CentER for Economic Research Discussion paper

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TILBURG UNIVERSITY

No. 9012

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February 1990

ISSN 0924-7815

THE EXISTENCE OF AN EQUILIBRIUM DENSITY FOR MARGINAL COST PRICES, AND THE SOLUTION TO THE SHIFTING-PEAK PROBLEM* †

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A result on the existence of a price density in Bewley's (1972) model of competitive equilibrium is used to set up a model of competitive equilibrium pricing for time-differentiated commodities (which are usually also differentiated over events of delivery), supplied either by price-taking, profit-maximizing industries or by public utilities pricing their products at exact marginal cost. This has applications to, e.g., the pricing of electricity and water. Since Bewley's "Exclusion Assumption" does not hold for firms using the differentiated commodity as an input, we use a weaker version of this assumption, which is shown to hold for firms with Mackey continuous production functions. In peak-load pricing problems, firms and households satisfy Mackey continuity assumptions if their consumption of the commodity in question is harmlessly interruptible. Under this assumption, we prove that the equilibrium time-profile of output has a peak plateau over which the marginal capacity cost, represented by a price density, is spread. This provides a formal setting in which a conjecture of Boiteux on the solution to the "shifting-peak problem" is true.

1. Introduction

Our purpose in this paper is to set up a continuous-time model of competitive equilibrium pricing for time-differentiated commodities (which are usually also differentiated over events of delivery and over locations) such as electricity and water, supplied either by price-taking, profit-maximizing industries or by public utilities pricing their products at exact marginal cost. To answer questions raised in the past on the existence of a market equilibrium in peak-load pricing problems, we formulate and prove an equilibrium existence result of the kind called for by Dreze (1964, pp. 16-17), and we examine within this context a conjecture of Boiteux (1964, pp. 81-82) on the form of a solution to the "shifting-peak problem".

Marginal cost pricing has long been recommended on the grounds of its theoretical allocative efficiency, and a number of tariffs for, e.g., electricity, has been constructed to approximate roughly marginal costs. However, pricing at exact marginal costs has until recently been impracticable. With advances in metering and computation, more detailed, "responsive" pricing has become feasible. As a result, there have been proposals for, and several experiments with, "spot" pricing which equilibrates supply and demand in real time: see, for example, Bohn, Golub, Tabors and Schweppe (1984). To provide a rigorous basis for such pricing, it is necessary to construct an equilibrium theory which not only takes account of the stochastic aspects of the problem and of the asymmetries of information which exist in such a market but also is set up in <u>continuous time</u>. The framework given in Section 2 below can be used as the basis for such a study, but in this paper, as far as peak-load pricing is concerned, we concentrate on the continuous-time aspects of the problem. For this reason, the example analyzed in Section 2 is a deterministic model of electricity generation. Even in this context the continuous-time treatment is desirable from a formal viewpoint, since, a priori, no natural periodization exists. Any approximate pricing solutions which may be required in practice for the deterministic problem, e.g., for the construction of tariffs with relatively few rating periods, should be obtained ex post, by simplifying the results of the continuous-time analysis.[1] Similar views are expressed by Boiteux (1964 , p. 81, lines 13-15) and by Gallant and Koenker (1984, p. 84, Footnote 1). The only proper alternative to modelling in continuous time would be to consider a sequence of discrete-time models (with the length of subperiods decreasing to zero in the limit), and to show that the solutions converge. The limit of such discrete-time equilibrium prices would be the continuous-time equilibrium price. Thus, the discrete-time approach offers no true simplification by comparison with the continuous-time approach if the dependence of the discrete-time solution on the periodization is taken into account, since conditions for the convergence of solutions should be established. (In technical terms, determining these requires a specification of the limiting commodity and price spaces, which are infinite-dimensional, and of the topologies in which convergence of discrete-time solutions takes place. This amounts to dealing with the same mathematical questions as those of the continuous-time analysis.)

In the context of peak-load pricing, doubts about the existence of an equilibrium have been expressed because of the so-called "shifting-peak problem", which can be described as follows (cf. Boiteux (1964, Section 3.4.3, p. 81) and Boiteux and Stasi (1964, p. 118)). For simplicity, we look at a deterministic, one-station model of electricity generation with a given unit capital cost, r, and a given unit fuel cost, w. The relevant time period, for one production cycle, is taken to be the unit time interval, $[0, 1]$. The long-run cost of generating a time-profile of output, $y(t)$ for $t \in [0, 1]$, is given by

$$C(y) = w \int_0^1 y(t)\,dt + r \sup_{t \in [0,1]} y(t),$$

where "sup" stands for the supremum (over time, t). Consider some time-profile of demand, y. An example of such a time-profile, with a peak of short duration, is shown in

Figure 1. Offpeak, the marginal cost, i.e., the cost, $p(t)$, of supplying an additional unit at time t, is constant and equal to the unit fuel cost, w. At every peak instant, the long-run marginal cost, $p(t)$, is higher than w, and the total excess of $p(t)$ over and above w, summed up over the peaks of y, is equal to the unit capital cost, r. Formally,

$$p(t) = w + r\nu(t)$$

for some nonnegative, integrable function, ν, with

$$\int_0^1 \nu(t)\,dt = 1,$$

and

$$\nu(t) = 0 \quad \text{for every } t \in [0, 1] \text{ with } y(t) < \sup y.$$

This is illustrated in Figure 2. Since the peak of y lasts only for a short time, peak marginal costs, $w + r\nu(t)$, are correspondingly high. Faced with these marginal costs as prices, electricity users may shift some of their consumption from high-price times to low-price times. If so, the new time-profile of demand, y', i.e., the demand at prices p, is likely to have its peak at times other than the peak times of y, in which case p is not a marginal cost price system for y'. That is, the peak shifts away from the original peak times, and, as a result, peak prices are charged at the wrong times. If a long-run marginal cost price system, p', for the new demand profile, y', is then tried, the peak may well shift again. The resulting iterative sequences of demands and marginal costs need not converge, and it is not clear whether an equilibrium, in which prices are equal to marginal costs, exists at all. Misgivings of this kind have never been completely cleared up, although Boiteux (1964, pp. 81-82) and Boiteux and Stasi (1964, p. 119) have made some progress towards sorting the problem out. Boiteux's conjecture is that in equilibrium the time-profile of output, y^*, has a "fairly long" peak plateau (perhaps consisting of a number of peak intervals). The peak charge, equal in total to r, is spread so as to sustain this plateau on the demand side, i.e., in equilibrium the peak-offpeak price difference is low enough not to cause the peak to shift. This is illustrated in Figures 3 and 4. The conjecture is formalized and proved in Section 2 (Example 2.3).

In the discrete-time set-up, the idea that in equilibrium the peak charge may be spread over more than one subperiod is also put forward by Steiner (1957). For a two-subperiod model and under the assumption of "independent demands" (i.e., cross-price independent demands), Steiner (1957, pp. 587-590) in effect shows the existence of an equilibrium long-run marginal cost tariff. However, Steiner's graphical argument is unsuitable for extensions either to the case of more than two subperiods or to the case of "interdependent demands" (i.e., cross-price dependent demands). Seeking to obtain such extensions, Steiner (1957, Appendix) reverts to the social surplus maximization framework, and only gives necessary and sufficient conditions for a surplus maximum. These shortcomings are pointed out by Dreze (1964, pp. 16-17) who stresses the need for a rigorous analysis of existence and uniqueness of peak-load pricing equilibria. In a continuous time set-up with "independent demands", peak-load pricing policies for a social surplus-maximizing monopoly and for a profit-maximizing monopoly are described by Takayama (1974, pp. 671-684). The issue of the existence of an optimum is not addressed in that work, either. This gap might be filled, but, as with Steiner's contribution, a basic flaw is that the Marshallian surplus concept is ill-defined except for very special cases. Although Marshallian surplus may be useful in practice for an approximate appraisal of the effect of small price and quantity changes, its use for global optimization, even were it correct, would be unnecessary for the purpose.

The equilibrium distribution, ν^*, of the peak charge is determined by the relative strength of demand at the peak instants, and, in general, it is not uniform (i.e., ν^* is not constant over the peak plateau), since, at different peak instants, different prices may be required to bring the demand down to the same (peak) level. Typically, a unique ν^* is singled out in this way, in which case the equilibrium marginal cost price system, p^*, is uniquely determined, despite there being many marginal costs at the equilibrium output, y^* (each corresponding to a possible spreading, ν, of the unit capital cost over the peak plateau of y^*). A similar observation is made by Littlechild (1970, p. 326, lines 3-7) in the context of a discrete-time model.

It can be shown that the equilibrium price, $p^*(t)$, is continuous over time, t, under the assumption that the marginal utility of each household's electricity consumption and the marginal productivity of each firm's electricity input are continuous (over time): see Horsley and Wrobel (1990a). This is because a discontinuous time-profile of price causes a "shifting-pattern problem", which can informally be described as follows. Since marginal utilities and productivities are continuous, an upward (discontinuous) jump in price at any time will bring about a downward jump in both household and input demand. This cannot be an equilibrium because the marginal cost, for a given output bundle, is not higher at a time when the output level is lower (i.e., for a time-profile of marginal cost, p, at an output bundle, y, and for every pair of instants, t and t', if $y(t) > y(t')$, then $p(t) \geq p(t')$).

To see what the price continuity result of Horsley and Wrobel (1990a) implies for the properties of the equilibrium output in peak-load pricing problems, consider a two-station model of electricity generation. From the continuity of the equilibrium price it follows that the equilibrium output has an off-peak plateau at the level equal to the total base-load capacity. This is because only during such a plateau is it possible for a gradual, continuous transition of the equilibrium price from the marginal operating cost of the first station, w_1, to that of the second station, w_2, to take place. This removes the "shifting-pattern" problem that would occur if, offpeak, the price could only be equal to w_1 or to w_2. Similarly, in an N-station model, generally there are $N-1$ off-peak plateaux in the equilibrium output, in addition to a peak plateau. A difference between the roles of the peak and the off-peak plateaux should be noted: whereas the peak plateau is necessary for the existence of an equilibrium price, in a multi-station model the off-peak plateaux are necessary for the equilibrium price to be continuous. It is worth observing that the price continuity result follows crucially from the property that cost is a symmetric function of the output bundle, i.e., the joint cost, $C(y)$, depends only on the distribution of the output level in the bundle y, and not on the way in which the values $y(t)$, for $t \in [0, 1]$, are arranged on the time interval. Cost symmetry is a characteristic of peak-load pricing problems: for details of the above arguments, see Horsley and Wrobel (1990a).

An implicit assumption of Boiteux's solution to the shifting-peak problem is that customers switch off briefly to avoid paying high charges levied during short periods of time. This implies that households' electricity consumption and the production processes of those firms using electricity as an input can be interrupted for a short time without much loss of utility or output. To state this assumption formally, denote by $f(z)$ a firm's output when the electricity input is z. If the set of times during which the firm switches off its power intake is denoted by A, then the firm's input is equal to $z(t)$ for $t \notin A$ and zero for $t \in A$. This new input can be written concisely as $z\chi_{[0,1]\setminus A}$, where $\chi_{[0,1]\setminus A}$ is the characteristic function of the complement of A, equal to 0 on A and to 1 on $[0, 1]\setminus A$. In these terms, the requirement on the production function is that $f(z\chi_{[0,1]\setminus A})$ increase to $f(z)$ as the duration of A decreases to zero (i.e., as the Lebesgue measure, $\mathrm{mes}(A)$, of A decreases to zero). This is illustrated in Figure 5. A similar continuity assumption must be made for each household's utility as a function of electricity consumption.

A general framework for the equilibrium analysis of pricing for differentiated commodities, based on a refinement of Bewley's (1972) model, is set up in Section 2. This framework is applicable, e.g., to peak-load pricing problems, also with a stochastic demand, and it is used for the cases of electricity and water by Horsley and Wrobel (1990a, 1990b, 1990c). Bundles of a differentiated commodity are modelled as (essentially) bounded functions on a set, Ξ, of commodity characteristics (which include the date). More precisely, there is a measure, μ, on a sigma-algebra, $\mathfrak{A}$, of subsets of Ξ, and the differentiated-commodity space is $L^\infty(\Xi, \mathfrak{A}, \mu)$, abbreviated to $L^\infty(\Xi)$. Prices are taken to be the norm-continuous linear functionals, $p \in L^{\infty *}(\Xi)$, on the commodity space. Generally, these have a singular part, p_f, as well as a density part, $p_c \in L^1(\Xi)$. The density term is a μ-integrable function on Ξ, and, therefore, it has a natural interpretation as a list of prices, with the value of any commodity bundle, $x \in L^\infty(\Xi)$, calculated as the integral, $\int_\Xi x(\xi) p_c(\xi) \mu(d\xi)$, of the quantity of each commodity, $\xi \in \Xi$, multiplied by its price. Strictly speaking, one cannot in an L^∞-economy identify singular elements of $L^{\infty *}$ as the prices of individual commodities. Formally the presence of a singularity in a price system means that the value of unit quantities of an arbitrarily small set of commodities is extraordinarily high in proportion to the size of this set of commodities (i.e., their value would not go down to zero with the measure of the set). In the case of our deterministic peak-load pricing example, for which the commodity space is $L^\infty[0, 1]$, the price has a singular component when the capital cost is concentrated on a peak of an "extremely short" duration. When, however, the peak lasts for a positive time, the peak charge is spread over the peak plateau, in which case it is mathematically represented by a density, i.e., an integrable function on $[0, 1]$. Thus, in this formalism, Boiteux's conjecture can be formulated as an assertion of the existence of an equilibrium price density, i.e., the singular component is absent from the equilibrium price. Technically, this is proved, in Theorem 2.1, under the assumption that both consumer preferences and production functions of those firms using the time-differentiated commodity as an input are Mackey continuous (for the duality between L^∞ and L^1). In fact, this assumption allows the largest possible class of continuous utility functions and production functions satisfying the requirement, stated above, that demand be harmlessly interruptible. This is because the Mackey topology is the strongest among those locally convex topologies on the commodity space $L^\infty[0, 1]$ in which the bundle $z\chi_{[0, 1] \setminus A}$, obtained by deleting the part of z with dates from a subset, A, of the time interval, converges to the original bundle, z, as the total duration of the deleted dates, mes(A), of A decreases to zero.

Under this assumption the equilibrium output, y^*, has a peak plateau, and the peak capacity cost, represented by a density, is levied during this time. This solution is illustrated in Figures 3 and 4 for the case of the simplest peak-load pricing problem, i.e., the deterministic, one-station model of electricity generation, discussed in Example 2.3.

Some explanation of the notion of "marginal cost" is due, since, in general, the cost of production, $C(y)$, is a nondifferentiable function of the output bundle, $y \in L^{\infty}_{+}(\Xi)$. To give a precise meaning to "marginal costs", we use the subdifferential, $\partial C(y)$, i.e., the set of all subgradients at y, as the concept of a generalized derivative. Since any equilibrium price, p^*, is a subgradient of the cost function, it follows that p^* automatically belongs to $L^1(\Xi)$ if the cost, $C(y)$, is a Mackey continuous function of the output bundle, y. However, in peak-load pricing problems, a part of the capital cost is proportional to the highest output level over the relevant time period, and it is Mackey lower semicontinuous but not Mackey continuous. Also, it should be noted that this cost term, viz., the supremum functional, $y \to \operatorname{ess\,sup}_{t\in[0,1]} y(t)$, is nondifferentiable. (There are multiple singular subgradients of $\operatorname{ess\,sup}$ at every $y \in L^{\infty}$, and subgradients with a density are either multiple -- if y has a peak plateau -- or non-existent: see Yamamuro (1974, (4.4.8) on p. 93 together with (4.1.4) on p. 77), Ioffe and Tihomirov (1979, Section 4.5.1, on p. 219), and Formula (2.11) below. Other cost terms may be nondifferentiable as well, e.g., in the multi-station model of electricity generation: see Horsley and Wrobel (1986a, 1988b).) In terms of subgradients, the equilibrium distribution of capital charges, ν^*, in Boiteux's solution is an element of $\partial \operatorname{ess\,sup}(y^*) \cap L^1[0,1]$.

For a multi-station model of electricity generation (to which Theorem 2.1 applies), the question of the existence of an equilibrium price density is not more complex, from the topological point of view, than in the one-station case. This is because the cost function, $C(y)$, has a decomposition as the sum of a term proportional to ess sup y and of a Mackey continuous term, $C_0(y)$, every subgradient of which has a density. (These densities are calculated by Horsley and Wrobel (1988b, Theorem 4) using the results of Horsley and Wrobel (1986b, 1987a).) However, with a multi-station technology the structure of the equilibrium price is more complicated because marginal operating costs are nonconstant, and, as a result, there are output plateaux on which marginal fuel costs are multiple, i.e., C_0 is nondifferentiable. As discussed above, it follows from the continuity of the equilibrium price density that such output plateaux occur in equilibrium. As a result, the equilibrium price system is determined <u>both</u> by an appropriate spreading of the peak capacity cost <u>and</u> by an appropriate variant of marginal operating costs (on those plateaux).

It should be noted, however, that in practice Mackey continuity assumptions about demand are by no means obviously correct. It seems that production and consumption processes are not always harmlessly interruptible, and, in practice, pointed peaks persist despite highly concentrated peak charges, when, apparently, it does not pay for firms and consumers to switch off even briefly. With point peaks, the equilibrium price includes a peak-load charge levied at the <u>peak instant</u>, and, as mentioned above, such a singularity in the price cannot be usefully represented if the commodity space is $L^\infty[0, 1]$. If, however, the commodity space is taken to be $\mathcal{C}[0, 1]$, the normed space of continuous functions on $[0, 1]$, with the dual equal to $\mathcal{M}[0, 1]$, the space of Borel measures on $[0, 1]$, then charges of this kind can be represented as scalar multiples of Dirac measures, each of which represents a charge per unit of demand at a peak instant: see Horsley and Wrobel (1986a, 1988a). This kind of equilibrium may occur if consumer preferences are norm-continuous but not Mackey-continuous: the simplest example for this is that of perfect complementarity in consumption over time, with each consumer's maximum demand occurring at a finite number of instants. The price space $\mathcal{M}[0, 1]$ is suitable for modelling this case because it contains pure point measures as well as measures with a density and, therefore, it can accommodate charges per unit of demand level (at specified times) as well as charges per unit of demand per unit of time. The difficulties with the use of $C[0, 1]$ as a commodity space, caused by the noncompactness of the unit ball (in any vector topology) are overcome by taking account of natural restrictions on the feasible demand bundles: see Horsley and Wrobel (1988a).

Results on the existence of an equilibrium price density, which are discussed in detail in Subsection 3.2, were first given by Bewley (1972). The case of exchange economies is simpler, and, under assumptions which include the Mackey lower semicontinuity of consumer preferences, Bewley shows that any equilibrium price is singularity-free. For production economies, equilibria with price singularities exist, and one can only show that, under appropriate assumptions on the production sets, the density part of every equilibrium price is itself an equilibrium price that supports the same equilibrium allocation. Bewley considers two examples with production: an intertemporal problem (in discrete time, with an infinite number of periods) and an uncertainty problem. He also offers a generalization of the relevant properties of production sets in these examples, which he calls the "Exclusion Assumption". For our purposes, however, Bewley's analysis is insufficient, since his "Exclusion Assumption" usually does not hold for firms that use a differentiated commodity as an input. This is because the loss of a part of the input bundle results in a lower output. Our point is that, in this context, it is sufficient to assume that the output (of a homogeneous commodity) is a Mackey continuous function of the input of the differentiated commodity: see Theorem 2.1. An abstract property of production sets assumed for an equilibrium price density result should include this example as a special case, and this requirement is met by the Elimination Property, given in Definition 3.3. The Elimination Property is, essentially, formulated by Back (1984, Properties E and M). However, Back (1984; 1988, Section 4) deals mainly with the structure of consumption sets, and does not give examples of production sets with the Elimination Property but not satisfying Bewley's Exclusion Assumption. By contrast, our main goal in this paper is to give an application of the result obtained by relaxing Bewley's assumption on the production sets. Such an application (Theorem 2.1) is a corollary to the more abstract Theorem 3.4, which is a somewhat streamlined version of the analysis of Back (1984, Theorems 1 and 2).

In each section (or appendix), the numbering of formulae, etc., is independent of other sections. For example, (2.1) is the first formula of Section 2, and (B.1) is the first formula of Appendix B. Each of Sections 2 and 3 has its own set of assumptions, with the first assumption for Section 2 is numbered as (a.2.1), etc. The other formal paragraphs (definitions, theorems, etc.) are numbered consecutively within each section (or appendix). Appendix A contains definitions and results about the space L^∞ needed for Sections 2 and 3. Appendix B contains proofs of the results of Sections 3 and 2 -- given in this order, since the set-up of Section 3 is a generalization of that of Section 2.

2. A Model of Equilibrium Pricing at Marginal Cost

We set up an equilibrium model for a time-differentiated commodity (which is usually also differentiated over events of delivery) such as electricity and water, supplied either by a price-taking, profit-maximizing industry or by a public utility pricing its products at exact marginal cost. The characteristics of the industry's products are taken to form a set, Ξ, with a sigma-finite, nonnegative measure, μ, on a sigma-algebra, $\mathfrak{A}$, of subsets of Ξ. Since sub-sigma-algebras of $\mathfrak{A}$ are also introduced below, it should be noted that the we use the abbreviation $L^\infty(\Xi)$ exclusively to mean $L^\infty(\Xi, \mathfrak{A}, \mu)$. The commodity space for the industry's products is $L^\infty(\Xi, \mathfrak{A}, \mu)$, abbreviated to $L^\infty(\Xi)$ or to L^∞, and its norm dual, $L^{\infty *}(\Xi, \mathfrak{A}, \mu)$, abbreviated to $L^{\infty *}(\Xi)$, is the price space for the industry's products. The value of a commodity bundle, x, at a price system, p, is denoted by $\langle x, p \rangle$. The subspace of the price space consisting of μ-integrable functions on Ξ is denoted by $L^1(\Xi)$, and it is of central importance in this model. By the Yosida-Hewitt decomposition, every $p \in L^{\infty *}(\Xi)$ can be uniquely represented as the sum of its <u>density</u> part (or countably additive part), $p_c \in L^1(\Xi)$, and of its <u>singular</u> part (or purely finitely additive part), $p_f \in L_s^{\infty *}(\Xi)$. Besides the norm, the commodity space $L^\infty(\Xi)$ is equipped with the Mackey topology for the duality with $L^1(\Xi)$, which, for brevity, is referred to as the "Mackey topology". This topology, which is usually denoted by $\tau(L^\infty, L^1)$, can be defined as the strongest locally convex topology in which the continuous dual of L^∞ is L^1. For our purposes, an essential property is that $\chi_{A_\alpha} \to 0$ in the Mackey topology as $\alpha \to \infty$ if $(A_\alpha)_{\alpha=1}^\infty$ is a μ-<u>vanishing sequence</u> of sets from $\mathfrak{A}$, i.e., a nonincreasing sequence of sets with an intersection of measure zero. Another useful property is that order-convergence in L^∞ implies convergence in the Mackey topology. These concepts and results are discussed in detail in Appendix A.

All commodities in the economy other than the given industry's products are taken to be homogeneous, i.e., nondifferentiated. It is assumed that their number is finite, and they are numbered by $n = 1, 2, \ldots, N$. Therefore, the full commodity space is $L^{\infty}(\Xi, \mathfrak{A}, \mu) \times R^N$, and a commodity bundle is written as a pair (x, m), where $x \in L^{\infty}(\Xi)$ and $m \in R^N$. A price system is written as (p, q), where $p \in L^{\infty *}(\Xi)$ and $q \in R^N$. A more abstract set-up is treated in Section 3, which can be read independently of this Section.

There is a finite number of of producers and of households (or consumers) in the economy. Households are numbered by $h = 1, 2, \ldots, H$. The set of feasible consumption plans for household h is taken to be a nonnegative orthant,

$$X_h = L_+^{\infty}(\Xi, \mathfrak{B}_h, \mu) \times R_+^N, \tag{2.1}$$

where $\mathfrak{B}_h$ is a sub-sigma-algebra of $\mathfrak{A}$, for each h. Note that, although the special case of $\mathfrak{B}_h = \mathfrak{A}$ is important, we do not assume that $\mathfrak{B}_h$ is equal to $\mathfrak{A}$ for each h, since this would be too restrictive for our applications to equilibrium with asymmetric observation of random events. The extent to which $\mathfrak{B}_h$ may differ from $\mathfrak{A}$ is, however, limited by Assumption (a.2.2) below. The initial endowment of household h is $(0, \bar{m}_h)$ with $\bar{m}_h \in R_+^N$, i.e., it consists of a nonnegative amount, $\bar{m}_{hn}$, of each homogeneous commodity. Preferences of household h are represented by a complete weak pre-order, i.e., a complete and transitive binary relation, $\lesssim_h$, in X_h. The strict order obtained as the asymmetric part of $\lesssim_h$ is denoted by $<_h$. One of the producers, referred to as "the industry", is the sole supplier of the differentiated commodity. As is usual in practical examples, the industry's production possibilities (or, equivalently, its production costs) are originally specified only for nonnegative product bundles. This is done in terms of a set, Y, consisting of commodity bundles,(y, a), each of which represents a nonnegative output of the differentiated commodity, $y \in L_+^{\infty}(\Xi)$, that the industry can produce from an input bundle, $-a \in R^N$, of the homogeneous commodities. (The possibility that the industry can also produce some of the homogeneous commodities, in which case some components of a are positive, is not excluded.) We assume that inaction is feasible, i.e., $(0,0) \in Y$. The asymptotic cone of Y, defined as the largest cone (with vertex at zero) contained in Y, is denoted by $As(Y)$. (In view assumption (a.2.7), this definition is equivalent to a number of other definitions of the asymptotic cone, also called "the recession cone".) With free disposal added, the industry's production set is the free-disposal hull of Y, i.e., it is the set

$$Y - L^{\infty}_{+}(\Xi) \times R^{N}_{+}. \tag{2.2}$$

By Part (vii) of Proposition A2, this set is equal to the set $Y^{\dagger}$, defined in Definition A1 of Appendix A. This equality is used to prove that the production set given by (2.2) is Mackey closed (if Y is).

Besides households, the industry's customers are the other producers, who are referred to as "firms" and numbered by $j = 1, 2, \ldots, J$. Firms use the differentiated commodity as an input into the production of the homogeneous commodities. The production set of firm j is denoted by Y_j, for each j. For each firm, j, its production possibilities are equivalently described by a production correspondence, F_j, from $L^{\infty}_{-}(\Xi, \mathfrak{C}_j, \mu)$ into R^N, where $\mathfrak{C}_j$ is a sub-sigma-algebra of $\mathfrak{A}$, for each j. Note that, although the special case of $\mathfrak{C}_j = \mathfrak{A}$ is important, we do not assume that $\mathfrak{C}_j$ is equal to $\mathfrak{A}$ for each j, since this would be too restrictive for our applications to equilibrium with asymmetric observation of random events. The extent to which $\mathfrak{C}_j$ may differ from $\mathfrak{A}$ is, however, limited by Assumption (a.2.4) below. For every input, $-z$, of the differentiated commodity, $F_j(z)$ is the set of those bundles, b, of the other commodities that firm j can produce from the input $-z$ of the differentiated commodity. Note that some of the homogeneous commodities may also be used by a firm as inputs, in which case they are represented by negative components of b. We assume that inaction is feasible, i.e., $0 \in F_j(0)$. The production set of firm j has the representation as the graph of F_j:

$$Y_j = \{(z, b) \in L^\infty_-(\Xi, \mathbf{C}_j, \mu) \times R^N \mid b \in F_j(z)\}, \tag{2.3}$$

for each j. For a single-product firm, its production possibilities can also be described by a concave production function, f_j, from $L^\infty_-(\Xi, \mathbf{C}_j, \mu) \times R^{N-1}_-$ into R, with $f_j(0, 0) = 0$. In this special case, the number $f_j(z, b_{-n_j})$ is the maximum output of the single, homogeneous commodity, n_j, that firm j can produce from the input $-z$ of the differentiated commodity and from the input

$$-b_{-n_j} = -(b_1, \ldots, b_{n_j - 1}, b_{n_j + 1}, \ldots, b_N)$$

of the other N-1 homogeneous commodities. (We follow the convention that the minus sign before a subscript to b denotes the vector obtained by deleting the corresponding component of b. Also, we write $b = (b_{-n_j}, b_{n_j})$.) On the assumptions of free disposal and that the output is nondecreasing in the amounts of inputs, the production set of a single-product firm has the representation as the hypograph of the firm's production function, f_j, i.e.,

$$Y_j = \{(z, b) \in L^\infty_-(\Xi, \mathbf{C}_j, \mu) \times (R^{N-1}_- \times R) \mid b_{n_j} \le f_j(z, b_{-n_j})\}, \tag{2.4}$$

or, equivalently, the firm's production correspondence is

$$F_j(z) = \{b \in R^{N-1}_- \times R \mid b_{n_j} \le f_j(z, b_{-n_j})\}, \tag{2.5}$$

for every $z \in L^\infty_-(\Xi, \mathbf{C}_j, \mu)$.

The share of household h in the profits of firm j is denoted by s_{hj}, with $s_{hj} \ge 0$ and $\Sigma_h s_{hj} = 1$ for each j. The share of household h in the industry is denoted by s_h, with $s_h \ge 0$ and $\Sigma_h s_h = 1$ (in the case of constant returns to scale, these shares are immaterial for the competitive equilibrium solution). The concepts of: an allocation, a competitive quasi-equilibrium, and a competitive equilibrium for this economy are defined in the usual way, which is given in Section 3 for a more general set-up. The correspondence of notation between this Section and Section 3 required for reference to Section 3 is: $(x_h, m_h) = \mathbf{x}_h$, $(0, \bar{m}_h) = \bar{\mathbf{x}}_h$, $(z_j, b_j) = \mathbf{y}_j$, $(p, q) = \mathbf{p}$, etc.

The following assumptions on households, firms and the industry are made for this Section. Assumptions (a.2.1), (a.2.5), (a.2.7) and (a.2.9) are the usual assumptions of: nonsatiation of households, convexity of preferences, convexity and closedness of production sets for firms, and boundedness of the set of feasible allocations, respectively. Their roles are explained, e.g., by Debreu (1962) and Bewley (1972, p. 520). Assumption (a.2.8), used together with Parts (vii) and (i) of Proposition A2, guarantees that the industry's production set -- with free disposal included, i.e., the set given by Formula (2.2) -- is Mackey closed. Assumption (a.2.10) is a rudimentary form of the adequacy (or survival) assumption. It guarantees that in any quasi-equilibrium all households have a positive income, and, therefore, that each quasi-equilibrium is an equilibrium. As usual, this is done by ensuring that each household's initial endowment has a positive value, and it follows that for the adequacy assumption to hold, all (or at least sufficiently many) of the productive factors should be explicitly included in the list of commodities, so that rents on fixed factors are modelled as endowment income rather than as profit. A weaker form of the adequacy assumption can be obtained by using the concept of an irreducible economy: see McKenzie (1959 and 1961). The rest of the assumptions, which relate to Mackey continuity, are crucial for the existence of an equilibrium price density. For clarity, the definitions of "semicontinuity" and "hemicontinuity" (for functions, orders, and correspondences) are given in Appendix A, since the use of these terms varies in the literature. The assumption of upper semicontinuity for preferences, included in (a.2.3), is needed for the existence of an equilibrium price in $L^{\infty *}$. For the existence of a density for equilibrium prices, the assumption of lower semicontinuity for preferences, included in (a.2.3), is needed, as is explained by Bewley (1972, p. 523). Assumption (a.2.2) is also needed for this purpose (in Bewley's set-up it holds trivially, since $\mathcal{B}_h = \mathfrak{A}$ for each h). Since, as well as consumer demand, there is also an input demand for the industry's products in our model, lower semicontinuity of firms' production functions is also needed for the price density result (just as, for households, semicontinuity of utility functions is needed), and it is assumed in (a.2.6). In our context, this assumption is significantly weaker than Bewley's "Exclusion Assumption", which is quoted in Formula (3.1) below. In our model, Bewley's "Exclusion Assumption" does not normally hold for firms, because it would mean that a small subset of commodities in the firm's input bundle of the differentiated commodity could be dropped without any loss of the firm's output at all. Instead of such a "no loss" assumption, which would usually be false, we make a "small loss" assumption, and this is (a.2.6). (Assumption (a.2.4) is also needed for the price density result, similarly as Assumption (a.2.2); in Bewley's set-up it holds trivially, since $\mathfrak{C}_j = \mathfrak{A}$ for each j.) In Section 3, we formulate an abstract property of production sets, which we call the Elimination Property, and which includes the example of a firm with a Mackey continuous production function as a special case. For the industry, which is a producer of the differentiated commodity (rather than its user), Bewley's "Exclusion Assumption" holds automatically.

(a.2.1) For each household, h, the preference relation, $\lesssim_h$, is: (i) Mackey locally nonsatiated, and (ii) convex. (Part (ii) of this assumption holds if preferences are represented by a quasi-concave utility function.)

(a.2.2) For every μ-vanishing sequence, $(A_\alpha)_{\alpha=1}^{\infty}$, of sets from $\mathfrak{A}$, and for each household, h, there exists a μ-vanishing sequence, $(A_\alpha^h)_{\alpha=1}^{\infty}$, of sets from $\mathcal{B}_h$ with $A_\alpha \subset A_\alpha^h$ for each $\alpha = 1, 2, \ldots$.

(a.2.3) For each household, h, the preference relation, $\lesssim_h$, is Mackey continuous. (This assumption holds if preferences are represented by a Mackey continuous utility function.)

(a.2.4) For every μ-vanishing sequence, $(A_\alpha)_{\alpha=1}^{\infty}$, of sets from $\mathfrak{A}$, and for each firm, j, there exists a μ-vanishing sequence, $(A_\alpha^j)_{\alpha=1}^{\infty}$, of sets from $\mathfrak{C}_j$ with $A_\alpha \subset A_\alpha^j$ for each $\alpha = 1, 2, \ldots$.

(a.2.5) For each firm, j, its production set, Y_j, is convex and Mackey closed (i.e., closed in the product of the Mackey topology on $L^\infty(\Xi)$ and the usual topology of R^N). Put in terms of the firm's production correspondence, F_j, this assumption means, by Formula (2.3), that the graph of F_j is convex and Mackey closed. (In the case of a single-product firm, with Y_j and F_j given by Formulae (2.4) and (2.5) in terms of the production function, f_j, this assumption is also equivalent to the concavity and Mackey upper semicontinuity of f_j.)

(a.2.6) For each firm, j, the production correspondence of firm j, $z \to F_j(z)$, is Mackey lower hemicontinuous. (In the case of a single-product firm -- with F_j defined in terms of f_j by Formula (2.5), and under Assumption (a.2.5) -- this is equivalent to the Mackey lower semicontinuity of the production function, f_j, in its first variable, i.e., in the input bundle of the differentiated commodity.)

(a.2.7) The industry's production set (before the inclusion of free disposal of output), Y, is a Mackey closed and convex subset of $L_+^\infty(\Xi) \times R^N$.

(a.2.8) (i) The set Y has the following monotonicity properties:

$$\text{If } (y, a) \in Y \text{ and } 0 \le y' \le y, \text{ then } (y', a) \in Y; \tag{2.6}$$

and

$$\text{If } (y, a) \in Y \text{ and } a' \le a, \text{ then } (y, a') \in Y. \tag{2.7}$$

(a.2.9) The set

$$Y \cap (L^\infty_+(\Xi) \times R^N_+ - \sum_j Y_j - \sum_h (0, \bar{m}_h))$$

and, for each $j' = 1, 2, \ldots, J$, the set

$$Y_{j'} \cap (L^\infty_+(\Xi) \times R^N_+ - Y - \sum_{j \neq j'} Y_j - \sum_h (0, \bar{m}_h))$$

are bounded (in the norm or, equivalently, in the weak* topology on $L^\infty(\Xi)$).

(a.2.10) (i) Each household is endowed with a positive amount of each homogeneous commodity, i.e., $\bar{m}_{hn} > 0$ for each h and n.

(ii) The industry can produce every output using production techniques with constant returns to scale, i.e., for every $y \in L^\infty_+(\Xi)$ there exists $a \in R^N$ with $(y, a) \in As(Y)$.

Theorem 2.1. Assume (a.2.1) to (a.2.10). Then there exists a competitive equilibrium with a price system for the differentiated commodity that is represented by a density, $p^* \in L^1(\Xi)$.

Production-supporting prices for the industry's outputs can be calculated as marginal costs. This is convenient when, as in Example 2.3 below, the relationship between the properties of the output prices and the properties of the output bundle is to be studied. For any input prices, $q \in R^N_+$, and for any nonnegative output bundle, $y \in L^\infty_+(\Xi)$, the production cost can be defined as

$$C(y, q) = \inf \{-\langle a, q\rangle \mid (y, a) \in Y\},$$

and this formula is extended to all $y \in L^\infty(\Xi)$ by

$$C^\dagger(y, q) = \inf \{-\langle a, q\rangle \mid (y, a) \in Y^\dagger\} = \inf \{-\langle a, q\rangle \mid (y^+, a) \in Y\}$$
$$= C(y^+, q).$$

In many examples the cost, $C(y, q)$, is a nondifferentiable function of the output bundle, y, e.g., the cost functions for electricity generation and for water supply, studied by Horsley and Wrobel (1986a, 1987b, 1990b). In the case of a nondifferentiable cost function, the notion of "marginal cost" requires clarification, and, to give it a precise meaning, we use the subdifferential, i.e., the collection of all subgradients, $\partial_1 C(y, q)$, as the concept of a generalized derivative, with respect to the first (vector) variable of C, at any point, (y, q). Properties of subdifferentials are discussed by, e.g., Ioffe and Tihomirov (1979).

Remark 2.2. (i) At any input prices, q, and any price system, p, for the industry's products, if a production plan, (y, a), maximizes the industry's profit, then $p \in \partial_1 C^\dagger(y, q)$. For this reason, if (p^*, q^*) is a competitive equilibrium price system, then p^* is termed an <u>equilibrium marginal cost price</u> system (for the differentiated commodity).

(ii) For the case of constant returns to scale, i.e., when Y is a cone, the industry's marginal costs can be characterized as follows. At any input prices, q, a price system for the industry's products, p, is a marginal cost at the zero output bundle if and only if the value of every feasible input-output bundle, calculated at prices (p, q), is nonpositive. Formally, $p \in \partial_1 C^{\dagger}(0, q)$ if and only if (p, q) belongs to the polar cone of $Y^{\dagger}$, usually denoted by $Y^{\dagger *}$. For any output, y, one has $p \in \partial_1 C^{\dagger}(y, q)$ if and only if $p \in \partial_1 C^{\dagger}(0, q)$ and $C^{\dagger}(y, q) = \langle y, p \rangle$.

Example 2.3. Theorem 2.1 can be applied as follows to solve the shifting-peak problem in peak-load pricing at marginal long-run cost. For simplicity, consider the deterministic, one-station model of electricity generation with constant returns to scale. Besides electricity, generating equipment and fuel, there may be other goods in the model (the number of which is $N-2$). In this example, the set of commodity characteristics, Ξ, is the unit interval of the real line, $[0, 1]$, which represents the relevant time period (usually a year). This interval is taken with the sigma-algebra of its Borel subsets, $\mathcal{B}$, and with the Lebesgue measure, mes, on $\mathcal{B}$. With the installed capacity, k, and the output level, $y(t)$, measured in, say, MW, and with the amount of fuel, v, measured in MWyears (one MWyear of fuel is defined as the amount needed to run a unit station continuously for a year), the production set, Y, is given by

$$Y = \{(y, (-k, -v), 0) \in L^{\infty}_{+}[0, 1] \times R^2 \times R^{N-2} \mid \operatorname*{ess\,sup}_{t \in [0,1]} y(t) \le k, \int_0^1 y(t)\, dt \le v\}, \qquad (2.8)$$

where "ess sup" stands for the essential supremum with respect to the Lebesgue measure. With the unit capital cost per period denoted by r, and the unit fuel cost denoted by w, the (long-run) cost function derived from the production set Y is given by

$$C(y; r, w) = w \int_0^1 y(t)\, dt + r \operatorname*{ess\,sup}_{t \in [0,1]} y(t), \qquad (2.9)$$

for nonnegative y, and its free-disposal extension to all $y \in L^{\infty}[0, 1]$ is

$$C^{\dagger}(y; r, w) = w\int_0^1 y^+(t)\,dt + r \operatorname*{ess\,sup}_{t\in[0,1]} y^+(t). \qquad (2.10)$$

The set Y is Mackey closed, since ess sup y is a Mackey lower semicontinuous function of y, and since the integral in Formula (2.8) is Mackey continuous in y. The set Y also satisfies the rest of Assumption (a.2.7), Assumption (a.2.8) and Part (ii) of Assumption (a.2.10). Therefore, with firms and households that satisfy the other assumptions of Theorem 2.1, there exists an equilibrium price system, $(p^*; r^*, w^*, \ldots)$, with $p^* \in L^1[0, 1]$. (The prices r^*, w^*, etc., are scalars.) The corresponding equilibrium output of electricity is $y^* \geq 0$. By Part (i) of Remark 2.2,

$$p^* \in \partial_1 C^{\dagger}(y^*; r^*, w^*).$$

We assume that $y^* \neq 0$ and, also, that $r^* > 0$ (this can readily be guaranteed by additional assumptions). For simplicity, suppose first that $y^*(t) > 0$ for (almost) every t; then $\partial_1 C^{\dagger}(y^*; r^*, w^*) = \partial_1 C(y^*; r^*, w^*)$, from (2.9)–(2.10). At every $y \in L^{\infty}[0, 1]$,

$$\partial \operatorname{ess\,sup}(y) \cap L^1[0, 1] = \{\nu \in L^1_+ \mid \int_0^1 \nu(t)\,dt = 1, \nu(t) = 0 \text{ if } y(t) < \operatorname{ess\,sup} y\}. \qquad (2.11)$$

This follows from the fact that, at each y, every subgradient of the supremum functional, $\nu \in \partial \operatorname{ess\,sup}(y)$, is supported, for every real number $\delta > 0$, by the set of approximate (up to δ) peaks of y, i.e., by the set

$$\{t \in [0, 1] \mid y(t) > \operatorname{ess\,sup} y - \delta\}. \qquad (2.12)$$

(Formula (2.11) is also given by, e.g., Ioffe and Tihomirov (1979, Section 4.5.1, on

p. 219).) By Formula (2.11), a subgradient of ess sup with a density exists if and only if $\text{mes}\{t \mid y(t) = \text{ess sup } y\} > 0$, i.e., if y has a peak plateau. Since the equilibrium price, p^*, has a density, i.e.,

$$p^* \in \partial_1 C^\dagger(y^*; r^*, w^*) \cap L^1[0,1] = \{w^* + r^*\nu \mid \nu \in \partial \, \text{ess sup}(y^*) \cap L^1[0,1]\},$$

it follows that the equilibrium output of electricity, y^*, has a peak plateau, over which the peak charge, equal in total to the unit capacity cost, r, is spread, i.e.,

$$p^*(t) = w^* + r^*\nu^*(t), \tag{2.13}$$

for some $\nu^* \in L^1[0,1]$ with

$$\int_0^1 \nu^*(t)\,dt = 1, \tag{2.14}$$

and

$$\nu^*(t) = 0 \quad \text{for (almost) every } t \in [0,1] \text{ with } y^*(t) < \text{ess sup } y^*. \tag{2.15}$$

If $y^* \neq 0$ but y^* is not strictly positive, then the constant w^* in Formula (2.13) has to be replaced by some (Borel-measurable) function, ϕ^*, on $[0,1]$ with the properties: $0 \leq \phi^*(t) \leq w^*$ for all t and $\phi^*(t) = w^*$ for all those t with $y^*(t) > 0$.

In the stochastic version of this Example, the peak plateau extends across states of the world as well as time.

Remark 2.4. (i) In the above peak-load pricing example <u>every</u> equilibrium price is singularity-free, unless the corresponding equilibrium output, y^*, is zero. To see this formally, take any equilibrium price, $(p^*; r^*, w^*, \ldots)$, with $p^* \in L^{\infty *}[0, 1]$. Then, by Theorem 2.1, the density part, $(p^*_c; r^*, w^*, \ldots)$ is also an equilibrium price which supports the same equilibrium allocation. Since C is linearly homogeneous in the output, it follows that $\langle y^*, p^* \rangle = C(y^*; r^*, w^*) = \langle y^*, p^*_c \rangle$. Hence, $\langle y^*, p^*_f \rangle = 0$. Since: $\operatorname{ess\,sup} y^* > 0$, $p^*_f \geq 0$, and p^*_f is supported by the set (2.13) for every positive δ, it follows, by taking a δ smaller than $\operatorname{ess\,sup} y^*$, that $p^*_f = 0$.

(ii) The equilibrium allocation can be supported by a price with a singular term in the degenerate case in which the equilibrium output, y^*, is zero. To give an example, assume, for simplicity, that in Example 2.3 the input prices, r and w, are fixed. This is an economy in which an electricity bundle, $y \in L^\infty[0, 1]$, can be produced at a cost, in terms of the numeraire commodity, given by Formula (2.9). There is one household, with an initial endowment, $\bar{m}$, of the numeraire commodity only, and with a utility function on $L^\infty_+[0, 1] \times R_+$ given by $U(x, m) = m + \int_0^1 u(t, x(t))\,dt$, where $u\colon [0, 1] \times R_+ \to R$ is assumed to be concave and nondecreasing in its second variable and to satisfy also the other conditions listed by Bewley (1972, p. 535). Furthermore, suppose that at zero consumption level the marginal utility of electricity, i.e., the partial derivative of u with respect to its second variable, is finite and integrable over t, i.e., $\int_0^1 D_2u(t, 0)\,dt < +\infty$. Then any price, p^*, with the density part $p^*_c(t) = D_2u(t, 0)$ and with any nonnegative singular part, p^*_f, of norm strictly less than $r - \int_0^1 (D_2u(t, 0) - w)^+\,dt$ (which can be made positive by the choice of r and w) supports the equilibrium allocation in which there is no production and the household consumes its initial endowment.

In much of the literature on marginal cost pricing it is assumed that the prices, q, of all commodities other than the given industry's products are fixed. In this case all commodities in the economy other than the industry's products can be aggregated into a homogeneous numeraire commodity (by positing that, to all intents and purposes, a unit of commodity n is equivalent to q_n units of the numeraire commodity). The full commodity space is then $L^\infty(\Xi) \times \mathbf{R}$, and the minimum production cost, $C(y)$, of any nonnegative output bundle, $y \in L^\infty_+(\Xi)$, is expressed in terms of the numeraire commodity. The industry's production set, Y, is equal to the hypograph of $-C$, i.e.,

$$Y = \{(y, a) \in L^\infty_+(\Xi) \times \mathbf{R} \mid a \le -C(y)\}. \tag{2.16}$$

In this context, Assumptions (a.2.7)-(a.2.8) and Part (ii) of (a.2.10) are implied by the following set of assumptions on the cost function: C, defined on $L^\infty_+(\Xi)$, is finite, convex, nondecreasing and Mackey lower semicontinuous, $C(0) = 0$, and the recession function[2] of C is norm-continuous. Under these assumptions, the free-disposal hull of the set Y given by Formula (2.16) is equal to the hypograph of $-C^\dagger$, i.e.,

$$Y - L^\infty_+(\Xi) \times \mathbf{R} = \{(y, a) \in L^\infty(\Xi) \times \mathbf{R} \mid a \le -C^\dagger(y)\}, \tag{2.17}$$

where $C^\dagger(y) = C(y^+)$. Since lattice operations (such as taking the nonnegative part) are Mackey continuous in L^∞, Mackey lower semicontinuity of C implies that of $C^\dagger$. It follows that $-C^\dagger$ is Mackey upper semicontinuous, or, equivalently, that its hypograph is Mackey closed. This is another way of phrasing the proof of the closedness of the free-disposal hull of Y, given in Parts (i) and (vii) of Proposition A2.

3. The Existence of an Equilibrium Price Density for Production Economies

In this Section we study the question of the existence of a density for equilibrium prices in a more abstract set-up than that of Section 2. For ease of reference, in this Section bold letters, e.g., $\mathbf{x}$ and $\mathbf{p}$, are used to denote commodity bundles and price systems, since in Section 2 the letters x and p denote bundles and price systems for the differentiated commodity only.

3.1 The Model

Consider an economy with the commodity space $L^{\infty}(M, \mathfrak{M}, \mu_M)$, where $(M, \mathfrak{M}, \mu_M)$ is a sigma-finite, nonnegative measure space. Its norm dual, $L^{\infty *}(M, \mathfrak{M}, \mu_M)$, is taken to be the price space. (The framework of Section 2 is a special case of this, with the underlying measure space equal to the direct sum of $(\Xi, \mathfrak{X}, \mu)$ and of the counting measure on the set $\{1, 2, ..., N\}$.) For brevity, the symbols M, $\mathfrak{M}$, and μ_M are suppressed except where their use helps clarity. All the concepts and results that we use about these spaces are given in Appendix A, including the Yosida-Hewitt decomposition of every $\mathbf{p} \in L^{\infty *}$ into the sum of its <u>density</u> part (or countably additive part), $\mathbf{p}_c \in L^1$, and of its <u>singular</u> part (or purely finitely additive part), $\mathbf{p}_f \in L^{\infty *}_s$. By the "Mackey topology" we always mean the Mackey topology on L^{∞} for the duality with L^1.

There is a finite number of households and producers in the economy. Households are numbered by $h = 1, 2, ..., H$. The set of feasible consumption plans for household h is denoted by X_h. The initial endowment of household h is denoted by $\bar{\mathbf{x}}_h$. The preferences of household h are represented by a complete weak pre-order, i.e., a complete and transitive binary relation, $\precsim_h$, in X_h. The strict order obtained from the weak pre-order $\precsim_h$ is denoted by $\prec_h$. Producers are numbered by $j = 0, 1, 2, ..., J$.

(For an application of the result of this Section in the framework of Section 2, it is convenient to number "the industry" of Section 2, with the production set $Y^{\dagger}$, as the 0-th producer.) The production set of producer j is denoted by Y_j. The share of household h in the profits of producer j is denoted by s_{hj}, with $s_{hj} \geq 0$ and $\Sigma_h s_{hj} = 1$ for each j. An allocation is a list of consumption plans, $x_h \in X_h$ for each household, h, and of production plans, $y_j \in Y_j$ for each producer, j. In summations, etc., we follow the convention that the range of a subscript (or a superscript) is the largest possible, with any restrictions specified; e.g., in Condition (i) of the following definition, h ranges from 1 to H and j ranges from 0 to J.

Definition 3.1. A pair consisting of an allocation, $((x^*_h)_{h=1}^{H}, (y^*_j)_{j=0}^{J})$, and of a price system, $p^* \in L^{\infty *}$, is termed a competitive quasi-equilibrium if:

(i) $\sum_h (x^*_h - \bar{x}_h) = \sum_j y^*_j$,

(ii) for each j, $\langle y^*_j, p^* \rangle = \sup \{\langle y_j, p^* \rangle \mid y_j \in Y_j\}$,

(iii) for each h, $\langle x^*_h, p^* \rangle \leq \langle \bar{x}_h + \sum_j s_{hj} y^*_j, p^* \rangle$,

(iv) for each h and for every $x \in X_h$, if $\langle x, p^* \rangle < \langle x^*_h, p^* \rangle$, then $x \precsim_h x^*_h$,

and

(v) $p^* \neq 0$.

Note that, in view of Condition (i), the inequality in Condition (iii) holds as an equality. An allocation and a price system, $((x^*_h), (y^*_j), p^*)$, is termed a competitive equilibrium if: it is a competitive quasi-equilibrium, and, in addition, (iv') for each h and for every $x \in X_h$, if $\langle x, p^* \rangle \leq \langle x_h^*, p^* \rangle$, then $x \lesssim_h x_h^*$.

Remark 3.2. (i) Assume that Y_0 is a cone, and denote its polar by Y_0°. Then for $j = 0$ Condition (ii) in Definition 1 can equivalently be replaced by the conditions that $\langle y_0^*, p^* \rangle = 0$ and that $p^* \in Y_0^\circ$, i.e., $\langle y^*, p^* \rangle \leq 0$ for all $y \in Y_0$.

(ii) If some household, h, is nonsatiated at x_h^*, then Condition (iv') in Definition 3.1 implies that $p^* \neq 0$.

3.2. A Review of Price Density Existence Problem

Results on the existence of an equilibrium price density were first given by Bewley (1972, Theorems 2 and 3). His first result deals with exchange economies, and his second is an extension to production economies, designed for applications to intertemporal problems (modelled in a sequence commodity space, ℓ^∞) and to uncertainty. For exchange economies, Bewley (1972, Theorem 2) shows that if: (i) the total initial endowment is assumed to be positive and bounded away from zero (with all the consumption sets taken to be equal to the nonnegative orthant, L_+^∞), and (ii) consumer preferences are Mackey lower semicontinuous, then any equilibrium price, p^*, is singularity-free, i.e., $p_f^* = 0$. Bewley (1972, p. 523) also offers the following heuristic explanation for the absence of a singular term in equilibrium prices: though one cannot in an L^∞-economy identify singular elements of $L^{\infty *}$ as the prices of <u>individual</u> <u>commodities</u>, formally their presence in a "mathematical" price system would mean that the total cost of unit quantities of an arbitrarily small set of commodities would be extraordinarily high in proportion to the size of this set of commodities (i.e., the cost would not go down to zero with the measure of the set), and consumers with

Mackey continuous preferences would want to trade such a set of commodities for cheaper ones. These commodities would then be in excess supply, since they are present in the total initial endowment, and this cannot be the case in an equilibrium. In formal terms, it is shown that $\langle x_h^*, p_f^* \rangle = 0$ for each consumer, h, where x_h^* is the equilibrium consumption bundle of consumer h. Since $\Sigma_h x_h^*$ is equal to the total initial endowment which, by assumption, is greater than some positive constant, and since $p_f^* \geq 0$, it follows that $p_f^* = 0$. (This formulation of the argument is given by Back (1988, the proof of Theorem 4, for the case $X_i = L_+^\infty$ and $\tilde{x}_i = 0$ for each consumer, i), who also extends Bewley's result to the case of more general consumption sets.)

For production economies, there is a number of cases to consider, which we now discuss to improve upon previous expositions and elucidate the matter. We retain the assumption that consumer preferences are Mackey lower semicontinuous. There is little problem if: (i) a singularity in a price system can only occur on a set of commodities that are not used as inputs by any producer, and (ii) the assumption that the total initial endowment is bounded away from zero is kept. In this case, for commodities with singular prices there is neither an input demand nor, by the argument given above for exchange economies, a consumer demand. It follows that, at singular prices, these commodities are in excess supply (since they are present in the total initial endowment), and the economy could not be in equilibrium. If Condition (i) above does not hold, then the total initial endowment of expensively priced commodities may be zero (or arbitrarily close to zero), and the position is more complicated. There are equilibria (an example is given in Remark 2.4) in which: (i) the price system has a singular component, (ii) demand for the expensively priced commodities is zero (since singular prices depress demand, as argued above), and (iii) the initial endowment of those commodities is zero, and so is their total supply because their equilibrium output is also zero. (Singular prices do not necessarily result in a positive output of the expensively priced commodities, unless their joint production cost is a Mackey continuous function of the output bundle. The point is that this function is usually not Mackey continuous but only Mackey lower semi-continuous, e.g., when there are capacity costs: see Section 2. Mackey lower semicontinuity of cost is, essentially, equivalent to Mackey closedness of the corresponding production set.) For peak-load pricing problems in which a price singularity can occcur only at peak output, it follows, however, that the only equilibria with singular prices are degenerate ones in which the output is zero: for a formal proof, see Part (i) of Remark 2.4.

Thus, when the total initial endowment is not bounded away from zero, it is not always true that every equilibrium price, p^*, is singularity-free. Instead, one proves that its density part, p^*_c, is itself an equilibrium price which supports the same equilibrium allocation, consisting of consumption plans, x^*_h, for each household, h, and of production plans, y^*_j, for each producer, j. The proof depends on showing that $\langle y^*_j, p^*_f \rangle \geq 0$ for each j. Given that p^*_f is nonnegative, this inequality holds automatically if p^*_f is concentrated on the set of outputs of producer j. This is automatically the case, e.g., for the production of a differentiated commodity from a finite number of homogeneous input commodities, as in the uncertainty example of Bewley (1972, p. 527, lines 12-20) and for "the industry" in the framework of Section 2 above. In some instances when commodities cannot a priori, i.e., before choosing a production bundle, be classed as either (net) inputs or outputs, e.g., in the intertemporal example of Bewley (1972, p. 527, lines 3-11), the inequality $\langle y^*, p^*_f \rangle \geq 0$ follows from the feasibility of inaction and free disposal of output. (A formal argument for that example, in which the commodity space is the sequence space ℓ^∞, goes as follows. Observe that, if $(y(t))_{t=1}^{\infty}$ is a feasible production plan, then the truncated plan, $y_{|T}$ (defined, for any natural number T, by $y_{|T}(t) = y(t)$ for $t < T$ and by $y_{|T}(t) = 0$ for $t \geq T$) is also feasible -- even if some of the discarded components, $y(t)$ for $t \geq T$, are negative. Next, note that $y^*_{|T}$ converges to y^* in the Mackey topology as $T \to \infty$, and, also, $\langle y^*_{|T}, p^*_f \rangle = 0$ for all T. Therefore, $\langle y^*, p^*_f \rangle < 0$ would imply that $\langle y^*_{|T}, p^* \rangle > \langle y^*, p^* \rangle$ for large enough T, which would contradict the profit-maximizing property of $\mathbf{y}^\#$ at prices $\mathbf{p}^\#$.) However, as we explain in the Introduction, Bewley's generalization of the relevant properties of production sets in these examples, i.e., his "Exclusion Assumption", usually does not hold (for firms) in the framework of Section 2 above. For this reason we formulate the Elimination Property (Definition 3.3 below), and we give Theorem 3.4, from which our main result, Theorem 2.1, follows as a special case. The proof of Theorem 3.4 is obtained by extending to the case of production economies the idea that, at singular prices, the "expensive commodities" would be in excess supply. One shows that the presence of a singularity in a mathematical price system would imply that producers with the Elimination Property (with respect to the zero bundle), and consumers with Mackey continuous preferences and consumption sets with Elimination Properties (with respect to the initial endowments), would not purchase the "extraordinarily expensive" commodities. In formal terms, this means that $\langle y_j^\#, \mathbf{p}_f^\# \rangle \geq 0$ for each producer, j, and that $\langle \mathbf{x}_h^\# - \bar{x}_h, \mathbf{p}_f^\# \rangle \leq 0$, where $\bar{x}_h$ is the initial endowment of consumer h, for each h. (These are Formulae (B.2) and (B.4), with the latter quoted here for the case $\hat{x}_h = \bar{x}_h$, in the proof of Theorem 3.4.)

Since, as stated by Aliprantis and Burkinshaw (1978, Exercise 4 on p. 163), the space L^∞ with the Mackey topology is a topological vector lattice, the general result about the existence of an equilibrium given by Richard (1989), who builds on the work of Mas-Colell (1986), is applicable to models with this commodity space. However, it yields a weaker result than that of Theorem 3.4, since, in addition to the fact that not all Mackey-continuous preferences are uniformly proper (as noted by Back (1988, p. 97-98), also on the production side Mackey uniform properness is a stronger assumption than the Elimination Property for production sets. For example, in the framework of Section 2, Mackey uniform properness for a firm using the differentiated commodity as an input not only implies that the firm's production function, f, is Mackey lower semicontinuous in this input, but also imposes a lower bound on the difference quotients[3] of f. Also, as pointed out in Remark 2.4, by using the Yosida-Hewitt decomposition, one obtains in many cases of interest the result that every equilibrium price is in L^1, which cannot be deduced from Richard's general result.

3.3 The Elimination Property and the Existence of an Equilibrium Price Density

To formulate our result on the existence of an equilibrium price density in the present, abstract framework, we next define a property which, when possessed by production sets, is a generalization of Mackey continuity of production functions (or correspondences), assumed in (a.2.6) for Theorem 2.1.

Definition 3.3. Let Z be a subset of, and $\hat{z}$ a point in, $L^\infty(M, \mathfrak{M}, \mu_M)$. Then:

(i) The set Z has the Weak Elimination Property with respect to $\hat{z}$ if: for every $p \in L^{\infty *}(M, \mathfrak{M}, \mu_M)$, for every $z \in Z$ and for every number $\delta > 0$, there exists $z' \in Z$ with $|\langle z' - z, p_c\rangle| < \delta$ and $|\langle z' - \hat{z}, p_f\rangle| < \delta$.

(ii) The set Z has the Elimination Property with respect to $\hat{z}$ if: for every $p \in L^{\infty *}(M, \mathfrak{M}, \mu_M)$, for every $z \in Z$, for every open neighbourhood, V, of z in the Mackey topology, and for every number $\delta > 0$, there exists $z' \in V \cap Z$ with $|\langle z' - \hat{z}, p_f\rangle| < \delta$.

(iii) The set Z has the Monotone Elimination Property with respect to $\hat{z}$ if: for every $p \in L^{\infty *}(M, \mathfrak{M}, \mu_M)$ and for every $z \in Z$, there exists a sequence, $(z^\alpha)_{\alpha=1}^\infty$, in Z with $z^\alpha \uparrow z$ as $\alpha \to \infty$ and $\langle z^\alpha - \hat{z}, p_f\rangle = 0$ for every $\alpha = 1, 2, \dots$.

The Monotone Elimination Property is stronger than the Elimination Property. The latter is formally stronger than the Weak Elimination Property, but, in our applications, the assumptions made imply that the relevant sets have at least the Elimination Property. (Our only reason for using the Weak Elimination Property is that it is weaker than the "Exclusion Assumption" of Bewley (1972): see Remark 3.5 below.) We can now state an equilibrium price density result due essentially to Back (1984).

Theorem 3.4. Assume that:

(a.3.1) For each household, h, the preference relation, $\precsim_h$, is Mackey lower semicontinuous, i.e., for every $x \in X_h$ the set $\{z \in X_h \mid z \precsim_h x\}$ is Mackey closed.

(a.3.2) For each household, h, the preference relation, $\precsim_h$, is Mackey locally nonsatiated, i.e., for every $x \in X_h$ the Mackey closure of the set $\{z \in X_h \mid x \prec_h z\}$ contains x.

(a.3.3) For some $\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_H$ with $\Sigma_h \hat{x}_h$ equal to the total initial endowment, $\Sigma_h \bar{x}_h$, the consumption set, X_h, has the Elimination Property with respect to $\hat{x}_h$, for each h; and

(a.3.4) For each producer, j, the production set Y_j has the Weak Elimination Property with respect to 0.

Then: (i) If $((x^*_h), (y^*_j), p^*)$ is a competitive quasi-equilibrium and $p^*_c \neq 0$, then $((x^*_h), (y^*_j), p^*_c)$ is also a competitive quasi-equilibrium; and

(ii) Assume, in addition, that $X_h \subset L^\infty_+$ and that X_h has the Monotone Elimination Property with respect to 0, for each h. If $((x^*_h), (y^*_j), p^*)$ is a competitive equilibrium and $p^* \geq 0$, then $((x^*_h), (y^*_j), p^*_c)$ is also a competitive equilibrium.

Remark 3.5. (i) The nonnegative orthant of the <u>whole</u> space, $L_+^\infty(M, \mathfrak{M}, \mu_M)$, has the Elimination Property with respect to each of its points; for a proof of this, see Back (1988, p. 96, lines 1-6 from below; however, unless μ is finite, the condition that $\mu(F_\nu) \to 0$ must be replaced by $\mu(\bigcap_{\nu=1}^\infty F_\nu) = 0$). Also, this orthant has the Monotone Elimination Property with respect to 0. Note that the orthant taken with respect to a sub-sigma-algebra, $\mathfrak{N}$, of $\mathfrak{M}$, considered as a subset of $L^\infty(M, \mathfrak{M}, \mu_M)$, need <u>not</u> have the Elimination Property. However, if $\mathfrak{N}$ and $\mathfrak{M}$ satisfy the condition obtained from Assumption (a.2.2) by substituting $\mathfrak{N}$ for $\mathfrak{B}_h$ and $\mathfrak{M}$ for $\mathfrak{A}$, then $L_+^\infty(M, \mathfrak{N}, \mu_M)$, considered as a subset of $L^\infty(M, \mathfrak{M}, \mu_M)$, has the Elimination Property with respect to each of its points and, also, the Monotone Elimination Property with respect to 0. (This is because, in this case, singular functionals on $L^\infty(M, \mathfrak{M}, \mu_M)$ remain singular when restricted to $L^\infty(M, \mathfrak{N}, \mu_M)$.) It follows that, in Section 2, under Assumption (a.2.2), the consumption set X_h defined by Formula (2.1) has the Elimination Properties, and this is used in the proof of Theorem 2.1.

(ii) If a set, Z, fulfils the condition:

For every $z \in Z$ and for every $p \in L^{\infty*}$ there exists a sequence of measurable sets, $(A_\alpha)_{\alpha=1}^\infty$, that supports p_f, with $p_c(A_\alpha) \to 0$ as $\alpha \to \infty$ and with $z\chi_{M\setminus A_\alpha} \in Z$ for each $\alpha = 1, 2, \ldots$; (3.1)

which is the "Exclusion Assumption" of Bewley (1972, p. 524), then Z has the Weak Elimination Property with respect to 0. (To show this, denote $z^\alpha = z\chi_{M\setminus A_\alpha}$. Then, firstly, $\langle z^\alpha - z, p_c\rangle \to 0$ as $\alpha \to \infty$, and, secondly, $\langle z^\alpha, p_f\rangle = 0$ for all α.) From this and from Part (i) of the Remark it follows that Theorem 3.4 is an extension of a result of Bewley (1972, Theorem 3). Also, in all examples given by Bewley (1972, pp. 527-528), the production sets satisfy a condition that is slightly stronger than (3.1), viz., For every $\mathbf{z} \in Z$ and for every $\mathbf{p} \in L^{\infty *}$ there exists a μ_M-vanishing sequence, $(A_\alpha)_{\alpha=1}^{\infty}$, of sets supporting $\mathbf{p}_f$, such that $\mathbf{z}\chi_{M \setminus A_\alpha} \in Z$ for each α; (3.2)

and this Condition is stronger than the Elimination Property for Z (with respect to 0).

(iii) Unlike the case of exchange economies, in Theorem 3.4 one does not show that $\mathbf{p}_f^* = 0$. This is because, for every equilibrium price to be in L^1, Bewley's assumption of a strictly positive total endowment, or a variant of it such as the "Adequacy Assumption" of Back (1988, p. 96), would be needed. In production economies, such an assumption would be very restrictive, since the initial endowment of a differentiated commodity may well be zero, as in our model of Section 2. (Note that, for production economies satisfying only the "Adequacy Assumption" of Bewley (1972, p. 520), an attempt at proving that all Pareto-optima can be supported by prices in L^1 would fail for the following reason. The condition that $\langle \mathbf{z}, \mathbf{p} \rangle \leq 0$ for every $\mathbf{z} \in Z$, where $\mathbf{p} \in L^{\infty}$ and Z is a closed, convex cone in L^{∞}, in general does <u>not</u> imply that $\langle \mathbf{z}, \mathbf{p}_f \rangle \leq 0$ for all $\mathbf{z} \in Z$, although for $Z = -L_+^{\infty}$ this implication holds. Were this implication true in general, one could use it with Z equal to the asymptotic cone of the aggregate production set.) However, since $\mathbf{p}_c^*$ is shown in Theorem 1 to support the same (quasi-) equilibrium allocation, and with the same producer profits, as $\mathbf{p}^*$, for some problems it does follow that $\mathbf{p}_f^* = 0$, e.g., for peak-load pricing problems: see Example 2.3 and Remark 2.4.

Appendix A: The Space L^∞, and the Yosida-Hewitt Decomposition of Its Norm Dual

Consider the space of essentially bounded functions, $L^\infty(M, \mathfrak{M}, \mu_M)$, on a sigma-finite, nonnegative measure space, $(M, \mathfrak{M}, \mu_M)$. Its norm dual is denoted by $L^{\infty *}(M, \mathfrak{M}, \mu_M)$. For brevity, the symbols M, $\mathfrak{M}$, and μ_M are suppressed except where their use adds clarity. Below we describe the Yosida-Hewitt decomposition of $L^{\infty *}$. Our terminology and notation is that of Dunford and Schwartz (1958, Chapters III and IV), except that we use the term "singular linear functional" in its more customary meaning, which is that of Castaing and Valadier (1977, Chapter VIII) and of Ioffe and Levin (1972, Appendix 1). All functions (including set functions) are taken to be real-valued. Also, we use the term "measure" to mean a "countably additive set function defined on a sigma-algebra of sets". In referring to other literature it should be borne in mind that the usage of some terms may differ from that adopted here.

A linear functional, $p \in L^{\infty *}$, is said to be supported by (or concentrated on) a measurable set, $A \in \mathfrak{M}$, if $p(x) = p(x\chi_A)$ for all $x \in L^\infty$, where χ_A is the characteristic function of A (equal to 1 on A and to 0 elsewhere). Also, p is termed a singular functional if and only if there exists a sequence of $\mathfrak{M}$-measurable sets, $(A_\alpha)_{\alpha=1}^\infty$, such that: (i) $A_{\alpha+1} \subset A_\alpha$ for every α, i.e., the sequence (A_α) is nonincreasing, (ii) $\mu_M(\bigcap_{\alpha=1}^\infty A_\alpha) = 0$, and (iii) p is supported by A_α for every α. Any sequence, (A_α), with properties (i) and (ii) is termed a μ_M-vanishing sequence of sets, and if, in addition, it has property (iii), then it is termed a μ_M-vanishing sequence of sets supporting p. (Also, if $\mu_M(M) < \infty$, then Condition (ii) is equivalent to $\mu_M(A_\alpha) \to 0$ as $\alpha \to \infty$.) The space of all singular functionals is denoted by $L_s^{\infty *}$; and the space of real-valued, μ_M-integrable functions on M is denoted by L^1. By the Yosida-Hewitt decomposition, every $p \in L^{\infty *}$ can be uniquely represented as the sum of its density part, $p_c \in L^1$ and of its singular part, $p_f \in L_s^{\infty *}$. (Under the isomorphism described in detail below, the density part of p is its countably additive part, and the singular part of p is its purely finitely additive part; hence the use of subscripts "c" and "f".) Furthermore, $L^{\infty*}$ is the direct sum of L^1 and $L_s^{\infty*}$, in the category of normed lattices; this also holds for the space of essentially bounded vector-valued mappings: see Castaing and Valadier (1977, Theorem VIII.5 on p. 236) or Ioffe and Levin (1972, Appendix 1, Theorem 3).

To derive the above decomposition of $L^{\infty*}$, the following representation of linear functionals on L^∞ as finitely additive set functions can be used. For brevity, we use the term "a set function" to mean "a bounded set function defined on $\mathfrak{M}$". The space of all finitely additive set functions is denoted by $ba(M, \mathfrak{M})$. The space $L^{\infty*}$ is isomorphic (in the category of normed lattices) to the space of all those set functions in $ba(M, \mathfrak{M})$ that vanish on all sets of μ_M-measure zero, which is denoted by $ba(M, \mathfrak{M}, \mu_M)$. The isomorphism is defined by the duality form $\langle x, p\rangle = \int_A x\, dp$, for all $x \in L^\infty(M, \mathfrak{M}, \mu_M)$ and for all $p \in ba(M, \mathfrak{M}, \mu_M)$, i.e., the isomorphic image of any $p \in ba(M, \mathfrak{M}, \mu_M)$ is the linear functional $\langle \cdot, p\rangle$: see Dunford and Schwartz (1958, Theorem IV.8.16 on p. 296). (For an exposition of the theory of integration with respect to finitely additive set functions, see Dunford and Schwartz (1958, Chapter III, Sections 1 and 2, pp. 95-119).) By a result of Yosida and Hewitt (1952, Theorems 1.23 and 1.24), every finitely additive set function, λ, can be uniquely decomposed into the sum of a countably additive set function (i.e., a measure), λ_c, and a purely finitely additive set function, λ_f. (A finitely additive set function is termed <u>purely finitely additive</u> if it is lattice-disjoint from every countably additive set function; by a result of Yosida and Hewitt (1952, Theorem 1.16), this definition is equivalent to that of Yosida and Hewitt (1952, Definition 1.13).)

To see that the above decomposition of $L^{\infty*}$ is the same as the decomposition of $ba(M, \mathfrak{M}, \mu_M)$ that results from the Yosida-Hewitt decomposition of the larger space $ba(M, \mathfrak{M})$, it remains to make two observations. First, the space L^1 is identified as the space of finite μ_M-continuous measures, since every such measure can be represented by its density with respect to μ_M: see Dunford and Schwartz (1958, Theorem III.10.2 on p. 176). (A measure is μ_M-continuous if and only if it vanishes on all μ_M-null sets: see Dunford and Schwartz (1958, Lemma III.4.13 on p. 131); the μ_M-continuity is also often termed "μ_M-absolute continuity".) Second, for any $p \in ba(M, \mathfrak{M}, \mu_M)$, p is purely finitely additive if and only if it is singular. (To see this, consider any $p \in ba(M, \mathfrak{M}, \mu_M)$. By Yosida and Hewitt (1952, Theorem 2.6), p is purely finitely additive if and only if it is lattice-disjoint from every element of $L^1(M, \mathfrak{M}, \mu_M)$. Take any finite measure, μ_M', with the same null sets as those of μ_M; such a measure exists because μ_M is sigma-finite. Note that p is lattice-disjoint from every element of $L^1(M, \mathfrak{M}, \mu_M)$ if and only if it is lattice-disjoint from μ_M'. This last condition is, by Yosida and Hewitt (1952, Theorem 1.22 with its proof), equivalent to the existence of a μ_M'-vanishing sequence of sets, (A_α), supporting p. Since $\mu_M'(\bigcap_{\alpha=1}^{\infty} A_\alpha) = 0$ if and only if $\mu_M(\bigcap_{\alpha=1}^{\infty} A_\alpha) = 0$, the argument is complete.) Thus, the space $L_s^{\infty *}$ is identified as the space of purely finitely additive set functions in $ba(M, \mathfrak{M}, \mu_M)$.

By "the Mackey topology" we mean the Mackey topology on L^∞ for the duality with L^1, which is usually denoted by $\tau(L^\infty, L^1)$. If $(A_\alpha)_{\alpha=1}^{\infty}$ is a μ_M-vanishing sequence of sets, then $x\chi_{A_\alpha} \to 0$ in the Mackey topology as $\alpha \to \infty$, for every $x \in L^\infty$; as Bewley (1972, Part (b) of (24) on p. 534) notes, this follows from Dunford and Schwartz (1958, Theorem IV.8.9 on p. 292). If $(x^\alpha)_{\alpha=1}^{\infty}$ is a sequence in L^∞ that order-converges to an $x \in L^\infty$, i.e., if there exist two sequences, $(y^\alpha)_{\alpha=1}^{\infty}$ and $(z^\alpha)_{\alpha=1}^{\infty}$, in L^∞ with $y^\alpha \le x - x^\alpha \le z^\alpha$ for all α and $y^\alpha \uparrow 0$ and $z^\alpha \downarrow 0$ as $\alpha \to \infty$, then $x^\alpha \to x$ in the Mackey topology: see, e.g., Aliprantis and Burkinshaw (1978, the equivalence of (i) and (ii) in Theorem 9.7).

In Part (vii) of Proposition A2 below we show that Definition A1 gives an equivalent way of incorporating free disposal into a production set. This is used in the proof of Theorem 2.1 to show that the free-disposal hull, $Y^\dagger$, of Y is Mackey closed (or, equivalently in view of its convexity, weak* closed). Our method is based on the continuity of the mapping $y \to y^+$, i.e., on the fact that L^∞ with its Mackey topology, $\tau(L^\infty, L^1)$, is a topological lattice. We note that the general results on the closedness of the sum of closed sets in topological vector spaces, such as those given by Khan and Vohra (1987), are insufficient for this purpose.

Definition A1: As in Section 2, let $(\Xi, \mathfrak{X}, \mu)$ denote a sigma-finite measure space, and let N be a natural number. For every $Y \subset L^\infty(\Xi, \mathfrak{X}, \mu) \times R^N$, define

$$Y^\dagger = \{(y, a) \mid (y^+, a) \in Y\}.$$

Proposition A2: (i) If Y is Mackey closed, then $Y^\dagger$ is Mackey closed.

(ii) If Y is a cone, then $Y^\dagger$ is also a cone.

(iii) $Y^{\dagger\dagger} = Y^\dagger$.

(iv) If Y is convex and has Property (2.6) of Section 2, then $Y^\dagger$ is also convex.

(v) If Y has Properties (2.6)-(2.7) of Section 2, then $Y^\dagger = Y^\dagger - L^\infty_+(\Xi) \times R^N_+$, which, with $Y^\dagger$ interpreted as a production set, means that $Y^\dagger$ includes free disposal.

(vi) If $Y \subset L^\infty_+(\Xi) \times R^N$, then $Y \subset Y^\dagger$.

(vii) Assume that Y has Properties (2.6)-(2.7) of Section 2 and that $Y \subset L^\infty_+(\Xi) \times R^N$. Then $Y^\dagger = Y - L^\infty_+(\Xi) \times R^N_+$.

Proof. Part (i) follows from the Mackey continuity of the mapping $y \to y^+$, i.e., from the fact that $L^\infty(\Xi)$ with the Mackey topology is a topological vector lattice, which is a special case of a result given, e.g., by Aliprantis and Burkinshaw (1978, Chapter 6, Exercise 4 on p. 163). (In that reference, the term "a locally convex-solid Riesz space" is used to mean "a locally convex lattice".) Parts (ii) and (iii) hold because

$(\lambda y)^+ = \lambda y^+$ for every nonnegative scalar, λ, and because $y^{++} = y^+$. Given their assumptions, Part (iv) holds by the convexity, and Part (v) by the monotonicity, of the mapping $y \to y^+$. Part (vi) follows directly from Definition A1. To prove Part (vii), note that the set on the right-hand side is contained in the set on the left-hand side by Parts (v) and (vi). The reverse inclusion follows directly from Definition A1 and from the fact that $y \leq y^+$. Q.E.D.

Definition A3: (i) A <u>correspondence</u> from a set X into a set Y is a mapping defined on X with values which are subsets of Y. If X and Y are topological spaces, $x_0 \in X$, and F is a correspondence from X into Y, then F is said to be <u>upper hemicontinuous</u> at x_0 if for every open set $U \subset Y$ there exists a neighbourhood, V, of x_0 such that the condition $F(x_0) \subset U$ implies that $F(x) \subset U$ for every $x \in V$. Also, F is said to be <u>lower hemicontinuous</u> at x_0 if for every open set $U \subset Y$ there exists a neighbourhood, V, of x_0 such that the condition $F(x_0) \cap U \neq \emptyset$ implies that $F(x) \cap U \neq \emptyset$ for every $x \in V$.

(ii) A weak pre-order, $\precsim$, in a topological space, X, is said to be <u>upper semicontinuous</u> if, for every $x \in X$, the set $\{z \in X \mid x \precsim z\}$ is closed, and it is said to be <u>lower semicontinuous</u> if, for every $x \in X$, the set $\{z \in X \mid z \precsim x\}$ is closed. (With this terminology, a preference pre-order represented by an upper (respectively, lower) semicontinuous utility function is upper (respectively, lower) semicontinuous.)

Appendix B: Proofs

Proof of Theorem 3.4. We first prove Part (i). For each producer, j, by the Elimination Property of the production set, i.e., by Assumption (a.3.4), for every $y \in Y_j$ and for every number $\delta > 0$ there exists $y' \in Y_j$ with $\langle y', p^*_c \rangle \geq \langle y, p^*_c \rangle - \delta$ and $\langle y', p^*_f \rangle \geq -\delta$. Since y^*_j maximizes profit on Y_j at prices $\mathbf{p}^*$,

$$\langle y^*_j, p^* \rangle \;\geq\; \langle y', p^* \rangle \;=\; \langle y', p^*_c \rangle + \langle y', p^*_f \rangle \;\geq\; \langle y, p^*_c \rangle - 2\delta,$$

and it follows that

$$\langle y^*_j, p^* \rangle \;\geq\; \langle y, p^*_c \rangle \tag{B.1}$$

for every $y \in Y_j$. By substituting y^*_j for y in (B.1), it follows that

$$\langle y^*_j, p^*_f \rangle \;\geq\; 0. \tag{B.2}$$

For any consumer, h, take any $x \in X_h$ with $x^*_h <_h x$. By Mackey lower semicontinuity of preferences and by the Elimination Property of the consumption set, i.e., by Assumptions (a.3.1) and (a.3.2), for every number $\delta > 0$ there exists $x' \in X_h$ with: $x^*_h <_h x'$, $\langle x', p^*_c \rangle \leq \langle x, p^*_c \rangle + \delta$ and $\langle x', p^*_f \rangle \leq \langle \hat{x}_h, p^*_f \rangle + \delta$. By Condition (iv) of Definition 3.1 and by the last two inequalities,

$$\langle x^*_h, p^* \rangle \;\leq\; \langle x', p^* \rangle \;=\; \langle x', p^*_c \rangle + \langle x', p^*_f \rangle \;\leq\; \langle x, p^*_c \rangle + \langle \hat{x}_h, p^*_f \rangle + 2\delta,$$

and it follows that

$$\langle x^*_h, p^* \rangle \;\leq\; \langle x, p^*_c \rangle + \langle \hat{x}_h, p^*_f \rangle. \tag{B.3}$$

By nonsatiation of preferences, i.e., by Assumption (a.3.2), for every number $\delta > 0$ there exists $z \in X_h$ with $x_h^* <_h z$ and $\langle z, p_c^* \rangle \le \langle x_h^*, p_c^* \rangle + \delta$. By substituting z for x in (B.3), it follows that $\langle x_h^*, p^* \rangle \le \langle x_h^*, p_c^* \rangle + \langle \hat{x}_h, p_f^* \rangle + \delta$ for every $\delta > 0$, i.e.,

$$\langle x_h^*, p_f^* \rangle \le \langle \hat{x}_h, p_f^* \rangle. \tag{B.4}$$

From Condition (i) in Definition 3.1, since $\Sigma_h \hat{x}_h = \Sigma_h \bar{x}_h$, it follows that equalities hold in (B.2) and (B.4), i.e., for each h and each j,

$$\langle x_h^* - \hat{x}_h, p_f^* \rangle = 0 \tag{B.5}$$

and

$$\langle y_j^*, p_f^* \rangle = 0. \tag{B.6}$$

By (B.1) and (B.6),

$$\langle y, p_c^* \rangle \le \langle y_j^*, p^* \rangle = \langle y_j^*, p_c^* \rangle, \tag{B.7}$$

for every $y \in Y_j$.

For every $x \in X_h$ with $x_h^* <_h x$, by (B.3) and by (B.5),

$$\langle x_h^*, p^* \rangle \le \langle x, p_c^* \rangle + \langle \hat{x}_h, p_f^* \rangle = \langle x, p_c^* \rangle + \langle x_h^*, p_f^* \rangle,$$

and it follows that

$$\langle x_h^*, p_c^* \rangle \le \langle x, p_c^* \rangle. \tag{B.8}$$

Since $p_c^* \neq 0$ by assumption, the proof that p_c^* is a quasi-equilibrium price is complete. This is because Formulae (B.5) and (B.6) and Condition (iii) of Definition 1 together imply Condition (iii) of Definition 3.1 with p_c^* substituted for p^*, and Formulae (B.7) and (B.8) mean that Conditions (ii) and (iv) of Definition 3.1, with p_c^* substituted for p^*, are fulfilled.

The proof of Part (ii) of the Theorem is obtained by the following modification of the argument. Take any $x \in X_h$ with $x_h^* <_h x$. By the Monotone Elimination Property, there exists a sequence, (x^α), in X_h with $x^\alpha \uparrow x$ as $\alpha \to \infty$ and $\langle x^\alpha, p_f^* \rangle = 0$ for every α. Since order-convergence in L^∞ implies convergence in the Mackey topology, it follows from Mackey lower semicontinuity of preferences, i.e., from Assumption (a.3.1), that $x_h^* <_h x^\alpha$ for all sufficiently large α. Since $p^* \geq 0$, one has $p_c^* \geq 0$ and $p_f^* \geq 0$. By using in succession: the nonnegativity of x_h^* and of p_f^* and Condition (iv') of Definition 3.1,

$$\langle x_h^*, p_c^* \rangle \leq \langle x_h^*, p^* \rangle < \langle x^\alpha, p^* \rangle = \langle x^\alpha, p_c^* \rangle. \qquad \text{(B.9)}$$

Since $x^\alpha \uparrow x$ and $p_c^* \geq 0$, the term on the right-hand side of the equality in (B.9) is nondecreasing in α, and it converges to $\langle x, p_c^* \rangle$ as $\alpha \to \infty$. It follows that

$$\langle x_h^*, p_c^* \rangle < \langle x, p_c^* \rangle,$$

as required. Q. E. D.

Proof of Theorem 2.1. The proof consists of two parts. In the first part we apply a result of Bewley (1972, Theorem 1) to show that there exists an equilibrium price system, $\mathbf{p}^* = (p^*, q^*)$, with prices for the differentiated commodity, p^*, in $L^{\infty*}(\Xi)$. In the second part we use Theorem 3.1, to conclude that the price system $\mathbf{p}^*_c = (p^*_c, q^*)$, for which $p^*_c \in L^1(\Xi)$, is also an equilibrium price. The first part of the argument can also be carried out by a "direct" proof of equilibrium existence, based on Florenzano's (1983) extension of the Gale-Nikaido-Debreu Lemma to infinite dimensions. As pointed out by Horsley and Wrobel (1988c), the advantage of this approach is in showing price-continuity of demand (in the infinite-dimensional commodity and price spaces), which may be useful for the setting of simplified tariffs.)

We verify those assumptions of Bewley (1972, Theorem 1) which are not made directly in our Theorem 2.1. The industry's production set, defined by (2.2), is equal to $Y^\dagger$, by Part (vii) of Proposition A2 and Assumption (a.2.8). Therefore, it is a Mackey closed, by Part (i) of Proposition A2 and Assumption (a.2.7), and it is convex, by Part (iv) of Proposition A2 and Assumptions (a.2.7)-(a.2.8). By Assumption (a.2.10), for each household, h, the initial endowment, $(0, -\bar{m}_h)$, belongs to the norm-interior (relative to the whole commodity space) of the cone $As(Y) - L^\infty_+(\Xi) \times R^N_+$ which is contained in $As(Y^\dagger)$, by Part (vii) of Proposition A2. This shows that the "Adequacy Assumption" of Bewley (1972, Theorem 1) holds. It follows that there exists of a competitive equilibrium, consisting of: consumption plans, $(x^*_h, m^*_h) \in X_h$ for each h, production plans of firms, $(z^*_j, b^*_j) \in Y_j$ for each j, a production plan for the industry, $(y^*, a^*) \in Y^\dagger$, and a nonnegative price system, (p^*, q^*), with $p^* \in L^{\infty*}_+(\Xi)$. Also, q^* is semi-positive, i.e., $q^*_n > 0$ for some n. (To show this, note that $\langle y, p^*\rangle + \langle a, q^*\rangle \le 0$ for every $(y, a) \in As(Y^\dagger)$. Since $(p^*, q^*) \ne 0$, one has $q^* \ne 0$ if $p^* = 0$. If $p^* \ne 0$, then there exists $y \in L^\infty(\Xi)$ with $\langle y, p^*\rangle > 0$. One has $(y, a) \in As(Y^\dagger)$ for some a, by Part (ii) of Assumption (a.2.10). It follows that $\langle a, q^*\rangle < 0$, so $q^* \ne 0$.)

To apply Theorem 3.4 and complete the proof, we verify the Elimination Property for the production sets and consumption sets. First, for the industry's production set, $Y^{\dagger}$, since the industry produces the differentiated commodity from a finite number of homogeneous inputs, this follows trivially from the free disposal property of $Y^{\dagger}$. However, for completeness, we give a formal argument. To show that Condition (3.2) holds, take any $\mathbf{y} \in Y^{\dagger}$; it can be written as $\mathbf{y} = (y, a)$, where $y \in L^{\infty}(\Xi)$ is the output bundle, and $-a \in R^N_+$ is the input bundle. It follows from Part (iii) of Proposition A2 that $(y, a) \in Y^{\dagger\dagger}$, i.e., $(y^+, a) \in Y^{\dagger}$. For any price system, $\mathbf{p} = (p, q) \in L^{\infty *}(\Xi) \times R^N$, take a μ-vanishing sequence, $(A_\alpha)_{\alpha=1}^{\infty}$, of sets supporting p_f, and define $\mathbf{y}^\alpha = (y\chi_{\Xi \setminus A_\alpha}, a)$. Since $\mathbf{y}^\alpha \leq (y^+, a) \in Y^{\dagger}$, it follows, by Part (v) of Proposition A2 that $\mathbf{y}^\alpha \in Y^{\dagger}$ for each α. Also, $\langle \mathbf{y}^\alpha, \mathbf{p}_f \rangle = \langle y\chi_{\Xi \setminus A_\alpha}, p_f \rangle = 0$. Thus, Condition (3.2) holds, and, by Part (ii) of Remark 3.5, $Y^{\dagger}$ has the Elimination Property with respect to 0.

Second, for each firm, j, take any $\mathbf{y}_j = (z_j, b_j) \in Y_j$, i.e., $b_j \in F_j(z_j)$, by (2.3). To verify the Elimination Property with respect to 0, take any neighbourhood, W, of b_j in R^N and any Mackey neighbourhood, V, of z_j in $L^{\infty}(\Xi)$. For any price system, $\mathbf{p} = (p, q)$, where $p \in L^{\infty *}(\Xi)$, take a μ-vanishing sequence, $(A^j_\alpha)_{\alpha=1}^{\infty}$, of sets that belong to $\mathfrak{C}_j$ and support p_f. (Such a sequence exists by Assumption (a.2.4).) Define $z_j^\alpha = z_j\chi_{\Xi \setminus A^j_\alpha}$. Since $z_j^\alpha \to z_j$ in the Mackey topology as $\alpha \to \infty$, by the Mackey lower hemicontinuity of F_j there exists an α' such that: $z_j^{\alpha'} \in V$, $b_j^{\alpha'} \in W$, and $b_j^{\alpha'} \in F_j(z_j^{\alpha'})$. Denote $\mathbf{y}_j' = (z_j^{\alpha'}, b_j^{\alpha'})$, this point belongs to Y_j and it has all the required properties, since $\langle \mathbf{y}_j', \mathbf{p}_f \rangle = \langle z_j^{\alpha'}, p_f \rangle = 0$.

Third, for each household, h, the orthant $L^{\infty}_+(\Xi, \mathcal{B}_h, \mu)$ has the Monotone Elimination Property, in the space $L^{\infty}(\Xi, \mathfrak{X}, \mu)$, with respect to 0, by Part (i) of Remark 3.5 and by Assumption (a.2.2). It follows that the household's consumption set, $L^{\infty}_+(\Xi, \mathcal{B}_h, \mu) \times R^N_+$, has the Monotone Elimination Property with respect to the household's initial endowment, $(0, \bar{m}_h)$, since this endowment contains none of the differentiated commodity. Therefore, Part (ii) of Theorem 3.4 is applicable, and the price system $p_c^* = (p_c^*, q^*)$, with $p_c^* \in L^1(\Xi)$, is an equilibrium price system.

Q. E. D.

Remark B1: Theorem 2.1 also holds with the initial endowment, $(0, \bar{m}_h)$, replaced by $(\bar{x}_h, \bar{m}_h)$, for any $\bar{x}_h \in L_+^\infty(\Xi, \mathcal{B}_h, \mu)$, i.e., when households have initially some of the differentiated commodity. In this case, the last part of the proof is modified as follows. By Part (i) of Remark 3.5 and by Assumption (a.2.2), the consumption set, $L_+^\infty(\Xi, \mathcal{B}_h, \mu) \times R_+^N$, has the Elimination Property with respect to the initial endowment, $(\bar{x}_h, \bar{m}_h)$. Therefore, Part (ii) of Theorem 3.4 is applicable, and it follows that (p_c^*, q^*) is a competitive quasi-equilibrium price system (with the same quasi-equilibrium allocation as for the equilibrium price system (p^*, q^*)). Since q^* is semi-positive and $\bar{m}_h$ is strictly positive, by Assumption (a.2.10), one has

$$\langle x_h^*, p_c^* \rangle + \langle m_h^*, q^* \rangle \geq \langle \bar{m}_h, q^* \rangle > 0 = \inf \{\langle x, p_c^* \rangle + \langle m, q^* \rangle \mid (x, m) \in X_h\}$$

for each h, and it follows that this quasi-equilibrium is an equilibrium, by the argument of Debreu (1962, p. 269, lines 6-9).

Proof of the equivalence noted in Assumption (a.2.6): We first show that the lower semicontinuity of f_j implies the lower hemicontinuity of F_j, defined by Formula (2.5). Define

$$G_j(z) = \{(b_{-n_j}, b_{n_j}) \in R_-^{N-1} \times R \mid b_{n_j} < f_j(z, b_{-n_j})\}.$$

Then $F_j(z)$ is equal to the closure of $G_j(z)$, for every z. (This is because, for every z, the function $f_j(z, \cdot)$ is upper semicontinuous by Assumption (a.2.5), so its hypograph, which is equal to $F_j(z)$ by (2.5) is closed.) Since the correspondence obtained by taking the closures of the values of a lower hemicontinuous correspondence is also lower hemicontinuous (see, e.g., Klein and Thompson (1984, Proposition 7.3.3 on p. 85)), to complete the argument it suffices to show that G_j is lower hemicontinuous. But this holds, since, by the lower semicontinuity of the function $f_j(\cdot, b_{-n_j})$ for every b_{-n_j}, the graph of G_j has open sections, i.e., for every b, the set $\{z \mid b \in G_j(z)\}$ is open (relative to $L^\infty_-(\Xi, \mathbf{C}_j, \mu)$). The proof that conversely, the lower hemicontinuity of F_j implies the lower semicontinuity of f_j, is even simpler. Note that, for each z, the half-line $(-\infty, f_j(z, b_{-n_j})]$ is equal to the section of $F_j(z)$ by b_{-n_j}. Therefore, this half-line, in its dependence on z, is a lower hemicontinuous correspondence, which, in other words, means that the function $z \to f_j(z, b_{-n_j})$ is lower semicontinuous. Q. E. D.

Proof of Formula (2.17). Denote $X = L^\infty(\Xi)$ for brevity. Since C is nondecreasing,

$$\begin{aligned} C^\dagger(y) &= C(y^+) = \inf\{C(y') \mid y' \geq 0,\ y' \geq y\} \\ &= \inf\{\, C(y') + \delta(y'' \mid X_-) \mid y' \geq 0,\ y'' = y - y'\}, \end{aligned} \qquad (B.10)$$

where $\delta(\cdot \mid X_-)$ is the indicator function of X_-, i.e., $\delta(y'' \mid X_-) = 0$ for $y'' \in X_-$ and $\delta(y'' \mid X_-) = +\infty$ for $y'' \in X \setminus X_-$. Since the last infimum in Formula (B.10) is attained (for $y' = y^+$ and $y'' = -y^-$), it follows that

$$\operatorname{epi} C^\dagger = \operatorname{epi} C + \operatorname{epi} \delta(\cdot \mid X_-) = \operatorname{epi} C + X_+ \times R_+,$$

which is equivalent to Formula (2.17). Q. E. D.

Remark B2: Formula (B.10) means that $C^{\dagger}$ is the infimal convolution of C and $\delta(\cdot \mid X_{-})$, where C is extended to X by setting $C(y) = +\infty$ for $y \in X \setminus X_{+}$.

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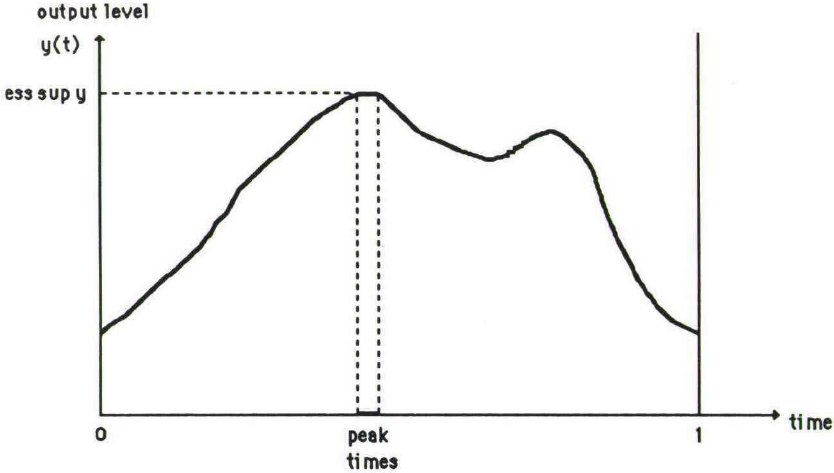

Figure 1. An electricity output, y, with a peak of a short duration.

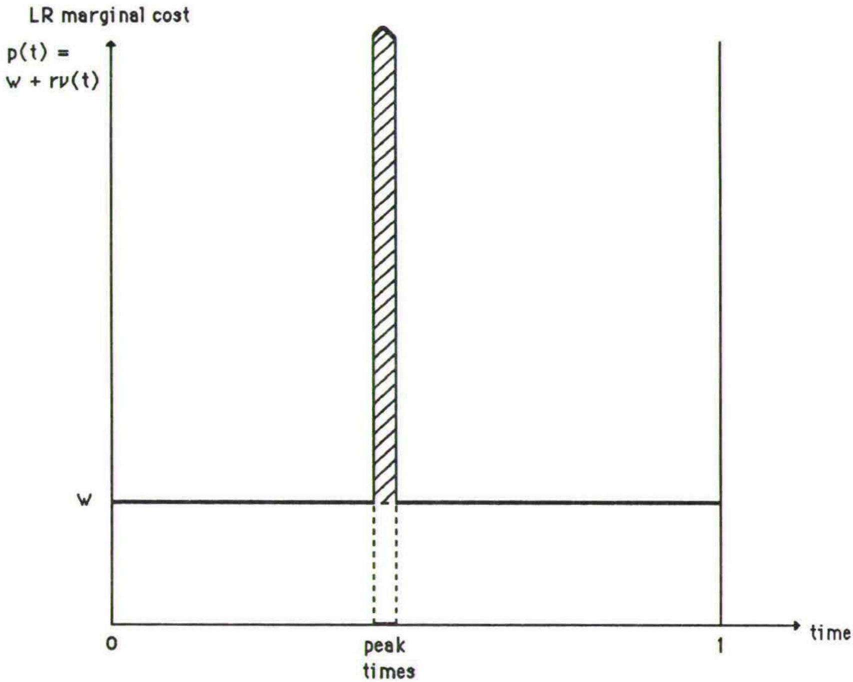

Figure 2. A long-run marginal cost, p, at the electricity output, y, of Figure 1, in the one-station model. The total of the peak charges, represented by the hatched area, is equal to r.

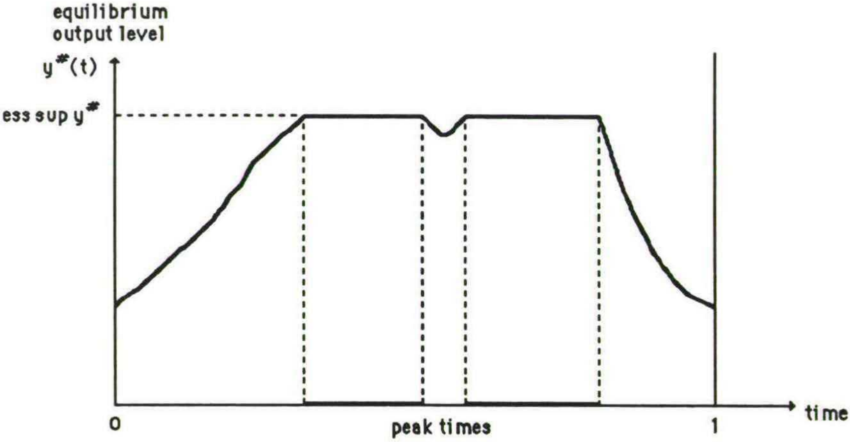

Figure 3. The long-run equilibrium output of electricity, y^*, has a peak plateau.

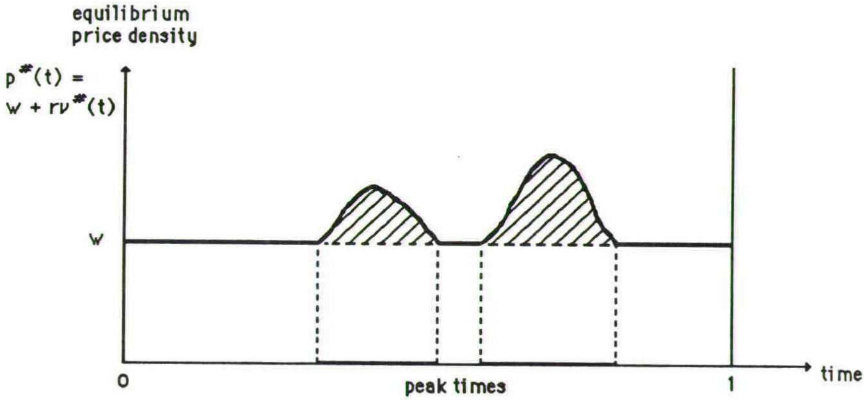

Figure 4. The long-run equilibrium price for electricity, p^*, corresponding to the equilibrium output, y^*, of Figure 3, in the one-station model. The hatched area is equal to r.

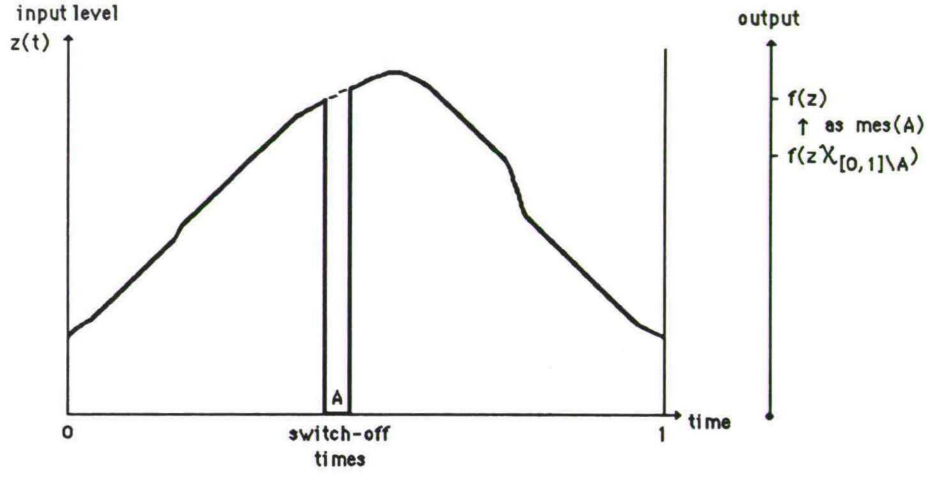

Figure 5. The continuity property of the production function, f, of a firm with an interruptible input demand for electricity.

FOOTNOTES:

* Financial support from BOC Limited and CentER during the course of this research is gratefully acknowledged.

† Journal of Economic Literature Classification: 021.

AMS 1980 Mathematics Subject Classification (1985): Primary 90A14.

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[1] A result of Horsley and Wrobel (1988c) that shows that demand is continuous in prices goes some way towards ensuring that such a simplification is, to some extent, possible.

[2] For the definition, see, e.g., Rockafellar (1970, p. 66).

[3] More precisely, a lower bound for the difference quotient $(f(z + \delta z') - f(z))/\delta$ that is uniform over all $z \in L^{\infty}_{-}(\Xi)$, all z' in some Mackey neighbourhood of zero in $L^{\infty}(\Xi)$ and all numbers δ such that $z + \delta z' \in L^{\infty}_{-}(\Xi)$.

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