



WORKING PAPERS

RESEARCH DEPARTMENT

**WORKING PAPER NO. 10-23
PAIRWISE CREDIT AND THE
INITIAL COST OF LENDING**

Daniel R. Sanches
Federal Reserve Bank of Philadelphia

June 2010

RESEARCH DEPARTMENT, FEDERAL RESERVE BANK OF PHILADELPHIA

Ten Independence Mall, Philadelphia, PA 19106-1574 • www.philadelphiafed.org/research-and-data/

Pairwise Credit and the Initial Cost of Lending

Daniel R. Sanches
Federal Reserve Bank of Philadelphia

June 2010

I thank Steve Williamson, Gaetano Antinolfi, James Bullard, Rody Manuelli, Thorsten Koepl, Cyril Monnet, William Roberds, Pedro Gomis-Porqueras, Randy Wright, Costas Azariadis, David Andolfatto, and Mitchell Berlin for helpful comments. I also thank participants at the 2nd European Economic Review Talented Economists Clinic in Florence, the 2009 Summer Workshop on Money and Banking at the Federal Reserve Bank of Chicago, and the Search and Matching Workshop at the Federal Reserve Bank of Philadelphia. Finally, I would like to thank seminar participants at the Federal Reserve Bank of Richmond, Federal Reserve Bank of Philadelphia, University of Kansas, University of Toronto, Federal Reserve Board, Bank of Canada, University of Miami, PUC-Rio, and FGV-EPGE.

Correspondence to Sanches at Research Department, Federal Reserve Bank of Philadelphia, Ten Independence Mall, Philadelphia, PA 19106-1574; phone: (215) 574-4358; Fax: (215) 574-4303; e-mail: Daniel.Sanches@phil.frb.org.

The views expressed in this paper are those of the author and do not necessarily reflect those of the Federal Reserve Bank of Philadelphia or the Federal Reserve System. This paper is available free of charge at www.philadelphiafed.org/research-and-data/publications/working-papers/

Abstract

We study the terms of credit in a competitive market in which sellers are willing to repeatedly finance the purchases of buyers by extending direct credit. Lenders (sellers) can commit to deliver any long-term credit contract that does not result in a payoff that is lower than that associated with autarky, while borrowers (buyers) cannot commit to any contract. A borrower's ability to repay a loan is privately observable. As a result, the terms of credit within an enduring relationship change over time, according to the history of trades. Two borrowers are treated differently by the lenders with whom they are paired because they have had distinct repayment histories. Although there is free entry of lenders in the credit market, each lender has to pay a cost to contact a borrower. A lower cost makes each borrower better off from the perspective of the contracting date, results in less variability in a borrower's expected discounted utility, and makes each lender uniformly worse off *ex post*. As this cost becomes small, borrowers get nearly the same terms of credit within their credit relationships with lenders, regardless of individual repayment histories.

Keywords: Bilateral Credit, Unsecured Loans, Dynamic Contracting, Initial Cost of Lending. JEL Classification Numbers: D8, E4, G2.

1 Introduction

The cost of starting a credit relationship has fallen significantly over the last few decades. For instance, Mester (1997) points out that the use of credit scoring has significantly reduced the time and cost in the loan approval process. Barron and Staten (2003) and Berger (2003) also provide evidence suggesting that advances in information technology have reduced the cost of processing information for lenders. An important question that needs to be addressed is the following: What is the impact of changes in the cost of starting a credit relationship on the supply of credit? Drozd and Nosal (2008) argue that such a drop in the initial cost of lending can account for several stylized facts in the market for unsecured loans, such as the significant increase in revolving lines of credit over the last two decades. To derive these results, they introduce a search friction into an incomplete markets model in which the terms of the contract offered by a lender are fixed. In a recent paper, Livshits, MacGee, and Tertilt (2009) also use an incomplete markets model to analyze the effect of technological progress on consumer credit.

Although both models are successful in reproducing some stylized facts of the market for unsecured loans, it is crucial to adopt a more fundamental approach by not restricting the space of contracts that can be offered by a lender in a competitive credit market. In this way, we can clearly analyze how changes in the initial cost of lending affect the endogenous credit contract offered by lenders. This is an essential aspect of the analysis because the dynamics of long-term credit arrangements is an important property of any model of credit. For instance, we observe in the real world that the terms of credit offered by lenders in the market for unsecured loans change over time and usually depend on a consumer's history of repayments within the credit relationship; see Bertaut and Haliassos (2006). In this paper, we emphasize precisely how changes in the initial cost of lending affect the dynamics of a bilateral credit relationship.

We study the impact of changes in the cost of starting a credit relationship on the terms of the contract in a decentralized credit market in which a seller is willing to repeatedly finance the purchases of a buyer by extending direct credit. Our approach is consistent with the endogenously incomplete markets literature (see Sleet (2008)) where trading arrangements are derived from primitive frictions instead of assumed. The frictions we choose to model are the following. First, lenders are asymmetrically informed with respect to a borrower's ability to repay a loan. Second, lenders can commit to some credit contracts while borrowers cannot commit to any contract. Specifi-

cally, lenders can commit to deliver any contract that does not result in a payoff at any moment that is lower than that associated with autarky, while borrowers have no ability to commit to long-term contracts. Third, transactions within each credit relationship are not publicly observable, which captures the idea that information is dispersed in the market for unsecured loans. Fourth, it is costly for a lender to contact a borrower in the credit market, as in Drozd and Nosal (2008) and Livshits, MacGee, and Tertilt (2009), and it is also costly for a lender to walk away from a contract with a borrower, as in Phelan (1995). Given these frictions, we derive the terms of the contract that lenders offer to borrowers in a competitive credit market.

We build on the model of perfect competition by Phelan (1995) in which a lender and a borrower engage in a dynamic credit relationship. In his model, there is a particular mechanism for price formation in the credit market: lenders post the terms of the contract. One important difference is that we assume that lenders need to pay a one-shot cost to make contact with a borrower in the credit market. This captures the idea that it is costly for a lender to start a credit relationship. Another crucial difference in our model is that we make the flow of payments associated with a credit contract explicit within each period, as opposed to net transfers. One important characteristic of a credit transaction is that settlement takes place at a future date: each credit transaction between a buyer and a seller necessarily creates a liability to the buyer that needs to be settled some time in the future. Given that buyers (borrowers) cannot commit to repay their loans, we obtain a series of *ex post* individual rationality constraints. Making the flow of payments explicit allows us to clearly characterize how the loan and repayment amounts within an enduring credit relationship evolve over time.

First, we characterize a lender's optimal contract in the market for unsecured loans. Because a lender does not observe a borrower's ability to repay a loan, the terms of credit associated with a lender's contract change over time, according to the history of transactions within the enduring credit relationship. In particular, a lender's optimal contract has the property of revolving credit, which is a contingency that allows a borrower to delay a payment to his lender. This is a mechanism through which the lender obtains more favorable terms of credit for future transactions within the credit relationship. This means that whenever a borrower delays the repayment of a loan, his lender extracts more surplus from the credit relationship by offering less favorable terms for the borrower in future transactions: smaller loan amounts and/or bigger state-contingent repayment amounts. In the model proposed in this paper, borrowers are treated differently by the lenders with whom they are paired because they have had distinct repayment histories.

At any point in time, two borrowers face different terms of credit to finance their purchases because they have made distinct repayments to their respective lenders in their past transactions.

After characterizing a lender's optimal contract, we perform the comparative statics exercise of changing the cost that each lender has to pay to contact a borrower in the credit market. A lower cost of starting a credit relationship has the following impact on the equilibrium outcome: (i) each borrower is better off from the perspective of the contracting date; (ii) a borrower's expected discounted utility fluctuates within a smaller set; and (iii) each lender is uniformly worse off *ex post*. A lower cost of entry in the credit market leads to more competition among lenders, which in turn results in more favorable terms of credit for each borrower. Another implication is that the terms of the contract are such that a borrower's expected discounted utility has less variability over time. The loan and repayment amounts associated with a lender's contract are such that the space of expected discounted utilities for a borrower shrinks as the initial cost of lending falls. Finally, a lender's cost function under an entry cost of $k' < k$ is uniformly above a lender's cost function under the cost k , which necessarily means that lenders are uniformly worse off *ex post* – after the decision of entering the credit market.

One important result is that the history of transactions within each credit relationship becomes less relevant to determine the terms of credit for future transactions as the initial cost of lending approaches zero. As this cost falls, the set of expected discounted utilities for a borrower shrinks, which means that the terms of credit associated with a lender's contract become very similar across the population of borrowers regardless of individual repayment histories. Finally, we show that a stationary equilibrium in the credit market may not exist if both the initial cost of lending and the cost of walking away from a credit contract are too small. In this case, there exists a lower bound on the initial cost of lending for which we can guarantee that a stationary equilibrium exists.

The model in this paper relates to decentralized models of credit, such as Diamond (1990), Temzelides and Williamson (2001), Nosal and Rocheteau (2006), Koepl, Monnet, and Temzelides (2008), and Andolfatto (2008), as opposed to centralized models of credit, such as Kehoe and Levine (1993) and Alvarez and Jermann (2000). The model also builds on search-theoretic models of money, such as Shi (1997) and Lagos and Wright (2005). However, we depart from these models by weakening the assumption that agents cannot engage in enduring relationships. Finally, the analysis builds on dynamic contracting. Important papers in this literature include Green (1987),

Thomas and Worrall (1990), Atkeson and Lucas (1992, 1995), Kocherlakota (1996), Aiyagari and Williamson (1999), and Krueger and Uhlig (2006).

2 The Model

Time is discrete and continues forever, and each period has two subperiods. There are two types of agents, referred to as borrowers and lenders. In the first subperiod, a lender is able to produce the unique perishable consumption good but does not want to consume, and a borrower wants to consume but cannot produce. In the second subperiod, we have the opposite situation: a borrower is able to produce but does not want to consume, and a lender wants to consume but cannot produce. Production and consumption take place within each subperiod. This generates a double coincidence of wants and, for this reason, we refer to the first subperiod as the transaction stage and to the second subperiod as the settlement stage. The types (borrower and lender) refer to the agent's role in the transaction stage. The production technology allows each agent to produce one unit of the good with one unit of labor. Each agent receives an endowment of $h > 0$ units of time in each subperiod.

A lender's utility in period t is given by $-q_t^l + x_t^l$, where q_t^l is production of the good in the transaction stage and x_t^l is consumption of the good in the settlement stage. A borrower's momentary utility from consuming q_t^b units of the consumption good in the transaction stage is given by $u(q_t^b)$. Assume that $u : \mathbb{R}_+ \rightarrow D \subset \mathbb{R}$ is increasing, strictly concave, and continuously differentiable. Let H denote the inverse of u , and let $w^a \equiv u(0)$ denote the value associated with autarky. Producing y_t^b units of the good in the settlement stage generates disutility y_t^b for a borrower. However, there is a friction that affects a borrower's ability to produce goods in the settlement stage. With probability π a borrower is unable to produce the consumption good and with probability $1 - \pi$ a borrower can produce the good using the linear production technology. This productivity shock is independently and identically distributed over time. Each borrower learns his productivity shock at the beginning of the settlement stage, which is privately observed. Finally, let $\beta \in (0, 1)$ be the common discount factor over periods.

Suppose that there is a large number of borrowers and lenders, with the set of lenders sufficiently large. There is a one-shot cost $k > 0$ in terms of the consumption good for a lender to post a credit contract in the credit market, which specifies consumption and production by each party as a function of the available information. As in Drozd and Nosal (2008) and

Livshits, MacGee, and Tertilt (2009), we motivate this assumption as the cost of contacting a borrower in the credit market and starting a credit relationship. We assume that each lender can have at most one borrower; it is infinitely costly for a lender to contact two borrowers at the same time. Each lender also has to pay a cost $\gamma > 0$ to walk away from her current contract. As in Phelan (1995), we motivate this assumption as a legal cost that a lender has to pay to effectively walk away from her current contract with a borrower.

Only the agents in a bilateral meeting observe the history of trades. Other agents in the economy observe a break in a particular match but do not observe the history of trades in that match. Notice that there are gains from trade since a lender can produce the consumption good for a borrower in the first subperiod (transaction stage) and a borrower can produce the good for a lender in the second subperiod (settlement stage). An important feature of the model is that, with probability π , a borrower is unable to produce the good in the second subperiod and settle his debt. This is equivalent to assuming that the settlement process involves a friction.

3 Equilibrium

In this section, we study an equilibrium allocation under a particular pricing mechanism: price posting by lenders. To enter the credit market, a lender needs to post a contract to attract a borrower and start a credit relationship. Although it is costly for a lender to make contact with a borrower, there is free entry of lenders in the credit market. We characterize the terms of the contract in each credit relationship in the economy and restrict attention to a symmetric, stationary equilibrium in which each borrower receives a market-determined credit contract offered by a lender that promises him expected discounted utility w^* , from the perspective of the contracting date. Each lender needs to provide incentives to induce the desired behavior by a borrower given that a borrower's ability to repay a loan is not observable.

We assume that lenders can commit to some credit contracts, while borrowers cannot commit to any contract. Specifically, each lender can commit to deliver any contract that does not result at any moment in an expected discounted utility that is lower than that associated with autarky; recall that a lender always has the option of remaining inactive. On the other hand, borrowers cannot commit to any contract and can walk away from a credit relationship at any moment without any pecuniary punishment. As we will see, a lender's optimal contract results in a long-term relationship

from which neither party wants to deviate.

The expected discounted utility w^* associated with the market contract must be such that it makes each lender indifferent between entering the credit market by posting a contract and remaining inactive, from the perspective of the contracting date. As a result, some lenders post a contract and successfully match with a borrower, while others do not post a contract and remain inactive. When offering her own contract, each lender takes as given the contracts offered by the other lenders. The only relevant characteristic about these contracts is the expected discounted utility w^* that each borrower associates with them. This is the utility that a borrower obtains by accepting a lender's contract, from the perspective of the signing date. The equilibrium is symmetric because every active lender offers the same credit contract.

The market contract must *always* result in an expected discounted utility for a borrower that is greater than or equal to w^* . If the market contract promises, in a given period, an expected discounted utility w' for a borrower which is less than w^* , the latter can do better by reneging on his current contract and starting a new credit relationship with another lender. Recall that inactive lenders observe the dissolution of a credit relationship and may be willing to enter the credit market. Given that there is free entry of lenders and limited commitment, we can have an equilibrium only if the lowest promised expected discounted utility at any moment is exactly w^* .

3.1 Recursive Formulation of the Contracting Problem

A contract specifies in every period a transfer of the good from the lender to the borrower in the transaction stage and a repayment – a transfer of the good from the borrower to the lender – in the settlement stage as a function of the available history of reports by the borrower. These are reports about a borrower's ability to produce goods in the settlement stage. Let $\eta^{t-1} = (\eta_0, \eta_1, \dots, \eta_{t-1}) \in \{0, 1\}^t$ denote a partial history of reports, where $\eta_\tau = 0$ means that a borrower is unable to produce the good in the settlement stage of period τ and $\eta_\tau = 1$ means that he is able to produce it in the settlement stage of period τ .

In equilibrium, each active lender chooses to offer a long-term contract, which means that she matches with a borrower at the first date and keeps him in the credit relationship forever. The long-term contract specifies quantities produced and transferred within each subperiod. We say that in each period t there is a *transaction* between a borrower and a lender that consists of a loan amount from the lender to the borrower in the first subperiod

(transaction stage) and a repayment amount in the second subperiod (settlement stage) contingent on the report of the productive state of nature ($\eta_t = 1$).

The optimal contracting problem has a recursive formulation in which we can use a borrower's expected discounted utility $w \in D$ as the state variable. The optimal contract minimizes the expected discounted cost for a lender of providing expected discounted utility w to a borrower subject to incentive compatibility. Let $C_{(w^*, \bar{w})} : [w^*, \bar{w}] \rightarrow \mathbb{R}$ denote the expected discounted cost for a lender that satisfies the following functional equation:

$$C_{(w^*, \bar{w})}(w) = \min_{\varphi \in \Upsilon_{(w^*, \bar{w})}(w)} \left\{ (1 - \beta) [H(u) - (1 - \pi) y_1] + \beta [\pi C_{(w^*, \bar{w})}(w_0) + (1 - \pi) C_{(w^*, \bar{w})}(w_1)] \right\}. \quad (1)$$

Here, the choices are given by $\varphi = (u, y_1, w_0, w_1)$, where u denotes a borrower's momentary utility of consumption in the transaction stage, y_1 denotes his production in the settlement stage given that he is able to produce the good, and w_η denotes his promised expected discounted utility at the beginning of the following period given that his report in the current period is $\eta \in \{0, 1\}$. Recall that $\eta = 0$ means that a borrower is unable to produce the good in the settlement stage and $\eta = 1$ means that he is able to produce it. The constraint set $\Upsilon_{(w^*, \bar{w})}(w)$ consists of all φ in $D \times [0, h] \times [w^*, \bar{w}]^2$ satisfying a borrower's individual rationality constraints,

$$w_0 \geq w^*, \quad (2)$$

$$-(1 - \beta) y_1 + \beta w_1 \geq \beta w^*, \quad (3)$$

a borrower's truth-telling constraint,

$$-(1 - \beta) y_1 + \beta w_1 \geq \beta w_0, \quad (4)$$

and the promise-keeping constraint,

$$(1 - \beta) [u - (1 - \pi) y_1] + \beta [\pi w_0 + (1 - \pi) w_1] = w. \quad (5)$$

It can be shown that, for any fixed lower bound w^* and upper bound \bar{w} , there exists a unique continuously differentiable, strictly increasing, and strictly convex function $C_{(w^*, \bar{w})} : [w^*, \bar{w}] \rightarrow \mathbb{R}$ satisfying the functional equation (1). Let $\hat{u} : [w^*, \bar{w}] \rightarrow D$, $y : [w^*, \bar{w}] \rightarrow [0, h]$, and $g : [w^*, \bar{w}] \times \{0, 1\} \rightarrow [w^*, \bar{w}]$ denote the associated policy functions, which can be shown to be continuous and bounded.

Given our transformation of the state space, a borrower's expected discounted utility w now summarizes his partial history of reports. As mentioned before, we say that there is a transaction between a lender and a borrower in the current period whose terms are given by $\{H[\hat{u}(w)], y(w)\}$. The quantity $H[\hat{u}(w)]$ gives the loan amount from the lender to the borrower in the transaction stage, and the quantity $y(w)$ gives the repayment amount in the settlement stage contingent on the report of the productive state of nature. Both quantities depend on w . This means that the terms of credit change over time according to the history of transactions, which is summarized by the statistic w .

Notice that a lender cannot commit to a contract that gives her at any moment an expected discounted utility that is lower than that associated with autarky. As a result, individual rationality for a lender requires that $C_{(w^*, \bar{w})}(w) \leq \gamma$ holds for all $w \in [w^*, \bar{w}]$. Recall that a lender needs to pay a cost $\gamma > 0$ in order to break up a credit relationship. As in Phelan (1995), we motivate this assumption as a legal cost that a lender has to pay to effectively walk away from her current contract with a borrower.

We show next that, for some values for the lower bound w^* , there exists an upper bound $\bar{w} = \bar{w}(w^*; \gamma)$ on the set of expected discounted utilities that gives the highest promised expected utility to which a lender can commit to deliver given that the lowest expected utility that can be promised is w^* . As we will see later, the market utility w^* is determined endogenously and is such that it makes each lender indifferent between entering the credit market by posting a contract and remaining inactive. Before we proceed, it is useful to define the following set: for any given value for γ , let $I(\gamma) = \{w : w \geq w^a \text{ and } C_{(w, w)}(w) \leq \gamma\}$. Notice that for any $w \geq w^a$ we have $C_{(w, w)}(w) = H(w) \geq 0$. This is the expected discounted cost for a lender of delivering expected discounted utility w to a borrower given that the lender is constrained to choose the same continuation values regardless of the state reported by the borrower. In this case, it is not possible to obtain a repayment from the borrower: the only incentive-feasible value for y_1 is zero. Because $H(w)$ is strictly increasing, we have that $I(\gamma)$ is an interval that shrinks as $\gamma \rightarrow 0$.

Lemma 1 *For any $w^* \in I(\gamma)$, there exists an upper bound $\bar{w}(w^*; \gamma)$ on the set of expected discounted utilities such that $C_{(w^*, \bar{w}(w^*; \gamma))}[\bar{w}(w^*; \gamma)] = \gamma$.*

Proof. Let $\bar{w}_F(\gamma)$ denote the expected discounted utility for a borrower such that the expected discounted cost for a lender of providing $\bar{w}_F(\gamma)$ given complete information equals γ . Define the function $\tau : [w^*, \bar{w}_F(\gamma)] \rightarrow$

$[w^*, \bar{w}_F(\gamma)]$ as follows. For any given $w \in [w^*, \bar{w}_F(\gamma)]$, if there is no $w' \in [w^*, w]$ such that $C_{(w^*, w)}(w') = \gamma$, then $\tau(w) = w^*$. Otherwise, $\tau(w)$ equals the highest point w' in $[w^*, w]$ for which $C_{(w^*, w)}(w') = \gamma$. Notice that $C_{(w^*, w^*)}(w^*) \leq \gamma$ by assumption, which implies that $\tau(w^*) = w^*$. For any other \bar{w} such that $\tau(\bar{w}) = \bar{w}$, it must be that $C_{(w^*, \bar{w})}(\bar{w}) = \gamma$.

Construct a sequence $\{w^k\}_{k=0}^\infty$ of candidates for the upper bound \bar{w} in the following way. Let $w^0 = \bar{w}_F(\gamma)$. We have that $C_{(w^*, w^0)}(w^0) \geq \gamma$, with strict inequality if the truth-telling constraint (4) binds. Also, notice that $\Upsilon_{(w^*, w^*)}(w^*) \subseteq \Upsilon_{(w^*, w^0)}(w^*)$, which implies that $C_{(w^*, w^0)}(w^*) \leq C_{(w^*, w^*)}(w^*) \leq \gamma$. The first inequality is strict if the truth-telling constraint binds. Continuity implies that there exists $w^1 \in [w^*, w^0]$ such that $C_{(w^*, w^0)}(w^1) = \gamma$. This means that $w^1 = \tau(w^0) \leq w^0$. We proceed in the same fashion to define w^2 . From the fact that $C_{(w^*, w^0)} \leq C_{(w^*, w^1)}$, it follows that $C_{(w^*, w^1)}(w^1) \geq C_{(w^*, w^0)}(w^1) = \gamma$. Given that $\Upsilon_{(w^*, w^*)}(w^*) \subseteq \Upsilon_{(w^*, w^1)}(w^*)$, we have that $C_{(w^*, w^1)}(w^*) \leq C_{(w^*, w^*)}(w^*) \leq \gamma$. Again, continuity implies that there exists $w^2 \in [w^*, w^1]$ such that $C_{(w^*, w^1)}(w^2) = \gamma$. This means that $w^2 = \tau(w^1) \leq w^1$. Notice then that $\{w^k\}_{k=0}^\infty$ is a non-increasing sequence on a closed interval. As a result, it converges to a point w^∞ in the interval $[w^*, \bar{w}_F(\gamma)]$. The Theorem of the Maximum guarantees that $\phi(w) \equiv C_{(w^*, w)}(w)$ moves continuously, which implies that w^∞ is the highest fixed point of τ . **Q.E.D.** ■

To ease notation, define $C_{w^*}(w) \equiv C_{(w^*, \bar{w}(w^*; \gamma))}(w)$ and define the set $D_{w^*} \equiv [w^*, \bar{w}(w^*; \gamma)]$. Given that $C_{w^*}(w)$ is strictly increasing in w , it follows that $C_{w^*}(w) \leq \gamma$ for all w in the set D_{w^*} . This means that, for any given lower bound w^* , D_{w^*} gives the set of promised expected discounted utilities that are actually incentive-feasible. If the truth-telling constraint binds, then it follows that $\bar{w}(w^*; \gamma) > w^*$ for any lower bound w^* satisfying $C_{(w^*, w^*)}(w^*) \leq \gamma$. We show next that the truth-telling constraint indeed binds for any w in D_{w^*} . But first notice that the truth-telling constraint (4), together with the constraint $0 \leq y(w) \leq h$, implies that $g(w, 1) \geq g(w, 0)$ for all $w \in D_{w^*}$, which means that the optimal contract needs to assign a higher promised expected discounted utility to a borrower contingent on the realization of the productive state of nature to effectively induce truthful reporting.

Lemma 2 *The truth-telling constraint (4) binds for any $w \in D_{w^*}$.*

Proof. Suppose that

$$-(1 - \beta)y_1 + \beta w_1 > \beta w_0 \quad (6)$$

holds at the optimum. This implies that

$$-(1 - \beta) y_1 + \beta w_1 > \beta w^* \quad (7)$$

must also hold at the optimum. Now, reduce the left-hand side of (6) and (7) by a small amount $\Delta > 0$ so that both inequalities continue to hold. Define $w'_1 = w_1 - \pi\Delta$ and $w'_0 = w_0 + (1 - \pi)\Delta$. Notice that $\pi w'_0 + (1 - \pi)w'_1 = \pi w_0 + (1 - \pi)w_1$ and $w'_1 - w'_0 < w_1 - w_0$. The strict convexity of C_{w^*} implies that

$$\pi C_{w^*}(w'_0) + (1 - \pi) C_{w^*}(w'_1) < \pi C_{w^*}(w_0) + (1 - \pi) C_{w^*}(w_1),$$

so that the value of the objective function on the right-hand side of (1) falls. Since all constraints continue to be satisfied, this implies a contradiction.

Q.E.D. ■

An immediate consequence of the previous result is that $w^* < \bar{w}(w^*; \gamma)$ for any given $w^* \in I(\gamma)$, that is, any $w^* \geq w^a$ such that $C_{(w^*, w^*)}(w^*) \leq \gamma$.

3.2 Existence and Uniqueness of Stationary Equilibrium

Now, we need to ensure that there exists a market-determined expected discounted utility w^* associated with a market contract that makes each lender indifferent between posting a contract and remaining inactive. This is equivalent to showing the existence of an equilibrium.

Formally, a stationary and symmetric equilibrium consists of a cost function $C_w : D_w \rightarrow \mathbb{R}$, policy functions $\hat{u} : D_w \rightarrow D$, $y : D_w \rightarrow [0, h]$, $g : D_w \times \{0, 1\} \rightarrow D_w$, and a market utility w^* such that: (i) C_{w^*} satisfies (1); (ii) (\hat{u}, y, g) are the optimal policy functions associated with (1); and (iii) w^* satisfies the free-entry condition:

$$C_{w^*}(w^*) + (1 - \beta)k = 0. \quad (8)$$

The market utility w^* gives the expected discounted utility for a borrower at the signing date. Due to limited commitment and free entry of lenders in the credit market, it is also the lower bound on the set of expected discounted utilities.

Lemma 3 *Given $\gamma > 0$, there exist $\underline{k}(\gamma)$ and $\bar{k}(\gamma)$, with $0 \leq \underline{k}(\gamma) < \bar{k}(\gamma)$, such that there exists a unique expected discounted utility w^* satisfying (8) provided that $k \in [\underline{k}(\gamma), \bar{k}(\gamma)]$.*

Proof. Notice that $C_{w^a}(w^a) < 0$. We need $k > 0$ to be such that $C_{w^a}(w^a) + (1 - \beta)k < 0$ and $C_{w^\gamma}(w^\gamma) + (1 - \beta)k \geq 0$, where $w^\gamma = \sup I(\gamma)$. If $C_{w^\gamma}(w^\gamma) \geq 0$, then the lower bound $\underline{k}(\gamma)$ equals 0 and the upper bound $\bar{k}(\gamma)$ is given by the value of k satisfying $C_{w^a}(w^a) + (1 - \beta)k = 0$. If $C_{w^\gamma}(w^\gamma) < 0$, then the lower bound $\underline{k}(\gamma)$ is given by the value of k satisfying $C_{w^\gamma}(w^\gamma) + (1 - \beta)k = 0$ and the upper bound is given by the value of k satisfying $C_{w^a}(w^a) + (1 - \beta)k = 0$. Given that $\hat{\phi}(w) \equiv C_w(w)$ is continuous in w , there exists $w^* \in [w^a, w^\gamma]$ such that $\hat{\phi}(w^*) + (1 - \beta)k = 0$ provided that $k \in [\underline{k}(\gamma), \bar{k}(\gamma)]$.

To show uniqueness, define the mapping $\sigma : [w^a, w^\gamma] \rightarrow [w^a, w^\gamma]$ as follows. If $C_w + (1 - \beta)k$ is always greater than zero on $[w, w^\gamma]$, then $\sigma(w) = w^a$. Otherwise, $\sigma(w)$ equals the point $w' \in [w, w^\gamma]$ for which $C_w(w') + (1 - \beta)k = 0$. We claim that σ is non-increasing in w . To verify this claim, we need to show first that $\bar{w}(w; \gamma)$ is non-increasing in w . Fix a lower bound w' in the set $[w^a, w^\gamma]$, and consider the associated upper bound $\bar{w}(w'; \gamma)$. Take another point $w'' > w'$ in the set $[w^a, \bar{w}(w'; \gamma)]$. Notice that $C_{(w', \bar{w}(w'; \gamma))} \leq C_{(w'', \bar{w}(w'; \gamma))}$. Thus, we have that $C_{(w'', \bar{w}(w'; \gamma))}[\bar{w}(w'; \gamma)] \geq \gamma$ given that $C_{(w', \bar{w}(w'; \gamma))}[\bar{w}(w'; \gamma)] = \gamma$ by the definition of $\bar{w}(w'; \gamma)$. This implies that $\bar{w}(w''; \gamma) \leq \bar{w}(w'; \gamma)$, and we conclude that $\bar{w}(w; \gamma)$ is indeed non-increasing in w . The fact that $\bar{w}(w; \gamma)$ is non-increasing then implies that raising the lower bound w only tightens the constraint set $\Upsilon_{(w, \bar{w}(w; \gamma))}(\cdot)$. As a result, the point at which $C_w + (1 - \beta)k$ equals zero is a non-increasing function of the lower bound w , which means that σ can have at most one fixed point. **Q.E.D.** ■

Notice that *ex ante* each lender gets zero expected discounted utility by posting a contract. *Ex post* a lender gets a higher utility, given that $C_{w^*}(w^*) < 0$. Moreover, as the contract is executed, there is no history of reports by a borrower that gives a lender an expected discounted utility that is lower than that associated with autarky. For this reason, neither a lender nor a borrower finds it optimal to renege on the credit contract.

If γ is sufficiently small, then it is possible to have $\underline{k}(\gamma) > 0$. This means that a stationary equilibrium may not exist if the initial cost of lending k is too small. To verify this claim, we first need to show that $\bar{w}(w; \gamma)$ is non-increasing in γ . Notice that $\bar{w}_F(\gamma') < \bar{w}_F(\gamma)$ for any $0 < \gamma' < \gamma$. Also, notice that $I(\gamma') \subset I(\gamma)$. If we fix $w^* \in I(\gamma')$ and construct the candidate sequences for the upper bound on the set of expected discounted utilities in the same way as in the proof of Lemma 1, we will find that $\bar{w}(w^*; \gamma) \leq \bar{w}(w^*; \gamma')$, which means that $\bar{w}(w; \gamma)$ is indeed non-increasing in γ . Second, notice that we must have $C_{w^a}(w^a) < 0$ for any given $\gamma > 0$.

This means that, for a sufficiently small value for γ , we have $C_{w^\gamma}(w^\gamma) < 0$, in which case $\underline{k}(\gamma) > 0$.

It is important to keep in mind the possibility that a stationary equilibrium may not exist when both the cost of walking away from a credit contract γ and the initial cost of lending k are small. In a later section, we will conduct the comparative statics exercise of changing the initial cost of lending k holding γ fixed. However, there may not exist a stationary equilibrium if we reduce the initial cost of lending too much when γ is small.

3.3 Properties of the Optimal Contract

In this subsection, we characterize the policy functions $\hat{u}(w)$, $y(w)$, $g(w, \eta)$ and establish some important properties of the optimal contract. We can rewrite the optimization problem on the right-hand side of (1) in the following way. The relevant constraints for the optimization problem are (2),

$$-(1 - \beta)y_1 + \beta w_1 = \beta w_0, \quad (9)$$

and

$$(1 - \beta)(u - y_1) + \beta w_1 = w. \quad (10)$$

Substituting (9) and (10) into (1), the optimization problem now consists of choosing y_1 and w_1 to minimize:

$$(1 - \beta) \left[H \left(\frac{w - \beta w_1}{1 - \beta} + y_1 \right) - (1 - \pi)y_1 \right] + \beta \left\{ \begin{array}{l} \pi C_{w^*} \left[w_1 - \frac{(1 - \beta)}{\beta} y_1 \right] + \\ (1 - \pi) C_{w^*}(w_1) \end{array} \right\},$$

subject to $w^* \leq w_1 \leq \bar{w}(w^*; \gamma)$, $0 \leq y_1 \leq h$, and

$$w_1 - \frac{(1 - \beta)}{\beta} y_1 \geq w^*. \quad (11)$$

The first-order conditions for the optimal choice of y_1 are

$$H' \left[\frac{w - \beta g(w, 1)}{1 - \beta} + y(w) \right] - \pi C'_{w^*} [g(w, 0)] + \frac{\lambda(w)}{\beta} \geq 1 - \pi, \quad (12)$$

if $y(w) < h$, and

$$H' \left[\frac{w - \beta g(w, 1)}{1 - \beta} + y(w) \right] - \pi C'_{w^*} [g(w, 0)] + \frac{\lambda(w)}{\beta} \leq 1 - \pi, \quad (13)$$

if $y(w) > 0$. The first-order condition for the optimal choice of w_1 is

$$H' \left[\frac{w - \beta g(w, 1)}{1 - \beta} + y(w) \right] \geq \left\{ \begin{array}{l} \pi C'_{w^*} [g(w, 0)] + \\ (1 - \pi) C'_{w^*} [g(w, 1)] - \frac{\lambda(w)}{\beta} \end{array} \right\}, \quad (14)$$

with equality if $g(w, 1) < \bar{w}(w^*; \gamma)$. Also, we have that

$$\lambda(w) \left[g(w, 1) - \frac{(1 - \beta)}{\beta} y(w) - w^* \right] = 0, \quad (15)$$

where $\lambda(w) \geq 0$ is the Lagrange multiplier associated with constraint (11). Finally, the envelope condition is given by

$$C'_{w^*}(w) = H' \left[\frac{w - \beta g(w, 1)}{1 - \beta} + y(w) \right], \quad (16)$$

for any value of w in the interior of the set D_{w^*} .

Now, we establish some properties of the optimal continuation value $g(w, \eta)$ for each $\eta \in \{0, 1\}$. These give a borrower's expected discounted utility at the beginning of the following period associated with the market contract as a function of his initially promised expected discounted utility w and his report in the settlement stage of the current period. If a borrower's expected discounted utility falls in the subsequent period relative to the current period, this means that the terms of the contract become less favorable for him – and as a result more favorable for the lender with whom he is paired.

Lemma 4 $g(w, 1) \geq w$ for all $w \in D_{w^*}$.

Proof. Suppose that $g(w, 1) < w$ for some w in the interior of D_{w^*} . Given that $g(w, 1) < w \leq \bar{w}(w^*; \gamma)$, it must be that

$$C'_{w^*}(w) = \pi C'_{w^*} [g(w, 0)] + (1 - \pi) C'_{w^*} [g(w, 1)] - \frac{\lambda(w)}{\beta}.$$

Recall that $g(w, 1) \geq g(w, 0)$ and that $C_{w^*}(w)$ is strictly convex in w . As a result, we have that

$$C'_{w^*}(w) < C'_{w^*}(w) - \frac{\lambda(w)}{\beta} \leq C'_{w^*}(w),$$

where the last inequality follows because $\lambda(w) \geq 0$. But this results in a contradiction. Hence, we conclude that $g(w, 1) \geq w$ for all w in the interior

of D_{w^*} . The fact that $g(w, 1)$ is continuous implies that $g(w, 1) \geq w$ holds for all $w \in D_{w^*}$ as claimed. **Q.E.D.** ■

A repayment by a borrower in the settlement stage results in at least the same terms of credit for future transactions within the credit relationship. If a borrower reports the productive state of nature in the settlement stage and as a result makes a repayment $y(w) > 0$ to his lender, his expected discounted utility at the beginning of the following period $g(w, 1)$ either rises or remains the same. This means that the terms of credit for all future transactions within the relationship either become more favorable or remain the same for him. This property of the optimal contract arises because a lender cannot observe a borrower's ability to repay a loan in the settlement stage. As a result, a lender needs to induce a repayment from a borrower who is currently productive in the settlement stage by promising him at least the same terms of credit for future transactions as those promised for the current transaction.

Lemma 5 *The function $g(w, 0)$ has the following properties: (i) $g(w, 0) < w$ for all $w > w^*$; (ii) $g(w^*, 0) = w^*$; and (iii) there exists $\delta > 0$ such that $g(w, 0) = w^*$ for all $w \in [w^*, w^* + \delta)$.*

Proof. First, notice that we must have $y(w) > 0$ for all $w \in D_{w^*}$. To verify this claim, suppose that $y(w) = 0$ for some $w \in (w^*, \bar{w}(w^*; \gamma))$. Then, we must have $g(w, 1) = g(w, 0)$ given that (4) holds with equality. Moreover, either $g(w, 1) = g(w, 0) = \bar{w}(w^*; \gamma)$ or $g(w, 1) = g(w, 0) < \bar{w}(w^*; \gamma)$. If $g(w, 1) = g(w, 0) = \bar{w}(w^*; \gamma)$, then (14) and (16) imply that $C'_{w^*}(w) \geq C'_{w^*}[\bar{w}(w^*; \gamma)]$, which results in a contradiction. Suppose now that $g(w, 1) = g(w, 0) < \bar{w}(w^*; \gamma)$. From (14) and (16), we conclude that $g(w, 1) = g(w, 0) = w$. Thus, we have that $C_{w^*}(w) = H(w) > H(w^a) = 0$, which also implies a contradiction. Therefore, we must have $y(w) > 0$ for all $w \in (w^*, \bar{w}(w^*; \gamma))$. Continuity then implies that $y(w^*) > 0$ and $y[\bar{w}(w^*; \gamma)] > 0$, so that $y(w) > 0$ for all $w \in D_{w^*}$ as claimed. As a result, $g(w, 1) > g(w, 0)$ for all $w \in D_{w^*}$.

Suppose that $g(w, 0) \geq w$ for some $w > w^*$. From (14) and (16), we have that

$$C'_{w^*}(w) \geq \pi C'_{w^*}[g(w, 0)] + (1 - \pi) C'_{w^*}[g(w, 1)] > C'_{w^*}(w),$$

where the last inequality follows from the fact that $C_{w^*}(w)$ is strictly convex in w and the fact that $g(w, 1) > g(w, 0)$. But we obtain a contradiction.

Hence, we must have $g(w, 0) < w$ for all $w > w^*$. Since $g(w, 0)$ is continuous in w , it follows that $g(w^*, 0) = w^*$.

Finally, to prove (iii), suppose that $g(w^* + \varepsilon, 0) > w^*$ for all $\varepsilon > 0$. Then, (14) and (16) require that

$$C'_{w^*}(w^* + \varepsilon) \geq \pi C'_{w^*}[g(w^* + \varepsilon, 0)] + (1 - \pi) C'_{w^*}[g(w^* + \varepsilon, 1)]$$

holds for all $\varepsilon > 0$, which in turn requires that $\lim_{\varepsilon \rightarrow 0} g(w^* + \varepsilon, 1) = w^*$. But this implies a contradiction. **Q.E.D.** ■

If the borrower delays a repayment to his lender in the settlement stage, then the terms of the contract become less favorable for him in all future transactions within the credit relationship. As a result of intertemporal allocation of resources by a risk-neutral lender, a delayed repayment is compensated by more favorable terms of credit for the lender in future transactions. In fact, we can interpret the state in which the borrower does not make a payment to the lender as the practice of revolving credit that we observe in the market for unsecured loans.

The contracts that we actually observe in this market take the form of a credit line. In the model, when the borrower truthfully reports the unproductive state – and as a result delays his repayment to the lender – we can say that such an action creates a liability for the borrower, which alters the terms of credit for the following transaction within the relationship. The fact that a lender offers a contract with this property is precisely because such a mechanism allows the lender to extract the maximum surplus from the borrower given the possible alternatives that the latter has. Hence, the property of revolving credit that we observe in a lender's optimal contract is part of an optimal mechanism through which the lender obtains more favorable terms of credit for future transactions within the relationship.

We want now to better characterize the loan amounts $H[\hat{u}(w)]$. Notice that the envelope condition (16) implies that the loan amount to which a borrower is entitled in the transaction stage is strictly increasing in his promised expected discounted utility w . As we have seen, the optimal provision of incentives by a lender results in a lower promised expected discounted utility for a borrower who reports the unproductive state and as a result fails to make a repayment. Thus, the loan amount that a borrower receives from a lender in the subsequent transaction stage shrinks, given that $H[\hat{u}(w)]$ is a strictly increasing function. This shows how the loan amount that a borrower receives from a lender in the current transaction depends on the history of trades within the credit relationship.

It is useful to define a statistic that summarizes the terms of credit within each enduring relationship. Notice that the expected return to a lender on the current transaction is given by

$$R(w) \equiv \frac{(1 - \pi)y(w)}{H[\hat{u}(w)]}, \quad (17)$$

which summarizes the terms of credit for the current transaction as a function of w . Since a borrower's expected discounted utility w evolves over time within the set D_{w^*} according to the history of transactions within the credit relationship, we expect $R(w)$ to fluctuate over time as a result.

Lemma 6 *The statistic $R(w)$ defined by (17) is strictly decreasing in w .*

Proof. It remains to be shown that $y(w)$ is non-increasing on D_{w^*} . To verify this claim, suppose that there is an interval $\tilde{D} \subset D_{w^*}$ on which $y(w)$ is strictly increasing. Then, there is an interval $\hat{D} \subseteq \tilde{D}$ on which $0 < y(w) < h$. Notice that (12)-(14) imply that $g(w, 1)$ is constant on \hat{D} . Then, (9) implies that $g(w, 0)$ must be strictly decreasing on \hat{D} . This necessarily means that $\lambda(w) = 0$ for all $w \in \hat{D}$. As a result, we must have

$$C'_{w^*}(w) = \pi C'_{w^*}[g(w, 0)] + 1 - \pi$$

for all $w \in \hat{D}$. But this implies a contradiction. Therefore, it must be that $y(w)$ is non-increasing on D_{w^*} as claimed. **Q.E.D.** ■

The statistic $R(w)$, which is depicted in Figure 1, captures the evolution of the terms of credit according to the history of transactions (summarized by w). This means that $R(w^*)$ gives the worst terms of credit for a borrower, while $R[\bar{w}(w^*; \gamma)]$ gives the best terms of credit. A lower value for w in D_{w^*} implies that $R(w)$ is relatively higher – closer to the upper bound $R(w^*)$. This means that the terms of credit for the current transaction are less favorable for the borrower – and more favorable for the lender – because he has had a weak repayment history within the relationship. Worse terms of credit for a borrower mean that he is entitled to a lower loan amount in the transaction stage and/or is required to make a bigger repayment in the settlement stage, contingent on the realization of the productive state.

Notice that the dynamics for $R(w)$ depend critically on the assumption that the productivity shock is privately observed by the borrower. It is precisely because a lender is unable to observe a borrower's ability to repay his loan that we observe the spreading of a borrower's continuation values associated with a lender's optimal contract.

4 Changes in the Initial Cost of Lending

An important parameter in the model is the cost $k > 0$ that a lender has to pay in order to post a contract in the credit market. We have seen that there exist $\underline{k}(\gamma)$ and $\bar{k}(\gamma)$, with $0 \leq \underline{k}(\gamma) < \bar{k}(\gamma)$, such that, for any $k \in [\underline{k}(\gamma), \bar{k}(\gamma)]$, there exists a unique market utility $w^*(k; \gamma)$ such that $\hat{\phi}[w^*(k; \gamma)] + (1 - \beta)k = 0$, where $\hat{\phi}(w) \equiv C_w(w)$. Again, $w^*(k; \gamma)$ gives a borrower's expected discounted utility from the perspective of the signing date. We are holding the cost of walking away from a contract γ fixed. Given that $\hat{\phi}(w)$ is a continuous function, for any k' in a neighborhood of k , there exists a unique $w^*(k'; \gamma)$ such that $\hat{\phi}[w^*(k'; \gamma)] + (1 - \beta)k' = 0$. Moreover, if $k' > k$, we have that $w^*(k'; \gamma) < w^*(k; \gamma)$; if $k' < k$, we have that $w^*(k'; \gamma) > w^*(k; \gamma)$. In the proof of Lemma 3, we have established that the upper bound $\bar{w}(w^*; \gamma)$ on the set of expected discounted utilities is a non-increasing function of the lower bound w^* . Thus, we have that $D_{w^*(k; \gamma)} \subset D_{w^*(k'; \gamma)}$ if $k' > k$ and that $D_{w^*(k'; \gamma)} \subset D_{w^*(k; \gamma)}$ if $k' < k$. This means that a lower value for k results in a smaller set of expected discounted utilities.

We have some important implications. First, a lower value for k makes each borrower better off from the perspective of the signing date because the expected discounted utility associated with the market contract rises: a lower cost of entry results in more competition in the credit market. Second, there is less variability in a borrower's expected discounted utility over time. The terms of the contract are such that a borrower's expected discounted utility fluctuates within a smaller set according to the history of trades. Third, a lender's cost function under $k' < k$ is uniformly above her cost function under k ; see Figure 2. This means that a lower value for k makes each lender uniformly worse off *ex post*.¹

We can interpret $k > 0$ as the initial cost of lending per customer for each lender in the market for unsecured loans. If technological progress drives the cost to nearly zero, we should expect small fluctuations over time in a borrower's expected discounted utility. As a result, the terms of credit that borrowers receive – measured by $R(w)$ – are nearly the same across the population of borrowers, regardless of their particular repayment histories. Another prediction of the model is that borrowers obtain more favorable terms of credit at the signing date as the initial cost of lending approaches zero.

¹*Ex post* means after the decision of entering the credit market by posting a contract. Recall that *ex ante* a lender is always indifferent in equilibrium.

If γ is very small, then a borrower's expected discounted utility fluctuates over a very small set as k approaches the lower bound $\underline{k}(\gamma)$. This means that at any point in time the difference between the expected discounted utility associated with the worst terms of credit, given by $R(w^*)$, and the expected discounted utility associated with the best terms of credit, given by $R[w(w^*; \gamma)]$, is very small across the population of borrowers. As a result, a borrower's repayment history does not much affect the terms of credit that are offered to him within a credit relationship. However, a stationary equilibrium may not exist when both k and γ are too small. In this case, there is a strictly positive lower bound on the initial cost of lending for which we can guarantee that a stationary equilibrium exists.

5 Discussion

A property of the equilibrium allocation is that at any moment borrowers are differentiated by lenders exclusively, according to their history of transactions – loan and repayment amounts – within a credit relationship. This means that two borrowers are treated differently by the lenders with whom they are paired only because they have had distinct repayment histories (due to different histories of productivity shocks). Recall that at the first date each lender offers the same contract to a borrower. Borrowers are *ex ante* identical and face variable terms of credit over time within their credit relationships with lenders as a result of different histories of productivity shocks. This is different from other theories of unsecured credit that assume that borrowers are *ex ante* heterogeneous with respect to some characteristic. For instance, in Livshits, MacGee, and Tertilt (2009), borrowers differ *ex ante* with respect to a characteristic that affects their ability to repay a loan; in Chatterjee, Corbae, and Ríos-Rull (2008), households differ *ex ante* with respect to the likelihood of a loss in their wealth.

In Drozd and Nosal (2008), borrowers are *ex ante* identical and differ *ex post* with respect to their wealth and income. In their analysis, the terms of the contract are fixed over time within each relationship between a borrower and a lender. This is a sufficient condition for obtaining default in equilibrium so that it is possible to interpret some results as bankruptcy. The contribution of our paper is to perform the comparative statics exercise of changing the initial cost of lending and to establish some properties of the equilibrium allocation in an environment where no restriction on the space of contracts is exogenously imposed. Although some properties that we obtain are similar – a lower initial cost of lending makes each borrower better

off – others arise precisely because the form of the contract is completely endogenous.

As we have seen, each borrower in the economy should get nearly the same terms of credit within his credit relationship with a lender as a result of any technological progress that drives the initial cost of lending to nearly zero. The history of transactions within each credit relationship becomes less relevant to determine the terms of credit for future transactions as the initial cost of lending approaches zero.

6 Long-Run Properties

In this section, we study the long-run properties of the equilibrium allocation. Specifically, we show that there exists a well-behaved long-run distribution of expected discounted utilities with mobility. Let $\Psi(D_{w^*}, \mathbf{D})$ be the space of all probability measures ψ on the measurable space (D_{w^*}, \mathbf{D}) , where \mathbf{D} is the collection of Borel subsets of D_{w^*} . Define the operator T^* on $\Psi(D_{w^*}, \mathbf{D})$ by

$$(T^*\psi)(D') = \pi \int_{\mathbf{Q}_0(D')} d\psi + (1 - \pi) \int_{\mathbf{Q}_1(D')} d\psi,$$

for each $D' \in \mathbf{D}$, where, for each $\gamma \in \{0, 1\}$, the set $\mathbf{Q}_\gamma(D')$ is given by

$$\mathbf{Q}_\gamma(D') = \{w \in D_{w^*} : g(w, \gamma) \in D'\}.$$

Notice that a fixed point of the operator T^* corresponds to an invariant distribution over D_{w^*} .

Lemma 7 *The operator T^* has a unique fixed point ψ^* , and for any probability measure ψ in $\Psi(D_{w^*}, \mathbf{D})$, $T^{*n}\psi$ converges to ψ^* in the total variation norm.*

Proof. Let ψ_w denote the probability measure that concentrates mass on the point w . I will show that there exist $N \geq 1$ and $\varepsilon > 0$ such that $(T^{*N}\psi_w)(w^*) \geq \varepsilon$ for all $w \in D_{w^*}$. From Lemma 5, there exists $k > 0$ such that either $g(w, 0) \leq w - k$ or $g(w, 0) = w^*$ for all $w \in D_{w^*}$. Now, choose an integer $N \geq 1$ large enough so that $\bar{w}(w^*; \gamma) - kN \leq w^*$. Then, the probability of moving from the point $\bar{w}(w^*; \gamma)$ to the point w^* in N steps is at least π^N . Since $g(w, 0)$ is non-decreasing in w , such a transition to w^* is at least as probable from any other point in D_{w^*} . Thus, if $\varepsilon = \pi^N$, then the

implied Markov process satisfies the hypotheses of Theorem 11.12 of Stokey, Lucas, and Prescott (1989), and the proof is complete. **Q.E.D.** ■

The existence of a non-degenerate long-run distribution derives from the fact that there is no absorbing point, which implies that the entire state space is an ergodic set. The role of limited commitment is to bound the set of promised utilities, which is necessary to obtain a non-degenerate long-run distribution. Specifically, the lower bound w^* on the set of expected discounted utility entitlements arises due to the fact that a borrower can defect from his current contract and sign with another lender at any moment. The upper bound $\bar{w}(w^*; \gamma)$ is the highest expected discounted utility to which a lender can commit to deliver to a borrower given that the lowest expected discounted utility that can be promised is w^* .

7 Conclusion

We have characterized the terms of the contract that a lender offers to a borrower in a competitive credit market with the following characteristics: lenders are asymmetrically informed about a borrower's ability to repay a loan; lenders can commit to some credit contracts, while borrowers cannot commit to any contract; the history of trades within each enduring credit relationship in the economy is not publicly observable; and it is costly for a lender to contact a borrower and to walk away from a contract. These frictions result in a market contract whose terms vary over time, according to the history of trades within each long-term credit relationship. A lender's optimal contract has the property of revolving credit, which is a contingency that allows a borrower to delay a repayment to his lender. This is a mechanism through which the lender obtains more favorable terms of credit for future transactions within the credit relationship.

If technological progress drives the cost to nearly zero, we should expect small fluctuations over time in a borrower's expected discounted utility. As a result, the terms of credit associated with a lender's contract become very similar across the population of borrowers, regardless of individual repayment histories. Another prediction of the model is that a borrower obtains more favorable terms of credit as the initial cost of lending approaches zero: a market contract is such that each borrower is better off from the perspective of the contracting date. Although we do not exploit the model's quantitative implications in this paper, we provide important properties of a lender's optimal contracting problem in the market for unsecured loans with the characteristics described above.

References

- [1] R. Aiyagari and S. Williamson. “Credit in a Random Matching Model with Private Information” *Review of Economic Dynamics* 2 (1999) 36-64.
- [2] F. Alvarez and U. Jermann. “Efficiency, Equilibrium, and Asset Pricing with Risk of Default” *Econometrica* 68 (2000) 775-797.
- [3] D. Andolfatto. “The Simple Analytics of Money and Credit in a Quasi-Linear Environment” *Working Paper* (2008), Simon Fraser University.
- [4] A. Atkeson and R. Lucas. “On Efficient Distribution with Private Information” *Review of Economic Studies* 59 (1992) 427-453.
- [5] A. Atkeson and R. Lucas. “Efficiency and Equality in a Simple Model of Efficient Unemployment Insurance” *Journal of Economic Theory* 66 (1995) 64-88.
- [6] J.M. Barron and M. Staten. “The Value of Comprehensive Credit Reports” in M.J. Miller, ed., *Credit Reporting Systems and the International Economy* (2003) 273-310, MIT Press.
- [7] A. Berger. “The Economic Effects of Technological Progress: Evidence from the Banking Industry” *Journal of Money, Credit, and Banking* 35 (2003) 141-176.
- [8] C. Bertaut and M. Haliassos. “Credit Cards: Facts and Theory” in G. Bertola, R. Disney, and C. Grant, eds., *The Economics of Consumer Credit* (2006) 181-238, MIT Press, Cambridge, MA.
- [9] S. Chatterjee, D. Corbae, and J.V. Ríos-Rull. “A Finite-Life Private-Information Theory of Unsecured Consumer Debt” *Journal of Economic Theory* 142 (2008) 149-177.
- [10] P. Diamond. “Pairwise Credit in Search Equilibrium” *Quarterly Journal of Economics* 105 (1990) 285-319.
- [11] L. Drozd and J. Nosal. “Competing for Customers: A Search Model of the Market for Unsecured Credit” *Working Paper* (2008), Columbia University and University of Wisconsin – Madison.

- [12] E. Green. “Lending and the Smoothing of Uninsurable Income” in E. Prescott and N. Wallace, eds., *Contractual Arrangements for Intertemporal Trade* (1987) 3-25, University of Minnesota Press, Minneapolis, MN.
- [13] T. Kehoe and D. Levine. “Debt-Constrained Asset Markets” *Review of Economic Studies* 60 (1993) 865-888.
- [14] N. Kocherlakota. “Implications of Efficient Risk Sharing without Commitment” *Review of Economic Studies* 63 (1996) 595-609.
- [15] T. Koepl, C. Monnet, and T. Temzelides. “A Dynamic Model of Settlement” *Journal of Economic Theory* 142 (2008) 233-246.
- [16] D. Krueger and H. Uhlig. “Competitive Risk Sharing Contracts with One-Sided Commitment” *Journal of Monetary Economics* 53 (2006) 1661-91.
- [17] R. Lagos and R. Wright. “A Unified Framework for Monetary Theory and Policy Analysis” *Journal of Political Economy* 113 (2005) 463-84.
- [18] I. Livshits, J. MacGee, and M. Tertilt. “Costly Contracts and Consumer Credit” *Working Paper* (2009), University of Western Ontario and Stanford University.
- [19] L. Mester. “What Is the Point of Credit Scoring?” *Business Review*, Federal Reserve Bank of Philadelphia (September/October 1997) 3-16.
- [20] E. Nosal and G. Rocheteau. “The Economics of Payments” *Policy Discussion Papers* 14, Federal Reserve Bank of Cleveland (2006).
- [21] C. Phelan. “Repeated Moral Hazard and One-Sided Commitment” *Journal of Economic Theory* 66 (1995) 488-506.
- [22] S. Shi. “A Divisible Search Model of Fiat Money” *Econometrica* 65 (1997) 75-102.
- [23] C. Sleet. “Endogenously Incomplete Markets: Macroeconomic Implications” in S. Durlauf and L. Blume, eds., *New Palgrave Dictionary of Economics* (2008).
- [24] N. Stokey, R. Lucas, and E. Prescott. “Recursive Methods in Economic Dynamics” Harvard University Press, Cambridge, 1989.

- [25] T. Temzelides and S. Williamson. “Payments Systems Design in Deterministic and Private Information Environments” *Journal of Economic Theory* 99 (2001) 297-326.
- [26] J. Thomas and T. Worrall. “Income Fluctuation and Asymmetric Information: An Example of a Repeated Principal-Agent Problem” *Journal of Economic Theory* 51 (1990) 367-390.

Figure 1 - Terms of Credit

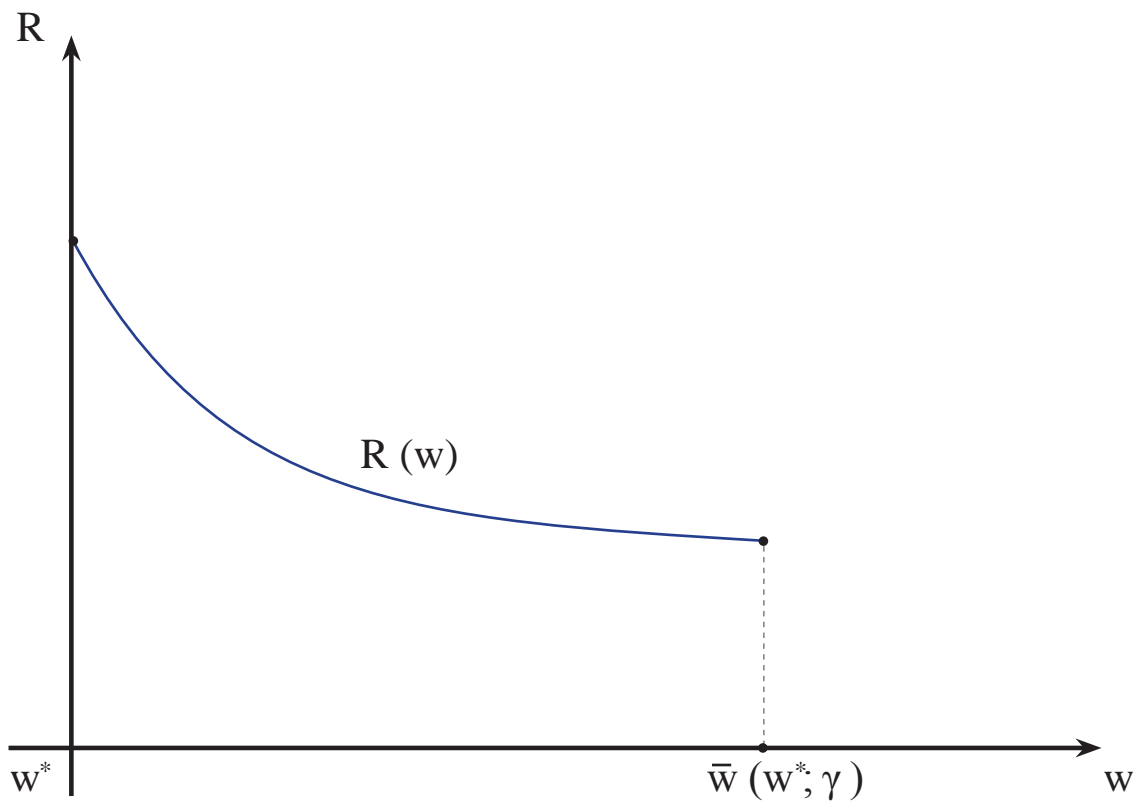


Figure 2 - Lender's Cost Function

