Collusion and Renegotiation in a Principal-Supervisor-Agent Relationship

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Abstract

This paper describes a principal-agent relationship with a supervisor who has information about the agent. The agent and the supervisor have the possibility to collude and misinform the principal. From the literature we know that there exists an optimal contract which excludes collusion in equilibrium. The optimal contract, however, is ex post inefficient and creates scope for renegotiation. If renegotiation is allowed then under some parameter constellations the optimal contract is a contract which necessarily induces collusion. The paper thus shows that the principal's behavior toward ex post inefficiencies may determine whether collusion occurs in equilibrium.

Keywords: Collusion, renegotiation, hierarchies, corruption.

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1 Introduction

In recent years economists have extended the standard hierarchical principal-agent model by including a third player: the supervisor. The extension can be used to analyze situations in which the principal is able to acquire information about the agent from other economic agents. The problem which arises in these situations is the manipulation of information. Since the principal may use the supervisor's information to discipline the agent, the agent has an incentive to collude with the supervisor and manipulate the information which is sent to the principal. An important question is then whether in equilibrium the principal will offer contracts which exclude such forms of collusion. This paper shows that the answer to this question may depend on the principal's behavior toward contracts which turn out to be inefficient ex post. If she is expected to renegotiate such contracts, then she might strictly prefer collusion to take place in equilibrium.

We develop a model in which renegotiation determines whether collusion will take place in equilibrium. If the principal can commit not to renegotiate then there exists an optimal contract for which collusion does not take place in equilibrium. Given a certain parameter constellation the optimal contract is, however, not ex post efficient. If the principal and the agent are able to renegotiate, they will adopt a different contract later in the game. Since rational players will anticipate the change, they will modify their behavior and the contract is no longer optimal. When we model the renegotiation explicitly, the principal strictly prefers to offer contracts which induce collusion to take place in equilibrium. The paper therefore shows that the principal's behavior towards ex post inefficiencies has a direct impact on whether collusion takes place in equilibrium.

The model we study is a procurement model with asymmetric information. The principal has a project of fixed size, which an agent can realize. The agent and a supervisor know the exact cost of the project, while the principal does not. The principal wants to elicit the information from the supervisor, in order to determine the appropriate transfer to the agent for realizing the project. If the contract is increasing with the cost reported by the supervisor, then the agent has an incentive to bribe the supervisor to report higher costs. If such bribery is possible and the principal is aware of this, then she is interested in signals which tell something about the likeliness that collusion occurred.[1] Even when these signals are imperfect, it is optimal for the principal to condition the contract on the signals, since this reduces the attractiveness of bribing. The imperfectness of the signal creates scope for renegotiation: Once the principal has ensured a truthful report, she prefers to change the contract. She does no longer want to condition the contract on the signal, since it may give wrong indications. Preventing

[1] In this paper bribery and collusion are treated as synonyms.

collusion and conditioning the contract on the external signal are incompatible and will lead to renegotiation. The principal has to choose between either allowing collusion to take place and to condition her contract on the external signal, or to prevent collusion and offer a contract which is not conditioned on the signal. We show that there exists a parameter constellation such that it is optimal for the principal to choose the former policy.

Before introducing the model we will briefly discuss related literature which addresses the occurrence of collusion in equilibrium. The literature on collusion in principal-supervisor-agent model was initiated by Tirole (1986). The paper studies an agency model with adverse selection and moral hazard, in which the supervisor may observe information about the agent's type. Tirole shows that there exists an optimal contract which does not induce collusion. Kofman and Lawarrée (1994) study a three tier hierarchy model in which there are two supervisors, who can be employed simultaneously. The first supervisor is costless to deploy, but sensitive to collusion. The second supervisor is uncorruptible, but expensive. Kofman and Lawarrée show that, depending on the cost of employing the second supervisor, collusion between the agent and the corruptible supervisor occurs with strictly positive probability in equilibrium. Scheepens (1995) shows that collusion takes place in equilibrium when the principal can monitor collusion, but when monitoring is unverifiable. The collusion problem is then transformed into a standard inspection game. When monitoring effort is contractible, there exists an optimal contract for which collusion does not take place in equilibrium. In Tirole (1992) it is shown that the principal may prefer to adopt contracts which induce collusion when there are different types of supervisors with different levels of scruple. By allowing collusion to take place by those types for which collusion is most costly to prevent the principal is able to screen between the different types. Depending on the parameter constellation screening may be optimal.

The fact that optimal long-term contracts which are efficient ex ante may turn out to be inefficient ex post in an adverse selection context was first recognized by Dewatripont (1986). He pointed out that when contracting parties are aware of the inefficiencies, then they may decide to renegotiate away the inefficiency to the benefit of all and adopt a new contract. Assuming that the principal will not renegotiate a contract when this is beneficial requires extreme commitment capabilities on part of the principal.

The paper is organized as follows. The next section introduces the general model. Section 3 derives the optimal contract of the game and points out the ex post inefficiency of the contract. Section 4 introduces and analyzes the game with renegotiation. Section 5 concludes.

2 The Model

The principal has a project which she values at R. A single agent can realize the project. Ex ante it is publicly known that the cost c of the project is c_l with probability ν and c_h with probability $1-\nu$, where $R > c_h > c_l$. In order to learn more about the costs the principal sends a supervisor to discuss the project with the agent. During the discussion the exact cost of the project is revealed to the agent and the supervisor. We have, therefore, two possible types of agents and supervisors: a high cost and a low cost agent and supervisor. After the discussions the supervisor reports the cost of the project to the principal. The report, specifying whether the cost is high or low, does not need to be truthful. The agent can bribe the supervisor in order to induce him to collude and falsify the report r. Collusion may be accompanied by a positive transfer from the agent to the supervisor, but not vice versa.[2] The transfer is then part of a side-contract between the supervisor and the agent, specifying a payment conditional on the supervisor's report. Transfers are costly and these costs are taken to be proportional to the size of the transfer. The parameter $k \in [0,1]$ expresses this cost. When the agent sends a transfer b, the supervisor receives only kb.[3]

We do not analyze the bargaining procedure by which the bribe b is determined. We assume that bargaining between the supervisor and the agent leads to an efficient outcome. Whenever there exists a surplus from colluding, collusion will indeed take place. Neither do we address the issue of enforceability of the side-contract. We merely assume enforceability and only hint that the difficulties of enforcing the contract may explain why bribing is costly.

After obtaining the report, the principal receives a signal $s \in \{b, n\}$, which is imperfectly correlated with collusion. When collusion has taken place the principal receives the signal $s = b$ with probability p and the signal $s = n$ with probability $1-p$. When collusion did not take place the signal $s = b$ is received with probability q, while the signal $s = n$ is received with probability $1-q$, where $0 < q < p < 1$. Since q is smaller than p a signal $s = b$ gives some indication that collusion has occurred. The signal s, therefore, contains information about the likeliness that collusion occurred. The parameters p and q are common knowledge.

We assume that the report r and the signal s are verifiable. The principal can, therefore, condition her contract on these observable variables. Consequently we may denote a contract to the agent as a vector $w \equiv (w_{ln}, w_{lb}, w_{hn}, w_{hb})$ and the contract to the

[2] The assumption that only the agent can send bribes is a simplifying assumption, which is not crucial for the analysis.

[3] The different transfer opportunities of the players can also be interpreted as if there exist different transfer technologies for the players: $k_P = 1$, $k_A = k < 1$ and $k_S = 0$ conform Laffont and Tirole (1991).

supervisor as a vector $t \equiv (t_{ln}, t_{lb}, t_{hn}, t_{hb})$, where the two subscripts denote the report r and the signal s respectively. Concerning admissible contracts we assume that the supervisor's liability is limited to zero: In none of the events the principal can force the supervisor to make a positive transfer. For the agent we assume a "no slavery condition": The agent cannot be contractually binded to execute the project. He can at any point in time decide to take the outside option not to undertake the project. Consequently, the contract w only specifies payments to the agent conditional on the realisation of the project.

In the following let U_P, U_A and U_S represent the payoff functions of the principal, agent and supervisor. We assume that all players are risk neutral and all outside options are normalized to zero.

Timing in the game is as follows:

t=1: Nature chooses the cost of the project and reveals this to the agent and supervisor.

t=2: The principal offers a contract $w \in \mathbb{R}^4_+$ to the agent and a contract $t \in \mathbb{R}^4_+$ to the supervisor. These contracts are public information.

t=3: The supervisor decides whether to accept the contract.

t=4: The supervisor and agent decide whether to collude.

t=5: The supervisor reveals his report r.

t=6: The signal s is revealed.

t=7: The agent decides whether to execute the project.

t=8: Payoffs are realized.

Note that stage 3 of the game is redundant. The supervisor does not incur any costs. Any contract $t \in \mathbb{R}^4_+$ is therefore individual rational and will be accepted by the supervisor.

In stage 7 the agent has to decide whether to execute the project. He will do so if the wage he gets for realizing the project outweighs the costs. At stage 7 the wage and the cost are perfectly known to him. Consequently, his decision is straightforward. The project is realized when the relevant wage w is larger than or equal to the cost c. We introduce the following two indicator functions, which we will later use for expressing the agent's decision.

$$I_l(x) \equiv \begin{cases} 1 & \text{if} \quad x \geq c_l \\ 0 & \text{otherwise} \end{cases} \qquad I_h(x) \equiv \begin{cases} 1 & \text{if} \quad x \geq c_h \\ 0 & \text{otherwise.} \end{cases}$$

3 The Optimal Contract

Benchmarks
Before deriving the optimal contract for the game, we will briefly comment on simplified versions of the model. This may help to develop some intuition for the more complicated game.

When the principal does not make use of a supervisor, she receives neither a report r nor the signal s. The optimal contract is given by a degenerated direct mechanism. The mechanism is degenerated in the sense that it prescribes identical schedules to both types of agents.[4] In the case that $R - c_h > \nu(R - c_l)$ it is optimal for the principal to offer a wage $w = c_h$ independent of the agent's announcement of his type. Under the parameter constellation $R - c_h < \nu(R - c_l)$ the optimal contract specifies a flat wage $w = c_l$.

The game is trivial when the players cannot forge the report. In this case the supervisor must truthfully reveal the cost of the project to the principal. The contract $t^* \equiv (0, 0, 0, 0)$, $w^*_{lb} \equiv w^*_{ln} \equiv c_l$ and $w^*_{hb} \equiv w^*_{hn} \equiv c_h$ gives the principal the payoff $U_P(w^*, t^*) = \nu(R - c_l) + (1 - \nu)(R - c_h)$. The contract (w^*, t^*) achieves the first best.

When the agent and supervisor are able to collude and forge the report, the contract (w^*, t^*) does no longer attain the first best. The low cost agent and the supervisor will collude in order to divide the surplus $\Delta c \equiv c_h - c_l$. This implies that under the contract (w^*, t^*) the principal receives a report $r = h$ whatever the cost of the project and has an expected payoff of $R - c_h$.

When collusion is possible, the principal has two options. She can design the contract (w, t) in such a way that there is no surplus from colluding. We will define such a contract as collusion-proof. The supervisor's report is truthful and the principal can make effective use of the report. A second option is to allow collusion to take place. In this case collusion will occur and the supervisor's report will not be truthful.[5] In the following we first show that we may assume without loss of generality that there exists an optimal contract which is collusion-proof and compute the optimal contract. Note that in this section we implicitely assume that the principal can fully commit to her contracts and renegotiation does not take place.

Collusion-proofness

[4] As Tirole (1992) notes the problem is identical to the classical pricing decision of a monopolist who faces two types of consumers with a different willingness to pay between which she cannot price-discriminate.

[5] A further option would be to allow collusion to take place with a certain probability. We here concentrate on pure actions only. Later we will come back to the issue of probabilistic collusion.

In order to ensure that collusion does not take place, the principal has to design the contract (w, t) in such a way that there does not exist a surplus between the agent and the supervisor from colluding. In principle the principal has to prevent two forms of collusion. First, collusion may occur between the low cost agent and the supervisor and, second, the high cost agent and the supervisor may collude. Since in the present model optimal contracts will be weakly monotonic increasing with the reported cost, the relevant threat of collusion comes from the low cost agent. A low cost agent will want to pass for a high cost agent in order to get a higher wage. We will therefore concentrate on collusion by the low cost agent and ex post check whether the obtained optimal contract does not induce collusion between the high cost agent and supervisor.

Whether collusion occurs depends on the effect which the report has on the payoffs of the agent and supervisor. Let the project be of the low cost $c = c_l$. Then, if the supervisor reports the cost truthfully, this results in an expected payoff to the agent and the supervisor of

$$U_A^T(w) \equiv (1-q)(w_{ln} - c_l)I_l(w_{ln}) + q(w_{lb} - c_l)I_l(w_{lb})$$
$$U_S^T(t) \equiv (1-q)t_{ln} + qt_{lb}.$$

Collusion, on the other hand, implies that a forged report $r = h$ is sent. The expected payoffs gross of the bribe are consequently,

$$U_A^F(w) \equiv (1-p)(w_{hn} - c_l)I_l(w_{hn}) + p(w_{hb} - c_l)I_l(w_{hb})$$
$$U_S^F(t) \equiv (1-p)t_{hn} + pt_{hb}.$$

By assumption collusion cannot be accompanied by a negative transfer from the agent to the supervisor. A necessary condition for collusion to take place is therefore $U_A^F(w) > U_A^T(w)$. In this case the agent is willing to send a non-negative bribe b of at most $b^{max} \equiv U_A^F(w) - U_A^T(w)$. In order for the supervisor to collude he has to receive a bribe of at least $b^{min} \equiv U_S^T(t) - U_S^F(t)$. It follows that collusion will not occur if

$$kb^{max} \leq b^{min}. \tag{1}$$

In this case the maximum transfer the agent is willing to give for collusion is not enough to induce the supervisor to cooperate. We can rewrite condition (1) as

$$\begin{aligned}(1-q)t_{ln} + qt_{lb} - (1-p)t_{hn} - pt_{hb} \geq k[(1-p)(w_{hn} - c_l)I_l(w_{hn}) \\ +p(w_{hb} - c_l)I_l(w_{hb}) - (1-q)(w_{ln} - c_l)I_l(w_{ln}) - q(w_{lb} - c_l)I_l(w_{lb})].\end{aligned} \tag{2}$$

This leads to the following definition of collusion-proofness.

Definition 1 *A contract (w,t) is collusion-proof if and only if it satisfies the collusion-proofness constraint (2).*

Proposition 1 *Any payoff associated with a contract (w,t) can also be attained by a contract which is collusion-proof.*

Proof: Consider a contract (w,t) which is not collusion-proof, then collusion occurs either when the project is low cost or when the project is high cost. If collusion takes place when the project is low cost then the principal always receives a report $r = h$. It follows that the principal will never receive a report $r = l$ and the wages w_{ln}, t_{ln} and w_{lb}, t_{lb} are irrelevant. Now consider the contract (w', t'), where $w' = (w_{hn}, w_{hb}, w_{hn}, w_{hb})$ and $t' = (t_{hn}, t_{hb}, t_{hn}, t_{hb})$. The contract (w', t') satisfies the collusion-proofness constraint (2) and is collusion-proof. The associated payoff to the principal of the contract (w', t') is the same as for the contract (w,t). A similar argument holds for the case in which the contract (w,t) is not collusion-proof with respect to the high cost project.

Q.E.D.

Proposition 1 implies that we may without loss of generality assume that there exists an optimal contract which is collusion-proof. Thus, an optimal contract is a contract $(\hat{w}, \hat{t})$ which maximizes

$$\begin{aligned} U_P(w,t) &= \nu((1-q)((R-w_{ln})I_l(w_{ln}) - t_{ln}) + q((R-w_{lb})I_l(w_{lb}) - t_{lb})) \\ &\quad +(1-\nu)((1-q)((R-w_{hn})I_h(w_{hn}) - t_{hn}) + q((R-w_{hb})I_h(w_{hb}) - t_{hb})). \quad (3) \end{aligned}$$

subject to the collusion-proofness constraint (2). Two observations regarding the optimal contract follow immediately. First, the optimal contract satisfies $\hat{t}_{hn} = \hat{t}_{hb} = 0$, since the principal's payoff is decreasing in t_{hn} and t_{hb} and the collusion-proofness constraint is given more slack when t_{hn} or t_{hb} decreases. It implies that in the optimum the supervisor is not paid for a report $r = h$. Second, the collusion-proofness constraint is binding at the optimum. If the collusion-proofness constraint does not bind then the principal is better off offering the supervisor a contract with a lower t_{ln} or t_{lb}, since $\partial U_P(w,t)/\partial t_{ln} = \partial U_P(w,t)/\partial t_{lb} = -\nu < 0$.[6] We can therefore treat the weak inequality in (2) as strict equality. Substituting (2) into the objective function (3) and rewriting the expression, leads to

[6] Due to the monotonicity of the optimal contract we have $U_A^F(w) - U_A^T(w) \geq 0$. This ensures that t_{ln} or t_{lb} has to be positive. Consequently the constraint $t_{ln}, t_{lb} \geq 0$ is not violated when the collusion-proofness constraint is satisfied in equality.

$$\max_{w} \quad U_P(w) = (1-\nu)(1-q)(R - w_{hn})I_h(w_{hn}) - \nu k(1-p)(w_{hn} - c_l)I_l(w_{hn}) \qquad (4)$$
$$+(1-\nu)q(R - w_{hb})I_h(w_{hb}) - \nu kp(w_{hb} - c_l)I_l(w_{hb})$$
$$+\nu(1-q)(R - (1-k)w_{ln} - kc_l)I_l(w_{ln})$$
$$+\nu q(R - (1-k)w_{lb} - kc_l)I_l(w_{lb}).$$

In the following the equilibrium will depend on the parameter constellation. To simplify notation we introduce the following function

$$S(a,b) \equiv a(1-\nu)(R - c_H) - b\nu\Delta c. \qquad (5)$$

Proposition 2 *The optimal contract* $(\hat{w}, \hat{t})$ *depends on the parameter constellation in the following way*

i) If $S(1-q,(1-p)k) > 0$ *then* $(\hat{w}_{ln}, \hat{w}_{lb}, \hat{w}_{hn}, \hat{w}_{hb}) = (c_l, c_l, c_l, c_l)$ *and* $\hat{t} = (0,0,0,0)$. *The principal's maximum payoff is* $U_P(\hat{w}, \hat{t}) = \nu(R - c_l)$.

ii) If $S(1-q,(1-p)k) \le 0$ *and* $S(q,pk) < 0$ *then* $(\hat{w}_{ln}, \hat{w}_{lb}, \hat{w}_{hn}, \hat{w}_{hb}) = (c_l, c_l, c_h, c_h)$ *and* $(\hat{t}_{ln}, \hat{t}_{lb}, \hat{t}_{hn}, \hat{t}_{hb}) = (k\Delta c, k\Delta c, 0, 0)$.[7] *The principal's maximum payoff is* $U_P(\hat{w}, \hat{t}) = \nu(R - c_l) + (1-\nu)(R - c_h) - k\nu\Delta c$.

iii) If $S(1-q,(1-p)k) \le 0$ *and* $S(q,pk) \ge 0$ *then* $(\hat{w}_{ln}, \hat{w}_{lb}, \hat{w}_{hn}, \hat{w}_{hb}) = (c_l, c_l, c_h, c_l)$ *and* $(\hat{t}_{ln}, \hat{t}_{lb}, \hat{t}_{hn}, \hat{t}_{hb}) = (kp\Delta c, kp\Delta c, 0, 0)$. *The principal's maximum payoff is* $U_P(\hat{w}, \hat{t}) = \nu(R - c_l) + (1-\nu)(1-q)(R - c_h) - \nu k(1-p)\Delta c$.

Proof: Note that the objective function in (4) is piece-wise linear in all w_i and has non-positive slopes. The function shows an upward jump at c_l for all w_i $(i = 1,2,3,4)$ and a second upward jump at c_h for w_{hn} and w_{hb}. From these observations we conclude that the optimal contract is found for $w_{ln} = w_{lb} = c_l$ and $w_{hn}, w_{hb} \in \{c_l, c_h\}$. We have four cases to consider: Case 1: $w = w^1 \equiv (c_l, c_l, c_l, c_l)$ with payoff $U_P(w^1) = \nu(R - c_l)$. Case 2: $w = w^2 \equiv (c_l, c_l, c_h, c_h)$ with an associated payoff of $U_P(w^2) = \nu(R - c_l) + (1-\nu)(R - c_h) - \nu k\Delta c$. Case 3: $w = w^3 \equiv (c_l, c_l, c_h, c_l)$ resulting in $U_P(w^3) = \nu(R-c_l) + (1-\nu)(1-q)(R-c_h) - k(1-p)\nu\Delta c$. Case 4: $w = w^4 \equiv (c_l, c_l, c_l, c_h)$ with $U_P(w^4) = \nu(R - c_l) + (1-\nu)q(R - c_h) - p\nu k\Delta c$.

First note that the contract w^4 cannot be optimal. For w^4 to achieve the maximum payoff it is required that $U_P(w^4) > U_P(w^1)$ and $U_P(w^4) > U_P(w^2)$, which is equivalent to the requirement that the two conditions $q(1-\nu)(R - c_h) > pk\nu\Delta c$ and $(1-q)(1-$

[7] Note that any contract t satisfying $(1-q)t_{ln} + qt_{lb} = k\Delta c$ is optimal. We take $t_{ln} = t_{lb}$, which has an intuitive interpretation: The principal is not interested in the signal s when the report is $r = l$. She then knows that the report is truthful and the signal s is non-informative.

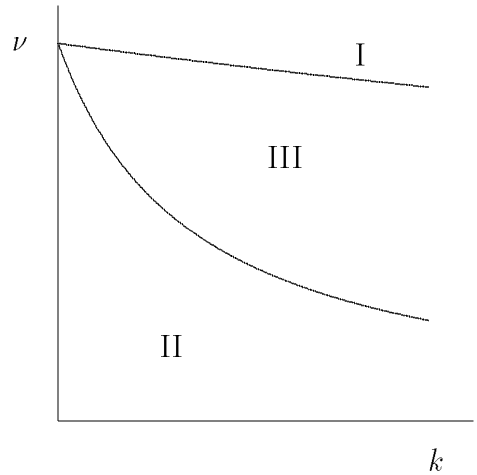

Figure 1: The optimal contracts

$\nu)(R-c_h) < (1-p)k\nu\Delta c$ hold. However, since $0 < q < p$ we have that $q(1-\nu)(R-c_h) > pk\nu\Delta c \Leftrightarrow (1-nu)(R-c_h) > k\nu\Delta c \Leftrightarrow (1-q)(1-\nu)(R-c_h) > (1-p)k\nu\Delta c$. The two conditions are therefore incompatible and w^4 cannot be optimal.

Comparing the different payoffs to the principal we arrive at the following conditions i) $U_P(w^1) > U_P(w^2) \Leftrightarrow \nu k\Delta c > (1-\nu)(R-c_h)$, ii) $U_P(w^3) > U_P(w^2) \Leftrightarrow p\nu k\Delta c > q(1-\nu)(R-c_h)$, and iii) $U_P(w^1) > U_P(w^3) \Leftrightarrow (1-p)\nu k\Delta c > (1-q)(1-\nu)(R-c_h)$. It follows that if $U_P(w^2) > U_P(w^1)$ then $U_P(w^3) > U_P(w^1)$ and the proposition is immediate. Finally note that collusion by the high cost agent and the supervisor will indeed not occur under the contracts which the proposition specifies.

Q.E.D.

The statement of proposition 2 is illustrated in figure 1. The diagram depicts the regions in which the different contracts are optimal. We will shortly discuss each of the regions. In region I the principal offers the agent a payment c_l independent of the supervisor's report r and the signal s. The principal, therefore, does not strictly benefit from employing the supervisor and receiving the signal s. Due to the monotonicity of the contract, the agent cannot gain from collusion and will not bribe the supervisor for misreporting. The principal, therefore, does not need to give additional incentives to the supervisor for reporting truthfully. Note that the project will only be executed in the case of low cost. Therefore, for the contract to be optimal the expected cost of foregoing the project needs to be small. This is the case when the probability that the project is low cost is high (ν close to one), and when the surplus from executing the high cost project is low ($R-c_h$ small).

In region II it is optimal for the principal to condition the agent's payment on the supervisor's report. This way the principal tries to discriminate between low and high cost projects. Since the agent's payment depends on the supervisor's report, there exists scope for collusion: The low cost agent would like to pass for a high cost agent. In order to prevent collusion the principal offers an incentive compatible contract to the supervisor to induce truthful reporting. The expected cost of this contract is $\nu k \Delta c$. The contract will be optimal when collusion is not too costly to prevent. This implies that k should be small. Since the principal conditions her contract only on the report r, we will refer to this contract as the partial-screening contract.

In region III it is optimal to condition the agent's payment not only on the supervisor's report but also on the signal s. By using this full-screening contract collusion becomes less attractive for the agent as compared to the partial-screening contract, which is optimal in region II. The expected costs of preventing collusion is thus reduced by a factor p to $p\nu k \Delta c$. The principal will prefer full-screening to partial-screening if collusion is relatively costly to prevent (i.e. a large k). The drawback of the contract is, however, that the agent does not execute the project when the cost is c_h and the signal is $s = b$. Therefore, for the contract to be optimal it must be that this event occurs with a reasonably small probability and when it occurs the principal should not care too much. This implies that the probability that a project is of high cost must be small (i.e. ν large) and that the signal s is sufficiently informative (i.e. the ratio q/p is small), while the principal's willingness to execute a high cost project $(R - c_h)$ must be not too high. These conditions are reflected by the constraint $S(q, pk) \leq 0$, which is the lower curve in the figure. On the other hand the principal's willingness to pay R should be large enough for the principal not to forgo the project entirely whenever its cost is c_h. This is ensured by the condition $S(1-q, (1-p)k) \geq 0$, represented in the figure by the upper curve.

An interesting observation is that the principal prevents collusion by setting the appropriate incentives to the supervisor rather than to the agent. The intuition behind this result is that it is cheaper to induce truthtelling by giving the incentives to the supervisor than to give these incentives to the agent. Referring to the collusion-proofness constraint (1), the principal can either prevent collusion by increasing the minimal bribe which is accepted by the supervisor (b^{min}), or by decreasing the maximal bribe which the agent is willing to give for inducing collusion (b^{max}). Due to the costly bribing technology the effect of reducing b^{max} by one unit is equal to increasing b^{min} with k units. Increasing b^{min} is, therefore, a factor k cheaper than reducing b^{max}.

Now assume that the parameter constellation is such that it is optimal for the principal to use full screening. In this case the principal does not make full use of the supervisor's information. When the principal receives a report $r = h$ and a signal $s = b$, she diverges from the supervisor's information and sets a transfer c_l to the agent. When the actual cost of the project is c_h then this gives rise to an inefficiency. The agent is promised a transfer c_l for realizing the project, but declines the offer, since his cost is larger than c_l. The agent does therefore not realize the project, even though the principal's willingness to pay R is greater than the cost of the project c_h. The optimal contract leaves scope for inefficiencies, which occur with a probability $(1 - \nu)q$.

4 Renegotiation

The ex-post inefficiency prompts us to look at renegotiation and commitment. The important observation is that when the principal uses full-screening and receives a report $r = h$ and a signal $s = b$ she knows that the actual cost of the project is c_h. To see this let the principal receive a report $r = h$ and a signal $s = b$. She then knows that the cost of the project must either be c_h or c_l. But since the contract is collusion-proof she knows that the report is truthful and she, therefore, must conclude that the cost of the project is indeed c_h. As a consequence she realizes that the agent will refuse to execute the project if she sticks to the transfer which is specified by the contract: $\hat{w}_{hb} = c_l$. After stage 6 she has an incentive to renegotiate the contract and raise w_{hb} to c_h. This change is also weakly preferred by the agent and is therefore a Pareto improvement.

We incorporate renegotiation by introducing an intermediate stage $6^1/_2$, where we allow the principal to propose a new contract w and in which the agent may decide to accept the new contract or to stick to the old contract. In stage 7 the agent decides whether to execute the project given the contract which is relevant at that stage.

At the renegotiation stage the principal forms a belief about the cost of the project. Let $\sigma(w, t, r, s)$ represent the principal's belief that the cost of the project is c_l given the contract (w, t), the report r and the signal s. Then we can define renegotiation-proofness in the following way.

Definition 2 *A contract (w, t) is renegotiation-proof if it satisfies for all $r \in \{h, l\}$ and $s \in \{b, n\}$*

(a) $(R - c_l)\sigma(w, t, r, s) > 0 \Rightarrow w_{rs} \geq c_l$

(b) $(R - c_l)\sigma(w, t, r, s) < R - c_h \Rightarrow w_{rs} \geq c_h$.

Condition a) states that when the principal attaches positive probability to the event that the project is of low cost then she should offer at least a payment c_l for executing the project. The second condition states that when the principal believes she receives a higher expected payoff from offering a wage c_h instead of offering less than c_h, then she should at least offer a payment c_h. It is obvious that when these conditions are not satisfied by a contract (w, t) then there exists a contingency in which the contract is renegotiated. When the two conditions are satisfied no such contingency exists.

Proposition 3 *Any payoff associated with a contract which is not renegotiation-proof can also be achieved by a contract which is renegotiation-proof.*

Proof: Consider a contract (w, t) which is not renegotiation-proof. Then it is common knowledge that this contract will be renegotiated into a contract (w', t) at a later stage. Rationality will ensure that all players act as if the relevant contract is the contract (w', t). Consequently the payoffs under the contract (w, t) and the contract (w', t) are identical.

Q.E.D.

It follows directly that we may assume without loss of generality that the optimal contract is renegotiation-proof.

In equilibrium the principal's beliefs should be consistent with the behavior of the agent and supervisor. Since the supervisor and agent do not collude given the contract $(\hat{w}, \hat{t})$, consistency of beliefs requires that $\sigma(\hat{w}, \hat{t}, r, s) = 0$ for $r = h$ and $s = n, b$. As a consequence the full-screening contract is not renegotiation-proof, since $\hat{w}_{hb} = c_l < c_h$. It follows that any contract (w, t) which is collusion-proof must specify $w_{hn} \geq c_h$ and $w_{hb} \geq c_h$ in order to be renegotiation-proof. This implies that independent of the parameter constellation the optimal collusion-proof contract which is also renegotiation-proof is the partial-screening contract $w_{lb} = w_{ln} = c_l$ and $w_{hb} = w_{hn} = c_h$.

Another implication of proposition 3 is that proposition 1 no longer needs to hold in the extended game with renegotiation. In order to prove proposition 1 we used the fact that for every non-collusion-proof contract one can find a collusion-proof contract which achieves the same payoff. It is, however, not ensured that one can find for every non-collusion-proof contract which is renegotiation-proof a collusion-proof contract which is also renegotiation-proof. We therefore can no longer assume that there exists an optimal contract which is collusion-proof and must also consider contracts which are not collusion-proof.

Proposition 4 *The optimal contract in the game with renegotiation depends on the parameter constellation in the following way:*

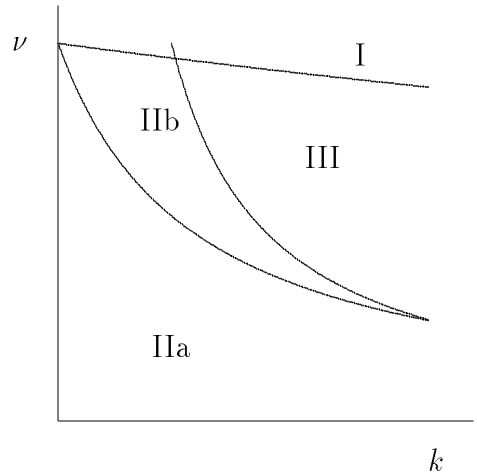

Figure 2: The optimal contracts with renegotiation

i) If $S(1-q,(1-p)k) < 0$ then the optimal contract is neither conditioned on the report r nor on the signal s. The principal's maximum payoff is $U_P = \nu(R - c_l)$.

ii) If $S(1-q,(1-p)k) \geq 0$ and $S(q,k+p-1) > 0$ then the optimal contract is collusion-proof. The optimal contract is conditioned on the report r, but not on the signal s. The principal's maximum payoff is $U_P = \nu(R-c_l) + (1-\nu)(R-c_h) - \nu k \Delta c$.

iii) If $S(1-q,(1-p)k) \geq 0$ and $S(q,k+p-1) \leq 0$ then the optimal contract is not collusion-proof. The optimal contract is conditioned on both the report r and the signal s. The principal's maximum payoff is $U_P = \nu p(R-c_l) + [\nu(1-p) + (1-\nu)(1-q)](R-c_h)$.

Proof: The proposition follows from a staightforward comparison between the maximum payoff which can be achieved in the set of renegotiation-proof contracts which are not collusion-proof (calculated in the appendix) and the maximum payoff which can be achieved in the set of renegotiation- and collusion-proof contracts.

Q.E.D.

Figure 2 depicts the regions for which the different contracts are optimal. In region I it is optimal for the principal to offer a flat wage independent of the report r and the signal s. The payoff associated with this contract is equal to the maximum payoff of the principal in the game without renegotiation. In fact under this parameter constellation the principal is not interested in employing a supervisor or receiving the signal s.

In region II the principal offers a collusion-proof contract in equilibrium. The region can be divided into two subregions. The area IIa depicts the region in which the optimal contract is identical to the optimal contract which we obtained in the game without renegotiation. Consequently, also the payoffs are the same in both games. In the region

IIb the optimal contract in the game with renegotiation differs from the optimal contract in the game without renegotiation. When the principal can commit not to renegotiate she offers a contract which also conditions the agent's contract on the signal s. The contract is, however, not ex-post efficient and therefore not renegotiation-proof. As a consequence, it will be renegotiated in the game with renegotiation and is no longer optimal. Instead it is optimal for the principal to offer a different collusion-proof contract. It follows that the maximum payoff of the principal is lower in the game with renegotiation than in the game without renegotiation.

The most interesting area is region III. For this region the optimal contract also differs from the optimal contract in the game without renegotiation. This implies that the principal's maximum payoff has also decreased. In the literature on renegotiation this is a general result. The reason being that in a game with renegotiation the principal has a smaller set of contracts to which she can commit. The interesting point here is, however, that the optimal contract changes from a collusion-proof contract to a contract which is not collusion-proof. The reason is that when the principal does offer the collusion-proof contract, which is optimal in the game without renegotiation, she may obtain information which will lead her to renegotiate the contract. This fact is common knowledge and induces the supervisor and the agent to change their behavior ex ante. The principal is better off ensuring that she does not obtain the information, in order not to change the behavior of the supervisor and the agent. Not obtaining the information means that she does not offer a collusion-proof contract.

Strong versus weak renegotiation-proofness

Note that in the above discussion we used the concept of strong renegotiation-proof contracts.[8] If we require contracts to be only weakly renegotiation-proof then also other equilibria in the renegotiation game exist. To see this, note that given the full-screening contract the supervisor and the agent are in fact indifferent between colluding and non-colluding. While we assumed that collusion would not take place given the contract (w, t), an alternative best response for the agent and the supervisor would have been to collude. In that case the contract (w, t) is renegotiation-proof.

Let the supervisor and the agent collude with probability π given the contract (w, t). In case the principal receives a report $r = h$ and a signal $s = b$ she updates her belief

[8]A contract is strong renegotiation-proof if there does not exist any equilibrium in the renegotiation game for which the contract is not renegotiation-proof. A weakly renegotiation-proof contract requires that there exist an equilibrium for which the contract is renegotiation-proof. See Maskin and Tirole (1992)

according to Bayes' rule

$$\sigma(w,t,h,b) = \frac{\pi p \nu}{\pi p \nu + q(1-\nu)}. \tag{6}$$

It follows that the contract (w,t) is renegotiation proof if and only if

$$\pi \geq \bar{\pi} \equiv \frac{(1-\nu)q(R-c_h)}{\nu p \Delta c}.$$

The associated payoff for the principal is

$$\begin{aligned} U_P(w,t,\pi) &= \nu(1-\pi)(R-c_l-k\Delta c(1-p)) + \nu\pi p(R-c_l) \\ &\quad + \nu\pi(1-p)(R-c_h) + (1-\nu)(1-q)(R-c_h). \end{aligned} \tag{7}$$

Proposition 5 *Let the parameter constellation be such that* $S(1-q,(1-p)k) \leq 0$ *and* $S(q,pk) \geq 0$ *then the full screening contract* (w,t) *with* $\pi > \bar{\pi}$ *is an equilibrium outcome in the game with renegotiation.*

Proof: We know that the full screening contract (w,t) is renegotiation-proof if $\pi > \bar{\pi}$. In order for the outcome (w,t,π) with $\pi > \bar{\pi}$ to be subgame perfect we need that $U_P(w,t,\pi)$ is larger than the maximum payoff of the non-collusion-proof contract (w^{ncp},t^{ncp}) for all $\pi > \bar{\pi}$. Since $U_P(w,t,\pi)$ is decreasing in π and $U_P(w^{ncp},t^{ncp}) = U_P(w,t,1)$ we have that $U_P(w^{ncp},t^{ncp}) < U_P(w,t,\pi)$ for all $\pi < 1$.

Q.E.D.

The fact that collusion has to occur with at least a probability $\bar{\pi}$ is again explained by referring to the informative content of the contract (w,t). In order for the principal not to renegotiate, she may not attach too high a probability to the state of the world being $c = c_h$ when she receives the report $r = h$ and the signal $s = b$. When she offers the contract (w,t) and the supervisor and agent collude with probability $\bar{\pi}$ her updated belief (6) makes her indifferent between proposing the partial-screening contract in the renegotiation-stage and not renegotiating. When the agent and the supervisor collude with a higher probability than $\bar{\pi}$, the principal's belief about the cost of the project is such that it is strictly better for her not to renegotiate.

Finally note that any equilibrium outcome with $\pi < 1$ gives the principal a strictly higher payoff than her payoff associated with the optimal contract in proposition 2.

5 Conclusion

This paper showed that when the principal renegotiates ex post inefficient contracts then under certain parameter conditions the optimal contract is necessarily not collusion-proof. When the principal can commit not renegotiate ex post inefficient contracts, then there exists an optimal contract which is collusion-proof regardless of the parameter constellation. The optimal contract may, however, produce ex post inefficiencies.

In order to explain the result we refer to two principles. First there exists the principle of renegotiation. This principle says that for every non-renegotiation-proof contract there exists a contract which is renegotiation-proof with identical payoffs. Second, there exists the principle of collusion-proofness. This principle says that for every non-collusion-proof contract there exists a contract which is collusion-proof with identical payoffs. We have shown that in the game without renegotiation the principle of collusion-proofness holds. As a direct consequence there exists an optimal contract which is collusion-proof. We have further shown that when the principal cannot commit not to renegotiate, then the principle of renegotiation takes precedence over the principle of collusion-proofness and the latter principle may fail to hold. The crux of the matter is that when one tries to find the collusion-proof counterpart of a non-collusion-proof contract, then the collusion-proof contract may not be renegotiation-proof even though the non-collusion-proof contract is renegotiation-proof. This explains why there may not exist an optimal contract which is collusion-proof.

We may give an alternative explanation for our result. It was shown that when a collusion-proof contract is offered the principal recognizes the inefficiency and wants to renegotiate. In contrast the principal is not certain enough about the inefficiency to renegotiate when she offers a contract which does induce collusion. Since the principal's attitude toward renegotiation is common knowledge, it affects the behavior of players ex ante. This worsens the principal's situation by such a degree that she is better off allowing collusion to take place instead of preventing it. It is the fact that a contract is collusion-proof which informs the principal about the actual state of the world and makes her fully aware of the inefficiency. The information embodied in the collusion-proof contract is harmful to the principal. Interpreting the result in this way it becomes clear that the result is closely linked to a general theme in game-theory that information may worsen a player's situation, when it is common knowledge that this player has information. Extra information changes the behavior of a player and this change is anticipated by other players in the game. In the present paper obtaining information is equivalent to offering a collusion-proof contract. Since the principal is aware that the extra information worsens her situation, she will not offer a collusion-proof contract. Instead she allows collusion to take place.

Appendix: The optimal non-collusion-proof contract

In this appendix we compute the optimal contract which is not collusion-proof. Note that this case does not reduce to the situation without a supervisor, since by employing the supervisor the principal still receives the signal s. First we calculate the optimal contract in the game without renegotiation. Then we show that this contract is also renegotiation-proof in the game with renegotiation.

If the agent and the supervisor collude then the report does not contain any information, because whatever the cost of the project the same report is sent. Consequently, the report is uninformative to the principal and she will offer the least costly contract which the supervisor accepts, i.e. $t = (0, 0, 0, 0)$. Without loss of generality we assume that it is the low cost agent and the supervisor who collude. This implies that the principal always receives a report $r = h$. The principal's payoff is

$$\begin{aligned} U_P(w) &= \nu p(R - w_{hn})I_l(w_{hn}) + \nu(1-p)(R - w_{hb})I_l(w_{hb}) \\ &\quad +(1-\nu)q(R - w_{hn})I_h(w_{hn}) + (1-\nu)(1-q)(R - w_{hb})I_h(w_{hb}) \end{aligned} \tag{8}$$

The principal's payoff does not depend on w_{ln} and w_{lb}. When the low cost agent always colludes, the principal will never receive a report $r = l$ and the wages w_{ln} and w_{lb} can be set arbitrarily. For reasons which will become clear later we set $w_{ln} = w_{lb} = c_l$. Note, however, that the wages w_{ln}, w_{lb} do affect the decision regarding collusion.

The function $U_P(w)$ is a discontinuous, piece-wise linear function. Again we have four cases to consider.

Case a: $w_{hn} = w_{hb} = c_l$. The principal does not try to screen between the high cost and the low cost project. Instead she offers a transfer c_l irrespective of the report and the signal. The project is executed if it is low cost. The payoff to the principal is $U_P^a \equiv \nu(R - c_l)$.

Case b: $w_{hn} = w_{hb} = c_h$. The contract results in a payoff of $U_P^b \equiv R - c_h$. The principal does not try to screen between the high and the low cost project. She offers a transfer c_h regardless the report r and the signal s. The project is always executed.

Case c: $w_{hn} = c_l$ and $w_{hb} = c_h$. The principal uses the signal s in order to screen between high and low cost projects. Under the contract w^b a project is not executed when it is of high cost and the principal observes $s = b$. This occurs with probability $(1-\nu)q$. The expected payoff is $U_P^c \equiv \nu p(R - c_l) + \nu(1-p)(R - c_h) + (1-\nu)(1-q)(R - c_h)$.

Case d: $w_{hn} = c_h$ and $w_{hb} = c_l$. In this case the principal also uses the signal s as a screening device. The expected payoff is $U_P^d \equiv \nu p(R - c_h) + \nu(1-p)(R - c_l) + (1-\nu)q(R - c_h)$. The project is not executed when it is of high cost and the principal observes the signal $s = n$. This occurs with probability $(1-\nu)(1-q)$.

Proposition 6 *In the game without renegotiation the contract* (w^{ncp}, t^{ncp}) *is optimal with respect to the set of contracts which are not collusion-proof, where* (w^{ncp}, t^{ncp}) *is defined as* $w_{ln}^{ncp} = w_{lb}^{ncp} = c_l$ *and* $t^{ncp} = (0,0,0,0)$*. The optimal values for* w_{hn}^{ncp} *and* w_{hb}^{ncp} *depends on the parameter constellation in the following way,*

i) If $S(1-q, 1-p) < 0$ *then* $w_{hn}^{ncp} = w_{hb}^{ncp} = c_l$*.*

ii) If $S(q,p) > 0$ *then* $w_{hn}^{ncp} = w_{hb}^{ncp} = c_h$*.*

iii) If $S(1-q, 1-p) \geq 0$ *and* $S(q,p) \leq 0$ *then* $w_{hn}^{ncp} = c_h$ *and* $w_{hb}^{ncp} = c_l$*.*

Proof: Comparing the principal's payoffs under the different wage contracts leads to the observation that the contract in case d) cannot be optimal. It would require that the principal's payoff is larger than in case a) and case b), which is equivalent to meeting the conditions $(1-q)(1-\nu)(R-c_h) < (1-p)\nu\Delta c$ and $q(1-\nu)(R-c_h) > p\nu\Delta c$. However, since $0 < q < p$ it holds that $q(1-\nu)(R-c_h) > p\nu\Delta c \Rightarrow (1-\nu)(R-c_h) > \nu\Delta c \Rightarrow (1-q)(1-\nu)(R-c_h) > (1-p)\nu\Delta c$. The two conditions are therefore incompatible.

From comparing the payoffs in the cases a,b and c we may conclude that i) $U_P^a > U_P^c \Leftrightarrow (1-q)(1-\nu)(R-c_h) < (1-p)\nu\Delta c$ and ii) $U_P^b > U_P^c \Leftrightarrow q(1-\nu)(R-c_h) > p\nu\Delta c$, while it holds that $S(q,p) > 0 \Rightarrow q(1-\nu)(R-c_h) > p\nu\Delta c \Rightarrow (1-q)(1-\nu)(R-c_h) > (1-p)\nu\Delta c \Rightarrow S(1-q, 1-p) > 0$. The proposition is then immediate.

Q.E.D.

Proposition 7 *The optimal contracts in proposition 6 are renegotiation-proof.*

Proof: It trivially holds that the contract specifying $w_{hn} = w_{hb} = c_h$ is renegotiation-proof. For the contract $w_{ln} = w_{lb} = w_{hb} = c_l$ and $w_{hn} = c_h$ it follows by Bayes' rule that

$$\sigma(w,t,h,b) = \frac{p\nu}{p\nu + q(1-\nu)}. \tag{9}$$

Note that the contract w^c is optimal when $p\nu\Delta c \geq q(1-\nu)(R-c_h)$. This condition together with equation (9) leads to the conclusion that when the contract w^c is optimal in the set of non-collusion-proof contracts then it is also renegotiation-proof. By the same argument one may ascertain that when the contract $w_{hn} = w_{hb} = c_l$ is the optimal non-collusion-proof contract then it is also renegotiation-proof.

Q.E.D.

References

Dewatripont, M. (1986), "Renegotiation and Information Revelation over Time in Optimal Labor Contracts" Chapter 1, *On the Theory of Commitment, with Applications to the Labor Market*, Ph.D. dissertation, Harvard University.

Kofman, F. and Lawarrée, J. (1993), "Collusion in Hierarchical Agency", *Econometrica* 61, p. 629-656.

Laffont, J.J. and Tirole, J. (1991), "The politics of Government Decision-Making: A Theory of Regulatory Capture", *Quarterly Journal of Economics* 106, p. 1087-1127.

Maskin, E. and Tirole, J. (1992), "The Principal Relationship with an informed Principal, II: Common Values", *Econometrica* 60, p. 1-42.

Scheepens, J. (1995), "Collusion and Hierarchy in Banking" Chapter 4, *Financial Intermediation and Corporate Finance*, Ph.D. dissertation, Tilburg University.

Tirole, J. (1986), "Hierarchies and Bureaucracies: On the Role of Collusion in Organizations", *Journal of Law, Economics and Organization* 2, p. 181-214.

Tirole, J. (1992), "Collusion and the Theory of Organizations", in: J.J. Laffont (ed.), *Advances in economic theory*, 6th World Congress of the Econometric Society, Vol. II, Cambridge, p. 151-206.