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Abstract

This paper points out some of the peculiarities that may arise not in the problematic and widely dealt with classical social choice context, but in the context of collective decision making in a team - a group of individuals who share the same interest but often possess different decisional skills. Employing the simple symmetric version of the uncertain, independent, dichotomous choice framework, we present a series of propositions about the somewhat unexpected violation of various forms of independence, monotonicity and symmetry properties.

1. Introduction

Numerous studies have been concerned with various aspects of the social choice problematics. To this day there remains considerable disagreement on the appropriate method for selecting an alternative when the social choice is based on heterogeneous preferences. Impossibility theorems abound and the shortcomings of alternative collective decision rules have often been demonstrated by showing that their use results in paradoxes. The purpose of the current paper is to complement this literature by pointing out some of the peculiarities - results that seem opposed to common sense or intuition (at least at first glance) - that may arise not in the problematic social choice context, but in the unproblematic[1] context of collective decision making in a team - a group of individuals (board of directors, committee or a panel of experts, court, jury) who share the same interest (preferences, objective function) but often possess different decisional skills. Employing a simple symmetric version of the uncertain, independent, dichotomous choice framework, we present a series of propositions about the somewhat unexpected violation of various forms of independence, monotonicity and symmetry properties. Some of these propositions (peculiarities) are reported without their proofs as these can be found in other works of one or the two of us. Most of the results, however, are novel. Their proofs are relatively non-technical and in some cases based on straighforward examples.

In the uncertain symmetric pairwise choice setting which is presented in the following section there always exists an optimal decision rule for the group. The optimal rule turns out to be a weighted majority rule, Nitzan and Paroush (1982), (1985), Shapley and Grofman (1984). The set of potentially optimal decision rules coincides therefore with the set of potentially optimal weighted majority rules.

The individual's optimal weight depends solely on his decisional skill. Nevertheless, his actual essentiality in the collective decision-making process does depend on the decisional skills and, in turn, on the optimal weights assigned to the other group members. Interdependence among individual effectivity levels is discussed in Section 3.

The optimal rule is determined by the group members' decisional skills. In Section 4 we discuss three types of monotonicity: monotonicity of the desirable extent of participation in group decision making with respect to inequality in the distribution of decisional skills; monotonicity of the individual's marginal effectivity on group performance with respect to his ranking in terms of decisional skills and monotonicity of group performance with respect to group size.

In our symmetric setting it seems plausible to expect that there are no "special" rules or "special" subsets of rules among the potentially optimal rules. In Section 5 we show that some rules are "special". The asymmetry among rules is manifested by the "specialness" of some of them. Another peculiarity which is discussed in this section is the fact that asymmetry is also manifested among rankings of rules. The last section contains a brief summary.

2. The framework and some basic results[2]

Consider a group of individuals $N = \{1,...,n\}$ that has to choose one of two mutually exclusive alternatives, a or b. The individual preferences are identical, however, their independent judgements concerning the alternative that ought to be chosen, the alternative that better suits their common objective, may differ. The decisional skill of individual i is represented by his probability p_i of choosing the correct alternative. With no loss of generality, it is assumed that $p_i \geq 0.5$ and that $i < j$ implies $p_i \geq p_j$. The decisional quality of the group, namely its probability of making a correct decision, π, depends on the collective decision rule it applies. Such a rule is a function f from the set of individual decisions $\{-1,1\}^n$ to the set of collective decisions $\{-1,1\}$ where -1 and 1 represent, respectively, decisions in favor of alternative a and alternative b. An element in the domain of the function f is an n-tuple of individual decisions or a decision profile $x = (x_1,\ldots,x_n)$, $x_i \in \{-1,1\}$. The set of all possible decision rules is denoted F. The number of decision rules in F, $|F|$, is 2^{2^n}. The optimal decision rule $\hat{f}$ corresponding to p is the solution of the problem[3]

$$\max_{f \in F} \pi(f(p)) \,. \tag{1}$$

The solution of this problem, Nitzan and Paroush (1982), Shapley and Grofman (1984), is given by a weighted majority rule (*WMR*) with individual weights that are equal to the logarithms of the individual odds of making a correct choice. That is

$$\hat{f}(p) = sign \left(\sum_{i=1}^{n} w_i x_i \right) \tag{2}$$

where $w_i = \ln \dfrac{p_i}{(1 - p_i)}$. Note that any *WMR* can be defined by a normalized system of weights $(w_1,\ldots,w_n)$ such that individual weights are non-negative integers and the sum of the weights is minimal. An individual is called inessential if his normalized weight is zero. The number of essential decision makers corresponding to $\hat{f}$ represents the optimal extent of participation in the group decision making and is denoted by $\hat{k}$. The *WMR* defined by the assignment of equal weight to the k most competent individuals, k being odd, and zero weight to the remaining individuals is called a restricted majority rule (*RMR*) of order k. The *RMR* of order 1 is the widely used expert rule. The *RMR* of order n is the common simple majority rule (*SMR*).

The proof of all the propositions (peculiarities) presented in this paper are based on the optimality condition (2) or on one of its following three corollaries: The necessary and sufficient conditions for the optimality of

(a) the expert rule,

(b) the *SMR*,

(c) the other *RMR's*,

are given, respectively, by (*see* Nitzan and Paroush (1982), (1984b) and Gradstein (1986)) (3a), (3b), (3c).

$$w_1 \geq \sum_{i=2}^{n} w_i \tag{3a}$$

$$\sum_{i=1}^{(n-1)/2} w_i \leq \sum_{i=(n+1)/2}^{n} w_i , \qquad n \text{ being odd} \tag{3b}$$

$$\sum_{i=1}^{(k-1)/2} w_i + \sum_{i=k+1}^{n} w_i \leq \sum_{i=(k+1)/2}^{k} w_i \tag{3c}$$

Table 1 (*see* Fishburn and Gehrlein (1977) and Nitzan and Paroush (1981)) lists all potentially optimal *WMR's* in n-member groups, $n \leq 5$.

Table 1: Potentially optimal *WMR's* in n-member groups, $n \leq 5$.

n	1	2	3	4	5
WMR's	(1)	(1,0)	(1,0,0)	(1,0,0,0)	(1,0,0,0,0)
represented by the			(1,1,1)	(1,1,1,0)	(1,1,1,0,0)
corresponding				(2,1,1,1)	(1,1,1,1,1)
distribution of					(2,1,1,1,0)
normalized weights					(2,2,1,1,1)
					(3,1,1,1,1)
					(3,2,2,1,1)

Table 2 (*see* Fishburn and Gehrlein (1977), Nitzan and Paroush (1981) and Isbell (1959)) presents the number of all possible collective decision rules, potentially optimal *WMR's* and potentially optimal *RMR's* for n-member groups, $n \leq 8$.

Table 2: Number of potentially optimal *RMR's*, $|RMR^*|$, potentially optimal *WMR's*, $|WMR^*|$ and collective decision rules, $|F|$, for n-member groups, $n \leq 8$.

n	1	2	3	4	5	6	7	8
$\mid RMR^* \mid$	1	1	2	2	3	3	4	4
$\mid WMR^* \mid$	1	1	2	3	7	21	135	2470
$\mid F \mid$	4	16	256	2^{16}	2^{32}	2^{64}	2^{128}	2^{256}

3. Interdependence

3.1 The effect of eliminating a decision maker on the essentiality of other decision makers

In our pairwise choice model decisions are independent. The optimal decision rule (2) entails that each individual's optimal weight is independent of the number and skills of the other decision makers; it depends just on his own decisional skill. However, the optimal decisional essentiality of the individual strongly depends on the skills and, in turn, on the optimal weights of the other members. The following three simple examples illustrate one aspect of this interdependence, namely, the possible decisive effect of eliminating from the group one of its members on the essentiality of more skilled or less skilled members.

Example 3.1: Consider the five-member group where $p = (0.9, 0.8, 0.7, 0.6, 0.6)$. By (2), the best rule is defined by the weights $(3,2,2,1,1)$, so every individual is essential, $\hat{k} = 5$. Suppose that individual 2 is excluded from the group. In the reduced four-member group $p' = (0.9, 0.7, 0.6, 0.6)$. By (3a), the expert rule is the optimal rule $\hat{k} = 1$. The example clearly illustrates that there are cases where the essentiality of less competent individuals is due to the presence of more competent ones.

Example 3.2: Consider the four-member group where $p = (0.95, 0.8, 0.8, 0.6)$. By (2), the optimal rule is a simple majority rule with a tie-breaking chairman. So all decision makers are essential. Suppose that individual 4 is excluded from the group. In the reduced three-member group $p' = (0.95, 0.8, 0.8)$ and, by (3a), the expert rule becomes the optimal rule. This example illustrates that there are cases where the essentiality of more competent members (in the example, individuals 2 and 3) is due to the presence of less competent ones.

Example 3.3: Consider the four-member group where $p = (0.9, 0.8, 0.7, 0.6)$. By (2), the optimal rule is a simple majority rule with a tie-breaking chairman. So all members are essential. It can be verified that the elimination of individual 2 makes individual 3 inessential and the elimination of individual 3 makes individual 2 inessential (in both cases the expert rule becomes the optimal rule in the reduced group). Hence, sometimes in the same group the essentiality of some member is due to the presence of less competent members and the essentiality of another member is due to the presence of more competent members.

To sum up,

(P.1) The essentiality of a decision maker may be due to the presence of more competent or less competent decision makers in the group.

3.2 Path dependence

The application of SMR within a committee, namely the use of a RMR, is very common. Often the committee is chosen sequentially. A single decision maker is assigned essentiality and then in each subsequent stage a decision is made on whether a given candidate pair of decision makers should be added to the committee. The question is whether the selection of a committee in such a manner is path dependent (depending on the order the candidate pairs of decision makers are introduced for consideration). If a certain pair is rejected does it make sense to reconsider its inclusion in the committee at a later stage? If a certain pair is admitted to the committee does it make sense at a later stage to consider its elimination? The answer to all these questions is positive, which is another aspect of the interdependence among individual optimal weights. This is illustrated in the example below.

Example 3.4: Suppose that individual 1 is assigned essentiality where $p_1 = 0.8$. There are three candidate pairs to joining the committee, A, B and C. Their respective decisional skills are $p^A = (0.75, 0.65)$, $p^B = (0.9, 0.9)$ and $p^C = (0.8, 0.8)$. At any stage a candidate pair is added to the m-member committee if

$$\frac{p_{m+1}}{(1-p_{m+1})} \cdot \frac{p_{m+2}}{(1-p_{m+2})} > \frac{\text{the probability to obtain a minimal majority in favor of the correct alternative in the } m\text{-member committee.}}{\text{the probability to obtain a minimal majority in favor of the incorrect alternative in the } m\text{-member committee.}}$$

(for a proof *see* Karotkin (1992a)). Applying this criterion, when the pair A is considered first, it is added to the one-member committee $\{1\}$. If pair B is considered in the next stage, it is added to the existing three-member committee. If the first decision regarding A can be reconsidered after the second decision has been made, the pair A will be eliminated from the five-member committee $\{\{1\} \cup A \cup B\}$. Given the committee $\{\{1\} \cup B\}$ the pair C is considered and added. If at this stage the pair A is given another chance it will be added. When decisions are irreversible and the candidate pairs to join the committee are introduced in the order $A \rightarrow B \rightarrow C$, all pairs will be admitted to the committee. If the order the candidate pairs are introduced is $B \rightarrow A \rightarrow C$, only B and C will be admitted. This makes clear that

(P.2) A committee applying a SMR which is obtained by using the sequential pair-selection process is path dependent.

4. Monotonicity?

4.1 Is the extent of participation in group decision making inversely related to the diversity in decisional skills?

If decisional skills are sufficiently heterogeneous such that condition (3a) is satisfied, the optimal number of essential decision makers is minimal, $\hat{k} = 1$, the expert rule being the optimal WMR. In a homogeneous group the optimal number of essential decision makers is maximal, $\hat{k} = n$, since by condition (3b) the SMR is the optimal WMR. A decrease in the decisional skill of the most competent decision maker clearly reduces the diversity in the distribution of decisional skills. A transfer of decisional skill from individual i to the less competent individual j, $i < j$, also seems to reduce the diversity in decisional skills.

It seems plausible to expect that $\hat{k}$ is inversely related to the diversity in decisional skills. The following examples clarify that such a relationship does not necessarily exist.

Example 4.1: Let $n = 5$ and $p = (0.97, 0.80, 0.80, 0.65, 0.60)$. By (2), the optimal rule is the *WMR* defined by the weights $w = (3,1,1,1,1)$ and $\hat{k} = 5$. Now suppose that the decisional skill of individual 1 declines to 0.85 and in turn $p' = (0.85, 0.80, 0.80, 0.65, 0.60)$. By (3c), the optimal rule now changes to the *RMR* of order 3, $\hat{k} = 3$, so under the more egalitarian distribution of decisional skills the optimal number of essential decision makers declines.

Example 4.2: Let $n = 3$ and $p = (0.95, 0.90, 0.70)$. By (2), the optimal rule in this case is the *SMR*, $\hat{k} = 3$. Now consider the rank-preserving equalization of decisional skills transforming p to $p' = (0.95, 0.80, 0.80)$. By (2), the optimal rule now changes to the expert rule, $\hat{k} = 1$. Under the more egalitarian distribution of decisional skills the number of essential decision makers declines.

To sum up, more egalitarian decisional skills and, in turn, more egalitarian optimal weights do not necessarily result in more egalitarian normalized weights. That is,

(P.3) The optimal number of essential decision makers does not necessarily increase when the distribution of decisional skills becomes more egalitarian.

An elitistic type of change in the structure of group decision making can be thus eliminated and justly rationalized by an egalitarian type of change in the distribution of decisional skills.

4.2 Is the individual's marginal effect on group performance declining with his relative ranking by decisional skill?

By (2), the decisional effectivity of an individual is positively related to his decisional quality. The marginal effect of the individual's decisional skill on group performance $\frac{\partial \pi(f(p))}{\partial p_i}$ is positive and declining. The question now is how does the individual's marginal effect on group performance relate to his relative ranking by skill. Is i 'th marginal contribution to the group decisional quality lower than that of j when $i < j$ (member i is more competent than member j)? Karotkin and Nitzan (1992b) have recently proved that this is not the case.

(P.4) If member i is more competent than member j, i 's marginal effect on the decisional quality of the group is higher than or equal to that of member j.

This finding has significant economic and ethical implications. For example, suppose that the group can afford sending just one member to a decision-making workshop that supposedly increases the decisional capability of the participants. If the marginal contribution of such a workshop to the individual decisional skill is identical for all individuals, then **(P.4)** implies that the most competent member should be sent to the workshop. The economically warranted policy of investment in human capital is not egalitarian - increasing the gap between the most competent member and the other members is desirable. Individual decisional skills can, of course, be related to income. In such a case, **(P.4)** may clearly justify an unequal income distribution generating unequal distribution of decisional skills that enhances efficiency, i.e., increases the quality of collective decision making (*see* Kolm (1990) for other examples of just unequal treatment of equals).

4.3 Are two or more better than one?

If all individuals share the same decisional skill p, the quality of a collective decision based on the decisions of two individuals is equal to the quality of a decision based on the decisional skill of just one of them $(p^2 + p(1 - p)0.5 + (1 - p)p0.5 = p)$. In other words, two equally skilled, valuable $(p > 0.5)$ decision makers are not better than one who solely determines the group decision. Two are also not better than one when they are added to a group consisting of an even number of members. Reinforcing the group by one or two individuals has the same effect on the performance of the group. However, two are better than one when they are added to an n-member group, n being odd, since such an addition improves the decisional quality of the group whereas the addition of just one of them has no effect on its performance. The above observations imply that, when $n = 1$, the substitution of the single decision maker with two even slightly less qualified decision makers is detrimental to the group performance. This latter assertion is also true for any homogeneous group with an odd number of members.[4] When n is even, the substitution of an individual in an n-member group with two less competent ones can be beneficial. Finally note that in a homogeneous population, three or more individuals are always better than one. This is due to the fact that, by (2), when individuals are equally skilled the *SMR* is optimal. In general, expansions of a group (reinforcement with valuable individuals) can be neutral, that is, have no effect on the performance of the group. The conditions ensuring such neutrality are provided in Karotkin and Nitzan (1992a).[5] To sum up,

(P.5) Two equally skilled valuable individuals are not better than one. Three or more are better than one. In general, reinforcement of a group with valuable members need not increase its performance.

5. Asymmetry

5.1 Are there "special" $WMR's$?

By (2), every WMR can be optimal, i.e., there exist configurations of decisional skills under which it is the best rule. In contrast, not every WMR can be the worst selection for the group, i.e., there exist $WMR's$ that never yield the lowest probability of a correct collective decision. More specifically, it has been recently shown, Karotkin (1992b), that the set of $RMR's$ is special since

(P.6) For any given distribution of decisional skills $(p_1,...,p_n)$, the worst rule among the potentially optimal $WMR's$ is always a RMR.

The number of $RMR's$ is relatively very small in comparison to the number of $WMR's$. For example, in a seven-member group there are 135 $WMR's$ but only four $RMR's$ (see Table 2). The common application of simple majority within a committee (a subset of the group) thus involves considerable risk. Even a seemingly insignificant change in the number of committee members may imply a move from the best to the worst WMR and in any event would be associated with a high likelihood of resulting in the worst rule.

Any doubt regarding the individual decisional skills clearly casts doubt on the rule selected for aggregating the group members' decisions. Suppose now that the actual decisions of the group members have been revealed.[6] The question arises whether one should still be disturbed by such a doubt. The answer to this question is negative if the collective decision is invariant to the WMR applied by the group, that is, if all $WMR's$ result in the same collective choice. It turns out that the set of $RMR's$ also has the following distinctive and useful diagnostic feature:

(P.7) For any given decision profile $x = (x_1,...,x_n)$, the collective choice is invariant to *WMR* selection if it is invariant to *RMR* selection.

To prove **(P.7)**, suppose that given the decision profile $x = (x_1,...,x_n)$ all *RMR's* result in the same collective decision, say alternative b. We need to show that alternative b is also selected by any other potentially optimal *WMR*, f^0, which is defined by the weights $w^0 = (w_1^0,...,w_n^0)$.

Since the *RMR* of order 1 selects alternative b, $x_1 = 1$. Since the *RMR* of order 3 also selects b, it must be the case that either x_2 or x_3 (or both) is (are) equal to 1. Similarly, by assumption, x_4 or x_5 must equal 1, etc. If n is odd, either x_{n-1} or x_n must equal 1. If n is even, either x_{n-2} or x_{n-1} must equal 1.

For an odd n,

$$\sum_{i=1}^{n} w_i^0 x_i = (w_1^0 x_1 + w_2^0 x_2) + (w_3^0 x_3 + w_4^0 x_4) + ... + (w_{n-2}^0 x_{n-2} + w_{n-1}^0 x_{n-1}) + w_n x_n .$$

Now even if $x_2 = -1$ and $x_3 = 1$, $x_4 = -1$ and $x_5 = 1$, ... and $x_{n-1} = -1$ and $x_n = 1$, the terms in brackets and the last term are non-negative and, therefore, $\sum_{i=1}^{n} w_i^0 x_i \geq 0$. One can readily verify that in all other possible cases $\sum_{i=1}^{n} w_i^0 x_i$ is also non-negative.

For an even n,

$$\sum_{i=1}^{n} w_i^0 x_i = (w_1^0 x_1 + w_2^0 x_2) + (w_3^0 x_3 + w_4^0 x_4) + ... + (w_{n-3}^0 x_{n-3} + w_{n-2}^0 x_{n-2}) + (w_{n-1}^0 x_{n-1} + w_n^0 x_n) .$$

Now even if $x_2 = -1$ and $x_3 = 1$, $x_4 = -1$ and $x_5 = 1$, ... and $x_{n-2} = -1$, $x_{n-1} = 1$ and $x_n = -1$, each of the terms in brackets is non-negative and therefore, $\sum_{i=1}^{n} w_i^0 x_i \geq 0$. The same result is obtained in all other possible cases. We have thus obtained that for any $f^0 \in \{WMR^*\}$, $f^0(x) = 1$.

Since the number of $RMR's$ is relatively very small, **(P.7)** provides us with a useful test for checking whether, given the information about the actual individual decisions, the problem of selecting the appropriate WMR is inconsequential. Obviously, when the decision profile is unanimous, the collective decision is invariant to the WMR selection. The following example illustrates a non-trivial situation where all $WMR's$ lead to the same outcome.

Example 5.1: Let $n = 7$ and $x = (-1,1,-1,-1,1,1,-1)$. So four members choose alternative a and the remaining three select alternative b. To check whether all 135 possible $WMR's$ result in the same collective decision, one needs to check the collective decision applying just the four possible $RMR's$ of order 1,3,5 and 7. The fact that $f^1(x) = f^3(x) = f^5(x) = f^7(x) = -1$ is sufficient to determine the invariance of collective decision to the WMR selected by the group.

When the collective choice is not invariant to rule selection, it is possible that, given the decisional skills p and the decision profile x, the group applies an unfortunate WMR that results in an incorrect decision whereas *all* other efficient $WMR's$[7] yield the correct collective decision. Such an unfortunate rule f must belong to the set of $RMR's$. For suppose that $f \notin \{RMR\}$ and that $f(x)$ results in a collective choice which is different than the one obtained by all other potentially optimal $WMR's$. This implies that all $RMR's$ select the same alternative. By **(P.7)**, all potentially optimal $WMR's$ including f select the same alternative. We obtain a contradiction, which proves that

(P.8) Given a decision profile $x = (x_1,...,x_n)$, an efficient *WMR* f can result in a choice which is different than the choice obtained by all other efficient *WMR's*, if and only if f is a *RMR*.

Example 5.2: Let $n = 7$ and $x = (1,1,-1,-1,-1,1,1)$.. In this case the *RMR* of order 5 is the only rule resulting in a decision which is different than the one obtained by any of the other 134 potentially optimal *WMR's*.

An interesting and useful property of the *RMR* of order n, namely the *SMR* is the robustness of its optimality to reductions of the group. The other side of the coin is the robustness of its non-optimality to extensions of the group. That is,

(P.9) (i) If *SMR* is optimal in group N, then it is also optimal in any odd-member subgroup of N.

(ii) If *SMR* is not optimal in group N, then it is also not optimal in any group containing N.

The necessary and sufficient condition for the optimality of the *SMR*, (3b), requires that,

$$w_1 + w_2 + ... + w_{\frac{n-1}{2}} \leq w_{\frac{n+1}{2}} + ... + w_n .$$

Each element on the left hand side of the inequality is larger than each element on the right hand side of the inequality. Since the number of elements on the right hand side is $\frac{n+1}{2}$ and since the number of elements on the left hand side is $\frac{n-1}{2}$, it must be the case that the sum of any two elements on the right hand side is larger than any element on the left hand side of the inequality. It is straightforward to show that this implies (i). The proof of (ii) is easily obtained using the reverse inequality (*see* Paroush and Karotkin (1989)).

(P.9) (i) implies that an optimal *SMR* is immune to absenteeism. That is, if $2m$ members are absent, $1 \leq m \leq \frac{n-1}{2}$, there is no need to reconsider the procedure for making group decisions as *SMR* remains optimal in the reduced group.

(P.9) (ii) implies that extending the group and, in particular, adding to the group less qualified individuals can never justify the employment of *SMR* if originally some other rule is optimal. 'Quantity without quality' cannot justify the employment of *SMR*.

Under complete inability of decisional skills verification, the *SMR* has another special and appealing property:

(P.10) Under complete inability of verifying decisional skills, the *SMR* is the best rule; it maximizes the expected probability of making the correct collective decision.

Interestingly, this result is valid independent of the available information regarding the characteristics of the distribution of individual decisional skills. For a three-member group this result is established in Nitzan and Paroush (1984a). Gradstein and Nitzan (1986) confirmed the result for $n < 10$. The general justification for the use of *SMR* under such circumstances for any group size is provided in Karotkin (1992c). This special advantage of the *SMR* is due to its symmetry in terms of the assignment of decisional effectivity to the group members and, in turn, to its nonvulnerability to the inability of verifying the individual decisional skills; even if decisional skills were verifiable this would not have caused any change in the assignment of weights as these are equal.

5.2 Are there "special" rankings of $WMR's$?

The fact that, regardless of group size and of individual decisional skills, one of the $RMR's$ is always the worst rule **(P.6)** implies that there is asymmetry among rankings of the $WMR's$ by their performance in the sense that some of these rankings are not feasible. **(P.6)** might be the tip of an iceberg and the question is can the iceberg be exposed? Put differently, is there a special pattern which is consistently (for any p) followed by the rankings of $WMR's$ according to their decisional quality? Such a pattern has been identified for small-member groups ($n \leq 5$) (*see* Karotkin et al. (1988)). The existence of an essential ordering over the efficient $WMR's$ in a five-member group implies that less than 200 rankings are feasible out of the 5040 = 7! possible rankings of the seven $WMR's$.[8] A generalization of this finding is a worthy task. We conclude our essay by making a partial step in this direction, namely, by establishing another general restriction on the feasible rankings of the efficient $WMR's$.

(P.11) If the expert rule is the optimal WMR in an n-member group, then in the second-best rule all decision makers are essential. More specifically, the second-best rule is defined by the weights $(n-2,1,\ldots,1)$.

In other words, in an organizational environment precluding a first-best rule associated with extreme concentration of decisional effectivity, namely the existence of a single essential decision maker, allowing just three, four, five, etc. essential decision makers is always inferior to allowing essentiality of all n group members.

To prove **(P.11)**, note that two *WMR's* are different if they result in a different collective choice for at least one decision profile. Since *WMR's* are neutral ($\forall f \in \{WMR\}$ and $\forall x, f(x) = -f(-x)$), two different *WMR's* yield different collective choices for at least two decision profiles, x^0 and $-x^0$. If the two rules differ only over x^0 and $-x^0$ the difference in the likelihood of obtaining the correct collective decision by the two *WMR's* is equal to $|Pr(x^0) - Pr(-x^0)|$. If one of the two rules is the expert rule, then it can be easily verified that $x^* = (1,-1,...,-1)$ is the solution to the problem

$$\min_x |Pr(x) - Pr(-x)| \ .$$

This implies that the second best rule to the expert rule is that *WMR* that differs from the expert rule only over x^* and $-x^*$. The normalized weights defining this rule are $(n-2,1,...,1)$. The number of essential decision makers corresponding to this rule is n, which proves **(P.11)**.

6. Summary

We pointed out two aspects of the interdependence among the individual optimal decisional effectivities. Specifically,

(P.1) The essentiality of a decision maker may be due to the presence of more competent or less competent decision makers in the group.

(P.2) A committee applying a *SMR* which is obtained by using the sequential pair-selection process is path dependent.

We considered three types of monotonicity and showed that,

(P.3) The optimal number of essential decision makers does not necessarily increase when decisional skills become more egalitarian.

(P.4) If member i is more competent than member j, then i's marginal effect on the decisional quality of the group is higher than or equal to that of j.

(P.5) Two equally skilled valuable individuals are not better than one. Three or more are better than one. In general, reinforcement of a group with valuable members need not increase its performance.

Within our symmetric group decision-making setting asymmetry is manifested by the existence of "special" rules and rankings of rules. Specifically,

(P.6) For a given distribution of decisional skills the worst potentially optimal *WMR* is always a *RMR*.

(P.7) Given a particular decision profile, the collective choice is invariant to *WMR* selection if it is invariant to *RMR* selection.

(P.8) Given a particular decision profile, a potentially optimal *WMR* can result in a collective choice which is different than the choice of any other potentially optimal *WMR*, if and only if it is a *RMR*.

(P.9) (i) If the *SMR* is optimal in some group, then it is also optimal in any of its odd-member subsets.

(ii) If the *SMR* is not optimal in some group, then it is also not optimal in any extension of the group.

(P.10) Under complete inability of verifying decisional skills, the *SMR* is the best rule; it maximizes the expected probability of making the correct collective decision.

(P.11) If the expert rule is the optimal *WMR* in an n-member group, then in the second-best rule all decision makers are essential.

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Footnotes

1. This context is unproblematic since an optimal collective decision rule always exists.

2. For more general frameworks of organizational decision making or decision making in teams based on the recognition that "to err is human" and thus errors must be aggregated in an optimal way, *see* Gradstein and Nitzan (1988), Heiner (1988), Nitzan and Paroush (1985), Sah (1991) and Sah and Stiglitz (1985), (1986), (1988).

3. In the current symmetric version of the uncertain pairwise choice model the two alternatives are assumed to be a priori equi-probable. Also the benefit (loss) associated with a correct (incorrect) choice is assumed to be independent of the particular alternative correctly (incorrectly) chosen. If decision making costs are disregarded, then the solution to our problem is also the solution to the problem of maximizing the expected benefit of the group (*see* Nitzan and Paroush (1982), (1984b)).

4. For $n > 1$, by (3b), the *SMR* is the optimal *WMR*. By (3c), after the substitution of one of the decision makers the optimal *WMR* for the expanded $(n + 1)$ member group is a *RMR* of order n. But this rule is in fact a *SMR* applied within an n-member group containing an individual with a reduced decisional skill. Clearly then the performance of the original group is reduced.

5. Karotkin and Nitzan (1992a) also provide sufficient conditions for the neutrality and inferiority of a substitution of a single individual by m less competent ones.

6. Note that the optimal rule which maximizes the *a priori* probability of reaching a correct collective decision given the individual decisional competencies also maximizes the *a posteriori* probability that the correct decision was reached given the decision profile of the group.

7. A ***WMR*** is called efficient if the weights are ordered by decisional skills, that is, $i < j$ implies $w_i \geq w_j$. Of course, a potentially optimal ***WMR*** must be efficient.

8. The essential ordering has a number of interesting applications (*see* Karotkin et al. (1988), Paroush (1990) and Karotkin and Paroush (1992)).

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