household's utility maximization problem, by Assumption (a.2), since $q^* \geq 0$ and $q^* \neq 0$. Hence, for each h there exists a number, λ_h, such that

$$\lambda_h p^*(\xi) = D_1 u_h(x_h^*(\xi), \xi), \tag{C.1}$$

for μ-almost every $\xi \in \Xi$. Also, $\lambda_h > 0$, by Assumption (a.7). (In equilibrium, also the marginal utilities of the homogeneous commodities are proportional to their equilibrium prices, q_n^*, but this is not used.)

Step 2 (The proof that $p^{\#}$ is essentially bounded):

Since $p^{\#} \geq 0$, it suffices to show that $p^{\#}$ is bounded from above. Take a number $\ell > 0$ with

(C.2) $$\mu\{\xi \in \Xi \mid y^{\#}(\xi) \geq \ell\} > 0.$$

(Such an ℓ exists, since $y^{\#}(\xi) > 0$ for (almost) every ξ, by (C.1) and by Assumption (a.8).) By (C.2), take a number $K < +\infty$ such that the set defined by

(C.3) $$B = \{\xi \in \Xi \mid y^{\#}(\xi) \geq \ell,\ p^{\#}(\xi) \leq K\}$$

has a positive measure, i.e., $\mu(B) > 0$. Take any positive number, ℓ', with $\ell' H < \ell$. By Assumption (a.5), take a number $M' < +\infty$ with

(C.4) $$D_1 u_h(\ell', \xi) \leq M', \text{ for every } \xi \in \Xi.$$

Then

(C.5) $$x_h^{\#}(\xi) < \ell' \quad \text{for every } \xi \text{ with } p^{\#}(\xi) > M'/\lambda_h.$$

(This is because the condition $x_h^{\#}(\xi) \geq \ell'$, together with (C.1), the concavity of u_h, and (C.4), implies that $\lambda_h p^{\#}(\xi) \leq M'$.) We now show that that $p^{\#}$ is essentially bounded from above by the number

(C.6) $$M'' = \max\{K, M'/\min_h \lambda_h\}.$$

Define

(C.7) $$A = \{\xi \in \Xi \mid p^{\#}(\xi) > M'/\min_h \lambda_h\}.$$

From (C.5) it follows that

(C.8) $$y^{\#}(\xi) = \Sigma_{h=1}^{H} x_h^{\#}(\xi) < \ell' H < \ell, \text{ for every } \xi \in A.$$

By (C.8) and (C.3), ess sup $_{\xi\in A} y^{\#}(\xi) < \ell \leq$ ess inf $_{\xi\in B} y^{\#}(\xi)$. Also by (C.3), since $\mu(B) > 0$, one has ess inf $_{\xi\in B} p^{\#}(\xi) \leq$ ess sup $_{\xi\in B} p^{\#}(\xi) \leq K$. Since $y^{\#}$ and $p^{\#}$ are similarly arranged, it follows (Definition A1) that ess sup $_{\xi\in A} p^{\#}(\xi) \leq K$, which means, by (C.6) and (C.7), that $p^{\#}$ is μ-almost everywhere bounded above by M".

Step 3 (The proof that $x_h^{\#}$ is bounded away from zero, for each h):

By Step 2 and Assumption (a.8), take a positive number, $\underline{\ell}_h$, with

(C.9) $$D_1 u_h(\underline{\ell}_h, \xi) > \lambda_h \text{ ess sup } p^{\#},$$

for every $\xi \in \Xi$. Then

$$x_h^{\#}(\xi) > \underline{\ell}_h,$$

for μ-almost every $\xi \in \Xi$. This is because the condition $x_h^{\#}(\xi) \leq \underline{\ell}_h$, the concavity of u_h, and (C.9), together imply that

$$D_1 u_h(x_h^{\#}(\xi), \xi) \geq D_1 u_h(\underline{\ell}_h, \xi) > \lambda_h p^{\#}(\xi),$$

which condradicts (C.1).

Step 4 (The proof that the lower and the upper essential limits of p^* are equal, i.e., that $\underline{p}^*(\xi) = \bar{p}^*(\xi)$ for every ξ):

Assume the contrary, and take a number $\delta > 0$ with

(C.10) $$\bar{p}^*(\xi_0) - \underline{p}^*(\xi_0) > \delta,$$

for some $\xi_0 \in \Xi$. By Step 3, ess inf $x^*_h > 0$, and it follows that there exists a neighbourhood, W, of ξ_0 such that for each h and for every pair, ξ and ξ', of points from W, and for every pair, ℓ and ℓ', with ess inf $x^*_h \leq \ell \leq \ell' \leq$ ess sup x^*_h one has

(C.11) $$D_1u_h(\ell', \xi') \leq D_1u_h(\ell, \xi') < D_1u_h(\ell, \xi) + \lambda_h\delta.$$

(The first inequality in (C.11) follows from the concavity of u_h in its first variable, and the second inequality in (C.11) follows from the continuity of D_1u_h in its second variable, uniformly over any compact range for the first variable.) By (C.10),

(C.12) $$\text{ess sup}_{\xi\in W}\, p^*(\xi) - \text{ess inf}_{\xi\in W}\, p^*(\xi) > \delta.$$

It follows directly from (C.12) that there exists a pair, A and A', of measurable subsets of W with $\mu(A) > 0$, $\mu(A') > 0$, and

(C.13) $$\text{ess inf}_{\xi\in A'}\, p^*(\xi) - \text{ess sup}_{\xi\in A}\, p^*(\xi) > \delta.$$

From (C.13) and (C.1), for each h and for μ-almost all $\xi \in A$ and $\xi' \in A'$,

(C.14) $$D_1u_h(x^*_h(\xi'), \xi') > D_1u_h(x^*_h(\xi), \xi) + \lambda_h\delta,$$

so, by (C.11),

$$x_h^*(\xi')) < x_h^*(\xi).$$

Since $y^* = \Sigma_h x_h^*$, it follows that

$$\text{(C.15)} \qquad \operatorname{ess\,sup}_{\xi \in A} y^*(\xi) > \operatorname{ess\,inf}_{\xi \in A'} y^*(\xi).$$

Since p^* and y^* are similarly arranged, Formulae (C.13) and (C.15) are contradictory. This proves that $\underline{p}^*(\xi_0)$ and $\overline{p}^*(\xi_0)$ are equal, and, by Lemma B2, their common value is a continuous function of ξ_0, equal to $p^*(\xi_0)$ for almost every $\xi_0 \in \Xi$. Q. E. D.

REFERENCES

Back, K. (1988): "Structure of consumption sets and existence of equilibria in infinite-dimensional spaces", Journal of Mathematical Economics 17 , 89-99.

Bewley, T. (1972): "Existence of equilibria in economies with infinitely many commodities", Journal of Economic Theory 4 , 514-540.

Boiteux, M. (1964): "Peak-load pricing", in: J. R. Nelson, ed., Marginal cost pricing in practice (Prentice-Hall, Engelwood Cliffs, N. J). (Translated from the original: "La tarification des demandes en pointe: application de la theorie de la vente au cout marginal", Revue Generale de l'Electricite 58 (1949), 321-340.)

Day, P. W. (1972): "Rearrangement inequalities", Canadian Journal of Mathematics 24, 930-943.

Day, P. W. (1973): "Decreasing rearrangements and doubly stochastic operators", Transactions of the American Mathematical Society 178, 383-392.

Dellacherie, C. and P. A. Meyer (1978): Probabilities and potential (North-Holland, Amsterdam-New York-Oxford).

Dreze, J. H. (1964): "Some postwar contributions of French economists to theory and public policy", American Economic Review 54, supplement (June, 1964), 1-64.

Hoffmann-Jorgensen, J. (1972): "Weak compactness and tightness of subsets of M(X)", Mathematica Scandinavica 31, 125-150.

Horsley, A. (1982): "Electricity pricing for large supply systems", Discussion Paper TE 82/48, (Suntory-Toyota International Centre for Economics and Related Disciplines, London School of Economics and Political Science).

Horsley, A., and A. J. Wrobel (1986a): "The formal theory of electricity pricing and investment. I: a continuous-time deterministic model of production", Discussion Paper TE 86/132, (Suntory-Toyota International Centre for Economics and Related Disciplines, London School of Economics and Political Science).

Horsley, A., and A. J. Wrobel (1986b): "The Mackey continuity of the monotone rearrangement", Proceedings of the American Mathematical Society 97, 626 - 628.

Horsley, A., and A. J. Wrobel (1987a): "The extreme points of some convex sets in the theory of majorization", Proceedings of the Nederlandse Koninklijke Akademie van Wetenschappen, Series A 90, 171-176.

Horsley, A. and A. J. Wrobel (1987b): "Water metering: A study of the cost structure of the UK Water Supply Industry and of adapted marginal expected cost tariffs", a conference paper presented at the Econometric Society European Meeting, Copenhagen 1987.

Horsley, A. and A. J. Wrobel (1988a): "Local compactness of choice sets, continuity of demand in prices, and the existence of a competitive equilibrium", Discussion Paper TE 88/168 (Suntory-Toyota International Centre for Economics and Related Disciplines, London School of Economics and Political Science).

Horsley, A. and A. J. Wrobel (1988b): "Subdifferentials of convex symmetric functions: An application of the inequality of Hardy, Littlewood and Polya", Journal of Mathematical Analysis and Applications 135, 462-475.

Horsley, A., and A. J. Wrobel (1988c): "Weak compactness of bounded parts of choice sets and the existence of competitive equilibrium", unpublished manuscript.

Horsley, A., and A. J. Wrobel (1989a): "The existence of an equilibrium price density for marginal cost pricing", Discussion Paper TE 89/186, Suntory-Toyota International Centre for Economics and Related Disciplines, London School of Economics and Political Science.

Horsley, A., and A. J. Wrobel (1989b): "The envelope theorem, joint costs, and equilibrium", Discussion Paper TE/89/199 (Suntory-Toyota International Centre for Economics and Related Disciplines, London School of Economics and Political Science).

Horsley, A., and A. J. Wrobel (1990a): "The existence of an equilibrium density for marginal cost prices, and the solution to the shifting-peak problem", CentER Discussion Paper 9012, Tilburg University.

Horsley, A., and A. J. Wrobel (1990b): "The closedness of the free-disposal hull of a production set", CentER Discussion Paper 9013, Tilburg University.

Horsley, A., and A. J. Wrobel (1990c): "The formal theory of electricity pricing and investment. IV: Equilibrium analysis", in preparation.

Horsley, A., and A. J. Wrobel (1990d): "The continuity of the equilibrium price density. II: The case of non-symmetric, additively separable cost", in preparation.

Ioffe, A.D. and V.M. Tihomirov (1979): Theory of extremal problems (North-Holland, Amsterdam-New York-Oxford).

Jones, L. E. (1984): "A competitive model of commodity differentiation", Econometrica 52, 507-530.

Luxemburg, W. A. J. (1967): "Rearrangement invariant Banach function spaces", Queen's Papers in Pure and Applied Mathematics 10 , 83-144.

Mas-Colell, A. (1986a): "The price equilibrium existence problem in a Banach lattice", Econometrica 54 , 1039-1053.

Mas-Colell, A. (1986b): "Valuation equilibrium and Pareto optimum revisited", in: Contributions to Mathematical Economics. In Honor of Gerard Debreu, W. Hildenbrand and A. Mas-Colell, eds, pp. 317-332 (North-Holland, Amsterdam -- New York -- Oxford -- Tokyo.

Richard, S. (1989): "A new approach to production equilibria in vector lattices", Journal of Mathematical Economics 18, 41-56.

Richard, S. and W. R. Zame (1986): "Proper preferences and quasi-concave utility functions", Journal of Mathematical Economics 15, 231-247.

Ryff, J. V. (1965): "Orbits of L^1-functions under doubly stochastic transformations", Transactions of the American Mathematical Society 117, 92-100.

Ryff, J. V. (1967): "Extreme points of some convex subsets of $L^1(0, 1)$", Proceedings of the American Mathematical Society 18, 1026-1034.

Valadier, M. (1969): "Sous-differentiels d'une borne superieure et d'une somme continue de fonctions convexes", Comptes Rendus Hebdomadaires des Seances de l'Academie des Sciences, Ser. A, 268, 39-42.

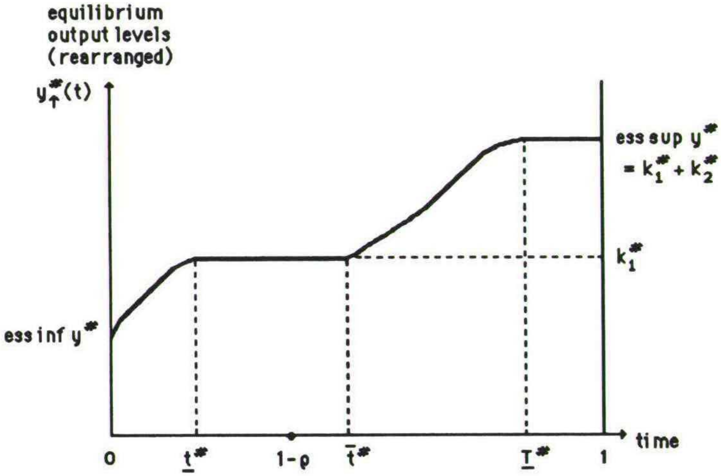

Figure 1: The nondecreasing rearrangement, $y_{\uparrow}^{*}$, of the long-run equilibrium output of electricity, y^{*}, illustrated in Figure 2 (Example 3.5).

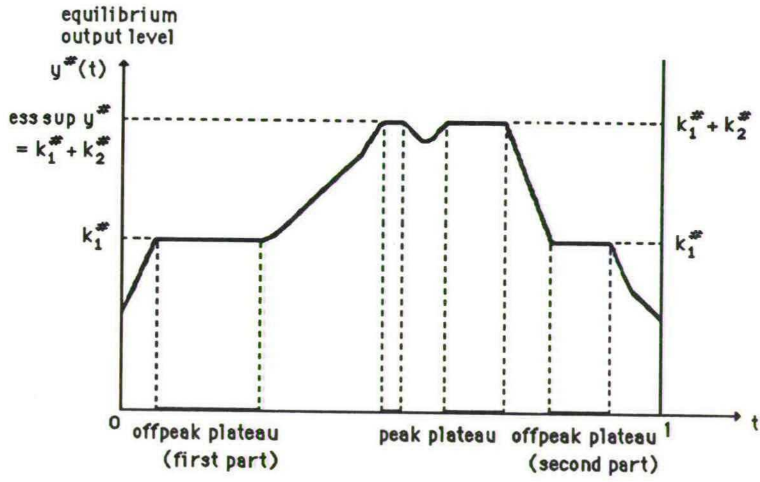

Figure 2: The long-run equilibrium output of electricity, y^{*} (Example 3.5). With a two-station technology, there are output plateaux both at peak and, offpeak, at the output level equal to the capacity of the base-load station, k_1^{*}.

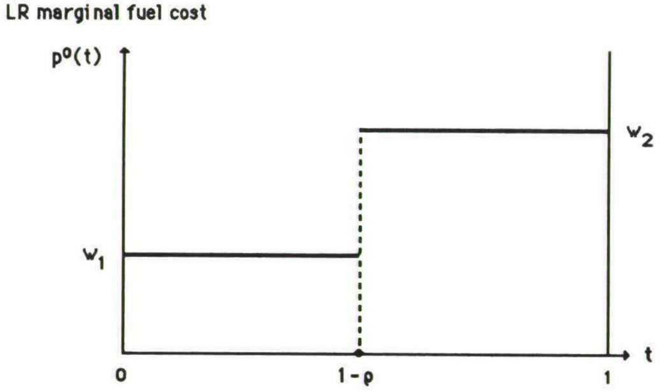

Figure 3: In the absence of a plateau containing the point $1-\rho$, the long-run marginal fuel cost (shown here for the case of a nondecreasing output), is unique and discontinuous.

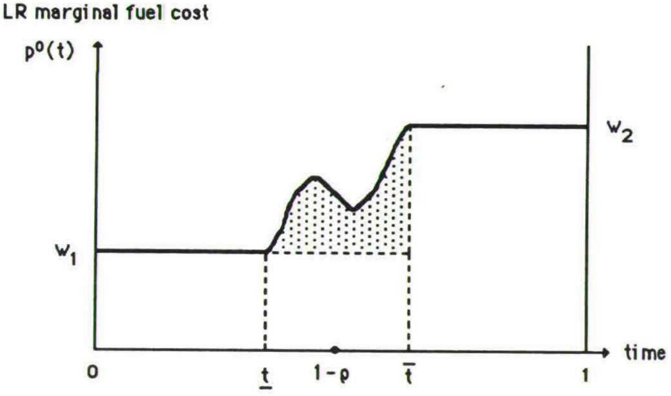

Figure 4: A continuous variant of marginal fuel cost that exists in the presence of a plateau (shown here for the case of a nondecreasing output, and extending from $\underline{t}$ to $\bar{t}$). The dotted area is equal to $(w_2 - w_1)(\bar{t} - (1-\rho))$.

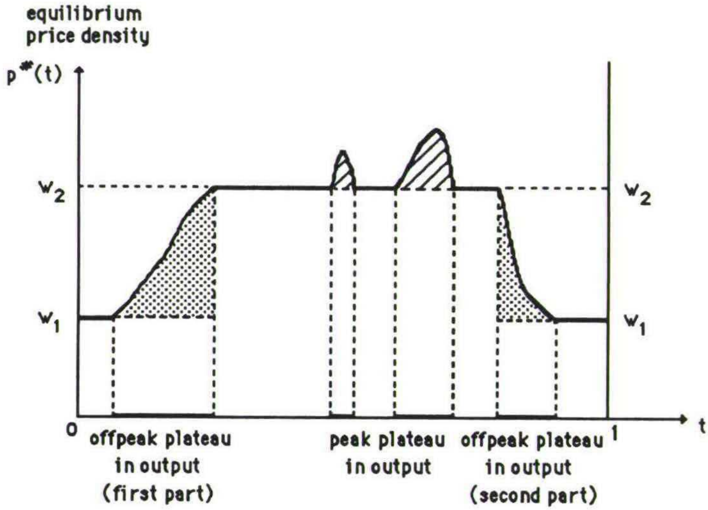

Figure 5: The long-run equilibrium price of electricity is continuous (Example 3.5). With a two-station technology, the dotted area is equal to $(w_2 - w_1)(\bar{t}^* - (1 - \rho))$, and the hatched area is equal to r_2 (cf. Figure 1).

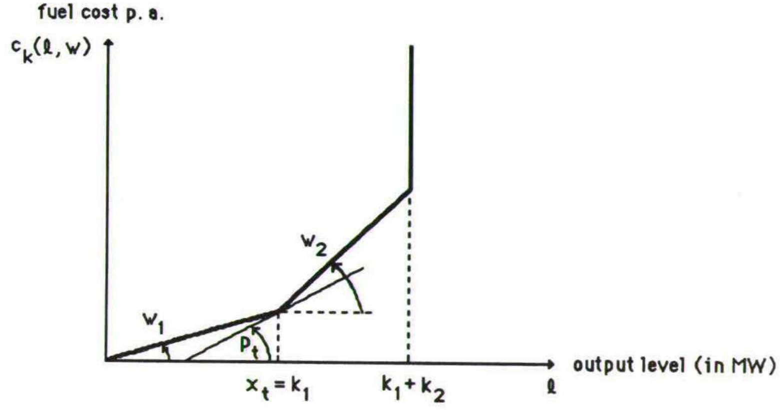

Figure 6: A short-run marginal fuel cost, p_t, of a plant mix, (k_1, k_2), at the output level equal to the total capacity, k_1, of the base-load type of station.

FOOTNOTES

* Journal of Economic Literature Classification: 021.

AMS 1980 Mathematics Subject Classification (1985): Primary 90A14.

1. Richard also shows that the uniform properness assumption for production sets can be simplified and weakened if L is a topological vector lattice. However, for $L = L^{\infty}$ with the Mackey topology, $\tau = \tau(L^{\infty}, L^{1})$, which is a topological lattice, Richard's (1989) theorem yields a weaker result than the result of Horsley and Wrobel (1989a, 1990a) combined with Bewley's (1972, Theorem 1) existence result for prices in the norm-dual, $L^{\infty *}$: in addition to the fact that not all Mackey-continuous preferences are uniformly proper, the Mackey uniform properness of production sets is a stronger assumption than the Elimination Property of Horsley and Wrobel (for example, in the case of a firm using the time-differentiated commodity as an input, it is stronger than the Mackey lower semicontinuity assumption on the firm's production function). Also, by using the Yosida-Hewitt decomposition, one proves that the density part of any equilibrium price is itself an equilibrium price supporting the same allocation, and, as Horsley and Wrobel (1989a, 1990a) point out, in many cases of interest it follows that <u>every</u> equilibrium price is in L^{1}, which cannot be deduced from Richard's general result.

2. Since little appears to be known about $\tau(\mathcal{M}, C)$ -- see the remarks of Hoffmann-Jorgensen (1972, p. 132) -- this seems to be an open question.

Discussion Paper Series, CentER, Tilburg University, The Netherlands:

No.	Author(s)	Title
8801	Th. van de Klundert and F. van der Ploeg	Fiscal Policy and Finite Lives in Interdependent Economies with Real and Nominal Wage Rigidity
8802	J.R. Magnus and B. Pesaran	The Bias of Forecasts from a First-order Autoregression
8803	A.A. Weber	The Credibility of Monetary Policies, Policy-makers' Reputation and the EMS-Hypothesis: Empirical Evidence from 13 Countries
8804	F. van der Ploeg and A.J. de Zeeuw	Perfect Equilibrium in a Model of Competitive Arms Accumulation
8805	M.F.J. Steel	Seemingly Unrelated Regression Equation Systems under Diffuse Stochastic Prior Information: A Recursive Analytical Approach
8806	Th. Ten Raa and E.N. Wolff	Secondary Products and the Measurement of Productivity Growth
8807	F. van der Ploeg	Monetary and Fiscal Policy in Interdependent Economies with Capital Accumulation, Death and Population Growth
8901	Th. Ten Raa and P. Kop Jansen	The Choice of Model in the Construction of Input-Output Coefficients Matrices
8902	Th. Nijman and F. Palm	Generalized Least Squares Estimation of Linear Models Containing Rational Future Expectations
8903	A. van Soest, I. Woittiez, A. Kapteyn	Labour Supply, Income Taxes and Hours Restrictions in The Netherlands
8904	F. van der Ploeg	Capital Accumulation, Inflation and Long-Run Conflict in International Objectives
8905	Th. van de Klundert and A. van Schaik	Unemployment Persistence and Loss of Productive Capacity: A Keynesian Approach
8906	A.J. Markink and F. van der Ploeg	Dynamic Policy Simulation of Linear Models with Rational Expectations of Future Events: A Computer Package
8907	J. Osiewalski	Posterior Densities for Nonlinear Regression with Equicorrelated Errors
8908	M.F.J. Steel	A Bayesian Analysis of Simultaneous Equation Models by Combining Recursive Analytical and Numerical Approaches

No.	Author(s)	Title
8909	F. van der Ploeg	Two Essays on Political Economy (i) The Political Economy of Overvaluation (ii) Election Outcomes and the Stockmarket
8910	R. Gradus and A. de Zeeuw	Corporate Tax Rate Policy and Public and Private Employment
8911	A.P. Barten	Allais Characterisation of Preference Structures and the Structure of Demand
8912	K. Kamiya and A.J.J. Talman	Simplicial Algorithm to Find Zero Points of a Function with Special Structure on a Simplotope
8913	G. van der Laan and A.J.J. Talman	Price Rigidities and Rationing
8914	J. Osiewalski and M.F.J. Steel	A Bayesian Analysis of Exogeneity in Models Pooling Time-Series and Cross-Section Data
8915	R.P. Gilles, P.H. Ruys and J. Shou	On the Existence of Networks in Relational Models
8916	A. Kapteyn, P. Kooreman and A. van Soest	Quantity Rationing and Concavity in a Flexible Household Labor Supply Model
8917	F. Canova	Seasonalities in Foreign Exchange Markets
8918	F. van der Ploeg	Monetary Disinflation, Fiscal Expansion and the Current Account in an Interdependent World
8919	W. Bossert and F. Stehling	On the Uniqueness of Cardinally Interpreted Utility Functions
8920	F. van der Ploeg	Monetary Interdependence under Alternative Exchange-Rate Regimes
8921	D. Canning	Bottlenecks and Persistent Unemployment: Why Do Booms End?
8922	C. Fershtman and A. Fishman	Price Cycles and Booms: Dynamic Search Equilibrium
8923	M.B. Canzoneri and C.A. Rogers	Is the European Community an Optimal Currency Area? Optimal Tax Smoothing versus the Cost of Multiple Currencies
8924	F. Groot, C. Withagen and A. de Zeeuw	Theory of Natural Exhaustible Resources: The Cartel-Versus-Fringe Model Reconsidered

No.	Author(s)	Title
8925	O.P. Attanasio and G. Weber	Consumption, Productivity Growth and the Interest Rate
8926	N. Rankin	Monetary and Fiscal Policy in a 'Hartian' Model of Imperfect Competition
8927	Th. van de Klundert	Reducing External Debt in a World with Imperfect Asset and Imperfect Commodity Substitution
8928	C. Dang	The D_1-Triangulation of R^n for Simplicial Algorithms for Computing Solutions of Nonlinear Equations
8929	M.F.J. Steel and J.F. Richard	Bayesian Multivariate Exogeneity Analysis: An Application to a UK Money Demand Equation
8930	F. van der Ploeg	Fiscal Aspects of Monetary Integration in Europe
8931	H.A. Keuzenkamp	The Prehistory of Rational Expectations
8932	E. van Damme, R. Selten and E. Winter	Alternating Bid Bargaining with a Smallest Money Unit
8933	H. Carlsson and E. van Damme	Global Payoff Uncertainty and Risk Dominance
8934	H. Huizinga	National Tax Policies towards Product-Innovating Multinational Enterprises
8935	C. Dang and D. Talman	A New Triangulation of the Unit Simplex for Computing Economic Equilibria
8936	Th. Nijman and M. Verbeek	The Nonresponse Bias in the Analysis of the Determinants of Total Annual Expenditures of Households Based on Panel Data
8937	A.P. Barten	The Estimation of Mixed Demand Systems
8938	G. Marini	Monetary Shocks and the Nominal Interest Rate
8939	W. Güth and E. van Damme	Equilibrium Selection in the Spence Signaling Game
8940	G. Marini and P. Scaramozzino	Monopolistic Competition, Expected Inflation and Contract Length
8941	J.K. Dagsvik	The Generalized Extreme Value Random Utility Model for Continuous Choice

No.	Author(s)	Title
8942	M.F.J. Steel	Weak Exogenity in Misspecified Sequential Models
8943	A. Roell	Dual Capacity Trading and the Quality of the Market
8944	C. Hsiao	Identification and Estimation of Dichotomous Latent Variables Models Using Panel Data
8945	R.P. Gilles	Equilibrium in a Pure Exchange Economy with an Arbitrary Communication Structure
8946	W.B. MacLeod and J.M. Malcomson	Efficient Specific Investments, Incomplete Contracts, and the Role of Market Alternatives
8947	A. van Soest and A. Kapteyn	The Impact of Minimum Wage Regulations on Employment and the Wage Rate Distribution
8948	P. Kooreman and B. Melenberg	Maximum Score Estimation in the Ordered Response Model
8949	C. Dang	The D_3-Triangulation for Simplicial Deformation Algorithms for Computing Solutions of Nonlinear Equations
8950	M. Cripps	Dealer Behaviour and Price Volatility in Asset Markets
8951	T. Wansbeek and A. Kapteyn	Simple Estimators for Dynamic Panel Data Models with Errors in Variables
8952	Y. Dai, G. van der Laan, D. Talman and Y. Yamamoto	A Simplicial Algorithm for the Nonlinear Stationary Point Problem on an Unbounded Polyhedron
8953	F. van der Ploeg	Risk Aversion, Intertemporal Substitution and Consumption: The CARA-LQ Problem
8954	A. Kapteyn, S. van de Geer, H. van de Stadt and T. Wansbeek	Interdependent Preferences: An Econometric Analysis
8955	L. Zou	Ownership Structure and Efficiency: An Incentive Mechanism Approach
8956	P.Kooreman and A. Kapteyn	On the Empirical Implementation of Some Game Theoretic Models of Household Labor Supply
8957	E. van Damme	Signaling and Forward Induction in a Market Entry Context

No.	Author(s)	Title
9001	A. van Soest, P. Kooreman and A. Kapteyn	Coherency and Regularity of Demand Systems with Equality and Inequality Constraints
9002	J.R. Magnus and B. Pesaran	Forecasting, Misspecification and Unit Roots: The Case of AR(1) Versus ARMA(1,1)
9003	J. Driffill and C. Schultz	Wage Setting and Stabilization Policy in a Game with Renegotiation
9004	M. McAleer, M.H. Pesaran and A. Bera	Alternative Approaches to Testing Non-Nested Models with Autocorrelated Disturbances: An Application to Models of U.S. Unemployment
9005	Th. ten Raa and M.F.J. Steel	A Stochastic Analysis of an Input-Output Model: Comment
9006	M. McAleer and C.R. McKenzie	Keynesian and New Classical Models of Unemployment Revisited
9007	J. Osiewalski and M.F.J. Steel	Semi-Conjugate Prior Densities in Multivariate t Regression Models
9008	G.W. Imbens	Duration Models with Time-Varying Coefficients
9009	G.W. Imbens	An Efficient Method of Moments Estimator for Discrete Choice Models with Choice-Based Sampling
9010	P. Deschamps	Expectations and Intertemporal Separability in an Empirical Model of Consumption and Investment under Uncertainty
9011	W. Güth and E. van Damme	Gorby Games - A Game Theoretic Analysis of Disarmament Campaigns and the Defense Efficiency-Hypothesis
9012	A. Horsley and A. Wrobel	The Existence of an Equilibrium Density for Marginal Cost Prices, and the Solution to the Shifting-Peak Problem
9013	A. Horsley and A. Wrobel	The Closedness of the Free-Disposal Hull of a Production Set
9014	A. Horsley and A. Wrobel	The Continuity of the Equilibrium Price Density: The Case of Symmetric Joint Costs, and a Solution to the Shifting-Pattern Problem

P.O. BOX 90153, 5000 LE TILBURG, THE NETHERLAND