THE REFORM AND DESIGN OF COMMODITY TAXES

IN THE PRESENCE OF TAX EVASION WITH

ILLUSTRATIVE EVIDENCE FROM INDIA

BY

RANJAN RAY[*]
Department of Economics
Delhi School of Economics
Delhi University
DELHI - 110007
INDIA

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ABSTRACT

The literature on tax evasion and its implication for optimal tax theory has concentrated on income tax evasion. The issue of commodity tax evasion has received relatively little attention even though it is important in many cases, especially in developing countries. This paper proposes a theory of marginal reform of indirect taxes that recognises the presence of commodity tax evasion. Illustrative evidence from Indian data confirm the sensitivity of the Pareto improving direction of marginal tax changes to alternative a priori assumptions on commodity tax evasion. The theory of marginal reform of commodity taxes is, then, extended to propose a theory of marginal reform of audits and penalties, and several propositions are derived. The underlying theory of tax design is also extended to include income tax design and income tax evasion, and a framework is proposed to allow the simultaneous analyses of both forms of tax evasion, and study of their impact on the "optimal mix" of direct and indirect taxes.

1. **<u>INTRODUCTION</u>**

The literature on tax evasion and its implications for optimal tax theory, pioneered independently by Allingham and Sandmo (1972) and Srinivasan (1973), has, until recently, concentrated almost exclusively on income rather than commodity taxes [see Cowell (1990) for a survey]. The problem of commodity tax evasion has received relatively little attention. Exceptions include Marrelli (1984), Schweizer (1984), Usher (1986), Virmani (1989), Kaplow (1990), Cremer and Gahvari (1992, 1993), and Kesselman (1993). Since in developing countries, e.g. India, indirect taxes play a much larger role than direct taxes, analysis of commodity tax evasion is of greater importance for these economies. To my knowledge, there is hardly any study on the extent of commodity tax evasion in LDCs but, if the evidence on China presented recently by Shu (1992) is any indication, then the issue of commodity tax evasion and its implication for tax policy is a substantive one, and deserves greater attention than it has received in the literature to date.

The limited literature on commodity tax evasion, referred to above, has mostly focussed attention on the production side of the economy. The papers by Schweizer (1984) and Cremer and Gahvari (1992, 1993) are among the very few to examine the welfare aspects of commodity tax evasion from the viewpoint of consumption. The present study is motivated by an attempt to present a theory of marginal commodity tax reforms that recognises the presence of tax evasion. We present illustrative Indian empirical evidence that confirms sensitivity of Pareto improving tax changes to the presence and extent of commodity tax evasion. Although the emphasis in this paper is on tax reform rather than tax design, the paper shows later that the elegant model of optimal commodity taxation under tax evasion, due to Cremer and Gahvari (1993), can be extended to include the case of income tax evasion. We, thus, provide an alternative to the expected utility maximization approach that has characterised much of the income tax evasion literature [see, for example, Sandmo (1981) and, recently, Lemieux, et. al. (1994)].

The plan of this paper is as follows. Section 2 develops the theory of marginal tax reforms, due to Ahmad and Stern (1984), to allow for commodity tax evasion, and extends the principle of Pareto improving tax changes to propose a theory of "marginal audit reforms". Section 3 provides illustrative empirical evidence for India that confirm sensitivity of direction of welfare improving tax changes to tax evasion. This section also contains evidence on the sensitivity of marginal tax reforms to alternative demand systems. The next two sections contain extensions of the optimal commodity tax/tax evasion model. Section 4 introduces a very simple model of fiscal federalism that allows commodity tax, audit probability and penalty for evasion to vary between regions. Section 5 extends the model to allow simultaneous treatment of commodity and income tax evasion. The paper ends on the concluding note of Section 6.

2. **THEORY OF COMMODITY TAX AND AUDIT REFORMS UNDER TAX EVASION**

We consider a competitive economy consisting of n industries producing n different commodities. The production technologies are assumed linear, and c_i is the constant marginal and average cost of good i. Let p, q, $\tilde{t}$ denote (n x 1) vectors of consumer prices, producer prices and nominal commodity taxes. Let A and u represent the n x n fixed input - output coefficients matrix and the n x 1 vector of inputs in the production of various commodities, respectively. If the commodity taxes are specific, we have

$$p = q + \tilde{t} \tag{1}$$

The competitive pricing conditions with commodity taxes are

$$q' = wu' + p'A \tag{2}$$

where w is the wage rate. Substituting (1) into (2), we have

$$p' = wu' (I - A)^{-1} + \tilde{t}' (I - A)^{-1}$$

or, alternatively,

$$p = c + t \tag{3}$$

where c is the n x 1 vector of average costs, and $t' = \tilde{t}' (I - A)^{-1}$ is the (1 x n) row vector of effective taxes (t_i) as in Ahmad and Stern (1984). The government revenue constraint with commodity taxes alone is given as follows:

$$\tilde{t}' Y = R \text{ or, } t' X = R \tag{4}$$

where Y, X represent (n x 1) vectors of gross output and final demands of commodities, respectively. Since in a static Leontief model, $Y = (I - A)^{-1} X$, we have the following relation between nominal and effective commodity taxes:

$$\tilde{t}' \, Y = t' \, X$$

In this section, expenditure and labour supply decisions are assumed separable, and direct taxation is ruled out. Let $v^h (p, \mu^h)$ denote individual h's (h = 1,..,H) indirect utility function where x^h denotes his vector of commodity demand, and $\mu^h = p'x^h$ his aggregate expenditure. Note that, since we are ignoring savings, the terms "income" and "expenditure" will be used synonymously in this paper. Let us define social welfare W over the individuals' indirect utilities, so that it is specified as a function of prices.

$$W (p, \mu) = W [v^1 (p, \mu^1), v^2 (p, \mu^2),..,v^H (p, \mu^H)] \tag{5}$$

If X(p) denotes the aggregate demand vector, then

$$X (p, \mu^1, ..., \mu^H) = \sum_{h=1}^{H} x^h (p, \mu^h) \tag{6}$$

The revenue constraint is given by

$$R = R_0 = \sum_{i=1}^{n} t_i \, X_i \tag{7}$$

where R_o is set exogenously by the authorities. Let us now introduce tax evasion into the model. Following Cremer and Gahvari (1993), let α_i denote the proportion of sales reported in industry i - in other words, $\alpha_i^* (= 1 - \alpha_i)$ is the proportion of actual sales evaded. We assume $0 < \alpha_i < 1$ to avoid the possibility of corner solutions - see Cremer and Gahvari (1992) for an analysis of the case where α can be 0 or 1. Let $G_i (\alpha^*)$, which is increasing and convex in α^*, be the firm's resource

cost of evading unit of output, so that $g_i (\alpha^*) = \alpha_i^* G_i (\alpha^*)$ is the cost of concealment per unit of output. The tax authorities audit a fraction of firms, β_i $(0 \leq \beta_i \leq 1)$. Firms caught cheating pay a fine τ proportional to the amount evaded. We initially assume the fine to be invariant across industries but relax it later.

The firm in industry i maximizes "expected" profits which, per unit of output, is given by

$$\pi_i^e = \{p_i - c_i - g_i(\alpha^*) - [(1 - \beta_i) \alpha_i t_i + \beta_i (t_i + (\tau - 1) (1 - \alpha_i) t_i)]\}$$

$$= p_i - c_i - g_i (\alpha^*) - (\alpha_i + \alpha_i^* \beta_i \tau) t_i \qquad (8)$$

Hence, the producer chooses α_i to minimise

$$g_i (\alpha_i^*) + (\alpha_i + \alpha_i^* \beta_i \tau) t_i \qquad (9)$$

The first and second order conditions for an optimal α_i^* are given by

$$g_i' (\alpha_i^*) = (1 - \beta_i \tau) t_i \qquad (9a)$$

$$g_i'' (\alpha_i^*) > 0 \qquad (9b)$$

where g_i', g_i'' denote the first and second derivatives of g_i with respect to α_i^*. (9a) implies that a necessary condition for interior solution, assumed to hold in this paper, is $\beta_i \tau < 1$. An economic rationale for this condition is as follows:

Expected gain from tax evasion per unit of sales

$$= t_i - (\alpha_i + \alpha_i^* \beta_i \tau) t_i = (1 - \alpha_i) (1 - \beta_i \tau) t_i \qquad (10)$$

Hence, $(1 - \beta_i \tau)$ is the expected gain from tax evasion as a proportion of the sales evaded. We, thus, require $\beta_i \tau < 1$ to ensure an incentive for tax evasion. In the empirical example considered later, we assume $g'(0) = 0$, $g'(1) = \infty$ to ensure interior solution.

Let $t_i^e = (\alpha_i + \alpha_i^* \beta_i \tau) t_i$ denote the "expected" tax payment per unit of output. Let the government's audit cost be denoted by $d(\beta)$ which is an increasing function of the audit probabilities, β.

Tax evasion requires equations (3), (7) to be modified as follows:

$$p = c + g + t^e \tag{11}$$

where both g and t^e, each a $n \times 1$ vector of g_i, t_i^e respectively, are evaluated at the optimal value of the tax evasion vector, α.

$$R = R_0 = \sum_i t_i^e X_i - d(\beta) \tag{12}$$

If λ_i^t $(i = 1,..,n)$ denotes the marginal social cost of raising an extra unit of revenue by taxing the i th. commodity, then

$$\lambda_i^t = - \frac{\partial W}{\partial t_i} \bigg/ \frac{\partial R}{\partial t_i} \tag{13}$$

If $\lambda_i^t \neq \lambda_j^t$, then social welfare can be increased by reducing taxes on commodities with higher λ_i^t s and raising taxes on others - in other words, the scope for welfare improving tax changes exists until the λ_i^t s are all equal, which characterises the state where commodity taxes are optimal. The first order conditions for optimal commodity taxes under tax evasion are given by

$$\sum_{h=1}^{H} \omega^h p_i x_i^h \frac{\partial p_i}{\partial t_i} = \lambda^t [p_i X_i \frac{\partial p_i}{\partial t_i} + \sum_j t_j^e X_j e_{ji} \frac{\partial t_i^e}{\partial t_i}] \tag{14}$$

where ω^h is the welfare weight of household h, and e_{ji} is the uncompensated price

elasticity of demand for j with respect to the price of item i. Note from (10) and (11) that the presence of tax evasion implies $\frac{\delta p_i}{\delta t_i}$, $\frac{\delta t_i^e}{\delta t_i} \neq 1$ unlike in the traditional formulation [see Atkinson and Stiglitz (1980)].

Differentiating (5) with respect to taxes and using Roy's identity, we have

$$\frac{\partial W}{\partial t_i} = - \sum_h \omega^h x_i^h \frac{\partial p_i}{\partial t_i} \tag{15}$$

where $\omega^h = \frac{\partial W}{\partial \mu^h}$ is as defined above. Assuming the social welfare function W to be additive in individual utilities, we have

$$W = \frac{1}{1 - \in} \sum_h v^{h^{1-\in}} \tag{16}$$

where $\in \geq 0$ denotes "inequality aversion". Normalising $\omega^h = 1$ for the poorest household (h = 1), the "social marginal utility of income" for individual h is given by

$$\omega^h = \left(\frac{\mu^h}{\mu^1} \right)^{\in} \tag{17}$$

where μ^1 is the aggregate expenditure of the poorest individual.

Differentiating both sides of the revenue constraint (7) with respect to the tax rate, we obtain

$$\frac{\partial R}{\partial t_i} = X_i \frac{\partial t_i^e}{\partial t_i} + \sum_{k=1}^n t_k^e \frac{\partial X_k}{\partial p_i} \frac{\partial p_i}{\partial t_i} \tag{18}$$

Substituting (15) and (18) into (13), we obtain

$$\lambda_i^t = \frac{\sum\limits_{h=1}^{H} \omega^h\, x_i^h\, \dfrac{\partial p_i}{\partial t_i}}{X_i\dfrac{\partial t_i^e}{\partial t_i} + \sum\limits_{k=1}^{n} t_k^e\, \dfrac{\partial X_k}{\partial p_i}\, \dfrac{\partial p_i}{\partial t_i}}$$

$$= \frac{\sum\limits_{h=1}^{H} \omega^h\, x_i^h}{X_i\left(\dfrac{\partial t_i^e}{\partial t_i} \Big/ \dfrac{\partial p_i}{\partial t_i}\right) + \sum\limits_{k} t_k^e\, \dfrac{\partial X_k}{\partial p_i}} \qquad (19)$$

Now, from equation (11), we have

$$\frac{\partial p_i}{\partial t_i} = (\alpha_i + \alpha_i^* \,\beta_i\, \tau) + (1 - \beta_i\, \tau)\, t_i\, \frac{\partial \alpha_i}{\partial t_i} - g_i'\, \frac{\partial \alpha_i}{\partial t_i} \qquad (20)$$

$$\frac{\partial t_i^e}{\partial t_i} = (\alpha_i + \alpha_i^* \,\beta_i\, \tau) + (1 - \beta_i\, \tau)\, t_i\, \frac{\partial \alpha_i}{\partial t_i} \qquad (21)$$

From the first order condition for optimal α_i^* [eqn. (9a)] we have

$$\frac{\partial \alpha_i}{\partial t_i} = -\frac{(1 - \beta_i\, \tau)}{g_i''} \qquad (22)$$

Substituting (22) into (20), (21), and the resulting expressions into the denominator of (19), and re-arranging terms, we obtain

$$\lambda_i^t = \frac{\sum_h \omega^h x_i^h}{X_i A_i + \sum_k t_k^e \frac{\partial X_k}{\partial p_i}} \tag{23}$$

$$\text{where } A_i = 1 - \frac{(1 - \beta_i \tau) g_i'}{(\alpha_i + \alpha_i^* \beta_i \tau) g_i''} \leq 1 \tag{23a}$$

Alternatively expressed in money expenditure and elasticity terms,

$$\lambda_i^t = \frac{\sum_h \omega^h p_i x_i^h}{E_i A_i + \sum_k (\alpha_k + \alpha_k^* \beta_k \tau) e_{ki} t_k^* E_k} \tag{24}$$

where $E_k = p_k X_k$ is aggregate expenditure on k, $t_k^* = \dfrac{t_k}{p_k}$ is the tax rate, and e_{ki} is the uncompensated price elasticity of k with respect to i.

Now, given the estimates of demand systems, income distributional weights (ω^h), the observed vectors of commodity demand, the tax rates and the magnitude of tax evasion represented by the vector α, we can compute the vector of marginal social costs, λ_i^t, from (24). The ranking of the λ_i^t s indicates the direction of welfare improving marginal tax reforms. This raises the issue of sensitivity of the λ_i^t rankings to (a) the estimated demand system used in calculating the price elasticities (e_{ki}), and (b) the estimate of tax evasion, α_k. Since neither of these behaviourial magnitudes is observed but have to be estimated or assumed, the sensitivity issue is of considerable policy significance. We present some illustrative evidence on Indian expenditure data in the next section.

If there is no "inequality aversion" ($\in = 0$), and the tax rates are uniform ($t_i^* = \emptyset$), then, using Cournot aggregation, (24) becomes

$$\lambda_i^t = \frac{1}{A_i - \emptyset \sum_k \tilde{\alpha}_k \left(\dfrac{E_k \, e_{ki}}{\sum_k E_k \, e_{ki}} \right)} \qquad (25)$$

where $\tilde{\alpha}_k = \alpha_k + \alpha_k^* \beta_k \tau$. Since the r.h.s. varies with i, $\lambda_i^t \neq \lambda_j^t$, and hence uniform tax rates will not be optimal in this case. This marks an important departure from the conventional case of no tax evasion ($A_i = 1$, $\tilde{\alpha}_k = 1$ for all k) and can be stated as the following proposition.

Proposition 1:

In the presence of commodity tax evasion, uniform tax rates will <u>not</u> generally be optimal even for a utilitarian ($\in = 0$) tax authority.

Equations (23a), (25) also tell us that for $\in = 0$, λ_i^t, will be the same across commodities if β_i, α_i and the g_i (.) function are commodity invariant. Alternatively, (24) implies that if there is no cost of sales concealment, i.e. $g_i = 0$, so that $A_i = 1$, then if $\tilde{\alpha}_i \, t_i^*$ is free of i, then, the λ_i^t s will equal one another. We, thus, have the following propositions:

Proposition 2:

If tax evasion (α_i^*), resource cost of evading unit of output (g_i), and audit probability (β_i) are the same for all commodities, then a utilitarian tax authority will find the uniform tax rate to be optimal.

Proposition 3:

If there is no resource cost of evasion, and the tax authority is utilitarian, then the "generalized tax rates", $\tilde{\alpha}_i \, t_i^*$ will be uniform.

$\tilde{\alpha}_i$, which is the ratio of "expected" tax rate (t_i^{*e}) to actual tax rate (t_i^*) is only partly determined by the government's action (namely, via the audit probability β_i). Since $\tilde{\alpha}_i = \alpha_i + \alpha_i^* \beta_i \tau$, hence uniform "generalized tax rate" implies for k, ℓ.

$$\frac{t_k^*}{t_\ell^*} = \frac{\tilde{\alpha}_\ell}{\tilde{\alpha}_k} \equiv \frac{\alpha_\ell + (1 - \alpha_\ell) \beta_\ell \tau}{\alpha_k + (1 - \alpha_k) \beta_k \tau} \tag{26}$$

Thus, if $\alpha_\ell = \alpha_k$, then $\beta_\ell > \beta_k \Rightarrow t_k^* > t_\ell^*$ or, alternatively, if $\beta_\ell = \beta_k$, then $\alpha_\ell > \alpha_k$ $\Rightarrow t_\ell^* < t_k^*$. This is formally stated as the following corollary to Proposition 3.

Proposition 3A:

If there is no resource cost of tax evasion, if rate of tax evasion is same in all industries and if the tax authority is utilitarian, then industries with higher audit probabilities should have lower tax rates; alternatively, if the audit probability is the same for all items, then industries with higher declaration (i.e. lower evasion) will attract lower tax rates.

If the uncompensated cross price elasticities are very small, i.e. $e_{ki} \simeq 0$ for $i \neq k$, so that $e_{ii} = -1$, then (25) implies

$$\lambda_i^t = \frac{\sum \omega^h p_i x_i^h}{E_i (A_i - \tilde{\alpha}_i t_i^*)} \tag{27}$$

If we keep in mind that t_i^* will be optimal if λ_i^t is invariant across commodities $(i = 1, ..., n)$, then (27) leads us to the following proposition:

Proposition 4:

If the uncompensated cross price elasticities are very small, then the optimal commodity tax rate will be given by

$$t_i^* = \frac{1}{\tilde{\alpha}_i} [A_i - \tilde{\phi} \frac{\sum_h \omega^h p_i x_i^h}{E_i}] \tag{28}$$

where $\tilde{\phi}$ (not indexed on i) is determined by a priori specified revenue constraint.

Let us consider the case of the Rawlsian planner for whom only the poorest individual matters i.e. $\omega^1 = 1$, $\omega^h = 0$ for $h \geq 2$. For such a planner, (27) implies

$$\frac{\lambda_\ell^t}{\lambda_k^t} = \frac{(X_k / x_k^1)}{(X_\ell / x_\ell^1)} \frac{(1 - \tilde{\alpha}_k t_k^*)}{(1 - \tilde{\alpha}_\ell t_\ell^*)} \tag{29}$$

where x_k^1, x_ℓ^1 denote the poorest individual's consumption of k, ℓ.

Hence, if $\dfrac{X_k}{x_k^1} = \dfrac{X_\ell}{x_\ell^1}$ and $\tilde{\alpha}_k = \tilde{\alpha}_\ell$, then $t_k^* = t_\ell^*$ implies $\lambda_k^t = \lambda_\ell^t$. This can be formally stated in the form of the following Proposition:

Proposition 5:

If there is no resource cost of tax evasion, if the uncompensated cross price elasticities are so small as to be negligible, if the tax evasion and audit probabilities are identical across commodities, and if the expenditure distribution is such that the ratio of aggregate to minimum consumption is the same for all items, then a Rawlsian planner will consider a uniform tax rate policy to be an optimal one.

This range of conditions is unlikely to hold so that, in practice, the Rawlsian planner will <u>not</u> favour a uniform tax-rate policy. In particular, if we relax only the last condition, then

$$\frac{X_k}{x_k^1} > \frac{X_\ell}{x_\ell^1} \quad => \quad t_k^* > t_\ell^*$$

The sign of $\dfrac{\partial \lambda_i^t}{\partial t_i}$ is of policy interest since the tax rates need to be changed to move the marginal social costs towards one another. The result $\dfrac{\partial \lambda_i^t}{\partial t_i} > 0$ in the conventional case of no tax evasion and which underlies the tax reform exercises of Ahmad and Stern (1984), Murty and Ray (1989) holds in the present case, if we

assume, as we do below, that A_i and the aggregate price responses $(\frac{\partial X_k}{\partial p_i})$ do not change with t_i. We demonstrate this below. Differentiating the r.h.s. of equation (23) and using the first order condition for optimal α_i [eqn. (9a)], we obtain after some rearrangement the following expression

$$\frac{\partial \lambda_i^t}{\partial t_i} = \lambda_i^t \, \tilde{\alpha}_i \, A_i \left[\frac{\sum_h \omega^h \frac{\partial x_i^h}{\partial p_i}}{\sum_h \omega^h \, x_i^h} - \frac{(1 + A_i) \frac{\partial X_i}{\partial p_i}}{X_i \, A_i + \sum_k t_k^e \frac{\partial X_k}{\partial p_i}} \right] \tag{30}$$

Since the terms outside the square bracket are all individually positive, hence ignoring the possibility of Giffen goods,

$$\frac{\partial \lambda_i^t}{\partial t_i} > 0 \text{ if}$$

$$\left[\frac{(1 + A_i) \frac{\partial X_i}{\partial p_i}}{X_i \, A_i + \sum_k t_k^e \frac{\partial X_k}{\partial p_i}} \right] \Big\rangle \left[\frac{\sum_h \omega^h \frac{\partial x_i^h}{\partial p_i}}{\sum_h \omega^h \, x_i^h} \right] \tag{31}$$

Since the numerator on l.h.s. will always be greater than that on r.h.s., condition (31) will be satisfied if

$$X_i \, A_i \; + \; \sum_k \; t_k^e \; \frac{\partial X_k}{\partial p_i} \; < \; \sum_h \; \omega^h \, x_i^h$$

$$\text{i.e.} \; \sum_h \; (A_i \; - \; \omega^h) \; x_i^h \; + \; \sum_k \; t_k^e \; \frac{\partial X_k}{\partial p_i} \; < 0 \qquad (32)$$

Since $A_i \leq 1$, condition (32) implies that if λ_i^t is increasing in t_i in the traditional case of no tax evasion ($A_i = 1$), then $\dfrac{\partial \lambda_i^t}{\partial t_i} > 0$ in the present case as well - in other words, tax evasion does not alter the qualitative nature of the relationship.

In the present context, there are three instruments at the disposal of the authorities - the tax rates (t_i^*), the audit probabilities (β_i) and fine (τ). The principle underlying the theory of marginal tax reform can be extended to a theory of "marginal audit reform". Analogous to λ_i^t, let us define λ_i^β as the marginal social cost of raising an extra unit of revenue by changing the audit probability in industry i. Then,

$$\lambda_i^\beta \; = \; - \; \frac{\partial W}{\partial \beta_i} \Big/ \frac{\partial R}{\partial \beta_i} \qquad (33)$$

The scope for Pareto improving marginal audit reform exists as long as $\lambda_i^\beta \neq \lambda_j^\beta$. The β_i s need to be so altered as to move the λ_i^β s towards one another. If we recall (5), (12), then (33) implies

$$\lambda_i^\beta = \frac{\displaystyle\sum_h \omega^h \, x_i^h \, \frac{\partial p_i}{\partial \beta_i}}{X_i \, \dfrac{\partial t_i^e}{\partial \beta_i} + \displaystyle\sum_k t_k^e \, \frac{\partial X_k}{\partial p_i} \, \frac{\partial p_i}{\partial \beta_i} - d_i}$$

$$= \frac{\displaystyle\sum_h \omega^h \, x_i^h}{X_i \, B_i + \displaystyle\sum_k t_k^e \, \frac{\partial X_k}{\partial p_i} - \dfrac{d_i}{\partial p_i \,/\, \partial \beta_i}} \tag{34}$$

where

$$B_i = \frac{\partial t_i^e}{\partial p_i} \Big/ \frac{\partial p_i}{\partial \beta_i}, \; d_i = \frac{\partial d}{\partial \beta_i}$$

Using equation (11)

$$\frac{\partial p_i}{\partial \beta_i} = g_i' \left(\frac{\partial \alpha_i^*}{\partial \beta_i} \right) + \frac{\partial t_i^e}{\partial \beta_i} \tag{35}$$

Using $\dfrac{\partial \alpha_i^*}{\partial \beta_i} = - \dfrac{t_i \, \tau}{g_i''}$ from (9a), and the definition of t_i^e, (35) yields after re-arrangement,

$$\frac{\partial p_i}{\partial \beta_i} = (1 - \alpha_i) \, \tau \, t_i$$

Substituting this into (34) yields the following expression for λ_i^β

$$\lambda_i^\beta = \frac{\displaystyle\sum_h \omega^h \, p_i \, x_i^h}{E_i \, B_i + \displaystyle\sum_k \left(\alpha_k + \alpha_k^* \, \beta_k \, \tau \right) e_{ki} \, t_k^* \, E_k - \dfrac{d_i}{(1 - \alpha_i) \, \tau \, t_i^*}} \tag{36}$$

From the definition of B_i and using (20), (21) and (35), it can be readily verified that

$$B_i = 1 + \frac{g_i'}{(1 - \alpha_i) \, g_i''} > 1 \qquad (37)$$

(36) implies that $\in = 0$, $B_i = B$, $e_{ki} = 0$ for $k \neq i$, and $d_i = 0$ provide a set of sufficient conditions for optimal audit probabilities to be uniform. This is stated in the following proposition.

Proposition 6:

If the uncompensated cross price elasticities are so small as to be negligible, if the cost of concealment function g_i is such that the ratio of its first and second derivatives (g_i' / g_i'') is directly proportional to concealment (α_i^*), and the audit costs are invariant to audit probabilities, then a Rawlsian planner will consider a uniform audit probability scheme to be the optimal one.

The above proposition serves as a benchmark case for optimally uniform audit probability and suggests that, in general, like for tax rates, a system of identical audit probabilities will not be an optimal one. (36) shows the potential sensitivity of directions of marginal audit reform to the welfare weights, the demand elasticities, the tax-rates, and the slope of the audit cost function.

The preceeding discussion of marginal tax and audit reforms has been based on a separate examination of the tax rates and audit probabilities. The directions of Pareto improving reforms, in either case, point to a state of internal optimality where there is no scope for improvement of either the tax structure or the audit probability scheme when taken in isolation from one another, i.e. $\lambda_k^t = \lambda_\ell^t$, $\lambda_k^\beta = \lambda_\ell^\beta$. The theories of marginal tax and audit reforms can, however, be extended to propose a theory of marginal fiscal reform that recognises the dependence of the tax rates and audit probabilities on one another via the common revenue constraint [eqn. (12)] that

binds them both. The theory of marginal fiscal reform is based on the idea of optimality of the tax and audit systems vis-a-vis one another, i.e. $\lambda_i^t = \lambda_i^\beta$, i = 1, ..., n - in other words, directions of Pareto improving fiscal reforms exist so long as $\lambda_i^t \neq \lambda_i^\beta$. If we recall the expressions for the marginal social costs of tax rates and audit probabilities given by eqns. (24), (36) respectively, then for optimality of a vector of tax rates <u>and</u> audit probabilities, we require

$$E_i \left(B_i - A_i \right) = \frac{d_i}{(1 - \alpha_i) \ \tau \ t_i^*} \tag{38}$$

Let us define

$$\phi_i = \left(\frac{\partial W \ / \ \partial t_i}{\partial R \ / \ \partial t_i} \right) \ / \ \left(\frac{\partial W \ / \ \partial \beta_i}{\partial R \ / \ \partial \beta_i} \right)$$

i.e.

$$\phi_i = \left(\frac{\partial W \ / \ \partial t_i}{\partial W \ / \ \partial \beta_i} \right) \ / \ \left(\frac{\partial R \ / \ \partial t_i}{\partial R \ / \ \partial \beta_i} \right)$$

$$= \frac{MRS_{t_{k,\beta k}}^{W}}{MRS_{t_{k,\beta k}}^{R}} \tag{39}, i = 1.., n$$

ϕ_i is the ratio of the m.r.s between t_i, β_i keeping social welfare constant to that between them keeping total revenue constant. In optimality, $\phi_i = 1$ for all i.

Now $\phi_i \lesseqqgtr 1$ according as $E_i \ A_i \gtreqqless E_i \ B_i - \frac{d_i}{(1 - \alpha_i) \ \tau \ t_i^*}$

i.e. $\frac{d_i}{(1 - \alpha_i) \ \tau \ t_i^*} \gtreqqless E_i \left(B_i - A_i \right)$

This, if we define $H_i = E_i(B_i - A_i) > 0$, then

$$\phi_i \lesseqgtr 1 \text{ according as } \frac{d_i}{\tau} \gtreqless (1 - \alpha_i) \, t_i^* \, H_i \tag{40}$$

Equation (40) gives us the rule for marginal fiscal reform based on the third policy instrument that has not been used so far, namely, the fine for evasion, τ. If we make them industry specific, τ_i, then the rule is as follows:

$$\text{If } \frac{d_i}{\tau_i} > (1 - \alpha_i) \, t_i^* \, H_i, \text{ then raise } \tau_i \tag{41a}$$

$$\text{If } \frac{d_i}{\tau_i} < (1 - \alpha_i) \, t_i^* \, H_i, \text{ then lower } \tau_i \tag{41b}$$

such that $\dfrac{d_i}{\tau_i}$ tends to $(1 - \alpha_i) \, t_i^* \, H_i$

To sum up the discussion in this section, the sequence of steps in the marginal reforms of taxes, audit probabilities and evasion penalties is as follows.

Step 1:

Change taxes t_i such that the λ_i^t s move towards one another

i.e. $\lambda_i^t \to \lambda^t$

Step 2:

Change audit probabilities β_i such that the λ_i^β s move towards one another, i.e. $\lambda_i^\beta \to \lambda^\beta$.

Step 3:

Change the fines, τ_i, such that $\phi_i \to 1$.

3.   TAX REFORM RESULTS FOR INDIA

This section investigates the sensitivity of the λ_i^t rankings, which determine the

direction of marginal tax reforms, to the estimated demand system and to alternative assumptions about commodity tax evasion. The estimated demand systems are LES and its alternative generalizations, namely, the Non linear Generalized CES (NGCES), and the QES due to Howe, Pollak and Wales (1979). NGCES is a new two parameter generalization of the LES proposed and estimated in this paper.

The alternative LES generalizations are expressed in budget share form w_i as follows

NGCES:

$$w_i = \gamma_i \left(\frac{p_i}{\mu}\right)^{\delta} + \frac{b_i \, p_i^{\rho}}{\sum_k b_k p_k^{\rho}} \left[1 - \sum_k \gamma_k \left(\frac{p_k}{\mu}\right)^{\delta} \right]$$

$$\sum_i b_i = 1, \; \delta > 0 \tag{42}$$

QES:

$$w_i = \gamma_i \left(\frac{p_i}{\mu}\right) + b_i \left[1 - \sum_k \gamma_k \left(\frac{p_k}{\mu}\right) \right]$$

$$+ \left[c_i \left(\frac{p_i}{\mu}\right) - b_i \sum_k c_k \left(\frac{p_k}{\mu}\right) \right] \prod_k p_k^{-2b_k} \left[\mu - \sum_k \gamma_k \, p_k \right]^2 \tag{43}$$

Note that $\{ b_i, \gamma_i, \delta, \rho \}$, $\{b_i, \gamma_i, c_i\}$ constitute the parameter sets of NGCES, QES respectively. NGCES specializes to LES if $\rho = 0$, $\delta = 1$ and QES to LES if $c_i = 0$ for all i.

The uncompensated price elasticity formulae for these demand systems, in the base year ($p_i = 1$ for all i), are given as follows.

NGCES:

$$e_{ik} = \frac{1}{w_i} \left[\delta \, \gamma_k \left(\frac{1}{\mu}\right)^{\delta} \left(\delta_{ik} - b_i\right) \right.$$
$$\left. + \rho \, b_i \left(\delta_{ik} - b_k\right) \left\{1 - \sum_j \gamma_j \left(\frac{1}{\mu}\right)\right\} \right] - \delta_{ik}$$

(44)

QES:

$$e_{ik} = \frac{1}{\mu \, w_i} \left[\gamma_k \left(\delta_{ik} - b_i\right) + \left(\mu - \sum_j \gamma_j\right)^2 \left\{ \left(c_i \, \delta_{ik} - b_i \, c_k\right) \right.\right.$$

$$\left.\left. - 2 \, b_k \left(c_i - b_i \sum c_j\right) \right\} - 2 \, \gamma_k \left(c_i - b_i \sum_j c_j\right) \right] - \delta_{ik}$$

(45)

where δ_{ik} is 'kronecker delta'.

The demand systems were estimated using the non linear maximum likelihood procedure of SHAZAM on a 6 item disaggregation of household expenditure from a time series of household budget surveys in India. These are collected by the National Sample Survey Organisation and published as NSS Reports. The present study is based on NSS 7th to 28 rounds (excluding the 26 th and 27 th rounds whose reports were not available) covering the period from 1953-54 to 1973-74 - see Ray (1985) for an analysis of the rural part of this data set using more complex demand systems. For each round, estimates of average per capita expenditures for three groups of population, namely, the poorest 30%, the middle 40% and the richest 30% have been used. The six item disaggregation is as follows. (1) Cereals (2) Milk and Milk Products (3) Other Food (4) Clothing (5) Fuel and light (6) Other Non Food.

The alternative sets of demand parameter estimates, along with their standard errors, are presented in the Appendix. The parameters are generally well determined, and the

estimates confirm significance of the LES generalizations. The aggregate uncompensated price elasticities, which along with the tax rates and welfare weights determine the λ_i^t s, were calculated for the base year using formulae (44, 45). The own price elasticities, presented in Table 1, exhibit considerable variation across demand systems, especially for 'Other Food' and 'Other Non Food' groups of items.

To simplify the tax reform calculations, we set $t_i^* = 0.1$ for all i - in other words, the λ_i^t s indicate Pareto improving directions of marginal tax reform from an assumed initial state of a uniform tax rate of 10% on each of the six groups of items. We additionally require estimates of the resource cost of evasion function, $g_i (\alpha^*)$ [see eqns. (23, 23a)]. The following functional form for $g_i (\alpha^*)$ was carefully chosen to satisfy the priori features mentioned in Section 2. Note that $g_i (0) = 0$ and $g_i (1) = \infty$ for intuitive interpretation as suggested by Cremer and Gahvari (1993).

$$g_i (\alpha^*) = (1 - \alpha_i^{*2})^{-\frac{1}{2}} - 1 \tag{46}$$

(46) implies

$$\frac{g_i^{'} (\alpha^*)}{g_i^{''} (\alpha^*)} = \frac{\alpha_i^* (1 - \alpha_i^{*2})}{1 + 2\alpha_i^{*2}} \tag{47}$$

Corresponding to the assumed values of α_i^* and t_i^*, (9a) with (47) gives us the estimate of $\beta_i \tau$ which, along with the social welfare weights and the estimated price elasticities, determine the λ_i^t s.

Tables 2, 3 provide evidence on the sensitivity of the λ_i^t rankings to ⓐ alternative demand systems, and ⓑ alternative assumptions on tax evasion. It is interesting to note that, for a utilitarian tax authority, the λ_i^t rankings seem much more robust to

changes in specification. This contrasts with the 'optimal tax' evidence for India presented in Ray (1986) - see Decoster and Schokkart (1990, p. 295) for a convincing explanation of this asymmetric result.

4. TAX EVASION AND FISCAL FEDERALISM

The optimal taxation model underlying the theory of marginal reforms of taxes, audits and penalties, outlined in Section 2, can be extended to include some of the key elements of a federal nation. For analytical and notational simplicity, we consider a federal nation with two provinces, and an individual residing in each province. The specific tax paid on item i by the individual consists of θ_i which accrues to the federal authority, and t_i^j which accrues to province j that the individual resides in. Assuming the tax evasion $\left(\alpha_i^*\right)$ and the evasion cost function $g_i(\alpha^*)$ to be invariant to the province where the product is sold, the producer's expected profits per unit output given before by eqn. (8) now becomes.

$$\pi_i^e = p_i - c_i - g_i(\alpha^*) - (\alpha_i + \alpha_i^* \tilde{\beta}_i \tilde{\tau}) \theta_i$$
$$- \sum_{j=1}^{2} \eta_i^j (\alpha_i + \alpha_i^* \beta_i^j \tau^j) t_i^j$$

(48)

where $\tilde{\beta}_i$, β_i^j are the federal and provincial audit probabilities, $\tilde{\tau}$, τ^j the corresponding penalties for evasion, and η_i^j is the share of item i that is sold in province j ($\sum_{j=1}^{2} \eta_i^j = 1$). The producer chooses α_i^* to minimise

$$g_i(\alpha_i^*) + (\alpha_i + \alpha_i^* \tilde{\beta}_i \tilde{\tau}) \theta_i + \sum_{j=1}^{2} \eta_i^j (\alpha_i + \alpha_i^* \beta_i^j \tau^j) t_i^j$$

which implies

$$g_i'(\alpha_i^*) = (1 - \tilde{\beta} \tilde{\tau}) \theta_i + \sum_{j=1}^{2} \eta_i^j (1 - \beta^j \tau^j) t_i^j$$

(49a)

$$g_i''(\alpha_i^*) > 0$$

(49b)

Note that interior solution requires

$$(1 - \tilde{\beta} \, \tilde{\tau}) \, \theta_i + \sum_{j=1}^{2} \eta_i^j \, (1 - \beta^j \, \tau^j) \, t_i^j > 0 \tag{50}$$

The federal and provincial revenue constraints are given by

$$R^0 = \sum_i \theta_i^e \, X_i - \tilde{d} \, (\beta) \tag{51a}$$

$$R^j = \sum_i t_i^{je} \, x_i^j - d^j \, (\beta^j) \tag{51b}$$

where $\theta_i^e = (\alpha_i + \alpha_i^* \, \tilde{\beta}_i \, \tilde{\tau})$, $t_i^{je} = (\alpha_i + \alpha_i^* \, \beta_i^j \, \tau^j) \, t_i^j$.

x_i^j is the consumption of i by the resident in province j, and $X_i = x_i^1 + x_i^2$ is the aggregate consumption of i in the country. The federal and provincial audit costs, d, are increasing functions of the corresponding audit probabilities $(\tilde{\beta}, \beta_i^j)$.

Following Gordon (1983, p. 573)'s federal model of 'fully coordinated decision making', optimal taxation and optimal audit scheme involve maximizing the social welfare function W with respect to $(\theta_i, t_i^j, \tilde{\beta}, \beta_i^j)$ subject to (51a, b).

The Lagrangean expression associated with this problem is

$$L = W \, (p, \mu) + \tilde{\lambda} \left[\sum_k \theta_k^e \, X_k - \tilde{d} \, (\beta) - R^0 \right]$$
$$+ \sum_{j=1}^{2} \lambda^j \left[\sum_{k=1}^{n} \tilde{t}_k^{je} \, x_k^j - d^j \, (\beta^j) - R^j \right] \tag{52}$$

The first order conditions for optimal commodity taxes are given as follows

$$\left(- \sum_{j=1}^{2} \frac{\omega^j}{\tilde{\lambda}} \, x_i^j + \tilde{A}_i X_i \right) + \sum_{j} \sum_{k} \left(\theta_k^e + \frac{\lambda^j}{\tilde{\lambda}} \, t_k^{je} \right) \frac{\partial x_k^i}{\partial p_i} = 0 \qquad (53a)$$

$$\left(- \frac{\omega^j}{\lambda^j} + A_i^j \right) x_i^j + \sum_{k} \left(\frac{\tilde{\lambda}}{\lambda^j} \, \theta_k^e + t_k^{je} \right) \frac{\partial x_k^j}{\partial p_i} = 0 \qquad (53b)$$

where ω^j is the welfare weight of the resident in province j, and $\tilde{A}_i$ A_i^j are given as follows.

$$\tilde{A}_i = 1 - \frac{(1 - \tilde{\beta}_i \, \tilde{\tau}) \, g_i'}{(\alpha_i + \alpha_i^* \, \tilde{\beta}_i \, \tilde{\tau}) \, g_i''} \qquad (54a)$$

$$A_i^j = 1 - \frac{(1 - \beta_i^j \, \tau^j) \, g_i'}{(\alpha_i + \alpha_i^* \, \beta_i^j \, \tau^j) \, g_i''} \qquad (54b)$$

Summing both sides of (53b) over j, subtracting from (53a) and re-arranging, we obtain

$$\frac{1 - \tilde{\beta}_i \, \tilde{\tau}}{\alpha_i + \alpha_i^* \, \tilde{\beta}_i \, \tilde{\tau}} = \frac{g_i''}{g_i'} - \sum_{j} \left[\frac{g_i''}{g_i'} - \frac{(1 - \beta_i^j \, \tau^j)}{\alpha_i + \alpha_i^* \, \beta_i^j \, \tau^j} \right] \left(\frac{\lambda^j}{\tilde{\lambda}} \right) \left(\frac{x_i^j}{X_i} \right) \qquad (55)$$

Re-arranging (55), we obtain an explicit expression for $\tilde{\beta}_i \, \tilde{\tau}$. This can be stated in the form of the following proposition.

Proposition 7:

In the centralized federal model with tax evasion, if commodity taxation is optimal, then the federal instruments of deterrence $(\tilde{\beta}_i, \tilde{\tau})$ can be expressed explicitly in terms of the provincial instruments (β_i^j, τ^j) and the consumption distribution as follows.

$$\tilde{\beta}_i \; \tilde{\tau} \; = \; \frac{1 \; - \; \alpha_i \; k_i \; (\alpha_i)}{1 \; - \; \alpha_i \; k_i \; (\alpha_i) \; + \; k_i \; (\alpha_i)} \tag{56}$$

$$k_i \; = \; \frac{g_i''}{g_i'} \; - \; \sum_j \; \left[\; \frac{g_i''}{g_i'} \; - \; \frac{(1 \; - \; \beta_i^j \; \tau^j)}{\alpha_i \; + \; \alpha_i^* \; \beta_i^j \; \tau^j} \; \right] \left(\frac{\lambda^j}{\tilde{\lambda}} \right) \left(\frac{x_i^j}{X_i} \right) \tag{57}$$

(56) - (57) imply that if $\beta^j \; \tau^j \; = \; \beta\tau$, and $\lambda^j \; = \; \tilde{\lambda}$, then $\beta \; \tau \; = \; \tilde{\beta} \; \tilde{\tau}$. This can be formally stated in the form of the following proposition.

Proposition 8:

If the instruments for deterrence of tax evasion (β^j, τ^j) are identical across the provinces, and if the marginal social cost of raising an extra unit of revenue is the same for each province and the federal authority, then the federal and provincial audit probabilities and fines must coincide.

An alternative representation of (55) is

$$\tilde{\lambda} \; = \; \sum_{j=1}^{2} \; \left(\frac{x_i^j \; \lambda^j}{X_i} \right) \frac{A^j}{\tilde{A}_i} \tag{58}$$

where, let us recall,

$$\tilde{A}_i \; = \; \frac{\partial \theta_i^e \; / \; \partial \theta_i}{\partial p_i \; / \; \partial \theta_i}$$

$$A_i^j \; = \; \frac{\partial t_i^{je} \; / \; \partial t_i^j}{\partial p_i \; / \; \partial t_i^j}$$

We, thus, have the following proposition.

Proposition 9:

In the centralized federal model of tax design with tax evasion, the federal and provincial instruments must satisfy (58) and, in the special case of no tax evasion, so that $\tilde{A}_i = A_i^j = 1$, the marginal social cost of federal revenue is a consumption weighted average of the marginal social costs of provincial revenue.

5. SIMULTANEOUS ANALYSIS OF COMMODITY AND INCOME TAX EVASION

The commodity tax design model underlying the theory of marginal reforms under tax evasion, outlined in Section 2, can be extended to include the design of income tax in the presence of income tax evasion. We consider here only the case of a single individual. The following framework, in the spirit of Dixit and Sandmo (1977) of treating labour services as just another commodity, can provide a useful basis for examining the issue of direct versus indirect taxes in presence of both forms of tax evasion. The absence of such a framework probably explains the lack of numerical evidence on the impact of tax evasion on the 'optimal mix' of direct and indirect taxes. Such evidence is of considerable value to the policy maker, especially in developing countries.

The consumer maximizes his direct utility function $U\ (x,\ \ell)$ defined over the commodity demand vector x and labour supply, ℓ, subject to the following augmented budget constraint.

$$\sum_i p_i\ x_i\ +\ g_\ell\ (\alpha_\ell^*)\ w\ell\ =\ w\ell\ -\ \theta^e\ w\ell \tag{59}$$

where w is the gross wage rate, $\alpha_\ell^*\ (=\ 1\ -\ \alpha_\ell)$ is the proportion of labour income that is evaded or, alternatively, α_ℓ is the proportion of labour income that is declared to the tax authorities. $g_i\ (\alpha_i^*)$ is the resource cost of income concealment that is increasing and convex in α_ℓ^*, and θ^e, the 'expected' tax rate on wage income (wℓ), is related to the actual tax rate (θ) as follows.

$$\theta^e\ =\ (\alpha_\ell\ +\ \alpha_\ell^*\ \beta_\ell\ \tau_\ell)\ \theta \tag{60}$$

β_ℓ is the audit probability of income tax declaration, and τ_ℓ the corresponding

penalty for income tax evasion. A linear income tax scheme (i.e. constant marginal tax rate) is being assumed for simplicity.

The consumer chooses x, ℓ, α_ℓ so as to maximize u (x, ℓ) subject to (59) and (60). Assuming the consumer's decision on work (ℓ) to be separable from that on how much wage earnings to declare (α_ℓ) so that the latter does not enter u (), α_ℓ is chosen so as to <u>maximize</u> $w^e \ell$, where the 'expected net wage rate', $w^e = (1 - \theta^e - g_\ell) w$. The first order condition for optimal α_ℓ^* is given by

$$g_\ell^/(\alpha_\ell^*) = (1 - \beta_\ell \tau_\ell) \theta \tag{61}$$

Interior solution requires

$$1 > \beta_\ell \tau_\ell \tag{61a}$$

The government's optimization problem involves maximization of the augmented indirect utility function with respect to t_i, θ, β_i, β_ℓ, τ, τ_ℓ, (i =, 1, ..,n) subject to the following revenue constraint.

$$R_0 \geq \sum_i t_i^e x_i + \theta^e w\ell - d (\beta, \beta_\ell) \tag{62}$$

where the audit cost d is increasing in the β_i s, β_ℓ.

The optimal commodity and income taxes are given by the following (n+1) first order conditions.

$$\left(- \frac{\omega}{\lambda} + A_i \right) x_i + \sum_k t_k^e \frac{\partial x_k}{\partial p_i} + \theta^e w \frac{\partial \ell}{\partial p_i} = 0 \qquad (63a)$$

$$\left(- \frac{\omega}{\lambda} + A_\ell \right) w \ell + \sum_k t_k^e \frac{\partial x_k}{\partial p_i} + \theta^e w \frac{\partial \ell}{\partial w^e} = 0 \qquad (63b)$$

where ω is now the marginal utility of 'full income', λ is the Lagrangean constraint multiplier, and A_i, A_ℓ are given as follows

$$A_i = \frac{\partial t_i^e / \partial t_i}{\partial p_i / \partial t_i} \qquad (64a)$$

$$A_\ell = \frac{\partial \theta^e / \partial \theta}{\partial w^e / \partial \theta} \qquad (64b)$$

If we denote the inverse of A_i, A_ℓ by A_i^*, A_ℓ^* respectively, then these latter terms can be interpreted as representing the impact of a unit change in expected tax rates t_i^e, θ^e on consumer price (p_i) and net expected wage rate (w^e), respectively, through a change in commodity tax rate (t_i) and income tax rate (θ). Note that in the absence of tax evasion, $A_i = A_i^* = 1$, and $A_\ell = A_\ell^* = - \frac{1}{w}$. Using (61) and, after some re-arrangement, an explicit expression for A_ℓ^* is given as follows.

$$A_\ell^* = \frac{w \left(\dfrac{g_\ell''}{g_\ell'} \right) \left(\alpha_\ell + \alpha_\ell^* \beta_\ell \tau_\ell \right)}{1 - \beta_\ell \tau_\ell - \left(\dfrac{g_\ell''}{g_\ell'} \right) \left(\alpha_\ell + \alpha_\ell^* \beta_\ell \tau_\ell \right)} \qquad (65)$$

Since the parameter A_ℓ^* is of some policy interest, it is useful to state the following implication of (65).

Proposition 10:

$A_\ell^* \gtreqless 1$ according as $\dfrac{(\alpha_\ell + \alpha_\ell^* \beta_\ell \tau_\ell) \, g_\ell'}{(1 - \beta_\ell \tau_\ell) \, g_\ell''} \gtreqless \dfrac{1}{1 + w}$ (66)

The first order conditions [eqns. (63a, b)] provide the estimating equations of the optimal commodity and income tax rates. We can say virtually nothing about their numerical magnitudes in the absence of complex calculations or a-priori assumptions. Equations (63a, b) together imply

$$A_\ell - A_i = \sum_k t_k^e \left[\frac{\partial x_k}{\partial p_i} \frac{1}{x_i} - \frac{\partial x_k}{\partial w^e} \frac{1}{w\ell} \right]$$

$$+ \, \theta^e w \left[\frac{\partial \ell}{\partial p_i} \frac{1}{x_i} - \frac{1}{w\ell} \frac{\partial \ell}{\partial w^e} \right] \quad (67)$$

If we assume the cross price and cross wage responses to be negligible, i.e.

$$\frac{\partial x_i}{\partial p_k} \simeq 0, \; i \neq k, \; \frac{\partial \ell}{\partial p_k} \simeq 0, \; \frac{\partial x_i}{\partial w^e} \simeq 0$$

then (67) implies

$$A_\ell - A_k = \frac{t_k^e}{p_k} e_{kk} - \frac{\theta^e}{w^e} e_{\ell w} \quad (68)$$

(68) can be alternatively expressed as follows.

$$\left(\frac{t_i^e}{p_i^e}\right) \Big/ \left(\frac{\theta^e}{w^e}\right) = \frac{e_{\ell w}}{e_{ii}} \left[1 + \frac{A_\ell - A_i}{\frac{\theta^e}{w^e} e_{\ell w}} \right] \qquad (69)$$

If we have, a-priori, numerical magnitudes on A_ℓ, A_k, then (68) or (69) provides a useful relation between optimal commodity and income taxes in the presence of tax evasion.

TABLE 1

Sensitivity of Own Price Elasticities to Demand System

Item	LES	NGCES	QES
1. Cereals	-.592	-.549	-.702
2. Milk and Milk Products	-.767	-2.742	-.925
3. Other Food	-.660	-1.533	-.729
4. Clothing	-.765	-2.526	-.910
5. Fuel and Light	-.366	-.674	-.447
6. Other Non Food	-.779	-1.615	-.867

TABLE 2[a]

Sensitivity of λ_i^t Rankings to Demand System and to Tax Evasion at $\in = 0$

ITEM	LES		NGCES		QES	
	$\alpha_k^* = 0, k = 1, 2, 3$ $= .2, k = 4, 5, 6$	$\alpha_k^* = .2, k = 1, 2, 3$ $= 0, k = 4, 5, 6$	$\alpha_k^* = 0, k = 1, 2, 3$ $= .2, k = 4, 5, 6$	$\alpha_k^* = .2, k= 1, 2, 3$ $= 0, k = 4, 5, 6$	$\alpha_k^* = 0, k= 1, 2, 3$ $= .2, k = 4, 5, 6$	$\alpha_k^* = .2, k= 1, 2, 3$ $= 0, k = 4, 5, 6$
1. Cereals	1 ↑	4 →	1 ←	2 ←	1 ←	4 →
2. Milk and Milk Products	4 ←	6 →	5 →	6 →	4 ←	6 →
3. Other Food	2 ←	5 →	3 ←	4 →	2 ←	5 →
4. Clothing	5 →	2 ←	6 →	5 →	6 →	3 ←
5. Fuel and Light	3 ←	1 ←	2 ←	1 ←	3 ←	1 ←
6. Other Non Food	6 →	3 ←	4 →	3 ←	5 →	2 ←
$\bar{\lambda}$	1.0925	1.0925	1.2280	1.2280	1.1053	1.1054

[a] An upward arrow indicates $\lambda_i^t < \bar{\lambda}$ so that t_i^* needs to be raised, and a downward arrow indicates the reverse.

TABLE 3[@]

Sensitivity of λ_i^t Rankings to Demand System and to Tax Evasion at $\in$ = 5.0

ITEM	LES		NGCES		QES	
	$\alpha_k^* = 0$, k= 1, 2, 3 = .2, k = 4, 5, 6	$\alpha_k^* = .2$, k= 1, 2, 3 = 0, k = 4, 5, 6	$\alpha_k^* = 0$, k= 1, 2, 3 = .2, k = 4, 5, 6	$\alpha_k^* = .2$, k= 1, 2, 3 = 0, k = 4, 5, 6	$\alpha_k^* = 0$, k= 1, 2, 3 = .2, k = 4, 5, 6	$\alpha_k^* = .2$, k= 1, 2, 3 = 0, k = 4, 5, 6
1. Cereals	6 ↓	6 ↓	5 ↓	6 ↓	6 ↓	6 ↓
2. Milk and Milk Products	1 ↑	1 ↑	1 ↑	1 ↑	1 ↑	1 ↑
3. Other Food	4 ↓	4 ↓	4 ↓	4 ↓	4 ↓	4 ↓
4. Clothing	2 ↑	2 ↑	3 ↑	3 ↑	2 ↑	2 ↑
5. Fuel and Light	5 ↓	5 ↓	6 ↓	5 ↓	5 ↓	5 ↓
6. Other Non Food	3 ↑	3 ↑	2 ↑	2 ↑	3 ↑	3 ↑
$\bar{\lambda}$	.1467	.1474	.1581	.1588	.1482	0.1490

[@] An upward arrow indicates $\lambda_i^t < \bar{\lambda}$ so that t_i^* needs to be raised, and a downward arrow indicates the reverse.

6.   **SUMMARY AND CONCLUSION**

This paper focusses attention on the much neglected issue of commodity tax evasion in the context of marginal tax reforms, and analyses some of the policy implications from the consumers viewpoint in a many person economy. The empirical evidence for India, which is purely illustrative, underlines the importance of the subject of this paper by confirming sensitivity of the Pareto improving direction of tax changes to the extent of commodity tax evasion in the economy. Much of the previous discussion on tax evasion has been concerned with income tax evasion and the recent papers, that do study commodity tax evasion, have mainly concentrated on the production implications of such evasion. The theory of marginal tax reforms needs modification to incorporate tax evasion before applying them in cases, especially the developing countries, where commodity taxes and tax evasion are more important than their direct counterpart. That is the chief motivation of this study. Moreover, the paper extends the theory of reform of commodity taxes to embrace reform of audit probabilities and penalty for evasion. We also derive several propositions that shed some light on the issue.

We show that the analytical model of commodity tax evasion can be extended to include income tax evasion, and provides a convenient framework for the simultaneous analysis of both forms of evasion and study of their impact on the "direct indirect" tax controversy. The numerical and analytical evidence of this paper points to the importance of getting reliable estimates of commodity tax evasion since this is a crucial determinant of tax reform. There is virtually no empirical study on commodity tax evasion, especially in LDCs, where the problem is particularly important. The paper makes a case for such studies. The subject of commodity tax evasion deserves a good deal more attention than it has received to date.

APPENDIX[a]

Demand Parameter Estimates (standard errors in brackets)

LES		NGCES		QES	
Parameter	**Estimate**	**Parameter**	**Estimate**	**Parameter**	**Estimate**
γ_1	4.89 (.64)	γ_1	1.41 (.13)	γ_1	4.37 (.41)
γ_2	.37 (.12)	γ_2	.04 (.05)	γ_2	.12 (1.44)
γ_3	.91 (.24)	γ_3	.28 (.04)	γ_3	.74 (.13)
γ_4	.43 (.09)	γ_4	.04 (.05)	γ_4	.17 (.07)
γ_5	.74 (.06)	γ_5	.20 (.02)	γ_5	.65 (.04)
γ_6	1.08 (.18)	γ_6	.12 (.07)	γ_6	.48 (.10)
b_1	.37 (.18)	b_1	-.03 (.08)	b_1	.48 (.02)
b_2	.06 (.03)	b_2	.31 (.04)	b_2	.10 (.01)
b_3	.17 (.08)	b_3	.16 (.02)	b_3	.19 (.02)
b_4	.03 (.02)	b_4	.33 (.04)	b_4	.07 (.01)
b_5	.03 (.02)	b_5	.005 (.01)	b_5	.05 (.003)
		δ	.39 (.05)	c_1	-.005 (.005)
				c_2	.003 (.001)
		ρ	-2.16 (.46)	c_3	-.0003 (.001)
				c_4	.004 (.0006)
				c_5	.0003 (.0005)
				c_6	.009 (.0009)
k	11	k	13	k	17
LL	804.7416	LL	922.1828	LL	946.7362

[a] k is number of 'free parameters', and LL is Log Likelihood.

REFERENCES

Ahmad, E. and N. Stern (1984), "The theory of reform and Indian indirect taxes", *Journal of Public Economics*, 25, 259-298.

Allingham, M.G. and A. Sandmo (1972), "Income tax evasion: A theoretical analysis", *Journal of Public Economics*, 1, 323-338.

Atkinson, A.B. and J.E. Stiglitz (1980), *Lectures on public economics*, Mc. Graw Hill, New York.

Cowell, F. (1990), *Cheating the government*, MIT Press, Cambridge MA.

Cremer, H. and F. Gahvari (1992), "Tax evasion and the structure of indirect taxes and audit probabilities", *Public Finance/Finance Publiques*, 47, 351-365.

Cremer, H. and F. Gahvari (1993), "Tax evasion and optimal commodity taxation", *Journal of Public Economics*, 50, 261-275.

Decoster, A. and E. Schokkaert (1990), "Tax reform results with different demand systems", *Journal of Public Economics*, 41, 277-296.

Dixit, A. and A. Sandmo (1977), "Some simplified formulae for optimal income taxation", *Scandinavian Journal of Economics*, 79, 417-423.

Gordon, R.H. (1983), "An optimal taxation approach to fiscal federalism", *Quarterly Journal of Economics*, 567-586.

Howe, H., R.A. Pollak and T.J. Wales (1979), "Estimation of the Quadratic Expenditure System", *Econometrica*, 47, 1231-1247.

Kaplow, L. (1990), "Optimal taxation with costly enforcement and evasion", *Journal of Public Economics*, 43, 221-236.

Kesselman, J. (1993), "Evasion effects of changing the tax mix", *The Economic Record*, 69, 131-148.

Lemieux, T., B. Fortin and P. Frechette (1994), "The effect of taxes on labour supply in the underground economy", *American Economic Review*, 84, 231-254.

Marrelli, M. (1984), "On indirect tax evasion", *Journal of Public Economics*, 25, 181-196.

Murty, M.N. and R. Ray (1989), "A Computational Procedure for calculating optimal commodity taxes with illustrative evidence from Indian budget data", *Scandinavian Journal of Economics*, 91, 655-670.

Ray, R., (1985), "A Dynamic Analysis of Expenditure Patterns in Rural India", *Journal of Development Economics*, 19, 283-297.

Ray, R.(1986), "Sensitivity of 'optimal' commodity tax rates to alternative demand functional forms: an econometric case study of India", *Journal of Public Economics*, 31, 253-268.

Sandmo, A. (1981), "Income tax evasion, labour supply, and the equity-efficiency trade off", *Journal of Public Economics*, 16, 265-288.

Schweitzer, V. (1984), "Welfare analysis of excise tax evasion", *Journal of Institutional and Theoretical economics*, 140, 247-258.

Shu, W. (1992), "Income Tax Evasion and Enforcement: a purely theoretical account of Chinese income tax system", *Public Finance/Finance Publiques*, 47, 287-302.

Srinivasan, T.N. (1973), "Tax evasion: a model", *Journal of Public Economics*, 2, 339-346.

Usher, D. (1986), "Tax evasion and the marginal cost of public funds", *Economic Inquiry*, 24, 563-586.

Virmani, A. (1989), "Indirect tax evasion and production efficiency", *Journal of Public Economics*, 39, 223-237.