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CentER
for
Economic Research

No. 9319

Multi-sided Pre-play Communication by Burning Money

by Sjaak Hurkens

March 1993

ISSN 0924-7815

Multi-sided pre-play communication by burning money

Sjaak Hurkens*†‡

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Abstract

We investigate the consequences of allowing some players to send a costly message before a game is played. Since messages have no literal meaning sending costly messages is also called 'burning money'. We consider a setting with n players among which k have the possibility to burn money. We show that equilibria in sets that are closed under rational behaviour yield all players that have the possibility to burn, their preferred outcome. Moreover, in such equilibria no money is actually burnt, the possibility alone suffices. For the special case with two players all results go through for persistent equilibria (Kalai and Samet (1984)) as well.

*CentER, Tilburg University P.O. Box 90153, 5000 LE Tilburg, The Netherlands.

†The author thanks Eric van Damme for many helpful comments.

‡This research was sponsored by the Foundation for the Promotion of Research in Economic Sciences, which is part of the Netherlands Organization for Scientific Research (NWO).

1 Introduction

Given a game in normal form, Ben-Porath and Dekel (1992) investigate the consequences of allowing some players to signal future actions by incurring costs before the game is played. They consider equilibria that survive repeated elimination of weakly dominated strategies. Their main result is that, in a certain class of two person games, if only one player can signal, then repeated elimination of weakly dominated strategies selects her most preferred outcome. Moreover, the player does not have to incur any cost to achieve this.

While this result is nice, it has some important limitations. First of all Ben-Porath and Dekel consider only two player games. As we will show later by example, repeated elimination of weakly dominated strategies need not work in games with more than two players. Furthermore, they show that if both players have the opportunity to signal (simultaneously), then signalling future actions is not possible, not even if the game has common interests. Finally, they need that a player can burn a considerable amount of money.

We consider a more general case. We extend n-person games by allowing k of the players to signal future actions by incurring costs. In order to obtain results similar to those of Ben-Porath and Dekel we work with a stronger solution concept. Van Damme (1989) showed that stable equilibria do not necessarily lead to efficient outcomes. We show that, if the refined notion of curb or curb* (Basu and Weibull (1991)) equilibrium is used, then signalling future actions is possible. Moreover, if there are two players then the notion of persistence (Kalai and Samet (1984)) gives the same result.

To be more precise, we get the following results in two person games:

- If only one player can signal, then, in the class of games considered by Ben-Porath and Dekel, persistence, curb and curb* select her most preferred outcome .
- If both players can signal, then persistence, curb and curb* select the mutually preferred outcome in common interest games.

In the general case with n players where k of them can send a message before the game is played, we obtain similar results. Roughly speaking, if the players that can burn money have common interests in the underlying game, then curb and curb* select their preferred outcome. Furthermore, in all models of pre-play communication, no costs are actually incurred.

It may be worth noting that our result on the model of one-sided pre-play communication is not a consequence of Ben-Porath and Dekel. They need that the player can burn a considerable amount of money whereas we only need that the player can choose to burn nothing or to burn a small amount.

In the next section we will consider a simple example of a two person game where only one player can signal. From this example the reader can develop some feeling for the solution concept we employ. Moreover, this example shows a difference between our approach and that of Ben-Porath and Dekel's. At the same time it shows that it is important that messages are costly.

In section 3 the formal model and the notions of curb and curb* are defined. In section 4 we prove the theorem for these concepts for the general case. In section 5 we consider the persistent equilibria in the special case of two person games. Moreover, it is shown that in games with more than two players persistence need not work. In section 6 we examine the consequences of analyzing the game in the agent normal form instead of the normal form. We close the paper with some concluding remarks.

2 An example: the battle of the sexes

Consider the battle of the sexes game represented by the following normal form. The woman (the row player) prefers to go to a soccer match ('S'), the man (the column player) prefers to go to the theatre ('t').

	s	t
S	9,5	0,4
T	4,4	6,7

figure 1: the battle of the sexes.

Suppose the woman can send one of two messages, m^0 or m^1. Message m^0 is costless and message m^1 costs c. Later we will consider the cases $c = 0$, $c = 1$ and $c = 2$. Let

m^iE denote the woman's strategy when she sends m^i and visits event E. Let e_0e_1 denote the man's strategy when he goes to event e_i if he receives message m^i. Then the game with one-sided pre-play communication can be represented by the following normal form.

	ss	st	ts	tt
m^0S	9,5	9,5	0,4	0,4
m^0T	4,4	4,4	6,7	6,7
m^1S	$9-c,5$	$-c,4$	$9-c,5$	$-c,4$
m^1T	$4-c,4$	$6-c,7$	$4-c,4$	$6-c,7$

figure 2.

First we will consider Ben-Porath and Dekel's approach. Consider the case $c = 2$. Action m^1T is dominated (weakly) by m^0T. So the man should go to the soccer match if he receives message m^1. This means that st and tt are dominated. If the woman knows that the man will play ss or ts, then the action m^0T is dominated by m^1S. So the man knows that the woman will go to the soccer match, hence he should go to the soccer match also (st is dominated by ss). If the woman knows that, she will send m^0 and go to the soccer match (m^1S is dominated by m^0S). Hence, repeated elimination of weakly dominated strategies selects the outcome where both go to the soccer match and no money is actually being burnt. Notice that this reasoning does not go through if $c = 0$ or if $c = 1$. In these cases no player has a dominated strategy. For example, if $c = 1$, m^1T can only be dominated by a strategy that puts at least weight $\frac{5}{6}$ on m^0T, and at least weight $\frac{1}{5}$ on m^0S.

We will look at minimal sets of strategies that are closed under best responses. (Basu and Weibull (1991) call this curb: closed under rational behaviour.) A set of strategy profiles is closed under best responses if with every strategy profile all its best replies are contained in the set.

Now look at our example. Suppose $R_1 \times R_2$ is closed under best responses. It is easy to check that we have the following implications (for all values of c under consideration).

$$m^1T \in R_1 \quad\Rightarrow\quad tt \in R_2 \quad\Rightarrow\quad m^0T \in R_1 \quad\Rightarrow\quad ts \in R_2 \quad\Rightarrow\quad m^1S \in R_1$$

$$m^1S \in R_1 \quad\Rightarrow\quad ss \in R_2 \quad\Rightarrow\quad m^0S \in R_1 \quad\Rightarrow\quad st \in R_2 \quad\Rightarrow\quad m^0S \in R_1$$

In the cases with $c > 0$ the set of strategy profiles where the woman plays m^0S and the man mixes between ss and st is closed under best responses. The serie of implications shows that this set is the unique minimal one.

If $c = 0$ the serie of implications continues.

$$m^0S \in R_1 \quad \Rightarrow \quad ss, st \in R_2 \quad \Rightarrow \quad m^1S \in R_1 \quad \Rightarrow \quad ts \in R_2$$

Since m^0T and m^1T are best replies against $\frac{1}{2}st + \frac{1}{2}ts$, we conclude that the only set that is closed under best responses is the set of all strategy profiles.

Hence, if talk is costless, then on the basis of curb no sharp predictions can be made.

3 The model

In this section we will formally set up the model of multi-sided pre-play communication in n-person games. Before we do that we recall the notions of curb and curb* (Basu and Weibull (1991)).

We denote the set of mixed strategies of player i by Σ_i. For a set of pure strategies X let $\mathrm{co}(X)$ denote the convex hull of this set, which is equivalent to the set of all probability distributions over X. If σ is a strategy profile then σ_{-i} denotes the profile of all players besides player i. The set of best replies of player i against σ_{-i} is denoted by $\mathrm{BR}_i(\sigma_{-i})$. Define $\mathrm{BR}(\sigma) = \prod_{i=1}^n \mathrm{BR}_i(\sigma_{-i})$. $\mathrm{UBR}(\sigma)$ denotes the set of best replies against σ that are not weakly dominated. A retract is a cartesian product $R = \prod_{i=1}^n R_i$ where $R_i \subset \Sigma_i$ is non-empty, convex and compact.

Definition 1 .

A retract R is said to have the curb property if for all $\sigma \in R$ $BR(\sigma) \subset R$.

A retract is said to have the curb property if for all $\sigma \in R$ $UBR(\sigma) \subset R$.*

A retract which is minimal with respect to the curb (curb) property is called a curb (curb*) retract.*

A Nash equilibrium that is contained in a curb (curb) retract is called a curb (curb*) equilibrium.*

These retracts are closely related and have some nice properties that are summarised in the following lemma.

Lemma 1. Every curb retract contains a curb* retract. Every curb* retract contains a Nash equilibrium. Every curb (curb*) retract is a cartesian product of convex hulls of pure strategies. The intersection of two retracts with the curb (curb*) property is either empty or it has the curb (curb*) property, hence different curb (curb*) retracts are disjoint.

Proof. Every curb retract has the curb* property and hence contains a curb* retract.

Suppose there exists an extreme point of a curb retract in which some player plays a mixed strategy. If this mixed strategy were a best reply against some strategy in the retract then all the pure strategies in its carrier would also be best replies, and hence in the retract. Since this contradicts the mixed strategy being an extreme point, every strategy that puts positive weight on this mixed strategy can be deleted from the retract. This yields a strictly smaller retract that still has the curb property. For curb* retracts the proof is similar.

Consider the reduced game where every player is constrained to play strategies from a curb* retract R. By the usual fixed point argument it can be shown that this reduced game has a Nash equilibrium. From the definition of a curb* retract it follows that this Nash equilibrium for the reduced game is also a Nash equilibrium of the original game.

The proof of the other assertions is trivial. □

Let $G = (S_1, \ldots, S_n, u_1, \ldots, u_n)$ be an n-person game with player set N. We split up the set of players into C and D. C is the set of players that can send a message in the pre-play communication stage (communicating players); D is the set of players that cannot (dumb players). Note that the players in D are dumb but not deaf. It is important that they can hear. We assume that C is not empty. D may be empty.

We will assume

Assumption 1 . *There exists $s^* \in S$ such that $u_i(s^*) > u_i(s)$, for all $i \in C$ and all $s \neq s^*$ and such that $u_j(s^*) > u_j(s^*_{-j}, s_j)$ for all $j \in D$ and all $s_j \neq s^*_j$.*

Notice that if $C = N$ then G is a common interest game (Aumann and Sorin (1989)). Assumption 1 says that the game has a strict Nash equilibrium that gives all communicating players their highest payoff.

We assume that all communicating players dispose of the same set of messages $M =$

$\{m^0, m^1, \ldots, m^L\}$ and that the cost of sending message m^p is $c(m^p)$ for all of them. These assumptions are made for notational convenience only. They do not play a role in the results we will derive. We assume that $c(m^0) = 0$, $c(m^1) > 0$ is 'small' and $c(m^p) > c(m^1)$ for all $p > 1$. In fact, $c(m^1)$ is so small that

$$c(m^1) < u_i(s^*) - u_i(s) \qquad \text{for all } i \in C \text{ and all } s \neq s^* \tag{3.1}$$

This implies that any communicating player that can induce the play of s^* by sending message m^1 will do so, unless sending m^0 induces the play of s^* also.

We denote the game with pre-play communication by $P_C(G) = (T_1, \ldots, T_n, v_1, \ldots, v_n)$.

$T_i = \{m_i f_i | m_i \in M \text{ and } f_i : M^{C\setminus\{i\}} \to S_i\} \quad (i \in C)$

$T_j = \{f_j | f_j : M^C \to S_j\} \quad (j \in D)$

(We use the following convention if $C = \{i\}$: $f_i : M^\emptyset \to S_i$ corresponds to a single element of S_i.) With some abuse of notation we have for $mf \in T = \prod_{l=1}^n T_l$

$v_i(mf) = u_i(f(m)) - c(m_i) \quad (i \in C)$

$v_j(mf) = u_j(f(m)) \quad (j \in D)$

Remark: The way we have defined the strategy space here means that we are looking at the reduced normal form (as Ben-Porath and Dekel did). In the normal form a communicating player's strategy also depends on his own message. It does not matter for our results whether we analyze the normal form or the reduced normal form. However, in the normal form the communicating players have a lot of equivalent strategies and this makes the proof for persistence quite tedious. We could also look at the agent normal form (Selten (1975)). We will do that in section 6.

4 Results for the general case

In this section we will prove the theorem for the curb and curb* equilibria. We will deal with the general case with n players among which the players in C can communicate.

Theorem 1 . *Every equilibrium of $P_C(G)$ that is contained in a curb or curb* retract yields player l $u_l(s^*)$ ($l \in N$).*

Proof. We will denote a vector with all coordinates equal to m^0 by $\widehat{m^0}$. (Depending on whether this vector is in the domain of a dumb or of a communicating player, it has $\#C$

or $\#C-1$ coordinates. No confusion will result.) Define

$$F = \{mf \in T | m = \widehat{m^0}\ ,\ f_l(\widehat{m^0}) = s_l^* \text{ for all } l \in N\}$$

Notice that F consists of all pure strategy profiles that yield the payoff vector $u(s^*)$.

We will show that every curb* retract has a non-empty intersection with F. If this is the case then every curb* retract is contained in $\bar{R} = \prod_{l=1}^{n} \text{co}(F_l)$, since $\bar{R}$ has the curb* property. Since every curb retract contains a curb* retract we know then that every curb retract has a non-empty intersection with $\bar{R}$. Since $\bar{R}$ has the curb property itself any curb retract is contained in $\bar{R}$. But then the theorem is proved since every equilibrium that is contained in $\bar{R}$ yields player l $u_l(s^*)$.

Let R be a curb* retract and let $\bar{m}\bar{f} \in R \setminus F$. Now player l has a lot of undominated best replies against $(\bar{m}\bar{f})_{-l}$. This is so because a player has a lot of freedom in how to react on tuples of messages that he does not receive. In particular we have

for all $i \in C$ there exists $m_i' f_i' \in R_i$ such that $f_i'(m_{-i}) = s_i^*$ for all $m_{-i} \neq \bar{m}_{-i}$

for all $j \in D$ there exists $f_j' \in R_j$ such that $f_j'(m) = s_j^*$ for all $m \neq \bar{m}$

If $m'f' \in F$ we are ready, because then $F \cap R \neq \emptyset$.

So suppose $m'f' \notin F$.

Case 1: $\bar{m} \neq \widehat{m^0}$. There exists $i \in C$ with $\bar{m}_i \neq m^0$. All best replies for i against $m'f'$ involve sending m^0. In particular, if $f_i''(m) = s_i^*$ for all m, then $m^0 f_i''$ is an undominated best reply and hence contained in R. Now for $j \in C, j \neq i$ all best replies against $(m^0 f_i'', m'f_{-i}')$ involve sending m^0. In particular, if $f_j''(m) = s_j^*$ for all m, then $m^0 f_j''$ is an undominated best reply and hence contained in R. Now it is shown that $F \cap R \neq \emptyset$.

Case 2: $\bar{m} = \widehat{m^0}$. Let $i \in C$. All best replies for i against $m'f'$ involve sending m^1. Hence, there exists $\bar{\bar{m}}\bar{\bar{f}} \in R$ with $\bar{\bar{m}} \neq \widehat{m^0}$. This brings us back to case 1. □

5 Further results for two player games

Let us recall the notion of persistent equilibria (Kalai and Samet (1984)).

Definition 2 .

A retract R is called absorbing if there exists an open neighbourhood $U \supset R$ such that for all $\sigma \in U$ we have that $BR(\sigma) \cap R \neq \emptyset$.

An absorbing retract is called persistent if it does not properly contain an absorbing retract.

An equilibrium that is contained in a persistent retract is called a persistent equilibrium.

For an elaborate discussion of the relation between persistent retracts and other solution concepts the reader is referred to Balkenborg (1992). Two interesting facts on persistent retracts we will state here, without proofs. Every curb* retract contains a persistent retract (Balkenborg (1992)) and every persistent retract contains a Nash equilibrium (Kalai and Samet (1984)).

Let us state the following lemma which will prove to be useful in the two player case.

Lemma 2. Let $G = (S_1, S_2, u_1, u_2)$ be a game. Let $F_i \subset S_i$ and $H_i = S_i \setminus F_i$. Let $F = F_1 \times F_2$ be such that

(i) There are $x_1, x_2 \in \mathbf{R}$ such that

1. for all $f \in F$ $u_i(f) = x_i$
2. for all $f_j \in F_j$, $h_i \in H_i$ $u_i(h_i, f_j) < x_i$

(ii) Every persistent retract has a non-empty intersection with F.

Then every persistent retract is contained in $\bar{R} = \mathrm{co}(F_1) \times \mathrm{co}(F_2)$.

Proof. Let R be a persistent retract, and let $V_1 \times V_2 \supset R$ be an open neighbourhood that is absorbed by R. Define

$$\begin{aligned}
m_i^+ &= \max u_i(s_1, s_2) \\
m_i^- &= \min u_i(s_1, s_2) \\
\delta(h_1) &= x_1 - \max_{f_2 \in F_2} u_1(h_1, f_2) \quad (h_1 \in H_1) \\
\delta(h_2) &= x_2 - \max_{f_1 \in F_1} u_2(f_1, h_2) \quad (h_2 \in H_2)
\end{aligned}$$

Notice that (i) implies $\delta(h_i) > 0$. Let $\delta_i = \min_{h_i \in H_i} \delta(h_i)$. Let $\epsilon_j > 0$ $(j = 3 - i)$ satisfy

$$(1 - \epsilon_j)(x_i - \delta_i) + \epsilon_j m_i^+ < (1 - \epsilon_j)x_i + \epsilon_j m_i^- \tag{5.2}$$

Let $\sigma_i(s_i)$ denote the weight that σ_i puts on s_i. Define $V_i(\epsilon_i) = \{\sigma_i \in V_i | \sum_{h_i \in H_i} \sigma_i(h_i) < \epsilon_i\}$. Then $V_1(\epsilon_1) \times V_2(\epsilon_2)$ is an open neighbourhood of $R \cap \bar{R}$. We claim that this neighbourhood is absorbed by $R \cap \bar{R}$.

Let $\sigma_2 \in V_2(\epsilon_2), h_1 \in H_1, f_1 \in F_1$. Now

$$\begin{aligned} u_1(h_1, \sigma_2) &\leq \sum_{F_2} \sigma_2(f_2)(x_1 - \delta(h_1)) + \sum_{H_2} \sigma_2(h_2) m_1^+ \\ &< (1 - \epsilon_2)(x_1 - \delta_1) + \epsilon_2 m_1^+ \\ &< (1 - \epsilon_2)x_1 + \epsilon_2 m_1^- \\ &\leq \sum_{F_2} \sigma_2(f_2)x_1 + \sum_{H_2} \sigma_2(h_2) u_1(f_1, h_2) = u_1(f_1, \sigma_2) \end{aligned}$$

Hence, there are no best replies against strategies in $V_2(\epsilon_2)$ in H_1. Hence, the best replies are in F_1. Reversing the roles of the players then proves the claim. Hence, every persistent retract is contained in $\bar{R}$. □

From now on we will assume

Assumption 2 . *No player has equivalent strategies in the game with pre-play communication.*

Then we have that every persistent retract of this game is a cartesian product of convex hulls of pure strategies. For a proof see Kalai and Samet (1984).

5.1 One-sided pre-play communication

First we will consider the case with one-sided pre-play communication, hence $C = \{1\}$. Assumption 1 is now equivalent to saying that the underlying game has a strict Nash equilibrium yielding player 1 a higher payoff than any other strategy profile.

Theorem 2 . *Every persistent equilibrium of $P_{\{1\}}(G)$ yields player l $u_l(s^*)$ $(l \in N)$.*

Proof. Let R be a persistent retract and let $ms_1 \in R_1$.

Case 1. $m \neq m^0$. Then there exists $g \in R_2$ with $g(m^0) = s_2^*$. This is so because all best replies against $(1-\epsilon)ms_1 + \epsilon m^0 s_1^*$ have this property. This implies that $m^0 s_1^* \in R_1$, since it is the unique best reply against g.

Case 2. $m = m^0$ and $s_1 \neq s_1^*$. Using the same trick as before we see that there exists $g \in R_2$ with $g(m^1) = s_2^*$. If $g(m^0) = s_2^*$, then $m^0 s_1^* \in R_1$. If $g(m^0) \neq s_2^*$, then $m^1 s_1^* \in R_1$. This brings us back to case 1.

Hence, $m^0 s_1^* \in R_1$. It is easy to check that the set of extreme points of

$$F = \{m^0 s_1^*\} \times \text{co}(\{g : M \to S_2 | g(m^0) = s_2^*\})$$

satisfies assumptions (i) and (ii) of lemma 2.

Then it follows from lemma 2 that $R \subset F$. Notice that every equilibrium in F yields player l $u_l(s^*)$ and the theorem has been proved. □

5.2 Two-sided pre-play communication

Now we turn our attention to the two-sided pre-play communication game, i.e. $C = \{1, 2\}$. Recall that assumption 1 now says that the underlying game has common interests.

Theorem 3 . *Every persistent equilibrium of $P_C(G)$ yields player l a payoff of $u_l(s^*)$.*

Proof. Let R be a persistent retract and let $mf \in R_i$. Let $j = 3 - i$.

Case 1. $m \neq m^0$. Then there exists $m'g \in R_j$ with $g(m^0) = s_j^*$. Hence, there exists $m^0 f' \in R_i$ with $f'(m^0) = s_i^*$.

Case 2. $m = m^0$ and $f(m^0) \neq s_i^*$. Then there exists $m'g \in R_j$ with $g(m^1) = s_j^*$. If $g(m^0) = s_j^*$, then there exists $m^0 f' \in R_i$ with $f'(m^0) = s_i^*$. If $g(m^0) \neq s_j^*$, then there exists $m^1 f'' \in R_i$. This brings us back to case 1.

Let $F_i = \{m^0 f | f(m^0) = s_i^*\}$. The two cases showed that $F_i \cap R_i \neq \emptyset$. It is easy to check that F_1 and F_2 satisfy assumptions (i) and (ii) in lemma 2. Hence, any persistent retract is contained in $\text{co}(F_1) \times \text{co}(F_2)$. Notice that any equilibrium in this set yields player i $u_i(s^*)$ and the theorem has been proved. □

5.3 A three player counterexample

The following example shows that persistence need not work in games with more than two players.

Example. Let $n = 3, C = \{3\}$. Player 1 chooses between rows R_1 and R_2, player 2 chooses between columns C_1 and C_2 and player 3 chooses A. (Player 3 has only one action.) The payoffs are given in figure 3.

	C_1	C_2
R_1	2,2,2	0,0,0
R_2	0,0,0	1,1,1

figure 3.

Now consider the following strategy profile: players 1 and 2 play their second strategy after all messages and player 3 sends the costless message and then 'chooses' A. This yields the strictly dominated payoff vector $(1,1,1)$. Nevertheless, this strategy profile is a persistent retract as a singleton and hence a persistent equilibrium. In a small neighbourhood of the retract player 3 has a unique best reply. Players 1 and 2 have a lot of best replies against the retract, but in a small neighbourhood outside the retract they have a unique best reply. This is due to the fact that players 1 and 2 have an interest in choosing the same action: in a small neighbourhood player 1 plays R_2 with a very high probability, after any message, and hence player 2 has to choose C_2, after any message.

Repeated elimination of weakly dominated strategies does not work either. In this example the messages are not important to signal actions, since player 3 has only one action. But the messages help the other players to coordinate on the efficient outcome, at least if the notion of curb or curb* is used.

6 Agent normal form

As remarked at the end of section 4 we have analyzed the pre-play communication game in the reduced normal form. In this section we will look at the agent normal form (Selten (1975)). The agent normal form is obtained by splitting every player into several agents, one agent corresponds to one information set of that player and each agent having the same utility function as that player. In the agent normal form different agents of the same player take their decisions independently.

Consider for example the one-sided pre-play communication game of the battle of the sexes of section 2. The extensive form of that game (when $c = 2$) is given in figure 4.

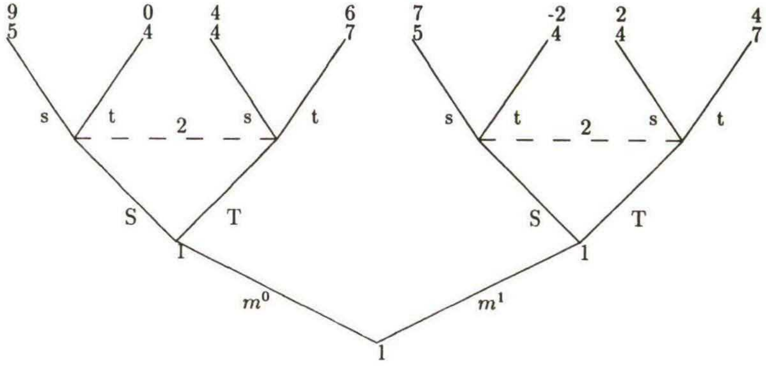

figure 4.

Player 1 has three information sets, namely $\emptyset$, $\{m^0\}$ and $\{m^1\}$. So in the agent normal form there are three agents for player 1; call them 1_0, $1(m^0)$ and $1(m^1)$. Player 2 has two information sets, namely $\{m^0S, m^0T\}$ and $\{m^1S, m^1T\}$. Call the agents $2(m^0)$ and $2(m^1)$. So the agent normal form of this game has 5 agents (or players).

It is well known by now that a lot of refinements of the Nash equilibrium concept may give different results in the normal form and in the agent normal form.

In the agent normal form of the game in figure 4 no agent has a weakly dominated strategy. Hence, repeated elimination of weakly dominated strategies does not work in the agent normal form.

For persistence we have similar problems. Consider the strategy profile where agent 1_0 sends m^0, agents $1(m^0)$ and $1(m^1)$ play T and agents $2(m^0)$ and $2(m^1)$ play t. One can check that this profile is a persistent retract.

It is not surprising that persistence and repeated elimination of weakly dominated strategies give different results in the agent normal form and in the normal form. By splitting up the players into agents we have made a five player game out of a two player game. We have already seen that these solution concepts do not work in games with more than two players.

In principle we could have the same problems with curb and curb*. In general it is not true that the curb and curb* equilibria in the normal form correspond to those in the agent normal form. However, the class of games considered in this paper is special. We have

Theorem 4 . *Every curb (curb*) equilibrium of the agent normal form of $P_C(G)$ yields all agents of player l $u_l(s^*)$ $(l \in N)$.*

Proof. For $i \in N$ let $i(m)$ denote the agent of player i with information set $\{m\}$. For $i \in C$ let i_0 denote the agent of player i that sends a message. The number of agents is $\#C + (\#N) \times (\#M)^{\#C}$. Let A be the set of agents. A strategy profile is denoted by ms. Let $\widehat{m^0}$ be the vector with $\#C$ coordinates, all equal to m^0.

Define

$$F = \{ms | m = \widehat{m^0}\ ,\ s_{i(\widehat{m^0})} = s_i^* \text{ for all } i \in N\}$$

F consists of all pure strategy profiles that yield the payoff vector $u(s^*)$.

We will show that every curb* retract has a non-empty intersection with F. If this is the case then every curb* retract is contained in $\bar{R} = \prod_{j \in A} \mathrm{co}(F_j)$, since $\bar{R}$ has the curb* property. Since every curb retract contains a curb* retract we know then that every curb retract has a non-empty intersection with $\bar{R}$. Since $\bar{R}$ has the curb property itself any curb retract is contained in $\bar{R}$. But then the theorem is proved since every equilibrium in $\bar{R}$ yields all agents of player l $u_l(s^*)$.

Let R be a curb* retract and let $\bar{m}\bar{s} \in R \setminus F$.

Case 1. $\bar{m} \neq \widehat{m^0}$.

For all agents $i(m)$ with $m \neq \bar{m}$ we have that s_i^* is an undominated best reply against ms. Hence, $ms_{i(\bar{m})}s^*_{-i(\bar{m})} \in R$. An agent i_0 with $m_{i_0} \neq m^0$ has a unique best reply against this profile, namely to send m^0. Hence, $m^0\bar{m}_{-i_0}s_{i(\bar{m})}s^*_{-i(\bar{m})} \in R$. Against this strategy agent j_0 has a unique best reply, namely to send m^0. Now it is shown that $R \cap F \neq \emptyset$.

Case 2. $\bar{m} = \widehat{m^0}$.

For all agents $i(m)$ with $m \neq \bar{m}$ we have that s_i^* is an undominated best reply against $\bar{m}\bar{s}$. Hence, $\bar{m}\bar{s}_{i(\bar{m})}s^*_{-i(\bar{m})} \in R$. Every agent i_0 has a unique best reply against this strategy, namely to send m^1. Hence, there is a strategy profile $\bar{\bar{m}}\bar{\bar{s}} \in R$ with $\bar{\bar{m}} \neq \widehat{m^0}$. This brings us back to case 1. □

7 Conclusion

We have generalised the result of Ben-Porath and Dekel into two directions. We consider games with more than two players, and there may be more than one player that has the possibility to burn money. We have shown that curb and curb* retracts select the equilibria that are preferred by the people that have the possibility to burn money.

This is in contrast with the unintuitive result that Ben-Porath and Dekel obtained in the case that both players can signal. In a two player game with common interests repeated elimination of weakly dominated strategies selects the mutually preferred outcome, if exactly one player can signal. It does not matter which player can signal. But if both can, then the efficiency result disappears.

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