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A NON-COOPERATIVE VIEW OF CONSISTENT BANKRUPTCY RULES

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A Non-Cooperative View of Consistent Bankruptcy Rules

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Abstract: We introduce a game form that captures a non-cooperative dimension of the consistency property of bankruptcy rules. Any consistent and monotone rule is fully characterized by a bilateral principle and consistency. Like the consistency axiom, our game form, together with a bilateral principle, yields the coresponding consistent bankruptcy rule as a result of a unique outcome of subgame perfect equilibria. The result holds for a large class of consistent and monotone rules, including the Constrained Equal Award, the Proportional and many other well-known rules. Moreover, for a large class of rules, all the subgame perfect equilibria are coalition-proof.

JEL classification numbers: C72 and D63.

Key words: Bankruptcy rules, Consistency, Monotonicity, Subgame Perfect Equilibrium, Coalition-Proof Equilibrium, Nash Program.

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1. INTRODUCTION

The consistency property has proved very powerful in characterizing some of the most important solution concepts in cooperative game theory (see, for example, the characterizations of the core and the pre-kernel by Peleg (1986), and of the Nash bargaining solution by Lensberg, 1988).[1]) However, consistency alone does not isolate a unique rule in bankruptcy problems, even after restricting attention to symmetric, scale-invariant and monotone rules. On the other hand, a monotone and consistent rule is completely characterized by a two-person rule and consistency.

Consistency has also been suggested as a valuable guide in designing non-cooperative mechanisms that implement some cooperative solutions (see, for example, Krishna and Serrano, 1990). Namely, extensive forms can be constructed whose subgames relate to the respective reduced cooperative problems. By concentrating on the subgame-perfect equilibria of such mechanisms, one can hope to implement the underlying consistent solution. This paper provides additional support to the idea that consistency is a useful tool in the Nash program for cooperative games.

Suppose that $n>2$ creditors, whose claims add up to more than the available estate, try to reach an agreement through decentralized negotiation instead of appealing to a court. We assume that society has agreed on a certain bilateral principle of justice to solve bankruptcy problems involving two creditors, but not on how to deal with multilateral problems; this may be so because the generalization to n of the bilateral principle is not straightforward, or because there might be several ways to generalize it. We present a consistency-based non-cooperative game that solves this problem for a large family of bankruptcy rules. In this game, "going to the bilateral court" can be interpreted as an outside option for the creditors. What should we expect from such negotiations?

Our game form generates a wide family of consistent bankruptcy rules presented in the axiomatic theory. It takes a two-person rule as an input and yields the unique consistent generalization of that rule as an output. The unique subgame-perfect equilibrium outcome of the game associated with a specific two-person rule is the allocation recommended by the unique consistent generalization of that rule. In this sense our game form operates like the consistency property in the axiomatic approach, capturing a non-cooperative dimension of consistency in the framework of bankruptcy problems.

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[1] For a good survey see Thomson (1990).

Like other games based on consistency, our game allows for "partial agreements," where a player cannot be prevented from getting her offered share if she is happy with it. The question arises whether such equilibria are coalitionally stable. Could the proposer offer a larger fraction of the pie to a creditor and then split it with him? When deviations are "coalitionally credible," the answer is no: we show that, for a large class of bankruptcy rules, all the subgame-perfect equilibria of the game associated with a two-person rule are also coalition-proof. Moreover, for any rule outside this class, there is a bankruptcy problem and a subgame-perfect equilibrium of the associated game, which is not coalition-proof.

Bankruptcy problems are legal problems. As such, their resolution should take into account only the legal rights of the creditors and the feasibility constraints. This means that no other consideration (such as creditors' risk attitudes, wealth and so on) should influence the final allocation. All bankruptcy rules presented in the literature satisfy this requirement. One further advantage of our game form is that its equilibrium split is independent of the creditors' utility functions, as long as these functions are strictly increasing in money. Other mechanisms (see, for example, Hart and Mas-Colell, 1992) do not satisfy this property when applied to bankruptcy problems.

The literature on law and economics concentrates on bankruptcy procedures that satisfy the principle of absolute priority among creditors.[2] It is argued there that higher claimants should have a more active role in the bankruptcy procedure because they are more keenly motivated to find a satisfactory solution to the problem. In our analysis this principle is derived as a result if one of our goals is the determinacy of the procedure: in general, only when the order of the proposers in the bargaining rounds is determined by the amount claimed does our mechanism yield a unique subgame-perfect equilibrium outcome. Hence, our paper provides a separate rationale for this principle.

We will assume throughout that the claims are known by everybody (including the court). As discussed above, our focus is the non-cooperative dimension of the consistency axiom in bankruptcy problems. In a companion piece, we will analyze the related problem of implementing bankruptcy rules when the claims are unknown to the court.

The paper is organized as follows: Section 2 is devoted to the axiomatic treatment of bankruptcy problems. Section 3 discusses the relation between bilateral principles of justice and consistency. The multilateral non-cooperative model and the main result are presented in Section 4. Coalition-proofness is discussed in Section 5. A result concerning strictly monotone rules is the object of Section 6, and Section 7 concludes.

2. THE AXIOMATIC BANKRUPTCY MODEL

A *bankruptcy problem* is a pair $(E;\mathbf{d})$ where $\mathbf{d}\in\mathbf{R}^I_+$ is a vector of non-negative real numbers (claims), indexed by some finite non-empty subset I of natural numbers (creditors), and $0\leq E\leq\Sigma_{i\in I}d_i:=D$. E is the estate to be allocated, and D is the sum of the claims.

An *allocation* in $(E;\mathbf{d})$ is a vector $\mathbf{x}\in\mathbf{R}^I_+$ such that $\Sigma_{i\in I}x_i = E$ and $x_i\leq d_i$ for all $i\in I$. The set of all allocations in $(E;\mathbf{d})$ is denoted by $\mathcal{A}(E;\mathbf{d})$.

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[2] See Aghion et al. (1992) and Bebchuk (1988).

Remark: For any list of claims $\mathbf{d}\in\mathbb{R}^{I}_{+}$, any vector $\mathbf{x}\in\mathbb{R}^{I}_{+}$ with $x_i \leq d_i$ is an allocation of the bankruptcy problem $(\Sigma_{i\in I}x_i;\mathbf{d})$. Therefore, when there is no danger of confusion, we shall call any such vector **x** an allocation without specifying the bankruptcy problem to which it refers.

A *rule* is a function that assigns to each bankruptcy problem a unique allocation.

Examples:

a) The *proportional rule*:

$$\mathrm{Pr}(E;\mathbf{d}) = \lambda\mathbf{d},$$

where $\lambda D = E$.

The proportional rule, widely applied nowadays, allocates awards in proportion to claim size. The proportionality principle was favored by the philosophers of ancient Greece, and Aristotle even considered it as equivalent to justice.

b) The *constrained equal award (CEA) rule*:

$$\mathrm{CEA}(E;\mathbf{d}) = \mathbf{x}$$

where $x_i = \min(\lambda, d_i)$ and λ solves the equation $\Sigma_{i\in I}\min(\lambda,d_i)=E$.[3]

This rule assigns the same sum to all creditors as long as it does not exceed each creditor's claim. This rule is also very ancient, and was adopted by important rabbinical legislators, including Maimonides.

c) The *constrained equal loss (CEL) rule*:

$$\mathrm{CEL}(E;\mathbf{d}) = \mathbf{x}$$

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[3] This equation has a unique solution when $D>E$. If $D=E$, any solution λ is greater than or equal to the maximum claim and therefore $x_i = d_i$ for all i.

where $x_i = \max(0, d_i-\lambda)$ and λ solves the equation $\Sigma_{i\in I}\max(0, d_i-\lambda)=E$.[4]

This rule assigns losses (d_i-x_i), in the same manner as the CEA assigns awards.

d) The *Pineles rule*:

$$\mathrm{Pin}(E;\mathbf{d}) = \mathrm{CEA}(\min\{D/2,E\};\mathbf{d}/2) + \mathrm{CEA}(\max\{E-D/2,0\};\mathbf{d}/2).$$

When the estate does not exceed half the sum of the claims, the Pineles rule assigns each creditor a fixed amount, as long as it does not exceed half his claim (otherwise, it assigns him half his claim). When the estate exceeds half the sum of the claims, it first gives each creditor half his claim and then divides the remainder (which, by definition, cannot exceed half the sum of the claims) according to the procedure described in the previous sentence. This rule appears in Pineles (1861, p. 64), and is an interpretation of a controversial mishna (Ketuboth 93).

e) The *Contested Garment Consistent (CGC) rule*:

$$\mathrm{CGC}(E;\mathbf{d}) = \mathrm{CEA}(\min\{D/2,E\};\mathbf{d}/2) + \mathrm{CEL}(\max\{E-D/2,0\};\mathbf{d}/2).$$

This rule was proposed by Aumann and Maschler (1985) as an alternative interpretation of the mishna mentioned above.

f) *Equal sacrifice rules*:

Let $U:\mathbf{R}_{++}\rightarrow\mathbf{R}$ be a continuous and strictly increasing function that satisfies $\lim_{x\to 0} U(x) = -\infty$. The equal sacrifice rule f relative to U satisfies

$$f(E;\mathbf{d}) = \mathbf{x} \Leftrightarrow \exists c\geq 0 \text{ such that } \forall i\in I \text{ with } d_i>0,\ U(d_i) - U(x_i) = c, \text{ when } E>0.$$

These rules assign awards so as to equalize absolute sacrifice evaluated according to a prespecified utility function. Note that the equal sacrifice rule with respect to the logarithmic function, is the proportional rule. The equal sacrifice principle in taxation appears in Mill (1848, Book V) and was axiomatically derived by Young (1988).

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[4] This equation has a unique solution when $E>0$. If $E=0$, any solution λ is greater than or equal to the maximum claim and therefore $x_i = 0$ for all i.

With a few axceptions that will be indicated, all these rules satisfy the properties discussed below. We begin with some basic ones and devote the next section to properties concerning the concept of consistency.

An allocation **x** in (E;**d**) is said to be *order preserving* if for all creditors i and j, if $d_i \leq d_j$ then $x_i \leq x_j$ and $d_i - x_i \leq d_j - x_j$. If we call $d_i - x_i$ the loss of creditor i, in any order preserving allocation, the order of creditors by claims, awards and losses is the same.

Remark: Order-preserving allocations are *symmetric* in the sense that if $d_i = d_j$ then $x_i = x_j$.

A rule is *order preserving* (*symmetric*) if it always assigns order-preserving (symmetric) allocations.[5]

A rule f is *consistent* if for any finite non-empty set I of creditors

$$\text{for all } (E;\mathbf{d}),\ \mathbf{d} \in \mathbb{R}^I_+, \text{ for all } \varnothing \neq J \subset I,$$

$$f(E;\mathbf{d}) = \mathbf{x} \Rightarrow \mathbf{x}|J = f(\Sigma_{i \in J} x_i,\ \mathbf{d}|J) \tag{1.1}$$

where when $\mathbf{y} \in \mathbb{R}^I_+$, $\mathbf{y}|J$ is the projection of **y** on $\mathbb{R}^J_+$.

A weaker condition is *bilateral consistency*, which requires (1.1) only for subsets J containing exactly two creditors. The interpretation of consistency is as follows. Suppose that a rule f assigns allocation **x** to the bankruptcy problem (E;**d**). Suppose also that some subset of creditors wants to reallocate the total amount $\Sigma_{i \in J} x_i$ assigned to them. If we apply the same rule f to allocate this amount among these creditors, each will get the amount originally assigned to him, provided f is consistent. Consistency in the setup of bankruptcy problems was first discussed by Aumann and Maschler (1985) and further analyzed by Young (1987, 1988).

A rule f is *monotone* if for all (E;**d**) and $0 \leq E' \leq E$, $f(E';\mathbf{d}) \leq f(E;\mathbf{d})$. Monotonicity says that a decrease in the estate does not benefit any creditor. A rule f is *strictly monotone* if for all $(E;\mathbf{d})$ and $0 \leq E' < E$, if $d_i > 0$ then $f_i(E';\mathbf{d}) < f_i(E;\mathbf{d})$. Strict monotonicity says that a decrease in the estate leaves every non-zero creditor worse off. The rules in the above examples, with the exception of the proportional and equal sacrifice rules, do not satisfy strict monotonicity.

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[5] Equal sacrifice rules with respect to non-concave utility functions are not necessarily order preserving in losses.

A rule f is *supermodular* if for all $(E;\mathbf{d})$ and $0 \leq E' \leq E$, if $d_i \leq d_j$ then $f_i(E;\mathbf{d})-f_i(E';\mathbf{d}) \leq f_j(E;\mathbf{d})-f_j(E';\mathbf{d})$. A supermodular rule allocates each additional shekel in an "order preserving" manner.[6]

The following lemmas will be useful in the rest of the paper.

LEMMA 2.1: Any supermodular rule f is order preserving.

Proof: Let $\mathbf{x} = f(E;\mathbf{d})$ and let i and j be two creditors with $d_i \leq d_j$.

$x_i = f_i(E;\mathbf{d}) = f_i(E;\mathbf{d}) - 0 = f_i(E;\mathbf{d}) - f_i(0;\mathbf{d}) \leq f_j(E;\mathbf{d}) - f_j(0;\mathbf{d}) = f_j(E;\mathbf{d}) = x_j$. The previous inequality follows from the supermodularity of f. Analogously we have

$d_i-x_i = f_i(D;\mathbf{d}) - f_i(E;\mathbf{d}) \leq f_j(D;\mathbf{d}) - f_j(E;\mathbf{d}) = d_j-x_j$.

LEMMA 2.2: Let $(E;\mathbf{d})$ be a bankruptcy problem and let i be a creditor with the highest claim. If f is supermodular and $0 \leq E' < E$, then $f_i(E;\mathbf{d}) > f_i(E';\mathbf{d})$. That is, i's award is strictly monotone in the estate.

Proof: Trivial.

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[6] Equal sacrifice rules relative to non-concave utility functions are not necessarily supermodular.

3. ON BILATERAL COMPARISONS, JUSTICE AND CONSISTENCY

Since every bankruptcy problem is a legal problem, its solutions should be guided by the principle of justice. Whatever form this principle may take, it should enable us to determine whether any one creditor received better or worse treatment than another at any given allocation. For example, if we believe, like Aristotle, that justice is proportionality, then we would say that i is treated better than j at allocation **x** if i receives a larger proportion of his claim than j does. According to this principle of justice, an allocation will treat i and j equally if they receive the same proportion of their claims. Obviously, we can think of other notions of justice, but in order to make these pairwise comparisons we clearly need only a bilateral principle.

A *bilateral principle* is a function that assigns a unique allocation to every two-person bankruptcy problem. We interpret this unique allocation as the just solution to the problem. We shall say that any other allocation in a two-person problem treats one creditor better than the other since it awards one creditor more than his "fair" share. Any rule, when applied to two-person problems, is an example of a bilateral principle. Conceptually, however, bilateral principles differ from two-person allocation rules. The former single out a just allocation that permits pairwise comparisons, while the latter allocate the estate in two-person problems.

Given a bankruptcy problem $(E;\mathbf{d})$ and a bilateral principle f, we shall say that an allocation **x** treats i and j f-*equally* if $(x_i,x_j)=f[x_i+x_j;(d_i,d_j)]$. An allocation in $(E;\mathbf{d})$ is said to be f-*just* if it treats every two creditors f-equally. Aumann and Maschler (1985) showed that if a bilateral principle f is monotone, then there is at most one f-just allocation for each bankruptcy problem. If a unique f-just allocation exists for any bankruptcy problem, then we can define the f-*just rule* to be the rule that assigns to each bankruptcy problem its unique f-just allocation.

We explore some relations between f-justice and consistency.

LEMMA 3.1: Let g be a monotone bilateral principle and f be the g-just rule, then f is consistent.

Proof: See Aumann and Maschler (1985).

LEMMA 3.2: Let f be a monotone and bilateral consistent rule, and let g be the bilateral principle induced by f. Then f is the g-just rule.

Proof: Trivial.

Lemmas 3.1 and 3.2 imply that *the g-just rule is the unique consistent rule that coincides with the bilateral principle g in two-creditor problems.*

Given a consistent, monotone and supermodular rule f, a list of claims **d** and an allocation $\mathbf{x} \leq \mathbf{d}$, we can define the following binary relation on the set of creditors I:

$$\succ_{\mathbf{x}} = \{(i,j) \in I \times I \mid f_i[x_i + x_j; (d_i, d_j)] < x_i\}.^{7}$$

$i \succ_{\mathbf{x}} j$ means that **x** treats i f-better than j. Note that in order to define the relation we only need the bilateral principle induced by f. Obviously, if $i \succ_{\mathbf{x}} j$ then $f_j[x_i + x_j; (d_i, d_j)] > x_j$. Note that if $i \succ_{\mathbf{x}} j$, then $i \succ_{\mathbf{y}} j$ for any other allocation **y** in which $y_i = x_i$ and $y_j = x_j$. That is, whether or not $i \succ_{\mathbf{x}} j$ is independent of the amounts assigned by **x** to other creditors.

We define the relations $\succsim_{\mathbf{x}}$ and $\sim_{\mathbf{x}}$ by replacing $<$ in the definition of $\succ_{\mathbf{x}}$ with $\leq$ and $=$ respectively. These relations have the obvious interpretation.

There are some interesting properties that the relations just defined satisfy:

LEMMA 3.3: Let f be a consistent and monotone rule, let (E;**d**) be a bankruptcy problem and

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[7] For $i=j$, we define $f_i[x_i + x_j; (d_i, d_j)]$ as x_i.

let **x*** be its f-just allocation. Let **x** be an allocation in (E;**d**) in which there are two creditors i and j with $x_i \leq x_i^*$ and $x_j \geq x_j^*$. Then, $j \succeq_x i$. Moreover, if both inequalities are strict, then $j \succ_x i$.

Proof:

Case 1: $x_i + x_j \geq x_i^* + x_j^*$. By monotonicity and consistency,

$$f_i[x_i + x_j;(d_i,d_j)] \geq f_i[x_i^* + x_j^*;(d_i,d_j)] = x_i^* \geq x_i. \text{ Hence, } j \succeq_x i.$$

Case 2: $x_i + x_j < x_i^* + x_j^*$. By monotonicity and consistency,

$$f_j[x_i + x_j;(d_i,d_j)] \leq f_j[x_i^* + x_j^*;(d_i,d_j)] = x_j^* \leq x_j. \text{ Hence, } j \succeq_x i.$$

This proves the first part of the claim. As for the second part, it is proved analogously and is left to the reader. □

This lemma says that the f-just allocation of a bankruptcy problem is a good benchmark for bilateral comparisons: if at some allocation **x** player i gets more than the f-just allocation assigns to him and if player j gets less than his f-just share, then at **x** i must be receiving better treatment than j.

LEMMA 3.4: Let f be a consistent and monotone rule, let (E;**d**) be a bankruptcy problem and let x be an allocation in (E;**d**). Then, $\succ_x$ is transitive.

Proof: Let i, j and k be three creditors such that $i \succ_x j$ and $j \succ_x k$. Define the following 3-creditor bankruptcy problem: $(E';\mathbf{d}') := [x_i + x_j + x_k;(d_i,d_j,d_k)]$. Define **x*** as the f-just allocation of this problem. It must be the case that $x_k < x_k^*$. Otherwise, since $j \succ_x k$, by lemma 3.3 $x_j > x_j^*$ and since $i \succ_x j$, by the same lemma $x_i > x_i^*$ contradicting the fact that $x_i + x_j + x_k = x_i^* + x_j^* + x_k^*$. Analogously, it must be that $x_i > x_i^*$. Hence by lemma 3.3 $i \succ_x k$. □

Lemma 3.4 says that the relation $\succeq_x$ is *quasi-transitive*. Hence, Sen's (1969) lemma 1 implies that $\succeq_x$ satisfies the following property: if $i \succeq_x j$ and $j \succ_x k$ or if $i \succ_x j$ and $j \succeq_x k$, then $i \succeq_x k$. (For convenience, we shall call this latter property quasi-transitivity.) On the other hand, note that when the rule f is not strictly monotone, $\succeq_x$ is not transitive. To see this, consider the following bankruptcy problem: $(E;\mathbf{d}):=[400;(300, 200, 100)]$ and the following allocation: $\mathbf{x}=(160, 140, 100)$. When f is the constrained equal award rule it is easy to see that $2 \succeq_x 3$, $3 \succeq_x 1$ but $1 \succ_x 2$.

LEMMA 3.5: Let $(E;\mathbf{d})$ be a bankruptcy problem, let f be a consistent and monotone rule and let x be an allocation in $(E;\mathbf{d})$. If there exists a creditor i such that for all j, $i \sim_x j$ then $x_i = f_i(E;\mathbf{d})$.

Proof: If $x_i > f_i(E;\mathbf{d})$ then there exists a creditor j with $x_j < f_j(E;\mathbf{d})$. Hence, by lemma 3.3, $i \succ_x j$ contradicting the assumption of the lemma. Analogously, if $x_i < f_i(E;\mathbf{d})$ there exists a creditor j with $j \succ_x i$. □

LEMMA 3.6: Let $(E;\mathbf{d})$ be a bankruptcy problem, let f be a consistent, monotone and supermodular rule and let x be an allocation in $(E;\mathbf{d})$. Let i be a creditor with the highest claim. If for all creditors j, $i \sim_x j$ then $\mathbf{x}=f(E;\mathbf{d})$.

Proof: By lemma 3.5, $x_i = f_i(E;\mathbf{d})$. Now assume there exists a creditor j with $x_j < f_j(E;\mathbf{d})$. By consistency, supermodularity and lemma 2.2, $i \succ_x j$ contradicting the assumption. So it must be that for all j, $x_j \geq f_j(E;\mathbf{d})$. Since $\mathbf{x}$ is an allocation, this implies that $\mathbf{x}=f(E;\mathbf{d})$. □

Lemma 3.6 says that if an allocation is such that all creditors are treated f-equally to one with a maximum claim, then this allocation is the f-just allocation. Thus, only n-1 equations are needed to calculate the f-just allocation of any n-creditor bankruptcy problem.

We have seen that the relation $\succeq_x$ is not an order. We now define a complete order on the set of creditors, which will be useful in the rest of the paper.

Let (E;**d**) be a given bankruptcy problem. We assume that the creditors are ordered by size of claim, that is, if $i<j$ then $d_i \geq d_j$. Without loss of generality we shall call the creditor with the lowest index "creditor 1". Let **x** be a given allocation in (E;**d**). We associate to **x** the following vector: $\mathbf{w} \in \mathbb{R}^I_+$ where $w_i := f_i[x_1+x_i;(d_1,d_i)]$. The amount w_i is what creditor i would get if we allocate x_1+x_i f-justly between creditors 1 and i. Note that w is not in general an allocation in (E;**d**). Now define the following binary relation on the set of creditors:

Definition: $iR_xj \Leftrightarrow w_i\text{-}x_i > w_j\text{-}x_j$ or, $w_i\text{-}x_i = w_j\text{-}x_j$ and $i \geq j$.

Clearly, R_x is a complete order, i.e., it is complete, transitive and antisymmetric.

In order to understand what the relation R_x means, note that $w_j\text{-}x_j$ is the amount that creditor 1 should add to x_j if we wanted to divide x_1+x_j f-equally between 1 and j. We then say that iR_xj if creditor 1 must give more money to creditor i than to creditor j when compensating them for the f-injustice inherent in the allocation **x**. If these monetary compensations are equal, then iR_xj means that $i \geq j$.

4. A MULTILATERAL NON-COOPERATIVE MODEL

Let (E;**d**) be a given bankruptcy problem. Throughout in this section and Section 5 we shall assume that the creditors are ordered by size of claim, that is, if $i<j$ then $d_i \geq d_j$. We are interested in defining an extensive form game for each bilateral principle f. The game, denoted by G^f(E;**d**), is defined recursively. If there is one creditor, he receives the whole estate and the

game is over. Assume the game is already defined for all bankruptcy problems with at most n-1 creditors, the game for n creditors is defined as follows: The first creditor (the one with the lowest index) proposes an allocation **x** in $\mathcal{A}(E;\mathbf{d})$; following this proposal all the other creditors have to respond simultaneously, either accepting or rejecting the offer. An accepting creditor i, gets $z_i=x_i$ and leaves the game. Let Y be the set of accepting creditors, N be the set of rejecting creditors, and 1 be the proposer. The proposer receives $z_1=(E-\Sigma_{i\in Y}x_i)-\Sigma_{i\in N}f_i[x_1+x_i;(d_1,d_i)]$. The rejecting creditors go on to the game corresponding to the bankruptcy problem $(E-\Sigma_{i\notin N}z_i;\mathbf{d}|N)$.

The interpretation of the rules of the game is as follows: the first creditor (one with a highest claim) proposes an allocation **x**. If it is unanimously accepted, the estate is divided according to **x**. Otherwise, those who accepted the offer get their shares and those who rejected it "renegotiate" bilaterally with the proposer. In these bilateral renegotiations, each rejecting creditor i "bargains" with the proposer 1 for his fair share of the amount $x_1 + x_i$. We assume that the bilateral principle f is commonly accepted in society, or that resorting to litigation is an outside option for the creditors: the proposer is held to be responsible for his offer, in the sense that all creditors who are unhappy with it have the right to be compensated in accordance to the bilateral principle. The rejecting creditors go on bargaining on the sum of the amounts awarded to them in the bilateral renegotiations. The proposer gets the remainder of the estate after all the other creditors received their shares.[8]

We also study an alternative version of the game that ensures feasibility. In this version, when the proposer's residual amount is negative, he receives 0 and the rejecting creditors keep bargaining over what is left after paying the acceptors. The analysis will concentrate on the first version. With some minor modifications, the uniqueness part of the proof of Theorem 1 will also apply to the second version. Existence requires a separate treatment.

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[8] This remainder can be negative. However, it is shown in lemma 4.1 below that the proposer can always guarantee a non-negative amount.

Although the payoffs are given in monetary terms, it is not necessary to assume that the creditors' utilities are linear in money. Our results are insensitive to the choice of the utility representation as long as it is strictly increasing in money, that is, risk preferences do not affect the result.

The introduction of the bilateral principle to calculate the proposer's payoff may seem arbitrary. However, we want to emphasize that our purpose is not to characterize a certain consistent rule or any bilateral principle; rather, we are interested more in the relations of the bilateral principles and their consistent generalizations than in characterizing any specific bilateral principle. For a model in which the bilateral principle does not appear in the extensive form game see Serrano (1993), who characterized the contested garment consistent rule.

Now we are ready to state the main result of this paper.

THEOREM 1: Let (E;**d**) be a bankruptcy problem and let f be a consistent, monotone and supermodular rule. The unique subgame-perfect equilibrium outcome of $G^f(E;\mathbf{d})$ is f(E;**d**).

Some remarks are in order.

1) The theorem holds for the case in which the proposers are ordered by the size of their claims. For other orders uniqueness is not obtained (see Example 6.1 in Section 6). Therefore, if the determinacy of the model is a *desideratum*, the principle of absolute priority among creditors should be applied to the choice of the order of proposers. However, as will be shown in Section 6, for strictly monotone rules, the order of proposers is of no importance.

2) The result does not use any refinement of the set of subgame-perfect equilibrium. This is

similar to results that relate non-cooperative models with pure bargaining problems, as in Binmore, Rubinstein and Wolinsky (1986). When the underlying cooperative model is more complex, as in general cooperative games, a refinement is usually needed in order to get uniqueness (see, for example, Hart and Mas-Colell, 1992; and Gul, 1989).

3) Unlike some other models, which provide a non-cooperative view of a cooperative solution concept, our result yields a non-cooperative view of a large *family of allocation rules* for bankruptcy problems. The two critical properties that characterize this family are consistency and monotonicity. These two properties guarantee that the allocation assigned by the consistent rule can be supported by a Nash equilibrium. These properties are the ones that drive the results of other consistency-based non-cooperative mechanisms (see, for example, Krishna and Serrano, 1990; and Chae and Yang, 1989).

4) Our result holds for the *whole family of bankruptcy problems*. This is in contrast to other models, such as those mentioned in the previous remark, where the unique subgame-perfect equilibrium outcomes converge to the Nash bargaining solution agreement for "dividing a dollar" bargaining problems, in which the Nash solution is monotone (see Chun and Thomson (1988)). The reason why these models do not yield a similar result in all bargaining problems becomes apparent: the Nash bargaining solution is not monotone in general.[9]

5) The unique equilibrium agreement is not achieved immediately in all the subgame-perfect equilibria of the game, even though there always exists an equilibrium in which the agreement is immediate. This feature of the model is consistent with the consistency principle. As we know, after applying a consistent rule any subset of agents is indifferent between accepting their

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[9] When the Nash solution is not monotone, the strategies proposed by Krishna and Serrano, or by Chae and Yang do not constitute even a Nash equilibrium. The proposer could find a profitable deviation by offering more than his equilibrium share to one of the responders, in the hope of benefiting from a bigger share in a smaller remaining pie.

shares and renegotiating among themselves.

6) The possibe emergence of a stepwise agreement in equilibrium is due to the fact that there is no cost for renegotiation, such as discounting, fixed renegotiation fees or the random elimination of players.

7) The fact that random devices are not used in the model has two advantages. Firstly, risk preferences do not affect the outcome, and secondly, the mechanism is more realistic (legal procedures in civilized economies typically do not use random devices).

Proof of Theorem 1: The proof follows from the following series of lemmas.

We denote f(E;**d**) by **x*** and equilibrium outcomes by **z**.

LEMMA 4.1: In any Nash equilibrium of $G^f(E;\mathbf{d})$, $z_1 \geq x_1^*$.

Proof: Creditor 1 can guarantee a payoff of x_1^* simply by proposing **x***. It is easy to see that whatever the replies, he will get exactly x_1^*.

We prove uniqueness by induction on the number of creditors. For one-creditor problems, the unique SPE outcome is x^*. Suppose then that Theorem 1 is true for all problems with at most n-1 creditors.

LEMMA 4.2: If Theorem 1 holds for all bankruptcy problems with less than n creditors, then for all bankruptcy problems with exactly n creditors there exists at most one subgame-perfect equilibrium outcome which is $\mathbf{x}^*=f(E;\mathbf{d})$.

Proof: Let (E;**d**) be an n-creditor bankruptcy problem and let σ be a subgame-perfect equilibrium with outcome **z**. Denote by **x** the equilibrium offer of the first proposer. Since **z** is an equilibrium outcome,

$$z_j \geq x_j \quad \text{for all } j \neq 1 \qquad (4.1),$$

which implies, with $\Sigma_{j\in I} z_j = \Sigma_{j\in I} x_j$ and lemma 4.1,

$$x_1 \geq z_1 \geq x_1^* \qquad (4.2).$$

Denote by N the set of creditors who reject the offer **x** and by Y the set of accepting creditors. If $N = \emptyset$, by lemma 4.1 $x_1 = z_1 \geq x_1^*$. If there exists a responding creditor k with $x_k < x_k^*$, then by supermodularity and monotonicity of f, $1 \succ_{\mathbf{x}} k$. Hence if he refuses offer **x**, his payoff will increase, which contradicts the assumption that k plays a best response. Hence $\mathbf{x} \geq \mathbf{x}^*$ and since x is an allocation, $\mathbf{z} = \mathbf{x} = \mathbf{x}^*$.

Now we turn to the case where $N \neq \emptyset$. In this case, by the induction hypothesis, $\mathbf{z}|N = f(\Sigma_{i\in N} w_i; \mathbf{d}|N) \geq \mathbf{x}|N$, where the last inequality follows from (4.1). Hence, by lemma A.3, $1 \succeq_{\mathbf{x}} i$ for all $i \in N$ and by lemma A.1 (see appendix) applied to the bankruptcy problem $(x_1 + \Sigma_{i\in N} x_i; \mathbf{d}|N \cup \{1\})$ we have

$$z_1 \leq f_1(x_1 + \Sigma_{i\in N} x_i; \mathbf{d}|N \cup \{1\}) \qquad (4.3),$$

which by (4.2) implies $x_1^* \leq z_1 \leq f_1(x_1 + \Sigma_{i\in N} x_i; \mathbf{d}|N \cup \{1\})$. Hence by consistency, monotonicity and supermodularity of f, $x_1 + \Sigma_{i\in N} x_i \geq x_1^* + \Sigma_{i\in N} x_i^*$. It also follows from (4.3) that $\Sigma_{i\in N} z_i \geq \Sigma_{i\in N} f_i(x_1 + \Sigma_{i\in N} x_i; \mathbf{d}|N \cup \{1\})$ and by consistency and monotonicity $\Sigma_{i\in N} z_i \geq \Sigma_{i\in N} f_i(x_1 + \Sigma_{i\in N} x_i; \mathbf{d}|N \cup \{1\}) \geq \Sigma_{i\in N} x_i^*$. Again, by the induction hypothesis, consistency and monotonicity we have $f(\Sigma_{i\in N} z_i; \mathbf{d}|N) = \mathbf{z}|N \geq \mathbf{x}^*|N$.

Next we show that $\mathbf{z}|Y := \mathbf{x}|Y \geq \mathbf{x}^*|Y$. Assume by contradiction that there exists $k \in Y$ such that $z_k = x_k < x_k^*$. Since $z_i \geq x_i^*$ for all $i \in N$, by lemma 3.3, $i \succeq_{\mathbf{z}} k$ for all $i \in N$. Consider the subgame in which k rejects the offer **x**. Denote by $\mathbf{v} \in \mathbb{R}^{N\cup\{k\}}$ the subgame-perfect equilibrium outcome after this deviation. By the induction hypothesis $\mathbf{v} = f(w_k + \Sigma_{i\in N} z_i; \mathbf{d}|N \cup \{k\})$. Since this is a deviation from subgame-perfect equilibrium, it must be that $v_k \leq z_k$. By lemma 3.3 and lemma 2.2, $1 \succ_{\mathbf{x}} k$, which implies $\Sigma_{i\in N\cup\{k\}} v_i > \Sigma_{i\in N\cup\{k\}} z_i$. This implies that $v_j > z_j$ for some $j \in N$.

Now we have, $v_j > z_j \geq x^*_j$ and $v_k \leq z_k < x^*_k$. Therefore, by lemma 3.3, $j \succ_v k$ which contradicts the fact that **v** is an f-just allocation. Hence $\mathbf{z}|Y \geq \mathbf{x}^*|Y$. Finally, we have $\mathbf{z} \geq \mathbf{x}^*$ but since $\Sigma_{i \in I} z_i = \Sigma_{i \in I} x^*_i$, $\mathbf{z} = \mathbf{x}^*$ which completes the proof.

LEMMA 4.3: If Theorem 1 holds for all bankruptcy problems with less than n creditors, then for all bankruptcy problems with exactly n creditors there exists a subgame-perfect equilibrium.

Proof: Let (E;**d**) be an n-creditor bankruptcy problem. We shall construct an equilibrium: The first proposer proposes f(E;**d**). To any proposal **x**, the responders answer according to the following algorithm:

[insert Figure 1 here]

For any proposal x, all creditors who, according to the algorithm, belong to the set N when the algorithm reaches the end, reject x and all the rest accept it. In any subsequent stage, the players play according to some prespecified subgame-perfect equilibrium of the game corresponding to the bankruptcy problem reduced to the set N, which will have less than n creditors.

Recall that for any proposal **x**, w_i is i's f-just share of the amount $x_1 + x_i$ when it is divided between creditors 1 and i. In our game w_i is the amount creditor i will contribute to the sum to be divided among all the rejecting creditors if he decides to reject the offer. At each stage of the algorithm Y is the set of potential acceptors, N is the set of virtual rejecters and M is a subset of Y which we call the "waiting list". In the beginning, all responding creditors are potential acceptors and no one is in the waiting list.

At any stage the algorithm chooses a candidate to join the set of rejecters among the potential acceptors who are not in the waiting list. The algorithm chooses candidate c to be the

R_x-maximal creditor in Y\M. He is then asked whether he wants to join the set of virtual rejecters under the assumption that all other creditors will remain in their respective sets, i.e., all creditors in N will reject **x** and all the other creditors in Y will accept it. Under this assumption, if candidate c joins N, he will get, given the equilibrium strategies in the following stages of the game, $g_c = f_c(w_c + \Sigma_{i \in N} w_i; \mathbf{d} | N \cup \{c\})$. If he does not join N, he will get x_c.

We assume that c joins the set of virtual rejecters (thereby leaving the set of potential acceptors) only if $g_c > x_c$. In this case the waiting list is reset to be empty. If the candidate does not join N, he joins the waiting list M. After c joines either M or N, another candidate is chosen according to the same criterion, and offered the opportunity to join the set of virtual rejecters. This process continues until there are no more potential accepting creditors or until all of them are on the waiting list. Then, according to the algorithm, all creditors in N reject x and all the creditors in Y accept it.

Note that a creditor who joined the set of virtual rejecters will never leave it. On the other hand, all those who could have joined N but did not do so are given a chance to reconsider only after some other creditor joined N. This, together with the fact that in every stage there is one creditor who joins either N or the waiting list, makes the algorithm end after a finite number of stages.

We claim that the n-tuple of strategies induced by the algorithm constitutes a subgame-perfect equilibrium. It is clear that all those who, according to the algorithm, accept x are playing a best response, since each was invited to join the rejecters separately and refused to do so.

As for the rejecters, in order to prove the optimality of their strategies, we need two lemmas.

LEMMA 4.3.1: Consider a stage in the algorithm where creditor c is asked whether he wants to join the set of rejecters. If he chooses to join the set of rejecters, proposal **x** does not treat him better than any creditor in the waiting list, i.e., if $f_c(w_c+\Sigma_{i\in N}w_i;\mathbf{d}|N\cup\{c\})>x_c$ then $j\succeq_x c$ for all $j\in M$.

Proof: Let j be a creditor in M and define the allocations **g** and **h** as follows:

$\mathbf{g} := f(w_c+\Sigma_{i\in N}w_i;\mathbf{d}|N\cup\{c\})$ and $\mathbf{h} := f(w_j+\Sigma_{i\in N}w_i;\mathbf{d}|N\cup\{j\})$.

By the assumption,

$$g_c > x_c. \tag{4.4}$$

Since **h** is the payoff that all members of N and j would have received had they been the only refusers of proposal **x**, and since j belongs to M, we conclude that he refused to join N when he was asked to,which means

$$h_j \leq x_j. \tag{4.5}$$

By construction of the algorithm, jR_xc. It must be that

$$w_j-h_j \geq w_j-x_j \geq w_c-x_c > w_c-g_c,$$

where the first inequality follows from (4.5), the second from the fact that jR_xc, and the last follows from (4.4). Hence

$$\Sigma_{i\in N}h_i=\Sigma_{i\in N}w_i+(w_j-h_j)>\Sigma_{i\in N}w_i+(w_c-g_c)=\Sigma_{i\in N}g_i.$$

Therefore, there exists a creditor k in N such that $h_k > g_k$.

Now define the following allocation: $(y_c, y_k, y_j):=(x_c, h_k, x_j)$.

Since $x_c < g_c$ and $g_k < h_k$, and since **g** is an f-just allocation, it follows from lemma 3.3 that $k \succ_y c$. Analogously, since $x_j \geq h_j$ and since **h** is an f-just allocation, it follows from lemma 3.3 that $j \succeq_y k$. Hence, by quasi-transitivity of $\succeq_y$, $j \succeq_y c$ which is equivalent to $j \succeq_x c$.

LEMMA 4.3.2: Consider a stage in the algorithm where creditor c is invited to join the set of rejecters. If he accepts (joins the set of rejecters), then $f_j(w_c+\Sigma_{i\in N}w_i;\mathbf{d}|N\cup\{c\})\geq x_j$ for all $j\in N$, that is, those who already decided to reject are still happy with their decision even after c joined them.

Proof: By induction on the number of creditors in N. If $N=\varnothing$ the statement is true. Assume now that $f(\Sigma_{i\in N}w_i;\mathbf{d}|N)\geq \mathbf{x}|N$. Let $\mathbf{g} := f(w_c+\Sigma_{i\in N}w_i;\mathbf{d}|N\cup\{c\})$.

Case 1: $w_c\geq g_c$. Then by consistency and monotonicity, $\mathbf{g}|N\geq f(\Sigma_{i\in N}w_i;\mathbf{d}|N)\geq \mathbf{x}|N$.

Case 2: $w_c<g_c$. Pick $j\in N$. Since **g** is an f-just allocation, it follows from lemma 3.3 that either $w_j\leq g_j$ or $j\succ_w c$.

We also know (by lemma A.3) that $w_j\geq x_j$. hence, if $g_j\geq w_j$ we conclude that $g_j\geq x_j$. Otherwise, if $j\succ_w c$, by lemma A.2 (see appendix), cR_xj, which implies that there was a stage in which j joined the set of rejecters and c was in the waiting list M. Hence, by lemma 4.3.1, $c\succeq_x j$. Since **g** is an f-just allocation and since $g_c>x_c$, lemma 3.3 implies that $g_j\geq x_j$.

Lemma 4.3.2 shows that all creditors in N play best responses by rejecting offer **x**, assuming that they are the only rejecters. This also holds for N when the algorithm reaches the end and shows that every rejecter is playing a best response to any offer **x** given the responses of the others.

As for the proposer, if he proposes $\mathbf{x}^*$ he will receive exactly x^*_1 since according to the algorithm every responder will accept his proposal. Hence we have to show that if he proposed $\mathbf{x}\neq\mathbf{x}^*$, the amount z_1 he would receive is not larger than x^*_1. Assume by contradiction that if he proposes **x** he receives $z_1>x^*_1$. Since we have already shown that all responders are playing best responses to any offer **x**, we can use arguments analogous to the ones used in lemma 4.2: Let N be the set of creditors that according to the algorithm reject offer **x**, and let Y be the set of accepting creditors. It can be shown that, since $z_1 > x_1^*$, it must be that N is non-empty and that $x_1^* < z_1 \leq f_1(x_1 + \Sigma_{i\in N} x_i; \mathbf{d} \mid N \cup \{1\})$. It follows that $x_1 + \Sigma_{i\in N} x_i > x_1^* + \Sigma_{i\in N} x_i^*$. This implies that there exists a creditor $k \in Y$ with $x_k < x_k^*$, which is impossible. □

Remark: Although the proof of uniqueness for the alternative model that ensures feasibility applies with minor modifications, the above algorithm does not necessarily yield a SPE. However, since monotone rules are continuous in the estate, we can show the existence of a SPE in which the responses may require randomizations. This is so because the existence problem amounts to asking whether the proposer's problem of finding the proposal that maximizes his payoff has a solution.

The following propositions outline some interesting properties of the SPE of our model. They will also be useful in the next section.

PROPOSITION 1: Let σ be a subgame-perfect equilibrium and let $\mathbf{x}$ be the equilibrium offer of creditor 1. Denote by N the set of creditors who, according to σ, reject $\mathbf{x}$. Then this equilibrium offer satisfies

$$w_i := f_i(x_1 + x_i; (d_1, d_i)) = x_i^* \text{ for all } i \in N.$$

Proof: Since $\mathbf{x}$ is an equilibrium offer and since by Theorem 1 the equilibrium outcome is $\mathbf{x}^*$, it must be that for all $i \neq 1$ $x_i \leq x_i^*$, and for creditor 1, $x_1 \geq x_1^*$. This implies, by consistency and monotonicity of f, that

$$w_i := f_i(x_1 + x_i; (d_1, d_i)) \geq x_i^* \text{ for all } i \neq 1. \tag{4.6}$$

Now, if we denote by $\mathbf{z}$ the equilibrium outcome, we have

$$\Sigma_{i\in N} z_i = \Sigma_{i\in N} w_i = \Sigma_{i\in N} x_i^*. \tag{4.7}$$

It follows directly from (4.6) and (4.7) that $w_i = z_i = x_i^*$ for all $i \in N$. □

PROPOSITION 2: Let σ be a subgame-perfect equilibrium and let $\mathbf{x}$ be any offer of creditor 1. Denote by N and Y the set of creditors who, according to σ, reject and accept the offer $\mathbf{x}$, respectively. Then for any creditor $k \in Y$ and any creditor $j \in N$, $k \succeq_x j$.

Proof: Let k be a creditor in Y and let j be a creditor in N. Assume by contradiction that $j \succ_x k$. It follows from Theorem 1 that the rejecters receive $\mathbf{z} = f(\Sigma_{i \in N} w_i, \mathbf{d} | N)$. Since σ is a subgame-perfect equilibrium, it must be that $\mathbf{z} \geq \mathbf{x} | N$. Thus, by lemma A.3 (in the appendix), $1 \succeq_x j$. Therefore, by quasi-transitivity $1 \succeq_x k$. Consider a deviation in which creditor k rejects $\mathbf{x}$. Denote by $\mathbf{z}'$ the payoffs of the rejecting creditors after this deviation. By Theorem 1, $\mathbf{z}' = f(w_k + \Sigma_{i \in N} w_i ; \mathbf{d} | N \cup \{k\})$. Since σ is a subgame-perfect equilibrium, $z'_k \leq x_k$. This, together with $1 \succeq_x k$ implies that $\Sigma_{i \in N} z'_i \geq \Sigma_{i \in N} w_i$. Hence, by consistency and monotonicity, $z'_j \geq z_j \geq x_j$. We therefore have $z'_k \leq x_k$ and $z'_j \geq x_j$ and since $\mathbf{z}'$ is an f-just allocation lemma 3.3 implies $k \succeq_x j$, contradicting the initial assumption. □

5. COALITIONAL STABILITY OF THE EQUILIBRIA

In the game presented in Section 4 creditors may exit with the share awarded to them simply by accepting the proposal. The question arises whether the equilibria are coalitionally stable. Could the proposer offer a larger share of the pie to a responder in the hope of profiting from a joint deviation? We first consider any kind of coalitional deviation and ask if the equilibria of our model are strong Nash (Aumann, 1959). This requires that no coalition of players has a joint deviation which leaves all its members better off. The subgame-perfect equilibria of our model are not strong. This is illustrated in the following example.

Example 5.1: Consider the bankruptcy problem $(E;\mathbf{d})=[99;(100, 100, 100)]$. Since this problem is symmetric, for all symmetric rules the game $G^f(E;\mathbf{d})$ is the same. Clearly, the f-just allocation in this problem is (33, 33, 33). Consider the following deviation by the first two creditors: The proposer offers $\mathbf{x}=(0, 98, 1)$, and the second creditor, who was offered 98 shekels, rejects it. In any subgame-perfect equilibrium of the game, the responders must accept this offer leaving the proposer with a payoff of 0. This shows that $\mathbf{x}$ is indeed an off-equilibrium offer and that rejecting it is an off-equilibrium response. This deviation yields the outcome $\mathbf{z}=(49, 49, 1)$, in which the deviating creditors receive 16 shekels more than they would have received in any subgame-perfect equilibrium.

Note that although this deviation improves creditor 2's payoff relative to the f-just allocation, he is playing a dominated strategy in the subgame that follows 1's offer. This makes the above deviation unstable. Examples of this sort motivated the alternative concept of coalitional stability known as coalition-proof Nash equilibrium, introduced by Bernheim, Peleg and Whinston (1987).

The following definition refers to games in normal form, $G=(I, (S_i)_{i\in I},(g^i)_{i\in I})$, where I is the set of players, S_i is the strategy set of player i and g^i is the payoff function of player i. Let J be a coalition, that is $\emptyset\neq J\subset I$. We denote $S_J=\Pi_{i\in J}S_i$. Also, if $\sigma\in S:=\Pi_{i\in I}S_i$ is a list of strategies, σ_J denotes the restriction of σ to coalition J. Given a game G, a list σ of strategies (one for each player) and a coalition J of players, an *internally consistent improvement of J upon* σ is defined by induction on $|J|$. If $J=\{i\}$ for some i in I, then $\tau_i\in S_i$ is an internally consistent improvement of J upon σ if $g^i(\tau_i,\sigma_{I\setminus\{i\}})>g^i(\sigma)$. If $|J|>1$ then $\tau_J\in S_J$ is an internally consistent improvement upon σ if (i) $g^i(\tau_J,\sigma_{I\setminus J})>g^i(\sigma)$ for all i in J, and (ii) no $T\subsetneq J$, $T\neq\emptyset$ has an internally consistent improvement upon $(\tau_J,\sigma_{I\setminus J})$. σ is a *coalition-proof Nash equilibrium* if no $J\subset I$, $J\neq\emptyset$, has an internally consistent improvement upon σ.

In contrast to strong Nash, coalition-proof Nash equilibrium requires that no coalition should have a profitable and self-enforcing deviation. Unfortunately, there are bankruptcy rules and bankruptcy problems for which not all the subgame-perfect equilibria of the associated game are coalition-proof. This is shown in the following example:

Example 5.2: Consider the 5-creditor bankruptcy problem $(E;\mathbf{d})=[160; (100, 100, 10, 10, 10)]$. Let f be the constrained equal loss rule. As can easily be calculated, the f-just allocation in this problem is (80, 80, 0, 0, 0). Consider the offer $\mathbf{x}=(100, 60, 0, 0, 0)$. Assume that the responders follow the algorithm presented in the previous section and always propose the f-just allocation when it is their turn to propose. This amounts to saying that the second creditor will reject the offer and the others will accept it. As the reader can verify, the offer x and these responses are the equilibrium path of a subgame-perfect equilibrium of our game that yields the f-just allocation as an outcome.

From these strategies, consider the joint deviation of the three smallest creditors in which all of them reject the equilibrium offer **x**. This deviation will lead to a subgame in which the four rejecting creditors will get $f[95;(100, 10, 10, 10)]=(91\frac{1}{4}, 1\frac{1}{4}, 1\frac{1}{4}, 1\frac{1}{4})$. Note that unlike the deviation in example 5.1, this one is self-enforcing.

In the above example, offer **x** treats all the responding creditors f-equally. Further, if a small creditor rejects the offer, he would gain an additional five shekels from the proposer in the bilateral renegotiations. However, in the next stage of the game, the large rejecting creditor would be the only one to benefit from this additional amount. Naturally, if f was strictly monotone, this could not happen. It turns out that a weaker property, which we call quasi-strict monotonicity, is sufficient to rule out such phenomena.

A rule f is *quasi-strictly monotone* if it is monotone and if for all bankruptcy problems $(E;\mathbf{d})$, $0 \leq E' < E$ and $f_i(E';\mathbf{d}) < d_i$ implies $f_i(E';\mathbf{d}) < f_i(E;\mathbf{d})$. Quasi-strict monotonicity says that an increase in the estate will benefit all creditors unless they already received their whole claim. Obviously, all strictly monotone rules are also quasi-strictly monotone. In addition, the CEA rule, for example, is quasi-strictly monotone as well.

This property ensures that all the subgame-perfect equilibria in our model are coalition-proof. Moreover, quasi-strict monotonicity is also a necessary condition for coalition-proofness of the subgame-perfect equilibria in our model. That is, if a rule is not quasi-strictly monotone, an example (similar to example 5.2) of a subgame-perfect equilibrium which is not coalition-proof can be constructed. This is formally stated as follows:

THEOREM 2: Let f be a consistent, monotone and supermodular rule. For all bankruptcy problems $(E;\mathbf{d})$, all the subgame-perfect equilibria of $G^f(E;\mathbf{d})$ are coalition-proof if and only if f is quasi-strictly monotone.

Proof of Theorem 2: Sufficiency: The proof is by induction. For 2-creditor problems all Nash equilibria are coalition-proof since in this case the game $G^f(E;\mathbf{d})$ is a constant-sum game. Assume that for all problems with less than n creditors all subgame-perfect equilibria are coalition-proof and let $(E;\mathbf{d})$ be an n-creditor bankruptcy problem. It is sufficient to show that for each subgame-perfect equilibrium σ, no coalition J has a profitable joint deviation that constitutes a Nash equilibrium in $G^f(E;\mathbf{d})|\sigma_{-J}$. First note that the grand coalition has no such deviation since $G^f(E;\mathbf{d})$ is a constant-sum game.

Lemma 5.1: Let σ be a subgame-perfect equilibrium of $G^f(E;\mathbf{d})$ and assume that there exists a coalition J that has a self-enforcing profitable deviation. Then, creditor 1 is a member of J.
Proof: Assume by contradiction that there is a coalition J, in which 1 is not a member, that has

a self-enforcing profitable deviation. Let **x** be the equilibrium offer of creditor 1 according to σ, let **z** be the respective equilibrium outcome and denote by Y and N the set of accepting and rejecting creditors of **x**, respectively, according to σ. By Proposition 1, $w_i=z_i=x_i^*$ for all $i\in N$. Since **x** is an equilibrium offer and since by Theorem 1 the equilibrium outcome is **x***, it must be that for all $i\neq 1$ $x_i\leq x_i^*$ and for creditor 1, $x_1\geq x_1^*$. This implies, by consistency and monotonicity of f that

$$w_i:=f_i(x_1+x_i;(d_1,d_i))\geq x_i^* \text{ for all } i\neq 1. \tag{5.1}$$

Now let $k\in Y$. It must be that $w_k=x_k^*$. For if $w_k>x_k^*$, then $x_k^*<d_k$. But then, quasi-strict monotonicity of f and Theorem 1 imply that k can profitably deviate by saying no. Hence we conclude that

$$w_i=z_i=x_i^* \text{ for all } i\neq 1. \tag{5.2}$$

Now, since $x_i\leq x_i^*$ for all $i\neq 1$ and creditor 1 is not a deviator, it must be that all deviators reject the offer. Thus, by the induction hypothesis and (5.2) it follows that after the deviation all rejecting creditors, including the deviators, will receive $\mathbf{z}'= f(\Sigma_{i\in N'}w_i;\mathbf{d}|N') = f(\Sigma_{i\in N'}x_i^*;\mathbf{d}|N') = \mathbf{x}^*|N'$, where N' is the set of creditors that actually rejected offer **x** in the deviation. This contradicts the assumption that the deviation was profitable.

Lemma 5.2: Let σ be a subgame-perfect equilibrium of $G^f(E;\mathbf{d})$ and assume that there exists a coalition J that has a self-enforcing profitable deviation. Then, creditor 1 is not a member of J.

Proof: Assume by contradiction that there is a coalition J, in which 1 is a member, that has a self-enforcing profitable deviation. Let **x** be 1's offer according to the deviation and let **z** be the outcome following the deviation. Denote by Y and N the set of accepting and rejecting creditors of **x**, respectively, according to σ and denote by Y' and N' the actual set of accepting and rejecting creditors, respectively.

First we claim that $w_i := f_i(x_1+x_i;(d_1,d_i)) \geq x_i$ for all $i \in N'$. By lemma A.3, this claim holds for all $i \in N$ and since the deviation is self-enforcing, by an argument similar to the one used in lemma A.3, it holds for the deviators too.

Further note that there must be a deviating creditor who belongs to N and therefore to Y'. To see this note that if all responding creditors stick to their equilibrium strategies, the proposer will receive no more than x_1^*. But since the proposer is a deviator he receives, after the deviation, more than x_1^*. Since for all rejecting creditors in N', $w_i \geq x_i$, this can only be possible if some of the creditors who were supposed to say No according to the equilibrium strategy say Yes.

All deviating creditors receive more than their equilibrium shares. This implies that $x_i > x_i^*$ for all deviating creditors in Y'. By Proposition 2, lemma 3.3, and the fact that there are some deviating creditors in Y', it follows that for all $i \in Y'$ $x_i \geq x_i^*$. Therefore, $\Sigma_{i \in Y'} x_i > \Sigma_{i \in Y'} x_i^*$. Hence, $x_1 + \Sigma_{i \in N'} x_i < x_1^* + \Sigma_{i \in N'} x_i^*$. Since $x_1 > x_1^*$, N' is non-empty. This together with the fact that $w_i \geq x_i$ for all $i \in N'$ and lemma A.3, implies that 1's payoff is no more than $f_1(x_1 + \Sigma_{i \in N'} x_i; \mathbf{d} | N' \cup \{1\})$ which by consistency and monotonicity cannot exceed x_1^*. This contradicts the assumption that 1 is a member of a coalition with a profitable deviation. This concludes the sufficiency part.

Necessity: Let f be a consistent, monotone and supermodular rule that is not quasi-strictly monotone. Then, there exist bankruptcy problems $(E;\mathbf{d})$ and $(E';\mathbf{d})$ with $E > E'$ and two creditors i and j such that $x_i := f_i(E;\mathbf{d}) = f_i(E';\mathbf{d}) := y_i < d_i$ and $x_j := f_j(E;\mathbf{d}) > f_j(E';\mathbf{d}) := y_j$. By consistency, $x_i = f_i(x_i+x_j;(d_i,d_j)) = f_i(y_i+y_j;(d_i,d_j)) = y_i$ and $x_j = f_j(x_i+x_j;(d_i,d_j)) > f_j(y_i+y_j;(d_i,d_j)) = y_j$. Note that supermodularity implies that $d_j > d_i$.

Let $$\alpha^* := \sup \{f_j(\alpha + x_i;(d_i,d_j)) \mid f_i(\alpha + x_i;(d_i,d_j)) = x_i\}.$$

α^* is the "maximum" amount that creditor j can get in any bankruptcy problem, given that creditor i receives exactly x_i. [α^* is $g(d_j,d_i,x_i)$ in Young, 1987 eq.(9)]. Note that supermodularity of f and lemma 2.1 implies that $\alpha^* \leq d_j-(d_i-x_i) < d_j$.

Pick δ that satisfies $0 < \delta < d_j-\alpha^*$, $\alpha^*-3\delta \geq 0$ and $\alpha^*-\delta \geq y_j$. Let $\beta := f_i(\alpha^*+\delta+x_i;(d_i,d_j))-x_i$. Since $\alpha^*+\delta+x_i > \alpha^*+x_i$, the definition of α^* and monotonicity of f imply that $\beta > 0$. By supermodularity of f, $\beta \leq \delta/2 < \delta$.

Pick a natural number n that satisfies $n\beta > \delta$. Consider the following (n+2)-creditor problem: $(E^*;\mathbf{d}^*) := [nx_i+2(\alpha^*-\delta);(d_j,d_j,d_i,d_i,\ldots,d_i)]$. Note that by construction, the f-just allocation of this problem is, $\mathbf{x}^* = (\alpha^*-\delta, \alpha^*-\delta, x_i, x_i,\ldots,x_i)$. Indeed, since $\alpha^* > \alpha^*-\delta \geq y_j$, it is easy to see that $\mathbf{x}^*$ treats any two creditors f-equally. Now consider the game $G^f(E^*;\mathbf{d}^*)$ and the following subgame-perfect equilibrium σ: The proposer, whose claim is d_j, proposes allocation $\mathbf{x} = (\alpha^*+\delta, \alpha^*-3\delta, x_i, x_i,\ldots,x_i)$. All the other creditors propose the f-just allocation of the underlying bankruptcy problem whenever they propose and respond to any offer according to the algorithm presented in section 4. Note that according to σ the largest creditor will reject the offer and the n small creditors will accept it. If a small creditor rejects offer **x**, he will neither benefit nor lose, but the large rejecting creditor will certainly benefit from the small creditor's deviation. We now show that this equilibrium is not coalition-proof. Consider the following joint deviation by all the small creditors: all of them reject offer **x**. By the choice of n, this deviation will benefit everybody but the proposer. In addition, since the large rejecting creditor will offer the f-just allocation of the bankruptcy problem associated with the next stage of the game, this deviation is self-enforcing. □

6. STRICTLY MONOTONE RULES

The uniqueness result in Theorem 1 is driven by the strict estate monotonicity of rule f with respect to the highest claim (lemma 2.2). This is why ordering the proposers by sizeof claim is important. If the proposer's component f_1 was not strictly monotone in the estate, multiplicity of subgame-perfect equilibrium outcomes might arise, as shown by the following example:

Example 6.1: Let $(E;\mathbf{d})=(100; (100,100,10))$. If f is the constrained equal award rule, $f(E;\mathbf{d})=(45,45,10)$. Suppose the order of proposers is (3,1,2). The reader can check that all the outcomes of the form $(45-a,45+a,10)$ for $-35\leq a\leq 35$ can be supported by SPE of the corresponding extensive form game.

If we confine ourselves to consistent and strictly monotone rules, the main result can be generalized to any order of proposers with a positive claim. In addition, since strictly monotone rules are also quasi-strictly monotone, all subgame-perfect equilibria of the generalized game are coalition-proof. These results are stated formally in Theorem 3. Let Q be a complete order of the set of creditors. The game $G^f_Q(E;\mathbf{d})$ is defined as in Section 4 with a minor change: the proposer is the Q-minimal creditor among the active creditors.

THEOREM 3: Let $(E;\mathbf{d})$ be a bankruptcy problem. Let Q be a complete order of the set of creditors (in which all zero creditors come after the non-zero creditors) and let f be a consistent and strictly monotone rule. The unique subgame-perfect equilibrium outcome of $G^f_Q(E;\mathbf{d})$ is $f(E;\mathbf{d})$. Moreover, all subgame-perfect equilibria of $G^f_Q(E;\mathbf{d})$ are coalition-proof.

Sketch of the Proof: Note that for strictly monotone rules, creditor 1 can be replaced by any non-zero creditor in lemma 3.6, in the definition of R_x and in lemmas A.2 and A.3. This modification, together with the proofs of Theorems 1 and 2 prove Theorem 3. □

7. CONCLUDING REMARKS

By giving a non-cooperative view of a wide class of bankruptcy rules, we believe we have provided additional support to the idea that the property of consistency is useful in the Nash Program for cooperative games. On the other hand, consistency alone, without the assistance of monotonicity, is insufficient to reach the results. Thus, construction of consistency based non-cooperative models that support consistent cooperative solution concepts which are not monotone seems to us a difficult task. Therefore there might be problems in supporting the nucleolus or the Nash bargaining solution on general pies by means of a non-cooperative model.[10]

In the bankruptcy model monotonicity is a natural requirement. Moreover, it is almost implied by consistency: Young (1987, lemma 1) showed that if a rule is symmetric, continuous and consistent, then it is also monotone.

It would be desirable to implement bankruptcy rules with a mechanism that does not require knowledge of the claims. However, it is not clear how to do this. With its demanding requirement of feasibility on and off the equilibrium path, the very notion of mechanism is in trouble when the feasible set is unknown to the planner. This is why the game forms proposed in the literature on implementation of social choice correspondences do not work when applied to our problem. The underlying assumption that this literature makes is that preferences are unknown, but the feasible set is perfectly known to the planner. In our case, preferences are known but the set of allocations (based on the claims) is not. We plan to analyze this problem in a companion paper.

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[10] Hart and Mas-Colell (1992) support the Nash bargaining solution for general pies via a noncooperative model, but their model is not "consistency based".

APPENDIX

LEMMA A.1: Let $(E;\mathbf{d})$ be a bankruptcy problem, let f be a consistent and monotone rule and let $\mathbf{x}$ be an allocation in $(E;\mathbf{d})$. Assume there exists a creditor i such that $i \succsim_{\mathbf{x}} j$ for all creditors j. Then $z_i := E - \Sigma_{j \neq i} f_j(x_i + x_j; (d_i, d_j)) \leq f_i(E;\mathbf{d})$.

Proof: Denote $\mathbf{x}^* = f(E;\mathbf{d})$.

Case 1: For all $j \neq i$, $x_i + x_j \geq x_i^* + x_j^*$. In this case, by consistency and monotonicity $f_j[x_i + x_j; (d_i, d_j)] \geq x_j^*$ for all $j \neq i$. Therefore $z_i := E - \Sigma_{j \neq i} f_j[x_i + x_j; (d_i, d_j)] \leq E - \Sigma_{j \neq i} x_j^* = x_i^*$.

Case 2: There exists $j \neq i$ such that $x_i + x_j < x_i^* + x_j^*$. In this case we have $z_i := E - \Sigma_{j \neq i} f_j(x_i + x_j; (d_i, d_j)) = [x_i + x_j - f_j(x_i + x_j; (d_i, d_j))] - \Sigma_{k \neq i,j}(f_k[x_i + x_k; (d_i, d_k)] - x_k)$. Note that $[x_i + x_j - f_j(x_i + x_j; (d_i, d_j))] = f_i[x_i + x_j; (d_i, d_j)] \leq x_i^*$ where the last inequality follows from consistency and monotonicity of f. Since $i \succsim_{\mathbf{x}} j$, by definition of $\succsim_{\mathbf{x}}$, $\Sigma_{k \neq i,j}[f_k(x_i + x_k; (d_i, d_k)) - x_k] \geq 0$ and therefore $z_i \leq x_i^*$. □

LEMMA A.2: Let $\mathbf{x}$ be an allocation and let i and j be two creditors different from 1. $iR_{\mathbf{x}}j$ implies $j \succsim_{\mathbf{w}} i$.

Proof: If $i=j$, the result is trivial. Otherwise, take the following allocations among i,j and 1: $\mathbf{z} := [f_1(x_1 + x_i; (d_1, d_i)], w_i, w_j)$ and $\mathbf{z}' := [f_1(x_1 + x_j; (d_1, d_j)), w_i, w_j]$.

Assume by contradiction that $i \succ_{\mathbf{w}} j$. This is equivalent to $i \succ_{\mathbf{z}} j$. By construction, $1 \sim_{\mathbf{z}} i$, so by quasi-transitivity of $\succsim_{\mathbf{z}}$, $1 \succsim_{\mathbf{z}} j$. Again by construction, $j \sim_{\mathbf{z}'} 1$. By monotonicity of f, $j \succsim_{\mathbf{z}} 1$. Therefore, $j \sim_{\mathbf{z}} 1$. By lemma 3.6, $\mathbf{z}$ is an f-just allocation among the three creditors, in contradiction to the assumption. □

LEMMA A.3: Let $(E;\mathbf{d})$ be a bankruptcy problem, let f be a consistent, monotone and supermodular rule and let $\mathbf{x}$ be an allocation in $(E;\mathbf{d})$. Assume there exists a set of creditors $N \subset I \backslash \{1\}$ such that $\mathbf{h} := f(\Sigma_{i \in N} w_i;\mathbf{d}|N) \geq \mathbf{x}|N$, then for all i in N, $1 \succeq_{\mathbf{x}} i$.

Proof: Assume by contradiction that there exists a creditor $j \in N$ such that $j \succ_{\mathbf{x}} 1$. Let k be the $R_{\mathbf{x}}$-minimal creditor in N, that is, for all $i \in N$ $w_k - x_k \leq w_i - x_i$. Since $j \succ_{\mathbf{x}} 1$ it follows that $x_k > w_k$. Since $\mathbf{h} \geq \mathbf{x}|N$, we have that $h_k > w_k$. By lemma A.2, $k \succeq_{\mathbf{w}} i$ for all $i \in N$. Since $\mathbf{h}$ is the f-just allocation of $(\Sigma_{i \in N} w_i;\mathbf{d}|N)$, it follows from lemma 3.3 that $h_i \geq w_i$ for all $i \in N$. Therefore we have $\Sigma_{i \in N} h_i > \Sigma_{i \in N} w_i$ which is a contradiction. □

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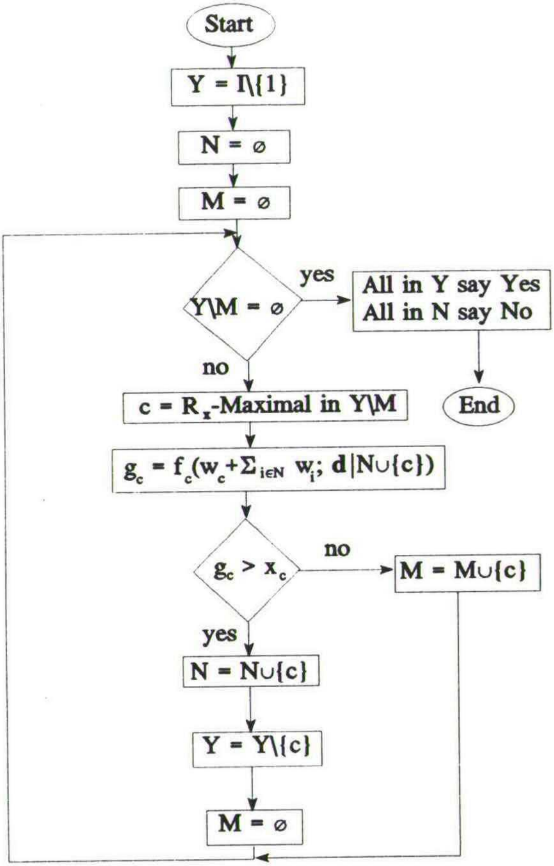
Start
Y = I\{1}
N = ∅
M = ∅
Y\M = ∅
yes
All in Y say Yes
All in N say No
End
no
c = Rx-Maximal in Y\M
gc = fc(wc+Σ i∈N wi; d|N∪{c})
gc > xc
no
M = M∪{c}
yes
N = N∪{c}
Y = Y\{c}
M = ∅

Figure 1

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