



BANCO CENTRAL DO BRASIL

Working Paper Series **196**

The role of macroeconomic variables in sovereign risk

Marco S. Matsumura and José Valentim Vicente

October, 2009

ISSN 1518-3548
CGC 00.038.166/0001-05

| | | | | | |
|----------------------|----------|--------|------|------|---------|
| Working Paper Series | Brasília | n. 196 | Oct. | 2009 | p. 1-57 |
|----------------------|----------|--------|------|------|---------|

Working Paper Series

Edited by Research Department (Depep) – E-mail: workingpaper@bcb.gov.br

Editor: Benjamin Miranda Tabak – E-mail: benjamin.tabak@bcb.gov.br

Editorial Assistant: Jane Sofia Moita – E-mail: jane.sofia@bcb.gov.br

Head of Research Department: Carlos Hamilton Vasconcelos Araújo – E-mail: carlos.araujo@bcb.gov.br

The Banco Central do Brasil Working Papers are all evaluated in double blind referee process.

Reproduction is permitted only if source is stated as follows: Working Paper n. 196.

Authorized by Mário Mesquita, Deputy Governor for Economic Policy.

General Control of Publications

Banco Central do Brasil

Secre/Surel/Cogiv

SBS – Quadra 3 – Bloco B – Edifício-Sede – 1º andar

Caixa Postal 8.670

70074-900 Brasília – DF – Brazil

Phones: +55 (61) 3414-3710 and 3414-3565

Fax: +55 (61) 3414-3626

E-mail: editor@bcb.gov.br

The views expressed in this work are those of the authors and do not necessarily reflect those of the Banco Central or its members.

Although these Working Papers often represent preliminary work, citation of source is required when used or reproduced.

As opiniões expressas neste trabalho são exclusivamente do(s) autor(es) e não refletem, necessariamente, a visão do Banco Central do Brasil.

Ainda que este artigo represente trabalho preliminar, é requerida a citação da fonte, mesmo quando reproduzido parcialmente.

Consumer Complaints and Public Enquiries Center

Banco Central do Brasil

Secre/Surel/Diate

SBS – Quadra 3 – Bloco B – Edifício-Sede – 2º subsolo

70074-900 Brasília – DF – Brazil

Fax: +55 (61) 3414-2553

Internet: <http://www.bcb.gov.br/?english>

The role of macroeconomic variables in sovereign risk

Marco S. Matsumura* José Valentim Vicente†

The Working Papers should not be reported as representing the views of the Banco Central do Brasil. The views expressed in the papers are those of the author(s) and not necessarily reflect those of the Banco Central do Brasil.

Abstract

We use a dynamic term structure model with default and observable factors to study the interaction between macro variables and the Brazilian sovereign yield curve. We also calculate the default probabilities implied from the estimated model and the impact of macro shocks on those probabilities. Our results indicate that the VIX is the most important macro factor affecting short-term bonds and default probabilities, while the American short-term rate is the most important factor affecting the long-term default probabilities. Regarding the domestic variables, only the slope of the local yield curve presents significant explanatory power for the sovereign rates and default probabilities.

JEL classification: C13, E44, G12.

Keywords: Macro-finance, credit risk, affine term structure models, emerging markets.

*IPEA, Av. Presidente Antônio Carlos 51 17th floor, Rio de Janeiro, Brazil, 55-21-3515-8533. E-mail: marco.matsumura@ipea.gov.br. Corresponding author.

†Central Bank of Brazil, Av. Presidente Vargas 730 7th floor, Rio de Janeiro, Brazil, 55-21-2189-5762. E-mail: jose.valentim@bcb.gov.br.

1 Introduction

Sovereign risk is a subtype of credit risk related to the possibility of a government failing to honor its payment obligations. It is a fundamental component of emerging countries' yield curves. Sovereign risk is also very important for emerging market firms, since the cost of foreign financing typically rises with the country risk. Accordingly, the following questions are of particular interest: What are the factors most affecting the sovereign yield curve? Which variables have greatest impact on default probabilities? This study presents an empirical investigation of these questions by using an affine term structure model with macroeconomic variables and default risk¹.

There are two main approaches in credit risk modeling: structural and reduced form models². While the former provides a link between the probability of default and firms' fundamental variables, the latter relies on the market as the only source of information regarding firms' credit risk structure. Black and Scholes (1973) and Merton (1974) proposed the initial ideas concerning structural models based on options theory. Black and Cox (1976) introduced the basic structural framework in which default occurs the first time the value of the firm's assets crosses a given default barrier. More recently, Leland (1994) extended the Black and Cox (1976) model, providing a significant contribution to the capital structure theory. In his model, the firm's incentive structure determines the default barrier endogenously. That is, default is determined as the result of an optimal decision policy carried out by equity holders.

All the papers cited above deal with the corporate credit risk case. However, the sovereign credit risk differs markedly from corporate risk³. For instance, it is not obvious how to model the incentive structure of a government and its optimal default decision, or what "assets" could be seized upon default. Moreover, post-default negotiating rounds regarding the recovery rate can be very complex and uncertain. Consequently, the use of structural models to assess the default risk of a country is a delicate question. Not

¹In this article, the term "macroeconomic (macro) variable" refers to any observable factor.

²Giesecke (2004) provides a short introductory survey of credit risk models.

³As discussed by Duffie et al. (2003), the main differences are: (i) a sovereign debt investor may not have recourse to a bankruptcy code at the default event. (ii) sovereign default can be a political decision. (iii) the same bond can be renegotiated many times. (iv) it may be difficult to collateralize debt with assets into the country. (v) the government can opt for defaulting on internal or external debt. (vi) in the case of sovereign risk, it is necessary to take into account the role played by key variables such as exchange rates, fiscal dynamics, reserves in strong currency, level of exports and imports, gross domestic product, and inflation.

surprisingly, it is difficult to find studies of sovereign debt pricing based on the structural approach⁴. Therefore, we opt to use reduced models, where the default time is a totally inaccessible stopping time that is triggered by the first jump of a given exogenous intensity process⁵. This means that the default always comes as a “sudden surprise”, which provides more realism to the model. In contrast, within the class of structural models, the evolution of assets usually follows a Brownian diffusion, in which there are no such surprises and the default time is a predictable stopping time.

Lando (1998), and Duffie and Singleton (1999) develop versions of reduced models in which the default risk appears as an additional instantaneous spread in the pricing equation. The spread can be modeled using state factors. In particular, it can be incorporated into the affine framework of Duffie and Kan (1996), a widely used model offering a good compromise between flexibility and numerical tractability⁶. Duffie et al. (2003) extend the reduced model to include the possibility of multiple defaults (or multiple “credit events”, such as restructuring, renegotiation or regime switches). The model is estimated in two steps. First, the risk-free reference curve is estimated. Next, the defaultable sovereign curve is obtained conditional on the first stage estimates. As an illustration, they apply their model to analyze the term structure of credit spreads for bonds issued by the Russian Ministry of Finance (MinFin) over a sample period encompassing the default on domestic Russian GKO bonds in August 1998. They investigate the determinants of the spreads, the degree of integration between different Russian bonds and the correlation between the spreads macroeconomic variables. Another paper applying reduced model to emerging markets is Pan and Singleton (2008), who analyze the sovereign term structures of Mexico, Turkey, and Korea through a dynamic approach.

Nevertheless, Duffie et al. (2003), and Pan and Singleton (2008) use a pure latent variables model. Thereby, the impact of macro factors changes on bond yields can be evaluated only indirectly through, for instance, a regression between observable and unobservable variables. Moreover, in pure latent models, the unobservable factors are abstractions that can, at best, be interpreted as geometric factors summarizing the yield curve movements, as shown by Litterman and Scheinkman (1991).

The modern literature linking the dynamics of the term structure with macro factors starts with Ang and Piazzesi (2003), who propose an ingenious

⁴Exceptions are Xu and Ghezzi (2002) and Moreira and Rocha (2004).

⁵A stopping time is totally inaccessible if it can never be announced by an increasing sequence of predictable stopping times (see Schönbucher, 2003).

⁶An affine model is a multifactor dynamic term structure model, such that the state process X is an affine diffusion, and the short short-term rate is also affine in X

solution to incorporate observable factors in the original framework of affine models. In their model, the macroeconomic factors affect the entire yield curve. However, the interest rates do not affect the macroeconomic factors, which means that monetary policy is ineffective. Similarly to Duffie et al. (2003), they employ a two-step estimation procedure, first determining the macro dynamics and then the latent dynamics conditional on the macro factors. Ang et al. (2007) estimate a dynamic macro-finance model using Markov Chain Monte Carlo (MCMC) technique in a single step procedure. Others studies that combine macro factors and no-arbitrage conditions are Rudebusch and Wu (2004) and Hördal et al. (2008).

Following the advances brought by these previous studies, we examine the impact of macro factors on a defaultable term structure through an affine model similar to that of Ang and Piazzesi (2003). We provide a comparison among a variety of specifications in order to determine the macro factors that most affect credit spreads and default probabilities of an emerging country. We also use impulse response and variance decomposition techniques to analyze the direct influence of observable macro factors on yields and default probabilities.

However, before estimating the parameters, one must choose an identification strategy. Not all parameters of the multifactor affine model can be estimated, since there are transformations of the parameter space preserving the likelihood. When sub-identified, parameters can be arbitrarily rotated, while over-identified specifications may distort the true response of the state variables. Based on the findings of Dai and Singleton (2000), we propose an identification procedure for affine models with macro factors and default⁷.

We select Brazil as the case study. The reason for this choice is that Brazil is one of the most important emerging countries with a rich history of credit events⁸. When using Brazilian data, one must take into account that frequent regime switches have occurred until recently, such as change from very high inflation to a stable economy (July 1994), change from fixed to floating exchange rate in a currency crisis in January 1999, and change of monetary policy to inflation targeting in July 1999. Thus, our sample comprises five and a half years of historical series. This sample size is compatible with that found in other recent academic studies containing data from emerging economies (see, for instance, Pan and Singleton, 2008, and Almeida and Vicente, 2009). Furthermore, following these authors, we de-

⁷Related to our specification analysis there is the work of Pericoli and Taboga (2008), who implement an identification of a default-free affine model with macro factors.

⁸Jointly with India, Russia and China, Brazil is considered as among the fastest growing developing economies in the world. Goldman Sachs refers to these countries as BRICs, an acronym for Brazil, Russia, India and China (see Goldman Sachs, 2007).

cided to employ continuous-time modeling with high-frequency data in order to avoid small-sample biases.

Our main model contains five state variables: one latent factor for the reference default-free curve, one external macro factor, one internal macro factor, and two latent factors for the Brazilian sovereign yield curve. We test the following observable variables: Fed interest rates, VIX (index of implied volatility of options in the Standard & Poor's index), Brazilian Real/US Dollar exchange rates, São Paulo Stock Exchange index (Ibovespa), and Brazilian interest rate swaps. In the estimation stage we follow common practice and use a two-step procedure as implemented by Duffie et al. (2003).

In a nutshell, we contribute to the finance literature in at least two aspects. First, we extend the works of Duffie et al. (2003) and Pan and Singleton (2008) by incorporating macro variables in a dynamic term structure model with default risk. Second, our model allows a full interaction between latent and observable sovereign factors, which in a sense extends the study of Ang and Piazzesi (2003)⁹.

Our main findings can be summarized as follows. First, VIX and Fed rates strongly affect the default probabilities in the short and in the long term, respectively. Second, VIX has a great effect on Brazilian sovereign yields, more than any investigated domestic macro indicator. This result agrees with one of Pan and Singleton's (2008) conclusions who report that VIX has the most explanatory power for Mexican credit default swap (CDS) spreads. Third, among the observable domestic factors only the slope of yield curve presents significant explanatory power of the Brazilian credit risk spread. Finally, a latent factor highly correlated with the level of the Brazilian sovereign curve predicts a substantial fraction of the yield and default probability movements. We also assert that the Brazilian spread is more sensitive to volatility of international markets (measured in our model by VIX) than local conditions. On the other hand, the moderate significance of the domestic yield curve slope indicates that expectations of Brazilian investors play a role in determining the sovereign yield and default probabilities.

The rest of this article is organized as follows. In Section 2 we present the model. Section 3 describes the dataset used. Section 4 details the estimation procedure. Section 5 presents the results of implementing the dynamic models. Section 6 offers concluding remarks. Auxiliary results are contained in the Appendices.

⁹Diebold et al. (2006), using a statistical model, find strong evidence of two-way interaction between latent and macro factors.

2 Affine Model with Default Risk and Macro Factors

Uncertainty in the economy is characterized by a filtered probability space $(\Omega, (\mathcal{F}_t)_{t \geq 0}, \mathcal{F}, \mathbb{P})$ where $(\mathcal{F}_t)_{t \geq 0}$ is a filtration generated by a standard N -dimensional Brownian motion $W^{\mathbb{P}} = (W_1^{\mathbb{P}}, \dots, W_N^{\mathbb{P}})$ defined on $(\Omega, \mathcal{F}, \mathbb{P})$ (see Duffie, 2001). We assume the existence of a pricing measure \mathbb{Q} under which discounted security prices are martingales with respect to $(\mathcal{F}_t)_{t \geq 0}$. The price P^D of a defaultable bond at time t that pays \$1 at maturity time T is given by

$$P^D(t, T) = E_t^{\mathbb{Q}} \left[\mathbf{1}_{[\tau_d > T]} e^{-\int_t^T r_u du} + Z_{\tau_d} \mathbf{1}_{[\tau_d \leq T]} e^{-\int_t^{\tau_d} r_u du} \right], \quad (1)$$

where $\mathbf{1}_A$ is the indicator function of the set A . The first part of the right-hand side of (1) represents what the bondholder receives if the maturity time comes before the default time τ_d , a totally inaccessible stopping time. In case of default, the investor receives the random variable Z_{τ_d} at the default time. Lando (1998), and Duffie and Singleton (1999) prove that if τ_d is *doubly stochastic* with intensity η_t , the recovery upon default is given by $Z_{\tau_d} = (1 - \ell_{\tau_d}) P^D(\tau_d, T)$, where ℓ_t is the loss rate in the market value, and if other technical conditions are satisfied, then

$$P^D(t, T) = E_t^{\mathbb{Q}} \left[\exp \left(- \int_t^T (r_u + s_u) du \right) \right], \quad (2)$$

where $s_t = \ell_t \eta_t$ is the spread due to the possibility of default.

We now briefly explain the concept of doubly stochastic stopping time (for more details, see Schönbucher, 2003 or Duffie, 2001). Define $N(t) = \mathbf{1}_{[\tau_d \leq t]}$ as the associated counting process. It can be shown that $N(t)$ is a submartingale. Applying the Doob-Meyer theorem (see Shiryaev, 1995), we know there exists a predictable, non-decreasing process $C(t)$ called the compensator of $N(t)$. One property of the compensator is to give information about the probabilities of the jump time. The expected marginal increments of the compensator $dC(t)$ are equal to the probability of the default occurring in the next increment of time: $E_t^{\mathbb{Q}} [C(t + \Delta t) - C(t)] = \mathbb{Q} [N(t + \Delta t) - N(t) = 1 | \mathcal{F}_t]$. An intensity process η_t for $N(t)$ exists if it is progressively measurable and non-negative, and $C(t) = \int_0^t \eta(u) du$. Under regularity conditions, it turns out that

$$\eta(t) = \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \mathbb{Q}[\tau_d \leq t + \Delta t | \tau_d > t]. \quad (3)$$

Thus, $\eta(t)$ represents the evolution of the instantaneous probability of defaulting by $t + dt$ if default has not occurred up to t . Finally, τ_d is said to be

doubly stochastic with intensity η if $N(t_2) - N(t_1) | \eta \sim \text{Poisson}\left(\int_{t_1}^{t_2} \eta(u) du\right)$. Therefore, in the reduced model, the default event is essentially given by the first jump of a Poisson process with stochastic intensity.

Our model is within the class of affine models analyzed by Duffie and Kan (1996). The state vector $X_t \in \mathbb{R}^N$ incorporates information about the United States, $X_t^{\text{US}} = (\theta_t^{\text{US}}, M_t^{\text{US}})$, and Brazil, $X_t^{\text{BR}} = (M_t^{\text{BR}}, \theta_t^{\text{BR}})$, that is, $X_t = (\theta_t^{\text{US}}, M_t^{\text{US}}, M_t^{\text{BR}}, \theta_t^{\text{BR}})$, where the variables $\theta_t = (\theta_t^{\text{US}}, \theta_t^{\text{BR}})$ and $M_t = (M_t^{\text{US}}, M_t^{\text{BR}})$ represent latent and observable factors, respectively. In the affine model with default, s_t and r_t are specified as affine functions of the state vector. In other words, we assume that $s_t = \delta_0^s + \delta_1^s \cdot X_t$ and $r_t = \delta_0^r + (\delta_1^{r,\text{US}}, \delta_1^{r,\text{BR}}) \cdot X_t = \delta_0^r + \delta_1^r \cdot X_t$, where $\delta_0^s, \delta_0^r \in \mathbb{R}$ and $\delta_1^s, \delta_1^r \in \mathbb{R}^N$. Then the default-adjusted short-rate process is

$$R_t = r_t + s_t = \delta_0^r + \delta_0^s + (\delta_1^r + \delta_1^s) \cdot X_t = \delta_0 + \delta_1 \cdot X_t. \quad (4)$$

The dynamics of the state variables is given by:

$$\begin{aligned} dX_t &= \begin{bmatrix} d\theta_t^{\text{US}} \\ dM_t^{\text{US}} \\ dM_t^{\text{BR}} \\ d\theta_t^{\text{BR}} \end{bmatrix} = \\ & \begin{bmatrix} K_{\theta,\theta}^{\text{US,US}} & 0 & 0 & 0 \\ K_{M,\theta}^{\text{US,US}} & K_{M,M}^{\text{US,US}} & 0 & 0 \\ K_{M,\theta}^{\text{BR,US}} & K_{M,M}^{\text{BR,US}} & K_{M,M}^{\text{BR,BR}} & K_{M,\theta}^{\text{BR,BR}} \\ K_{\theta,\theta}^{\text{BR,US}} & K_{\theta,M}^{\text{BR,US}} & K_{\theta,M}^{\text{BR,BR}} & K_{\theta,\theta}^{\text{BR,BR}} \end{bmatrix} \left(\begin{bmatrix} \xi_{\theta}^{\text{US}} \\ \xi_M^{\text{US}} \\ \xi_M^{\text{BR}} \\ \xi_{\theta}^{\text{BR}} \end{bmatrix} - \begin{bmatrix} \theta_t^{\text{US}} \\ M_t^{\text{US}} \\ M_t^{\text{BR}} \\ \theta_t^{\text{BR}} \end{bmatrix} \right) dt \\ & + \begin{bmatrix} \Sigma_{\theta,\theta}^{\text{US,US}} & 0 & 0 & 0 \\ \Sigma_{M,\theta}^{\text{US,US}} & \Sigma_{M,M}^{\text{US,US}} & 0 & 0 \\ \Sigma_{M,\theta}^{\text{BR,US}} & \Sigma_{M,M}^{\text{BR,US}} & \Sigma_{M,M}^{\text{BR,BR}} & 0 \\ \Sigma_{\theta,\theta}^{\text{BR,US}} & \Sigma_{\theta,M}^{\text{BR,US}} & \Sigma_{\theta,M}^{\text{BR,BR}} & \Sigma_{\theta,\theta}^{\text{BR,BR}} \end{bmatrix} \begin{bmatrix} dW_{\theta}^{\mathbb{P},\text{US}}(t) \\ dW_M^{\mathbb{P},\text{US}}(t) \\ dW_M^{\mathbb{P},\text{BR}}(t) \\ dW_{\theta}^{\mathbb{P},\text{BR}}(t) \end{bmatrix} \\ & = K(\xi - X_t)dt + \Sigma dW^{\mathbb{P}}(t), \end{aligned} \quad (5)$$

where K and Σ are $N \times N$ matrices and $\xi \in \mathbb{R}^N$. That is, X follows an affine process with constant volatility. Similar to Duffie et al. (2003), we set a “block-triangular” form for the dynamics of the state variables. The zeros above the main diagonal of Σ and K imply that the American yield curve factors affect the Brazilian yield curve factors, but not vice versa. Furthermore, unlike Ang and Piazzesi (2003), we allow the macro and yield factors to interact fully.

The connection between martingale probability measure \mathbb{Q} and objective probability measure \mathbb{P} is given by Girsanov’s Theorem with a time-varying risk premium:

$$dW_t^{\mathbb{P}} = dW_t^{\mathbb{Q}} - (\lambda_0 + \lambda_1 X_t)dt, \quad (6)$$

where $\lambda_0 = (\lambda_0^{\text{US}}, \lambda_0^{\text{BR}}) \in \mathbb{R}^N$ and λ_1 is $N \times N$ matrix given by

$$\lambda_1 = \begin{bmatrix} \lambda_1^{\text{US,US}} & 0 \\ \lambda_1^{\text{BR,US}} & \lambda_1^{\text{BR,BR}} \end{bmatrix}$$

As a result, the price P^{BR} of a defaultable bond is exponential affine, that is, $P^{\text{BR}}(t, T) = \exp(a^{\text{BR}}(\tau) + b^{\text{BR}}(\tau)X_t)$, where $\tau = T - t$, and a^{BR} and b^{BR} solve a system of Riccati differential equations:

$$\begin{aligned} b^{\text{BR}}(\tau)' &= -(\delta_1^r + \delta_1^s) - K^* b^{\text{BR}}(\tau) \\ a^{\text{BR}}(\tau)' &= -(\delta_0^r + \delta_0^s) + \xi^* K^* b^{\text{BR}}(\tau) + \frac{1}{2} b^{\text{BR}}(\tau)' \Sigma \Sigma' b^{\text{BR}}(\tau), \end{aligned} \quad (7)$$

with $K^* = K + \Sigma \lambda_1$ and $\xi^* = K^{*-1}(K\xi - \Sigma \lambda_0)$. An explicit solution for this system of differential equations exists only in some special cases, such as diagonal K . However, the Runge-Kutta method provides accurate numerical approximations. Thus, the yield at time t with time to maturity τ is given by

$$Y_t^{\text{BR}}(\tau) = A^{\text{BR}}(\tau) + B_{\theta}^{\text{BR,US}}(\tau) \theta_t^{\text{US}} + \quad (8)$$

$$B_M^{\text{BR,US}}(\tau) M_t^{\text{US}} + B_M^{\text{BR,BR}}(\tau) M_t^{\text{BR}} + B_{\theta}^{\text{BR,BR}}(\tau) \theta_t^{\text{BR}}.$$

If the loss given default rate is constant, i.e. $\ell_t = \ell$ for all t , then the term structure of default probabilities is given by (see Schönbucher, 2003):

$$\Pr(t, \tau) = 1 - E_t^{\mathbb{P}} \left[\exp \left(- \int_t^{t+\tau} \frac{s_u}{\ell} du \right) \right], \quad (9)$$

which can be calculated similarly to the conditional expectation contained in the pricing equation, with the objective measure replacing the martingale measure. It turns out that $\Pr(t, \tau) = 1 - \exp(a^{\text{Pr}}(\tau) + b^{\text{Pr}}(\tau)X_t)$, where a^{Pr} and b^{Pr} are again solutions of Riccati differential equations:

$$\begin{aligned} b^{\text{Pr}}(\tau)' &= -\delta_1^s / \ell - K' b^{\text{Pr}}(\tau), \\ a^{\text{Pr}}(\tau)' &= -\delta_0^s / \ell + \xi' K' b^{\text{Pr}}(\tau) + \frac{1}{2} b^{\text{Pr}}(\tau)' \Sigma \Sigma' b^{\text{Pr}}(\tau). \end{aligned} \quad (10)$$

We close this section with two remarks. First, the reduced model can be replaced by a standard term structure model with macro factors: it suffices to let the US factors take the role of macro factors for the defaultable bonds. However, the interpretation of the spread as the instantaneous expected loss

and the computation of model implied default probabilities are no longer possible. Second, all the models in this article are in the class of Gaussian models, the simplest specification of the affine family. The inclusion of macro variables and default substantially complicates the model and its estimation. Therefore, we follow the standard macro-finance approach and decide not to use a model with stochastic volatility¹⁰. However, note that macro factors such as the VIX volatility can approximately play the role of stochastic volatility of the non-Gaussian affine models. Furthermore, models with constant volatility are the best choice matching some stylized facts (as shown, for instance, by Duffee, 2002, and Dai and Singleton, 2002) and to describe corporate CDS spreads (see Berndt et al., 2004).

3 Data

Our sample consists of a daily series of the following variables: (i) constant maturity zero-coupon term structure of US yields provided by the Federal Reserve (Fed); (ii) constant maturity zero-coupon term structure of Brazilian sovereign yields constructed by Bloomberg¹¹; (iii) the implied volatility of S&P 500 index options measured by the Chicago Board Options Exchange Volatility Index - VIX; (iv) Brazilian Real/US Dollar exchange rate, (v) São Paulo Stock Exchange index - Ibovespa¹², (vi) Brazilian domestic zero-coupon yields extracted from ID *x* Pre swaps obtained from Brazilian Mercantile and Futures Exchange (BM&F)¹³. The first two data sets are used as basic yields and the others play the role of observed (macro) factors in our model.

The sample begins on February 17, 1999, and ends on September 15, 2004, with a total of 1320 days. The sample starts one month after the change of the exchange rate regime from fixed to floating in January 1999, forced by a devaluation crisis. The maturities of the US and Brazilian sovereign yields are the same, namely 3 and 6 months, 1, 2, 3, 5, 7, 10, and 20 years, while the maturities of the Brazilian domestic yields are 1, 3, and 36 months. Figure 1 depicts the US and Brazilian sovereign yields. Figure 2 shows the observed variables. Note that the American yield curve is almost flat in the beginning

¹⁰An exception of this common practice is Spencer (2008), who generalizes the homoscedastic macro-finance model by allowing for stochastic volatility process.

¹¹The dataset of sovereign yields provided by Bloomberg is extracted from Brazilian Global bonds.

¹²Ibovespa is the main Brazilian stock market index.

¹³The ID rate is the average one-day interbank borrowing/lending rate, calculated by CETIP - OTC Clearing House every business day. The ID rate is expressed in effective rate per annum, based on 252 day-year. For more information about the ID rate and ID *x* Pre swaps, see the websites <http://www.cetip.com.br> and <http://www.bmf.com.br>.

of the sample. After January 2001, short-yields decline over time and the shape of the term structure changes to upward sloping. In end of 2002, there is a stress movement in the Brazilian market due to a presidential succession process in which the candidate of the opposition won the election.

4 Estimation

The parameters are estimated via the maximum likelihood method. Although it is possible to make one-step estimations of the US and Brazilian sovereign yield curves, it is computationally more interesting to work with a simpler technique using a two-step procedure, as in Duffie et al. (2003). We use the US term structure as the reference curve (default-free curve) for our analysis. In the first step we estimate the reference curve using only latent factors. Then, conditional on the parameters and state vector of the US curve, we estimated the Brazilian sovereign yield curve.

We now describe the procedure adopted for a model with macro variables and default. The estimation of US parameters is a particular case of this general framework. By stacking the parameters and state variables, the yield of a defaultable bond (Equation 8) can be written as

$$Y_t^{\text{BR}}(\tau) = A^{\text{BR}}(\tau) + B^{\text{BR}}(\tau)X_t, \quad (11)$$

where the dynamics of X_t is given by Equation 5.

The likelihood is the joint probability density function of the sequence of observed Brazilian sovereign yields $Y_t^{\text{BR}} = (Y_{t_1}^{\text{BR}}, \dots, Y_{t_n}^{\text{BR}})$ and macro factors M_t . It is possible to show that the transition density of $X_{t_i}|X_{t_{i-1}}$, denoted by f_X , is normally distributed with mean $\mu_i^{\text{BR}} = e^{-K(t_i-t_{i-1})}X_{t_{i-1}} + (I_N - e^{-K(t_i-t_{i-1})})\xi$ and variance $(\sigma_i^{\text{BR}})^2 = \int_{t_{i-1}}^{t_i} e^{-K(t_i-u)}\Sigma\Sigma'e^{-K(t_i-u)'}du$ (see, for instance, Fackler, 2000).

Suppose first the vectors θ_t^{BR} and Y_t^{BR} have the same dimension, that is, we observe as many yields as latent variables. Then we can invert a linear equation and find the unobserved factors θ_t^{BR} as a function of yields Y_t^{BR} and observable factors M_t^{BR} . Using change of variables, the log-likelihood function can be written as

$$\mathcal{L}(Y_t, M_t, \Psi) = \sum_{t=2}^H \log f_X(X_t|X_{t-1}, \Psi) + (H-1)\log \det |Jac|,$$

where H is the sample size, $\Psi = (\delta_0, \delta_1, K, \xi, \Sigma, \lambda_0, \lambda_1)$ is a vector stacking the model parameters, and the Jacobian matrix is

$$Jac = \bar{B}^{\text{BR}}(\tau_1, \dots, \tau_{N^{\text{BR}}}) = \begin{bmatrix} B^{\text{BR}}(\tau_1) \\ \vdots \\ B^{\text{BR}}(\tau_{N^{\text{BR}}}) \end{bmatrix}, \quad (12)$$

where $\tau_1, \dots, \tau_{N^{\text{BR}}}$ are the time to maturities of the observable Brazilian yields.

If we want to use additional yields, direct inversion is not possible. This is known as “stochastic singularity”. One solution is to follow Chen and Scott (1993), and add measurement errors to the extra yields. Let $N_{\text{obs}}^{\text{BR}}$ be the number of Brazilian sovereign yields observed on each day, $N_{\text{obs}}^{\text{BR}} > N^{\text{BR}}$ where N^{BR} is the size of X_t^{BR} . We select N^{BR} yields to be priced without error. The other $(N_{\text{obs}}^{\text{BR}} - N^{\text{BR}})$ are priced with independent normal measurement errors. Therefore, the log-likelihood function is

$$\mathcal{L}(Y_t, M_t, \Psi) = \sum_{t=2}^H \log f_X(X_t | X_{t-1}, \Psi) + (H-1) \log \det |Jac| + \frac{1}{2} \sum_{t=2}^H u_t' \Omega^{-1} u_t,$$

where u_t is the vector of yield measurement errors and Ω represents the covariance matrix for u_t , estimated using the sample covariance matrix of the u_t 's implied by the extracted state vector, and $Jac = \bar{B}^{\text{BR}}(\tau_1, \dots, \tau_{N_{\text{obs}}^{\text{BR}}})$.

In order to complete the estimation procedure, it is necessary to identify the model. If the model is sub-identified then there are more than one set of parameters that generate the same likelihood. Therefore, not all parameters can be estimated. On the other hand, over-identified models produce sub-optimal results that may distort the impulse response functions. However, identification of parameters in a state-space system is tricky. In Appendix A we provide identification strategies for some specifications of our model, based on the results of Dai and Singleton (2000).

5 Results

In this section we analyze the results of three different specifications of our model estimated by the maximum likelihood method described in Section 4. We begin with a simple macro-to-yield without default specification. In order to avoid local maxima, many trial numerical optimizations are performed using the Nelder-Mead Simplex algorithm until stable results are obtained. Then, taking advantage of these results, we select starting vectors for the estimation of two higher dimensional models with default. After that, other independent trial maximization starting from random vectors are performed.

Finally we choose the best results. Although this procedure may be path-dependent, the “curse of dimensionality” does not allow the use of a complete grid of random starting points, as would be desirable.

5.1 Macro-to-yield without default

The simplest specification of our model is characterized by a macro-to-yield dynamics without default. It is exactly the model of Ang and Piazzesi (2003) applied to the Brazilian yield curve. The absence of default implies that American latent factors (θ^{US}) are unnecessary. In a macro-to-yield model the observable factors affect the latent factors but not vice versa. This means that $K_{M,\theta}^{\text{BR,BR}}$ is a matrix of zeros.

The macro-to-yield without default specification presents three state variables, $X = (M, \theta_1^{\text{BR}}, \theta_2^{\text{BR}})$. It serves to indicate the relevant macro factors for the sovereign yield curve, which are then selected for use in the other models. To extract Brazilian latent factors, we set the 3-month and 5-year sovereign yields to be flawless. Nine versions are estimated, each having a different observed factor M : (1) VIX; (2) logarithm of the Brazilian Real/US Dollar exchange rate (LEX); (3) logarithm of the Ibovespa (LIBOV); (4) BM&F 1-month yield (B1m); (5) BM&F 3-year yield (B3y); (6) BM&F slope (Bsl) = B3y - B1m; (7) Fed 1-month yield (F1m); (8) Fed 10-years yield (F10y), and (9) Fed slope (Fsl) = F10y - F1m.

Table 1 presents the log-likelihood divided by the number of observations (\mathcal{L}/H) and the mean (for the nine maturities) of the absolute measurement errors in basis points (MAE) for all specifications. These measures can be used to evaluate the different versions of a model. Table 1 also presents the correlations between factor 1 (θ_1^{BR}) and the slope of the Brazilian sovereign term structure ($\rho_{1,s}$) and between factor 2 (θ_2^{BR}) and the level of the Brazilian sovereign term structure ($\rho_{2,l}$). The likelihood does not vary significantly, but the specifications that included US rates show slightly higher values. The mean absolute measurement error is around 60 basis points. The latent factor θ_2^{BR} represents the level, since it is highly correlated with this factor in all cases, while θ_1^{BR} can be interpreted as the slope due to its positive correlation with the slope of the yield curve.

In order to measure the relative contributions of the macro and latent factors to forecast variances we perform variance decompositions¹⁴. Table 2 presents the proportion of the 1-month and 9-month ahead forecast variance of the {3m, 3y, 20y}-yields attributable to each observable factor used in

¹⁴Appendix B presents some mathematical details about the variance decomposition of our model.

each of the nine versions. This provides a comparison of the importance of the different macro variables for the sovereign yield curve by showing the macro participation in the variance of the yields one and nine months after the shock. The order of the impact can be summarized as follows: VIX and BM&F slope present the largest effect, accounting for up to 69% and 79% of the 20-year yields nine months after the shock. Although still significant, the contribution of Brazilian Real/US Dollar exchange rate, 10-years Fed yield, Fed slope, and Ibovespa are much smaller. Finally, BM&F 1-month and 3-years yield, and Fed 1-month yield show negligible effect.

5.2 Macro-to-yield with default

In this subsection, we introduce default risk into the previous specification. Again, we assume that the state variables follow a macro-to-yield dynamics. There is a need for another latent factor besides the macro factor and the two Brazilian latent factors. The job of this new factor is to capture the US term structure, which represents the reference curve. The parameters corresponding to the US latent factor are estimated in a first step, while the other parameters are estimated conditional on the first step. The American latent factor is obtained from the yield with 3 months maturity while the Brazilian latent factors are obtained from the sovereign yields with maturities of three months and five years.

In view of the results of the previous subsection, we divide the observable factors into three groups. The first one is composed of the VIX and BM&F slope which are the factors that have the largest impact on the yields. The intermediate group consists of the Brazilian Real/US Dollar exchange rate, 10-year Fed yield, Fed slope, and Ibovespa. The third group presents little effect on yields, being formed of BM&F 1-month and 3-year yields, and Fed 1-month yield. In order to understand the impact of macro variables on the yields in a model with default, we use both factors of the first group, one factor of the second group (Fed slope), and one factor of the third group (BM&F 3-year yield)¹⁵.

Table 3 summarizes the results of some versions of the macro-to-yield with default model. It shows the likelihood, correlations and measurement errors of the yields of each specification. The first column refers to the *yields only* model (y.o.) in which only latent factors are used. The others are macro-to-yield models with VIX, BM&F slope, Fed slope, and BM&F 3-year yield as observable factors. The inclusion of the US reference curve produces

¹⁵Models with other observable factors from the second and third groups were also tested, providing similar qualitative results.

a gain in likelihood and in fit, because the measurement errors are lower. The latent factor θ_2 remains highly linked to the level of the sovereign yields.

Table 4 presents the variance decomposition of the {3m, 3y, 20y}-yield for one and nine months ahead. We see that the VIX is still very important, contributing up to 70% of the 20-year yield variation. Other variables accounted for less, but still some effect can be attributed to them. Furthermore, in the y.o. version the US factor seems to be insignificant.

We also calculate the variance decompositions of the logarithm of the default probabilities, which can be seen in Table 5. All results presented in this paper are obtained using a fixed loss given default $\ell = 50\%$. This particular choice is, of course, arbitrary, however there is empirical evidence that the mean of the loss rate is around this value (see, for instance, Moody's, 2008)¹⁶. The VIX is responsible for the greatest effect, especially in the short-term. According to the model, in the 1- and 9-month horizon, VIX accounts, respectively, for 54% and 61% of the 3-month default probability. The BM&F and Fed slopes and BM&F 3-year yield explain 5%, 4% and 8% for 1-month ahead, and 9%, 18% and 25% for 9-month ahead, respectively, of the 3-month default probability. On the other hand, the Fed slope has the highest explanatory power for long-term default probability among the macro factors.

5.3 Bilateral models

In this subsection we present our main model. It has one American latent factor, one American macro factor (VIX), one Brazilian macro factor and two Brazilian latent factors. The Brazilian macro factor has a bilateral interaction with the Brazilian sovereign factors, that is, the macro factors and the sovereign yield curves fully interact. This means that $K_{M,\theta}^{\text{BR,BR}} \neq 0$. Once more, the American latent factor is obtained from the yield with maturity of three months while Brazilian latent factors are extracted considering that sovereign yields with maturities of three months and five years are priced without error.

We fix VIX as the American macro factor since it presents the best explanatory power for the simpler models analyzed in the previous subsections. We test four specifications, which only differ with respect to the Brazilian macro factor. The first specification takes the BM&F slope as the Brazilian macro factor. This is a very natural choice because this slope is the observable Brazilian factor that best explains the yield variations according to the

¹⁶In order to verify the sensitivity of the results to the loss rate, we tested other values ($\ell = 25\%$ and $\ell = 75\%$) in the macro-to-yield with default model. From a qualitative point of view the results were very similar.

macro-to-yields models. The second use the logarithm of the Ibovespa in US Dollars. This variable combines in single factor the information of two sources of uncertainty that present fairly good explanatory power in the macro-to-yield without default framework. Finally, although Brazilian domestic yields present little effect, we consider the 3-month and 3-year Brazilian yields as domestic factors just to implement a robustness test.

Table 6 contains statistical measures of some versions of the bilateral model. Their likelihoods have increased in relation to the previous models, which indicates that the second macro factor and the bilateral dynamics add information and improve the in-sample fit, with the specification containing the Ibovespa presenting slightly higher likelihood. Also, the mean measurement errors of yields decreased to about 50 basis points. The unobservable factor θ_2 can still be interpreted as the level of the sovereign curve, but θ_1 is in some cases uncorrelated to the slope.

Table 7 reports the variance decomposition of {1m, 3y, 20y}-yields for forecast horizons of one and nine months ahead. In line with the preliminary models, the VIX is again the most important macro factor influencing the yields. The effect is stronger on the long end of the curve. Among the domestic variables, only the BM&F slope presents significant explanatory power. Note that the latent factor related with the level of the sovereign curve is responsible for a large amount of yield variations. This suggests the existence of idiosyncratic sources of uncertainty in the sovereign yield curve that are not explained by the observable factors used in our model. This result is in agreement with the findings of Ang and Piazzesi (2003) and Diebold et al. (2006).

Table 8 presents the variance decomposition of the default probabilities. We now analyze in more details the 9-month horizon decomposition, since in this case the effect of the initial condition is attenuated. Note that in all specifications, the US latent factor (approximately the Fed short rate) shows almost no effect on short-term default probabilities. However, for the long-term (20 years), it is the principal factor, explaining around 80% of changes of implied default probabilities nine months ahead. The effect of the VIX is smaller over the long-term, but about 50% of changes in implied short-term default probabilities are attributable to changes in this observable factor. Among the domestic factors, only the slope of the Brazilian local term structure has a relatively important effect, accounting for 11% of changes in implied short-term default probabilities. Thus, we can conclude that, given our model and sample, the domestic rates, and also the Ibovespa are not relevant sources driving default probability movements.

Figure 3 compares the evolution of the 1-year survival probabilities (one minus default probabilities) over the sample period. It can be seen that

changing the domestic macro factor does not significantly alter the probabilities. Observe that all versions capture the Brazilian electoral crisis in the second half of 2002, with the y.o. model having the largest impact on survival probability. The 1-year ahead survival probabilities fell from an average of 85% to around 70%, recovering later to around 90%.

In order to gauge the response of yields due to an unexpected change in state variables, we calculate impulse response functions¹⁷. Figures 4, 5 and 6 show the effect of a shock to US latent factor (θ_1^{US}), VIX and observable domestic factors (BM&F yields and slope and Ibovespa in US Dollars), respectively, on the Brazilian {3m, 3y, 20y}-yields up to 18-months after the shock. The size of the shock is one standard deviation of a monthly variation of a state variable. In the next three months after a shock on VIX, yields rise about 1% and then fall. Changes in either the domestic short or long rate do not result in changes of the sovereign yields. The same is true for the domestic stock exchange index (Ibovespa). However, a positive BM&F slope shock causes an increase in the yields. This may indicate a change of expectations of a future rise in inflation.

We now turn to survival probabilities. Figures 7, 8, 9 show the impact of a one deviation increase of a monthly variation of the US latent factor, VIX and observable domestic factors, respectively, on the survival probabilities in the next three months, three years and twenty years. It shows that the survival probability falls by up to 4% in relative terms due to a shock in the Fed short rate. An increase in VIX also decreases the survival probability about 1.5% in relative terms. Among the domestic factors, only the BM&F slope has some impact, decreasing the long-term survival probability by about 0.7% in relative terms.

6 Conclusion

We proposed a model that combines an affine yield dynamics with macro factors and credit risk. The model was estimated in two steps using the US and Brazilian sovereign yield curves. The credit spreads, the macro factors and the US yield curve have contemporaneous and lagged interaction. We were able to test how selected domestic and external macro factors such as the Brazilian Real/US Dollar exchange rate, VIX (volatility index of S&P), Ibovespa (São Paulo stock exchange index) and domestic yield curve influence the spreads and default probabilities. The model was identified before making restrictions motivated by economic assumptions. Our findings indi-

¹⁷Appendix B presents some results concerning the impulse response functions applied to our model.

cate that the VIX and US yield curve are the most important factors driving the Brazilian sovereign term structure and default probabilities. This result is consistent with the fact that credit risk premia of sovereign bond are highly correlated with the US economic conditions. The VIX has a high impact on 20-year bond yields and on short-term default probabilities, while the fed fund rate has high explanatory power on the long-term default probabilities. Among the domestic factors, only the slope of the local yield curve shows a significant effect on the Brazilian credit spread. However, a significant portion of variations in yields and default probabilities are explained by an unobservable factor highly correlated with the level of the Brazilian sovereign curve. Due to lack of an extensive historical dataset, we estimated a continuous-time version with daily observations, which limited the choices of macro variables. Future work can test monthly models, allowing the use of important variables such as Central Bank reserves, real activity and inflation.

Appendix A - Model identification

Here, we show how to identify the parameters of a Gaussian affine model with macro factors and credit spreads. This approach is based on the study of Dai and Singleton (2000).

First we consider the default-free case. Suppose there are p macro variables M and q latent variables θ . The vector $X = (M, \theta)$ follows a Gaussian affine dynamics:

$$\begin{aligned} dX_t = \begin{bmatrix} dM_t \\ d\theta_t \end{bmatrix} &= \begin{bmatrix} K_{M,M} & K_{M,\theta} \\ K_{\theta,M} & K_{\theta,\theta} \end{bmatrix} \left(\begin{bmatrix} \xi_M \\ \xi_\theta \end{bmatrix} - \begin{bmatrix} M_t \\ \theta_t \end{bmatrix} \right) dt \\ &+ \begin{bmatrix} \Sigma_{M,M} & \Sigma_{M,\theta} \\ \Sigma_{\theta,M} & \Sigma_{\theta,\theta} \end{bmatrix} \begin{bmatrix} dW_M^{\mathbb{P}}(t) \\ dW_\theta^{\mathbb{P}}(t) \end{bmatrix} = K(\xi - X_t)dt + \Sigma dW^{\mathbb{P}}(t). \end{aligned} \quad (13)$$

The instantaneous short-term rate is given by $r_t = \delta_0 + \delta_1 \cdot X_t$ while the market price of risk obeys Equation 6. Hence, the dynamics of X in the risk-neutral measure is $dX = K^*(\xi^* - X_t)dt + \Sigma dW^{\mathbb{Q}}(t)$ and the yield curve is an affine function of X , $Y_t(\tau) = A(\tau) + B^M(\tau)M_t + B_t^\theta(\tau)\theta = A(\tau) + B(\tau)X_t$. The parameter vector is denoted by $\Psi = (\delta_0, \delta_1, K, \xi, \lambda_0, \lambda_1, \Sigma)$.

Some of the above parameters must be arbitrarily fixed, otherwise there are multiple solutions to the estimation problem since we can define operators that preserve the likelihood as shown below.

Let $L \in \mathbb{R}^{(p+q) \times (p+q)}$ be a non-singular matrix and $v \in \mathbb{R}^{p+q}$ a vector such that

$$L = \begin{pmatrix} \mathbf{I} & 0 \\ \alpha & \beta \end{pmatrix} \text{ and } v = \begin{pmatrix} 0 \\ v_\theta \end{pmatrix},$$

where $\mathbf{I} \in \mathbb{R}^{p \times p}$ is the identity matrix, $\alpha \in \mathbb{R}^{q \times p}$, $\beta \in \mathbb{R}^{q \times q}$, and $v_\theta \in \mathbb{R}^q$. Consider the following maps:

$$\begin{aligned} T_{L,v}\{\Psi, X\} &= \\ &\{(\delta_0 - \delta_1' L^{-1}v, (L')^{-1}\delta_1, LKL^{-1}, v + L\xi, \lambda_0 - \lambda_1 L^{-1}v, \lambda_1 L^{-1}, L\Sigma), LX + v\} \end{aligned} \quad (14)$$

and

$$T_O\{\Psi, X\} = \{(\delta_0, \delta_1, K, \xi, \lambda_0, \lambda_1, \Sigma O'), X\}, \quad (15)$$

where $O \in \mathbb{R}^{(p+q) \times (p+q)}$ is a rotation matrix.

Proposition 1 *The operators $T_{L,v}$ and T_O preserve the likelihood of the affine model defined above under the Chen-Scott (1993) estimation procedure.*

Proof

The log-likelihood \mathcal{L} of the affine model under the Chen-Scott (1993) inversion is

$$\begin{aligned}
\mathcal{L}(\Psi, X) &= \log f_Y(Y_{t_1}, \dots, Y_{t_H} | \Psi, X) = \\
&\log f_X(X_{t_1}, \dots, X_{t_H} | \Psi) + \log f_u(u_{t_1}, \dots, u_{t_H}) + \log |\det Jac|^{H-1} = \\
&(H-1) \log |\det \bar{B}^\theta| + \sum_{t=2}^H \log f_{X_t | X_{t-1}}(X_t | \Psi) + \log f_u(u_t) = \\
&(H-1) \log |\det \bar{B}^\theta| - \frac{1}{2}(H-1) \log \det \left[\Delta t (e^{-K\Delta t}) \Sigma \Sigma' (e^{-K\Delta t})' \right] \\
&+ \sum_{t=2}^H \log f_u(u_t) - \frac{1}{2} \sum_{t=2}^H (X_t - \mu)' \left[\Delta t (e^{-K\Delta t}) \Sigma \Sigma' (e^{-K\Delta t})' \right]^{-1} (X_t - \mu),
\end{aligned}$$

where $\mu = e^{-K\Delta t}\xi + (1 - e^{-K\Delta t})X_{t-1}$, $\Delta t = t_i - t_{i-1} \forall i$, H is the sample size, and $\bar{B}^\theta(\cdot)$ is evaluated at the time to maturities of yields without measurement errors (see Equation 12).

We begin by proving that $\mathcal{L}(\Psi, X) = \mathcal{L}(T_{L,v}(\Psi, X))$. The strategy of the proof is to analyze what happens with each of the four terms of the log-likelihood when the operator $T_{L,v}$ is applied. First, note that the expression under the last summation symbol is preserved. The transformation of μ is

$$\begin{aligned}
\mu(T_{L,v}(\Psi, X)) &= e^{-LKL^{-1}\Delta t}L\xi + \left(1 - e^{-LKL^{-1}\Delta t}\right)LX_{t-1} = \\
&Le^{-K\Delta t}L^{-1}L\xi + (1 - Le^{-K\Delta t}L^{-1})LX_{t-1} = L\mu.
\end{aligned}$$

Then, applying $T_{L,v}$ on the last summation expression of the log-likelihood, we have

$$\begin{aligned}
&(LX_t - L\mu)' \left[\left(e^{-LKL^{-1}\Delta t}L\Sigma\sqrt{\Delta t} \right) \left(e^{-LKL^{-1}\Delta t}L\Sigma\sqrt{\Delta t} \right)' \right]^{-1} (LX_t - L\mu) \\
&= (X_t - \mu)' L' \left[\left(Le^{-K\Delta t}L^{-1}L\Sigma\sqrt{\Delta t} \right) \left(Le^{-K\Delta t}L^{-1}L\Sigma\sqrt{\Delta t} \right)' \right]^{-1} L(X_t - \mu) \\
&= (X_t - \mu)' L' \left[L'^{-1} \left(e^{-K\Delta t}\Sigma\sqrt{\Delta t} \right)'^{-1} \left(e^{-K\Delta t}\Sigma\sqrt{\Delta t} \right)^{-1} L^{-1} \right] L(X_t - \mu) \\
&= (X_t - \mu)' \left[\left(e^{-K\Delta t}\Sigma\sqrt{\Delta t} \right)'^{-1} \left(e^{-K\Delta t}\Sigma\sqrt{\Delta t} \right)^{-1} \right] (X_t - \mu).
\end{aligned}$$

The second term of the log-likelihood changes to

$$\begin{aligned}
& -\frac{1}{2}(H-1)\log \det \left[\left(e^{-LKL^{-1}\Delta t} L \Sigma \sqrt{\Delta t} \right) \left(e^{-LKL^{-1}\Delta t} L \Sigma \sqrt{\Delta t} \right)' \right] = \\
& -\frac{1}{2}(H-1) \left\{ \log \det \left[\left(e^{-K\Delta t} \Sigma \sqrt{\Delta t} \right) \left(e^{-K\Delta t} \Sigma \sqrt{\Delta t} \right)' \right] + 2\log \det L \right\} = \\
& -\frac{1}{2}(H-1) \left[\log \det \left[\left(e^{-K\Delta t} \Sigma \sqrt{\Delta t} \right) \left(e^{-K\Delta t} \Sigma \sqrt{\Delta t} \right)' \right] - (H-1)\log \det L \right].
\end{aligned} \tag{16}$$

It is easy to see that

$$\begin{aligned}
(H-1)\log |\det \bar{B}^\theta| (T_{L,v}(\Psi, X)) &= (H-1)\log |\det \beta^{-1} \bar{B}^\theta| \\
&= (H-1)\log |\det \bar{B}^\theta| + (H-1)\log |\det \beta^{-1}|.
\end{aligned}$$

Since $\det L = \det \beta$, the last term that appeared in (16) cancels out with the last term in the expression above.

Moreover, it is also easy to see that u_t does not change under the transformation $T_{L,v}$.

Finally, $\mathcal{L}(\Psi, X) = \mathcal{L}(T_O(\Psi, X))$ since the only expression affected by the rotation is preserved:

$$\begin{aligned}
& \left[\left(e^{-K\Delta t} \Sigma O' \sqrt{\Delta t} \right) \left(e^{-K\Delta t} \Sigma O' \sqrt{\Delta t} \right)' \right]^{-1} \\
&= \left[\left(e^{-K\Delta t} \Sigma \sqrt{\Delta t} \right) \left(e^{-K\Delta t} \Sigma \sqrt{\Delta t} \right)' \right]^{-1}.
\end{aligned}$$

□

Therefore, there are infinite parameter vectors with the same likelihood. Hence, before estimation through the maximum likelihood method, some parameters must be fixed. On the other hand, the imposition of over-identifying restrictions may produce sub-optimal results that distort the impulse response functions. The model can be considered identified if all the degrees of freedom of the model, which are given by α , β , v_θ and O , are eliminated.

Note that v_θ can always be used to set $\xi_\theta = 0$. In addition, the rotation O implies that Σ must be a triangular matrix for a given state vector order. Hence, we choose $\Sigma_{\theta,\theta}$ and $\Sigma_{M,M}$ to be lower triangular and $\Sigma_{M,\theta} = 0$. Finally, α and β can be set so that $\Sigma_{\theta,\theta} = \mathbf{I}$, $\Sigma_{M,\theta} = 0$, and $K_{\theta,\theta}$ is lower triangular. This completes the identification of the default-free case.

We now turn to the case with default. Formally speaking, the reduced credit risk model of Duffie and Singleton (1999) is simply a higher-dimensional affine model and the same identification procedure can be applied. There are, however, two subtleties involved.

The first is that there are natural restrictions that can be placed to the default model coming from economic considerations. For instance, we have considered that the American yield curve and macro factors affect the Brazilian curve, but not vice versa. However, the model must be first identified from the econometric point of view before additional restrictions are imposed, otherwise the same parameters might be fixed twice, leaving unresolved degrees of freedom.

The second point is that in the default-free case was illustrated supposing that the macro factors are “more endogenous” than the latent factors. In the default case, $X = (\theta^{\text{US}}, M^{\text{US}}, M^{\text{BR}}, \theta^{\text{BR}})$, thus the American latent factors come before the Brazilian factors, which would in principle change the operator $T_{L,v}$ and consequently the degrees of freedom. The other inversion, namely the American macro vector coming after the latent vector, is due to the fact that only the VIX is considered and it does not interfere with the identification procedure.

However, since we use a two-step procedure, the parameters and state factors related to the American term structure are estimated first. So, we can think of the American latent factors as if they were “macro” factors and proceed to the identification considering that $\widetilde{M}^{\text{BR}} = (\theta^{\text{US}}, M^{\text{US}}, M^{\text{BR}})$ is in fact the macro vector for the default case.

In summary, the economic restrictions impose that $\delta_1^r = (\delta_1^{r,\text{US}}, 0)$ and that the matrix K is block-triangular, which means that Brazilian factors do not affect American factors. Therefore the identified Σ is given by:

$$\begin{pmatrix} \widetilde{\Sigma}_{MM} & 0 \\ 0 & \mathbf{I} \end{pmatrix}, \text{ where } \widetilde{\Sigma}_{MM} = \begin{pmatrix} \mathbf{I} & 0 & 0 \\ 0 & \Sigma_{M,M}^{\text{US,US}} & 0 \\ \Sigma_{M,\theta}^{\text{BR,US}} & \Sigma_{M,M}^{\text{BR,US}} & \Sigma_{M,M}^{\text{BR,BR}} \end{pmatrix}.$$

Appendix B - Impulse Response and Variance Decomposition

One way to evaluate the impact of macro shocks on the term structure of interest rates and default probabilities is through impulse response functions (IRF) and variance decompositions (VD). In continuous time, the evolution

of the state vector is given by

$$X_{t_i|t_{i-k}} = e^{-K(t_i-t_{i-k})}X_{i-k} + \sum_{l=0}^{k-1} \int_{t_{i-k+l}}^{t_{i-k+l+1}} e^{-K(t_i-u)}\Sigma dW_u^{\mathbb{P}}.$$

The stochastic integral is Gaussian with zero mean and variance

$$E \left[\int_{t_{i-1}}^{t_i} e^{-K(t_i-u)}\Sigma dW_u^{\mathbb{P}} \right]^2 = \int_{t_{i-1}}^{t_i} e^{-K(t_i-u)}\Sigma\Sigma'(e^{-K(t_i-u)})'du. \quad (17)$$

When $\Delta t = t_i - t_{i-1}$ is small, the variance is approximately $e^{-K\Delta t}\Sigma\Sigma'(e^{-K\Delta t})'\Delta t$. Hence, the response of X_t to a shock ε_t in a time interval of Δt is

$$\begin{array}{ccccccc} \Sigma\sqrt{\Delta t}\varepsilon_t & e^{-K\Delta t}\Sigma\sqrt{\Delta t}\varepsilon_t & e^{-2K\Delta t}\Sigma\sqrt{\Delta t}\varepsilon_t & e^{-3K\Delta t}\Sigma\sqrt{\Delta t}\varepsilon_t & \dots & & \\ t+0 & t+1 & t+2 & t+3 & \dots & & \end{array} \quad (18)$$

Similarly, the response of the yield $Y_t = A + BX_t$ is given by

$$\begin{array}{ccccccc} B\Sigma\sqrt{\Delta t}\varepsilon_t & Be^{-K\Delta t}\Sigma\sqrt{\Delta t}\varepsilon_t & Be^{-2K\Delta t}\Sigma\sqrt{\Delta t}\varepsilon_t & Be^{-3K\Delta t}\Sigma\sqrt{\Delta t}\varepsilon_t & \dots & & \\ t+0 & t+1 & t+2 & t+3 & \dots & & \end{array} \quad (19)$$

and the response of the logarithm of the survival probability, $\log \Pr(t, \tau) = a^{\text{Pr}} + b^{\text{Pr}}X_t$, is

$$\begin{array}{ccccccc} b^{\text{Pr}}\Sigma\sqrt{\Delta t}\varepsilon_t & b^{\text{Pr}}e^{-K\Delta t}\Sigma\sqrt{\Delta t}\varepsilon_t & b^{\text{Pr}}e^{-2K\Delta t}\Sigma\sqrt{\Delta t}\varepsilon_t & b^{\text{Pr}}e^{-3K\Delta t}\Sigma\sqrt{\Delta t}\varepsilon_t & \dots & & \\ t+0 & t+1 & t+2 & t+3 & \dots & & \end{array} \quad (20)$$

In Section 5 we work with a shock of one standard deviation of a monthly variation of a factor. This means that $\sqrt{\Delta t} = \sqrt{21/252}$ considering a 252 day-year.

To find the variance decomposition, we must calculate the mean squared error (MSE) of h -periods ahead error $X_{t+h} - EX_{t+h|t}$:

$$MSE = \int_t^{t+h} e^{-K(t+h-u)}\Sigma\Sigma'(e^{-K(t+h-u)})'du.$$

Hence, the contribution corresponding to the j^{th} factor in the variance decomposition of X_{t+h} , $Y_{t+h}(\tau)$ and $\log \Pr(t+h, \tau)$ at time t are

$$VD_j(X) = \int_t^{t+h} e^{-K(t+h-u)}\Sigma_j\Sigma_j'(e^{-K(t+h-u)})'du,$$

$$VD_j(Y) = B'(\tau) \left(\int_t^{t+h} e^{-K(t+h-u)}\Sigma_j\Sigma_j'(e^{-K(t+h-u)})'du \right) B(\tau), \quad (21)$$

$$VD_j(\log \Pr) = b^{\text{Pr}'}(\tau) \left(\int_t^{t+h} e^{-K(t+h-u)}\Sigma_j\Sigma_j'(e^{-K(t+h-u)})'du \right) b^{\text{Pr}}(\tau).$$

References

- [1] Almeida, C. and J. Vicente (2009). Identifying volatility risk premia from fixed income Asian options. *Journal of Banking & Finance*, **33**, pp. 652-661.
- [2] Ang, A. and M. Piazzesi (2003). A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables, *Journal of Monetary Economics*, **50**, pp. 745-787.
- [3] Ang, A., S. Dong, and M. Piazzesi (2007). No-arbitrage Taylor rules, *NBER Working paper* 13448.
- [4] Berndt, A., R. Douglas, D. Duffie, M. Ferguson, and D. Schranzk (2004). Measuring default risk premia from default swap rates and EDFs. Working Paper, Stanford University.
- [5] Black, F. and J. Cox (1976). Valuing corporate securities: some effects of bond indenture provisions, *Journal of Finance*, **31** (2), pp. 351-367.
- [6] Black, F. and M. Scholes (1973). The pricing of options and corporate liabilities, *Journal of Political Economy*, **81** (3), pp. 637-654.
- [7] Chen, R. and L. Scott (1993). Maximum likelihood estimation for a multi-factor equilibrium model of the term structure of interest rates, *Journal of Fixed Income*, **3** (4), pp. 14-31.
- [8] Dai, Q. and K. Singleton (2000). Specification analysis of affine term structure models, *Journal of Finance*, **55**, pp. 1943-1978.
- [9] Dai, Q. and K. Singleton (2002). Expectation puzzles, time-varying risk premia, and affine models of the term structure. *Journal of Financial Economics*, **63**, pp. 415-441.
- [10] Diebold, F., G. Rudebusch and S. Aruoba (2006). The macroeconomy and the yield curve: a dynamic latent factor approach. *Journal of Econometrics*, **131**, pp. 309-338.
- [11] Duffee, G. (2002). Term premia and interest rates forecasts in affine models. *Journal of Finance*, **57**, pp. 405-443.
- [12] Duffie, D. (2001) *Dynamic asset pricing theory*, Third edition, Princeton University Press.

- [13] Duffie, D. and R. Kan (1996) A yield-factor model of interest rates, *Mathematical Finance*, **6**, pp. 379-406.
- [14] Duffie, D., L. Pedersen and K. Singleton (2003). Modeling sovereign yield spreads: a case study of Russian debt, *Journal of Finance*, **58**, pp. 119-159.
- [15] Duffie, D. and K. Singleton (1999). Modeling term structures of defaultable bonds, *Review of Financial Studies*, **12**, pp. 687-720.
- [16] Fackler, P. (2000). Moments of affine diffusions, *Working paper*, North Carolina State University.
- [17] Giesecke, K. (2004). Credit risk modeling and valuation: an introduction, in *Credit Risk Models and Management*, Vol. 2, Ed. D. Shimko, Riskbooks.
- [18] Hördal, P., O. Tristani, and D. Vestin. (2006). A Joint Econometric Model of Macroeconomic and Term Structure Dynamics. *Journal of Econometrics*, **127**, pp. 405-44.
- [19] Goldman Sachs Global Economics Department (2007). *BRICs and beyond*.
- [20] Lando, D. (1998). On Cox processes and credit risky bonds, *Review of Derivatives Research*, **2**(2), pp. 99-120.
- [21] Leland, H. (1994). Risky debt, bond covenants and optimal capital structure, *Journal of Finance*, **49** (4), pp. 1213-1252.
- [22] Litterman, R. and J. Scheinkman (1991). Common factors affecting bond returns, *Journal of Fixed Income* **1**, pp. 54-61.
- [23] Moody's (2008). Sovereign default and recovery rates, 1983-2007, Working Paper, Moody's Global Credit Research. Available at <http://www.moody.com/>.
- [24] Merton, R. (1974). On the pricing of corporate debt: the risk structure of interest rates, *Journal of Finance* **29** (2), pp. 449-470.
- [25] Moreira, A. and K. Rocha (2004). Two-factor structural model of determinants of Brazilian sovereign risk, *Journal of Fixed Income*, **14** (2).
- [26] Pan, J. and K. Singleton (2008). Default and recovery implicit in the term structure of sovereign CDS spreads. *Journal of Finance*, **63** (5), pp. 2345-2384.

- [27] Pericoli, M. and M. Taboga (2008). Canonical term-structure models with observable factors and the dynamics of bond risk premia, *Journal of Money, Credit and Banking*, **40** (7), pp. 1471-1488.
- [28] Rudebusch, G. and T. Wu (2004). A Macro-finance model of the term structure, monetary policy, and the economy. Proceedings of the Federal Reserve Bank of San Francisco.
- [29] Schönbucher, P. (2003). *Credit derivatives pricing models*, Wiley Finance.
- [30] Shiryaev, A. (1995). *Probability*, Springer Verlag.
- [31] Spencer, P. (2008). Stochastic volatility in a macro-finance model of the U.S. term structure of interest rates 1961-2004. *Journal of Money, Credit and Banking*, **40** (6), pp. 1177-1215.
- [32] Xu, D. and P. Ghezzi, (2002). From fundamentals to spreads: A fair spread model for high yield emerging market sovereigns, *Deutsche Bank Global Markets Research*.

| | VIX | LEX | LIBOV | B1m | B3y | Bsl | F1m | F10y | Fsl |
|-----------------|------|------|-------|------|------|------|------|------|------|
| \mathcal{L}/H | 44.7 | 44.3 | 44.3 | 44.8 | 44.8 | 45.0 | 47.5 | 47.5 | 47.1 |
| $\rho_{1,s}$ | 0.20 | 0.37 | 0.29 | 0.59 | 0.56 | 0.57 | 0.66 | 0.69 | 0.61 |
| $\rho_{2,l}$ | 0.99 | 0.83 | 0.98 | 0.94 | 0.94 | 1.00 | 0.94 | 0.84 | 0.94 |
| MAE | 54 | 66 | 56 | 62 | 62 | 58 | 62 | 61 | 62 |

Table 1: Summary of results of the macro-to-yield without default model.

This table presents the log-likelihood divided by the number of observations (\mathcal{L}/H), the mean (for the nine maturities) of the absolute measurement errors in basis points (MAE), and the correlations between factor 1 (θ_1^{BR}) and the slope of the Brazilian sovereign yield curve ($\rho_{1,s}$) and between factor 2 (θ_2^{BR}) and the level of the Brazilian sovereign yield curve ($\rho_{2,l}$). The macro-to-yield without default model presents only one observable factor in each specification. They are (1) VIX; (2) logarithm of the BR Real/US Dollar exchange rate (LEX); (3) logarithm of the Ibovespa (LIBOV); (4) BM&F 1-month yield (B1m); (5) BM&F 3-year yield (B3y); (6) BM&F slope (Bsl) = B3y - B1m; (7) Fed 1-month yield (F1m); (8) Fed 10-year yield (F10y), and (9) Fed slope (Fsl)= F10y - F1m.

| Yields | 1-month ahead | | | | | | | | |
|--------|----------------|-----|-------|-----|-----|-----|-----|------|-----|
| | VIX | LEX | LIBOV | B1m | B3y | Bsl | F1m | F10y | Fsl |
| 3m | 15 | 7 | 1 | 0 | 0 | 16 | 0 | 0 | 0 |
| 3y | 23 | 9 | 0 | 0 | 0 | 23 | 0 | 4 | 0 |
| 20y | 54 | 9 | 6 | 0 | 0 | 50 | 0 | 8 | 0 |
| Yields | 9-months ahead | | | | | | | | |
| | VIX | LEX | LIBOV | B1m | B3y | Bsl | F1m | F10y | Fsl |
| 3m | 31 | 7 | 22 | 0 | 0 | 46 | 0 | 0 | 6 |
| 3y | 46 | 11 | 13 | 0 | 0 | 61 | 0 | 10 | 7 |
| 20y | 69 | 14 | 21 | 0 | 0 | 79 | 0 | 16 | 7 |

Table 2: Variance decompositions of the macro-to-yield without default model.

This table presents the proportion (in percent) of the 1-month and 9-month ahead forecast variance of the {3m, 3y, 20y}-yields attributable to each observable factor. The macro-to-yield without default model presents only one observable factor in each specification. They are (1) VIX; (2) logarithm of the BR Real/US Dollar exchange rate (LEX); (3) logarithm of the Ibovespa (LIBOV); (4) BM&F 1-month yield (B1m); (5) BM&F 3-year yield (B3y); (6) BM&F slope (Bsl) = B3y - B1m; (7) Fed 1-month yield (F1m); (8) Fed 10-year yield (F10y), and (9) Fed slope (Fsl) = F10y - F1m.

| | y.o. | VIX | Bsl | Fsl | B3y |
|-----------------|------|------|------|-------|------|
| \mathcal{L}/H | 42.1 | 47.0 | 48.2 | 49.9 | 47.8 |
| $\rho_{1,s}$ | 0.28 | 0.37 | 0.42 | -0.17 | 0.19 |
| $\rho_{2,l}$ | 0.99 | 0.97 | 0.98 | 0.96 | 0.86 |
| MAE | 68 | 53 | 50 | 59 | 54 |

Table 3: Summary of results of the macro-to-yield with default model.

This table presents the log-likelihood divided by the number of observations (\mathcal{L}/H), the mean (for the nine maturities) of the absolute measurement errors in basis points (MAE), and the correlations between factor 1 (θ_1^{BR}) and the slope of the Brazilian sovereign yield curve ($\rho_{1,s}$) and between factor 2 (θ_2^{BR}) and the level of the Brazilian sovereign yield curve ($\rho_{2,l}$). The macro-to-yield with default model presents one observable factor, one latent factor driving the US curve and two latent factors driving the Brazilian curve. The observable factors are (1) VIX; (2) BM&F slope (Bsl) = B3y - B1m, (3) Fed slope (Fsl) = F10y - F1m, and (4) BM&F 3-year yield (B3y). The y.o. model refers to a specification in which only yields are used, that is, a specification without observable factors.

| Model | | y.o. | | VIX | | Bsl | | Fsl | | B3y | |
|------------------------|-------|------|----|-----|----|-----|----|-----|----|-----|----|
| Factor | Yield | 1m | 9m | 1m | 9m | 1m | 9m | 1m | 9m | 1m | 9m |
| θ^{US} | 3m | 0 | 1 | 0 | 4 | 0 | 1 | 0 | 6 | 1 | 4 |
| | 3y | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 5 | 1 | 2 |
| | 20y | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 5 | 1 | 2 |
| Macro | 3m | - | - | 15 | 37 | 1 | 7 | 2 | 14 | 1 | 11 |
| | 3y | - | - | 25 | 50 | 2 | 8 | 1 | 16 | 0 | 2 |
| | 20y | - | - | 56 | 70 | 5 | 9 | 1 | 17 | 4 | 3 |
| θ_1^{BR} | 3m | 13 | 26 | 10 | 8 | 25 | 14 | 51 | 23 | 30 | 32 |
| | 3y | 1 | 20 | 1 | 2 | 8 | 11 | 24 | 17 | 79 | 89 |
| | 20y | 18 | 12 | 9 | 5 | 7 | 11 | 2 | 13 | 85 | 91 |
| θ_2^{BR} | 3m | 87 | 74 | 75 | 51 | 74 | 77 | 47 | 57 | 68 | 53 |
| | 3y | 99 | 80 | 74 | 47 | 89 | 80 | 75 | 63 | 20 | 7 |
| | 20y | 82 | 88 | 35 | 24 | 88 | 80 | 96 | 65 | 10 | 4 |

Table 4: Variance decompositions of the yields of the macro-to-yield with default model.

This table presents the proportion (in percent) of the one month and nine months ahead forecast variance of the {3m, 3y, 20y}-yields attributable to each observable factor in the macro-to-yield with default model. The macro-to-yield with default model presents one observable factor, one latent factor driving the US curve and two latent factors driving the Brazilian curve. The observable factors are (1) VIX; (2) BM&F slope (Bsl) = B3y - B1m, (3) Fed slope (Fsl) = F10y - F1m, and (4) BM&F 3-year yield (B3y). The y.o. model refers to a specification in which only yields are used, that is, a specification without observable factors.

| Model | | y.o. | | VIX | | Bsl | | Fsl | | B3y | |
|------------------------|------|------|----|-----|----|-----|----|-----|----|-----|----|
| Factor | Term | 1m | 9m | 1m | 9m | 1m | 9m | 1m | 9m | 1m | 9m |
| θ^{US} | 3m | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 4 | 0 | 1 |
| | 3y | 0 | 1 | 19 | 60 | 0 | 0 | 4 | 4 | 12 | 25 |
| | 20y | 0 | 1 | 69 | 92 | 5 | 6 | 3 | 3 | 51 | 65 |
| Macro | 3m | - | - | 54 | 61 | 5 | 9 | 4 | 18 | 8 | 25 |
| | 3y | - | - | 48 | 26 | 9 | 10 | 20 | 22 | 10 | 26 |
| | 20y | - | - | 19 | 5 | 9 | 9 | 22 | 23 | 9 | 9 |
| θ_1^{BR} | 3m | 22 | 28 | 9 | 7 | 15 | 12 | 34 | 19 | 12 | 21 |
| | 3y | 27 | 29 | 6 | 3 | 12 | 11 | 17 | 15 | 11 | 18 |
| | 20y | 27 | 29 | 2 | 1 | 11 | 11 | 15 | 15 | 11 | 24 |
| θ_2^{BR} | 3m | 78 | 72 | 37 | 30 | 80 | 79 | 62 | 60 | 70 | 56 |
| | 3y | 73 | 71 | 27 | 11 | 79 | 78 | 60 | 59 | 45 | 31 |
| | 20y | 73 | 71 | 10 | 02 | 74 | 74 | 59 | 59 | 16 | 6 |

Table 5: Variance decompositions of the default probabilities of the macro-to-yield with default model.

This table lists the contribution (in percent) of each factor to the one month and nine months ahead forecast of the {3m, 3y, 20y} default probabilities within the macro-to-yield with default model. The macro-to-yield with default model presents one observable factor, one latent factor driving the US curve and two latent factors driving the Brazilian curve. The observable factors are (1) VIX; (2) BM&F slope (Bsl) = B3y - B1m, (3) Fed slope (Fsl) = F10y - F1m, and (4) BM&F 3-year yield (B3y). The y.o. model refers to a specification in which only yields are used, that is, a specification without observable factors.

| | VIX Bsl | VIX LIBOV-EX | VIX B3m | VIX B3y |
|-----------------|---------|--------------|---------|---------|
| \mathcal{L}/H | 52.5 | 55.6 | 52.9 | 52.9 |
| $\rho_{1,s}$ | 0.48 | 0.86 | 0.08 | 0.04 |
| $\rho_{2,l}$ | 0.93 | 0.96 | 0.92 | 0.93 |
| MAE | 47 | 51 | 46 | 46 |

Table 6: Summary of results of bilateral model with default.

This table presents the log-likelihood divided by the number of observations (\mathcal{L}/H), the mean (for the nine maturities) of the absolute measurement errors in basis points (MAE), and the correlations between factor 1 (θ_1^{BR}) and the slope of the Brazilian sovereign yield curve ($\rho_{1,s}$) and between factor 2 (θ_2^{BR}) and the level of the Brazilian sovereign yield curve ($\rho_{2,l}$). The bilateral model with default presents one observable American factor (VIX), one latent factor driving the US curve, two latent factors driving the Brazilian curve and one observable Brazilian factor. The observable Brazilian factors are (1) BM&F slope (Bsl); (2) logarithm of the Ibovespa in US Dollars (LIBOV-EX) (3) BM&F 3-month yield (B3m), and (4) BM&F 3-year yield (B3y).

| Model | | VIX Bsl | | VIX LIBOV-EX | | VIX B3m | | VIX B3y | |
|------------------------|-------|---------|----|--------------|----|---------|----|---------|----|
| Factor | Yield | 1m | 9m | 1m | 9m | 1m | 9m | 1m | 9m |
| θ^{US} | 3m | 0 | 2 | 1 | 3 | 1 | 2 | 1 | 3 |
| | 3y | 0 | 1 | 0 | 5 | 0 | 1 | 0 | 1 |
| | 20y | 0 | 0 | 0 | 4 | 0 | 0 | 0 | 0 |
| M^{US} | 3m | 2 | 20 | 2 | 21 | 4 | 26 | 4 | 32 |
| | 3y | 2 | 23 | 4 | 21 | 5 | 33 | 5 | 39 |
| | 20y | 27 | 38 | 36 | 16 | 20 | 48 | 20 | 53 |
| M^{BR} | 3m | 2 | 7 | 0 | 0 | 0 | 2 | 1 | 1 |
| | 3y | 3 | 10 | 0 | 1 | 0 | 2 | 0 | 1 |
| | 20y | 3 | 9 | 0 | 0 | 0 | 1 | 1 | 2 |
| θ_1^{BR} | 3m | 38 | 22 | 68 | 50 | 76 | 12 | 17 | 9 |
| | 3y | 13 | 9 | 19 | 52 | 3 | 4 | 2 | 1 |
| | 20y | 2 | 4 | 5 | 70 | 0 | 1 | 0 | 1 |
| θ_2^{BR} | 3m | 58 | 48 | 29 | 25 | 19 | 57 | 77 | 56 |
| | 3y | 81 | 57 | 76 | 21 | 91 | 61 | 93 | 58 |
| | 20y | 67 | 49 | 59 | 9 | 80 | 50 | 79 | 43 |

Table 7: Variance decompositions of the yields of the bilateral model with default.

This table presents the proportion (in percent) of the one month and nine months ahead forecast variance of the {3m, 3y, 20y}-yields attributable to each observable factor in the bilateral model with default. The bilateral model with default presents one observable American factor (VIX), one latent factor driving the US curve, two latent factors driving the Brazilian curve and one observable Brazilian factor. The observable Brazilian factors are (1) BM&F slope (Bsl); (2) logarithm of the Ibovespa in US Dollars (LIBOV-EX) (3) BM&F 3-month yield (B3m), and (4) BM&F 3-year yield (B3y).

| Model | | VIX Bsl | | VIX LIBOV-EX | | VIX B3m | | VIX B3y | |
|------------------------|------|---------|----|--------------|----|---------|----|---------|----|
| Factor | Term | 1m | 9m | 1m | 9m | 1m | 9m | 1m | 9m |
| θ^{US} | 3m | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 |
| | 3y | 7 | 21 | 21 | 59 | 8 | 29 | 9 | 29 |
| | 20y | 47 | 73 | 80 | 93 | 51 | 79 | 52 | 78 |
| M^{US} | 3m | 19 | 32 | 16 | 31 | 29 | 42 | 34 | 56 |
| | 3y | 31 | 31 | 26 | 18 | 36 | 33 | 55 | 51 |
| | 20y | 18 | 11 | 7 | 3 | 20 | 10 | 29 | 16 |
| M^{BR} | 3m | 8 | 11 | 0 | 0 | 1 | 3 | 1 | 1 |
| | 3y | 11 | 10 | 1 | 1 | 3 | 3 | 2 | 2 |
| | 20y | 6 | 3 | 0 | 0 | 1 | 1 | 1 | 1 |
| θ_1^{BR} | 3m | 21 | 12 | 61 | 49 | 11 | 9 | 6 | 3 |
| | 3y | 11 | 6 | 38 | 16 | 9 | 7 | 1 | 0 |
| | 20y | 6 | 2 | 10 | 3 | 5 | 2 | 0 | 0 |
| θ_2^{BR} | 3m | 52 | 44 | 22 | 18 | 59 | 46 | 59 | 39 |
| | 3y | 41 | 31 | 14 | 6 | 43 | 29 | 33 | 17 |
| | 20y | 24 | 11 | 3 | 1 | 23 | 8 | 18 | 5 |

Table 8: Variance decompositions of the default probabilities of bilateral model with default.

This table lists the contribution (in percent) of each factor to the one month and nine months ahead forecast of the {3m, 3y, 20y} default probabilities within the bilateral model with default. The bilateral model with default presents one observable American factor (VIX), one latent factor driving the US curve, two latent factors driving the Brazilian curve and one observable Brazilian factor. The observable Brazilian factors are (1) BM&F slope (Bsl); (2) logarithm of the Ibovespa in US Dollars (LIBOV-EX) (3) BM&F 3-month yield (B3m), and (4) BM&F 3-year yield (B3y).

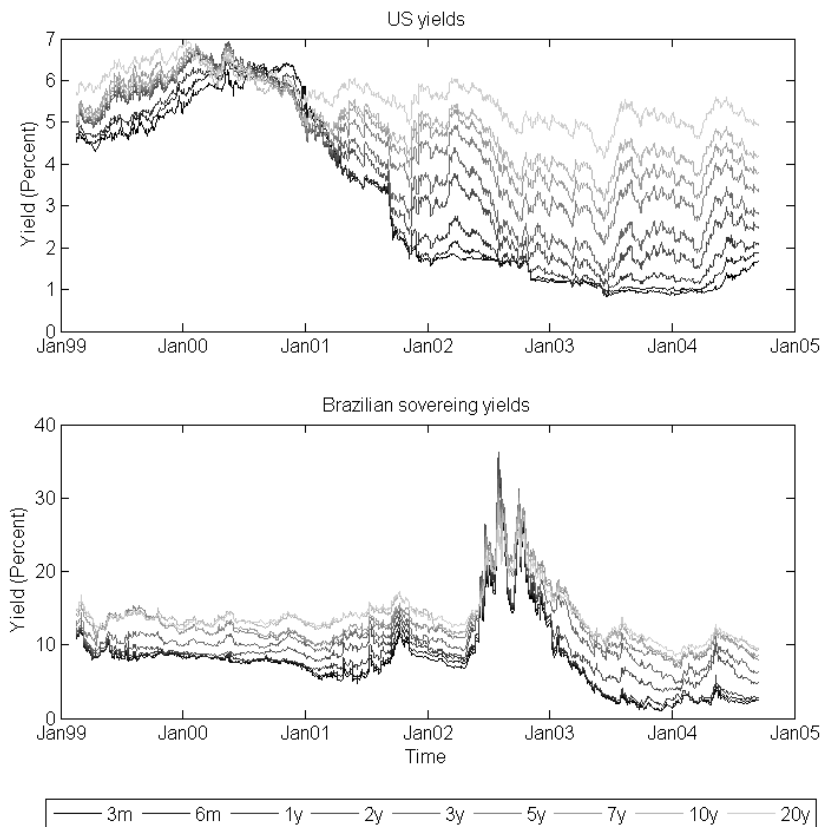


Figure 1: US and Brazilian sovereign yields.

This figure contains time series of US (top panel) and Brazilian sovereign (bottom panel) yields with time to maturity of 3 and 6 months, 1, 2, 3, 5, 7, 10 and 20 years between February 17, 1999 and September 15, 2004.

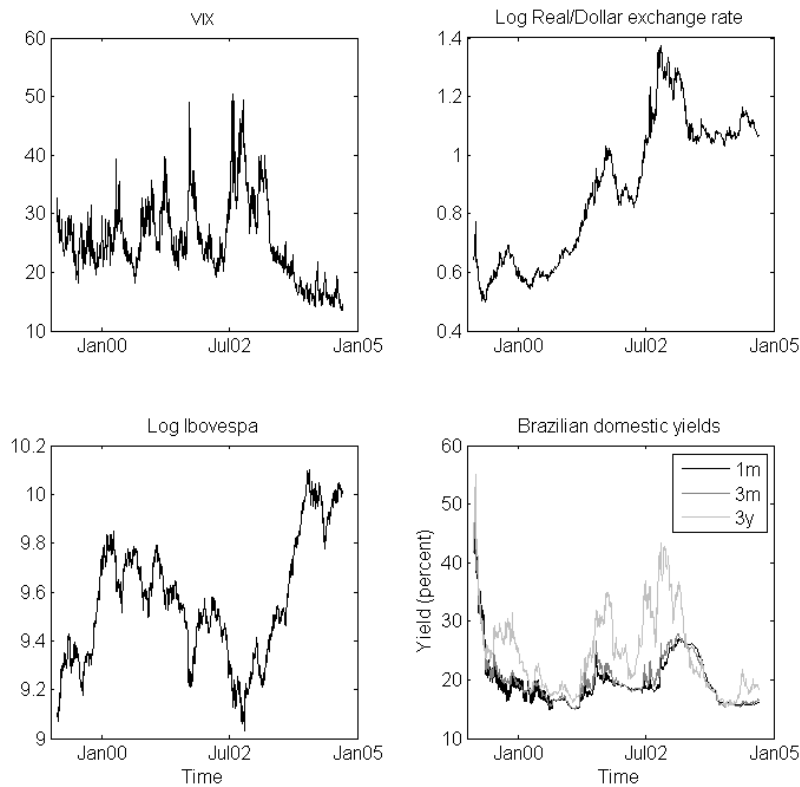


Figure 2: Observable variables.

This figure contains time series of variables used as observable factors in our model between February 17, 1999 and September 15, 2004. The upper left panel shows the evolution of the VIX (implied volatility of S&P 500 index options). The upper right panel presents the logarithm of the Brazilian Real/US Dollar exchange rate. The lower left panel presents the logarithm of the Ibovespa, and the lower right panel shows the Brazilian domestic zero-coupon yields with time to maturity of 1, 3 and 36 months.

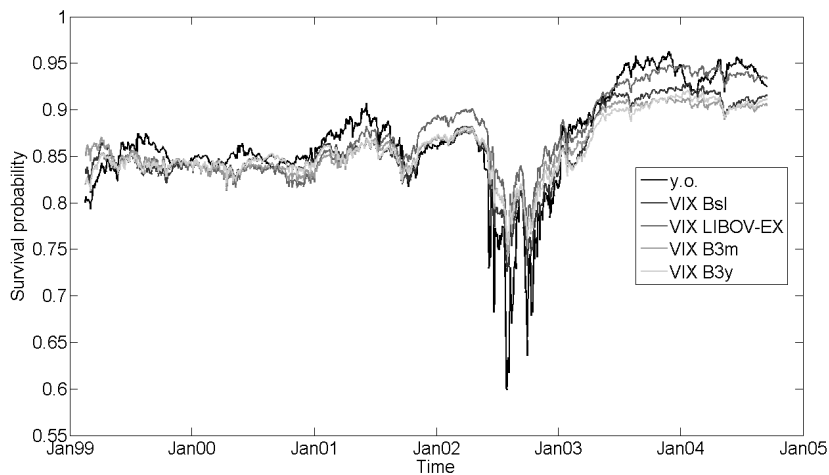


Figure 3: Survival probabilities.

This figure shows the 1-year survival probabilities extracted from some versions of the bilateral model and from y.o. model between February 17, 1999 and September 15, 2004. The bilateral model with default presents one observable American factor (VIX), one latent factor driving the US curve, two latent factors driving the Brazilian curve and one observable Brazilian factor. The observable Brazilian factors are (1) BM&F slope (Bsl); (2) logarithm of the Ibovespa in US Dollars (LIBOV-EX) (3) BM&F 3-month yield (B3m), and (4) BM&F 3-year yield (B3y). The y.o. model refers to a specification in which only yields are used, that is, a specification without observable factors.

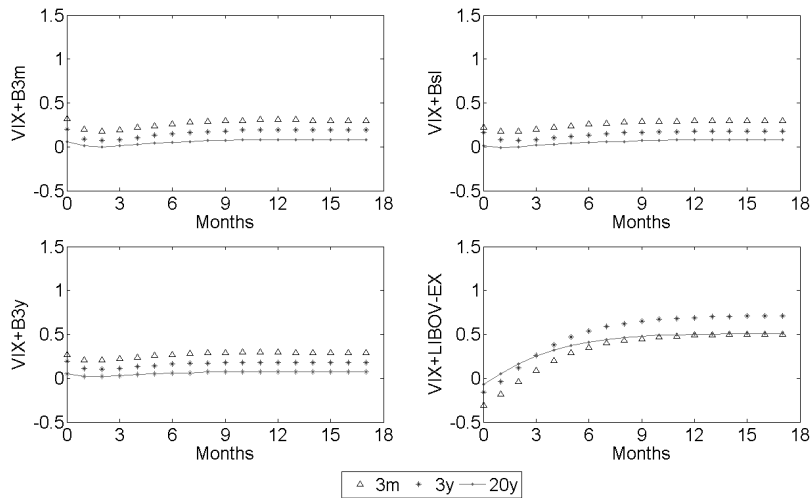


Figure 4: Impulse response of shocks to Fed factor on yields.

This figure shows the effect of a shock to Fed factor (θ_1^{US}) on the Brazilian sovereign yields with maturities of three months, three years and twenty years up to 18-month after the shock. The size of the shock is one standard deviation of a monthly variation of the Fed factor. The responses are evaluated considering the bilateral model. The bilateral model with default presents one observable American factor (VIX), one latent factor driving the US curve, two latent factors driving the Brazilian curve and one observable Brazilian factor. The observable Brazilian factors are (1) BM&F slope (Bsl); (2) logarithm of the Ibovespa in US Dollars (LIBOV-EX) (3) BM&F 3-month yield (B3m), and (4) BM&F 3-year yield (B3y).

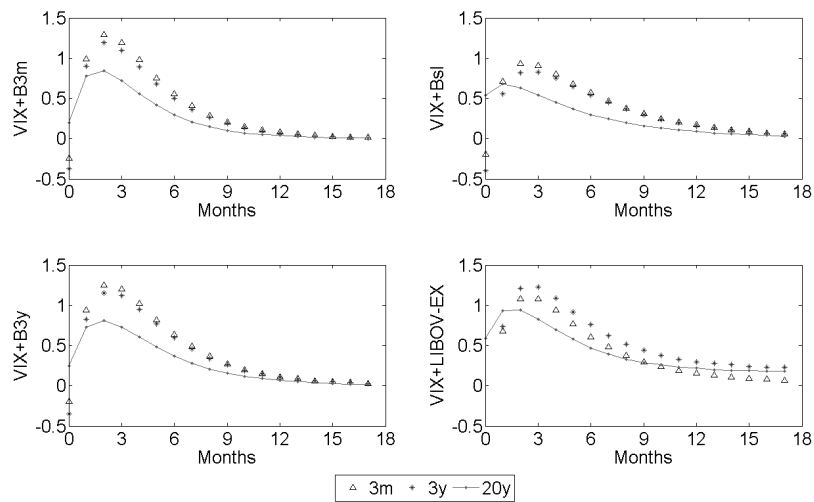


Figure 5: Impulse response of shocks to the VIX on yields. This figure shows the effect of a shock to the VIX on the Brazilian sovereign yields with maturities of three months, three years and twenty years up to 18-month after the shock. The size of the shock is one standard deviation of a monthly variation of the VIX. The responses are evaluated considering the bilateral model. The bilateral model with default presents one observable American factor (VIX), one latent factor driving the US curve, two latent factors driving the Brazilian curve and one observable Brazilian factor. The observable Brazilian factors are (1) BM&F slope (Bsl); (2) logarithm of the Ibovespa in US Dollars (LIBOV-EX) (3) BM&F 3-month yield (B3m), and (4) BM&F 3-year yield (B3y).

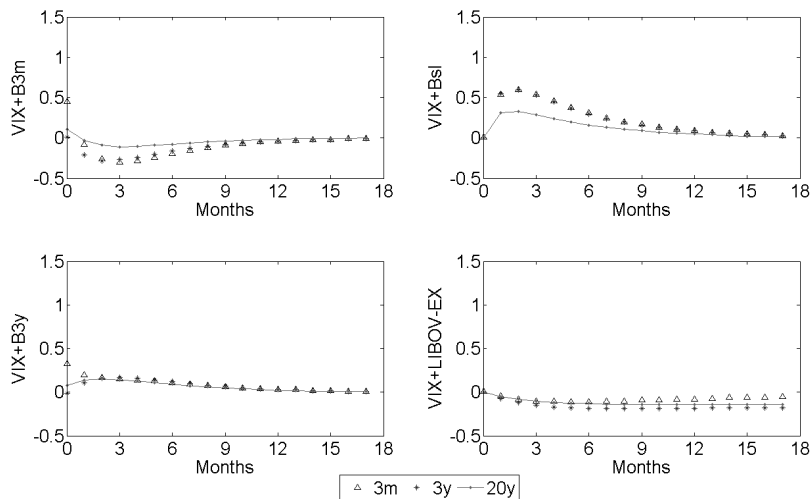


Figure 6: Impulse response of shocks to observable Brazilian factors on yields.

This figure shows the effect of a shock to observable Brazilian factors on the Brazilian sovereign yields with maturities three months, three years and twenty years up to 18-month after the shock. The size of the shock is one standard deviation of a monthly variation of the observable factor. The responses are evaluated considering the bilateral model. The bilateral model with default presents one observable American factor (VIX), one latent factor driving the US curve, two latent factors driving the Brazilian curve and one observable Brazilian factor. The observable Brazilian factors are (1) BM&F slope (Bsl); (2) logarithm of the Ibovespa in US Dollars (LIBOV-EX) (3) BM&F 3-month yield (B3m), and (4) BM&F 3-year yield (B3y).

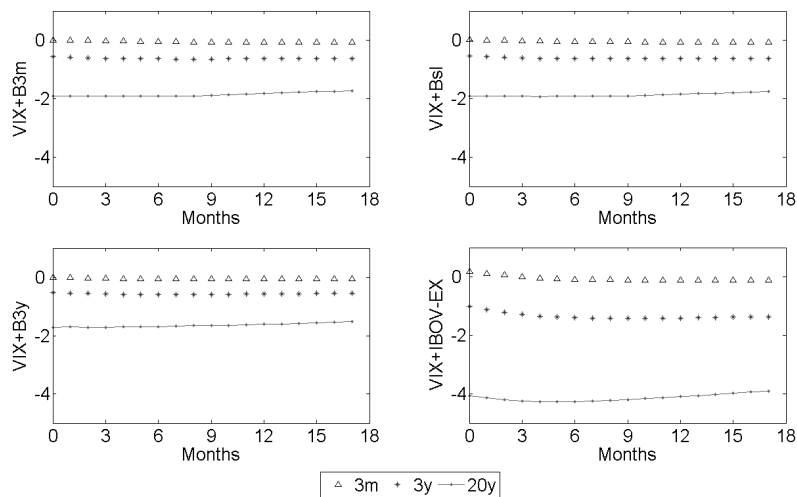


Figure 7: Impulse response of shocks to Fed factor on survival probabilities.

This figure shows the effect of a shock to Fed factor (θ_1^{US}) on the three months, three years and twenty years survival probabilities up to 18-month after the shock. The size of the shock is one standard deviation of a monthly variation of the Fed factor. The responses are evaluated considering the bilateral model. The bilateral model with default presents one observable American factor (VIX), one latent factor driving the US curve, two latent factors driving the Brazilian curve and one observable Brazilian factor. The observable Brazilian factors are (1) BM&F slope (Bsl); (2) logarithm of the Ibovespa in US Dollars (LIBOV-EX) (3) BM&F 3-month yield (B3m), and (4) BM&F 3-year yield (B3y).

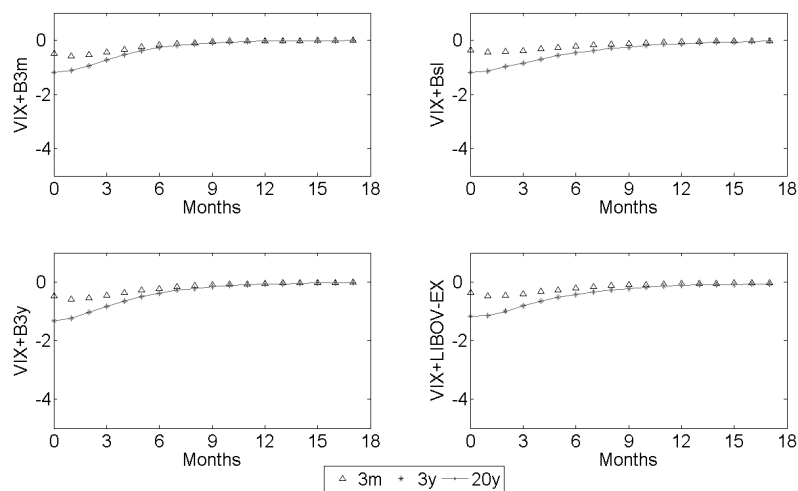


Figure 8: Impulse response of shocks to the VIX on survival probabilities.

This figure shows the effect of a shock to the VIX on the three months, three years and twenty years survival probabilities up to 18-month after the shock. The size of the shock is one standard deviation of a monthly variation of the VIX. The responses are evaluated considering the bilateral model. The bilateral model with default presents one observable American factor (VIX), one latent factor driving the US curve, two latent factors driving the Brazilian curve and one observable Brazilian factor. The observable Brazilian factors are (1) BM&F slope (Bsl); (2) logarithm of the Ibovespa in US Dollars (LIBOV-EX) (3) BM&F 3-month yield (B3m), and (4) BM&F 3-year yield (B3y).

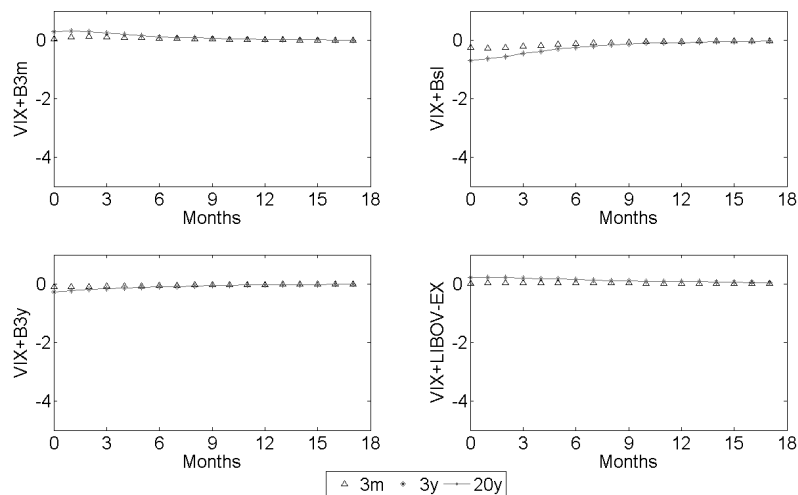


Figure 9: Impulse response of shocks to observable Brazilian factors on survival probabilities.

This figure shows the effect of a shock to observable Brazilian factors on the three months, three years and twenty years survival probabilities up to 18-month after the shock. The size of the shock is one standard deviation of a monthly variation of the observable factor. The responses are evaluated considering the bilateral model. The bilateral model with default presents one observable American factor (VIX), one latent factor driving the US curve, two latent factors driving the Brazilian curve and one observable Brazilian factor. The observable Brazilian factors are (1) BM&F slope (Bsl); (2) logarithm of the Ibovespa in US Dollars (LIBOV-EX) (3) BM&F 3-month yield (B3m), and (4) BM&F 3-year yield (B3y).

Banco Central do Brasil

Trabalhos para Discussão

Os Trabalhos para Discussão podem ser acessados na internet, no formato PDF, no endereço: <http://www.bc.gov.br>

Working Paper Series

Working Papers in PDF format can be downloaded from: <http://www.bc.gov.br>

- | | | |
|----|---|----------|
| 1 | Implementing Inflation Targeting in Brazil <i>Joel Bogdanski, Alexandre Antonio Tombini and Sérgio Ribeiro da Costa Werlang</i> | Jul/2000 |
| 2 | Política Monetária e Supervisão do Sistema Financeiro Nacional no Banco Central do Brasil <i>Eduardo Lundberg</i> | Jul/2000 |
| | Monetary Policy and Banking Supervision Functions on the Central Bank <i>Eduardo Lundberg</i> | Jul/2000 |
| 3 | Private Sector Participation: a Theoretical Justification of the Brazilian Position <i>Sérgio Ribeiro da Costa Werlang</i> | Jul/2000 |
| 4 | An Information Theory Approach to the Aggregation of Log-Linear Models <i>Pedro H. Albuquerque</i> | Jul/2000 |
| 5 | The Pass-Through from Depreciation to Inflation: a Panel Study <i>Ilan Goldfajn and Sérgio Ribeiro da Costa Werlang</i> | Jul/2000 |
| 6 | Optimal Interest Rate Rules in Inflation Targeting Frameworks <i>José Alvaro Rodrigues Neto, Fabio Araújo and Marta Baltar J. Moreira</i> | Jul/2000 |
| 7 | Leading Indicators of Inflation for Brazil <i>Marcelle Chauvet</i> | Sep/2000 |
| 8 | The Correlation Matrix of the Brazilian Central Bank's Standard Model for Interest Rate Market Risk <i>José Alvaro Rodrigues Neto</i> | Sep/2000 |
| 9 | Estimating Exchange Market Pressure and Intervention Activity <i>Emanuel-Werner Kohlscheen</i> | Nov/2000 |
| 10 | Análise do Financiamento Externo a uma Pequena Economia Aplicação da Teoria do Prêmio Monetário ao Caso Brasileiro: 1991–1998 <i>Carlos Hamilton Vasconcelos Araújo e Renato Galvão Flôres Júnior</i> | Mar/2001 |
| 11 | A Note on the Efficient Estimation of Inflation in Brazil <i>Michael F. Bryan and Stephen G. Cecchetti</i> | Mar/2001 |
| 12 | A Test of Competition in Brazilian Banking <i>Márcio I. Nakane</i> | Mar/2001 |

| | | |
|-----------|--|----------|
| 13 | Modelos de Previsão de Insolvência Bancária no Brasil <i>Marcio Magalhães Janot</i> | Mar/2001 |
| 14 | Evaluating Core Inflation Measures for Brazil <i>Francisco Marcos Rodrigues Figueiredo</i> | Mar/2001 |
| 15 | Is It Worth Tracking Dollar/Real Implied Volatility? <i>Sandro Canesso de Andrade and Benjamin Miranda Tabak</i> | Mar/2001 |
| 16 | Avaliação das Projeções do Modelo Estrutural do Banco Central do Brasil para a Taxa de Variação do IPCA <i>Sergio Afonso Lago Alves</i> | Mar/2001 |
| | Evaluation of the Central Bank of Brazil Structural Model's Inflation Forecasts in an Inflation Targeting Framework <i>Sergio Afonso Lago Alves</i> | Jul/2001 |
| 17 | Estimando o Produto Potencial Brasileiro: uma Abordagem de Função de Produção <i>Tito Nícias Teixeira da Silva Filho</i> | Abr/2001 |
| | Estimating Brazilian Potential Output: a Production Function Approach <i>Tito Nícias Teixeira da Silva Filho</i> | Aug/2002 |
| 18 | A Simple Model for Inflation Targeting in Brazil <i>Paulo Springer de Freitas and Marcelo Kfoury Muinhos</i> | Apr/2001 |
| 19 | Uncovered Interest Parity with Fundamentals: a Brazilian Exchange Rate Forecast Model <i>Marcelo Kfoury Muinhos, Paulo Springer de Freitas and Fabio Araújo</i> | May/2001 |
| 20 | Credit Channel without the LM Curve <i>Victorio Y. T. Chu and Márcio I. Nakane</i> | May/2001 |
| 21 | Os Impactos Econômicos da CPMF: Teoria e Evidência <i>Pedro H. Albuquerque</i> | Jun/2001 |
| 22 | Decentralized Portfolio Management <i>Paulo Coutinho and Benjamin Miranda Tabak</i> | Jun/2001 |
| 23 | Os Efeitos da CPMF sobre a Intermediação Financeira <i>Sérgio Mikio Koyama e Márcio I. Nakane</i> | Jul/2001 |
| 24 | Inflation Targeting in Brazil: Shocks, Backward-Looking Prices, and IMF Conditionality <i>Joel Bogdanski, Paulo Springer de Freitas, Ilan Goldfajn and Alexandre Antonio Tombini</i> | Aug/2001 |
| 25 | Inflation Targeting in Brazil: Reviewing Two Years of Monetary Policy 1999/00 <i>Pedro Fachada</i> | Aug/2001 |
| 26 | Inflation Targeting in an Open Financially Integrated Emerging Economy: the Case of Brazil <i>Marcelo Kfoury Muinhos</i> | Aug/2001 |
| 27 | Complementaridade e Fungibilidade dos Fluxos de Capitais Internacionais <i>Carlos Hamilton Vasconcelos Araújo e Renato Galvão Flôres Júnior</i> | Set/2001 |

- 28 **Regras Monetárias e Dinâmica Macroeconômica no Brasil: uma Abordagem de Expectativas Racionais** Nov/2001
Marco Antonio Bonomo e Ricardo D. Brito
- 29 **Using a Money Demand Model to Evaluate Monetary Policies in Brazil** Nov/2001
Pedro H. Albuquerque and Solange Gouvêa
- 30 **Testing the Expectations Hypothesis in the Brazilian Term Structure of Interest Rates** Nov/2001
Benjamin Miranda Tabak and Sandro Canesso de Andrade
- 31 **Algumas Considerações sobre a Sazonalidade no IPCA** Nov/2001
Francisco Marcos R. Figueiredo e Roberta Blass Staub
- 32 **Crises Cambiais e Ataques Especulativos no Brasil** Nov/2001
Mauro Costa Miranda
- 33 **Monetary Policy and Inflation in Brazil (1975-2000): a VAR Estimation** Nov/2001
André Minella
- 34 **Constrained Discretion and Collective Action Problems: Reflections on the Resolution of International Financial Crises** Nov/2001
Arminio Fraga and Daniel Luiz Gleizer
- 35 **Uma Definição Operacional de Estabilidade de Preços** Dez/2001
Tito Nícias Teixeira da Silva Filho
- 36 **Can Emerging Markets Float? Should They Inflation Target?** Feb/2002
Barry Eichengreen
- 37 **Monetary Policy in Brazil: Remarks on the Inflation Targeting Regime, Public Debt Management and Open Market Operations** Mar/2002
Luiz Fernando Figueiredo, Pedro Fachada and Sérgio Goldenstein
- 38 **Volatilidade Implícita e Antecipação de Eventos de Stress: um Teste para o Mercado Brasileiro** Mar/2002
Frederico Pechir Gomes
- 39 **Opções sobre Dólar Comercial e Expectativas a Respeito do Comportamento da Taxa de Câmbio** Mar/2002
Paulo Castor de Castro
- 40 **Speculative Attacks on Debts, Dollarization and Optimum Currency Areas** Apr/2002
Aloisio Araujo and Márcia Leon
- 41 **Mudanças de Regime no Câmbio Brasileiro** Jun/2002
Carlos Hamilton V. Araújo e Getúlio B. da Silveira Filho
- 42 **Modelo Estrutural com Setor Externo: Endogenização do Prêmio de Risco e do Câmbio** Jun/2002
Marcelo Kfoury Muinhos, Sérgio Afonso Lago Alves e Gil Riella
- 43 **The Effects of the Brazilian ADRs Program on Domestic Market Efficiency** Jun/2002
Benjamin Miranda Tabak and Eduardo José Araújo Lima

| | | |
|-----------|--|----------|
| 44 | Estrutura Competitiva, Produtividade Industrial e Liberação Comercial no Brasil <i>Pedro Cavalcanti Ferreira e Osmani Teixeira de Carvalho Guillén</i> | Jun/2002 |
| 45 | Optimal Monetary Policy, Gains from Commitment, and Inflation Persistence <i>André Minella</i> | Aug/2002 |
| 46 | The Determinants of Bank Interest Spread in Brazil <i>Tarsila Segalla Afanasieff, Priscilla Maria Villa Lhacer and Márcio I. Nakane</i> | Aug/2002 |
| 47 | Indicadores Derivados de Agregados Monetários <i>Fernando de Aquino Fonseca Neto e José Albuquerque Júnior</i> | Set/2002 |
| 48 | Should Government Smooth Exchange Rate Risk? <i>Ilan Goldfajn and Marcos Antonio Silveira</i> | Sep/2002 |
| 49 | Desenvolvimento do Sistema Financeiro e Crescimento Econômico no Brasil: Evidências de Causalidade <i>Orlando Carneiro de Matos</i> | Set/2002 |
| 50 | Macroeconomic Coordination and Inflation Targeting in a Two-Country Model <i>Eui Jung Chang, Marcelo Kfoury Muinhos and Joaúlio Rodolpho Teixeira</i> | Sep/2002 |
| 51 | Credit Channel with Sovereign Credit Risk: an Empirical Test <i>Victorio Yi Tson Chu</i> | Sep/2002 |
| 52 | Generalized Hyperbolic Distributions and Brazilian Data <i>José Fajardo and Aquiles Farias</i> | Sep/2002 |
| 53 | Inflation Targeting in Brazil: Lessons and Challenges <i>André Minella, Paulo Springer de Freitas, Ilan Goldfajn and Marcelo Kfoury Muinhos</i> | Nov/2002 |
| 54 | Stock Returns and Volatility <i>Benjamin Miranda Tabak and Solange Maria Guerra</i> | Nov/2002 |
| 55 | Componentes de Curto e Longo Prazo das Taxas de Juros no Brasil <i>Carlos Hamilton Vasconcelos Araújo e Osmani Teixeira de Carvalho de Guillén</i> | Nov/2002 |
| 56 | Causality and Cointegration in Stock Markets: the Case of Latin America <i>Benjamin Miranda Tabak and Eduardo José Araújo Lima</i> | Dec/2002 |
| 57 | As Leis de Falência: uma Abordagem Econômica <i>Aloisio Araujo</i> | Dez/2002 |
| 58 | The Random Walk Hypothesis and the Behavior of Foreign Capital Portfolio Flows: the Brazilian Stock Market Case <i>Benjamin Miranda Tabak</i> | Dec/2002 |
| 59 | Os Preços Administrados e a Inflação no Brasil <i>Francisco Marcos R. Figueiredo e Thaís Porto Ferreira</i> | Dez/2002 |
| 60 | Delegated Portfolio Management <i>Paulo Coutinho and Benjamin Miranda Tabak</i> | Dec/2002 |

| | | |
|----|---|-----------|
| 61 | O Uso de Dados de Alta Frequência na Estimação da Volatilidade e do Valor em Risco para o Ibovespa <i>João Maurício de Souza Moreira e Eduardo Facó Lemgruber</i> | Dez/2002 |
| 62 | Taxa de Juros e Concentração Bancária no Brasil <i>Eduardo Kiyoshi Tonooka e Sérgio Mikio Koyama</i> | Fev/2003 |
| 63 | Optimal Monetary Rules: the Case of Brazil <i>Charles Lima de Almeida, Marco Aurélio Peres, Geraldo da Silva e Souza and Benjamin Miranda Tabak</i> | Fev/2003 |
| 64 | Medium-Size Macroeconomic Model for the Brazilian Economy <i>Marcelo Kfoury Muinhos and Sergio Afonso Lago Alves</i> | Fev/2003 |
| 65 | On the Information Content of Oil Future Prices <i>Benjamin Miranda Tabak</i> | Fev/2003 |
| 66 | A Taxa de Juros de Equilíbrio: uma Abordagem Múltipla <i>Pedro Calhman de Miranda e Marcelo Kfoury Muinhos</i> | Fev/2003 |
| 67 | Avaliação de Métodos de Cálculo de Exigência de Capital para Risco de Mercado de Carteiras de Ações no Brasil <i>Gustavo S. Araújo, João Maurício S. Moreira e Ricardo S. Maia Clemente</i> | Fev/2003 |
| 68 | Real Balances in the Utility Function: Evidence for Brazil <i>Leonardo Soriano de Alencar and Márcio I. Nakane</i> | Fev/2003 |
| 69 | r-filters: a Hodrick-Prescott Filter Generalization <i>Fabio Araújo, Marta Baltar Moreira Areosa and José Alvaro Rodrigues Neto</i> | Fev/2003 |
| 70 | Monetary Policy Surprises and the Brazilian Term Structure of Interest Rates <i>Benjamin Miranda Tabak</i> | Fev/2003 |
| 71 | On Shadow-Prices of Banks in Real-Time Gross Settlement Systems <i>Rodrigo Penaloza</i> | Apr/2003 |
| 72 | O Prêmio pela Maturidade na Estrutura a Termo das Taxas de Juros Brasileiras <i>Ricardo Dias de Oliveira Brito, Angelo J. Mont'Alverne Duarte e Osmani Teixeira de C. Guillen</i> | Maio/2003 |
| 73 | Análise de Componentes Principais de Dados Funcionais – uma Aplicação às Estruturas a Termo de Taxas de Juros <i>Getúlio Borges da Silveira e Octavio Bessada</i> | Maio/2003 |
| 74 | Aplicação do Modelo de Black, Derman & Toy à Precificação de Opções Sobre Títulos de Renda Fixa <i>Octavio Manuel Bessada Lion, Carlos Alberto Nunes Cosenza e César das Neves</i> | Maio/2003 |
| 75 | Brazil's Financial System: Resilience to Shocks, no Currency Substitution, but Struggling to Promote Growth <i>Ilan Goldfajn, Katherine Hennings and Helio Mori</i> | Jun/2003 |

| | | |
|-----------|--|-----------|
| 76 | Inflation Targeting in Emerging Market Economies <i>Arminio Fraga, Ilan Goldfajn and André Minella</i> | Jun/2003 |
| 77 | Inflation Targeting in Brazil: Constructing Credibility under Exchange Rate Volatility <i>André Minella, Paulo Springer de Freitas, Ilan Goldfajn and Marcelo Kfoury Muinhos</i> | Jul/2003 |
| 78 | Contornando os Pressupostos de Black & Scholes: Aplicação do Modelo de Precificação de Opções de Duan no Mercado Brasileiro <i>Gustavo Silva Araújo, Claudio Henrique da Silveira Barbedo, Antonio Carlos Figueiredo, Eduardo Facó Lemgruber</i> | Out/2003 |
| 79 | Inclusão do Decaimento Temporal na Metodologia Delta-Gama para o Cálculo do VaR de Carteiras Compradas em Opções no Brasil <i>Claudio Henrique da Silveira Barbedo, Gustavo Silva Araújo, Eduardo Facó Lemgruber</i> | Out/2003 |
| 80 | Diferenças e Semelhanças entre Países da América Latina: uma Análise de Markov Switching para os Ciclos Econômicos de Brasil e Argentina <i>Arnildo da Silva Correa</i> | Out/2003 |
| 81 | Bank Competition, Agency Costs and the Performance of the Monetary Policy <i>Leonardo Soriano de Alencar and Márcio I. Nakane</i> | Jan/2004 |
| 82 | Carteiras de Opções: Avaliação de Metodologias de Exigência de Capital no Mercado Brasileiro <i>Cláudio Henrique da Silveira Barbedo e Gustavo Silva Araújo</i> | Mar/2004 |
| 83 | Does Inflation Targeting Reduce Inflation? An Analysis for the OECD Industrial Countries <i>Thomas Y. Wu</i> | May/2004 |
| 84 | Speculative Attacks on Debts and Optimum Currency Area: a Welfare Analysis <i>Aloisio Araujo and Marcia Leon</i> | May/2004 |
| 85 | Risk Premia for Emerging Markets Bonds: Evidence from Brazilian Government Debt, 1996-2002 <i>André Soares Loureiro and Fernando de Holanda Barbosa</i> | May/2004 |
| 86 | Identificação do Fator Estocástico de Descontos e Algumas Implicações sobre Testes de Modelos de Consumo <i>Fabio Araujo e João Victor Issler</i> | Maio/2004 |
| 87 | Mercado de Crédito: uma Análise Econométrica dos Volumes de Crédito Total e Habitacional no Brasil <i>Ana Carla Abrão Costa</i> | Dez/2004 |
| 88 | Ciclos Internacionais de Negócios: uma Análise de Mudança de Regime Markoviano para Brasil, Argentina e Estados Unidos <i>Arnildo da Silva Correa e Ronald Otto Hillbrecht</i> | Dez/2004 |
| 89 | O Mercado de Hedge Cambial no Brasil: Reação das Instituições Financeiras a Intervenções do Banco Central <i>Fernando N. de Oliveira</i> | Dez/2004 |

- 90 **Bank Privatization and Productivity: Evidence for Brazil** Dec/2004
Márcio I. Nakane and Daniela B. Weintraub
- 91 **Credit Risk Measurement and the Regulation of Bank Capital and Provision Requirements in Brazil – a Corporate Analysis** Dec/2004
Ricardo Schechtman, Valéria Salomão Garcia, Sergio Miki Koyama and Guilherme Cronemberger Parente
- 92 **Steady-State Analysis of an Open Economy General Equilibrium Model for Brazil** Apr/2005
Mirta Noemi Sataka Bugarin, Roberto de Goes Ellery Jr., Victor Gomes Silva, Marcelo Kfoury Muinhos
- 93 **Avaliação de Modelos de Cálculo de Exigência de Capital para Risco Cambial** Abr/2005
Claudio H. da S. Barbedo, Gustavo S. Araújo, João Maurício S. Moreira e Ricardo S. Maia Clemente
- 94 **Simulação Histórica Filtrada: Incorporação da Volatilidade ao Modelo Histórico de Cálculo de Risco para Ativos Não-Lineares** Abr/2005
Claudio Henrique da Silveira Barbedo, Gustavo Silva Araújo e Eduardo Facó Lemgruber
- 95 **Comment on Market Discipline and Monetary Policy by Carl Walsh** Apr/2005
Maurício S. Bugarin and Fábria A. de Carvalho
- 96 **O que É Estratégia: uma Abordagem Multiparadigmática para a Disciplina** Ago/2005
Anthero de Moraes Meirelles
- 97 **Finance and the Business Cycle: a Kalman Filter Approach with Markov Switching** Aug/2005
Ryan A. Compton and Jose Ricardo da Costa e Silva
- 98 **Capital Flows Cycle: Stylized Facts and Empirical Evidences for Emerging Market Economies** Aug/2005
Helio Mori e Marcelo Kfoury Muinhos
- 99 **Adequação das Medidas de Valor em Risco na Formulação da Exigência de Capital para Estratégias de Opções no Mercado Brasileiro** Set/2005
Gustavo Silva Araújo, Claudio Henrique da Silveira Barbedo, e Eduardo Facó Lemgruber
- 100 **Targets and Inflation Dynamics** Oct/2005
Sergio A. L. Alves and Waldyr D. Areosa
- 101 **Comparing Equilibrium Real Interest Rates: Different Approaches to Measure Brazilian Rates** Mar/2006
Marcelo Kfoury Muinhos and Márcio I. Nakane
- 102 **Judicial Risk and Credit Market Performance: Micro Evidence from Brazilian Payroll Loans** Apr/2006
Ana Carla A. Costa and João M. P. de Mello
- 103 **The Effect of Adverse Supply Shocks on Monetary Policy and Output** Apr/2006
Maria da Glória D. S. Araújo, Mirta Bugarin, Marcelo Kfoury Muinhos and Jose Ricardo C. Silva

- 104 Extração de Informação de Opções Cambiais no Brasil** Abr/2006
Eui Jung Chang e Benjamin Miranda Tabak
- 105 Representing Roommate's Preferences with Symmetric Utilities** Apr/2006
José Alvaro Rodrigues Neto
- 106 Testing Nonlinearities Between Brazilian Exchange Rates and Inflation Volatilities** May/2006
Cristiane R. Albuquerque and Marcelo Portugal
- 107 Demand for Bank Services and Market Power in Brazilian Banking** Jun/2006
Márcio I. Nakane, Leonardo S. Alencar and Fabio Kanczuk
- 108 O Efeito da Consignação em Folha nas Taxas de Juros dos Empréstimos Pessoais** Jun/2006
Eduardo A. S. Rodrigues, Victorio Chu, Leonardo S. Alencar e Tony Takeda
- 109 The Recent Brazilian Disinflation Process and Costs** Jun/2006
Alexandre A. Tombini and Sergio A. Lago Alves
- 110 Fatores de Risco e o Spread Bancário no Brasil** Jul/2006
Fernando G. Bignotto e Eduardo Augusto de Souza Rodrigues
- 111 Avaliação de Modelos de Exigência de Capital para Risco de Mercado do Cupom Cambial** Jul/2006
Alan Cosme Rodrigues da Silva, João Maurício de Souza Moreira e Myrian Beatriz Eiras das Neves
- 112 Interdependence and Contagion: an Analysis of Information Transmission in Latin America's Stock Markets** Jul/2006
Angelo Marsiglia Fasolo
- 113 Investigação da Memória de Longo Prazo da Taxa de Câmbio no Brasil** Ago/2006
Sergio Rubens Stancato de Souza, Benjamin Miranda Tabak e Daniel O. Cajueiro
- 114 The Inequality Channel of Monetary Transmission** Aug/2006
Marta Areosa and Waldyr Areosa
- 115 Myopic Loss Aversion and House-Money Effect Overseas: an Experimental Approach** Sep/2006
José L. B. Fernandes, Juan Ignacio Peña and Benjamin M. Tabak
- 116 Out-Of-The-Money Monte Carlo Simulation Option Pricing: the Joint Use of Importance Sampling and Descriptive Sampling** Sep/2006
Jaqueline Terra Moura Marins, Eduardo Saliby and Josete Florencio dos Santos
- 117 An Analysis of Off-Site Supervision of Banks' Profitability, Risk and Capital Adequacy: a Portfolio Simulation Approach Applied to Brazilian Banks** Sep/2006
Theodore M. Barnhill, Marcos R. Souto and Benjamin M. Tabak
- 118 Contagion, Bankruptcy and Social Welfare Analysis in a Financial Economy with Risk Regulation Constraint** Oct/2006
Aloísio P. Araújo and José Valentim M. Vicente

| | | |
|-----|---|----------|
| 119 | A Central de Risco de Crédito no Brasil: uma Análise de Utilidade de Informação <i>Ricardo Schechtman</i> | Out/2006 |
| 120 | Forecasting Interest Rates: an Application for Brazil <i>Eduardo J. A. Lima, Felipe Ludovice and Benjamin M. Tabak</i> | Oct/2006 |
| 121 | The Role of Consumer's Risk Aversion on Price Rigidity <i>Sergio A. Lago Alves and Mirta N. S. Bugarin</i> | Nov/2006 |
| 122 | Nonlinear Mechanisms of the Exchange Rate Pass-Through: a Phillips Curve Model With Threshold for Brazil <i>Arnildo da Silva Correa and André Minella</i> | Nov/2006 |
| 123 | A Neoclassical Analysis of the Brazilian "Lost-Decades" <i>Flávia Mourão Graminho</i> | Nov/2006 |
| 124 | The Dynamic Relations between Stock Prices and Exchange Rates: Evidence for Brazil <i>Benjamin M. Tabak</i> | Nov/2006 |
| 125 | Herding Behavior by Equity Foreign Investors on Emerging Markets <i>Barbara Alemanni and José Renato Haas Ornelas</i> | Dec/2006 |
| 126 | Risk Premium: Insights over the Threshold <i>José L. B. Fernandes, Augusto Hasman and Juan Ignacio Peña</i> | Dec/2006 |
| 127 | Uma Investigação Baseada em Reamostragem sobre Requerimentos de Capital para Risco de Crédito no Brasil <i>Ricardo Schechtman</i> | Dec/2006 |
| 128 | Term Structure Movements Implicit in Option Prices <i>Caio Ibsen R. Almeida and José Valentim M. Vicente</i> | Dec/2006 |
| 129 | Brazil: Taming Inflation Expectations <i>Afonso S. Bevilaqua, Mário Mesquita and André Minella</i> | Jan/2007 |
| 130 | The Role of Banks in the Brazilian Interbank Market: Does Bank Type Matter? <i>Daniel O. Cajueiro and Benjamin M. Tabak</i> | Jan/2007 |
| 131 | Long-Range Dependence in Exchange Rates: the Case of the European Monetary System <i>Sergio Rubens Stancato de Souza, Benjamin M. Tabak and Daniel O. Cajueiro</i> | Mar/2007 |
| 132 | Credit Risk Monte Carlo Simulation Using Simplified Creditmetrics' Model: the Joint Use of Importance Sampling and Descriptive Sampling <i>Jaqueline Terra Moura Marins and Eduardo Saliby</i> | Mar/2007 |
| 133 | A New Proposal for Collection and Generation of Information on Financial Institutions' Risk: the Case of Derivatives <i>Gilneu F. A. Vivan and Benjamin M. Tabak</i> | Mar/2007 |
| 134 | Amostragem Descritiva no Apreçamento de Opções Europeias através de Simulação Monte Carlo: o Efeito da Dimensionalidade e da Probabilidade de Exercício no Ganho de Precisão <i>Eduardo Saliby, Sergio Luiz Medeiros Proença de Gouvêa e Jaqueline Terra Moura Marins</i> | Abr/2007 |

- 135 **Evaluation of Default Risk for the Brazilian Banking Sector** May/2007
Marcelo Y. Takami and Benjamin M. Tabak
- 136 **Identifying Volatility Risk Premium from Fixed Income Asian Options** May/2007
Caio Ibsen R. Almeida and José Valentim M. Vicente
- 137 **Monetary Policy Design under Competing Models of Inflation Persistence** May/2007
Solange Gouvea e Abhijit Sen Gupta
- 138 **Forecasting Exchange Rate Density Using Parametric Models: the Case of Brazil** May/2007
Marcos M. Abe, Eui J. Chang and Benjamin M. Tabak
- 139 **Selection of Optimal Lag Length in Cointegrated VAR Models with Weak Form of Common Cyclical Features** Jun/2007
Carlos Enrique Carrasco Gutiérrez, Reinaldo Castro Souza and Osmani Teixeira de Carvalho Guillén
- 140 **Inflation Targeting, Credibility and Confidence Crises** Aug/2007
Rafael Santos and Aloísio Araújo
- 141 **Forecasting Bonds Yields in the Brazilian Fixed income Market** Aug/2007
Jose Vicente and Benjamin M. Tabak
- 142 **Crises Análise da Coerência de Medidas de Risco no Mercado Brasileiro de Ações e Desenvolvimento de uma Metodologia Híbrida para o Expected Shortfall** Ago/2007
Alan Cosme Rodrigues da Silva, Eduardo Facó Lemgruber, José Alberto Rebello Baranowski e Renato da Silva Carvalho
- 143 **Price Rigidity in Brazil: Evidence from CPI Micro Data** Sep/2007
Solange Gouvea
- 144 **The Effect of Bid-Ask Prices on Brazilian Options Implied Volatility: a Case Study of Telemar Call Options** Oct/2007
Claudio Henrique da Silveira Barbedo and Eduardo Facó Lemgruber
- 145 **The Stability-Concentration Relationship in the Brazilian Banking System** Oct/2007
Benjamin Miranda Tabak, Solange Maria Guerra, Eduardo José Araújo Lima and Eui Jung Chang
- 146 **Movimentos da Estrutura a Termo e Critérios de Minimização do Erro de Previsão em um Modelo Paramétrico Exponencial** Out/2007
Caio Almeida, Romeu Gomes, André Leite e José Vicente
- 147 **Explaining Bank Failures in Brazil: Micro, Macro and Contagion Effects (1994-1998)** Oct/2007
Adriana Soares Sales and Maria Eduarda Tannuri-Pianto
- 148 **Um Modelo de Fatores Latentes com Variáveis Macroeconômicas para a Curva de Cupom Cambial** Out/2007
Felipe Pinheiro, Caio Almeida e José Vicente
- 149 **Joint Validation of Credit Rating PDs under Default Correlation** Oct/2007
Ricardo Schechtman

| | | |
|------------|--|----------|
| 150 | A Probabilistic Approach for Assessing the Significance of Contextual Variables in Nonparametric Frontier Models: an Application for Brazilian Banks <i>Roberta Blass Staub and Geraldo da Silva e Souza</i> | Oct/2007 |
| 151 | Building Confidence Intervals with Block Bootstraps for the Variance Ratio Test of Predictability <i>Eduardo José Araújo Lima and Benjamin Miranda Tabak</i> | Nov/2007 |
| 152 | Demand for Foreign Exchange Derivatives in Brazil: Hedge or Speculation? <i>Fernando N. de Oliveira and Walter Novaes</i> | Dec/2007 |
| 153 | Aplicação da Amostragem por Importância à Simulação de Opções Asiáticas Fora do Dinheiro <i>Jaqueline Terra Moura Marins</i> | Dez/2007 |
| 154 | Identification of Monetary Policy Shocks in the Brazilian Market for Bank Reserves <i>Adriana Soares Sales and Maria Tannuri-Pianto</i> | Dec/2007 |
| 155 | Does Curvature Enhance Forecasting? <i>Caio Almeida, Romeu Gomes, André Leite and José Vicente</i> | Dec/2007 |
| 156 | Escolha do Banco e Demanda por Empréstimos: um Modelo de Decisão em Duas Etapas Aplicado para o Brasil <i>Sérgio Mikio Koyama e Márcio I. Nakane</i> | Dez/2007 |
| 157 | Is the Investment-Uncertainty Link Really Elusive? The Harmful Effects of Inflation Uncertainty in Brazil <i>Tito Nícias Teixeira da Silva Filho</i> | Jan/2008 |
| 158 | Characterizing the Brazilian Term Structure of Interest Rates <i>Osmani T. Guillen and Benjamin M. Tabak</i> | Feb/2008 |
| 159 | Behavior and Effects of Equity Foreign Investors on Emerging Markets <i>Barbara Alemanni and José Renato Haas Ornelas</i> | Feb/2008 |
| 160 | The Incidence of Reserve Requirements in Brazil: Do Bank Stockholders Share the Burden? <i>Fábia A. de Carvalho and Cyntia F. Azevedo</i> | Feb/2008 |
| 161 | Evaluating Value-at-Risk Models via Quantile Regressions <i>Wagner P. Gaglianone, Luiz Renato Lima and Oliver Linton</i> | Feb/2008 |
| 162 | Balance Sheet Effects in Currency Crises: Evidence from Brazil <i>Marcio M. Janot, Márcio G. P. Garcia and Walter Novaes</i> | Apr/2008 |
| 163 | Searching for the Natural Rate of Unemployment in a Large Relative Price Shocks' Economy: the Brazilian Case <i>Tito Nícias Teixeira da Silva Filho</i> | May/2008 |
| 164 | Foreign Banks' Entry and Departure: the recent Brazilian experience (1996-2006) <i>Pedro Fachada</i> | Jun/2008 |
| 165 | Avaliação de Opções de Troca e Opções de Spread Europeias e Americanas <i>Giuliano Carrozza Uzêda Iorio de Souza, Carlos Patrício Samanez e Gustavo Santos Raposo</i> | Jul/2008 |

- 166 **Testing Hyperinflation Theories Using the Inflation Tax Curve: a case study** Jul/2008
Fernando de Holanda Barbosa and Tito Nícias Teixeira da Silva Filho
- 167 **O Poder Discriminante das Operações de Crédito das Instituições Financeiras Brasileiras** Jul/2008
Clodoaldo Aparecido Annibal
- 168 **An Integrated Model for Liquidity Management and Short-Term Asset Allocation in Commercial Banks** Jul/2008
Wenersamy Ramos de Alcântara
- 169 **Mensuração do Risco Sistêmico no Setor Bancário com Variáveis Contábeis e Econômicas** Jul/2008
Lucio Rodrigues Capelletto, Eliseu Martins e Luiz João Corrar
- 170 **Política de Fechamento de Bancos com Regulador Não-Benevolente: Resumo e Aplicação** Jul/2008
Adriana Soares Sales
- 171 **Modelos para a Utilização das Operações de Redesconto pelos Bancos com Carteira Comercial no Brasil** Ago/2008
Sérgio Mikio Koyama e Márcio Issao Nakane
- 172 **Combining Hodrick-Prescott Filtering with a Production Function Approach to Estimate Output Gap** Aug/2008
Marta Areosa
- 173 **Exchange Rate Dynamics and the Relationship between the Random Walk Hypothesis and Official Interventions** Aug/2008
Eduardo José Araújo Lima and Benjamin Miranda Tabak
- 174 **Foreign Exchange Market Volatility Information: an investigation of real-dollar exchange rate** Aug/2008
Frederico Pechir Gomes, Marcelo Yoshio Takami and Vinicius Ratton Brandi
- 175 **Evaluating Asset Pricing Models in a Fama-French Framework** Dec/2008
Carlos Enrique Carrasco Gutierrez and Wagner Piazza Gaglianone
- 176 **Fiat Money and the Value of Binding Portfolio Constraints** Dec/2008
Mário R. Páscoa, Myrian Petrassi and Juan Pablo Torres-Martínez
- 177 **Preference for Flexibility and Bayesian Updating** Dec/2008
Gil Riella
- 178 **An Econometric Contribution to the Intertemporal Approach of the Current Account** Dec/2008
Wagner Piazza Gaglianone and João Victor Issler
- 179 **Are Interest Rate Options Important for the Assessment of Interest Rate Risk?** Dec/2008
Caio Almeida and José Vicente
- 180 **A Class of Incomplete and Ambiguity Averse Preferences** Dec/2008
Leandro Nascimento and Gil Riella
- 181 **Monetary Channels in Brazil through the Lens of a Semi-Structural Model** Apr/2009
André Minella and Nelson F. Souza-Sobrinho

| | | |
|------------|---|-----------|
| 182 | Avaliação de Opções Americanas com Barreiras Monitoradas de Forma Discreta <i>Giuliano Carrozza Uzêda Iorio de Souza e Carlos Patrício Samanez</i> | Abr/2009 |
| 183 | Ganhos da Globalização do Capital Acionário em Crises Cambiais <i>Marcio Janot e Walter Novaes</i> | Abr/2009 |
| 184 | Behavior Finance and Estimation Risk in Stochastic Portfolio Optimization <i>José Luiz Barros Fernandes, Juan Ignacio Peña and Benjamin Miranda Tabak</i> | Apr/2009 |
| 185 | Market Forecasts in Brazil: performance and determinants <i>Fabia A. de Carvalho and André Minella</i> | Apr/2009 |
| 186 | Previsão da Curva de Juros: um modelo estatístico com variáveis macroeconômicas <i>André Luís Leite, Romeu Braz Pereira Gomes Filho e José Valentim Machado Vicente</i> | Maio/2009 |
| 187 | The Influence of Collateral on Capital Requirements in the Brazilian Financial System: an approach through historical average and logistic regression on probability of default <i>Alan Cosme Rodrigues da Silva, Antônio Carlos Magalhães da Silva, Jaqueline Terra Moura Marins, Myrian Beatriz Eiras da Neves and Giovanni Antonio Silva Brito</i> | Jun/2009 |
| 188 | Pricing Asian Interest Rate Options with a Three-Factor HJM Model <i>Claudio Henrique da Silveira Barbedo, José Valentim Machado Vicente and Octávio Manuel Bessada Lion</i> | Jun/2009 |
| 189 | Linking Financial and Macroeconomic Factors to Credit Risk Indicators of Brazilian Banks <i>Marcos Souto, Benjamin M. Tabak and Francisco Vazquez</i> | Jul/2009 |
| 190 | Concentração Bancária, Lucratividade e Risco Sistêmico: uma abordagem de contágio indireto <i>Bruno Silva Martins e Leonardo S. Alencar</i> | Set/2009 |
| 191 | Concentração e Inadimplência nas Carteiras de Empréstimos dos Bancos Brasileiros <i>Patricia L. Tecles, Benjamin M. Tabak e Roberta B. Staub</i> | Set/2009 |
| 192 | Inadimplência do Setor Bancário Brasileiro: uma avaliação de suas medidas <i>Clodoaldo Aparecido Annibal</i> | Set/2009 |
| 193 | Loss Given Default: um estudo sobre perdas em operações prefixadas no mercado brasileiro <i>Antonio Carlos Magalhães da Silva, Jaqueline Terra Moura Marins e Myrian Beatriz Eiras das Neves</i> | Set/2009 |
| 194 | Testes de Contágio entre Sistemas Bancários – A crise do <i>subprime</i> <i>Benjamin M. Tabak e Manuela M. de Souza</i> | Set/2009 |
| 195 | From Default Rates to Default Matrices: a complete measurement of Brazilian banks' consumer credit delinquency <i>Ricardo Schechtman</i> | Oct/2009 |