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Multiplicative uncertainty, central bank transparency and optimal degree of conservativeness

Meixing Dai
BETA, University of Strasbourg, France

Abstract

This paper extends the results of Kobayashi (2003) and Ciccarone and Marchetti (2009) by considering the optimal choice of central bank conservativeness. It is shown that the government can choose a sufficiently populist but opaque central banker so that higher multiplicative uncertainty improves the social welfare only when the society is very conservative.

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1. Introduction

In a model without inflationary bias, Kobayashi (2003) has shown that an increase in Brainard-type multiplicative uncertainty in the transmission mechanism of monetary policy improves social welfare if the degree of central bank opacity is high enough, i.e., if the weights attached to its objectives are sufficiently uncertain for the public, and *vice versa*. This is because the influence of preference uncertainty on inflation and output could be reduced through the Brainard's (1967) conservatism principle. Ciccarone and Marchetti (2009) argue that this result holds only in a specific case where society as well as the central bank on average are "conservative", i.e., in the social welfare loss, the weight attached to output is lower than unity. They show that there is a statistical bound on opacity, preventing an increase in multiplicative uncertainty from lowering enough the variability of output and hence the social loss.

This paper assumes that the government, which represents the society, delegates the monetary policy decisions to an independent central bank with optimal expected degree of conservativeness. We re-examine in this framework how is plausible the possibility that an increase in multiplicative uncertainty improves the social welfare.

2. The model without optimal delegation

Kobayashi (2003) assumes that the monetary policy is delegated to a central banker who is randomly selected from society. The central bank minimizes the loss function:

$$L^{CB} = (1+\alpha)\pi^2 + (\lambda - \alpha)y^2, \qquad \lambda > 0, \tag{1}$$

where y is output in log terms, π is inflation rate, and λ denotes the relative weight assigned by the central bank on average to the output objective. A higher λ corresponds to a lower expected degree of central bank conservativeness in the sense of Rogoff (1985). $\alpha \in [-1, \lambda]$ is a random variable and hence its value is unobservable for the private sector *ex ante,* with expected value $E(\alpha) = 0$ and variance σ_α^2 , which represents the degree of opacity about central bank preferences.

The social welfare loss is given by:

$$L^S = \pi^2 + \lambda y^2 . \tag{2}$$

Since the weights attached by the society to objectives equal the expected ones by the central banker, i.e. $E(1+\alpha) = 1$ and $E(\lambda - \alpha) = \lambda$, we have $E[L^{CB}] = E[L^S]$.

Aggregate supply is given by a conventional Lucas's supply function with the natural rate of output normalized to zero:

$$y = \pi - \pi^e - \varepsilon . \tag{3}$$

where π^e is expected inflation rate and ε a supply shock with zero mean and variance σ_ε^2.

The central bank sets the money growth rate (m) to control inflation, subject to multiplicative uncertainty in the transmission mechanism of monetary policy:

$$\pi = (1+v)m , \tag{4}$$

where v is a random variable with mean zero and variance σ_v^2. All shocks are assumed to be mutually uncorrelated, so that $E[\alpha\varepsilon] = E[\alpha v] = E[v\varepsilon] = 0$.

The timing of the game is as follows. 1) The private sector forms rational expectations. 2) Supply shock ε is observed. 3) The policymaker sets *m*. 4) Shock v occurs, and then inflation and output are realized. The game is solved by backward induction. The minimization of Eq. (1) subject to Eqs. (3) and (4) yields the reaction function $m = \frac{(\lambda-\alpha)\pi^e + (\lambda-\alpha)\varepsilon}{(1+\lambda)(1+\sigma_v^2)}$. Substituting it into Eq. (4) gives $\pi = \frac{(1+v)[(\lambda-\alpha)\pi^e + (\lambda-\alpha)\varepsilon]}{(1+\lambda)(1+\sigma_v^2)}$. Rational expectations imply $\pi^e = 0$. Thus, the equilibrium solutions for m, π and y are given by

$$m = \frac{(\lambda-\alpha)\varepsilon}{(1+\lambda)(1+\sigma_v^2)}, \tag{5}$$

$$\pi = \frac{(1+v)(\lambda-\alpha)\varepsilon}{(1+\lambda)(1+\sigma_v^2)}, \tag{6}$$

$$y = \frac{(1+v)(\lambda-\alpha) - (1+\lambda)(1+\sigma_v^2)}{(1+\lambda)(1+\sigma_v^2)}\varepsilon. \tag{7}$$

The expected social loss is hence evaluated as:

$$E[L^S] = \overbrace{\frac{(\lambda^2+\sigma_\alpha^2)\sigma_\varepsilon^2}{(1+\lambda)^2(1+\sigma_v^2)}}^{\sigma_\pi^2} + \lambda \overbrace{\frac{[(\lambda^2+\sigma_\alpha^2) - 2(1+\lambda)\lambda + (1+\lambda)^2(1+\sigma_v^2)]\sigma_\varepsilon^2}{(1+\lambda)^2(1+\sigma_v^2)}}^{\sigma_y^2}. \tag{8}$$

According to Eq. (8), an increase in σ_v^2 reduces the variance of inflation, σ_π^2, while it has ambiguous effect on the variance of output, σ_y^2. Deriving $E[L^S]$ with respect to σ_v^2 yields:

$$\frac{\partial E[L^S]}{\partial \sigma_v^2} = \frac{\lambda^2 - \sigma_\alpha^2}{(1+\lambda)(1+\sigma_v^2)^2}\sigma_\varepsilon^2. \tag{9}$$

The conclusion obtained by Kobayashi is that if $\lambda^2 < \sigma_\alpha^2$, an increase in σ_v^2 reduces social loss. For sufficiently high degree of opacity, the effect of an increase in multiplicative uncertainty on the variance of inflation will be greater than that on the variance of output. Brainard's conservatism principle implies that the central bank varies its instrument less under multiplicative uncertainty than it would under multiplicative certainty (see Eq. (5)), improving hence the social welfare.

Knowing that the probability distribution of α ensuring the highest variance is the one that assigns positive probability values only to the extrema of α (-1 and λ) and zero elsewhere, Ciccarone and Marchetti (2009) have shown that the maximal value for σ_α^2 is constrained, i.e. $\sigma_{\alpha,\max}^2 = \lambda$.[1] This implies that, $\forall \lambda > 1$, i.e. the society is strictly "populist" (and the central bank is "populist" on average), the condition $\lambda^2 < \sigma_\alpha^2$ will not be verified since $\sigma_\alpha^2 \le \lambda < \lambda^2$,. In this case, increases in multiplicative uncertainty always reduce the expected social welfare. In the opposite case where $\lambda < 1$, i.e. the society is strictly "conservative" (and the central bank is "conservative" on average), higher multiplicative uncertainty could reduce or increase expected social welfare according to the sign of the term $(\lambda^2 - \sigma_\alpha^2)$ in Eq. (9).

In the following, we examine how the optimal choice of the expected degree of central bank conservativeness affects the effects of multiplicative uncertainty on the social welfare, in particular when $\lambda < 1$.

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[1] See also Ciccarone *et al.* (2007).

3. Optimal degree of conservativeness

We consider a policy game where monetary policy is delegated to an independent central bank. The government will optimally choose the expected degree of central bank conservativeness by minimizing the social loss function:

$$L^S = \pi^2 + \delta y^2 , \qquad \delta < 0 , \qquad (10)$$

where δ is the weight assigned by the society to the output objective and it is different from λ, i.e., the expected weight assigned by the central bank to the same objective. Higher values of δ and $(\lambda - \alpha)$ signify that the society and the central bank are more liberal or populist. The government is assumed, as the private sector, to be imperfectly informed about $(\lambda - \alpha)$ and considers the latter as a random variable with mean λ and variance σ_α^2.

The central banker is assumed to be nominated by the government before the private sector forms inflation expectations. The timing of other events defined before is kept unchanged. Using backward induction, the optimal value of λ, i.e. λ^*, is obtained by minimizing the social loss function (10) subject to Eqs. (6)-(7). The government's minimization problem is written, using Eqs. (6)-(7) in Eq. (10), as:

$$\min_\lambda E[L^S] = \frac{(\lambda^2 + \sigma_\alpha^2)\sigma_\varepsilon^2}{(1+\lambda)^2(1+\sigma_v^2)} + \delta \frac{(\lambda^2 + \sigma_\alpha^2) - 2(1+\lambda)\lambda + (1+\lambda)^2(1+\sigma_v^2)}{(1+\lambda)^2(1+\sigma_v^2)} \sigma_\varepsilon^2 . \qquad (11)$$

The first-order condition is:

$$\frac{\partial L^S}{\partial \lambda} = 0 \Rightarrow \overbrace{\frac{2(\lambda - \sigma_\alpha^2)\sigma_\varepsilon^2}{(1+\lambda)^3(1+\sigma_v^2)}}^{\partial\sigma_\pi^2/\partial\lambda} - \delta \times \overbrace{\frac{2(1+\sigma_\alpha^2)\sigma_\varepsilon^2}{(1+\lambda)^3(1+\sigma_v^2)}}^{\partial\sigma_y^2/\partial\lambda} = 0 . \qquad (12)$$

It follows that[2]

$$\lambda^* = \delta + (1+\delta)\sigma_\alpha^2 . \qquad (13)$$

Proposition 1: The optimal expected weight attached by the central bank to the output objective is greater than that of the society, i.e., $\lambda^* > \delta$. It is always greater than the degree of central bank opacity, i.e. $\lambda^* > \sigma_\alpha^2$, and increases with the latter.

Proof: It follows directly from Eq. (13). ■

The higher are δ and λ, the less conservative (more liberal) become the society and the central bank (on average). Therefore, the condition $\lambda^* > \delta$ implies that the society is more conservative (less liberal) than the central bank on average. When the central bank is fully transparent, λ^* equals δ in the absence of inflation bias. For $\sigma_\alpha^2 > 0$, the government has an incentive to nominate a relatively more liberal (less conservative) central banker to reduce the

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[2] The second-order condition is verified since $\left.\frac{\partial^2 L^S}{\partial \lambda^2}\right|_{\lambda=\lambda^*} = \frac{2(1+\delta)(1+\sigma_\alpha^2)}{(1+\lambda^*)^4(1+\sigma_v^2)}\sigma_\varepsilon^2 > 0$.

negative effects of opacity on the social welfare, so that λ^* must be higher than δ to neutralise the joint effects of inflation shocks and opacity. When λ equals σ_α^2, an increase in λ reduces both σ_π^2 and σ_y^2 (see Eq. (12)), implying that λ^* is always greater than σ_α^2.

The result $\lambda^* > \sigma_\alpha^2$ is similar to the statistical higher bound on opacity found by Ciccarone and Marchetti (2009), i.e., $\sigma_{\alpha,\max}^2 = \lambda$. Meanwhile, their significations are quite different. The inequality $\lambda^* \geq \sigma_\alpha^2$ implies that the optimal expected weight attached by the central bank to the output objective must never be smaller than the degree of opacity, while the statistical bound on opacity $\sigma_{\alpha,\max}^2 = \lambda$ is a technical condition implied by probability considerations.

Proposition 2: For given degree of opacity, if the society is sufficiently conservative, i.e. $\delta < \frac{\sqrt{\sigma_\alpha^2} - \sigma_\alpha^2}{1+\sigma_\alpha^2}$, the government can choose a relatively more liberal central banker to ensure that an increase in multiplicative uncertainty improves the social welfare. Inversely, if $\delta > \frac{\sqrt{\sigma_\alpha^2} - \sigma_\alpha^2}{1+\sigma_\alpha^2}$, the government cannot avoid that higher multiplicative uncertainty worsens the social welfare.

Proof: Substituting λ^* given by Eq. (13) in the social loss function (11) and deriving the resulting function with respect to σ_v^2 yield:

$$\frac{\partial E[L^S]}{\partial \sigma_v^2} = \overbrace{\frac{-(\lambda^2 + \sigma_\alpha^2)\sigma_\varepsilon^2}{(1+\lambda)^2(1+\sigma_v^2)^2}}^{\partial\sigma_\pi^2/\partial\sigma_v^2} + \delta \times \overbrace{\frac{[2(1+\lambda)\lambda - (\lambda^2 + \sigma_\alpha^2)]\sigma_\varepsilon^2}{(1+\lambda)^2(1+\sigma_v^2)^2}}^{\partial\sigma_y^2/\partial\sigma_v^2} = \frac{\overbrace{[\delta + (1+\delta)\sigma_\alpha^2)]^2}^{\lambda^{*2}} - \sigma_\alpha^2}{(1+\lambda)(1+\sigma_v^2)^2}\sigma_\varepsilon^2. \quad (14)$$

Imposing $\frac{\partial E[L^S]}{\partial \sigma_v^2} < 0$ leads to $-\sigma_\alpha^2 + [\sigma_\alpha^2 + \delta(1+\sigma_\alpha^2)]^2 < 0$. Solving the inequality for δ gives:

$$\delta < \frac{\sqrt{\sigma_\alpha^2} - \sigma_\alpha^2}{(1+\sigma_\alpha^2)}. \quad (15)$$

The opposite result, i.e. $\frac{\partial E[L^S]}{\partial \sigma_v^2} > 0$, is obtained when the condition (15) is reversed. ■

Proposition 2 implies that if the society is sufficiently conservative, i.e. δ is lower enough, the government can nominate a relatively less conservative (more liberal) central banker so that $\frac{\partial E[L^S]}{\partial \sigma_v^2} < 0$. In effect, an increase in opacity reinforces the negative effects of multiplicative uncertainty on the social welfare (see Eq. (14)), which can be counterbalanced by a decrease in the expected degree of central bank conservativeness (see Eq. (12)). If the society is very conservative, the government will be able to choose a central banker who is liberal enough compared to the society to more than neutralize the negative effects of higher multiplicative uncertainty and hence to improve the social welfare.

Since Eq. (13) implies that $\lambda^* \geq \sigma_\alpha^2$, then for $\sigma_\alpha^2 > 1$, we always have $\lambda^{*2} > \sigma_\alpha^2$. This implies that, when the central bank is highly opaque and liberal, an increase in multiplicative uncertainty will always deteriorate the social welfare, i.e. $\frac{\partial E[L^S]}{\partial \sigma_v^2} > 0$, according to Eq. (14).

Proposition 3: If the degree of conservativeness of the society is low enough so that δ is greater than $\delta_{\max} \approx 0{,}2071$, the government will never be able to choose a relatively more liberal central banker to ensure that an increase in multiplicative uncertainty improves the social welfare.

Proof: To determine the higher bound on δ under which an increase in multiplicative uncertainty improves the social welfare under opacity, one maximizes the function $\delta_{\text{sup}} = \frac{\sqrt{\sigma_\alpha^2} - \sigma_\alpha^2}{1+\sigma_\alpha^2}$. The first-order condition is:

$$\frac{\partial \delta_{\text{sup}}}{\partial \sigma_\alpha^2} = \frac{1 - \sigma_\alpha^2 - 2\sqrt{\sigma_\alpha^2}}{2(1+\sigma_\alpha^2)^2} = 0\,.$$

Solving $1 - \sigma_\alpha^2 - 2\sqrt{\sigma_\alpha^2} = 0$ leads to $\sigma_\alpha^2 \approx 0.1716$. Hence, the maximal value for δ_{sup} is $\delta_{\max} \approx 0{,}2071$.[3] ■

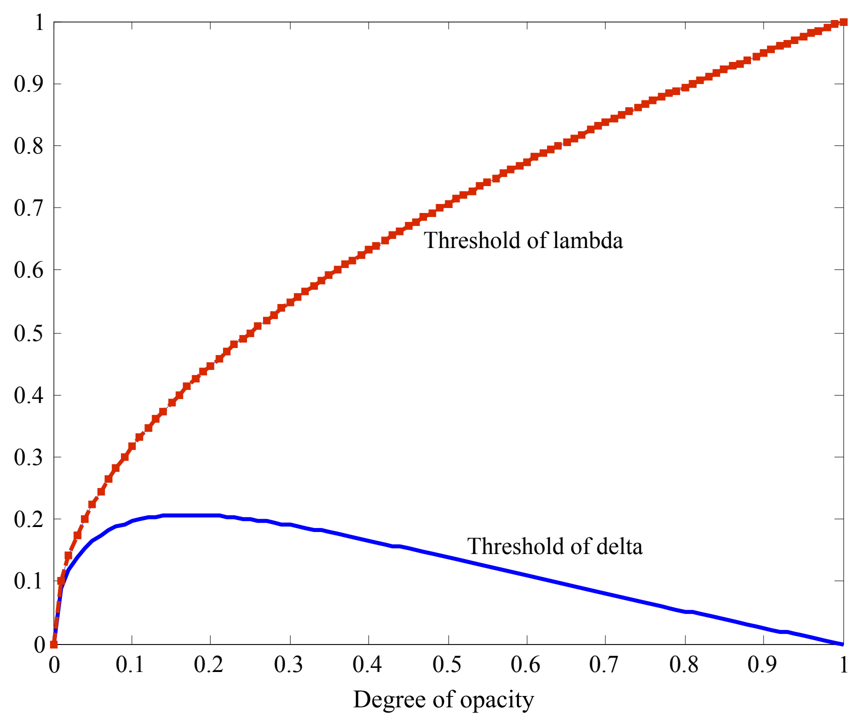

Figure 1: The thresholds of liberalism for the society and the central bank.

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[3] The second-order condition is verified since $\left.\frac{\partial^2 \delta_{\text{sup}}}{(\partial \sigma_\alpha^2)^2}\right|_{\sigma_\alpha^2 \approx 0{,}1716} \approx \frac{-(1+\sqrt{\sigma_\alpha^2})}{2(1+\sigma_\alpha^2)^2\sqrt{\sigma_\alpha^2}} < 0\,.$

Figure 1 draws two curves representing respectively the threshold of liberalism of the society (i.e. $\delta_{\sup} = \frac{\sqrt{\sigma_\alpha^2} - \sigma_\alpha^2}{1+\sigma_\alpha^2}$, bold lower line) and that of the central bank (i.e. $\lambda_{\sup}^* = \delta_{\sup} + (1+\delta_{\sup})\sigma_\alpha^2 = \sqrt{\sigma_\alpha^2}$, dashed higher line) under which an increase in multiplicative uncertainty improves the social welfare. The curve representing $\delta_{\sup}$ is concave, increasing in σ_α^2 if $\sigma_\alpha^2 < 0{,}1716$ and *vice versa*. The curve corresponding to $\lambda_{\sup}^*$ is increasing in σ_α^2. The difference between them, i.e. $(\lambda_{\sup}^* - \delta_{\sup}) = \frac{(1+\sqrt{\sigma_\alpha^2})\sigma_\alpha^2}{1+\sigma_\alpha^2}$, is increasing in σ_α^2 at the rate $\frac{\partial(\lambda_{\sup}^* - \delta_{\sup})}{\partial\sigma_\alpha^2} = \frac{\sigma_\alpha^2\sqrt{\sigma_\alpha^2} + 3\sqrt{\sigma_\alpha^2} + 2}{2(1+\sigma_\alpha^2)^2} > 0$.

Proposition 3 and Figure 1 suggest that for high or very low degrees of opacity, the society must be very conservative for being able to choose a relatively more liberal (less conservative) central bank to implement a monetary policy which allows an increase in multiplicative uncertainty to improve the social welfare.

4. Conclusion

In this paper, we have examined the effects of multiplicative uncertainty on the social welfare in a framework where the government can optimally choose the central banker in office under imperfect information about the latter's preferences. It is shown that the optimal expected weight assigned by the central bank to the output objective (the inverse of which is a measure of central bank conservativeness) is always greater than that of the society as well as the degree of central bank opacity. In general, the society must be very conservative for allowing the choice of a relatively more liberal but opaque central bank to improve the social welfare when multiplicative uncertainty becomes higher.

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