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Dunia López-Pintado and Juan D. Moreno-Ternero

CORE
Voie du Roman Pays 34
B-1348 Louvain-la-Neuve, Belgium.
Tel (32 10) 47 43 04
Fax (32 10) 47 43 01
E-mail: corestat-library@uclouvain.be
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The principal's dilemma

Dunia LÓPEZ-PINTADO[1] and Juan D. MORENO-TERNERO[2]

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Abstract

A recurrent dilemma in team management is to select between a team-based and an individual-based wage scheme. We explore such a dilemma in a simple model of production in teams, in which the team members may differ in their effort choices and qualification. We show that, in spite of enhancing output as the basis for payment, a team-based wage scheme might be less profitable for the principal than an individual-based wage scheme. We also highlight a deep misalignment between designing optimal (output-based) incentives for a team and treating its members impartially. Finally, upon introducing the possibility of liquidity constraints in our model, we provide rationale for the so-called "rich get richer" hypothesis.

Keywords: team production, management, incentives, effort.

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[1] Universidad Pablo de Olavide, Spain.

[2] Universidad de Malaga and Universidad Pablo de Olavide, Spain; Université catholique de Louvain, CORE, B-1348 Louvain-la-Neuve, Belgium.

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1 Introduction

A large theoretical literature has emphasized how employers design compensation contracts to induce employees to operate in their interests (see, for instance, Prendergast (1999) and the references cited therein). It is typically assumed that workers respond to incentives and that, in particular, paying on the basis of output induces workers to supply more output (e.g., Lazear, 2000). When firms can accurately measure the contribution of individual workers simple piece-rate incentive plans have proven to be effective, provided there is careful measurement of output (e.g., Baker, 1992; Nagin et al., 2002). There are some features of employment relationships, though, that limit the effectiveness of simple piece-rate incentive pay plans and that force managers to consider other forms of incentive pay (e.g., Ichniowski and Shaw, 2003). For instance, in many production processes, output is a function not of the effort of a single worker, but of the combined effort of many workers, and the reward for participation in such teams is likely to be some form of group-based pay (e.g., Holmstrom and Milgrom, 1991).

In this paper, we present a simple model of production in teams to explore the dilemma for a principal between guaranteeing (deserving) workers a fixed wage, or making wages contingent on collective (besides individual) performance. The dilemma can be interpreted as a choice between a team-based or an individual-based wage scheme for the management of teams.

More precisely, imagine a project that has to be managed by a team of agents each of whom is responsible for a different task. Agents (who might differ in their skills) decide whether to exert effort or not in order to perform their tasks. Exerting effort is a costly action and the higher the skill, the lower the cost. The overall project succeeds with a probability which is an increasing function of the number of agents exerting effort.

The principal (who knows each agent's skill and observes each agent's effort) chooses between two management scenarios. In the first one, the principal designs a mechanism rewarding agents exerting effort only if the project ends successfully. In the second one, the scheme is not contingent and therefore the principal rewards (deserving) agents independently of the success of the overall project.

We show in this paper that the dilemma is tilted in favor of the second management scenario, which is typically more profitable for the principal. More precisely, we show that, under general conditions, the expected benefits of a principal are lower under the contingent scenario than under the non-contingent one. As a byproduct of our analysis, we will also show that agents have the dual preferences over the two scenarios. That is, they prefer the contingent scenario, as their expected benefits will be lower under the contingent scenario than under the non-contingent one.

These results have implications that might even be seen as counterintuitive at first sight. On the one hand, we obtain that, when it comes to the management of teams, naive stimulus measures, such as making payments contingent to the overall success of the team project, might not necessarily be a good option for the principal. On the other hand, agents would typically prefer a scheme that, albeit risky, would enhance incentives further, rather than a scheme guaranteeing them a secure wage.

Our analysis will also provide rationale for the so-called "rich get richer" hypothesis. In a market economy, there is no clear implication as to whether economic activities will tend to reduce or else to widen initial wealth disparities (e.g., Durham et al., 1998). The so-called Paradox of Power (e.g., Hirshleifer, 1991) is the observation that poorer or weaker contestants improve their position relative to richer or stronger opponents. Nevertheless, in some social and economic contexts the reverse occurs, i.e., initially richer and/or more powerful contestants do exploit weaker rivals and thus the *rich get richer*.[1] Our model and results take a side on this debate upon endorsing the latter instance. To elaborate on this, one just has to assume the existence of liquidity constraints in our model. For instance, think of the case of start-up companies without enough stock resources to face wages if there is a team failure (i.e., rewards could not exceed the revenues of the team). In such a case, the principal would be forced to the contingent management option described above. Our results would therefore tell us that a principal without liquidity constraints is likely to obtain higher expected benefits than a principal with liquidity constraints, which is to be interpreted as an instance of the "rich get richer" hypothesis.

As it can be inferred from the above, this paper deals with team management, a topic that has been the object of intense study in economics since Marschak (1955).[2] Holmstrom (1982) initiated the interest on moral-hazard problems within teams, an aspect that has received considerable attention

[1]For instance, the accruing of greater increments of recognition for particular scientific contributions to scientists of considerable repute and the withholding of such recognition from scientists who have not yet made their mark, was already reported in the sociology of science long time ago (e.g., Merton, 1968). Similarly, in the literature on networks, the counterpart to this hypothesis refers to the idea that nodes gain new links with probabilities that are proportional to the number of links that they already have. This hypothesis is widely accepted as the explanation for the occurrence of node connectivities following a power-law distribution in systems as diverse as genetic networks, citation networks or the World Wide Web (e.g., Barabàsi and Albert, 1999).

[2]See Marschak and Radner (1972) for a comprehensive survey.

ever since (e.g., McAfee and McMillan, 1991; Itoh, 1993; Che and Yoo, 2001; Winter, 2004). In our case, the moral-hazard problem is absent as agents' effort is observable. This feature allows us to scrutinize the robustness of some of the results obtained in the mentioned literature. For instance, Winter (2004) argues that even when agents are identical and act simultaneously (i.e., with no information among peers) the principal may gain by discriminating among them. Nevertheless, this feature happens in Winter's model if and only if technology functions exhibit *decreasing returns of scale*, whereas, as we shall show later, in our model this feature occurs without imposing additional conditions whatsoever on the technology functions.

Our benchmark model relies on some key assumptions, such as agents' neutrality to risk; a flat (as opposed to hierarchical) organization of the team; the viability of wage schemes, or the use of the Nash equilibrium concept to design them. We, nonetheless, explore the extensions of our benchmark model in each of the corresponding directions that arise when relaxing each of these assumptions. In doing so, we test the robustness of our results.

The rest of the paper is organized as follows. In Section 2, we set up the benchmark model. In Section 3, we obtain the main results of the paper. In Section 4 we address some extensions of the benchmark model and their corresponding results. We conclude in Section 5.

2 The benchmark model

There is a project involving n activities performed by n agents of a team. Each agent decides simultaneously whether to exert effort (invest) or not towards the performance of her activity.[3] We denote by $\delta_i \in \{0, 1\}$ the effort decision of agent i, where $\delta_i = 1$ (0) if player i does (not) exert effort. The cost of exerting effort of agent i is c_i. This parameter is to be interpreted as a sign of the agent's skill (i.e., the lower the cost of exerting effort, the higher the skill). We assume, without loss of generality, that $c_1 \leq c_2 \leq \cdots \leq c_n$. An agent will invest if and only if her expected benefits (i.e., her expected wage minus her cost) are non-negative.[4] The project's technology is a strictly increasing function $p : \{0, 1, \ldots, n\} \rightarrow [0, 1]$ specifying the probability of success for any given number of agents exerting effort. In doing so, we are

[3]This is a way of modeling the fact that the team has a flat (rather than hierarchical) organization. In doing so, we are implicitly assuming that communication among agents does not necessarily exist (perhaps reflecting geographical constraints) and that individual effort choices might not be observed by the other agents.

[4]We assume all agents are risk neutral. We will elaborate on this assumption later in the text.

implicitly assuming that agents' efforts are equally valuable for the success of the project.

A principal observes agents' skills and effort decisions, and designs the wage scheme for the team with the aim of maximizing her benefits. Let $\beta > 0$ denote the proceeds for the principal if the project is successful and assume that an unsuccessful project yields 0. Agents are subject to limited liability, which means that the principal cannot impose negative wages on them.[5] Let $\omega_i \geq 0$ denote agent i's wage, which will obviously depend on i's effort decision.[6] The principal will have two options to design the scheme $\{\omega_i\}_{i\in N}$. One in which wages are conditional on the success of the project, which can therefore be considered as a team-based scheme, and another in which they are not, which can therefore be considered as an individual-based scheme. Under each option, the principal will have an optimal group of agents $K \subseteq N$ she would like to see exerting effort. If the principal obeys the vNM axioms, K is obtained, respectively, by solving the programs

$$\max_{k=0,1,\ldots,n} \; p(k) \cdot \left(\beta - \sum_{i\in K} \omega_i \right), \tag{1}$$

and

$$\max_{k=0,1,\ldots,n} \; p(k) \cdot \beta - \sum_{i\in K} \omega_i, \tag{2}$$

where, in each case, k denotes the cardinality of K.[7] The value of k solving a program of this sort will be referred to as the *optimal size of the team.*

In order to solve the above optimization problems, we shall need to construct the mechanism that induces agents within a given group (and only them) to exert effort at the minimum possible cost. A mechanism achieving such an aim will be called an *optimal investment-inducing mechanism.* Note that any given wage scheme defines a game. An investment-inducing mechanism for a group of agents K would be a wage scheme for which its corresponding game would have a unique Nash equilibrium in which all agents in K, and only them, exert effort. The optimal investment-inducing mech-

[5] Limited liability of the agents may arise from workers' having the freedom to quit or from institutional constraints such as laws banning firms' exacting payments from workers. In any case, dropping this assumption would not alter the message of our results.

[6] Note that, given the assumptions, it is natural to assume that $\omega_i = 0$ for each i such that $\delta_i = 0$.

[7] At the risk of stressing the obvious, note that the values of ω_i in (1) and (2) will not coincide and that, therefore, the corresponding optimal sets (and their cardinalities) in each program need not be the same.

anism for K would be the least expensive investment-inducing mechanism for K.

Once the above optimization problems are solved, the corresponding wage schemes are easily described. More precisely, let K^1 and K^2 denote the optimal groups obtained from (1) and (2) respectively, and $\{\omega_i^1\}_{i\in N}$ and $\{\omega_i^2\}_{i\in N}$ denote the corresponding optimal investment-inducing mechanism for these groups. Then, the wage schemes for each option would be, respectively,

$$\omega_i = \begin{cases} \omega_i^1 & \text{if } \delta_i = 1,\ i \in K^1, \text{ and the project is successful,} \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\omega_i = \begin{cases} \omega_2^1 & \text{if } \delta_i = 1 \text{ and } i \in K^2, \\ 0 & \text{otherwise.} \end{cases}$$

3 The main results

We start this section exploring the principal's behavior under both scenarios.

Proposition 1 *The following statements hold:*

1. *If wages are contingent on the project's success, the principal solves*

$$\max_{k=0,1,\ldots,n} \; p(k) \cdot \left(\beta - \sum_{i=1}^{k} \frac{c_i}{p(i)} \right). \tag{3}$$

2. *If wages are not contingent on the project's success the principal solves*

$$\max_{k=0,1,\ldots,n} \; p(k) \cdot \beta - \sum_{i=1}^{k} c_i. \tag{4}$$

Proof. Since the second statement of the proposition is straightforward, we focus on the first one. Let $(\omega_1, \omega_2, \ldots, \omega_n)$ be a scheme inducing a game whose unique Nash equilibrium is $(\delta_1, \delta_2, \ldots, \delta_n) = (1, 1, \ldots, 1)$. Then, in particular, $(\delta_1, \delta_2, \ldots, \delta_n) = (0, 0, \ldots, 0)$ is not a Nash equilibrium for that game, which implies that there exists, at least, an agent $i_1 \in N$ wanting to deviate by investing. In other words, there exists $i_1 \in N$ for which $p(1)\omega_{i_1} \geq c_{i_1}$.[8] Equivalently,

$$\omega_{i_1} \geq \frac{c_{i_1}}{p(1)}.^{9} \tag{5}$$

Let us consider now the profile $(\delta_1, \delta_2, \ldots, \delta_n)$ in which $\delta_{i_1} = 1$ and $\delta_i = 0$ otherwise.[10] Since this profile cannot be an equilibrium either, it follows that there exists an agent $i_2 \in N \setminus \{i_1\}$ wanting to deviate by investing. In other words, there exists $i_2 \in N \setminus \{i_1\}$ for which $p(2)\omega_{i_2} \geq c_{i_2}$.[11] Equivalently,

$$\omega_{i_2} \geq \frac{c_{i_2}}{p(2)}.$$

This argument can be subsequently repeated for the remaining profiles to show, in the end, that there exists some permutation π of the set $\{1, \ldots, n\}$, for which

$$(\omega_1, \omega_2, \ldots, \omega_n) \geq (\omega_1^\pi, \ldots, \omega_n^\pi) = \left(\frac{c_1}{p(\pi(1))}, \ldots, \frac{c_n}{p(\pi(n))}\right).$$

Now, since p is increasing, and $c_1 \leq c_2 \leq \cdots \leq c_n$, it is straightforward to show that

$$\sum_{i=1}^{n} \frac{c_i}{p(i)} \leq \sum_{i=1}^{n} \omega_i^\pi,$$

for any permutation π, which shows that

$$(\omega_1, \omega_2, \ldots, \omega_n) = \left(\frac{c_1}{p(1)}, \ldots, \frac{c_n}{p(n)}\right),$$

is *the* optimal investment-inducing mechanism for N (together with the schemes obtained by permuting indices corresponding to agents with a same cost, which would also generate the same overall wage).

[8]Note that if i_1 deviates there would only be one agent exerting effort, which would make $p(1)$ the probability of success and therefore the probability for agent i_1 of getting a positive wage ω_{i_1}.

[9]Note that this condition guarantees that exerting effort is a dominant strategy for agent i_1.

[10]Note that (5) not only guarantees that the profile in which no agent exerts effort is not a Nash equilibrium, but also that no profile in which only one agent (different from i_1) exerts effort constitutes a Nash equilibrium either.

[11]Note that if i_2 deviates there would be only two agents exerting effort, which would make $p(2)$ the probability of success and therefore the probability for agent i_2 of getting a positive wage ω_{i_2}.

Thus, it follows from there that the optimal wage scheme to guarantee that agents in K (and only them) exert effort is given by

$$\omega_i = \frac{c_i}{p(\sigma(i))},$$

for all $i \in K$, where $\sigma(i)$ denotes the rank of i in K, and $\omega_i = 0$ for all $i \notin K$. Therefore, among the sets with the same cardinality of K, the *optimal* one for the principal would be $\{1, 2, \ldots, k\}$. Consequently, the objective program (1) translates into

$$p(k) \cdot \left(\beta - \sum_{i=1}^{k} \frac{c_i}{p(i)} \right),$$

as desired. ■

We now compare the objective functions in the statement of Proposition 1. It is straightforward to show that

$$p(k) \cdot \left(\beta - \sum_{i=1}^{k} \frac{c_i}{p(i)} \right) \leq p(k) \cdot \beta - \sum_{i=1}^{k} c_i,$$

with a strict inequality for any $k > 1$. This proves the following corollary:

Corollary 1 *The principal gets higher expected benefits when wages are not contingent on the project's success.*

Corollary 1 shows the superiority of what we called the non-contingent option. In other words, the principal would prefer guaranteeing workers a salary with no risk whatsoever rather than enhancing incentives further upon linking wages to (collective) performance. Corollary 1 also says, in particular, that when the principal faces liquidity constraints in the design of the wage scheme (i.e., she is forced to make wages contingent on the project's success) then she typically obtains lower benefits than without liquidity constraints (and therefore avoiding contingency on the project's success). Thus, Corollary 1 can provide rationale for the so-called "rich get richer" hypothesis, as a "rich" principal (e.g., a company with enough stock resources) increases her benefits with respect to a "poor" principal (e.g., a start-up company) without the option of offering a wage scheme deprived of contingencies.

Proposition 1 also provides information regarding the preferences of the team members. More precisely, it shows that the expected benefits for each agent are always 0 under the non-contingent management strategy, whereas under the contingent strategy this is only the case if the agent does not belong to the *optimal* set $K = \{1, 2, \ldots, k\}$. Otherwise, agent $i \in K$ gets

$$p(k)\omega_i - c_i = c_i \left(\frac{p(k)}{p(i)} - 1 \right) \geq 0.$$

Thus, we have the following corollary:

Corollary 2 *Agents get higher expected benefits when wages are contingent on the project's success.*

Corollary 2 illustrates how principal and agents have opposite preferences regarding management strategies. It says that (risk-neutral) agents prefer a scheme that, albeit risky, would enhance incentives further, rather than a scheme guaranteeing them a secure wage.

The information provided by Proposition 1 regarding agents' (expected) wages shows another interesting feature. On the one hand, we observe that with the optimal non-contingent wage scheme agents would end up receiving the same (actually, zero) benefits despite having different skills (and therefore receiving different wages). Thus, it would be, ex-post, an (extremely) egalitarian scheme. Things, however, differ with the optimal contingent wage scheme. In such a case, the expected benefits in equilibrium of an agent i exerting effort, would be

$$p(k)\omega_i - c_i = c_i \left(\frac{p(k)}{p(i)} - 1 \right) \geq 0,$$

where, recall that $K = \{1, 2, \ldots, k\}$ is the set of agents exerting effort, and therefore $i \in K$. This implies that agents would typically end up receiving not only different wages, but also different (and positive) benefits. In other words, the optimal contingent wage scheme is not only inegalitarian *ex-ante*, but also *ex-post*. As a matter of fact, the inegalitarian aspect of this scheme is obvious as it violates the most fundamental notion of horizontal equity:

Corollary 3 *The contingent wage scheme violates equal treatment of equals both from an ex-ante and an ex-post viewpoint.*

More precisely, Corollary 3 says that, under the optimal contingent wage scheme, equal agents (in terms of their skills) might not only receive different wages, but also enjoy different benefits.

We conclude the analysis of the benchmark model by showing another difference between both management options.

Corollary 4 *The optimal size of the team when the principal uses the contingent management option is never higher than when using the non-contingent management option.*

Proof. Let us denote by $\overline{k}$ ($\widehat{k}$) the optimal size of the team for a principal using the contingent management (non-contingent) management option. Formally, $\overline{k}$ is the value where (3) is maximized, whereas $\widehat{k}$ is the value where (4) is maximized. We show that $\overline{k} \leq \widehat{k}$. By contradiction, assume that the opposite holds. Then, since $\overline{k}$ maximizes (3), it follows that

$$p(\overline{k}) \cdot \left(\beta - \sum_{i=1}^{\overline{k}} \frac{c_i}{p(i)}\right) \geq p(\widehat{k}) \cdot \left(\beta - \sum_{i=1}^{\widehat{k}} \frac{c_i}{p(i)}\right),$$

and therefore,

$$p(\overline{k})\beta \geq p(\widehat{k})\beta + \sum_{i=1}^{\overline{k}} \frac{p(\overline{k})}{p(i)} c_i - \sum_{i=1}^{\widehat{k}} \frac{p(\widehat{k})}{p(i)} c_i.$$

Thus, it follows from here that

$$\begin{aligned}
& p(\overline{k})\beta - \sum_{i=1}^{\overline{k}} c_i \\
\geq\ & p(\widehat{k})\beta + \sum_{i=1}^{\overline{k}} \frac{p(\overline{k})}{p(i)} c_i - \sum_{i=1}^{\widehat{k}} \frac{p(\widehat{k})}{p(i)} c_i - \sum_{i=1}^{\overline{k}} c_i \\
=\ & p(\widehat{k})\beta - \sum_{i=1}^{\widehat{k}} c_i + \sum_{i=1}^{\overline{k}} \frac{p(\overline{k})}{p(i)} c_i - \sum_{i=1}^{\widehat{k}} \frac{p(\widehat{k})}{p(i)} c_i - \sum_{i=\widehat{k}+1}^{\overline{k}} c_i \\
\geq\ & p(\widehat{k})\beta - \sum_{i=1}^{\widehat{k}} c_i + \sum_{i=1}^{\overline{k}} \frac{p(\overline{k})}{p(i)} c_i - \sum_{i=1}^{\widehat{k}} \frac{p(\widehat{k})}{p(i)} c_i - \sum_{i=\widehat{k}+1}^{\overline{k}} \frac{p(\overline{k})}{p(i)} c_i \\
=\ & p(\widehat{k})\beta - \sum_{i=1}^{\widehat{k}} c_i + \sum_{i=1}^{\widehat{k}} \left(\frac{p(\overline{k})}{p(i)} - \frac{p(\widehat{k})}{p(i)}\right) c_i \\
>\ & p(\widehat{k})\beta - \sum_{i=1}^{\widehat{k}} c_i.
\end{aligned}$$

Thus,

$$p(\overline{k})\beta - \sum_{i=1}^{\overline{k}} c_i > p(\widehat{k})\beta - \sum_{i=1}^{\widehat{k}} c_i,$$

which represents a contradiction, as $\widehat{k}$ maximizes (4). ■

Corollary 4 says that the non-contingent management option typically induces more agents to exert effort. Thus, if the principal values per se that members of the team exert effort, Corollary 4 shows an additional advantage of the non-contingent management strategy.

4 Further results

In this section we explore several directions in which our benchmark model could be extended and obtain the corresponding results.

4.1 Intermediate management strategies

Our benchmark model only considers two extreme cases of management strategies in which wages are either fully contingent on the project's success or not contingent at all. A natural question arising from here (and our previous analysis) is whether our results would extend for *intermediate* cases in which wages are only partially contingent on the project's success. More precisely, assume now that the principal faces some liquidity constraints and, as a result, only has some stock resources (S), although maybe not enough to face the salaries of all workers if the project is not successful. In this context, a *semi-contingent* management option consists of a scheme guaranteeing to each deserving agent a fraction $s_i \leq c_i$ from the stock for sure, as well as a wage ω_i contingent on the project's success. That is, a principal using this option, and obeying the vNM axioms, would maximize the following function:

$$S - \sum_{i \in K} s_i + p(k) \cdot \left(\beta - \sum_{i \in K} \omega_i \right), \tag{6}$$

where K is the group of agents exerting effort and k its cardinality.

A similar argument to the one at the proof of Proposition 1, allows us to show that, provided $(s_1, \ldots, s_n)$ is the allocation of stock resources S, then the wage scheme to guarantee that agents in K (and only them) exert effort would be given by

$$\omega_i = \frac{c_i - s_i}{p(\sigma(i))}, \tag{7}$$

for all $i \in K$, where $\sigma(i)$ denotes the rank of i in K, and $\omega_i = 0$ for all $i \notin K$.

This implies that maximizing (6) is equivalent to maximizing

$$S - \sum_{i=1}^{k} s_i + p(k) \cdot \left(\beta - \sum_{i=1}^{k} \frac{c_i - s_i}{p(i)} \right), \tag{8}$$

where $(s_1, \ldots, s_n)$ is such that $S = \sum_{i=1}^{n} s_i$ and the *optimal* set K is $K = \{1, 2, \ldots, k\}$. Ultimately, this amounts to maximizing

$$\sum_{i=1}^{k} \alpha_i \cdot s_i, \tag{9}$$

where $\alpha_i = \left(\frac{p(k)}{p(i)} - 1\right)$ for all $i = 1, \ldots, k$. It is straightforward to show that $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_k$. Thus, the optimal semi-contingent investment-inducing mechanism for $K = \{1, \ldots, k\}$ is given by

$$(s_i, \omega_i) = (c_i, 0) \text{ for all } i \in \{1, \ldots i_0\},$$

$$(s_i, \omega_i) = \left(r, \frac{c_{i_0+1} - r}{p(i_0 + 1)}\right) \text{ for } i = i_0 + 1,$$

$$(s_i, \omega_i) = \left(0, \frac{c_i}{p(i)}\right) \text{ for all } i \in \{i_0 + 2, \ldots k\},$$

where i_0 is such that $1 \leq i_0 \leq k$, $\sum_{i=1}^{i_0} c_i \leq S < \sum_{i=1}^{i_0+1} c_i$ and $r = S - \sum_{i=1}^{i_0} c_i$.

It is straightforward to show from here that, for $S = 0$, we obtain the (contingent) mechanism at statement 1 of Proposition 1, whereas for $S > \sum_{i=1}^{k} c_i$ we obtain the (non-contingent) mechanism at statement 2 of Proposition 1. One can easily conclude that principals prefer non-contingent mechanisms to semi-contingent mechanisms and these ones to (fully) contingent mechanisms, hence supporting further the "rich get richer" hypothesis mentioned above.

4.2 Equity constraints

We have shown that optimal (contingent) wage schemes typically sacrifice the idea of equal treatment of equals. This feature can be seen as another instance of the so-called equality-efficiency trade-off (e.g., Okun, 1975). Nevertheless, a wide number of advanced democracies have passed, in the last decades, bills promoting different forms of equality in wage schemes. Thus, it might be reasonable to impose in our model some sort of equity constraints in the design of wage schemes. In this section, we compute the efficiency loss that would arise as a consequence of imposing these constraints.

Let $i_1, \cdots, i_k$ be the agents for which the cost ranking has a discontinuity, i.e., $i_1, \cdots, i_k$ are such that $i_j \leq i_j + 1$ for all j and

$$c_1 = c_2 = \cdots = c_{i_1 - 1} < c_{i_1} = c_{i_1+1} = \cdots = c_{i_2-1} < c_{i_2} \leq \cdots \leq c_n.$$

Then, an analogous argument to the proof of Proposition 1 allows us to show that the following scheme constitutes the optimal contingent mechanism, out of those preserving equal treatment of equals.

$$\omega_i = \frac{c_i}{p(1)} \text{ for all } i \in \{1, \ldots i_1 - 1\},$$

$$\omega_i = \frac{c_i}{p(i_1)} \text{ for all } i \in \{i_1, \ldots i_2 - 1\},$$

$$\cdots$$

$$\omega_i = \frac{c_i}{p(i_k)} \text{ for all } i \in \{i_k, \ldots n\}.$$

Thus, the existence of equality constraints can exacerbate the effect of liquidity constraints widening the initial wealth disparities that might exist among principals.

As one might expect, the more demanding is the notion of equality being considered, the higher the burden for a principal forced to adopt the contingent wage scheme. Instances of more demanding options would be the so-called *weak equity axiom* introduced by Sen (1973), which imposes a positive discrimination towards *disabled* individuals (to be interpreted in this model as those with higher costs) or, more generally, the so-called *priority axiom*, introduced by Moreno-Ternero and Roemer (2006), which imposes a reasonable limit on the scope of Sen's axiom, by endorsing the view that the discrimination in favor of the disabled should never be to the extent of making the disabled better-off than abler individuals, after the allocation takes place.[12]

[12]It is straightforward to show that the non-contingent wage scheme we have described would satisfy the priority axiom. As for the contingent wage scheme, it can actually be shown that, for a given technology function, it satisfies the priority axiom if and only if the distribution of skills in the team is sufficiently dispersed. Thus, in order to be prioritarian with teams having a low level of skill heterogeneity, the contingent wage scheme should be modified (at the expense of the principal) which, in other words, says that the imposition of a more demanding notion of equality (such as the priority axiom), at least under the presence of low skill heterogeneity, would intensify the effect of liquidity constraints.

4.3 Risk attitudes

Our results are based on the implicit assumption that agents are all risk neutral. The next result summarizes our findings for other risk attitudes.

Proposition 2 *The following statements hold:*

1. *If agents are risk averse, the principal gets higher expected benefits when wages are not contingent on the project's success.*
2. *There is a critical threshold of agents' risk lovingness, above (below) which the principal gets lower (higher) expected benefits when wages are not contingent on the project's success.*

Proof. We start focussing on the case in which agents are (equally) risk averse. Formally, let $u(\cdot)$ be each agent's (strictly concave) utility function over sure prospects. Let $(\omega_1, \omega_2, \ldots, \omega_n)$ be the scheme inducing a game in this case, whose unique Nash equilibrium is $(\delta_1, \delta_2, \ldots, \delta_n) = (1, 1, \ldots, 1)$. In particular, $(\delta_1, \delta_2, \ldots, \delta_n) = (0, 0, \ldots, 0)$ is not a Nash equilibrium for that game, which implies that there exists, at least, an agent $i_1 \in N$ wanting to deviate by investing. In other words, there exists $i_1 \in N$ for which

$$p(1)u_{i_1}(\omega_{i_1} - c_{i_1}) + (1 - p(1))u_{i_1}(-c_{i_1}) \geq u_{i_1}(0).$$

Now, due to the strict concavity of $u_{i_1}(\cdot)$, we have that

$$p(1)u_{i_1}(\omega_{i_1} - c_{i_1}) + (1 - p(1))u_{i_1}(-c_{i_1}) < u_i(p(1)\omega_{i_1} - c_{i_1})$$

Thus,

$$\omega_{i_1} > \frac{c_{i_1}}{p(1)}.$$

If we proceed iteratively, we show that, for any set of agents K, and $i \in K$,

$$\omega_i > \frac{c_i}{p(\sigma(i))},$$

where $\sigma(i)$ denotes the rank of i in K. Thus, for a given size of the firm (k), the objective function for the principal in this case satisfies that

$$p(k) \cdot \left(\beta - \sum_{i \in K} \omega_i \right) \leq p(k) \cdot \left(\beta - \sum_{i=1}^{k} \frac{c_i}{p(i)} \right), \tag{10}$$

which shows that the wage scheme would be more expensive under the assumption of risk neutral agents, and which, combined with Corollary 1, implies that the principal gets higher expected benefits when wages are not contingent on the project's success.

We now move to the case in which agents are (equally) risk loving. Formally, let $u(\cdot)$ be each agent's (strictly convex) utility function over sure prospects. Analogously to the previous case, there exists $i_1 \in N$ for which $p(1)u_{i_1}(\omega_{i_1} - c_{i_1}) + (1 - p(1))u_{i_1}(-c_{i_1}) \geq u_{i_1}(0)$. Now, due to the strict convexity of $u_{i_1}(\cdot)$, we have that

$$p(1)u_{i_1}(\omega_{i_1} - c_{i_1}) + (1 - p(1))u_{i_1}(-c_{i_1}) > u_{i_1}(p(1)\omega_{i_1} - c_{i_1}).$$

Thus, by continuity, if follows that there exists $\epsilon_{i_1} > 0$ for which, if

$$\omega_{i_1} = \frac{c_{i_1}}{p(1)} - \epsilon_{i_1},$$

then

$$p(1)u_{i_1}(\omega_{i_1} - c_{i_1}) + (1 - p(1))u_{i_1}(-c_{i_1}) = u_{i_1}(0).$$

Obviously, ϵ_{i_1} is correlated to the agent's degree of risk lovingness in a positive way, i.e., the higher the degree of risk lovingness, the higher ϵ_{i_1}. If we proceed iteratively, we obtain that the optimal wage scheme to guarantee that agents in K (and only them) exert effort is given by

$$\omega_i = \frac{c_i}{p(\sigma(i))} - \epsilon_i,$$

for all $i \in K$, where $\sigma(i)$ denotes the rank of i in K, and $\omega_i = 0$ for all $i \notin K$. This implies that, in this case, the principal solves

$$\max_{k=0,1,\ldots,n} \; p(k) \cdot \left(\beta - \sum_{i=1}^{k} \frac{c_i}{p(i)} - \epsilon_i \right), \tag{11}$$

which, combined with Corollary 1, shows that the resulting wage scheme would be more (less) expensive than the non-contingent wage scheme for low (high) values of ϵ_i, i.e., low (high) degrees of risk lovingness. Incidentally, the argument also shows that there is a sequence of thresholds $\{\epsilon_i\}$, or, for that matter, a precise degree of agents' risk lovingness, for which the principal would be indifferent between both management options. ■

4.4 Optimistic principals

In our analysis, while designing (contingent) investment-inducing mechanisms, we have imposed that the profile in which all agents in a group (and only them) exert effort be the only existing (Nash) equilibrium of the game induced by the corresponding wage scheme. Another (less demanding) option would be to find investment-inducing mechanisms where the profile in which all agents in a group (and only them) exert effort is an equilibrium, but not necessarily the only one. An interpretation for this alternative option is that the principal is *optimistic* and believes that agents will coordinate on the *right* equilibrium and therefore does not need to worry about the other existing equilibria. In other words, the principal is not concerned with the strategic uncertainty induced by the presence of multiple equilibria and, more precisely, by the existence of an equilibrium in which no agent exerts effort. We show next that this change alters our results substantially.

Let us start by noting that if a principal in this new setting commits to reward agents independently of the project's success, nothing changes, i.e., the optimal way to do so is also by rewarding each (deserving) agent within the optimal set with her cost and giving nothing to all other agents. Formally, the principal would solve

$$\max_{k=0,1,\ldots,n} \; p(k) \cdot \beta - \sum_{i=1}^{k} c_i, \tag{12}$$

and if $\widehat{k}$ denotes the solution to this program, the ensuing wage scheme would be

$$\omega_i = \begin{cases} c_i & \text{if } \; \delta_i = 1 \text{ and } i \in \{1, \ldots, \widehat{k}\}, \\ 0 & \text{otherwise.} \end{cases}$$

However, if the principal makes wages contingent on the project's success, then she would solve the following program:

$$\max_{k=0,1,\ldots,n} \; p(k) \cdot \left(\beta - \sum_{i=1}^{k} \frac{c_i}{p(k)} \right). \tag{13}$$

If $\widetilde{k}$ denotes the solution to this program, the ensuing wage scheme would be

$$\omega_i = \begin{cases} \frac{c_i}{p(\widetilde{k})} & \text{if } \; \delta_i = 1, \, i \in \{1, \ldots, \widetilde{k}\}, \text{ and the project is successful,} \\ 0 & \text{otherwise.} \end{cases}$$

It is not difficult to show that there is indeed strategic uncertainty for the above wage scheme as two equilibria arise; namely, the one in which all agents shirk and the one in which all agents receiving a positive wage exert effort.[13] It is also easy to show that this is indeed the cheapest wage scheme supporting the existence of an equilibrium in which all agents in a given group exert effort.

It follows from the above that the objective function in programs (12) and (13) is the same, which implies that the expected benefits for an *optimistic* principal are the same no matter whether she makes wages contingent on the project's success or not. In particular, this shows that the effect of liquidity constraints would be mitigated at the cost of assuming strategic uncertainty in the design of contingent wage schemes.

4.5 Flat Vs. Hierarchical structures

Our benchmark model has assumed a flat organization for the team. As mentioned above, this could be interpreted as a way of assuming that communication among agents does not necessarily exist (perhaps reflecting geographical constraints) and that, therefore, individual effort choices might not be observed by the other agents. Nevertheless, an alternative option would be to assume a hierarchical organization in which agents instead of performing their tasks simultaneously do so sequentially. That is, agents would decide sequentially (instead of simultaneously) whether to exert effort or not towards the performance of their activity. In such a case, the natural equilibrium notion to be used, while designing mechanisms, would be the so-called *subgame perfect Nash* equilibrium.

As before, if the principal aims to induce agents exert effort (at the minimum possible cost) commiting to reward them independently of the project's success, then the optimal way to do so would be by rewarding each (deserving) agent within the optimal set with her cost and giving nothing to all other agents.

However, if the principal makes wages contingent on the project's success, then the optimal wage scheme to guarantee that agents in K (and only them) exert effort is given by

$$\omega_i = \frac{c_i}{p(k)}, \tag{14}$$

for all $i \in K$, where k denotes the cardinality of K, and $\omega_i = 0$ for all $i \notin K$.[14]

[13] Note that an agent i would deviate from the profile in which all agents shirk only if $p(1)\omega_i \geq c_i$ or, equivalently, $\omega_i \geq \frac{c_i}{p(1)}$. Now, if $\widetilde{k} \geq 2$ (otherwise the scheme is identical to the one in the benchmark model) and since p is strictly increasing, it follows that $\frac{c_i}{p(1)} > \frac{c_i}{p(k)}$, which shows that no deviation would occur.

[14] Note that the location of agents in the hierarchy does not affect this result.

Thus, an analogous argument to the one in the previous section would allow us to show that the objective function the principal faces to determine the optimal set is the same for both cases, which therefore implies that the expected benefits for a principal of a hierarchical organization would be independent of the fact that wages are contingent on the project's success. Again, as before, this shows that the effect of liquidity constraints in our model of team production would be alleviated were principals allowed to freely design the architecture of their firms.

4.6 More flexible management strategies

Our analysis of contingent wage schemes has imposed that individual contracts depend only on the individual effort decision and the success (or failure) of the joint venture, which could be considered as a public signal. This might partially be justified on the grounds that agents (ex post) neither observe their peers' decisions nor the realization of their wages, and therefore would not find credible contingent contracts depending on additional aspects to the ones just mentioned.

Nevertheless, one might think of alternative contexts of team production in which more flexibility is allowed while designing wage schemes, and additional information (e.g., private signals that only the principal observes) might be considered. If so, as we show next, and similarly to what we obtain in the previous sections, the principal would be indifferent between a contingent scheme and a non-contingent scheme and, therefore, the effect of liquidity constraints would also be mitigated. More precisely, consider the following (contingent) scheme. Let $\widetilde{k}$ denote the solution of the following program:

$$\max_{k=0,1,\dots,n} \; p(k) \cdot \left(\beta - \sum_{i=1}^{k} \frac{c_i}{p(k)} \right),$$

and let $\widehat{k}$ denote the total number of agents exerting effort within the team (i.e., $\widehat{k} = \sum_{i \in N} \delta_i$). Then, consider the following wage scheme, described by a menu of options depending on $\widehat{k}$,

$$\text{For each } \widehat{k} = 1, 2, \dots, n,$$

$$\omega_i = \begin{cases} \frac{c_i}{p(\widehat{k})} & \text{if } \; \delta_i = 1,\, i \in \{1, \dots, \widetilde{k}\}, \text{ and the project is successful,} \\ 0 & \text{otherwise.} \end{cases}$$

Note that, with this scheme, individual contracts not only depend on the individual effort decision and the success (or failure) of the team, but also on the number of agents in the team who actually exerted effort, as described in the above menu. It is clear that, on the equilibrium path, i.e., when the agents exerting effort are those in $\{1, \ldots, \widetilde{k}\}$, and therefore, $\widehat{k} = \widetilde{k}$, the scheme coincides with the one described in Section 4.4. However, the strategic uncertainty inherent there does not occur here, thanks to the off-equilibrium path. More precisely, an agent i would deviate from the profile in which all agents shirk only if $p(1)\omega_i \geq c_i$ or, equivalently, $\omega_i \geq \frac{c_i}{p(1)}$, which is precisely the amount that this scheme guarantees to agent i, provided she is the only one exerting effort. A similar argument allows to show that no other profile with some agents in $\{1, \ldots, \widetilde{k}\}$ shirking is an equilibrium, and that the profile in which all agents in $\{1, \ldots, \widetilde{k}\}$ exert effort is indeed the unique equilibrium (being the scheme described above the cheapest one to achieve that goal).

It then follows that the overall wage the principal faces turns out to be the same than the one under the non-contingent scenario, which shows that allowing more flexibility to design contingent contracts (in particular, to consider the menu described above) also vanishes the comparative advantage of the individual-based wage scheme we have considered throughout this paper.

5 Discussion

We have analyzed in this paper a simple model of organization and shown that an optimal management strategy for team production may involve guaranteeing workers a fixed wage, rather than linking wages to collective (besides individual) performance.[15] This implies, in particular, that when the principal faces liquidity constraints (and therefore is forced to link wages to the team's performance) then she is expected to obtain lower benefits than without liquidity constraints. Thus, we provide rationale for the so-called "rich get richer" hypothesis. Our finding can also be interpreted as an argument to endorse individual-based wage schemes rather than team-based wage schemes for the management of teams in which agents differ in their qualification and effort choices. This is in line with some related literature in which it has been suggested that team-based compensation gives rise to problems when workers vary in their ability (e.g., Prendergast, 1999; Meidinger et al., 2003).

Furthermore, we show a deep misalignment between optimal team-based

[15] Our dichotomy is also reminiscent of the so-called make-or-buy decision in the theory of the firm (e.g., Milgrom and Roberts, 1994).

compensation schemes and an impartial treatment of its members. More precisely, in our model, an optimal team-based compensation scheme violates equal treatment of equals. In other words, equally talented (and deserving) agents might well receive different wages and end up with different benefits. Discriminating among equals in a team production model is a feature already obtained by Winter (2004) in the case in which principals only care about making *all* agents exert effort. Nevertheless, Winter (2004) only obtains this feature for the case in which production functions exhibit *increasing returns of scale*, whereas we obtain it here without imposing additional conditions on the production function.[16]

We have also shown that if the team is managed under a profit-sharing plan, less agents are expected to exert effort, which might be considered as an additional advantage of individual-based wage schemes for team production. The lack of success of profit-sharing plans in fostering individual effort, within a context of teams, has been observed, for instance, in medical practices (e.g., Newhouse, 1973) or partnerships in law firms (e.g., Leibowitz and Tollison, 1980).

Our main finding is robust to the extension of our benchmark model in three directions; namely, the existence of intermediate management options (with semi-contingent mechanisms), agents' aversion to risk, and equity constraints. On the other hand, it is not robust to the extension in three other directions. More precisely, one amounts to assume a hierarchical (rather than a flat) organization for the team. This, however, seems to be an unrealistic assumption nowadays, where hierarchies are being challenged from below or are transforming themselves from top-down structures into more horizontal and collaborative ones (e.g., Friedman, 2007). Another amounts to assume the existence of optimistic principals, who might be satisfied with guaranteeing that all agents exerting effort is an equilibrium, but not the only one. The flaw of this option is its inherent strategic uncertainty induced by the existence of multiple equilibria (and, in particular, the equilibrium in which no agent exerts effort).[17] A third one amounts to assume more flexibility in the design of contingent wage schemes upon allowing contracts to offer a menu of options depending on the number of agents exerting effort,

[16] It is worth remarking that Winter (2004) analyzes a different model in which the principal does not observe agents' effort decisions and hence the moral-hazard problem becomes the priority of the analysis.

[17] This feature is somehow reminiscent of the literature on public goods (see, for instance, Ledyard (1995) for an excellent survey) in which voluntary contributions mechanisms, that might yield strategic uncertainty, are considered as a way of combating the free-riding equilibrium.

which might be plausible in some instances, although not always. Altogether, it can be safely argued that even though liquidity constraints can have an important effect in the rise of initial wealth disparities, the more freedom we provide a principal with (either by allowing different team architectures, strategic uncertainty, or more flexibility in the design of contracts) the more she can mitigate their effect.

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