

Localized competition, multimarket operation and collusive behavior

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Abstract

This paper studies collusive behavior in a repeated oligopoly model with localized competition, also reinterpreted as a model of multimarket operation. Private information about the rivals' past actions naturally arises from these product market structures. The resulting communication problems imply that firms should not adopt strategies with too severe punishments. Infinite grim punishments may be too severe, for large discount factors. The standard stick-and-carrot punishments from the perfect public information model are always too severe, for all discount factors. Modified stick-and-carrot punishments can still be used, though for a smaller range of discount factors than the standard stick-and-carrot punishments.

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1 Introduction

As has been recognized at least since Stigler (1964), the essential problem of enforcing collusive agreements in an oligopoly consists of detecting and punishing past deviations. In a repeated game framework the recent oligopoly literature has explored several aspects of this enforcement problem in detail. The present paper aims to link the enforcement problem explicitly to the structure of the product market. I consider an industry with localized competition, also reinterpreted as an industry with multimarket operation. Problems of private information about the rival firms' past actions endogenously arise in such product markets. It is shown that these information problems create serious difficulties in both detecting and punishing deviations from a collusive agreement.

In the absence of a formal information exchange mechanism, such as a trade association, firms cannot easily observe their rivals' past actions. In some product markets they can nevertheless make sufficient *inferences* about these actions, based on their own past realized profits. This is the case in a homogeneous goods oligopoly with certain demand, in which all firms compete directly with each other. In such a simple market an unexpectedly low price or high output by one of the firms results in a profit reduction to *each* other firm, from which the unexpected action can be inferred. Most product markets, however, are more complicated and contain aspects of *localized competition* is localized: different firms may compete directly with different sets of rivals. The firms' profits then do not depend on the actions of all firms in the market. As a result, the possibility arises that firms make different inferences about their rivals' past actions. In particular, they may make different inferences about the incidence of cheating in a collusive agreement. This creates the following *communication problem*. A firm who has privately observed cheating may not want to punish the cheater: this would avoid a further punishment and allow the firm to at least continue to collude with the other firms, who did not observe the cheating.

In a repeated game model that captures the essential properties of localized competition, I analyze this communication problem in detail. For simplicity, I focus on symmetric punishments, using a symmetric model of demand.¹ The results of the analysis

¹On the one hand, the used model of demand is more general than Abreu's (1986) model because it relaxes the homogeneous goods assumption. On the other hand, the used model is more specific, because it assumes a linear demand structure.

stress the importance of appropriately chosen punishments for collusion to be sustainable. When firms are patient, i.e. for large discount factors, some punishments may be too severe to induce communication of privately observed cheating. In particular, it is shown that the frequently studied infinite grim punishments, as in Friedman (1971), are too severe for sufficiently large discount factors. For this range of discount factors, *less severe* punishments then become necessary to sustain collusion, for example finite grim punishments. In addition, it is shown that the "standard" (symmetric) stick-and-carrot punishments of the perfect public information model, as in Abreu (1986, 1988), are *always* too severe, for *all* discount factors. This result suggests that the communication problem due to localized competition hinders collusion in a nontrivial sense.² To obtain more concreteness about the reduced collusive possibilities, I construct "modified" stick-and-carrot punishments, which take into account the communication problem and are correspondingly less severe than Abreu's standard stick-and-carrot punishments. These modified punishments sustain collusion for a larger range of discount factors than Friedman's grim punishments. However, they cannot sustain collusion for the entire range of discount factors covered by Abreu's standard stick-and-carrot punishments in the public information model.

The analysis is further reinterpreted in a model of *multimarket operation absent multimarket contact*, a situation where a multimarket firm meets different single-market competitors in different markets. Multimarket operation absent multimarket contact may then hinder collusion due to the same kind of communication problems. This is in sharp contrast with the established theoretical results on *multimarket contact*, a situation where two or more multimarket firms meet each other in several markets. Bernheim and Whinston (1990) have shown that multimarket contact may facilitate collusion, due to a pooling of the firms' incentive constraints. Because of crucial communication problems their results therefore do not turn out to generalize to economic situations with multimarket operation but without multimarket contact. It would be interesting to investigate whether this theoretical implication can be verified in the growing empirical work on multimarket competition.³

²This is based on Abreu's (1986) Theorem 14, stating that there are no other (symmetric) punishments than the standard stick-and-carrot punishments that can enforce a better collusive outcome in the public information model.

³More references on the theoretical and empirical multimarket literature are given in section 5.

The results of the analysis explain the presence of various information exchange mechanisms in industries with a complicated market structure. Such mechanisms make the firms' private information about past actions public, and may hence eliminate the communication problem. An interesting example of an information exchange mechanism in practice is the ATP computer system, commonly owned by a group of airlines in the US. The ATP system provides instant information of every price change (or future price change) to each of the subscribing airlines. See Barla (1993) for a more detailed discussion. If the sole reason for these information exchange mechanisms is the elimination of the communication problem, then the above analysis makes a strong case for making them *per se* illegal. The debate on information exchange in antitrust is still going on, both in the US and in Europe. Scherer and Ross (1990, p. 347-352) discuss the US antitrust debate, based on several past legal cases. Philips (1988) discusses the "woodpulp decision", made by the European Commission in 1984.

Much of the previous literature on collusive behavior in a repeated oligopoly has assumed *perfect public* information about the firms' past actions.⁴ A firm can obtain perfect information about the relevant aspects of its competitors' past actions either directly or indirectly through information on the past realized prices or its own past realized profits. The most notable exception to the perfect public information literature is Green and Porter (1984). They allow for random aggregate demand shocks so that firms cannot infer whether a low realized price or profit was due to a low demand or due to cheating by one of the firms. In the Green-Porter model, however, all firms receive the same information. Competition is nonlocalized, so that an unexpectedly low realized price or profit is shared by all firms. In sum, the Green-Porter model and its subsequent generalizations⁵ are models of *imperfect public* information. In contrast, as explained above, the present model with localized competition, and its reinterpretation as a model of multimarket operation absent multimarket contact, generates *private* information about the firms' past actions. To the best of my knowledge, this is the first model in which such problems of private information arise naturally from the product market structure. In this sense the results contribute to the growing Industrial Organization

⁴Because firms move simultaneously during each period, and consequently do not observe each others' *current* actions, these games are sometimes called games of *almost* perfect information.

⁵See especially Fudenberg, Levine and Maskin (1994).

literature, formalizing the links between market structure, conduct and performance.

A few game-theoretic papers have already addressed some aspects of private information about the players' actions in a repeated game. Fudenberg and Levine (1991) assume players are only epsilon-rational. If players are patient, they may not mind behaving suboptimally for some time.⁶ Ben-Porath and Kahneman (1993) and Kandori and Matsushima (1994) consider rational players. However, these papers assume that players can make explicit public announcements at certain periods in time about their obtained private information. In contrast with these previous contributions, the present paper considers rational players who are not able to make explicit public announcements about their own private information. Furthermore, plausible economic environments are considered from which the private information problems naturally emerge.

Though the paper focuses on the sustainability of one specific outcome, the joint-profit maximizing outcome, it seems that the analysis would readily extend to the sustainability of the whole set of outcomes for which all firms obtain more than their one-shot Nash profits. This then implies a Nash-threat folk theorem. It is not so clear, however, whether a minmax folk theorem would still apply.

Section 2 presents the model of localized competition. Section 3 provides some preliminary observations and section 4 obtains the main results. In section 5 the model is reinterpreted as a model of multimarket operation. Conclusions and suggested extensions follow in section 6.

2 The model of localized competition

There are several ways to model localized competition. The best-known model is the Hotelling model with price-setting firms. In this model firms are located on a line, representing a one-dimensional product space in which firms are differentiated. With three firms located at different points on the line competition is localized: although the firm in the middle competes directly with both the firm to the left and the right, the left and right firm do not directly compete with each other. Localized competition is

⁶Furthermore, Fudenberg and Levine only consider informationally connected games, i.e. games in which a message can always be passed from one player to another, regardless of which single player might try to interfere. In models of localized competition, in particular in the model developed in this paper, this property may not hold.

also present in more complicated spatial models with price competition. It is possible to allow for more than three firms, possibly located on a circle rather than a line. The product space may consist of more than one dimension, provided there are enough firms relative to the number of differentiating dimensions.⁷

In this paper I will consider a nonspatial setting to model localized competition. Demand is determined by a representative consumer with preferences for three goods. The preferences for two of these goods are independent. There are three firms who compete in quantities. This specific model generates localized competition of a similar form as the Hotelling triopoly model with price-setting firms. It has, however, two main advantages for the purposes of this paper. First, it is analytically more tractable because of some convenient symmetry properties and because of the continuity of the reaction functions.⁸ Second, and more importantly, it allows for a close comparison with the literature on collusion in oligopolies with perfect public information, in particular with Abreu's results in a homogenous goods oligopoly and with Deneckere's (1983) results in a differentiated goods oligopoly. Despite the advantages of the used model, it remains an interesting future research topic to consider the robustness of the results in alternative models that capture aspects of localized competition.

2.1 Demand and profits

There is a representative consumer with the following quadratic utility function:

$$U(q_0, q_1, q_2, q_3) = \alpha (q_1 + 2q_2 + q_3) - \frac{\beta}{2} (q_1^2 + 2q_2^2 + q_3^2) - \gamma (q_1 q_2 + q_2 q_3) + q_0$$

where q_i , $i = 0, 1, 2, 3$, is the quantity consumed of good i . Assume $\alpha > 0$, $\beta > 0$, $\gamma \neq 0$ and $\beta \geq \gamma \geq -\beta$. Good 0 is the outside good: the utility from its consumption is independent of the consumption of all other goods. The utilities from the consumption of good 1 and good 3 are independent of each other. The utilities from the consumption of good 1 and 2, and of good 2 and 3 are, however, interdependent: good 1 and 2, and good 2 and 3, are substitutes for $\gamma > 0$, they are complements for $\gamma < 0$. Solving the representative consumer's utility maximization problem subject to the budget constraint

⁷Anderson, de Palma and Thisse (1989) provide sufficient conditions on the dimension of the product space for competition to be localized.

⁸Spatial models with price-setting firms typically generate discontinuous reaction functions, making the analysis more tedious.

$y \geq \sum_{i=0}^3 p_i q_i$, where y is income and p_i is the price of good i , $p_0 = 1$, gives the inverse demand equations for good 1, 2 and 3:

$$\begin{aligned} p_i &= \alpha - \beta q_i - \gamma q_2, & i \neq 2 \\ p_2 &= 2[\alpha - \beta q_2 - \gamma(q_1 + q_3)/2]. \end{aligned}$$

There are three firms. Each firm i chooses to produce the quantity of a single good i , q_i , given the quantity chosen by the other firms. Normalizing marginal cost to zero and ignoring fixed costs, firm i 's realized profit π_i is given by

$$\begin{aligned} \pi_i &= \pi_i(q_i, q_2) \equiv (\alpha - \beta q_i - \gamma q_2) q_i \\ \pi_2 &= \pi_2(q_1, q_2, q_3) \equiv 2[\alpha - \beta q_2 - \gamma(q_1 + q_3)/2] q_2. \end{aligned} \tag{1}$$

These profit equations reveal that competition is localized in the following sense: firm i 's profit, $i \neq 2$, directly depends only on its own quantity choice and the quantity of firm 2, and not on the quantity choice of the third firm. In other words, firm 1 and firm 3 do not directly compete with each other; they only compete indirectly through the interaction with the common competitor firm 2. This structure of localized competition is similar to the standard Hotelling model with price-setting firms. Using this analogy, one may call firms 1 and 2, and firms 2 and 3, "neighboring" firms, whereas firms 1 and 3 are "non-neighbors". This analogy should not, of course, be taken literally.

Despite the localized nature of competition, the specific functional form of the profit equations generates some convenient symmetry properties. These properties will be exploited below.

2.2 Information and beliefs

In the one-shot game firms interact for only one period. A strategy for firm i is then simply a quantity choice independent of the past. In the infinite horizon game, firms interact for an infinite number of periods. In every period t each firm may then condition its own quantity choice on the observed sequence of the firms' past quantity choices. This fact lies at the basis of potential information problems in the infinite horizon game.

Define firm i 's *history* h_i^{t-1} at the end of period $t - 1$, $t \geq 2$, as the sequence of its past quantity choices, $h_i^{t-1} \equiv (q_i^1, \dots, q_i^{t-1})$. Firm i 's *null-history* at the end of period 0 is defined as $h_i^0 \equiv \emptyset$. At the beginning of period t , each firm i observes its own history

h_i^{t-1} and the sequence of its own realized profits $(\pi_i^1, \dots, \pi_i^{t-1})$. From this information each firm can make some inferences about the other firms' history. Due to the localized structure of competition, these inferences are necessarily imperfect. Upon observing their own history and their own past realized profits, firm 1 and firm 3 can infer firm 2's history, with whom they directly compete, but they cannot infer each others' history. In contrast, firm 2 can infer a sequence $(q_1^1 + q_3^1, \dots, q_1^{t-1} + q_3^{t-1})$. Assume for simplicity that firm 2 can identify the quantity q_1^τ from q_3^τ .⁹ This then gives the following information for each firm i .

- At the beginning of period $t \geq 1$, firm i 's *observed history*, $i \neq 2$, is (h_i^{t-1}, h_2^{t-1}) .
- At the beginning of period $t \geq 1$, firm 2's *observed history* is $(h_1^{t-1}, h_2^{t-1}, h_3^{t-1})$.

Hence, firm 2 who is directly competing with all other firms, is perfectly informed about the history of all firms. Firm 1 and 3 are only imperfectly informed. Clearly, the localized structure of competition generates this private information structure. It is in sharp contrast with the standard approach with public information.

Given their imperfect information, firm 1 and firm 3 must have a *belief* about each other's history at each period t and after every possible observed history. The equilibrium concept will require these beliefs to be consistent with the strategies.

2.3 Strategies and machines

A strategy for firm i is a sequence of quantity functions $\{f_i^t\}_{t=1}^\infty$, where each function f_i^t assigns a quantity q_i^t to every possible observed history. In particular, for firm i , $i \neq 2$, $f_i^t(h_i^{t-1}, h_2^{t-1})$ is the quantity to be chosen in period t when its observed history is (h_i^{t-1}, h_2^{t-1}) ; for firm 2, $f_2^t(h_1^{t-1}, h_2^{t-1}, h_3^{t-1})$ is the quantity to be chosen in period t when its observed history is $(h_1^{t-1}, h_2^{t-1}, h_3^{t-1})$. Assuming firms discount the future at a common factor δ , the (normalized) present value of each firm i 's continuation profit from following its strategy, given that the other firms follow their strategy, is at any period t

⁹This can be justified if firm 2 can decide in each period whether or not to spend an amount ϵ upon observing a certain (unexpected) sequence $(q_1^1 + q_3^1, \dots, q_1^{t-1} + q_3^{t-1})$ in order to identify $q_1^\tau + q_3^\tau$ for some τ . In the Hotelling model of localized competition, firm 2 could be assumed to be able to inspect the actual customers that he served in the past in order to identify the prices charged by the left neighbor from the prices observed by the right neighbor.

after every observed history:

$$(1 - \delta) \sum_{\tau=t}^{\infty} \delta^{\tau-t} \pi_i [f_i^{\tau}(h_i^{\tau}, h_2^{\tau}), f_2^{\tau}(h_1^{\tau}, h_2^{\tau}, h_3^{\tau})] , i \neq 2,$$

$$(1 - \delta) \sum_{\tau=t}^{\infty} \delta^{\tau-t} \pi_2 [f_1^{\tau}(h_1^{\tau}, h_2^{\tau}), f_2^{\tau}(h_1^{\tau}, h_2^{\tau}, h_3^{\tau}), f_3^{\tau}(h_3^{\tau}, h_2^{\tau})].$$

The normalization factor $(1 - \delta)$ serves to measure the one-shot profits and the continuation profits in the same units.

It will be more convenient to use the language of machines to describe strategies in the repeated game. Each firm i 's machine is intended as an abstraction of the process by which firm i executes its strategy $\{f_i^t\}_{t=1}^{\infty}$.¹⁰ Firm i 's machine has the following components:

- a set of states $S_i = \{c_i\} \cup M_i \cup \{p_i^1, \dots, p_i^J\}$; $M_i = \{m_i^i\}$, $i \neq 2$; $M_2 = \{m_2^1, m_2^2, m_2^3\}$;
- an initial state $c_i \in S_i$;
- a quantity function $f_i : S_i \rightarrow \mathbb{R}_+$, assigning a quantity to each state s_i ;
- a transition function $g_i : S_i \times \mathbb{R}_+^2 \rightarrow S_i$, $i \neq 2$, assigning a state to each vector (s_i, q_1, q_2)
 $g_2 : S_2 \times \mathbb{R}_+^3 \rightarrow S_2$, assigning a state to each vector (s_2, q_1, q_2, q_3) .

Call the state c_i the collusive state of firm i 's machine; the state m_i^j the j -th communication state of firm i 's machine, and the state p_i^j the j -th punishment state of firm i 's machine. The reason for this terminology will become clear below.

Each firm i 's machine induces a sequence of states and a sequence of quantity choices. The actual state of firm i 's machine, $i \neq 2$, at period t is given by the transition function $s_i^t = g_i[s_i^{t-1}, (q_i^{t-1}, q_2^{t-1})]$, and similarly for firm 2. Hence, the actual state s_i^t recursively summarizes the "relevant" elements of firm i 's observed history. The quantity to be chosen in period t is then $f_i(s_i^t)$, i.e. it is based on the relevant elements of firm i 's observed history. In this sense, firm i 's machine executes its strategy. In principle, machines and their resulting strategies can be very complex, containing a large set of possible states, and sophisticated quantity and transition functions. In practice, however, some restrictions on the firms' machines can be imposed. In this paper I explicitly impose restrictions on the machines' transitions *from* the collusive and the communication states.

¹⁰See Osborne and Rubinstein (1994) for an overview.

This approach can be viewed as a way of modelling the firms' bounded rationality. Alternatively, it may be viewed as a first step towards a better understanding of the collusion problem when not all firms' actions are observed due to localized competition. Abbreviate firm i 's quantity choice when it is in the collusive state by $q_i^c \equiv f_i(c_i)$. The transitions from the collusive and communication states are now restricted as follows:

Transitions from c_i , for all i :

Firm i , $i \neq 2$: stay in c_i unless

- only $q_i \neq q_i^c$, in which case move to m_i^i
- at least $q_2 \neq q_2^c$, in which case move to p_i^1 .

Firm 2: stay in c_2 unless

- only $q_i \neq q_i^c$, $i \neq 2$, in which case move to m_2^i
- only $q_1 \neq q_1^c$ and $q_3 \neq q_3^c$, in which case move to m_2^2
- at least $q_2 \neq q_2^c$, in which case move to p_i^1 .

Transitions from m_i^j , for all i, j :

Firms 1,2 and 3: stay in m_i^j unless at least $q_2 \neq q_2^c$, in which case move to p_i^1 .

These transitions are illustrated in Figure 1. In contrast with models of public information, a firm i who is in the collusive state cannot always immediately move to the first punishment state p_i^1 after one of the firms has deviated from its specified collusive quantity. This is only possible in case the deviation occurred by firm 2, who is publicly observed. In case firm 1 or firm 3 deviated, an intermediate communication state is required.¹¹ A firm who is in the communication state remains in this state until firm 2 has chosen a quantity different from its collusive quantity q_2^c , in which case all firms move to the first punishment state.

¹¹Note that firm 2 may go to three different communication states m_2^1, m_2^3 and m_2^2 , depending on whether firm 1, firm 3 or both deviated.

3 Preliminary observations

3.1 One-shot best-response functions

An important element in the analysis of equilibria in both the one-shot game and the infinite horizon game is the derivation of the firms' one-shot best-response functions. Given the profit functions derived above, firm i 's *unilateral* one-shot best-response quantity is:

$$\begin{aligned} q_i &= q_i^U(q_2) && \equiv \frac{(\alpha - \gamma q_2)}{2\beta} && , \quad i \neq 2 \\ q_2 &= q_2^U(q_1, q_3) && \equiv \frac{\alpha - \gamma(q_1 + q_3)/2}{2\beta} . \end{aligned} \quad (2)$$

Due to the localized structure of competition firm i 's best-response quantity, $i \neq 2$, does only depend on firm 2's quantity. Firm 2's best-response quantity, in contrast, depends on both firm 1's and firm 3's quantity.

Firm i and firm 2's simultaneous, or *bilateral*, one-shot best-response quantities to the third firm j 's quantity q_j , $j \neq i, 2$, are given by the solution to the system $q_i = q_i^U(q_2)$, and $q_2 = q_2^U(q_1, q_3)$:

$$\begin{aligned} q_i &= q_i^B(q_j) && \equiv \frac{2\alpha(2\beta - \gamma) + \gamma^2 q_j}{8\beta^2 - \gamma^2} && , \quad i \neq 2, \quad j \neq i, 2 \\ q_2 &= q_2^B(q_j) && \equiv \frac{\alpha(4\beta - \gamma) - 2\gamma\beta q_j}{8\beta^2 - \gamma^2} && , \quad j \neq i, 2. \end{aligned} \quad (3)$$

Finally, firm 1, firm 2 and firm 3's *trilateral* one-shot best-response quantities to each other are given by the solution to $q_1 = q_1^U(q_2)$, $q_2 = q_2^U(q_1, q_3)$ and $q_3 = q_3^U(q_2)$:

$$q_i^T = q^T \equiv \frac{\alpha}{2\beta + \gamma} && , \quad \text{for each } i. \quad (4)$$

The solution (q^T, q^T, q^T) is simply the unique Cournot-Nash equilibrium for the one-shot game. Notice that this solution is symmetric in the present model.¹²

3.2 Necessary equilibrium conditions

In the one-shot game an appropriate solution concept is the Cournot-Nash equilibrium. In the infinite horizon game with private information developed above, an appropriate

¹²The intuition for the symmetry between firm 1 and firm 3 is straightforward. The additional symmetry with firm 2 follows from an exact balance of two forces. On the one hand, firm 2 has a higher "intrinsic" demand, inducing it to set higher quantities. On the other hand, firm 2 has two competitors instead of just one, inducing it to set lower quantities.

refinement is the concept of perfect Bayesian equilibrium [Fudenberg and Tirole (1991)]. This solution concept requires a consistent specification of firm 1 and 3's beliefs about each others' history, at every period t after every observed history. A specification of beliefs is consistent if it can be derived from the strategies using Bayes' rule whenever possible. The following simple specification of beliefs is consistent with the strategies in this paper, and will henceforth be used: at each period t after every observed history, firm 1 (firm 3) believes that firm 3 (firm 1) has chosen the quantity prescribed by its equilibrium strategy.¹³

A perfect Bayesian equilibrium then results if no firm has an incentive to deviate from its strategy in any state, given the other firms' strategies and given its consistent beliefs. Using the transitions from c_i and m_i^j described above, and using w_i to denote firm i 's (normalized) continuation profit once play has moved to the first punishment state p_i^1 , it is already possible to obtain the firms' no-deviation constraints in the collusive state and in the communication states. Call these constraints briefly the collusion and communication constraints. These constraints are necessary for all punishments considered in this paper. Observation 1 shows that the collusion and the communication constraints can hold only if the firms choose their one-shot best-response quantity in the communication state. More precisely:

OBSERVATION 1. Given the transitions from c_i and m_i^j described above, a perfect Bayesian equilibrium obtains only if:

$$\begin{aligned} f_i(m_i^i) &= q_i^B(q_j^c), \quad i \neq 2, j \neq i, 2 \\ f_2(m_2^i) &= q_2^B(q_j^c), \quad i \neq 2, j \neq i, 2; \quad f_2(m_2^2) = q_2^U \left[q_1^B(q_3^c), q_3^B(q_1^c) \right]. \end{aligned}$$

PROOF: See Appendix.

The intuition for this observation is simple. First, in equilibrium firm 2 does not choose q_2^c in its communication state, because then firm i , $i \neq 2$, would always have an incentive

¹³Consider firm 1's period t beliefs after an observed history (h_1^{t-1}, h_2^{t-1}) . Suppose first that firm 1 did not observe a period $t-1$ defection by firm 2 from its equilibrium strategy. It can then use Bayes' rule to infer that firm 3's period $t-1$ quantity choice was according to its equilibrium strategy. Suppose next that firm 1 did observe a period $t-1$ defection by firm 2 from its equilibrium strategy. It can then have any belief about firm 3's period $t-1$ quantity choice. Specify these beliefs such that firm 1 believes firm 3 did not defect in period $t-1$. A similar specification of beliefs applies to firm 3.

to deviate in its collusive state (as this deviation would never be "communicated"). Next, given that firm 2 chooses a quantity different from q_2^c in the communication states, all firms in their communication states expect the punishment state to obtain independent of their specific quantity choice, so that the one-shot best-response quantity choice remains the single candidate equilibrium quantity choice in the communication state. It is now possible to write the collusion and communication constraints as follows, after some rearrangements.

Collusion constraints for firm 1, 2 and 3:

$$\begin{aligned} c_i, i \neq 2 : \quad \pi_i(q_i^c, q_2^c) &\geq (1 - \delta)\pi_i[q_i^U(q_2^c), q_2^c] + \delta(1 - \delta)\pi_i[q_i^B(q_j^c), q_2^B(q_j^c)] + \delta^2 w_i \\ c_2 : \quad \pi_2(q_1^c, q_2^c, q_3^c) &\geq (1 - \delta)\pi_2[q_1^c, q_2^U(q_1^c, q_3^c), q_3^c] + \delta w_2. \end{aligned} \quad (5)$$

Communication constraints for firm 2:¹⁴

$$\begin{aligned} m_2^1 : \quad (1 - \delta)\pi_2[q_1^B(q_3^c), q_2^B(q_3^c), q_3^c] + \delta w_2 &\geq \pi_2[q_1^B(q_3^c), q_2^c, q_3^c], i \neq 2. \\ m_2^3 : \quad (1 - \delta)\pi_2[q_1^c, q_2^B(q_1^c), q_3^B(q_1^c)] + \delta w_2 &\geq \pi_2[q_1^c, q_2^c, q_3^B(q_1^c)]. \end{aligned} \quad (6)$$

These constraints are incomplete in the sense that the continuation profit w_i at the start of the first punishment state is not yet determined. The next section considers alternative, perfect Bayesian punishments to calculate w_i . Nevertheless, it is instructive to already have a first look at the collusion and communication constraints. Only firm 2's collusion constraint is standard from the perfect public information models. It says that the (normalized) present value from colluding forever should be at least as high as the profit from (optimally) deviating once and then going to the first punishment state. The other constraints differ from the public information models.

First, consider firm i 's collusion constraint, $i \neq 2$. Firm i realizes that, after deviating, there is an intermediate communication state before the first punishment state. *A priori* it is not clear how this intermediate communication state affects firm i 's collusion constraint. Depending on the severity of the punishment, it turns out that the intermediate communication state may either relax or tighten firm i 's, $i \neq 2$ collusion constraint, as compared to the public information models.

Second, consider firm 2's communication constraints. These potentially binding constraints are not present in public information models. Take firm 2's communication

¹⁴The communication constraints for firm 1 and 3 (states m_1^1 and m_3^3) and for firm 2 in state m_2^2 are trivially satisfied.

constraint in state m_2^1 , i.e. when firm 2 is supposed to communicate a deviation by firm 1 to firm 3. This constraint says that firm 2's (normalized) profit from communicating once and then going to the first punishment state should be at least as high as the present value of the payoff from *never* communicating firm 1's defection to firm 3, and hence at least keeping collusion with firm 3. A similar intuition holds for 2's communication constraint in state m_2^3 .

The collusion constraints are more likely to be satisfied the lower the w_i , i.e. the more severe the punishments. The standard literature on public information has emphasized one bound on the severity of the punishments: the punishment should not be so severe that firms have an incentive to deviate from their strategy in any of their punishment states. The present model with private information identifies a new potential bound on the severity of the punishment: the punishment should not be so severe that firm 2 has an incentive not to communicate a privately observed deviation. From (??), if the punishment is too severe (i.e. w_2 is too low), firm 2 may prefer not to communicate a deviation by one of the other firms in order to at least continue to collude with the other firm, who did not observe the deviation. This demonstrates the even greater importance of appropriately chosen punishments when there is private information. Section 4 discusses alternative punishments in greater detail.

3.3 Symmetric collusion, symmetric punishments

In the infinite horizon game many outcomes different from the static Cournot outcome may generally be sustainable as a perfect Bayesian equilibrium. Attention is restricted here to the set of *symmetric collusive* outcomes, defined as the set of symmetric outcomes (q^c, q^c, q^c) that give all firms strictly greater profits than the Cournot-Nash profits. This restriction can be defended as follows. As is easily seen, for all symmetric outcomes (q, q, q) , the firms' profits are in the same, constant proportion, i.e. $\pi_1(q, q) = \pi_2(q, q, q)/2 = \pi_3(q, q)$. Furthermore, the one-shot Cournot-Nash outcome (q^T, q^T, q^T) derived above is symmetric. Consequently, in the present model any symmetric collusive outcome (q^c, q^c, q^c) in the infinite horizon game increases the firms' profits proportionally, a property frequently imposed in the literature.¹⁵

¹⁵Friedman and Thisse (1993), for example, define the collusive solution as the Pareto-optimal solution that yields profits that are in the same proportion as in the one-shot equilibrium. See also Schmalensee

The *joint-profit maximizing* outcome, i.e. the outcome that maximizes the sum of the firms' profits, is given by:

$$q_i^c = q^c \equiv \frac{\alpha}{2(\beta + \gamma)}, \quad \text{for each } i. \quad (7)$$

Because the joint-profit maximizing outcome (q^c, q^c, q^c) is symmetric, it is an element of the set of symmetric collusive outcomes. Furthermore, it is also a Pareto-optimal outcome. These facts make the joint-profit maximizing solution a particularly attractive candidate equilibrium outcome. Therefore most of the attention will be focused on the joint-profit maximizing outcome (q^c, q^c, q^c) .

Attention is further restricted to *symmetric* punishments, defined as punishments such that all firms choose the same quantity in a given j -th communication state, i.e. $f_1(p_1^j) = f_2(p_2^j) = f_3(p_3^j)$ for all j , so that $w_1 = w_2/2 = w_3$. Given symmetric collusive outcomes, symmetric punishments reduce all firms' profits proportionally after a publicly observed deviation. This generalizes Abreu's (1986) notion of symmetric punishments.

It is now possible to make the following observation, determining which of the collusion and communication constraints is nonbinding.

OBSERVATION 2. Given a symmetric collusive outcome and a symmetric punishment, firm 1 and 3's collusion constraints coincide. Moreover, firm 2's communication constraint in m_2^1 and in m_2^3 coincide. Finally, firm i 's, $i \neq 2$, collusion constraint is nonbinding if $\pi_i[q_i^B(q^c), q_2^B(q^c)] - w_i \leq 0$; otherwise, firm 2's collusion constraint is nonbinding.

PROOF: Check the collusion and communication constraints (??) and (??), noting that $\pi_1(q^c, q^c) = \pi_2(q^c, q^c, q^c)/2 = \pi_3(q^c, q^c)$, $\pi_1[q_1^U(q^c), q^c] = \pi_2[q^c, q_2^U(q^c, q^c), q^c]/2 = \pi_3[q^c, q_3^U(q^c)]$, $\pi_2[q_1^B(q^c), q_2^B(q^c), q^c] = \pi_2[q^c, q_2^B(q^c), q_3^B(q^c)]$, $\pi_2[q_1^B(q^c), q^c, q^c] = \pi_2[q^c, q^c, q_3^B(q^c)]$, and $w_1 = w_2/2 = w_3$. \square

Using observation 2, it suffices to consider only (i) either firm 1 or firm 2's collusion constraint depending on the sign of $\pi_i[q_i^B(q^c), q_2^B(q^c)] - w_i$, (ii) firm 2's first communication constraint, and (iii) the no-deviation constraints in the punishments states, or

(1987) for a general discussion of Pareto-optimal solutions that are not necessarily inconsistent with the collusive solution defined here.

shortly, the punishment constraints. These punishment constraints are derived below for alternative punishments, completing the description of the strategies.

4 Alternative punishments

I consider two classes of (symmetric) punishments: grim punishments and stick-and-carrot punishments. These punishments are relatively simple, and they allow for a close comparison with existing results.

4.1 Grim punishments

First consider *infinite* grim punishments, as in Friedman (1971). These are defined as follows. For each firm i , there is only one punishment state, p_i^1 . In p_i^1 firm i produces its Cournot-Nash quantity q^T . Transitions from p_i^1 are very simple: once in p_i^1 , firms always stay in p_i^1 .

With infinite grim punishments, the punishment constraints are trivially satisfied for each firm i , given that its rivals produce their Cournot-Nash quantity. Necessary and sufficient conditions for a perfect Bayesian equilibrium are then firm 2's first communication constraint in (??), and either firm 1 or firm 2's collusion constraints (??), where $w_1 = w_2/2$ is substituted by $\pi_1(q^T, q^T) = \pi_2(q^T, q^T, q^T)/2$. Using the joint profit maximizing outcome as the collusive solution, straightforward calculations¹⁶ show that firm 1's collusion constraint is satisfied if and only if $\delta \geq \underline{\delta}_g^1$; firm 2's collusion constraint is satisfied if and only if $\delta \geq \underline{\delta}_g^2$; and firm 2's communication constraint is satisfied if and only if $\delta \leq \bar{\delta}_g$, where $\underline{\delta}_g^1$ is the largest solution to quadratic

$$-4\sigma(32 + 32\sigma - 4\sigma^2 - 5\sigma^3)(\underline{\delta}_g^1)^2 + (2 + \sigma^2)^2(4 - \sigma)(32 + 16\sigma - 8\sigma^2 - \sigma^3)\underline{\delta}_g^1 - (8 - \sigma^2)^2(2 + \sigma)^2 = 0,$$

where

$$\underline{\delta}_g^2 \equiv \frac{(2 + \sigma)^2}{(2 + \sigma)^2 + 4(1 + \sigma)}$$

$$\bar{\delta}_g \equiv \frac{(2 + \sigma)^2}{(2 + \sigma)^2 + \sigma(4 + \sigma)(8 - \sigma^2)/(4 - \sigma)^2},$$

¹⁶The calculations follow from substituting the payoffs given in the Appendix in the various constraints. They are available from the author on request.

and where $\sigma \equiv \gamma/\beta$ is the substitution parameter. Which of these constraints is actually binding, turns out to depend crucially on whether the goods are substitutes, $\sigma > 0$, or complements, $\sigma < 0$. Applying observation 2, it turns out that $\delta \geq \underline{\delta}_g^1$ is irrelevant for $\sigma > 0$, and that $\delta \geq \underline{\delta}_g^2$ is irrelevant for $\sigma < 0$. Furthermore, $\bar{\delta}_g > 1$ for $\sigma < 0$, and $\bar{\delta}_g < 1$ for $\sigma > 0$. To economize on notation, define a new variable $\underline{\delta}_g \equiv \underline{\delta}_g^1$ for $\gamma < 0$, and $\underline{\delta}_g \equiv \underline{\delta}_g^2$ for $\gamma > 0$. The earlier findings then yield:

PROPOSITION 1. Suppose firms use infinite grim punishments. (i) If $\sigma > 0$, the joint-profit maximizing outcome is sustainable as a perfect Bayesian equilibrium if and only if $\delta \in [\underline{\delta}_g, \bar{\delta}_g]$.
(ii) If $\sigma < 0$, the joint-profit maximizing solution is sustainable if and only if $\delta \in [\underline{\delta}_g, 1]$.

Proposition 1 is illustrated on Figure 2, plotting the range of feasible discount factors, delineated by the critical discount factors $\underline{\delta}_g^1$, $\underline{\delta}_g^2$, and $\bar{\delta}_g$, as a function of the substitution parameter σ . It is straightforward to show that $\partial \underline{\delta}_g^1 / \partial \sigma < 0$ for $\sigma < 0$, $\partial \underline{\delta}_g^2 / \partial \sigma > 0$ for $\sigma > 0$, and $\partial \bar{\delta}_g / \partial \sigma < 0$ for $\sigma > 0$, as drawn. This shows that, when firms use infinite grim punishments, the joint-profit maximizing solution is most likely to be sustainable when goods are very independent, i.e. σ close to zero. The more the goods become complements or substitutes, the smaller the range of discount factors for which the joint-profit maximizing solution is sustainable.

These findings may be compared with the hypothetical case in which there would be perfect public information. In this case only firm 2's collusion constraint would be relevant,¹⁷ so that the joint-profit maximizing outcome would be sustainable if and only if $\delta \geq \underline{\delta}_g^2$. This turns out to be exactly the same condition as derived by Deneckere (1983), in a related model without localized competition.¹⁸ The analysis then stresses that when there is private information due to localized competition and when goods are substitutes, $\sigma > 0$, there exists a binding *upper bound* on the discount factor, $\bar{\delta}_g$, above which the joint-profit maximizing outcome is not sustainable, in addition to the

¹⁷To see this, note that when there is public information, no intermediate communication state is required in case firm 1 or firm 3 deviated; firms can then immediately move to the first punishment state. Firm 1 and 3's collusion constraint consequently would coincide with firm 2's collusion constraint, and firm 2's communication constraints would be absent.

¹⁸The results on product differentiation and collusion differ when firms set prices rather than quantities. See Deneckere (1983), Chang (1991) and Ross (1992).

standard lower bound from the public information model. The upper bound arises from firm 2's incentive problem to communicate privately observed deviations by firm 1 or firm 3. If firm 2 is very patient, then it prefers to continue to collude with even just one firm, rather than to communicate and then to be punished. This suggests that the infinite grim punishment may sometimes be *too severe*, rather than insufficiently severe as argued in the literature on public information [e.g. Abreu (1986)]. The question now arises whether the upper bound may be relaxed if firms follow alternative, less severe punishments.¹⁹

Consider therefore the following less severe punishments: finite grim punishments. These punishments specify J punishment states, i.e. $\{p_i^1, \dots, p_i^J\}$ for each firm i . In each state p_i^j firm i produces its Cournot-Nash quantity q^T . The transition from p_i^j , $j = 1, \dots, J - 1$, is always to p_i^{j+1} ; the transition from p_i^J is to c_i . In other words, after a deviation by firm 2, each firm i reverts to the Cournot quantity for only a *finite* number of periods, J , and then goes back to the collusive state.²⁰

The punishment constraints in all states p_i^j , $j = 1, \dots, J$ are trivially satisfied for each firm i , given that its rivals produce their Cournot quantity. Necessary and sufficient conditions for a perfect Bayesian equilibrium are then the collusion and communication constraints (??) and (??), where $w_1 = w_2/2$ is now substituted by $\pi_1(q^T, q^T) + \delta^J[\pi_1(q^c, q^c) - \pi_1(q^T, q^T)]$. Clearly, as $J \rightarrow \infty$ the case of infinite grim punishments obtains; as J decreases, the punishment payoff increases and hence the punishment becomes less severe. This generates a great deal of flexibility on the punishments.

Figure 3 plots the range of discount factors that sustain the joint-profit maximizing outcome for alternative punishment lengths, $J \rightarrow \infty$, $J = 50$ and $J = 10$. As could be

¹⁹An alternative way to relax the upper bound may seem to be to increase firm 2's profit from communicating. Notice however that firm 2 is already producing a one-shot best-response strategy against the other firms. Hence, the only way to increase firm 2's communication profit is by changing firm i , $i \neq 2$ quantity when it is in the communication state. By observation 1, however, this is not possible.

²⁰Notice that finite grim punishments can be considerably more "complex" than infinite grim punishments in the sense that the firms' machines need J states to execute the punishment rather than just one state under infinite grim punishments. Consequently, firms who are boundedly rational in the sense that their machines can only process a small number of states, may not be able to execute these strategies for large J .

expected, reducing the punishment length, and thus softening the punishment, has two effects: the collusion constraint is tightened and the communication constraint is relaxed. For high discount factors, the collusion constraint is nonbinding and the communication constraint is binding, so that a reduction of the punishment length is in fact appropriate. The question arises whether it is possible to vary the punishment length such that the joint-profit maximizing outcome becomes sustainable for *all* discount factors above $\bar{\delta}_g$. The following proposition establishes that this is indeed the case.

PROPOSITION 2. There are grim punishments, finite or infinite, such that the joint-profit maximizing outcome is sustainable as a perfect Bayesian equilibrium for $\delta \in [\underline{\delta}_g, 1]$.

PROOF. If $\sigma < 0$, then $J \rightarrow \infty$ supports collusion for any $\delta \in [\underline{\delta}_g, 1]$ by proposition 1. If $\sigma > 0$, then $J \rightarrow \infty$ supports collusion for $\delta \in [\underline{\delta}_g, \bar{\delta}_g]$ by proposition 1. Now consider $J = 2$. Straightforward algebra using the collusion and communication constraints (??) and (??) with the appropriate substitutions shows that a grim punishment with length $J = 2$ supports collusion if $\delta \in [\underline{\delta}_g^*, 1]$, where

$$\underline{\delta}_g^* \equiv \frac{1}{2} \left[\sqrt{1 + \frac{(\sigma + 2)^2}{\sigma + 1}} - 1 \right].$$

Furthermore, for $\sigma > 0$, $\partial \underline{\delta}_g^* / \partial \sigma > 0$, $\partial \bar{\delta}_g / \partial \sigma < 0$; and at $\sigma = 1$, $\underline{\delta}_g^* = .673 < \bar{\delta}_g = .698$. Therefore, for any $1 \geq \sigma > 0$, $\underline{\delta}_g^* < \bar{\delta}_g$. This implies that the regions $[\underline{\delta}_g, \bar{\delta}_g]$ and $[\underline{\delta}_g^*, 1]$ overlap so that the range $[\underline{\delta}_g, 1]$ is covered by either infinite grim punishments or grim punishments with length $J = 2$. This suffices to show the proposition. \square

Proposition 2 shows that *less severe* punishments than infinite grim punishments may sometimes be necessary, when some of the firms' deviations are not publicly observed due to localized competition. This contrasts with the literature on public information [Abreu(1986)], which criticizes infinite punishments as being not sufficiently severe. A surprising part in the proof of proposition 2 is that, in the present model, a very soft punishment, with length $J = 2$, already suffices to cover the whole range of discount factors above $\bar{\delta}_g$. This implies that the strategies do not actually need to be too "complex" (in the sense of requiring a large number of states) in order to sustain collusion when the discount factor is high.

4.2 Stick-and-carrot punishments

The previous subsection established that appropriate punishments can be found such that the joint-profit maximizing outcome is sustainable for $\delta \geq \underline{\delta}_g$. Do there exist punishments such that this outcome is sustainable for some discount factors *below* $\underline{\delta}_g$? This requires relaxing the collusion constraints (??), which should be done by making the punishments *more severe* than the infinite grim punishments considered above. To find the true lower bound on the discount factor, the most severe credible punishments should be found. In a perfect public information model Abreu (1986) has shown that the most severe punishments, within the class of symmetric punishments, take the simple form of stick-and-carrot punishments: a very severe one-period punishment after which collusion is restored. In Abreu's public information model, the severity of the punishment is limited solely by a simple no-deviation constraint in the punishment state. In the present model with private information the severity of the punishment may be further limited due to the communication problems discussed above. I now investigate how these communication problems precisely influence the severity of the punishment, and correspondingly the lower bound on the discount factor. To avoid complications I restrict attention to the case in which goods are substitutes, i.e. $\sigma > 0$.²¹ This case is also the most frequently analyzed case in the literature on collusion.²²

In the present model with private information due to localized competition, stick-and-carrot punishments may be defined as follows. For each firm i , there is one punishment state, p_i^1 . In this punishment state firm i chooses a symmetric punishment quantity $q^p \equiv f_i(p_i^1)$ yet to be determined. Transitions from the punishment state are analogous to transitions from the collusive state as described above:

Firm i , $i \neq 2$: move to c_i unless

- only $q_i \neq q^p$, in which case move to m_i^i
- at least $q_2 \neq q^p$, in which case move to p_i^1 .

Firm 2: move to c_2 unless

²¹In the alternative case in which goods are complements corner solutions where the punishment quantities are zero, may arise.

²²I will also assume here that firms operate at a sufficiently large marginal cost $\alpha > c > 0$. This rules out cases in which the punishment quantities cause prices to be negative. In this section, the parameter α should therefore be interpreted net of marginal cost.

- only $q_i \neq q^p$, $i \neq 2$, in which case move to m_2^i
- only $q_1 \neq q^p$ and $q_3 \neq q^p$, in which case move to m_2^2
- at least $q_2 \neq q^p$, in which case move to p_2^1 .

In contrast to Abreu's model with public information, each firm i who is in the punishment state p_i^1 cannot always immediately move back to the first punishment state p_i^1 in the event one of the firms would have deviated from the punishment quantity q^p . This is only possible in case the deviation occurred by firm 2, who is publicly observed. In case firm 1 or firm 3 deviated, an intermediate communication state is required.

To determine the punishment quantity q^p in the stick-and-carrot punishment, note that firm i 's (normalized) continuation profit at the start of the punishment state, w_i , is now given by

$$w_1 = w_2/2 = w_3 \equiv (1 - \delta)\pi_1(q^p, q^p) + \delta\pi_1(q^c, q^c) \quad (8)$$

and that the punishment constraints for firms 1, 2 and 3 are:²³

$$\begin{aligned} w_i &\geq (1 - \delta)\pi_i[q_i^U(q^p), q^p] + (1 - \delta)\delta\pi_i[q_i^B(q^c), q^c] + \delta^2 w_i, \quad i \neq 2 \\ w_2 &\geq (1 - \delta)\pi_2[q^p, q_2^U(q^p, q^p), q^p] + \delta w_2. \end{aligned} \quad (9)$$

The punishment quantity q^p in the stick-and-carrot punishment is then determined as follows: q^p minimizes the punishment payoff (??) subject to *all* no-deviation constraints.

The following observation is useful to determine which of the punishment constraints is nonbinding.

OBSERVATION 3. Given a symmetric collusive outcome and symmetric stick-and-carrot punishments, firm 1 and 3's punishment constraints coincide. Furthermore, firm i 's, $i \neq 2$, punishment constraint is nonbinding if $\pi_i[q_i^B(q^c), q_2^B(q^c)] - w_i \leq 0$; otherwise firm 2's punishment constraint is nonbinding.

PROOF: Check (??), noting that $\pi_1[q_1^U(q^p), q^p] = \pi_2[q^p, q_2^U(q^p, q^p), q^p]/2 = \pi_3[q_3^U(q^p), q^p]$, $\pi_1[q_1^B(q^c), q^c] = \pi_3[q_3^B(q^c), q^c]$ and $w_1 = w_2/2 = w_3$. \square

From observation 2 and 3, it suffices to consider only (i) firm 2's first communication constraint, and (ii) either firm 1's collusion and punishment constraints or firm 2's collusion and punishment constraints depending on the sign of $\pi_i[q_i^B(q^c), q_2^B(q^c)] - w_i$.

²³In contrast to grim punishments, these constraints are not trivially satisfied.

It is straightforward to show that, for $\sigma > 0$, q^p is the *largest* quantity that solves *either* firm 1's punishment constraint in (??), *or* firm 2's punishment constraint (??), *or* firm 2's communication constraint (??) as an equality without violating any of the other no-deviation constraints. Call these solutions respectively \hat{q}^p , \bar{q}^p and \tilde{q}^p .

If there would be public information, as in Abreu's model, it would be easy to determine q^p because communication constraint (??) would be absent and because firm 1's punishment constraint in (??) would coincide with firm 2's punishment constraint. Under public information q^p is then the largest solution to firm 2's punishment constraint in (??), i.e.

$$q^p = \bar{q}^p \equiv \frac{\alpha}{\beta + \gamma} \left(\frac{\beta + \gamma - \sqrt{\beta(\beta + \gamma)\delta}}{2\beta + \gamma - 2\sqrt{\beta(\beta + \gamma)\delta}} \right).$$

Call the stick-and-carrot punishment using this quantity \bar{q}^p the *standard* stick-and-carrot punishment, and the stick-and-carrot punishments using either \hat{q}^p or \tilde{q}^p the *modified* stick-and-carrot punishments. Under public information it has been shown that the standard stick-and-carrot punishment is the most severe punishment in the class of symmetric punishments.²⁴ It is therefore also the optimal symmetric punishment under public information. The corresponding lower bound on the discount factor is found from firm 2's collusion constraint (??), after substituting w_2 given in (??) with $q^p = \bar{q}^p$:

$$\delta \geq \underline{\delta}_s \equiv \frac{(2 + \sigma)^2}{16(1 + \sigma)}$$

which is always below the grim punishment lower bound $\underline{\delta}_g$, as could be expected.

The first question is whether the standard stick-and-carrot punishment, being the most severe and hence optimal punishment of the public information model, is still feasible in the private information model, or whether, to the contrary, the communication problems make this punishment infeasible. This question is answered in the following proposition:

PROPOSITION 3. Consider $\sigma > 0$. When firms use the standard stick-and-carrot punishment, in which $q^p = \bar{q}^p$, the joint-profit maximizing outcome is not sustainable as a perfect Bayesian equilibrium.

PROOF: See Appendix.

²⁴This was shown by Abreu for the case in which $\sigma = 1$; it immediately generalizes to $\sigma < 1$.

The intuition for Proposition 3 is simple: the standard stick-and-carrot punishment, in which firms produce the punishment quantity \bar{q}^p of the public information model, is too severe. For all $\sigma \in (0, 1]$ firm 1 and firm 3 do not have an incentive to carry out the punishment, because by deviating from the punishment an intermediate communication period occurs²⁵ before they are punished for their deviation. Furthermore, for most σ , i.e. $\sigma \in (0, 0.795]$ firm 2 does not have an incentive to communicate a deviation by firm 1 or 3 from their prescribed collusive or punishment quantity, because firm 2 prefers to at least collude with the nondeviating firm rather than engaging in a one-period communication state and a very severe punishment afterwards.

Given that the standard stick-and-carrot punishment, in which $q^p = \bar{q}^p$, cannot sustain the joint-profit maximizing outcome under private information, the next question is for which discount factors the modified stick-and-carrot punishments, in which $q^p = \hat{q}^p$ or $q^p = \tilde{q}^p$, sustain the joint-profit maximizing outcome. To answer this question, let the value of the discount factor for which firm 1's collusion constraint (??) is just satisfied with equality be $\hat{\delta}_s$ when $q^p = \hat{q}^p$, and $\tilde{\delta}_s$ when $q^p = \tilde{q}^p$. We then have:

PROPOSITION 4. Consider $\sigma > 0$ and suppose firms use modified stick-and-carrot punishments. (i) For $\sigma \in (0, 0.117]$, the joint-profit maximizing outcome is sustainable as a perfect Bayesian equilibrium if and only if $\delta \geq \hat{\delta}_s$, where $\underline{\delta}_s < \hat{\delta}_s < \underline{\delta}_g$. (ii) For $\sigma \in (0.117, 1]$, the joint-profit maximizing outcome is sustainable as a perfect Bayesian equilibrium if and only if $\delta \geq \tilde{\delta}_s$, where $\underline{\delta}_s < \tilde{\delta}_s < \underline{\delta}_g$.

PROOF: See Appendix.

Intuitively, for $\sigma \in (0, 0.117]$, firms produce \hat{q}^p , the largest quantity such that firm 1 and firm 3 are just indifferent between punishing and deviating from the punishment when in p_1^1 and p_3^1 . For $\sigma \in (0.117, 1]$, firms produce \tilde{q}^p , the largest quantity such that firm 2 is just indifferent between communicating and not communicating when in m_2^1 or m_2^3 . In other words, for $\sigma \in (0, 0.117]$ the firms are constrained by firm 1 and 3's punishment constraint; for $(0.117, 1]$ the firms are constrained by firm 2's communication constraint. Note that, for $\sigma \in (0, 0.117]$, \tilde{q}^p is more severe than \hat{q}^p , but this would violate firm 1's

²⁵The payoff in this communication period is larger than the punishment, in contrast with the case of grim punishments and $\sigma > 0$.

punishment constraint for discount factors close to the lower bound $\hat{\delta}_s$ in this range of σ . Furthermore, for $\sigma \in (0.117, 1]$, \hat{q}^p is more severe than \tilde{q}^p , but this would violate either firm 1's collusion constraint or firm 2's communication constraint for any discount factor in this range of σ .

Propositions 3 and 4 are illustrated on Figure 4. Note that the lower bounds $\hat{\delta}_s$ and $\tilde{\delta}_s$ are much below $\underline{\delta}_g$, the lower bound when grim punishments are used. They are also, however, much above $\underline{\delta}_s$, the lower bound when the standard stick-and-carrot punishments from the public information case would have been feasible. This then demonstrates that the communication problems due to the localized nature of competition may be a significant factor hindering collusion.

5 Multimarket operation

The above model analyzed the stability of collusion when there is private information due to localized competition. This section reinterprets the model in an alternative way, as a model of multimarket operation. This exercise is useful, because antitrust policy has often been concerned with the possibility of collusion when firms operate in several markets.

There is a growing theoretical and empirical literature on the relationship between multimarket operation and collusion.²⁶ From a theoretical perspective, Bernheim and Whinston (1990) provide the most significant contribution. They analyze *multimarket contact*, a situation in which two or more firms meet each other in several markets. They show that multimarket contact serves to pool the firms' incentive constraints and identify conditions under which this pooling strictly enhances the firms' ability to collude. In their concluding section Bernheim and Whinston ask whether their results would extend to the alternative setting of *multimarket operation* absent multimarket contact, i.e. a situation in which only one firm operates in several markets, meeting single-market competitors in each of these markets. They question the possibility that their results will indeed extend to such a setting, in part because the single-market competitors may not even observe outcomes in the markets where they do not operate.

²⁶See Bernheim and Whinston (1990), van Witteloostuijn and van Wegberg (1992), Barla (1993), Evans and Kessides (1994), Gimeno (1994).

In fact, a simple reinterpretation of the model developed in this paper allows to provide a formal answer to this question. Consider two markets, A and B . In each market k , there is one representative consumer with the standard quadratic utility function

$$U_k(q_{0k}, q_{1k}, q_{2k}) = \alpha (q_{1k} + q_{2k}) - \frac{\beta}{2} (q_{1k}^2 + q_{2k}^2) - \gamma q_{1k} q_{2k} + q_{0k}$$

where q_{ik} , $i = 0, 1, 2$, is the quantity consumed of good i in market k . Assume $\alpha > 0$, $\beta > 0$, $\gamma \neq 0$ and $\beta \geq \gamma \geq -\beta$. Good 0 is the outside good. In each market k , goods 1 and 2 are substitutes for $\gamma > 0$, they are complements for $\gamma < 0$. The inverse demand equations for good 1 and 2 in market k can be derived from solving the representative consumer's utility maximization problem:

$$p_{ik} = \alpha - \beta q_{ik} - \gamma q_{jk}, \quad i = 1, 2; j = 1, 2; i \neq j.$$

Consider the following situation of multimarket operation absent multimarket contact. There are three firms, firm 1A, firm 1B and firm 2, competing in quantities for an infinite number of periods, discounting the future at a factor δ . Firm 1A and 1B produce the quantity of good 1 in market A and B, respectively. Firm 2 produces the quantity of good 2 in both markets A and B. Hence there is multimarket operation by firm 2, but there is no multimarket contact: firm 2 faces two different single-market competitors in each market. (Think of a multinational firm facing a domestic firm in two different countries.) Normalizing marginal cost to zero, profits are given by

$$\begin{aligned} \pi_{1k} &= \pi_{1k}(q_{1k}, q_{2k}), \quad k = A, B \\ \pi_2 &= \pi_{2A}(q_{2A}, q_{1A}) + \pi_{2B}(q_{2B}, q_{1B}) \end{aligned} \tag{10}$$

where

$$\pi_{ik}(q_{ik}, q_{jk}) \equiv (\alpha - \beta q_{ik} - \gamma q_{jk}) q_{ik}, \quad i = 1, 2; j = 1, 2; i \neq j \tag{11}$$

Consider the following three cases.

Case 1: Public information on outcomes in both markets.

In this case the three firms observe the quantity choices of each firm in each market, even firms 1A and 1B who are only active in one of the markets. Consequently, Bernheim and Whinston's analysis applies: firm 2's no-deviation constraints can be pooled over the two markets. This allows firm 2 to transfer possible slack on its no-deviation constraints from one market to the other. In the present model Bernheim and Whinston's irrelevance

result would hold: due to the symmetry of demand and technology in both markets the pooling of firm 2's no-deviation constraints does not actually facilitate collusion.²⁷ In contrast, if there would be some asymmetry across the markets, the pooling of firm 2's no-deviation constraints could, in fact, facilitate collusion.²⁸

Case 2: Information only on outcomes in market where active; no "inherent" linkages between markets.

In this case firm 1A and 1B do not observe outcomes in the market in which they do not operate. Only firm 2 observes the outcomes in both markets. Consequently firm 2's no-deviation constraints cannot be pooled over the two markets, unlike in the previous case.

Furthermore, in this case there are no demand- or cost-based linkages across the markets, nor any other constraints influencing firm 2, who operates in both markets. Consequently, firms 1A, 1B and 2 behave as if there is only a single market, and the no-deviation constraints from the standard single-market model apply.

Case 3: Information only on outcomes in market where active; firm 2 constrained to produce $q_{2A} = q_{2B} = q_2$.

As in the previous case, due to the information problems, firm 2's no-deviation constraints cannot be pooled over the two markets. Furthermore, there is an inherent linkage between market *A* and *B*: firm 2 is constrained to produce the same quantity in both markets. One interpretation of this constraint is as follows: firm 2 can only control the total quantity of its good produced, but it cannot control in which market this quantity will actually be marketed. While this constraint may seem somewhat artificial, it should not be taken too literally. It merely serves to illustrate a more general potential problem with multimarket operation and collusion in exactly the same framework of the previous sections. To see this, simply rewrite firm 2's profit equation, given by (??) and (??), after substituting $q_{2A} = q_{2B} = q_2$:

$$\pi_2 = 2[\alpha - \beta q_2 - \gamma(q_{1A} + q_{1B})/2]q_2.$$

²⁷See Bernheim and Whinston (1990), Proposition 1, p.6.

²⁸Asymmetry may be obtained by allowing the parameters δ and $\sigma \equiv \gamma/\beta$ to differ across markets, or by allowing firms to differ in marginal costs.

The profit equations of firm 1A, 1B and 2 then are isomorphic to the profit equations of firm 1, 3 and 2 in the model of localized competition analyzed in the previous sections. Consequently, the same incentive problems apply. In particular, multimarket firm 2 may not have the necessary incentive to punish deviations by firm 1A or 1B, because this will trigger a break-down of collusion in the other market, given that firm 2 is constrained to $q_{2A} = q_{2B}$.

Case 3 illustrates a more general potential problem of collusion under multimarket operation: (i) if the single-market competitors are not informed about the outcomes in the other markets, and (ii) if the actions of the multimarket firm necessary to sanction deviations in one market cause an *externality* on the uninformed single-market competitors in the other markets, then the multimarket firm may actually no longer be willing to execute the necessary sanctions out of fear for triggering a break-down of collusion in these other markets.

A nice economic example of this potential collusion problem under multimarket operation is the simple two-market duopoly model as described above, but with *price-setting* firms as in Bernheim and Whinston. The analogous constraint of case 3 would then be $p_{1A} = p_{2A} = p_2$, i.e. multimarket firm 2 should set the same price in both market A and B. This may be nicely interpreted as a standard arbitrage constraint. A more detailed analysis of this model, following the approach of the present paper, could then show that Bernheim and Whinston's conclusion that multimarket competition facilitates collusion may actually be reversed: (i) due to the information problems, pooling of the multimarket firm's no-deviation constraints is no longer possible; (ii) due to the arbitrage constraint, an externality arises reducing the multimarket firm's incentive to sanction privately observed deviations in some markets.

Given the large empirical literature on multimarket contact and collusive behavior, it would be interesting to also empirically investigate in more detail the relationship between multimarket operation absent multimarket contact and collusive behavior. From a theoretical perspective, the above analysis shows that multimarket operation turns out to be distinct from multimarket contact.

6 Conclusion

This paper has studied collusive behavior in a repeated oligopoly model with localized competition, also reinterpreted as a model of multimarket operation. Private information about the rivals' past actions naturally arises from these product market structures. The resulting communication problems imply that firms should not adopt strategies with too severe punishments. Infinite grim punishments may be too severe, for large discount factors. The standard stick-and-carrot punishments from the perfect public information model are always too severe. Modified stick-and-carrot punishments can still be used, though for a smaller range of discount factors than the standard stick-and-carrot punishments.

The specific economic model chosen made it possible to formalize information problems generated by the product market structure in a relatively simple way. The model also allowed for a close comparison with the collusion literature under perfect public information, and for a reinterpretation as a model of multimarket operation. Nevertheless, it would be desirable to investigate how sensitive the results are to some of the specific assumptions. Do the results extend to alternative economic models where similar private information problems arise from the product market structure?

A first alternative worth investigating is a triopoly model with price-setting firms rather than quantity-setting firms. One possibility is the well-known Hotelling model of localized competition with three price-setting firms. In this model three firms are located on a line, representing a one dimensional product space.²⁹ An alternative possibility is a model of multimarket operation with price-setting firms and a cross-market arbitrage constraint, as already suggested above. Both possible models will generate exactly the same information structure as the model developed in this paper. However, the specific equilibrium conditions will differ, and it is not clear *a priori* how this will affect the precise results.

A second alternative is a model where the private information structure is no longer exactly the same as in the present paper. One interesting possibility is the Salop circle model with more than three firms. In this version of the Hotelling model firms are

²⁹See Kats and Neven (1990) for an analysis of the static version of this model.

located on a circle, rather than on a line.³⁰ This generates a rather different information structure. A firm realizes that he cannot unilaterally "block" its information about a past deviation: choosing whether or not to communicate a past deviation, he realizes that independent of his action the whole industry will eventually become informed about the deviation anyway, through the equilibrium communications taking place at the other side of the deviator. More generally speaking, the Salop circle model is a convenient model to illustrate communication problems in markets where firms can only *delay* an eventual punishment, but not completely *prevent* it as in the present model. Similar communication problems, with delay instead of complete prevention of the punishment, may occur in markets with multidimensional product differentiation.

³⁰The line model with more than three firms does not seem to generate interesting new insights.

A Appendix

Proof of Observation 1. First it is shown that $f_2(m_2^i) \neq q_2^c$. Then it is shown that given that $f_2(m_2^i) \neq q_2^c$, firms should produce their one-shot best-response quantity in the communication states.

Suppose $f_2(m_2^i) = q_2^c$. Then firm i 's, $i \neq 2$, collusion constraint (state c_i) is given by

$$\pi_i(q_i^c, q_2^c) \geq (1 - \delta)\pi_i(q_i^U(q_2^c), q_2^c) + \delta\pi_i[f_i(m_i^i), q_2^c].$$

For this inequality to hold, it is necessary, given that $\pi_i[q_i^U(q_2^c), q_2^c] > \pi_i(q_i^c, q_2^c)$ for $q_i^c \neq q_i^U(q_2^c)$, that

$$\pi_i(q_i^U(q_2^c), q_2^c) > \pi_i[f_i(m_i^i), q_2^c] \quad (\text{A.1})$$

Furthermore, if $f_2(m_2^i) = q_2^c$, then firm i 's, $i \neq 2$, communication constraint (state m_i^i) is given by:

$$(1 - \delta)\pi_i[f_i(m_i^i), q_2^c] + \delta\pi_i[f_i(m_i^i), q_2^c] \geq (1 - \delta)\pi_i[q_i^U(q_2^c), q_2^c] + \delta\pi_i[f_i(m_i^i), q_2^c]$$

so that it is necessary that $\pi_i[f_i(m_i^i), q_2^c] \geq \pi_i[q_i^U(q_2^c), q_2^c]$, which contradicts (??). This demonstrates that $f_2(m_2^i) \neq q_2^c$. Given that $f_2(m_2^i) \neq q_2^c$, the communication constraints for firm 1 and 3, and for firm 2 in m_2^1 are as follows:

$$m_i^i, i \neq 2 : \quad (1 - \delta)\pi_i[f_i(m_i^i), f_2(m_2^i)] + \delta w_i \geq (1 - \delta)\pi_i[q_i^U(f_2(m_2^i)), f_2(m_2^i)] + \delta w_i$$

$$m_2^1, i \neq 2 : \quad (1 - \delta)\pi_2[f_1(m_1^1), f_2(m_2^1), q_3^c] + \delta w_2 \geq$$

$$\max \left\{ (1 - \delta)\pi_2[f_1(m_1^1), q_2^U(f_1(m_1^1), q_3^c), q_3^c] + \delta w_2, \right. \\ \left. (1 - \delta)\pi_2[f_1(m_1^1), q_2^c, q_3^c] + \delta(1 - \delta)\pi_2[f_1(m_1^1), f_2(m_2^1), q_3^c] + \delta^2 w_2 \right\}$$

For these inequalities to hold it is necessary (and for firm i , $i \neq 2$, also sufficient) that $\pi_i[f_i(m_i^i), f_2(m_2^i)] \geq \pi_i[q_i^U(f_2(m_2^i)), f_2(m_2^i)]$ and $\pi_2[f_1(m_1^1), f_2(m_2^1), q_3^c] \geq \pi_2[f_1(m_1^1), q_2^U(f_1(m_1^1), q_3^c), q_3^c]$. A similar argument holds for the communication states m_2^2 and m_2^3 . This then shows Observation 1. \square

Proof of Proposition 3.

By definition of the standard stick-and-carrot punishment firm 2's punishment constraint in (??) is just satisfied with equality. It may be verified that firm 1's punishment constraint is satisfied if and only if $\pi_1[q_1^U(q^p), q^p] \geq \pi_1[q_1^B(q^c), q_2^B(q^c)]$, which holds after some calculations if and only if

$$\delta \leq \frac{\sigma^2}{(4 - \sigma)^2(1 + \sigma)}. \quad (\text{A.2})$$

It can be easily verified that this condition is violated for $1 \geq \sigma > 0$, given that (??) should hold for firm 2's collusion constraint to be satisfied. This is sufficient to prove the proposition. (Note that it can analogously be checked that firm 2's communication constraint (??) is violated for $.795 \geq \sigma \geq 0$, given that (??) should hold for firm 2's collusion constraint to be satisfied.) \square

Proof of Proposition 4.

Given that the standard stick-and-carrot punishment in which $q^p = \bar{q}^p$ is not a perfect Bayesian equilibrium by Proposition 3, only the stick-and-carrot punishments in which $q^p = \hat{q}^p$ or $q^p = \tilde{q}^p$ remain to be considered. (a) Consider \hat{q}^p , so that firm 1's punishment constraint (??) is just satisfied with equality. For firm 1's collusion constraint to be satisfied, it turns out to be necessary that $\pi_1[q_1^B(q^c), q_2^B(q^c)] - w_1 \geq 0$. Therefore by observations 2 and 3 firm 2's collusion and punishment constraints are nonbinding. Hence, given $q^p = \hat{q}^p$, it remains to check only firm 1's collusion constraint and firm 2's communication constraint. These two inequality constraints are extremely tedious, but fortunately they depend on only two parameters. Hence it is possible to characterize these constraints fully, using numerical simulations. Numerical simulations show that for $\sigma \geq 0.117$ these constraints cannot be simultaneously satisfied, and that for $\sigma < 0.117$ these constraints are satisfied if and only if $\delta \in [\hat{\delta}_s, \hat{\delta}'_s]$, where $\underline{\delta}_s < \hat{\delta}_s < \underline{\delta}_g$, and where $\hat{\delta}'_s$ is the upper bound on the discount factor for firm 2's communication constraint to be satisfied. (b) Consider \tilde{q}^p , so that firm 2's communication constraint (??) is just satisfied with equality. For firm 1's collusion constraint to be satisfied, it turns out to be necessary that $\pi_1[q_1^B(q^c), q_2^B(q^c)] - w_1 \geq 0$. Therefore by observations 2 and 3 firm 2's collusion and punishment constraints are nonbinding. Hence, given $q^p = \tilde{q}^p$, it remains to check only firm 1's collusion constraint and firm 1's punishment constraint. Numerical simulations show that for $\sigma \geq 0.117$ these constraints are satisfied if and only if $\delta \geq \tilde{\delta}_s$, where $\underline{\delta}_s < \tilde{\delta}_s < \underline{\delta}_g$, and for $\sigma < 0.117$ these constraints are satisfied if and only if $\delta \geq \tilde{\delta}'_s$,

where $\tilde{\delta}'_s$ is the lower bound on the discount factor for firm 1's punishment constraint to be satisfied.

Combining (a) and (b), part (ii) of Proposition 4 immediately follows. Further numerical simulations show that, for $\sigma \geq .117$, $\hat{\delta}_s < \tilde{\delta}'_s < \hat{\delta}'_s$, from which part (i) of Proposition 4 follows. \square

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