Can Habit Formation be Reconciled with Business Cycle Facts? *

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Abstract

Many asset pricing puzzles can be explained when habit formation is added to standard preferences. We show that utility functions with a habit then gives rise to a puzzle of consumption volatility in place of the asset pricing puzzles when agents can choose consumption and labor optimally in response to more fundamental shocks. We show that the consumption reaction to technology shocks are too small by an order of magnitude when a utility includes a habit. Alternative models with consistent and exogenous but stochastic labor input are considered. A model with persistent technology shocks and stochastic labor is shown to be potentially consistent with substantial consumption variability as well as procyclical labor input and labor productivity even when a habit is present.

1 Introduction

In recent years, models with habit formation[1] have been quite successful in linking consumption with asset prices. In particular, Constantinides (1990) has shown that once a habit is added to the standard model with power utility and lognormal distribution, the equity premium puzzle of Mehra and Prescott (1985) disappears. More recently, Campbell and Cochrane (1995) present a different habit formation model that avoids some of the drawbacks of earlier models, such as a high and very volatile risk free rate. See also Weil (1992) for a discussion on how habit formation changes the volatility bounds of Hansen and Jagannathan (1991). Typically these models specify an exogenously given consumption process and use the first-order condition of a representative consumer to derive the implication for asset prices. This approach leaves the following question open: How does the consumption path look like when consumers choose consumption optimally in response to some more fundamental shock in the presence of habit formation?

In this spirit we study versions of Hansen's (1985) real business cycle model with shocks to technology and preferences which include habit formation following Campbell and Cochrane (1995). One feature of the model is that agents can adjust consumption and labor input in response to technology shocks. We find that this labor-leisure channel provides an avenue for adjusting to the aggregate shock, enabling the agent to drastically smooth consumption. The intuition is that the habit formation makes the agent (locally) very risk averse, which implies a very low (local) elasticity of substitution. Hence the agents want to smooth consumption extremely, making consumption very unresponsive to shocks. This low elasticity of substitution has also an effect on the optimal labor choice after a positive technology shock. There are two effects. First, labor is more productive, hence wages are higher. Thus induces the worker to work more now to take advantage of the higher wages as long as the technology shock has not died out yet. Second, workers are induced to work less because they earn more per unit worked and the low elasticity of substitution implies that they do not want to adjust consumption by much. Hence, they can reduce their labor input. The sign of the net effect depends on parameters of the models, such as the persistence of the technology shock and elasticity of substitution for consumption and labor. When the technology shock is very persistent and the intertemporal elasticity of substitution of labor is not too large, we find that labor input decreases after a positive technology shock. Moreover, the consumption responses are still very small when the persistence of technology shocks is high or risk

[1] The term habit refers here to external habits, or in Abel's (1990) words: "keeping up with the Joneses."

aversion is decreased.

The key insight is that agents use labor input to smooth consumption extremely. Since the Hansen (1985) model is very restrictive in assuming that labor enters the utility function in a linear fashion, we consider several extensions of the benchmark model. First we specify a power utility in labor which allows us to control the intertemporal elasticity of labor substitution. However, this utility function does not produce qualitatively different consumption reactions in the model with habit. As a sidenote we present some interesting reactions in labor input. Next, we consider a model in which labor input is fixed and hence cannot be used by the agent to smooth consumption. This model generates quite substantial consumption responses as long as the technology shock is highly persistent. As an alternative we study a model where labor is exogenously varying over time which can be motivated by some models with labor market frictions. When combined with persistent technology shocks, this version of the model produces reasonable consumption variability as well as procyclical labor input and labor productivity. The consumption reaction to reasonable values once when we assume that labor is fixed exogenously increases. This illustrates the importance of the ability to use labor to smooth consumption in this model. Finally, we study a model with technology shocks and exogenously fluctuating labor input. We show that this version of the model with habit produces approximately the same consumption variability as the data suggest as long as technology shocks are every persistent.

After completing an earlier version of this paper, two related working papers came to our attention: Jerman (1993) and Boldrin, Christiano and Fisher (1995). Both papers consider real business cycle models with habit formation as do we, but they focus on the implications for the equity premium instead of the variability of consumption as in our paper. Jerman (1993) looks at a model where labor input is fixed and there are adjustment costs in capital accumulation. He finds that the equity premium is fairly large as long as the adjustment costs are substantial. Boldrin, Christiano and Fisher (1995) study a two-sector model with limited resource flexibility across sectors. They find that the model with habit formation is not able to match the high equity premium in the data even when capital goods cannot be moved between sectors and the labor inputs are predetermined before the shock is realized. Our paper differs from theirs in several ways. First, they consider a special case of Constantinides' (1990) habit, which is known to produce a high and volatile risk free rate. We consider the 'state-of-the-art' habit of Campbell and Cochrane (1995) which matches the asset pricing data better. Second, we look at a one-sector model while Boldrin et.al. consider a two-sector model. Both papers have in common that agents have to be restricted in their labor, investment and/or counsumption choice in order to get a high equity premium or large consumption fluctuations. Boldrin et.al accomplish this through rigidities between sectors while we concentrate on the labor market. Third, in the last version of their model, consumption, which is the focus of our study, is essentially determined by the technology shock alone since all substitution between sectors is switched off and labor input is chosen before the shock is realized.

The next section contains the specification of the utility function. Section 3 describes the model. Section 4 contains the results in form of a comparison to a version of the model without habit formation. Section 5 contains models with alternative utility functions in labor. The last section concludes.

2 Specifying the utility function

2.1 Habit formation following Campbell and Cochrane

The specification of the habit in the utility function follows Campbell and Cochrane (1995). Capital letters denote levels, and small letters natural logs of a variable. Let $(X_t)_{t=0}^{\infty}$ denote a (stochastic) sequence of habits. X_t is a function of past consumption and will be defined below. Define a discount factor $0 < \beta < 1$ and a curvature parameter $\gamma > 0$. The utility of an individual agent for a stochastic sequence of individual consumption $(C_t)_{t=0}^{\infty}$ is given by

$$U((C_t)_{t=0}^{\infty}) = E_0 \sum_{t=0}^{\infty} \beta^t \ \ u(C_t; X_t) \tag{1}$$

where

$$u(C; X) = \frac{(C - X)^{1-\gamma} - 1}{1 - \gamma}.$$

The stochastic sequence of habits $(X_t)_{t=0}^{\infty}$ is regarded as exogenous by the individual agents and tied to the stochastic sequence of aggregate consumption $(C_t)_{t=0}^{\infty}$ as follows (note that we use the same symbol for individual as well as for aggregate consumption, as we are only going to study environments with a representative agent). Let

$$S_t = \frac{C_t - X_t}{C_t}$$

denote the surplus consumption ratio and $s_t = \log S_t$ its natural logarithm.

Let $c_t = \log(C_t), \Delta c_{t+1} = c_{t+1} - c_t$, let g be the average consumption growth rate, $g = E[\Delta c_{t+1}]$ and let σ_ν^2 denote the conditional variance of consumption growth, $\sigma_\nu^2 = \text{Var}_t[\Delta c_{t+1}]$.

Finally, let the evolution of X_t or, equivalently, of s_t be given by

$$s_{t+1} = (1-\phi)\bar{s} + \phi s_t + \lambda(s_t)(\Delta c_{t+1} - g), \tag{2}$$

where $0 < \phi < 1$ is a parameter, and $\lambda(s)$ defines a sensitivity function as follows

$$\lambda(s) = \begin{cases} \frac{1}{\bar{S}}\sqrt{1 - 2(s - \bar{s})} - 1, & s \leq s_{max} \\ 0, & s \geq s_{max}. \end{cases} \tag{3}$$

Let

$$\bar{S} = \sigma_\nu \sqrt{\frac{\gamma}{1-\phi}},$$
$$\bar{s} = \log \bar{S},$$

and

$$s_{max} = \bar{s} + \frac{1}{2}(1 - \bar{S}^2).$$

Note that by construction, $E[s_t] = \bar{s}$ unconditionally. It is useful to calculate the marginal rate of substitution:

$$\begin{aligned} M_{t+1} &= \beta \frac{u'(C_{t+1}; X_{t+1})}{u'(C_t; X_t)} \\ &= \beta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma} \\ &= \beta \exp\left(-\gamma\left((1-\phi)(\bar{s} - s_t) + (\lambda(s_t) + 1)(\Delta c_{t+1} - g)\right) + g. \end{aligned} \tag{4}$$

This equation can be used to price assets. Using the basic pricing relationship $E_t[M_{t+1}R_{t+1}] = 1$ when R_{t+1} is a (gross) asset return. The risk-free rate, for example, is given by the log of the risk-free return, $r_t^f = -\log(E_t[M_{t+1}])$. If Δc_{t+1} is unconditionally normally distributed, then the logarithm of the average risk-free return is given by

$$\bar{r}^f = -\log E[M_{t+1}] = -\log(\beta) + \gamma g - \frac{\gamma}{2}(1-\phi).$$

If additionally, log consumption follows a random walk, then the risk free rate in each period is the same, $r_t^f = \bar{r}^f$. This in fact is the reason for Campbell and Cochrane to specify $\lambda(s)$ as stated above. In the context of the real business cycle model to be examined below, however, there is no a priori reason to expect log-consumption to follow a random walk and to demand the risk free rate to be constant over the cycle.

Using this habit formulation, Campbell and Cochrane (1995) are able to generate a set of asset pricing relations which are consistent with the data while avoiding some of the problems of earlier habit models. They get a constant and low risk free rate (see Weil (1989)), counter-cyclical risk prices, autocorrelated variances of stock return, among other things.

2.2 The role of the sensitivity function $\lambda(s)$

The function $\lambda(s)$ in (3) controls how changes in consumption affect the habit s. Campbell and Cochrane (1995) choose $\lambda(s)$ so that the risk free rate r_t^f is constant for all s. Moreover, $\lambda(s)$ plays an important role in their paper in that the price of risk also depends on $\lambda(s)$. Hence a non-constant $\lambda(s)$ is needed to get state-dependent risk prices. However, apart from the constant risk free rate, there is no a priori reason to use the specification in (2). Thus, this subsection asks how an alternative choice of $\lambda(s)$ affects the results in Campbell and Cochrane. Note, that in the real business cycle model which we study in the next section, all equations have to be (log) linearized to solve the model. This leads us to study the following linear process for the log surplus ratio which uses the linearized λ-function evaluated at the steady state value of s:

$$s_{t+1} = (1-\phi)\bar{s} + \phi s_t + \left(\frac{1}{\bar{S}} - 1\right)(\Delta c_{t+1} - g).$$

As mentioned before, the choice of a constant λ function comes at the price of less rich asset pricing implications. On the other hand, the implications for the risk free rate are not very dramatic. The average risk free rate does not change much and it is not too volatile. Using the same parameters on Campbell and Cochrane ($g = 0.0044, \sigma_v = 0.00555, \phi = 0.97, \gamma = 2.372, \delta = 0.973$) the average risk free rate is 0.23% per quarter with a quarterly variance of 0.09%. Moreover, the correlation of the two alternative habit formulations is around 0.99 in stimulation, hence the habits are virtually indistinguishable. These facts make us confident that not too much is lost when the constant λ-function is chosen.

3 A real business cycle model

3.1 Describing the model

For the real business cycle world, we modify Hansen's (1985) benchmark RBC model. A characteristic feature of that model is that utility is linear in labor provided as a consequence of indivisibilities in the labor market. This implies that the intertemporal elasticity of substitution of labor is quite high, thus the agent is quite willing to adjust labor input over time. We will relax the linearity assumption later in Section 5, where we will assume that the agent has a power utility for leisure. The representative agent solves

$$\max_{C_t, K_t, N_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma} - AN_t \right]$$

s.t.

$$C_t + K_t = (d_t + (1-\delta))K_{t-1} + w_t N_t + \pi_t$$

where K_t denotes the capital stock chosen at date t and owned by the agents, d_t are dividends per unit of old capital, N_t is labor, w_t are wages, π_t are firm profits, and δ is the depreciation rate.

The representative firm maximizes profit

$$\pi_t = \max_{(K^d_{t-1}, N_t)} Y_t - d_t K^d_{t-1} - w_t N^d_t$$

where

$$Y_t = Z_t (K^d_{t-1})^{\rho} (N^d_t)^{1-\rho}$$

is output and K^d_{t-1}, N^d_t are demanded capital and labor. Market clearing requires that $C_t + K_t = Y_t + (1-\delta)K_{t-1}, N^d_t = N_t$ and that $K^d_{t-1} = K_{t-1}$. The externally given stochastic sequences for the habit X_t and productivity Z_t are assumed to be given by equation (3) for X_t (or, equivalently, s_t) and

$$z_t = \tilde{z} + \psi z_{t-1} + \epsilon_t, \ \epsilon_t \sim \text{ i.i.d. } \mathcal{N}(0; \sigma^2_{\epsilon}) \quad (5)$$

for $z_t = \log Z_t$, where $0 < \psi \leq 1$, and some $\tilde{z} = \log \tilde{Z}$.

3.2 Solving the model

Competitive markets require that

$$d_t = \rho \frac{Y_t}{K_{t-1}},$$

$$w_t = (1-\rho)\frac{Y_t}{N_t}$$

and $\pi_t = 0$. Thus, the equations characterizing the equilibrium are given by

$$\begin{aligned}
C_t + K_t &= Y_t + (1-\delta)K_{t-1} \\
Y_t &= Z_t K_{t-1}^{\rho} N_t^{1-\rho} \\
1 &= \beta E_t \left[\left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{t+1} \right] \\
R_{t+1} &= \rho \frac{Y_{t+1}}{K_t} + 1 - \delta \\
A &= (S_t C_t)^{-\gamma}(1-\rho)\frac{Y_t}{N_t}
\end{aligned}$$

as well as equations (4) and (5).

We restrict attention to the case $\psi < 1$, which makes all variables stationary and thus local analysis valid. We will consider the case of a near-unit root of ψ in order to analyse the effects of permanent technological change. Let $\bar{z} = \frac{\tilde{z}}{1-\psi}$ and $\bar{Z} = \exp(\bar{z})$. The steady state is given by the equations

$$\begin{aligned}
\bar{C} + \delta \bar{K} &= \bar{Y} \\
\bar{Y} &= \bar{Z} \bar{K}^{\rho} \bar{N}^{1-\rho} \\
1 &= \beta \bar{R} \\
\bar{R} &= \rho \frac{\bar{Y}}{\bar{K}} + 1 - \delta \\
A &= (\bar{S}\bar{C})^{-\gamma}(1-\rho)\frac{\bar{Y}}{\bar{N}}.
\end{aligned}$$

This system of equations allows us to calibrate the model. We normalize $\bar{N} = \bar{Z} = 1$. given parameter choices for ρ, β, δ the steady-state values can be calculated. Finally, specifying values for γ and $\bar{S}$ ties down the parameter A: Campbell and Cochrane suggest $\gamma = 2.372, \sigma_\nu = 0.0056$, and $\phi = 0.97$, yielding $\bar{S}$. Once we have solved for the dynamic properties of the model, one can also solve for σ_ν and thus $\bar{S}$ endogenously.

Note that we do not adopt the suggestion in Campbell and Cochrane to set $\beta = 0.973$. Their suggestion is the result of matching a risk free rate of 0.25% per quarter in an environment, where consumption grows 0.44% per quarter. Since we are considering a stationary world without permanent growth here, the appropriate choice would be to set $\beta = 0.9975$ here: we experimented with that choice as well. For the dynamic analysis to follow, one also needs to choose ψ and σ_ϵ. We follow Hansen (1985) and set $\psi = 0.95$ and $\sigma_\epsilon^2 = 0.00712$.

To analyze the dynamic implications, we loglinearize the equations characterizing equilibrium, a technique proposed in particular by Campbell (1994). To that end, let small letters denote log-deviations from steady state rather than logarithm of the level variables, e.g. $x_t = \log(X_t) - \log(\bar{X})$ for a variable X_t. Loglinearizing the other equations leads to the following linearized system of equations, describing the dynamic system (see also Uhlig (1995) for a further description of the techniques):

$$\begin{aligned}
\bar{C}c_t + \bar{K}k_t &= \bar{Y}y_t + (1-\delta)\bar{K}k_{t-1} \\
y_t &= z_t + \rho k_{t-1} + (1-\rho)n_t \\
0 &= E_t[-\gamma(\Delta s_{t+1} + \Delta c_{t+1}) + r_{t+1}] \\
\bar{R}r_{t+1} &= \rho\frac{\bar{Y}}{\bar{K}}(y_{t+1} - k_t) \\
0 &= -\gamma(s_t + c_t) + y_t - n_t,
\end{aligned}$$

together with the log-linearized equation for the habit s_t from section 2.2 with $g = 0$ since we consider a stationary economy.

These equations can now be solved, using the method of undetermined coefficients with the techniques in Uhlig (1995). The state of the economy is given by the vector $[k_{t-1}, s_{t-1}, c_{t-1}, z_t]$. The solution for this dynamic system is a (linear) vector function f: $[k_t, s_t, c_t, y_t, nt] = f([k_{t-1}, s_{t-1}, c_{t-1}, z_t])$. More on the details can be found in the in Uhlig (1995). Of particular interest for us are the reactions of consumption and labor following a technology shock. Let η_c and η_n denote the respective elasticities with respect to technology. We will report the values for η_c and η_n in the following tables.

Standard frequency domain techniques can be used to compute the complex matrix spectral density function of all variables, see Uhlig and Xu (1995) for details. Hence, no simulations are necessary to obtain results for HodrickPrescott filtered series.

We compare this model also to a model without habit formation. This can simply be achieved by setting $\bar{S} = 1$. The resulting model is essentially Hansen's (1985) real business cycle model except that the coefficient of risk aversion is varied as well.

4 Results

The results of our analysis can be found in the following table and figures, comparing the reaction of consumption in particular to a shock in ϵ_t. We compare the instantaneous reaction of consumption in the model with and without habit formation in one set of tables and the full impulse response functions in the figures. We also present results for Hodrick-Prescott filtered series in another set of tables. We compare the model data to the relevant numbers from the US economy which we take from Cooley and Prescott (1995). The data is taken quarterly from 1954:I to 1992:II.

Table 1A: Impulse Responses
Labor variable

		with habit			without habit		
γ	ψ	η_c	η_n	figure	η_c	η_n	figure
2.372	0.95	0.020%	0.162%	2	0.300%	0.800 %	1
47.635	0.95				0.020%	0.146%	3
1.0	0.95	0.030%	0.192%	4	0.470%	1.472%	5
0.118	0.95	0.082%	0.349%	6	-2.86%	3.717%	
2.372	0.80	0.001%	1.463%		0.156%	1.795%	
2.372	0.99	0.028%	-0.984%		0.427%	-0.036%	
2.372	0.999	0.032%	-1.414%	7	0.475%	-0.350%	8

Table 1B: Hodrick-Prescott filter
Labor variable
US data: $\sigma_c = 0.86\%$, $\sigma_n = 1.59\%$, $\sigma_y = 1.72\%$

		with habit			without habit		
γ	ψ	σ_c	σ_n	σ_y	σ_c	σ_n	σ_y
2.372	0.95	0.021%	1.99%	1.83%	0.325%	1.88%	1.94%
47.635	0.95	0.005%	2.01%	1.83%	0.021%	2.00%	1.83%
1.0	0.95	0.031%	1.98%	1.84%	0.558%	1.99%	2.13%
0.118	0.95	0.086%	1.94%	1.85%	3.736%	3.49%	3.28%
2.372	0.80	0.012%	2.10%	2.08%	0.199%	2.11%	2.14%
2.372	0.99	0.028%	2.44%	1.77%	0.430%	1.94%	1.76%
2.372	0.999	0.031%	2.71%	1.83%	0.471%	2.05%	1.73%

First, consider the benchmark case, where we have used the "standard" parameters stated in the previous section. A positive 1% shock ϵ_t moves the technology parameter and therefore output at the steady state values given K_{t-1} and N_t up by 1%. If the agent decided to never change its level of capital and labor, the entire output change would be consumed, resulting in an increase of consumption by 1.36%, compared to the steady state level.[2] The models imply, of course, that the agent will usually not leave his gross investment levels and his labor input unchanged. The effect on consumption is decreased, if gross investment is increased or if the agent takes the opportunity of higher productivity to enjoy more leisure, i.e. if the labor input is decreased. The first row of the table lists the actual reaction of consumption in the model with and without habit at the standard parameters. In the model without habit consumption moves up by 0.3% rather than 1.36%. The effect is more dramatic by an order of magnitude in the model with habit, however. There, consumption moves up by merely 0.02%, not even a tenth of the movement in the model without habit formation. Figures 1 and 2, which correspond to these parameter choices, show what happens: capital is increased by about the same amount in both models, but the essential difference is in the labor input.

Insert Figures 1 and 2 approximately here.

While the agent in the model without habit formation (Figure 1) uses the opportunity of increased productivity to work a lot harder to build up capital, the agent in the model with habit formation will do so a lot less. Intuitively, that agent does not expect to change his consumption by much in the future: thus, why should he work very hard now? The tiny reaction of consumption in Figure 2 is no surprise, of course. With the habit formulation, the intertemporal elasticity of substitution is reduced strongly and is locally around the steady state close to $\bar{S}/\gamma$.[3] Since $\bar{S} = 0.0498$, this means that a version of the model without habit formation but with $\gamma = 2.372/0.0498 = 47.635$ should

[2]Note, by the way, that these numbers do not depend on choices for γ or ψ, or whether one considers the steady state in the model with or without habit formation.

[3]To see this, examine the derivative of the per-period utility function, given by $u'(C_t; X_t) = (C_t - X_t)^{-\gamma}$. Write $C_t = \bar{C}e^{c_t}$, let $X_t \equiv \bar{X}$ and take a first-order Taylor approximation in c_t to find $\log u'(C_t; X_t) \approx -\frac{\gamma}{\bar{S}} c_t$ and $u'(C_t; X_t) \approx (C_t/\bar{C})^{\frac{-\gamma}{\bar{S}}}$.

show a similar responsiveness of consumption to a technology shock, and it indeed does: compare Figure 3 to Figure 1 and row 2 to row 1 in the table.

Insert Figure 3 approximately here.

These results are also reflected by the Hodrick-Prescott filtered series presented in Table 1B. The standard deviation of consumption is too small by an order of magnitude in the model with habit and the benchmark parameter values. Even the model exuding habit produces not enough variation in consumption. The standard deviation for labor and output are fairly close to the US data for both models. As mentioned in the last paragraph, the habit model with $\gamma = 2.372$ behaves approximately similarly to the non-habit model with $\gamma = 47.635$.

Given these insights, one might therefore try to reduce γ in the habit formation model. A reduction to $\gamma = 1$ is given in row 3 of the tables and in Figures 4 and 5.

Insert Figures 4 and 5 approximately here.

The value of $\gamma = 1$ is an important benchmark in many other contributions to the real business cycle literature, and it is also the only parameter consistent with balanced growth. The instantaneous reaction in the model with habit grows to 0.03%, which is still not much. The reason for such a muted response to a change in γ is that habit moves endogenously as well: Figure 4 shows that s_t increases dramatically. This becomes even clearer when γ is reduced all the way to $2.372 \bullet \bar{S} = 0.118$, see row 4 and Figure 6.

Insert Figure 6 approximately here.

At $\gamma = 0.118$ and a habit level $X_t \equiv \bar{X}$ fixed at the steady state level, the intertemporal elasticity of substitution locally around the steady state is now 2.372 as in Figure 2. However, the response of consumption to a shock in z_t is still a lot smaller in the habit model with $\gamma = 0.118$ than in the no-habit-model with $\gamma = 2.372$: consumption rises by puny 0.08% rather than 0.30%. The reason is that the habit does not remain at the steady state level: the reaction in s_t was so large that we found it wise to leave it off

Figure 6. Thus, lowering γ does not repair the low responsiveness of consumption in the habit model easily. And even if it would, it would simply destroy all the interesting asset pricing implications which Campbell and Cochrane fought so hard to obtain. The HP filter result confirm again the impulse responses. Even with an extremely low value for γ produces the habit model consumption paths which are too smooth by an order of magnitude. Note, that the effect of a lower γ is very pronounced in the non-habit model. The variability of all relevant variables, consumption, labor and output, increases to very large values.

One may suspect that consumption does not react much in the habit models, since perhaps the shocks are still too transitory. Shouldn't it make more sense to raise consumption levels even in the habit model, if productivity changes are permanent? After all, Hall (1978) has taught us that a one per cent increase in permanent income should be accompanied by a one per cent increase in permanent consumption. Thus, we have increased the persistence of the technology shock in Figures 7 and 8 and rows 6 and 7 of the table to the near unit-root values $\psi = 0.99$ and even $\psi = 0.999$.

Insert Figures 7 and 8 approximately here.

Obviously, the response of consumption to a technology shock is still disappointingly low. The figures immediately make clear, why this is the case: in the model with habit, the agent does not expect to change his consumption much in the future. With a permanent increase in productivity, he thus simply takes this opportunity to increase his consumption of leisure. While this effect is certainly present even without habit (Figure 8), it is even more dramatic in the model with habit (Figure 7). While Hall's logic still holds true, the rise in productivity simply does not correspond to a rise in income. An interesting feature of high persistence of shocks is that the labor input reaction is much smaller than with less persistent shocks. With a risk aversion coefficient of 2.372, $\eta_n = -0.036$ for $\gamma = 0.99$, and $\eta_n = -0.350$ for $\gamma = 0.999$ in the model without habit. When a habit is included, the numbers are even more negative. Hence, the persistence of shocks must not be too large when labor input should be procyclical as is suggested by the data. Again, the HP filtered standard deviations confirm the intuition. Increasing the persistence of the technology shock has only a minor effect on consumption variability in the habit model. Since consumption is not varying much, the corresponding numbers for labor and output are not too sensitive to changes in ψ as well.

For completeness we also have included the number for less persistent shocks, i.e. $\gamma = 0.80$ (see row 5). The labor reaction is fairly large in both models with and without habit as the agent wants to take advantage of the high productivity as long as it is high. However, the consumption path is not changed by much.

In the next section, we consider various changes from the benchmark model in order to check whether it is possible to increase the variability of the consumption path.

5 Can the habit formation model be saved?

5.1 Power utility in labor

In this subsection we relax the assumption that utility is linear in labor. Instead we assume that the agent has a power utility in labor:

$$U(C, N; X) = \frac{(C - X)^{1-\gamma} - 1}{1 - \gamma} + A\frac{(1 - N)^{1-\gamma_n} - 1}{1 - \gamma_n},$$

where $1 - N$ is the amount of leisure enjoyed by the consumer.[4] Note, that the special cases $\gamma_n = 0$ and $\gamma_n = 1$ correspond to a model with linear utility in labor and a model with fixed labor, respectively. As usual, a high curvature parameter γ_n implies that the agent is very risk averse in labor or equivalently that he has a low elasticity of intertemporal substitution for labor (or leisure).

The only difference to the model with linear labor utility is the first-order condition for the labor-leisure choice, which becomes now in log-linearized form:

$$-\gamma(s_t + c_t) + y_t - n_t = \gamma_n \frac{\bar{N}}{1 - \bar{N}} n_t.$$

Following Prescott (1986) and Campbell (1994) we pick $\bar{N} = 1/3$ since households allocate about one-third of their time to labor. For the risk aversion coefficient in consumption, we pick the Campbell and Cochrane (1995) value of $\gamma = 2.372$. The persistence parameter of technology shocks is set to the benchmark value of $\psi = 0.95$. Table 2A presents the immediate reaction of consumption and labor to a unit technology shock while Figures 9-12 present to complete set of impulse responses.

[4] King, Plosser and Rebello (1988) show that log utility for consumption is needed to obtain balanced growth. We nevertheless decided to stick to the general power utility framework to be able to compare our results more directly to the Campbell and Cochrane (1995) paper.

Table 2A: Impuls Responses
Power utility in labor
$\gamma = 2.372, \psi = 0.95$

	with habit			without habit		
γ_n	η_c	η_n	figure	η_c	η_n	figure
0.5	0.022%	-0.065%	9	0.298%	0.480%	
1	0.024%	-0.163%	10	0.300%	0.334%	
10	0.050%	-0.257%	11	0.333%	0.039%	
1000	0.110%	-0.008%	12	0.355%	0.000%	

Table 2B: Hodrick-Prescott filter
Power utility in labor
$\gamma = 2.372, \psi = 0.95$
US data: $\sigma_c = 0.86\%$, $\sigma_n = 1.59\%$, $\sigma_y = 1.72\%$

	with habit			without habit		
γ_n	σ_c	σ_n	σ_y	σ_c	σ_n	σ_y
0.5	0.022%	1.28%	1.41%	0.317%	1.14%	1.53%
1	0.024%	0.97%	1.23%	0.317%	0.82%	1.35%
10	0.049%	0.34%	0.90%	0.342%	0.14%	1.01%
1000	0.108%	0.01%	0.95%	0.359%	0.01%	0.97%

Several aspects of these results are worth noting. First, note that the consumption reaction in the model with habit is fairly small as long as γ_n is not too large. Only when γ_n becomes very large, i.e. the agents labor intertemporal elasticity is so low that labor input is essentially fixed, is the consumption reaction a bit higher (see row 4). Second, the consumption reaction in the model without habit does not depend very much on γ_n. η_c is between 0.3 and 0.35 for all γ_n. The labor input response, η_n is however varying quite a bit with γ_n. This shows that consumption is essentially determined by other variables in the model. Third, note the on first sight counterintuitive ranking of the η_n's for $\gamma_n = 0.5,\ 1,\ 10$ in the model with habit. The elasticity of intertemporal labor substitution is decreasing as γ_n is increasing. Nevertheless η_n is becoming more and more negative. Only when γ_n is very large, moves η_n towards zero. The pattern becomes clear when Figures 9-12 and Table 2B are consulted. Despite the fact that the initial reaction is less negative for smaller γ_n, the labor reaction is less smooth for these values. The minimum point in the labor reaction is reached after about four years. The minimum is more negative for low values of γ_n despite the smaller initial reaction. The results in Table 2B confirm that the total variability of labor is decreasing as γ_n is increasing. The intuition is that a higher γ_n makes the agent want to smooth labor more over the course of the business cycle. But a smooth labor path requires a fairly substantial initial (negative) reaction so that labor does not need to be changed too much in the future. This is not the case for lower value of γ_n.

We conclude that a model with nonlinear utility in labor is not able to increase the variability of consumption when a habit is present. Moreover it produces countercyclical labor input which is not in accordance with the data. Only when the elasticity of intertemporal labor substitution is essentially zero, i.e. labor is essentially fixed, increases the consumption reaction to plausible values. This model with fixed labor is studied next in more detail.

5.2 A model with fixed labor input

It should be clear that the flexibility of adjusting labor is crucial to our argument. We find this appealing: after all, cyclical output fluctuations are foremost fluctuations in hours worked. Thus, the adjustability of leisure should be an essential feature of any model that attempts to derive consumption fluctuations endogenously. As long as the agent can achieve a high degree of consumption smoothing by adjusting leisure, he will choose to do so, and we are left with consumption paths which are too smooth to be in accordance with observations. Despite this argument, it is still interesting to see what happens once labor input is fixed exogenously in the RBC model. How big is the consumption reaction once agents are not able to use labor as device to smooth out consumption? See Table 2 and Figures 13 and 14. We fix $\gamma = 2.372$ at the benchmark level and vary the persistence parameter ψ. For $\psi = 0.95$ it can already be seen that the consumption reaction in the fixed labor care are larger than in the flexible labor care (0.112% compared to 0.020%). Once the persistence parameter is increased to 0.999 the initial consumption reaction is now even up to 0.435 compared to 0.032. Note also that the difference between the habit and no-habit models are much smaller with fixed labor input. This illustrates the importance of labor in this model. Hence, if one is willing to accept the view that labor is more or less fixed exogenously and not by the first-order condition of the representative agent, then the habit model is able to generate reasonable consumption fluctuations when technology is persistent. On the other hand, it is well known that a highly variable labor supply is needed to get a elasticity of output with respect to technology which exceeds one (see e.g. Campbell (1994)).

HP filtered consumption of the habit model is about half as variable as US consumption when technology is almost a random walk. The no-habit model almost matches the value in the data in this case. Note, however that output is smoother than in the data for both models. Interestingly, changing the persistence of the shock does not affect the variability of the HP filtered output sereis much. The reason is that changes in output are almost permanent due to the persistent technology but the HP filter cancels out this permanent component.

Table 3A: Impuls Responses
Labor fixed

	with habit				without habit		
γ	ψ	η_c	η_n	figure	η_c	η_n	figure
2.372	0.95	0.112%	0.00%	13	0.355%	0.00%	
2.372	0.99	0.284%	0.00%		0.680%	0.00%	
2.372	0.999	0.435%	0.00%	14	0.861%	0.00%	

Table 3B: Hodrick-Prescott filter
Labor fixed
US data: $\sigma_c = 0.86\%$, $\sigma_n = 1.59\%$, $\sigma_y = 1.72\%$

		with habit			without habit		
γ	ψ	σ_c	σ_n	σ_y	σ_c	σ_n	σ_y
2.372	0.95	0.110%	0.00%	0.95%	0.362%	0.00%	0.96%
2.372	0.99	0.277%	0.00%	0.95%	0.671%	0.00%	0.96%
2.372	0.999	0.424%	0.00%	0.95%	0.845%	0.00%	0.96%

5.3 A model with stochastic labor input

The model with fixed labor supply might be regarded as extremely restrictive and unappealing. As an alternative, we offer a model in which labor input is given exogenously but is varying over time. We consider a very simple model where labor input follows an AR(1) process. The model is intentionally kept as simple as possible because we want to work out the principal channel by which movements in other variables can influence the consumption choice of the individual. Since this model yields interesting results, which are discussed below, it would be interesting to find deeper model-theoretic justifications for what are now exogenous fluctuations in labor input. Possible avenues to pursue are frictions on the labor supply side or institutional constraints, such as union power, or efficiency wages. It is beyond the scope of the paper to pursue these issues here.

We assume that labor input follows an AR(1) process with AR-parameter ψ_n:

$$n_t = \psi_n n_{t-1} + e_{n,t}, \quad \epsilon_{n,t} \sim \mathcal{N}(0, \sigma_n^2).$$

This equation replaces the first-order condition for labor choice in the dynamic system. It turns out that this model is equivalent to the fixed-labor model with technology shocks in which the technology z_t is replaced by $(1-\rho)n_t$. Hence, we can use that model and multiply the results by $1-\rho = .64$ to get the results for the AR(1) labor model. However, it seems not plausible that shocks to labor are as persistent as technology shocks, hence we choose a lower AR coefficient. The first order autocorrelation of quarterly GNP is about 0.85 in post-war data (see Cooley and Prescott (1993)). Since output and labor input are highly, but not perfectly, correlated we pick 0.80 as reasonable persistence parameter for the labor process.

Figure 15 with $\gamma = 2.372$ presents the impulse response function for the model with habit following a positive 1% shock in labor. For comparison, the initial consumption adjustment in the model without habit is 0.1681% whereas with habit this reduces to 0.029%. Note, that the numbers are smaller than those in Table 3A since the persistence of the shock is smaller. Hence, despite the fact that agents cannot use labor to smooth consumption, the savings channel dominates the consumption effect. Moreover, there is another feature that makes this simple model unappealing. As can be seen from Figure 15 labor productivity $y_t - n_t$ is strongly countercyclical which is counterfactual. In fact the initial labor productivity reaction is equal to $-\rho$.

5.4 A model with technology and labor shocks

Next we consider a model with both technology shock and shocks to the labor input. It is shown that this version of the RBC model can be consistent with procyclical labor productivity as well as fairly large consumption fluctuation. Assume that technology follows a AR(1) process with a parameter close to unity. As we have shown in the previous section, this model yields reasonably high consumption responses to technology shocks. Additionally, assume that labor input also follows an AR(1) with a smaller persistence parameter. Hence, labor will be procyclical and labor productivity mildly procyclical. The persistent technology shock produces a substantial variability in consumption even with habit formation since the agent cannot use labor to smooth it, see Table 4A. Table 4B presents the results for the HP series. Since we want to study the consumption reaction in a realistic setting, we choose the standard deviations of the shocks to approximately match the HP filtered standard deviations of labor and output. Moreover, since high persistence of technology shocks is required to get a variable consumption path, we pick $\psi_z = 0.999$. ψ_n is set to 0.8. Given these values, the model with habit produces a variability of consumption which almost as high as in the data: 0.61 versus 0.86. On the other hand, the model without habit produces a consumption path which is too variable. Hence, we conclude that an RBC model with habit formation is able to generate reasonable consumption variability when one is willing to accept large labor market frictions and highly persistent technology shocks.

Table 4A: Impuls Responses
Technology and labor shocks
$\gamma = 2.372$
$\psi_z = 0.999$, $\sigma_z = 1.1\%$
$\psi_n = 0.8$, $\sigma_n = 1.3\%$

	with habit			without habit		
	η_c	η_n	figure	η_c	η_n	figure
1% shock in z	0.44%	0.00%		0.86%	0.00%	
1% shock in n	0.02%	1.00%		0.08%	1.00%	

Table 4B: Hodrick-Prescott filter
Technology and labor shocks
$\gamma = 2.372$
$\psi_z = 0.999$, $\sigma_z = 1.1\%$
$\psi_n = 0.8$, $\sigma_n = 1.3\%$
US data: $\sigma_c = 0.86\%$, $\sigma_n = 1.59\%$, $\sigma_y = 1.72\%$

with habit			without habit		
σ_c	σ_n	σ_y	σ_c	σ_n	σ_y
0.61%	1.57%	1.67%	1.23%	1.57%	1.69%

To sum up, standard real business cycle models with shocks to technology tend to produce very small variability in consumption when agents can use labor to smooth consumption. Only when this channel is effectively shut and labor input is determined exogenously, is the RBC model able to produce reasonable consumption fluctuation.

6 Conclusions

In this paper we studied how habit formation in the utility function affects the optimal responses of consumption, labor input and output in response to exogenous shocks. We chose the habit formulation of Campbell and Cochrane (1995) because it is able to explain a wide range of asset pricing puzzles when consumption is assumed to be exogenous. We show that once a habit is included in Hansen's (1985) RBC model with adjustable labor, consumption is extremely smooth and unresponsive to shocks. The intuition is that the habit makes the consumer (locally) very risk averse hence lowering the elasticity of intertemporal substitution of consumption dramatically. Since agents can choose their labor input they decide to consume more leisure following a positive technology shock. Thus agents do not have to adjust consumption by much. We consider various extensions to this benchmark model. First, we generalize the utility function in labor to be able to control the intertemporal elasticity of labor substitution. We find that this does not change the results qualitatively as long as the intertemporal elasticity of labor substitution is not so small that labor is essentially fixed. Since the labor channel is crucial in the model, we study a model in which labor is fixed exogenously. Hence agents cannot adjust their labor input after a shock. When coupled with very persistent technolgy shock, this model is able to create substantial consumption fluctuations even in the presence of a habit. Since a model with fixed labor input produces counterfactual labor and labor productivity results, we consider next a model with fluctuating but exogenous labor input. We view this as a extreme model in which there are substantial frictions in the labor market which are not modelled in this paper. A model with persistent technology shocks coupled with exogenous labor fluctuations produces a consumption path which is almost as volotile as US consumption. We view this as a partial success of the habit model. The labor frictions in this paper are very extreme and are not explicitely modelled. However, it is clear that the labor channel has to be effectively closed to the agents to get a reasonable consumption variability in the presence of habit formation.

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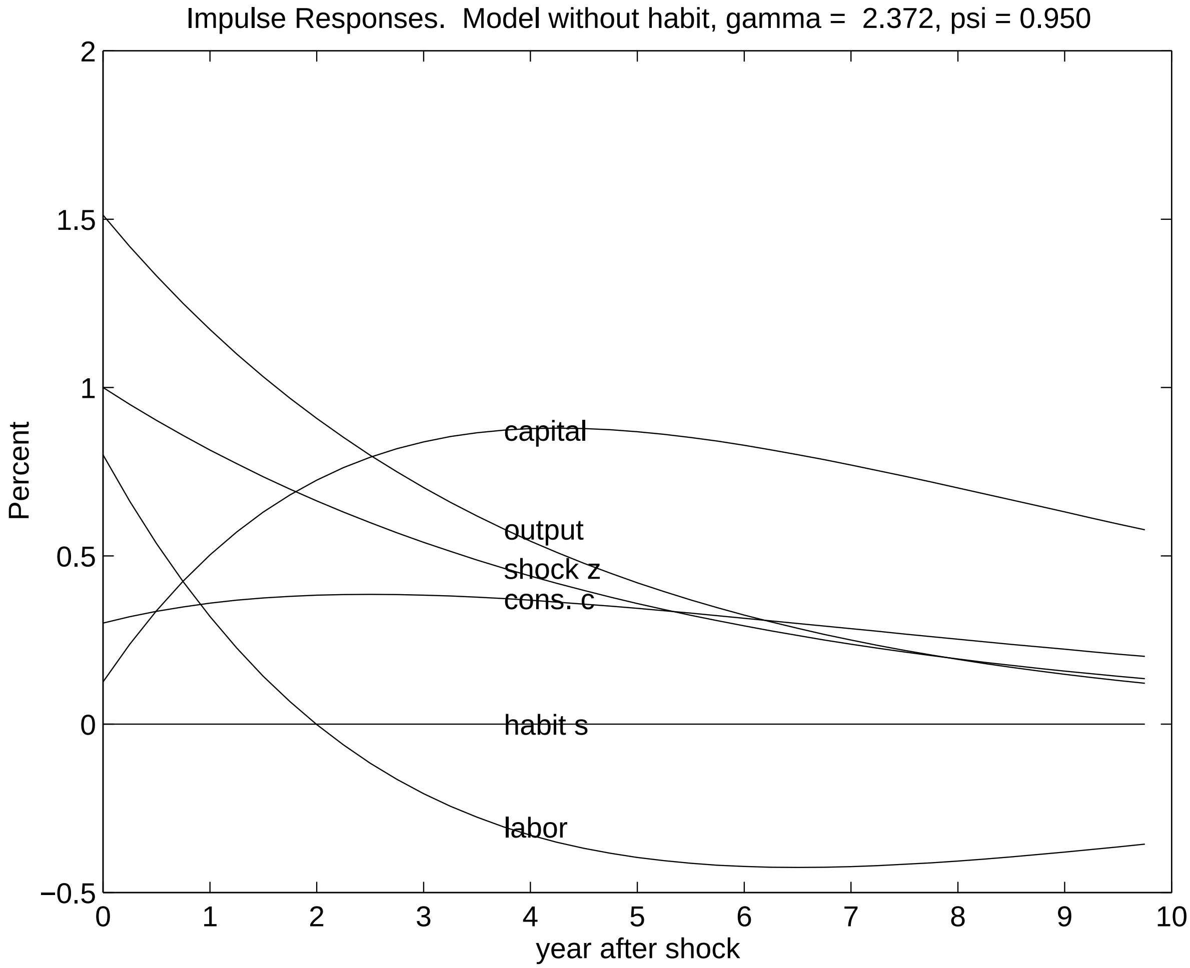
Figure 1


Impulse Responses. Model without habit, gamma = 2.372, psi = 0.950
Percent
year after shock
capital
output
shock z
cons. c
habit s
labor
2
1.5
1
0.5
0
−0.5
0
1
2
3
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5
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10

Figure 2

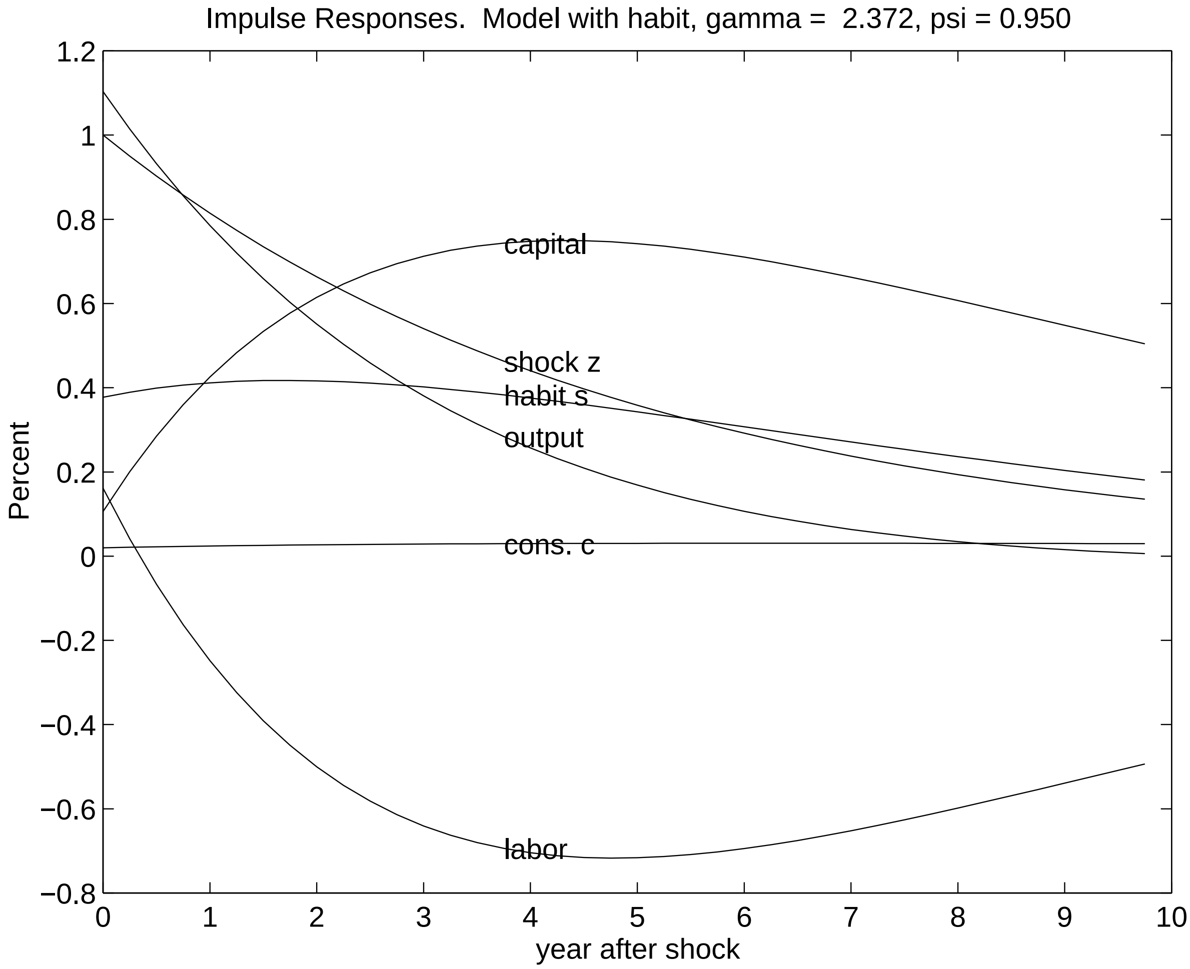
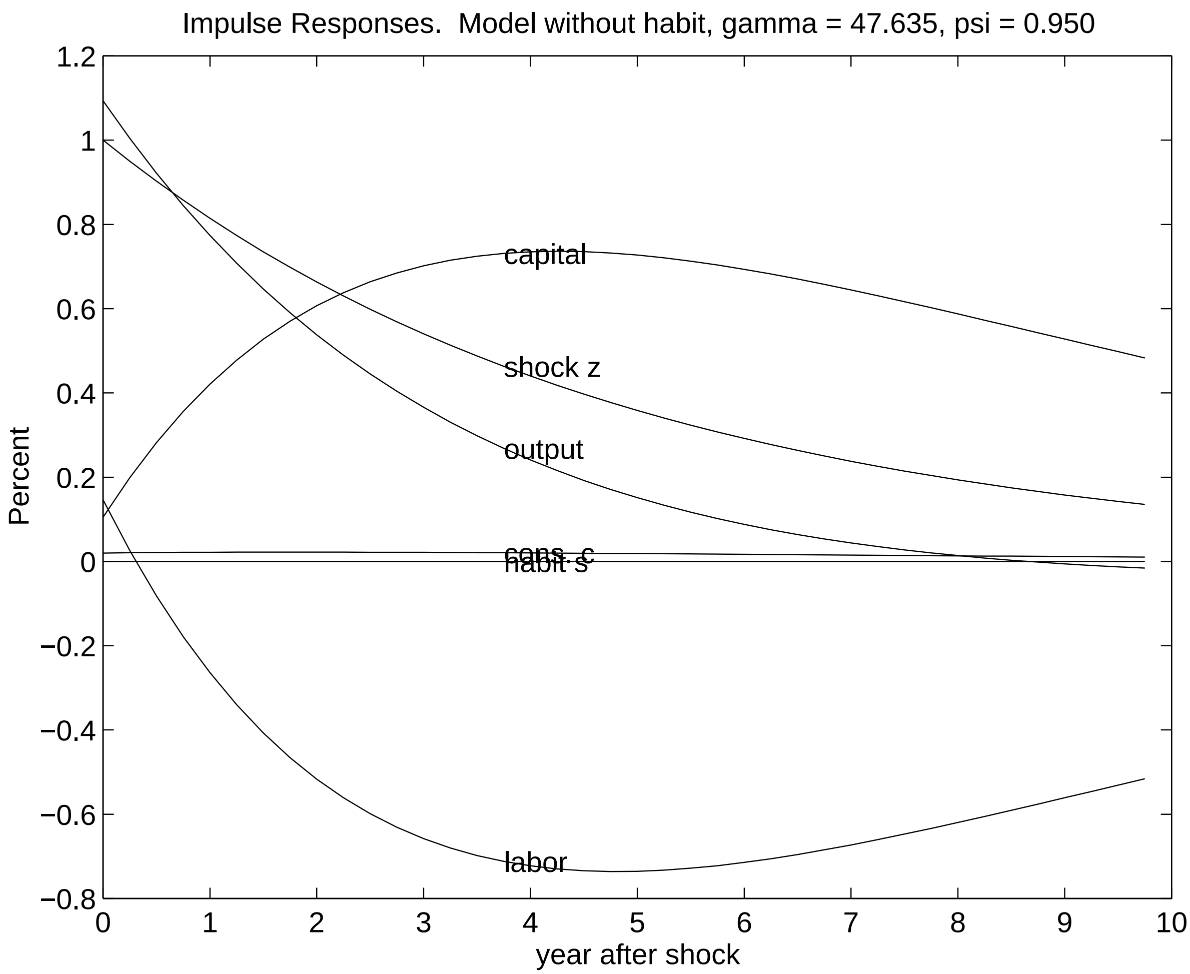

Impulse Responses. Model with habit, gamma = 2.372, psi = 0.950
Percent
year after shock
capital
shock z
habit s
output
cons. c
labor
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
0
1
2
3
4
5
6
7
8
9
10

Figure 3


Impulse Responses. Model without habit, gamma = 47.635, psi = 0.950
Percent
year after shock
capital
shock z
output
cons. c
habit s
labor
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
0
1
2
3
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9
10

Figure 4

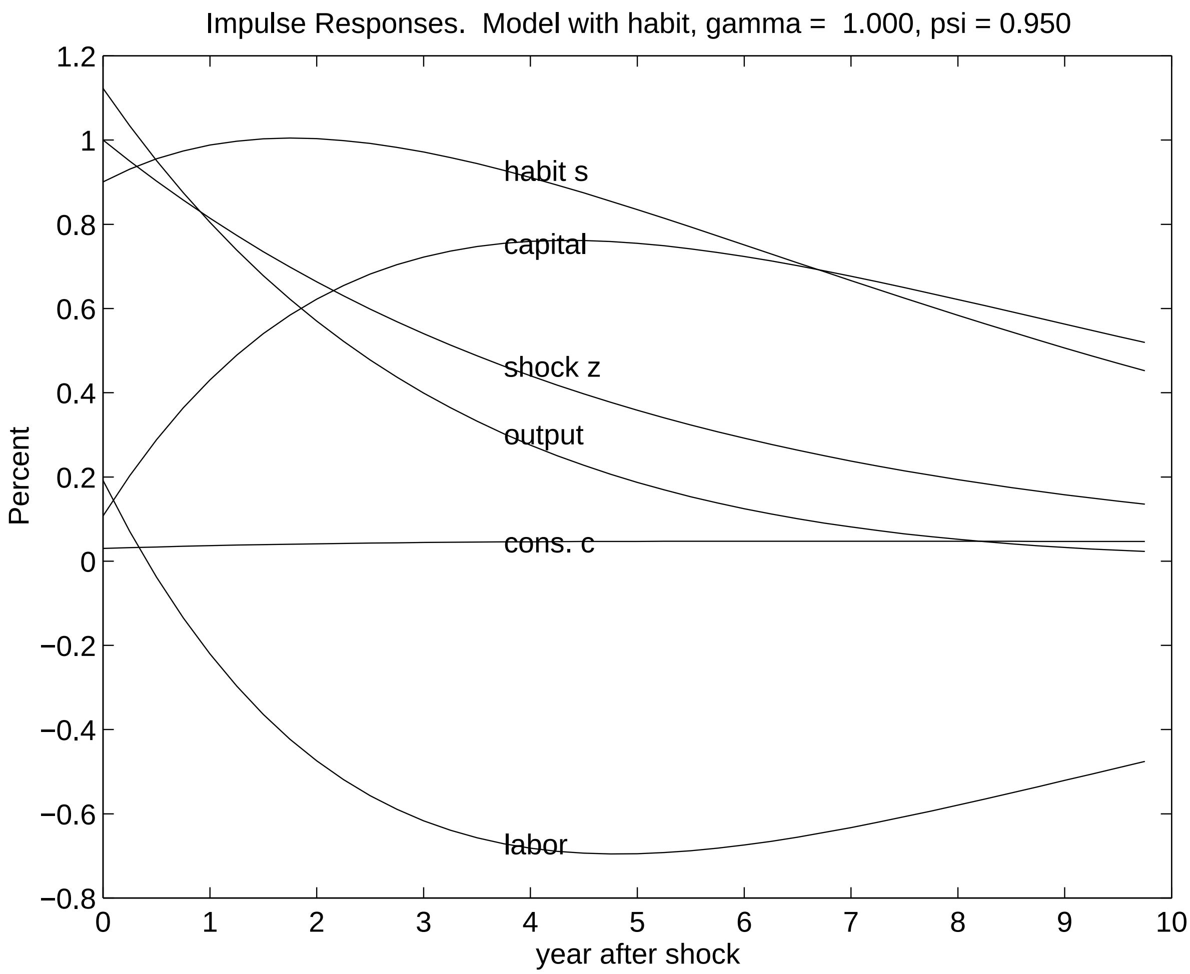
Impulse Responses. Model with habit, gamma = 1.000, psi = 0.950
habit s
capital
shock z
output
cons. c
labor
Percent
year after shock
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
0
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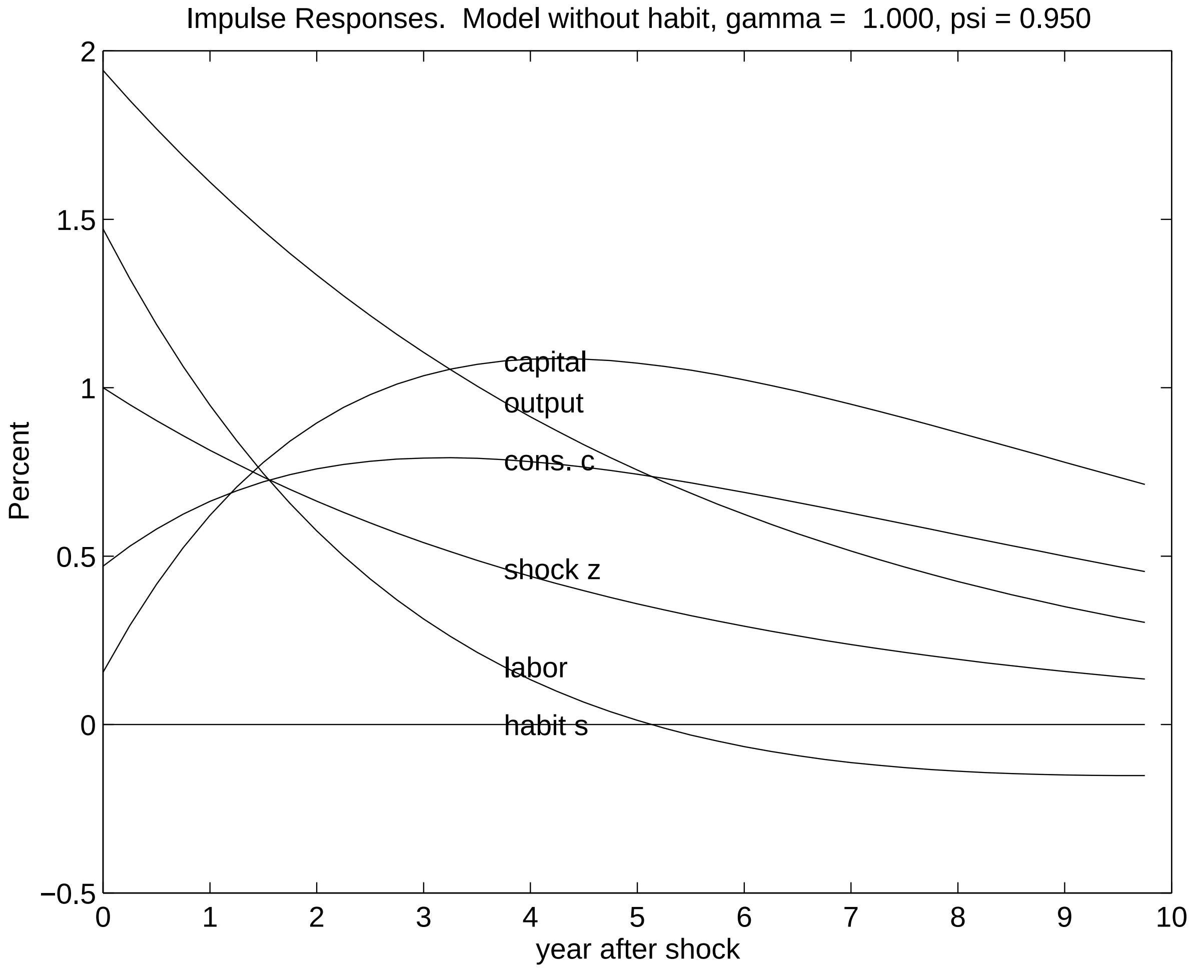
Figure 5


Impulse Responses. Model without habit, gamma = 1.000, psi = 0.950
capital
output
cons. c
shock z
labor
habit s
Percent
year after shock
2
1.5
1
0.5
0
−0.5
0
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2
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4
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7
8
9
10

Figure 6

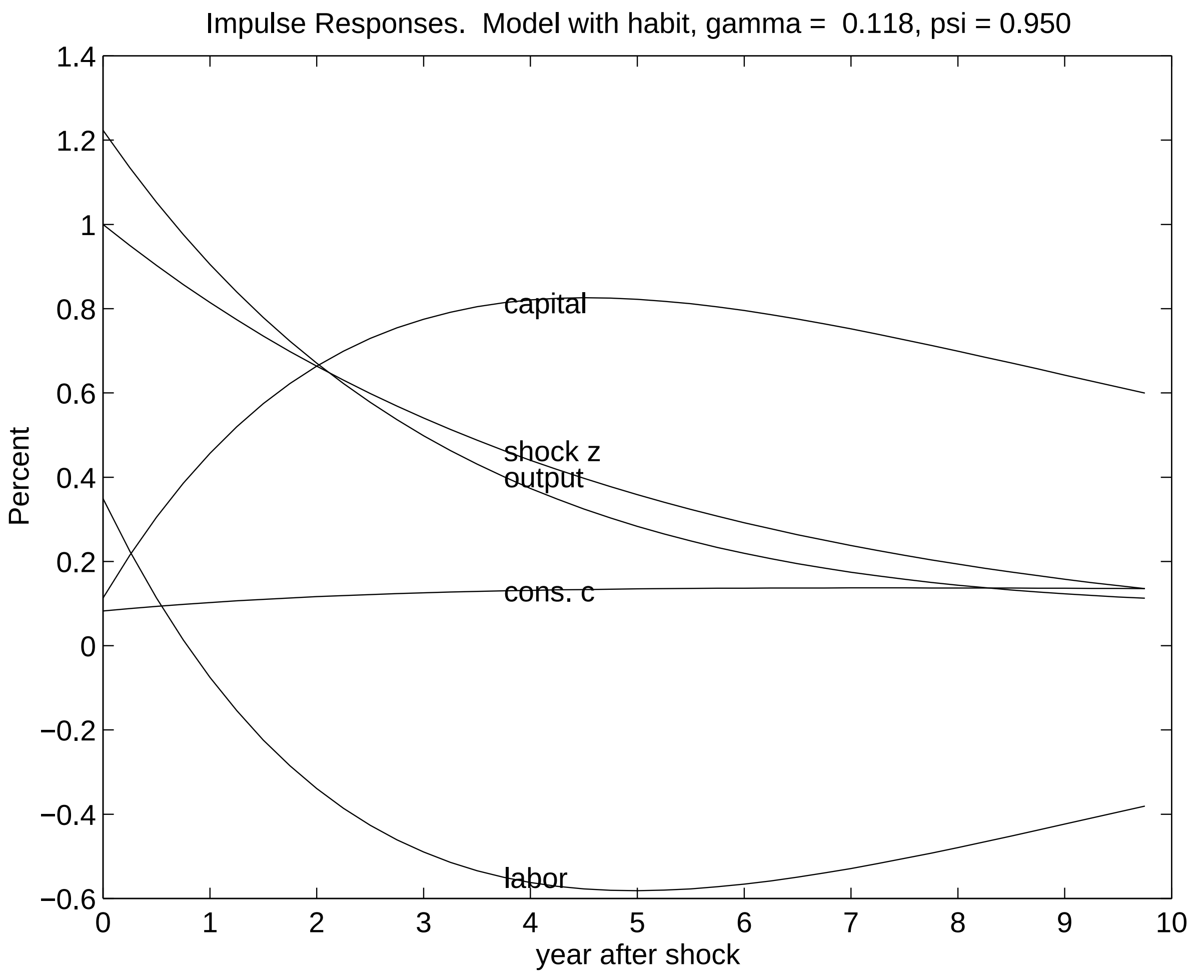
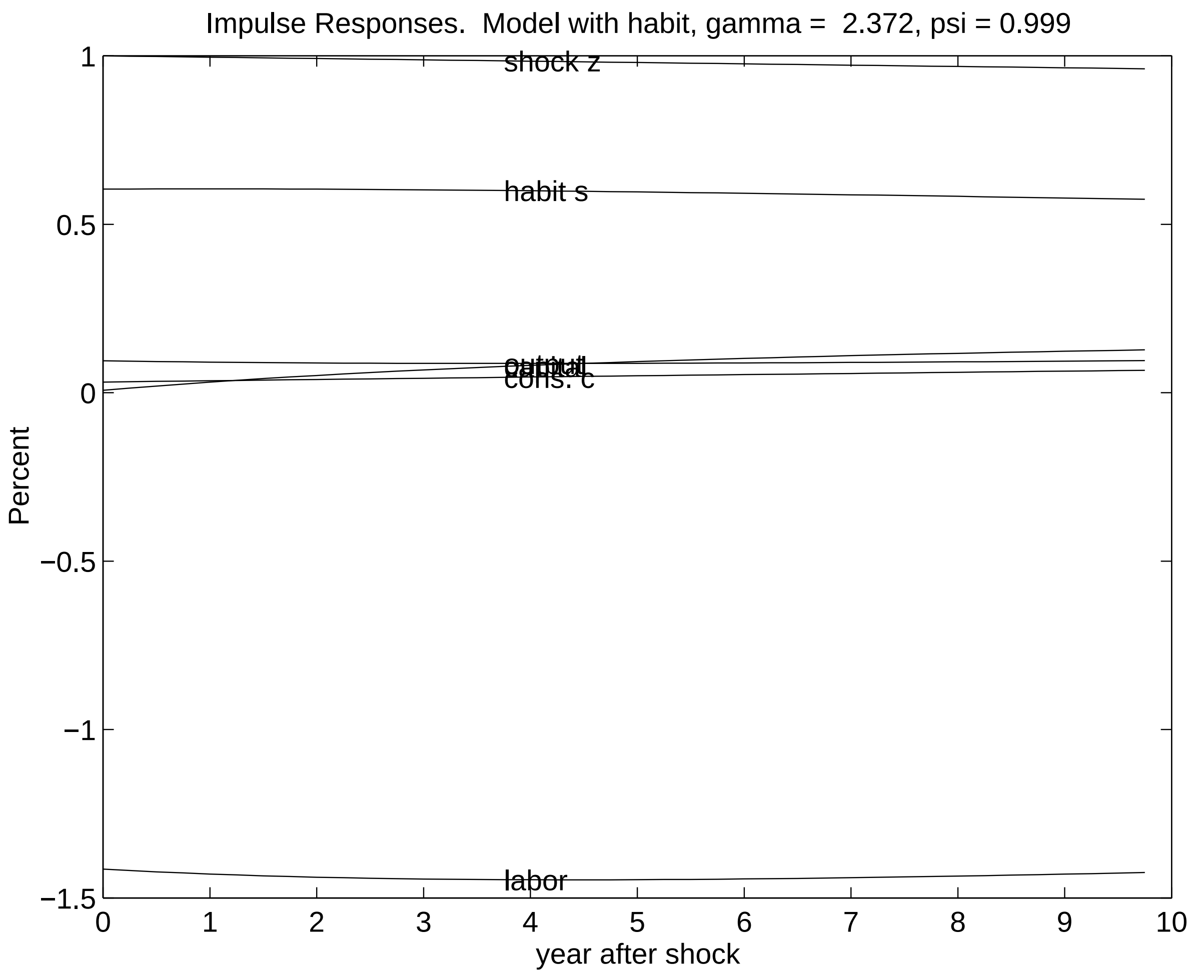

Impulse Responses. Model with habit, gamma = 0.118, psi = 0.950
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
Percent
0
1
2
3
4
5
6
7
8
9
10
year after shock
capital
shock z
output
cons. c
labor

Figure 7


Impulse Responses. Model with habit, gamma = 2.372, psi = 0.999
shock z
habit s
output
capital
cons. c
labor
Percent
year after shock
1
0.5
0
−0.5
−1
−1.5
0
1
2
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4
5
6
7
8
9
10

Figure 8

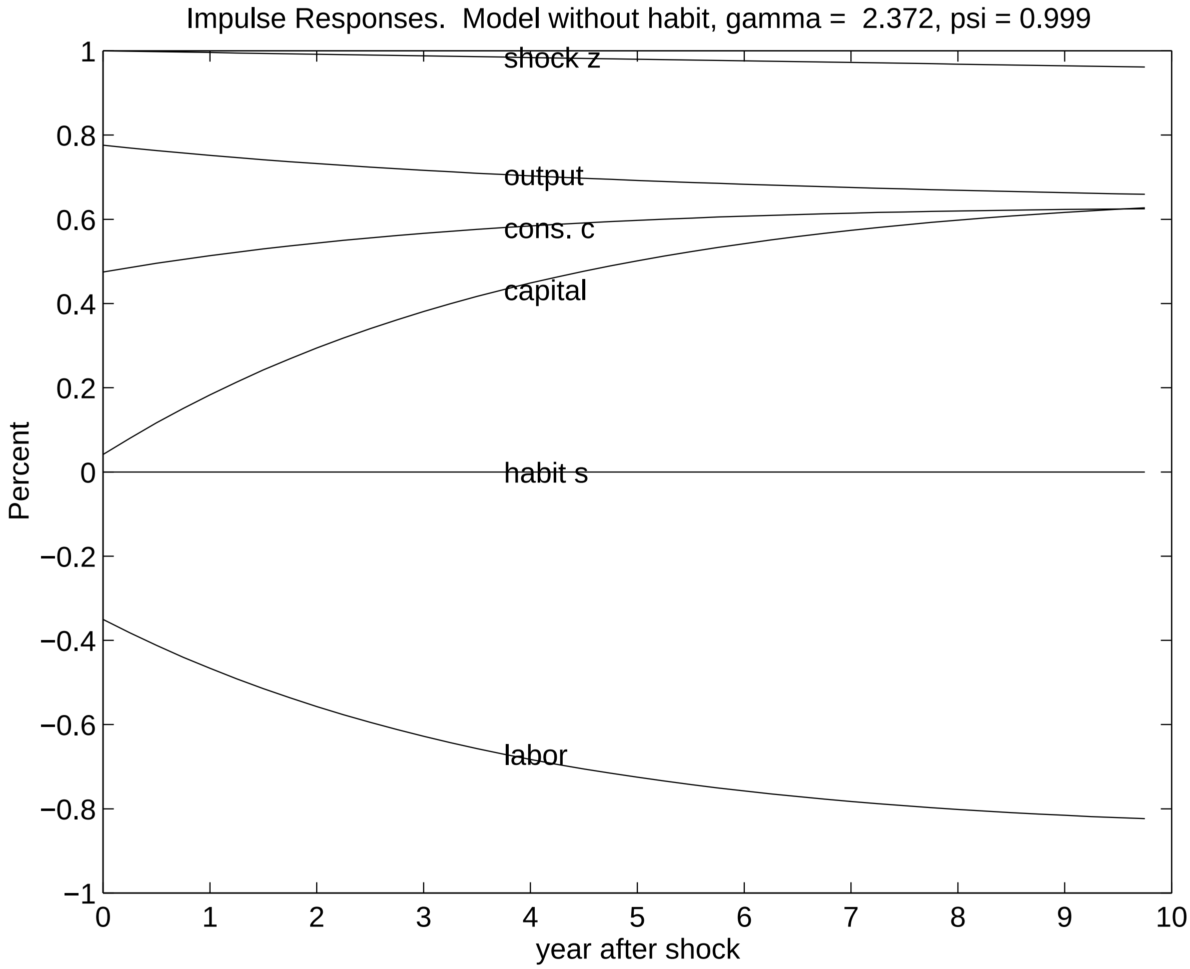
Impulse Responses. Model without habit, gamma = 2.372, psi = 0.999
Percent
year after shock
shock z
output
cons. c
capital
habit s
labor
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0
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Figure 9

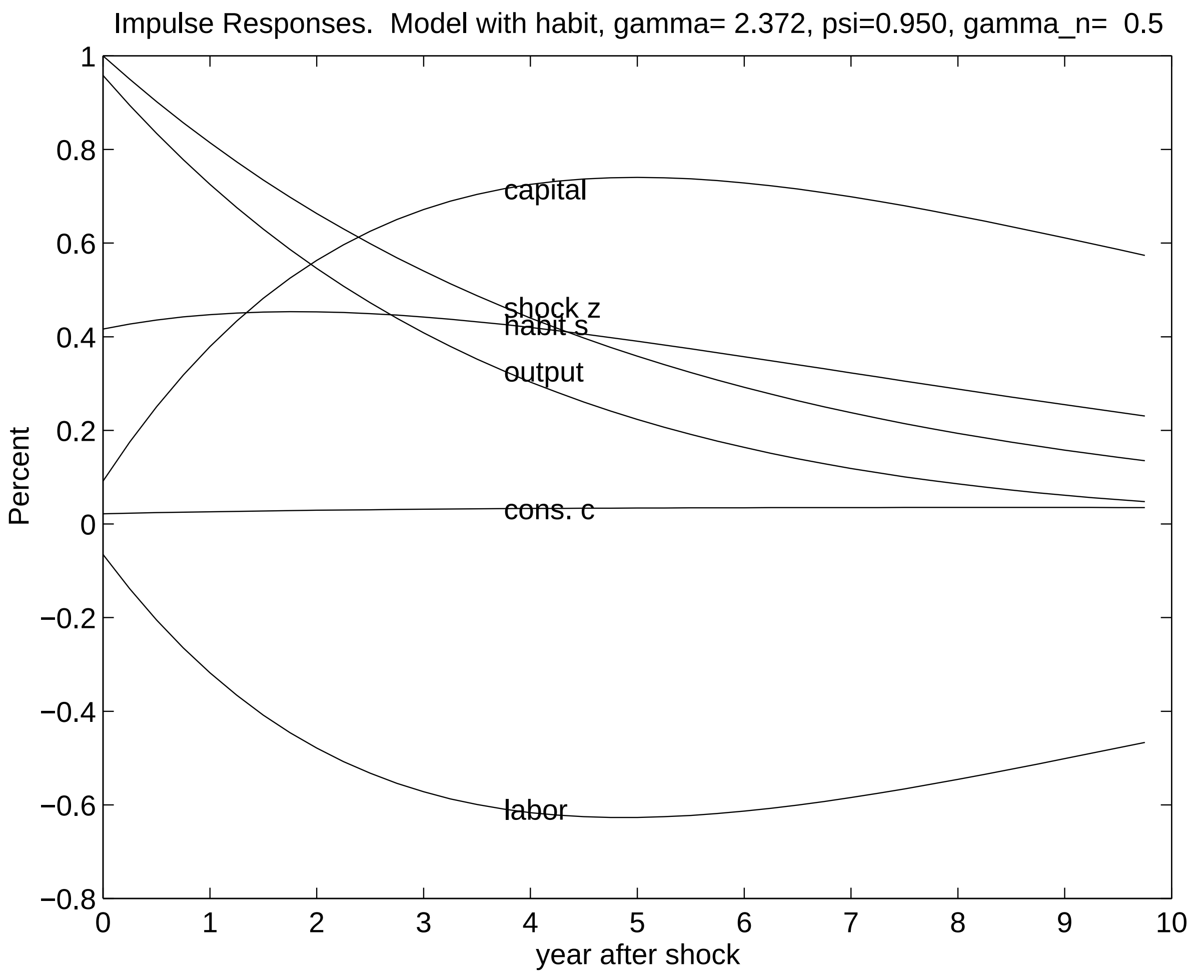
Impulse Responses. Model with habit, gamma= 2.372, psi=0.950, gamma_n= 0.5
Percent
year after shock
capital
shock z
habit s
output
cons. c
labor
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
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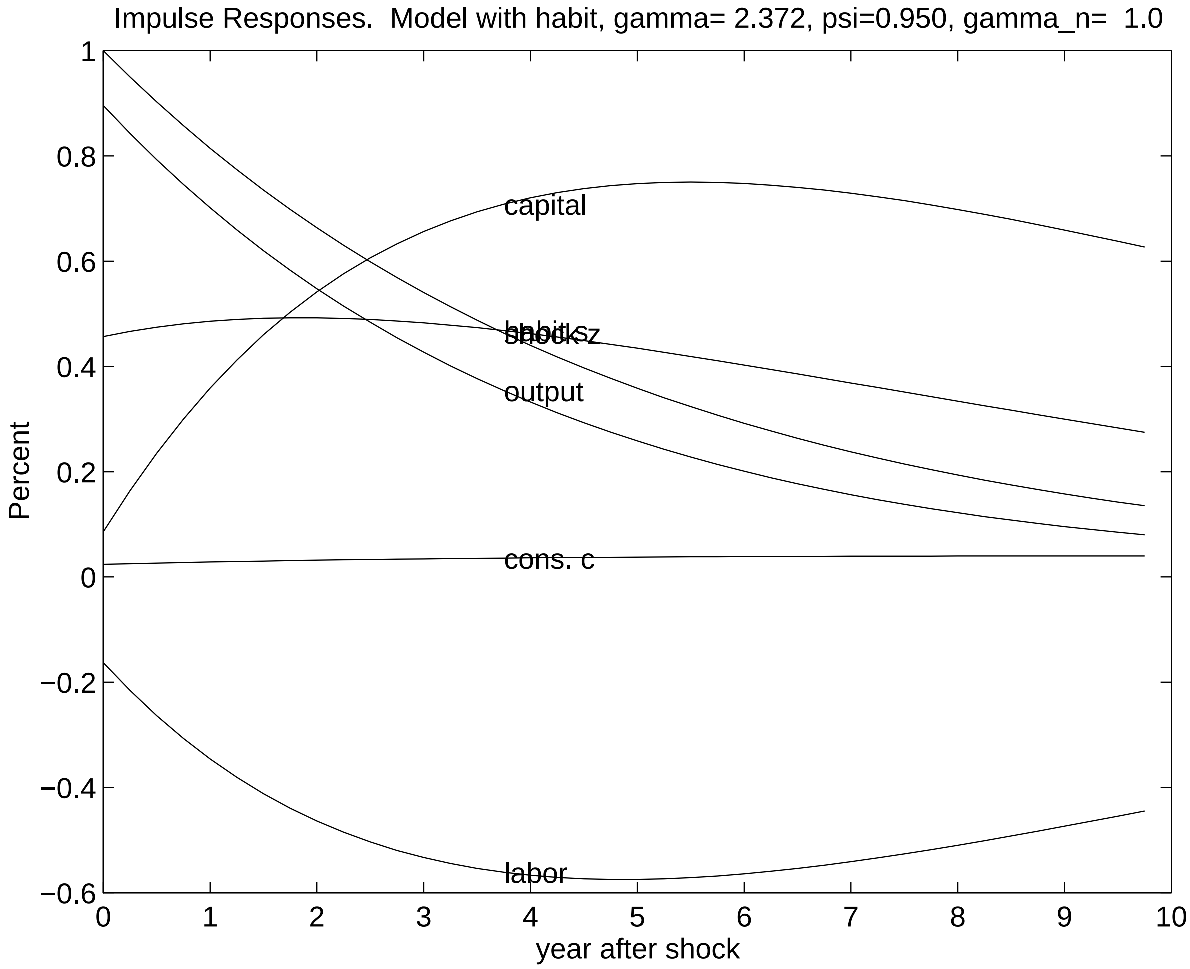
Figure 10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capital
habit s
shock z
output
cons. c
labor
Percent
year after shock
1
0.8
0.6
0.4
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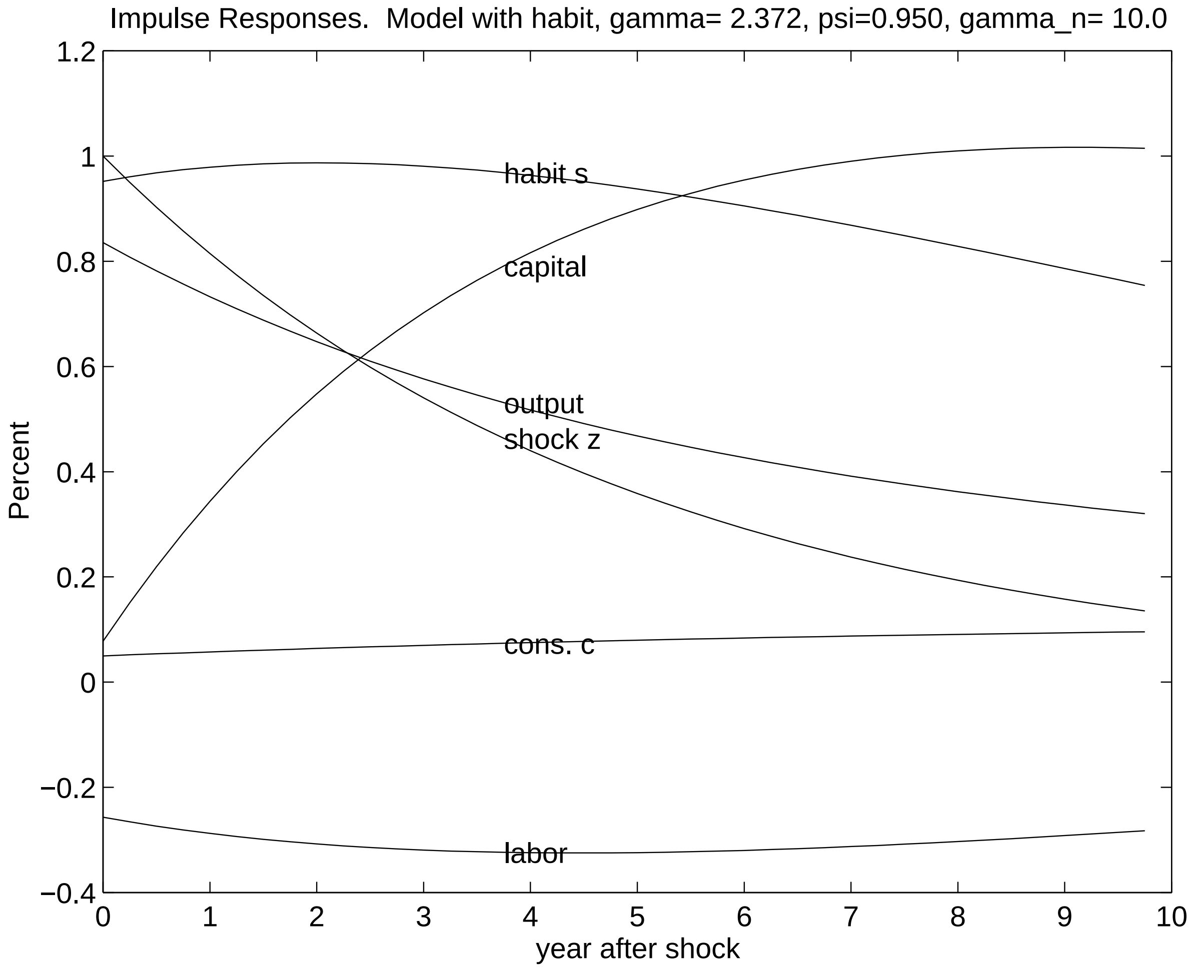
Figure 11


Impulse Responses. Model with habit, gamma= 2.372, psi=0.950, gamma_n= 10.0
1.2
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0.8
0.6
0.4
0.2
0
−0.2
−0.4
Percent
habit s
capital
output
shock z
cons. c
labor
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10
year after shock

Figure 12

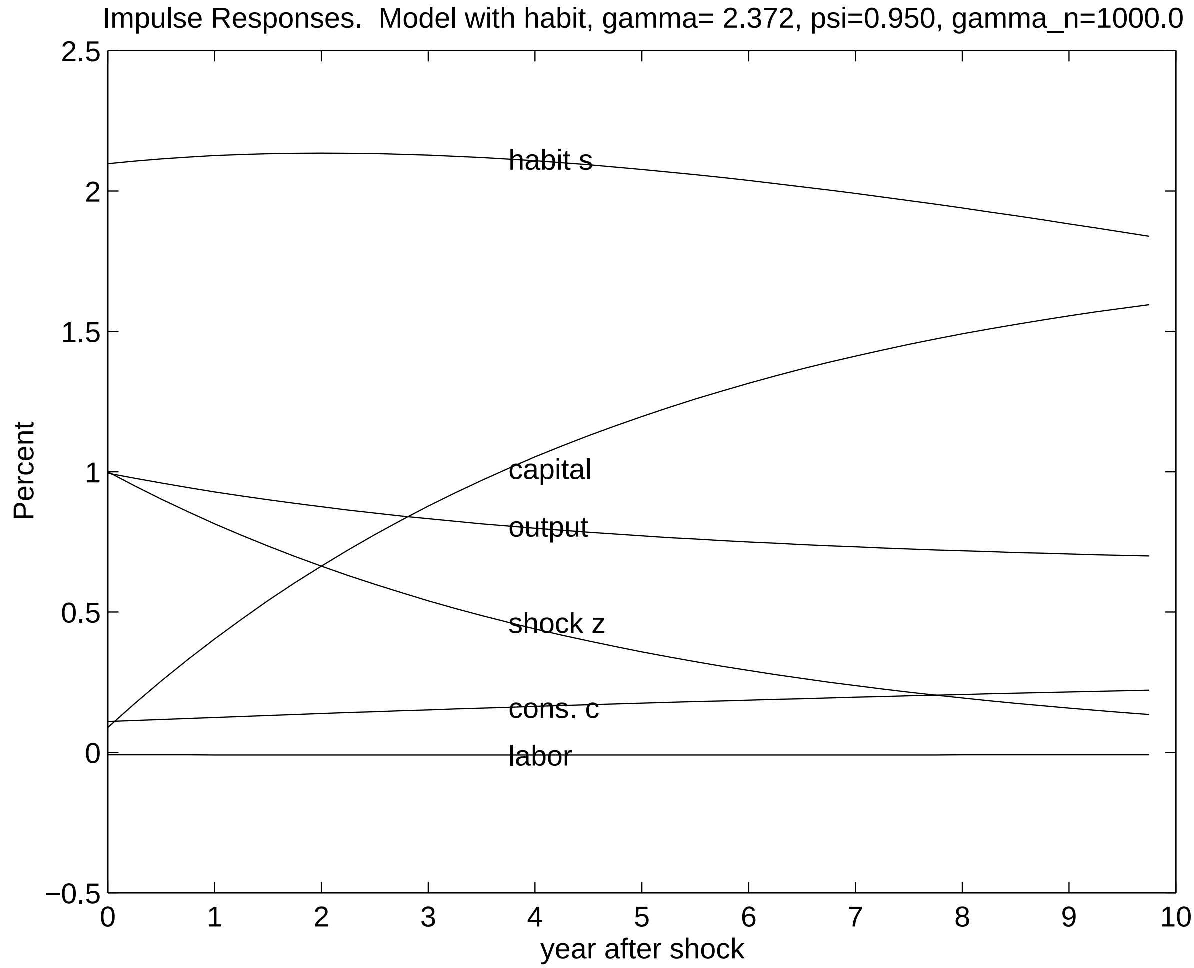
Impulse Responses. Model with habit, gamma= 2.372, psi=0.950, gamma_n=1000.0
habit s
capital
output
shock z
cons. c
labor
Percent
year after shock
2.5
2
1.5
1
0.5
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Figure 13

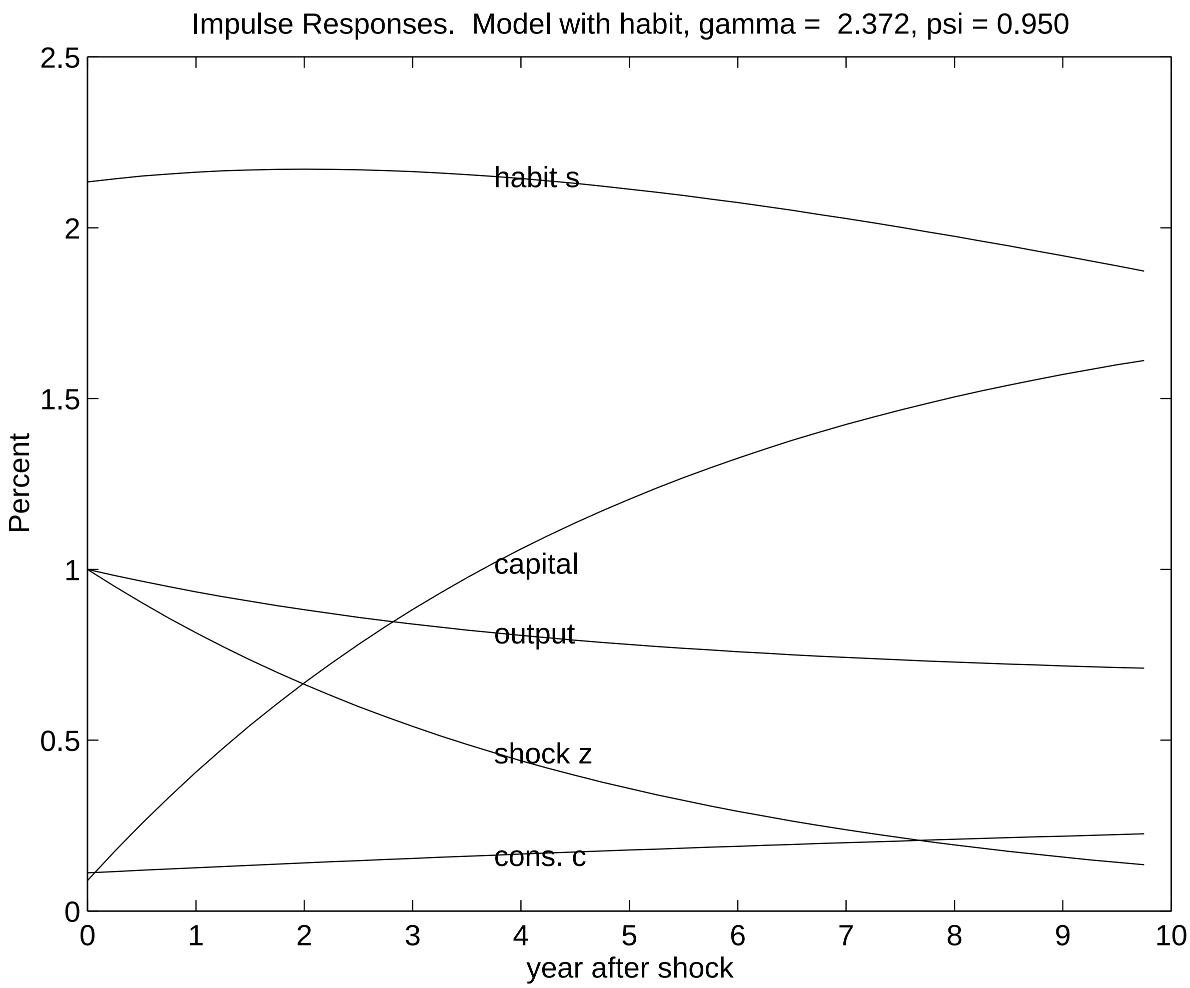
Impulse Responses. Model with habit, gamma = 2.372, psi = 0.950
Percent
year after shock
habit s
capital
output
shock z
cons. c
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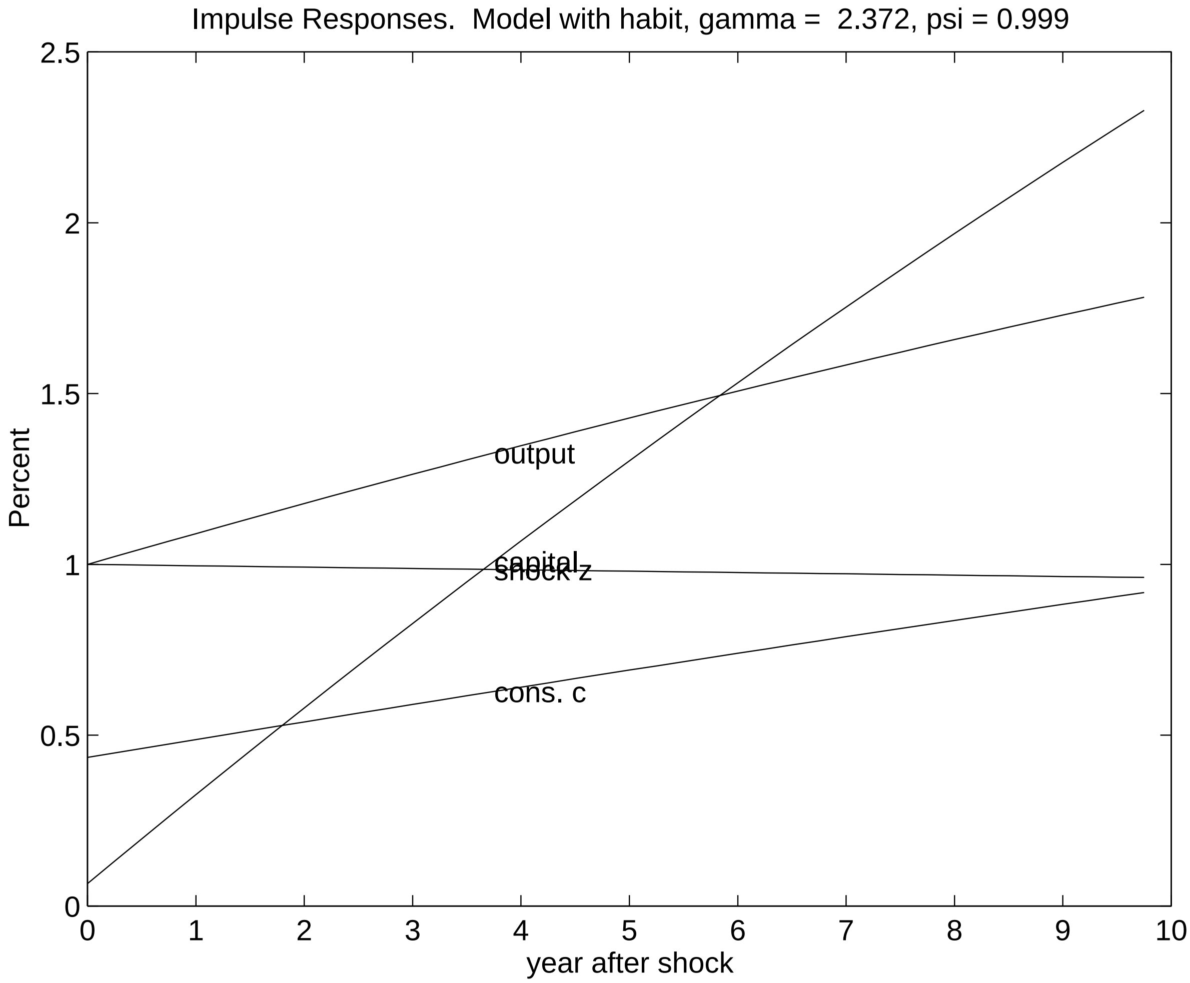
Figure 14


Impulse Responses. Model with habit, gamma = 2.372, psi = 0.999
Percent
year after shock
output
capital
shock z
cons. c
0
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1.5
2
2.5
0
1
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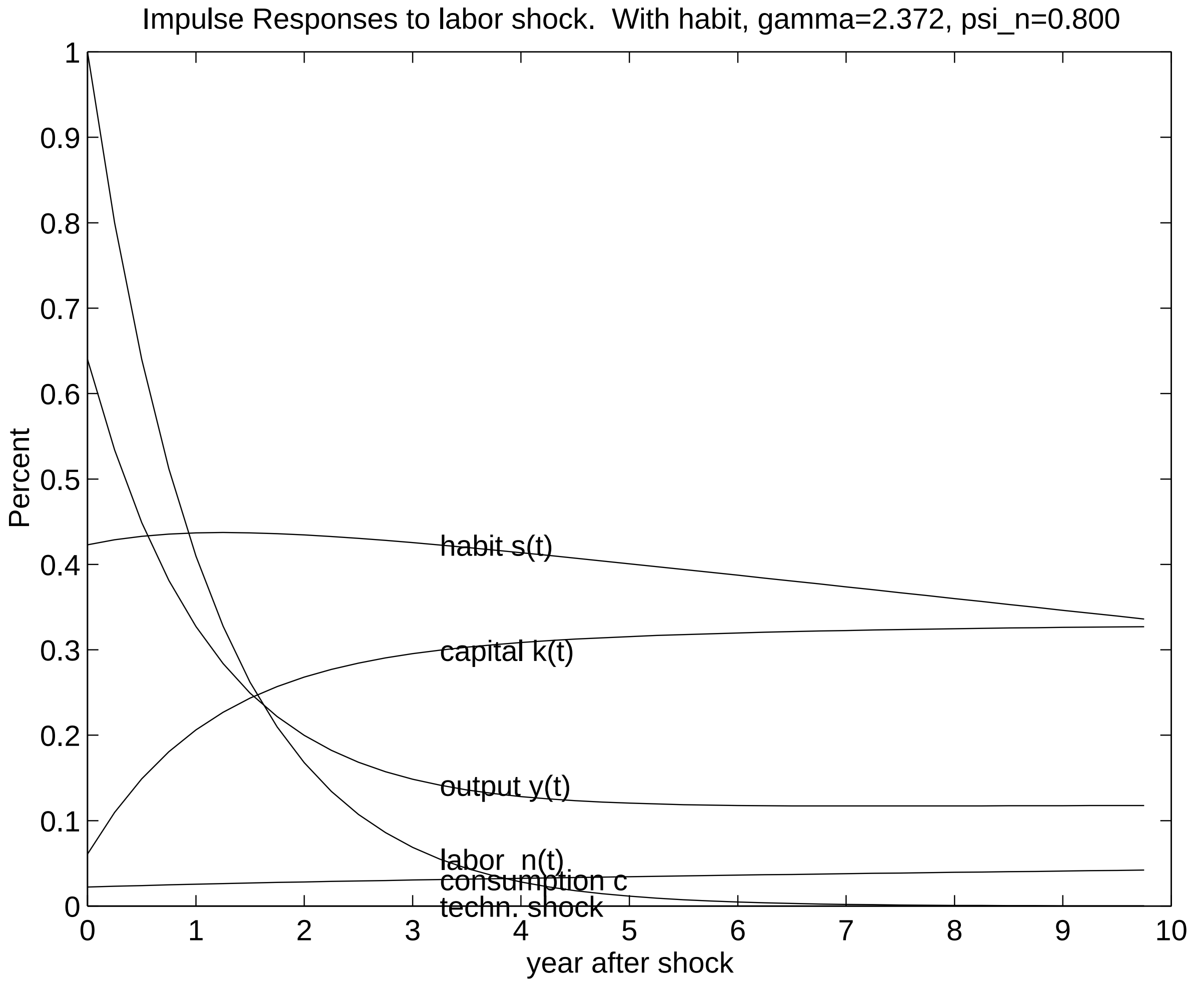
Figure 15


Impulse Responses to labor shock. With habit, gamma=2.372, psi_n=0.800
Percent
year after shock
habit s(t)
capital k(t)
output y(t)
labor n(t)
consumption c
techn. shock
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
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5
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7
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9
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